In your own words, describe what the Central Limit Theorem (CLT) is and what it does. Discuss the significance and applications of the CLT in statistics and explain how it enables researchers to make inferences about populations. Provide examples and insights into the conditions required for the Central Limit Theorem to hold. (200 to 400 words)

Central Limit Theorem tells us that the average of the independent and identically distributed sample distribution follows normal distribution when the sample size is large regardless of the population distribution. That is, as we draw more samples, the mean of sample means is very close to population mean (standard deviation of the sample mean equals the population standard deviation divided by the sample size). This allows researchers making assumptions on population mean based on the sample mean when they don’t know true population distribution. Also they can do hypothesis testing or construct confidence intervals. For example, you want to know the average number of M%Ms per bag. Obviously, it is impossible to buy all M%M bags and know the exact distribution of the number. Therefore, you can collect as many samples as possible and assume the mean number of M&Ms will follow the normal distribution as you collect more samples.

Explain the concept of Maximum Likelihood Estimation (MLE) in statistics and its importance in parameter estimation. Discuss the underlying principles and assumptions of MLE, and provide a detailed example of how MLE is applied in a specific statistical or scientific context. Additionally, explore the strengths and limitations of MLE and how it compares to other estimation methods in various real-world applications. (200 to 400 words)

Maximum Likelihood Estimation is a statistical technique to estimate the parameters given the data. It is finding a parameter that maximizing the likelihood function which is a function of a parameter given data. It is important when you are trying to tell how the parameters of a model represents the observed data. MLE converges to a true population parameter when the sample size gets larger, yet it doesn’t always converge when maximum likelihood is obtained in multiple points. Also it is sensitive to outliers that can leads to biased estimations as we are underpredicting the population variance. Compared with other estimation methods, MLE is robust yet computationally more complex. Bayesian estimation works better in small sample sizes and dealing with outliers. Using the M&M example and assuming the number of M%Ms follow normal distribution, we can estimate the mean. We opened 10 bags and each bag has 6,6,7,8,6,6,7,8,9,7 respectively. Knowing that MLE of mean in normal distribution, we can estimate the mean by taking the sample mean of those 10 bags, which is 7.