Chapter 4 Statistical Inference

4.1 Probability

Key Concepts

- **Statistical Inference**: Drawing conclusions about a population using data from a sample.
- **Probability**: A mathematical framework for reasoning about uncertainty.

Probability Experiment

- A **probability experiment** must be repeatable and allow the listing of all possible outcomes.
  Example: Tossing a coin twice:
  - Sample space: {HH, HT, TH, TT}.
  - An **event** is a subset of the sample space, e.g., "at least one tail" = {HT, TH, TT}.

Probability Rules

1. $0 \leq P(E) \leq 1$.
2. $P(S) = 1$ (Probability of the entire sample space is 1).
3. If events E and F are mutually exclusive:
   $P(E \cup F) = P(E) + P(F)$.

Uniform Probability

- Equal probability for all outcomes:

$$P(\text{each outcome}) = \frac{1}{N}$$

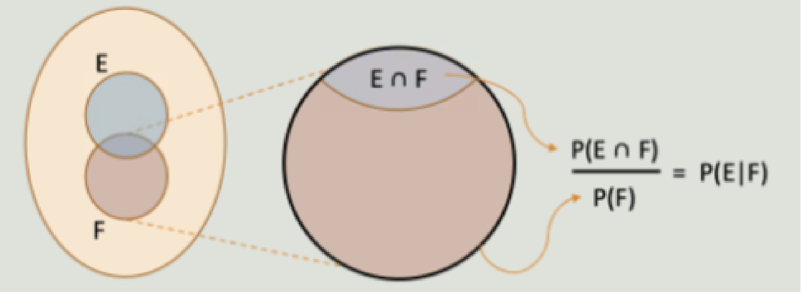
  Example: Rolling a fair die once:
  Sample space = {1, 2, 3, 4, 5, 6}, $P(\text{each face}) = \frac{1}{6}$.

4.2 Conditional Probability and Independence

Conditional Probability

- $P(E \mid F)$: Probability of E given F occurred.

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



Example: In a lucky draw with 500 participants, 280 males:
- E: Winner is John.
- F: Winner is male.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{500}}{\frac{280}{500}} = \frac{1}{280}.$$

Independence

- Two events A and B are **independent** if:

$$P(A \cap B) = P(A) \cdot P(B)$$

  - Example: Rolling two dice. Probability of rolling a 4 on die 1 and a 6 on die 2:

$$P(4 \cap 6) = P(4) \cdot P(6) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}.$$

Random sampling	Corresponds to	Probability experiment
Sampling frame	Corresponds to	Sample space
A subgroup A of the sampling frame	Corresponds to	An event A of the sample space
The rate of A, rate(A)	Corresponds to	The probability of A, $P(A)$

4.3 Common Fallacies

1. **Prosecutor's Fallacy**: Confusing $P(A \mid B)$ with $P(B \mid A)$.
   Example: Sally Clark case where $P(\text{Evidence} \mid \text{Innocent}) \neq P(\text{Innocent} \mid \text{Evidence})$.
2. **Conjunction Fallacy**: Believing $P(A \cap B) > P(A)$.
   Fact: $P(A \cap B) \leq P(A)$.
3. **Base Rate Fallacy**: Ignoring base rates when calculating conditional probabilities.
   Example: Breathalyzer test.
   - False positive rate: 5%.
   - Drunk driving rate: 0.1%.
   - $P(\text{Drunk} \mid \text{Positive}) \approx 2\%$.

4.4 Confidence Intervals

Definition

- A confidence interval gives a range likely to contain a population parameter with a specified confidence level (e.g., 95%).

For Proportion

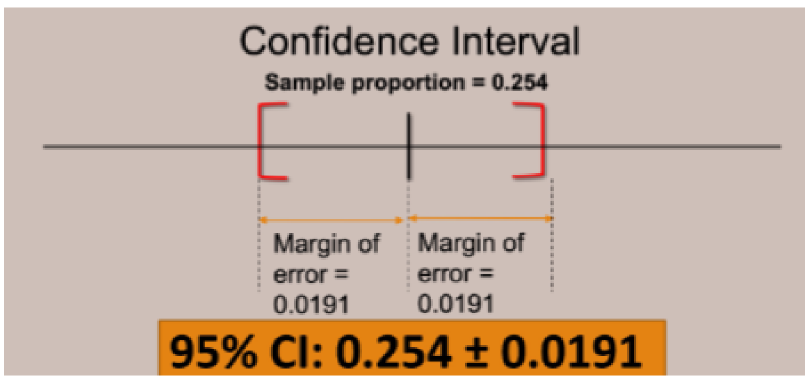
$$p^* \pm z^* \cdot \sqrt{\frac{p^*(1-p^*)}{n}}$$

Where:

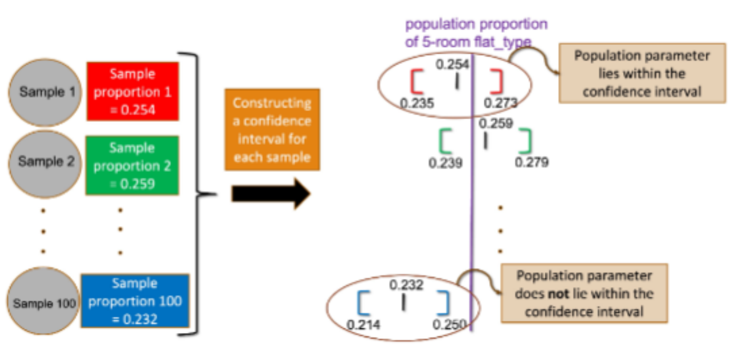
- p^*: Sample proportion.
- z^*: Z-value for the chosen confidence level (e.g., 1.96 for 95%).
- n: Sample size.

Example: 95% confidence interval for a sample proportion of 0.254 (sample size = 2000):

$$0.254 \pm 1.96 \cdot \sqrt{\frac{0.254(1-0.254)}{2000}} = 0.254 \pm 0.0191.$$



We are 95% confident that the population proportion (the parameter in this case) of resale flat transactions in 2020 that are 5-room, lies within the confidence interval.



For Mean

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

Where:

- $\bar{x}$: Sample mean.
- t^*: T-value based on confidence level and sample size.
- s: Sample standard deviation.

4.5 Hypothesis Testing

Key Steps

1. State Hypotheses:
   - Null hypothesis H_0: No effect or difference.
   - Alternative hypothesis H_1: Effect or difference exists.
2. Set Significance Level: Commonly $\alpha = 0.05$.
3. Calculate Test Statistic: Based on sample data.
4. Compute p-value: Probability of observing test results as extreme as the sample, assuming H_0 is true.
5. Conclusion:
   - If $p \leq \alpha$: Reject H_0.
   - If $p > \alpha$: Fail to reject H_0.

p-value $<$ significance level	p-value $\geq$ significance level
Sufficient evidence to reject null hypothesis in favour of the alternative hypothesis	Insufficient evidence to reject the null hypothesis. **The hypothesis test is inconclusive.** This *does not* mean that we *accept* the null hypothesis.

Example: Hypothesis Test for Proportion

- Question: Is the proportion $p = 0.5$?
- Observed: Sample proportion $p^* = 0.335$, $n = 200$.
- Compute p-value: $p < 0.001$.
- Conclusion: Reject H_0, proportion is likely less than 0.5.

Example: Hypothesis Test for Mean

- $H_0 : \mu = 69$, $H_1 : \mu > 69$.

- Sample mean: $\bar{x} = 70.345$.
- $p = 0.093$ (not significant at $\alpha = 0.05$).
- Conclusion: Insufficient evidence to reject H_0.

Summary

1. Probability is essential for reasoning about uncertainty and forming statistical inferences.
2. Confidence intervals provide a range for population parameters.
3. Hypothesis testing evaluates claims about population parameters using p-values.
4. Avoid common fallacies (e.g., base rate and prosecutor’s fallacies) by using structured approaches like contingency tables and precise formulas.