Chapter 9 Linear Momentum and Collisions

Linear Momentum

- The **linear momentum** of a particle of an object can be modeled as a particle of mass m moving with a velocity v, thus $p = mv$
- Linear momentum is a vector quantity
  - Its direction is the same as the direction of v
- SI Unit of momentum are $kg \cdot m/s$
- Can be expressed in component form
  - $p_x = mv_x$ or $p_y = mv_y$ or $p_z = mv_z$

Newton and Momentum

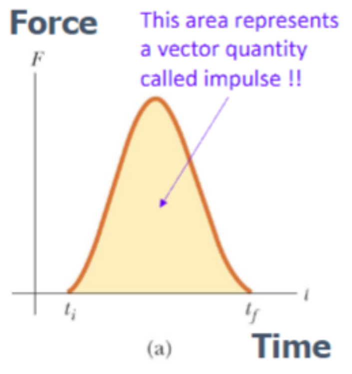
- Newton called the product mv the *quantity of motion* of the particle
- Newton's Second Law can be used to relate the momentum of a particle to the resultant force acting on it
  - $\sum F = ma = m\frac{dv}{dt} = \frac{d(mv)}{dt} = \frac{dp}{dt}$
- Net force is also equal to the time rate of change of linear momentum
  - $F = ma = m(\frac{dv}{dt}) = \frac{m \times dv}{dt} = \frac{mv_f - mv_i}{dt}$

Conservation of Linear Momentum

- Whenever two or more particles in an *isolated system* interact, the total momentum of the system remains constant
  - The momentum of the *system* is conserved, but the momentum of individual particle may not necessarily be conserved
  - The *total* momentum of an isolated system equals its *initial* momentum
- Conservation of momentum
  - $p_{total} = p_1 + p_2 = constant$
  - $p_{1i} + p_{2i} = p_{1p} + p_{2f}$
- In component form for the various directions, the total momentum in each direction is *independently* conserved
  - $p_{ix} = p_{fx}$ or $p_{iy} = p_{fy}$ or $p_{iz} = p_{fz}$
- Conservation of momentum can be applied to systems with any number of particles

Impulse

- Impulse is a vector quantity
- The magnitude of the *impulse is equal* to the area under the force-time curve
- Impulse is *not a property of the particle*, but a measure of the change in momentum of the particle

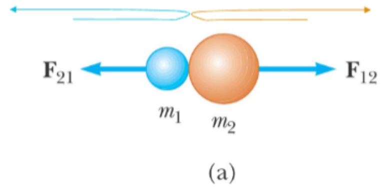


Impulse = Change in momentum = Force x time
$I = \Delta p = p_f - p_i = F \times \Delta t$
$F = \frac{\Delta p}{\Delta t}$

Collisions

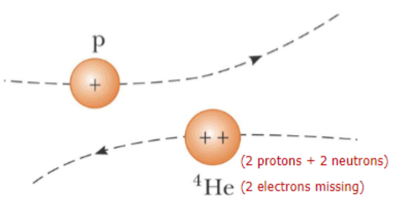
We use the term *collision* to represent an event during which two particles come close to each other and interact by means of forces
The time interval during which the velocity changes from its initial to final values is assumed to be very short but measurable
Momentum is conserved in any type of collusions *as long as **no** net external force is affecting the system during the contact*

Contact



Collisions may be result of direct contact where its momentum is conserved

Non-contact

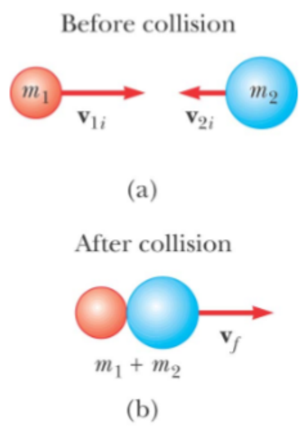


Non-contact collisions, such as between a proton and an alpha particle, involve strong electrostatic forces without physical contact and can be analyzed like contact collisions.

Types of Collisions

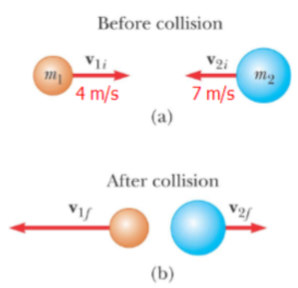
- In an **inelastic** collision kinetic energy is not conserved, although momentum is
    - If the objects stick together after the collision, it is a *perfectly inelastic* collision
- In an **elastic** collision, momentum and kinetic energy are conserved
    - Perfectly elastic collisions occurs on a microscopic level

Perfectly Inelastic Collisions



Since the object stick together, they share the same velocity after the collision.
$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$
$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$
In perfectly inelastic collisions (no separation), there will be a *loss* in their kinetic energy (this is the **price for the reconciliation**)

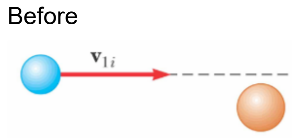
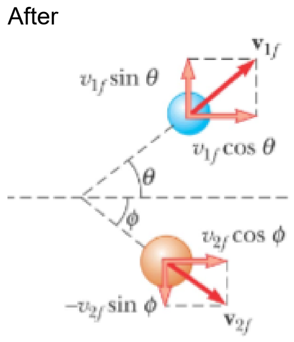
Elastic Collisions



Both **momentum** and *kinetic energy* are conserved for any elastic collisions
$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

Two-Dimensional Collisions

The momentum is **conserved in all directions**
If the collisions is elastic, use the conservation of kinetic energy as a second equation

> Before
>
> 
>
>
> - Particle 1 is moving at velocity v_{1i} and particle 2 is at rest
> - In the *x*-direction, the initial momentum is $m_1 v_{1i}$
> - In the *y*-direction, the initial momentum is 0
>   After
>
> 
>
>
> - After the collision, the momentum in the *x*-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
> - After the collision, the momentum in the *y*-direction is $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$

If collision is **inelastic**, kinetic energy of the *system is not conserved*, and should consider the work done by the *non-conservative forces like friction*, where these can be exhibited as form of heat energy
If collision is **perfectly inelastic**, where the *objects stick together*, the final velocities of the two objects are equal
If collision is **elastic**, the kinetic energy *of the system is conserved*, equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain more information on the relationship between the velocities, *formula for relative velocities*, $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$