Chapter 3 Vectors

Coordinate Systems

- Used to describe the position of a point in space
- With reference to the origin
  - (0, 0) for 2D
  - (0, 0, 0) for 3D
- Specific axes with scales and labels

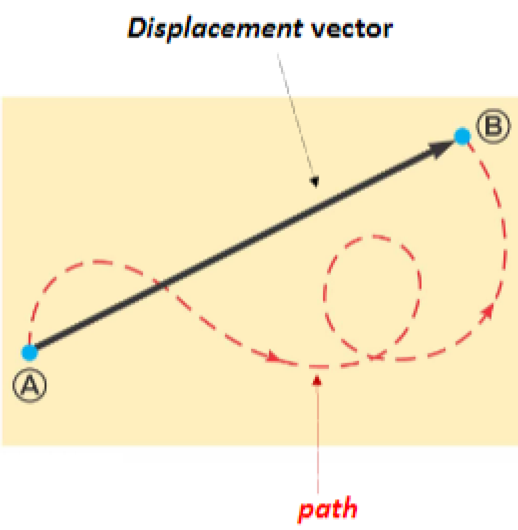
Cartesian Coordinate System

- Rectangular coordinate system
- x-axis and y-axis intersect at the origin
- (x, y)

Vectors

Written as $\vec{A}$ or **A**
Magnitude of a vector written as $|A|$ or *A*

Example



The displacement vector is independent of the path.

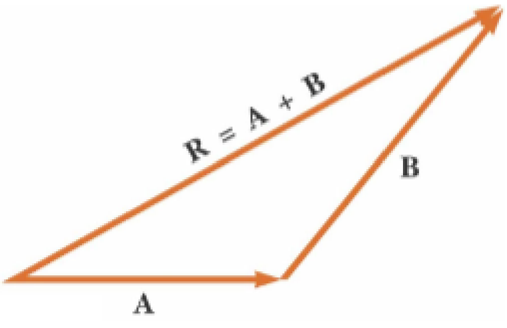
Equality of Two Vectors

- Two vectors are **equal** if they have the same **magnitude and direction**.
- $\vec{A} = \vec{B}$ and $|A| = |B|$, they point along parallel lines in the same direction

Adding Vectors

- Direction must be taken into account
- Units must be the same

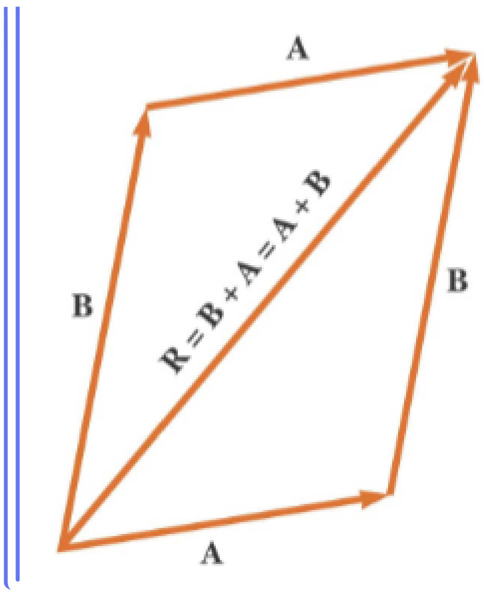
Adding Vectors Graphically



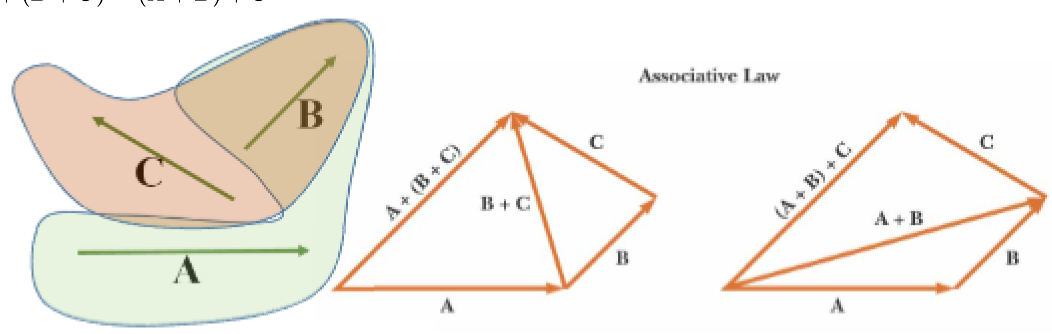
draw the vectors from "head-to-tail".

Rules

> **Commutative Law of Addition**: When two vectors are added, the sum is independent of the order of the addition.
> $\vec{A} + \vec{B} = \vec{B} = \vec{A}$



Associative Property of Addition: When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped
$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$



All units must be the same, cm cannot be added to m

Negative of a Vector

Same magnitude but point to the opposite direction $-\vec{A}$
$\vec{A}+(-\vec{A})=0$ if added to the original vector will result 0

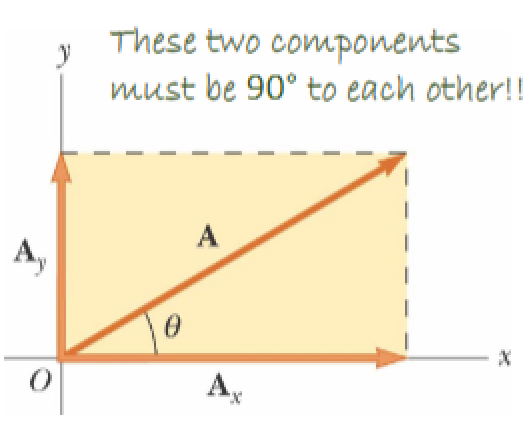
Subtracting Vectors

instead of $\vec{A}-\vec{B}$, use $\vec{A}+(-\vec{B})$

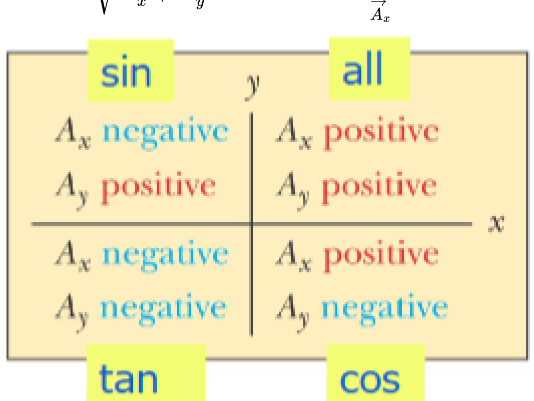
Multiplying or Dividing a Vector by a Scalar

$2\times\vec{A}=2\vec{A}, -0.8\times\vec{A}=-0.8\vec{A}$
if the scalar is negative, the resultant vector will also be negative

Components of a Vector



Parts of a vector, one along the x-axis and one along the y-axis
$\vec{A}_x$ and $\vec{A}_y$ are the component vector of $\vec{A}$
The components are scalar.
the projections are $\vec{A}_x=\vec{A}\cos\theta$ and $\vec{A}_y=\vec{A}\sin\theta$, thus $\vec{A}=\vec{A}_x+\vec{A}_y$
and $\vec{A}=\sqrt{\vec{A}_x^2+\vec{A}_y^2}$ and $\theta=\tan^{-1}\frac{\vec{A}_y}{\vec{A}_x}$

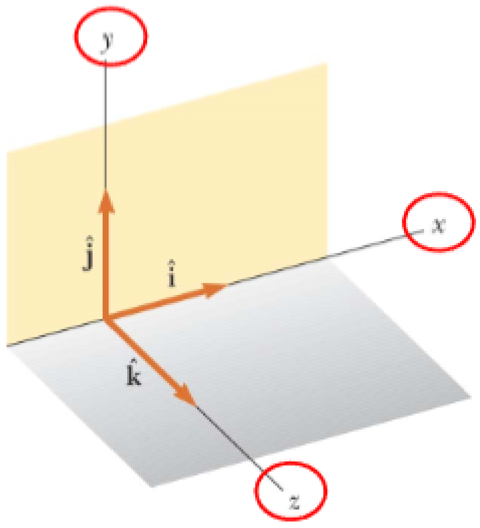


Components can be positive and negative and will have the same unit as the original vector, vector in the 3rd quadrant, both the x and y will be negative.

Unit Vectors

- A **unit vector** is a *dimensionless* vector with a magnitude of exactly 1.
- They are only used to specify a direction.
- Symbols are $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$

- They follow *right-hand rules*

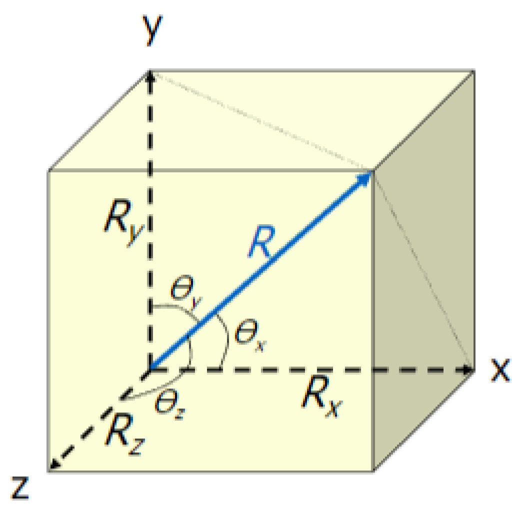


- Since they are dimensionless, thus $\vec{A_x} \cdot \hat{i}$ is the same as $\vec{A_y} \cdot \hat{j}$
- And thus, $\vec{A} = \vec{A_x}\,\hat{i} + \vec{A_y}\,\hat{j} + \vec{A_z}\,\hat{k}$

Adding Vectors Using Unit Vectors

Using resultant vector
$R = \vec{A} + \vec{B}$
Then,
$R = (\vec{A_x}\,\hat{i} + \vec{A_y}\,\hat{j}) + (\vec{B_x}\,\hat{i} + \vec{B_y}\,\hat{j})$
$R = (\vec{A_x} + \vec{B_x})\,\hat{i} + (\vec{A_y} + \vec{B_y})\,\hat{j}$
$R = R_x + R_y$
So,
$R_x = A_x + B_x$ and $R_y = A_y + B_y$
and,
$R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1} \frac{R_y}{R_x}$

Angle of Vector in 3D



$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$
$\theta_x = \cos^{-1} \frac{R_x}{R}$
$\theta_y = \cos^{-1} \frac{R_y}{R}$
$\theta_z = \cos^{-1} \frac{R_z}{R}$