Chapter 4 Motion in Two Dimensions

Projectile Motion

An object moving in both x and y directions simultaneously are called a **projectile motion**.

Assumptions

- The free-fall acceleration g is constant over the whole range of motion.
- g is always going downwards and has the average magnitude of $9.8\,m/s^2$
- Air friction is negligible

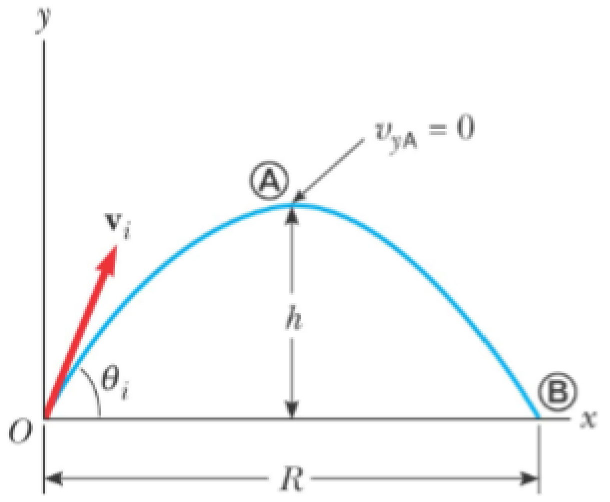
With all these assumptions, an object in a projectile motion will follow a **parabolic path** called a *trajectory*

Acceleration components are $a_y = -g$ and $a_x = 0$
Initial velocity components are $v_{xi} = v_i \cos\theta$ and $v_{yi} = v_i \sin\theta$

Parabolic Curves

Range and Maximum Height of a Projectile

- Range R is the horizontal distance of the projectile
- Maximum height is the projectile reaches is h



With these we can derive:

$$h = \frac{v_i^2 \sin^2\theta_i}{2\,g} \; and \; R = \frac{v_i^2 \sin(2\theta_i)}{g}$$

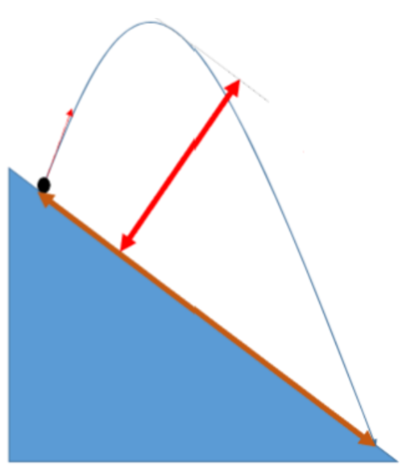
This equation is valid only for symmetric motion

Maximum value of R

$\sin(2\theta) = 1,\ 2\theta = 90^\circ,\ \theta = 45^\circ$
Thus,
$R = \frac{u^2}{g}(\sin 90^\circ)$
$\therefore R = \frac{u^2}{g}$

- So the max range will occur when $\theta_i = 45^\circ$
- Complementary angles $(90^\circ - \theta_i)$ will produce the same range.
- But the max height will be different for the two angles.
- The time of flight will be different for the two angles, thou the range are the same.

Non Symmetric Trajectory

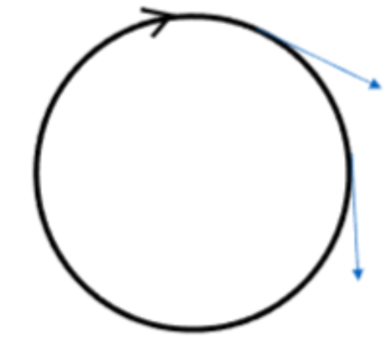


- Follows the general rules for projectile motion
- The x and y positions now can be positive or negative
- Another way to solve this is to rotate the x and y axis.

$$y = (\tan\theta_i)\,x - \frac{gx^2}{2v_i^2\cos^2\theta_i}$$

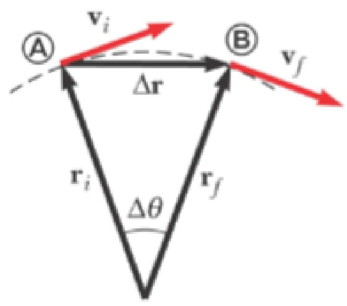
Uniform Circular Motion

- **Uniform circular motion** occurs when an object moves in a circular path with a *constant speed*
- An acceleration exists since the direction of the motion is changing, this change in velocity is related to an acceleration
- The velocity vector is always tangent to the path of the object



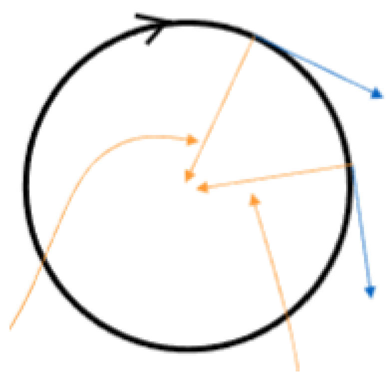
Changing Velocity in Uniform Circular Motion

The change in the velocity vector is due to the change in direction



Centripetal Acceleration

- The acceleration is always perpendicular to the path of the motion
- The acceleration always points towards the center of the circle of the motion



Magnitude of Velocity is a Constant

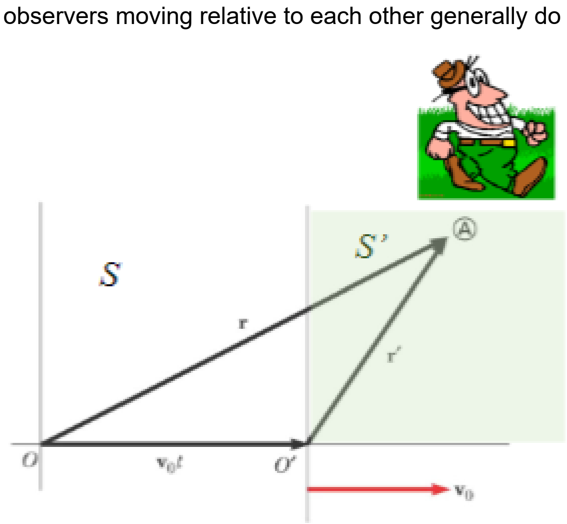
There must be acceleration; otherwise, the ball's direction won’t change, and it won’t return to the horizontal circle. This acceleration points toward the circle’s center, so it’s called centripetal acceleration.
Its magnitude $a_c = \frac{v^2}{r}$

Period

- The period, T, is the time required for one complete revolution/cycle
- Thus a period is $T = \frac{2\pi r}{v}$

Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment



- Reference frame S is stationary
- Reference frame S' is moving at v_0 as observed from a stationary frame (like S)
  - This also means that S moves at $-v_0$ relative to S'
- Define time $t = 0$ as that time when the two origins coincide.
- The positions as seen from the two reference frames (r is the position from moving frame) are related through the velocity
  - $r' = r - v_0 t$ or $r = r' + v_0 t$
- The derivative of the position equation will give the velocity equation
  - $v' = v - v_0$
- The acceleration of the particle measured by an observer in one frame of reference *is the same as* that measured by any other observer moving at a *constant velocity* relative to the first frame.

- v_0 is a constant, so v_0 has no net effect on the change in velocity, $a' = a$