

# Chapter 29 Sources of the Magnetic Field

## Introduction to Magnetic Field Sources

- **Focus:** Unlike the previous chapter that discussed magnetic fields as causes of force, this chapter explores what generates magnetic fields themselves.

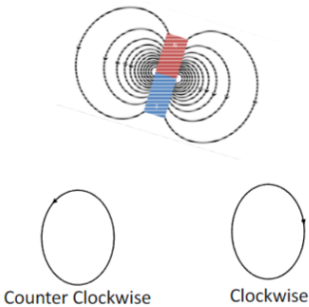
## Biot-Savart Law

- **Law Overview:**
  - Describes the magnetic field  $d\vec{B}$  generated by a small segment of a current-carrying conductor.
  - The direction of  $d\vec{B}$  is perpendicular to both the current segment  $d\vec{s}$  and the position vector  $\hat{r}$  from the segment to the observation point.
- **Mathematical Form:**

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{r}}{r^2}$$

- $\mu_0$ : Permeability of free space,  $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
- $I$ : Current in the wire
- $d\vec{s}$ : Small segment of the wire
- $\hat{r}$ : Unit vector from  $d\vec{s}$  to point  $P$
- $r$ : Distance between  $d\vec{s}$  and point  $P$

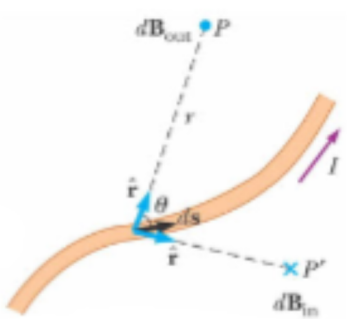
How to specify a magnetic field?



## Total Magnetic Field

- **Integration:**
  - To find the total magnetic field  $\vec{B}$  due to a current distribution, integrate  $d\vec{B}$  over the entire current path:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{s} \times \hat{r}}{r^2}$$

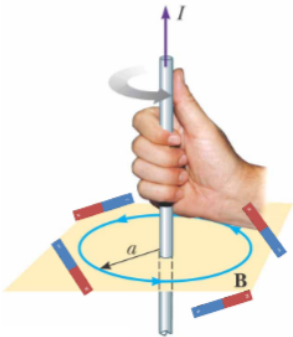


## Magnetic Field Due to a Long, Straight Conductor

- **Result:**
  - For a straight conductor carrying current  $I$ , the magnetic field at a distance  $a$  from the wire is given by:

$$B = \frac{\mu_0 I}{2\pi a}$$

- **Direction:**
  - Determined by the right-hand rule: thumb in the direction of current, fingers curl in the direction of  $\vec{B}$ .

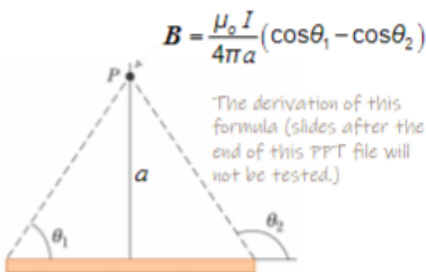
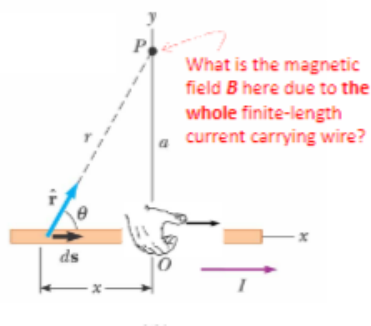


# Magnetic Field for a Long, Straight Conductor

- A thin, straight wire is carrying a constant current and we want to calculate the magnetic field from this whole finite-length wire. We already know

$$d\mathbf{B} = \left(\frac{\mu_0}{4\pi}\right) \frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

due to small length segment  $ds$

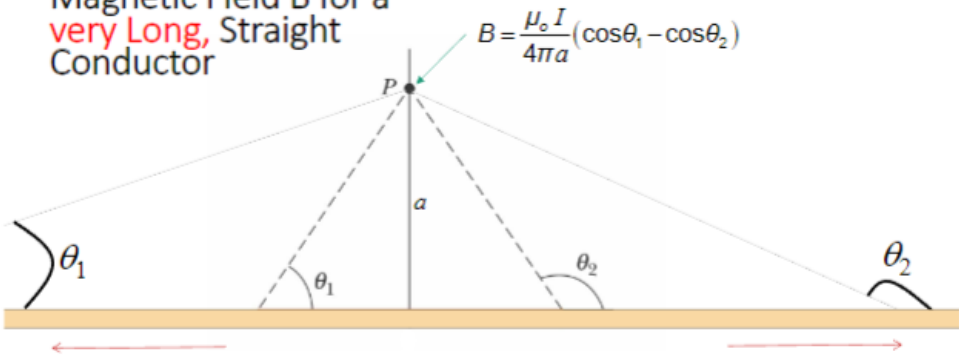


- If  $a$  is fixed for Point  $P$ , the length of the wire can be represented by  $\theta_1$  and  $\theta_2$ . We can derive that

$$B = \frac{\mu_0 I}{4\pi a} (\cos\theta_1 - \cos\theta_2)$$

at point  $P$  due to the whole length of current-carrying wire.

## Magnetic Field B for a very Long, Straight Conductor



If the conductor is an infinitely long and straight wire, we let  $\theta_1 = 0^\circ$  and  $\theta_2 = 180^\circ$ , the magnetic field becomes

$$B = \frac{\mu_0 I}{4\pi a} (\cos 0^\circ - \cos 180^\circ)$$
$$= \frac{\mu_0 I}{2 \cdot 4\pi a} (1 - (-1)) = \boxed{\frac{\mu_0 I}{2\pi a}}$$

Magnetic field due to long straight conductor:  
 $B = \frac{\mu_0 I}{2\pi a}$  where  
 $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$

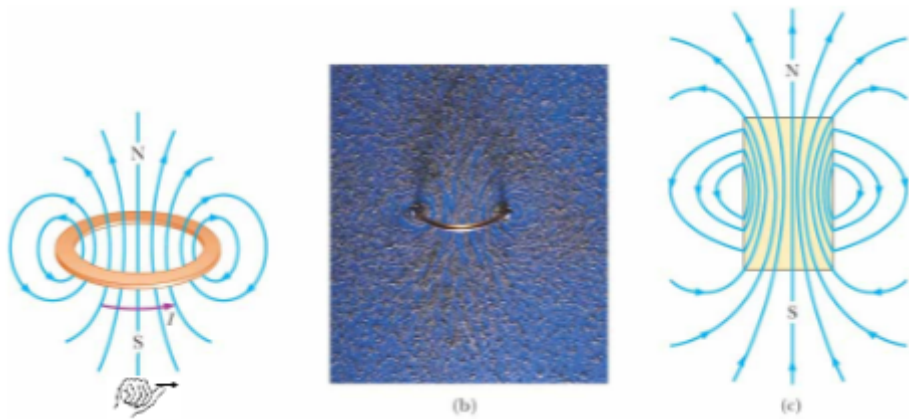
Magnetic Field B due to infinitely Long and Straight Conductor

## Magnetic Field of a Circular Loop

- At the Center of a Loop:

$$B = \frac{\mu_0 I}{2R}$$

- $R$ : Radius of the loop
- Direction:
  - Follows the right-hand rule for loops: curl of fingers in current direction, thumb points to  $\vec{B}$  at the center.



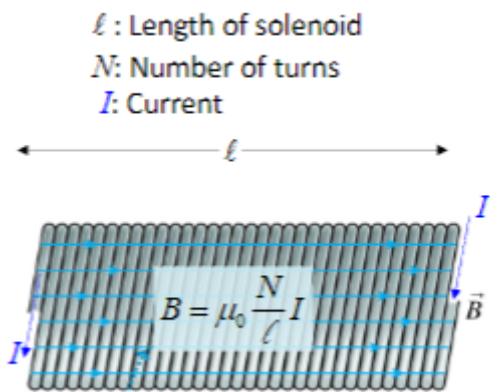
## Ampère's Law

- Law Statement:
  - For any closed loop path, the sum of  $B \cdot ds$  around the loop is proportional to the total current enclosed:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$

- Applications:

- Used for calculating  $\vec{B}$  in symmetrical situations such as solenoids and toroids.



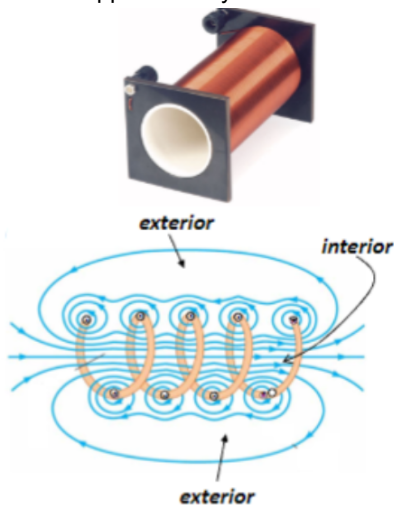
## Magnetic Field of a Solenoid

- **Uniform Field:**
  - Inside a long solenoid:

$$B = \mu_0 n I$$

- $n$ : Number of turns per unit length
- $I$ : Current

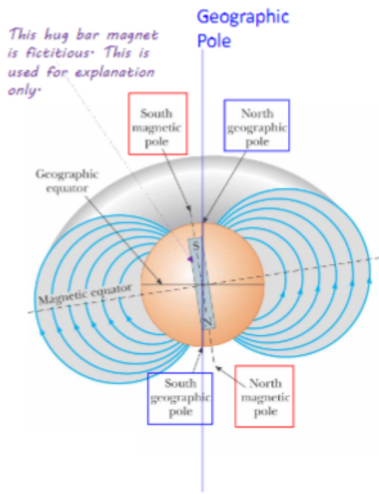
- **Characteristics:**
  - Field is approximately uniform inside and zero outside.



- **Ideal Solenoid**
  - Infinitely long and the coils are closely packed
  - Many turns
  - Internal magnetic field is regarded as uniform and its magnitude is a constant
  - The external magnetic field is regarded as 0

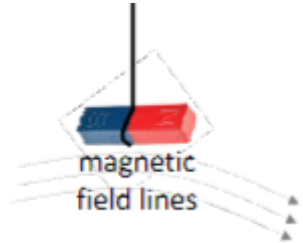
## Earth’s Magnetic Field Overview

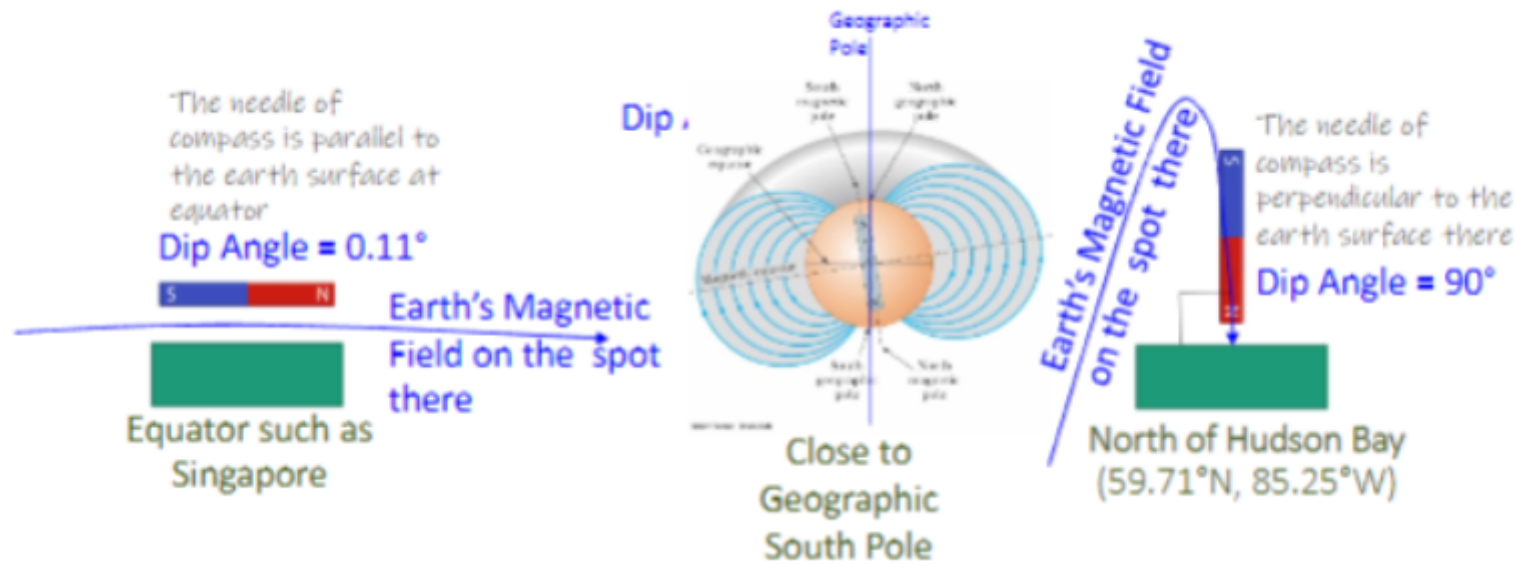
- **Model:** The Earth’s magnetic field resembles that produced by a large bar magnet buried within the Earth, although this is a simplified model for explanation.
- **Poles:**
  - The **south magnetic pole** is located near the **north geographic pole**.
  - The **north magnetic pole** is located near the **south geographic pole**.
- **Important Note:** The actual magnetic field is more complex than the bar magnet model due to variations and deviations.



## Dip Angle of Earth’s Magnetic Field

- **Definition:**
  - The **dip angle** is the angle between the horizontal surface at a location and the direction of the magnetic field at that point.
- **Examples:**
  - At the **equator**, the dip angle is nearly **0°**, meaning the compass needle stays parallel to the Earth's surface.
  - Near **Hudson Bay** in Canada, the dip angle reaches **90°**, where the compass needle points directly down.





## Magnetic Declination

- **Definition:**
  - The **magnetic declination** is the angle difference between **true north** (geographic north) and **magnetic north** as indicated by a compass.
- **Variation:**
  - Declination varies based on location on the Earth's surface and can be crucial for navigation.



## Source of the Earth's Magnetic Field

- **Explanation:**
  - The core of the Earth is too hot for permanent magnetization due to temperatures around **5200°C**.
  - The most likely source is **convection currents** in the molten metal part of the Earth's core.
  - The Earth's rotation may also contribute to generating the magnetic field.

## Reversals of the Earth's Magnetic Field

- **Phenomenon:**
  - The direction of the Earth's magnetic field has reversed multiple times over millions of years.
  - Evidence is found in **basalt rocks** formed from volcanic activity, which contain records of the Earth's past magnetic orientations.
- **Uncertainty:**
  - The exact cause of these reversals is not fully understood.

