Calculus Formulas: Derivatives and Integrals

Derivatives

$$
\begin{aligned}
(f(x))^n \quad & \text{Derivative:} \quad n f'(x)(f(x))^{n-1} \\
\cos(f(x)) \quad & \text{Derivative:} \quad -f'(x)\cdot\sin(f(x)) \\
\sin(f(x)) \quad & \text{Derivative:} \quad f'(x)\cdot\cos(f(x)) \\
\tan(f(x)) \quad & \text{Derivative:} \quad f'(x)\cdot\sec^2(f(x)) \\
\sec(f(x)) \quad & \text{Derivative:} \quad f'(x)\cdot\sec(f(x))\tan(f(x)) \\
\csc(f(x)) \quad & \text{Derivative:} \quad -f'(x)\cdot\csc(f(x))\cot(f(x)) \\
\cot(f(x)) \quad & \text{Derivative:} \quad -f'(x)\cdot\csc^2(f(x)) \\
e^{f(x)} \quad & \text{Derivative:} \quad f'(x)\cdot e^{f(x)} \\
\ln(f(x)) \quad & \text{Derivative:} \quad \frac{f'(x)}{f(x)} \\
\sin^{-1}(f(x)) \quad & \text{Derivative:} \quad \frac{f'(x)}{\sqrt{1-(f(x))^2}} \\
\cos^{-1}(f(x)) \quad & \text{Derivative:} \quad -\frac{f'(x)}{\sqrt{1-(f(x))^2}} \\
\tan^{-1}(f(x)) \quad & \text{Derivative:} \quad \frac{f'(x)}{1+(f(x))^2}
\end{aligned}
$$

Derivative Rules

$$
\begin{aligned}
\frac{d}{dx}(x^n) &= nx^{n-1} \\
\frac{d}{dx}(\ln x) &= \frac{1}{x} \\
\frac{d}{dx}(a^x) &= a^x \ln a \\
\frac{d}{dx}(e^x) &= e^x \\
\frac{d}{dx}(\sin x) &= \cos x \\
\frac{d}{dx}(\cos x) &= -\sin x \\
\frac{d}{dx}(\tan x) &= \sec^2 x \\
\frac{d}{dx}(\cot x) &= -\csc^2 x \\
\frac{d}{dx}(\sec x) &= \sec x \tan x \\
\frac{d}{dx}(\csc x) &= -\csc x \cot x \\
\frac{d}{dx}(\sin^{-1} x) &= \frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}(\cos^{-1} x) &= -\frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}(\tan^{-1} x) &= \frac{1}{x^2+1}
\end{aligned}
$$

Derivatives of Powers of Trigonometric Functions

$$
\begin{aligned}
&\frac{d}{dx}(\sin^2 x) = 2\sin x \cos x = \sin(2x) \\
&\frac{d}{dx}(\cos^2 x) = 2\cos x(-\sin x) = -\sin(2x) \\
&\frac{d}{dx}(\tan^2 x) = 2\tan x \sec^2 x \\
&\frac{d}{dx}(\cot^2 x) = 2\cot x(-\csc^2 x) = -2\cot x \csc^2 x \\
&\frac{d}{dx}(\sec^2 x) = 2\sec x \sec x \tan x = 2\sec^2 x \tan x \\
&\frac{d}{dx}(\csc^2 x) = 2\csc x(-\csc x \cot x) = -2\csc^2 x \cot x
\end{aligned}
$$

Chain Rule Examples for Powers of Trigonometric Functions

$$
\begin{aligned}
&\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \cdot \cos x \\
&\frac{d}{dx}(\cos^4 x) = 4\cos^3 x \cdot (-\sin x) = -4\cos^3 x \sin x
\end{aligned}
$$

Product, Quotient, and Chain Rules

Product Rule:

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Integral Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int 1 \, dx = x + C$$

$$\int \sin(kx) \, dx = -\frac{\cos(kx)}{k} + C$$

$$\int \cos(kx) \, dx = \frac{\sin(kx)}{k} + C$$

$$\int \sec^2(x) \, dx = \tan(x) + C$$

$$\int \csc^2(x) \, dx = -\cot(x) + C$$

$$\int \sec(x) \tan(x) \, dx = \sec(x) + C$$

$$\int \csc(x) \cot(x) \, dx = -\csc(x) + C$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \sin^2(x) \, dx = \int \frac{1-\cos(2x)}{2} \, dx = -\frac{\sin(2x) - 2x}{4} + C$$

$$\int \cos^2(x) \, dx = \int \frac{1+\cos(2x)}{2} \, dx = \frac{x - \frac{\sin(2x)}{2}}{2} + C$$

Standard Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C \quad (n \neq -1)$$

$$\int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \sin(ax+b) \, dx = -\frac{1}{a} \cos(ax+b) + C$$

$$\int \cos(ax+b) \, dx = \frac{1}{a} \sin(ax+b) + C$$

$$\int \tan(ax+b) \, dx = \frac{1}{a} \ln|\sec(ax+b)| + C$$

$$\int \sec(ax+b) \, dx = \frac{1}{a} \ln|\sec(ax+b) + \tan(ax+b)| + C$$

$$\int \csc(ax+b) \, dx = -\frac{1}{a} \ln|\csc(ax+b) + \cot(ax+b)| + C$$

$$\int \cot(ax+b) \, dx = -\frac{1}{a} \ln|\csc(ax+b)| + C$$

$$\int \sec^2(ax+b) \, dx = \frac{1}{a} \tan(ax+b) + C$$

$$\int \csc^2(ax+b) \, dx = -\frac{1}{a} \cot(ax+b) + C$$

$$\int \sec(ax+b) \cdot \tan(ax+b) \, dx = \frac{1}{a} \sec(ax+b) + C$$

$$\int \csc(ax+b) \cdot \cot(ax+b) \, dx = -\frac{1}{a} \csc(ax+b) + C$$

$$\int \frac{1}{a^2 + (x+b)^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x+b}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2 - (x+b)^2}} \, dx = \sin^{-1}\left(\frac{x+b}{a}\right) + C$$

$$\int \frac{-1}{\sqrt{a^2 - (x+b)^2}} \, dx = \cos^{-1}\left(\frac{x+b}{a}\right) + C$$

$$\int \frac{1}{a^2 - (x+b)^2} \, dx = \frac{1}{2a} \ln\left|\frac{x+b+a}{x+b-a}\right| + C$$

$$\int \frac{1}{\sqrt{(x+b)^2 + a^2}} \, dx = \ln\left|(x+b) + \sqrt{(x+b)^2 + a^2}\right| + C$$

Integration by Parts

The formula for integration by parts is derived from the product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$\int u\,dv = uv - \int v\,du$$

Integration by Substitution

The method of substitution is used when an integral contains a composite function. Let $u = g(x)$, then $du = g'(x)\,dx$. The integral becomes:

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$