Chapter 27 Direct-Current Circuits

Conservation Principles

- **Conservation of Charges**: At any junction in a circuit, the total current entering must equal the total current leaving.
- **Conservation of Energy**: For any closed loop in a circuit, the sum of changes in potential energy must be zero.

Definition of Direct Current

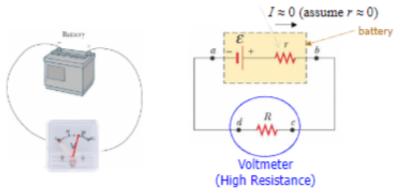
- **Direct Current (DC)**: A type of electrical current that maintains a constant magnitude and direction.
- Batteries generate DC because the potential difference between their terminals remains steady.
- The electromotive force (emf), denoted as ϵ, represents the maximum voltage a battery can supply between its terminals.

Key Points on E.M.F.

- **E.M.F. is not a force** but a measure of energy provided per unit charge by a source.
- The voltage across an open circuit equals the emf.
- Terminal voltage ΔV can be expressed as:

$$\Delta V = \epsilon - Ir$$

  where r is the internal resistance of the battery and I is the current.

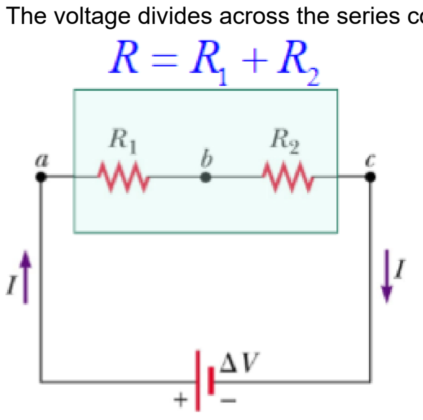


Resistors in Series

- Resistors are in series if connected end-to-end, and the same current flows through each resistor.
- The total or equivalent resistance for series resistors:

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \dots$$

- The voltage divides across the series components proportionally.

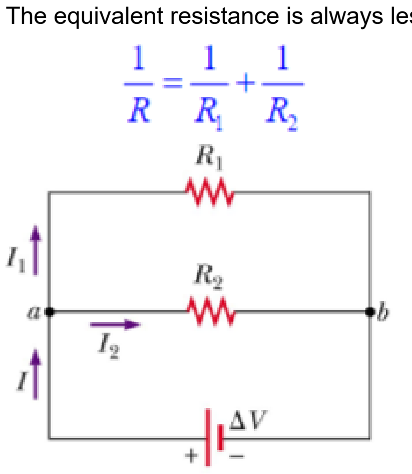


Resistors in Parallel

- Each parallel resistor shares the same potential difference but divides the total current.
- Equivalent resistance for parallel resistors:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

- The equivalent resistance is always less than the smallest individual resistance.



Kirchhoff's Rules

Junction Rule

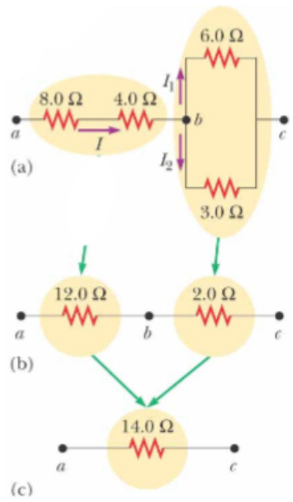
- States that the sum of currents at any junction is zero:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Loop Rule

- States that the sum of the potential differences around any closed loop must be zero:

$$\sum \Delta V_{\text{loop}} = 0$$



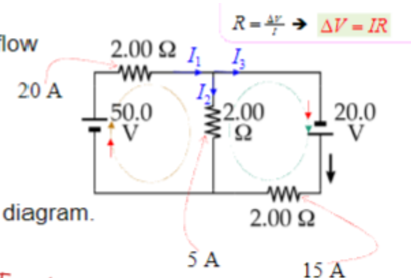
Problem-Solving Strategy

1. **Diagram**: Sketch the circuit and label all known and unknown values.
2. **Direction of Current**: Assign current directions arbitrarily but remain consistent.
3. **Apply Rules**: Use the junction rule for current and the loop rule for potential differences as needed.
4. **Solve Equations**: Simultaneously solve the equations for unknown quantities.
    - A negative current result means the current flows in the opposite direction to what was assigned.

Example. Calculate the current flow in each resistor shown in figure.

Answer:

Let the currents be as shown.



We apply Kirchhoff's rules to the diagram.

$50 - 2I_1 - 2I_2 = 0$ … (1)

$20 - 2I_3 + 2I_2 = 0$ … (2)

$I_1 = I_2 + I_3$ … (3)

Traditional approach presented in Textbook!!

These are algebra!!

Sub (3) to (1) → $50 - 2(I_2 + I_3) - 2I_2 = 0$

→ $50 - 4I_2 - 2I_3 = 0$ → $-2I_3 = 4I_2 - 50$ … (4)

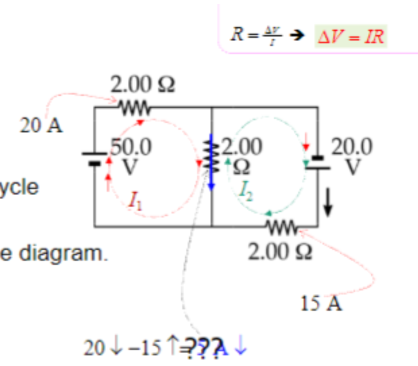
Sub (4) to (2) → $20 + (4I_2 - 50) + 2I_2 = 0$ → $6I_2 - 30 = 0$ → $I_2 = \boxed{5\text{ A}}$

(1) → $2I_1 = 50 - 2I_2 = 50 - 2(5) = 40$ A → $I_1 = \boxed{20\text{ A}}$

(3) → $I_3 = I_1 - I_2 = 20 - 5 = \boxed{15\text{ A}}$

Simplified Approach I have learnt from MIT Professor Walter Lewin from YouTube video:

Let the partial current in each cycle be as shown.



We apply Kirchhoff's rules to the diagram.

$50 - 2I_1 + 2(-I_1 + I_2) = 0$

→ $50 - 4I_1 + 2I_2 = 0$ … (1)

$20\downarrow - 15\uparrow = ??? \downarrow$

$20 - 2I_2 + 2(-I_2 + I_1) = 0$

→ $20 - 4I_2 + 2I_1 = 0$ … (2)

Simplified approach

These are algebra!!

(2) x 2: $40 - 8I_2 + 4I_1 = 0$ … (3)

(1) + (3): $90 - 6I_2 = 0$ → $I_2 = \boxed{15\text{ A}}$

(2): $20 - 4 \times 15 + 2I_1 = 0$ → $I_1 = \boxed{20\text{ A}}$

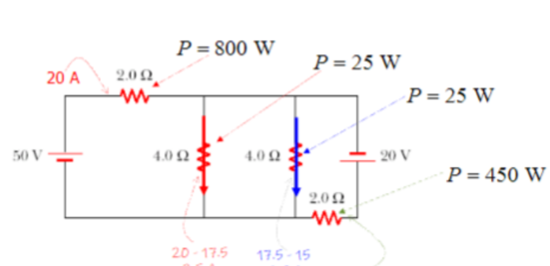
Power Distribution

- Total power output of a battery:

$$P = I\epsilon$$

- Power delivered to an external resistor:

$$P = I^2R$$



To find the power delivered to each resistor, we apply $P = I^2R$ to each resistor:

$(2.00\ \Omega)$: $P = (20\text{ A})^2(2\ \Omega) = \boxed{800\text{ W}}$

$(4.00\ \Omega)$: $P = (2.5\text{ A})^2(4.00\ \Omega) = \boxed{25.0\text{ W}}$

$(2.00\ \Omega)$: $P = (15.0\text{ A})^2(2.00\ \Omega) = \boxed{450\text{ W}}$