

National University of Singapore
MA1511 Engineering Calculus

Semester 1: AY2022/23 Examination

Time allowed: 1 hour 30 minutes

INSTRUCTIONS TO STUDENTS

1. Please write only your Student Number. Do not write your name.
2. This examination paper contains **TEN** questions and comprises **FOUR** printed pages.
3. Students are required to answer **ALL** questions.
4. Students should write the answers for each question on a new page.
5. This is a **CLOSED BOOK** (with authorized material) examination. Students are only allowed to bring into the examination hall one A4 double side help sheet.
6. You may use any calculator. However, you should lay out systematically the various steps in the calculations.

Question 1 [10 marks]

Find $\frac{\partial f}{\partial x}$ if $f(x, y, z) = (z + x^3) \sin(xy + z) + y \exp(zx^2)$, where $\exp w = e^w$.

Question 2 [10 marks]

Use the method of Lagrange multipliers to find the local extreme value of

$$f(x, y, z) = x^2 + 4y^2 + 16z^2$$

subject to the given constraint $xyz = 1$.

(Zero marks will be awarded if the method of Lagrange multipliers is not used.)

Question 3 [10 marks]

Find the exact value of the integral $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1} \, dy dx$.

Question 4 [10 marks]

Use polar coordinates to find the exact value of the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} \, dy dx$.

Question 5 [10 marks]

Let C be the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = y$. Find the exact value of the line integral

$$\int_C f(x, y, z) \, ds,$$

where $f(x, y, z) = \frac{z}{\sqrt{2x^2 + y^2}}$.

Question 6 [10 marks]

It is known that the vector field

$$F(x, y) = (e^x + 2xy)\mathbf{i} + (x^2 + \cos y)\mathbf{j}$$

is a conservative vector field. Find a potential function f for F .

Question 7 [10 marks]

Use Green's Theorem to find the exact value of the line integral

$$\oint_C \frac{x}{x^2 + y^2} dy - \frac{y}{x^2 + y^2} dx,$$

where C is the rectangular curve with vertices $(2, -3)$, $(2, 3)$, $(-2, 3)$ and $(-2, -3)$, taken in a counterclockwise direction. Justify your answer.

Hint: You may use the following identity without proof

$$\oint_{C_1} \frac{x}{x^2 + y^2} dy - \frac{y}{x^2 + y^2} dx = 2\pi,$$

where C_1 is the circle $x^2 + y^2 = 1$, taken in a counterclockwise direction.

(Zero marks will be awarded if Green's Theorem is not used.)

Question 8 [10 marks]

Find the exact value of the line integral

$$\int_C y dx + z dy + x dz,$$

where C is the vertical line segment from $(3, 4, 5)$ to $(3, 4, 0)$.

Question 9 [10 marks]

(a) Find the exact value of $\lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{n+1} \right)^{n+1} \left(\frac{2n+1}{n+2} \right) \right]$.

(b) Let $g(x) = \ln \left[\left(\frac{1+2x}{1-2x} \right)^2 \right]$, $-1 < 2x < 1$. Find the exact value of $g^{(3)}(0)$.

Question 10 [10 marks]

(a) Find the radius of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{6n}{2^n + 4^n} (5x + 1)^{2n-1}.$$

(b) Let $f(x) = \sum_{n=2}^{\infty} \frac{1}{(n-1)!} x^{2n}$. Find the exact value of $f(2)$.

– END OF PAPER