NATIONAL UNIVERSITY OF SINGAPORE

MA1301 –Introductory Mathematics

(Semester 1 : AY 2018/2019)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write your Student Number only. Do not write your name.
2. This assessment paper contains **Four (4)** questions and comprises **Five (5)** printed pages.
3. Students are to write the answers for each question on a new page.
4. Students are required to answer **ALL** questions.
5. This is a **closed book** assessment.
6. Students are only allowed to bring into the examination hall ONE piece A4 size help-sheet which can be used on both sides.
7. Students may use any non-programmable calculators.

Question 1 [25 marks]

(a) [5 marks]

Find $\dfrac{dy}{dx}$ if $y=\left(\sin(x^2+x)\right)^2$.

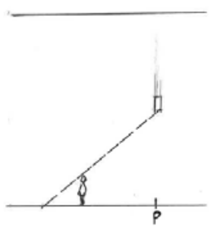
(b) [5 marks]

Suppose the equation of the line tangent to the curve $x^2+xy-y^3=7$ at $(3,2)$ is $y=mx+c$. Find the values of m and c.

(c) [5 marks]

Suppose $x(t)=\dfrac{1}{2}t^2,\ y(t)=t,$ where $t>0$. Find $\dfrac{d^2y}{dx^2}$.

(d) [10 marks]



A person 6 ft tall stands 10 ft from the point P directly beneath a lantern hanging 30 ft above the ground, as shown in the above figure. The lantern starts to fall, thus causing the person's shadow to lengthen. Given that the lantern falls $16t^2$ ft in t sec, how fast will the shadow be lengthening when $t=1$? The length of the lantern is negligible.

(Hint: Let $S(t)=16t^2$ and the length of the shadow be $L(t)$. Find $\dfrac{dL}{dt}$ when $t=1$.)

Question 2 [25 marks]

(a) [5 marks]

Prove the following identity by mathematical induction.

$$\sum_{r=1}^{n} 2^r = 2^{n+1}-2 \text{, for all natural number } n.$$

(Zero mark will be awarded if induction method is not used.)

Not tested

(b) [5 marks]

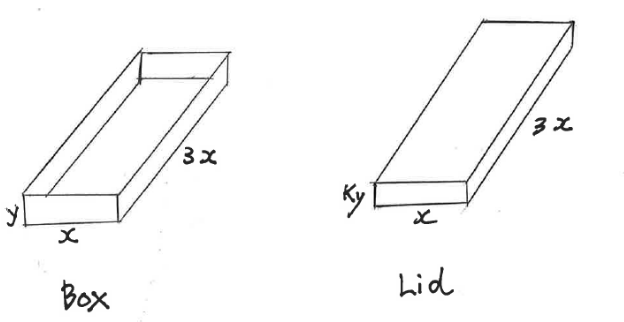
Let $f(x)=x^9-2x+1$. Use the linear approximation to approximate the value of $f(1.05)-f(1.00)$. Give your answer correct to two decimal places.

(Zero mark will be awarded if the linear approximation is not used.)

(c) [5 marks]

Let $f(x)$ be a function with the derivative $f'(x)=\dfrac{(x+1)(x-2)^2(7-2x)}{x^2+8}$. At what points (x-coordinates), if any, does the graph of f have a local minimum, local maximum, or a saddle point? Justify your answers.

(d) [10 marks]



A company requires a box made of cardboard of negligible thickness to hold 300 cm^3 of powder when full. The length of the box is $3x$ cm, the width is x cm and the height is y cm. The lid has depth ky cm, where $0<k\le 1$ (see the above diagram).

(i) Use differentiation to find, in terms of k, the value of x which gives a minimum total external surface area of the box and the lid.

(ii) Find the value of k for which the box has square ends in part (i).

Question 3 [25 marks]

(a) [5 marks]

Find $\displaystyle\int (1+2x)(x+x^2)^{\frac{-1}{2}}\,dx$.

(b) [5 marks]

Find $\displaystyle\int x^8 \ln x^9 \; dx$.

(c) [5 marks]

Let $\displaystyle\int_1^u x^{-1}\sin x\, dx = G(u)$. Find, in term of π , the exact value of the derivative $G'(\pi/2)$.

Not tested

(d) [10 marks]

(i) It is known that the area A of the region bounded by $x = 2y^2$ and $x + 2y = 4$ is

$$A = \int_{-2}^{1} f(y)dy = \int_{0}^{2} g(x)dx + \int_{2}^{8} h(x)dx$$

and the points of intersection of the above two curves are $(2,1)$ and $(8,-2)$ respectively. Find the functions $f(y)$, $g(x)$, and $h(x)$.

(ii) A torus (doughnut) is formed when the circle of radius 2 centered at (3,0) is revolved about the y-axis. Suppose V is the volume of the torus and

$$V = \pi \int_{-2}^{2} f(y)dy \ .$$

Find the function $f(y)$.

Question 4 [25 marks]

(a) [5 marks]

Solve the following differential equation.

$$\frac{dy}{dx} - \frac{e^{y^2+8x}}{y} = 0 \ .$$

(b) [5 marks]

Let $A(0,0,1)$, $B(2,0,0)$, and $C(0,3,0)$ be three points in three- dimensional space.

(i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(ii) Suppose $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ is the equation of the plane passing through the points $A(0,0,1)$, $B(2,0,0)$, and $C(0,3,0)$. Find the values of a, b, and c .

(c) [5 marks]

Suppose the following pair of lines intersects.

$L_1 : r = (i + 2j - 3k) + \lambda(2i - 4j + k)$

$L_2 : r = (-2i + j + 3k) + \mu(i + 5j + \alpha k)$

Find the value of α .

(d) [10 marks]

Suppose a plane $\prod_1$ passes through the origin and the point $(6,-3,2)$, and it is perpendicular to a plane $\prod_2$: $4x - y + 2z = 8$. If the equation of the plane $\prod_1$ is $2x + by + cz = d$, find the values of b, c, and d.

Formulae are given on page 5

Formulae

Table of integrals

1. $\int (ax+b)^{\alpha}dx = \dfrac{(ax+b)^{\alpha+1}}{(\alpha+1)a} + C$, where $\alpha \neq -1$.

2. $\int \cos(ax+b)\ dx = \dfrac{1}{a}\sin(ax+b) + C$.

3. $\int \sin(ax+b)\ dx = -\dfrac{1}{a}\cos(ax+b) + C$

4. $\int \sec^2(ax+b)\ dx = \dfrac{1}{a}\tan(ax+b) + C$

5. $\int e^{ax+b}\ dx = \dfrac{1}{a}e^{ax+b} + C$

6. $\int \dfrac{1}{ax+b}\ dx = \dfrac{1}{a}\ln\left|ax+b\right| + C$

7. $\int \dfrac{1}{\sqrt{a^2-(x+b)^2}}dx = \sin^{-1}\left(\dfrac{x+b}{a}\right) + C$

8. $\int \dfrac{1}{a^2+(x+b)^2}dx = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x+b}{a}\right) + C$

– END OF PAPER –