Chapter 15 Oscillatory Motion

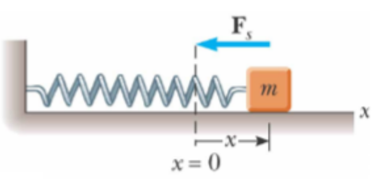
Periodic Motion

Periodic motion is motion that regularly repeats a pattern, with the object returning to a specific position after a set time interval. In mechanical systems, if the force on the object is proportional to its position relative to an equilibrium and always directed toward it, this motion is called **simple harmonic motion (SHM)**.

Conditions for Force in Simple Harmonic Motion

- In simple harmonic motion, the farther the object is from the equilibrium position, the larger the force acting on it, and the closer it is, the smaller the force.
- The force always acts in the opposite direction of the object's position.

Acceleration



- The force described by Hooke's Law is the net force in Newton's Second Law

$$\begin{aligned} F_{\text{Hooke}} &= F_{\text{Newton}} \\ -kx &= ma_x \\ a_x &= -\frac{k}{m}x \end{aligned}$$

- The magnitude of acceleration is proportional to the magnitude of the displacement of the block from equilibrium
- The direction of the **acceleration is opposite** to the position from equilibrium
- An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the position from equilibrium
- The acceleration is **not** a constant, as the spring force keeps changing (means kinematics equations cannot be applied)
- If the block is released from some position $x = A$, then the initial acceleration is $-\frac{k}{m}A$
- When the block passes through the equilibrium position, $a = 0$
- The block continues to $x = -A$ where its acceleration is $+\frac{k}{m}A$

Motion of the Block

- The block continues to oscillate between $-A$ and $+A$ (these are the turning points of the motion)
- The force is conservative
- In the absence of friction, the motion will continue *forever* (but real system has frictions)

Vertical Spring

- When the block is hung from a vertical spring, its weight will cause the spring to stretch a distance e
- If the *resting position* of the spring (*after* the block is attached) is defined as $x = 0$, then it will be same as a horizontal spring

SHM Equations

- Acceleration $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$
- We let $\omega^2 = \frac{k}{m}$, or $\omega = \sqrt{\frac{k}{m}}$
- Then $a = -\omega^2 x$
- Note: ω represents the angular frequency or angular velocity
- $x(t) = A\cos(\omega t + \phi)$ (ϕ is the phase constant or the *initial* phase angle)
- $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$
- $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Period and Frequency

- The frequency and the period depend only on the mass of the particle and the force constant of the spring
- They do not depend on the parameters of motion, *only amplitude is affected*
- But if replaced by a *stiffer* spring or a larger value of k, and/or *decreased* the mass, the **frequency will be larger**

Maximum Values of v and a

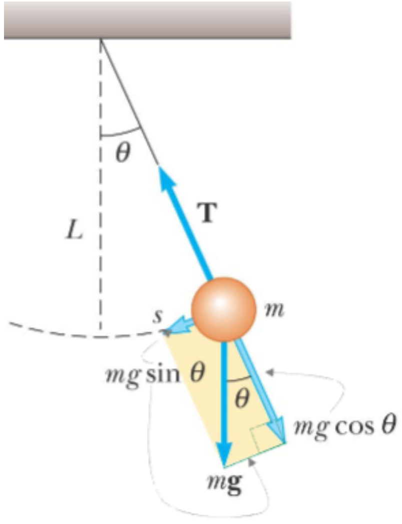
$v_{max} = -\omega A \times (-1) = \sqrt{\frac{k}{m}}A$
$v_{max} = -\omega^2 A \times (-1) = \frac{k}{m}A$

Energy of the SHM Oscillator

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$
$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$
$K + U = \frac{1}{2}kA^2$

- The total mechanical energy is constant
- The total mechanical energy $\propto$ square of the amplitude
- Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the block

Simple Pendulum



- The forces acting on the bob are T and mg
  - T is the force exerted on the bob by the string
  - mg is the gravitational force
- The tangential component of the gravitational force is a restoring force for SHM
- Take going *right to be positive*
- In the tangential direction, $F_t = -mg\sin\theta = ma = m\frac{d^2s}{dt^2}$
- The length, L of the pendulum is constant
- $\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta$
- $\theta = \theta_{max}\cos(\omega t + \phi)$
- $\omega = \sqrt{\frac{g}{L}}$
- $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$
- The period and frequency of a simple pendulum depends only on the length of the string and acceleration due to gravity
- The period is **independent of the mass**, so all simple pendula that are of equal length are at the same location oscillate with the *same period*

Damped Oscillation

- Decreasing amplitude
- Absorb energy due to vibration