Chapter 24 Electric Potential

Concept of Electric Potential

- Electric potential (V) at a point is the **electric potential energy per unit charge**:

$$V = \frac{U}{q_0}$$

  - U: Potential energy of the charge-field system.
  - q_0: Test charge.
- Electric potential is a **scalar quantity** and depends on:
  - The location relative to the source charge.
  - The sign and magnitude of the source charge.
- The electric potential decreases with distance from a source charge.

Key Properties

1. Potential at a location far from the source charge ($r \to \infty$) is set to $V = 0$.
2. Positive charges create **positive potential** values.
3. Negative charges create **negative potential** values.

- The unit of V is volts ($1\,V = 1\,J/C$).

Work and Potential Difference

- The work done by an electric field to move a charge from A to B:

$$W = q_0 \Delta V = q_0(V_B - V_A)$$

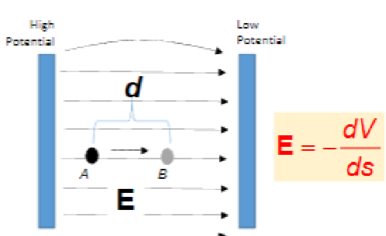
- The electric field E relates to the rate of change of potential:

$$E = -\frac{dV}{ds}$$

Electron-Volt (eV)

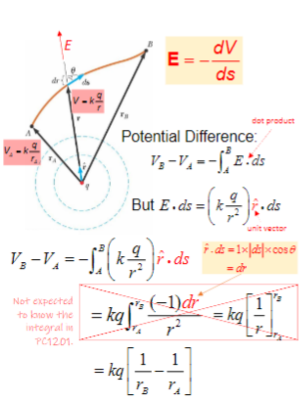
- Energy unit used in atomic/nuclear physics.
- $1\,eV = 1.602 \times 10^{-19}\,J$.

Potential in Uniform Electric Fields

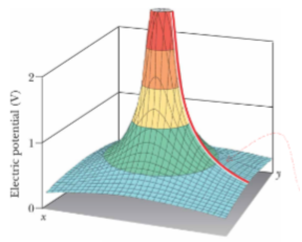


- For a uniform field E, the potential difference (ΔV) between points A and B is:
- $$\Delta V = -E \cdot d$$
  - E: Electric field strength.
  - d: Distance between points along the field direction.
- The direction of E is from **high potential** to **low potential**.

Potential of Point Charges



- Electric potential (V) due to a single point charge (q) at distance r from the charge:
- $$V = k\frac{q}{r}$$
  - $k = 8.9875 \times 10^9\,N \cdot m^2/C^2$.
- Potential difference (ΔV) between two points A and B:
- $$\Delta V = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



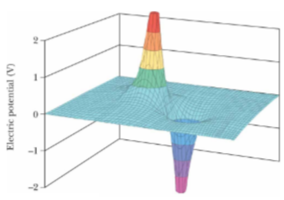
Electric Potential with Multiple Charges

- The potential at a point due to multiple charges is the **algebraic sum** of individual potentials:

$$V = \sum_i k\frac{q_i}{r_i}$$

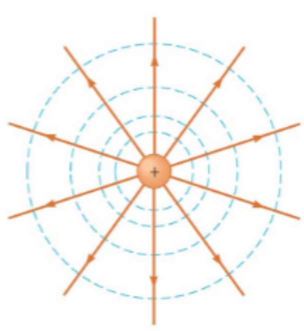
- Apply the **superposition principle**.

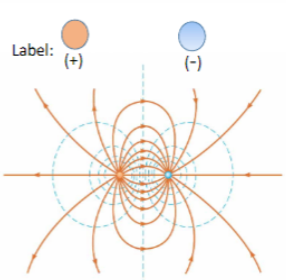
Potential of an Electric Dipole



- A dipole consists of two charges $+q$ and $-q$ separated by a distance d.
- The potential at a point on the dipole axis decreases as the distance from the dipole increases.

Equipotential Surfaces



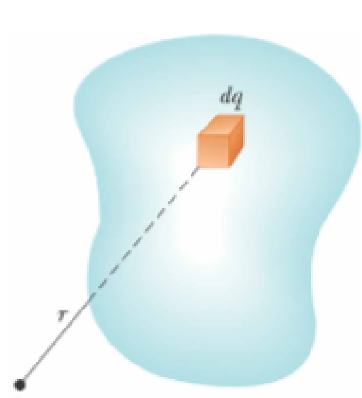


- An **equipotential surface** is a 3D surface where the electric potential is constant.
- Properties:
  1. Equipotential surfaces are **perpendicular** to electric field lines.
  2. No work is done moving a charge along an equipotential surface.

Examples

1. For a **point charge**, surfaces are concentric spheres.
2. For a **uniform field**, surfaces are parallel planes.

Continuous Charge Distributions



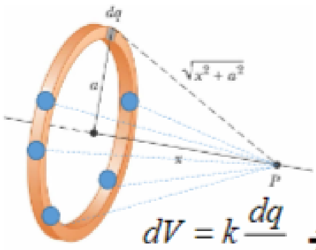
- The potential V due to a continuous charge distribution is calculated by integration:

$$V = k\int \frac{dq}{r}$$

- dq: Charge element.
  - r: Distance between dq and the point of interest.

Special Case: Charged Ring

- For a ring with radius a and total charge Q, the potential at a point P on its axis (x) is:
- $$V = \frac{kQ}{\sqrt{x^2 + a^2}}$$



- P is located on the perpendicular central axis of the uniformly charged ring
  - The ring has a radius a and a total charge Q

(Potential is a scalar)

$$dV = k\frac{dq}{r} \rightarrow V = k\int\frac{dq}{r} = k\int\frac{dq}{\sqrt{x^2+a^2}}$$

But any dq on the ring is at the same distance from point P. So, $V = \frac{k}{\sqrt{x^2+a^2}}\int dq = \boxed{\frac{kQ}{\sqrt{x^2+a^2}}}$ $x \geq 0$

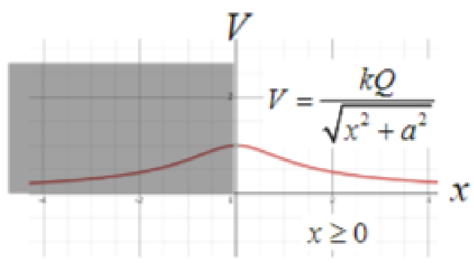
Not expected to know the derivative in PC1201.

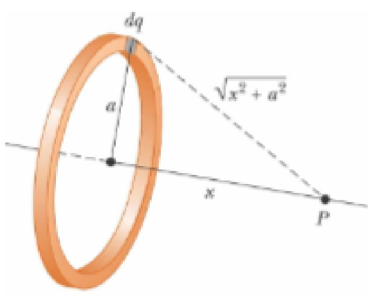
Electric field at point P:

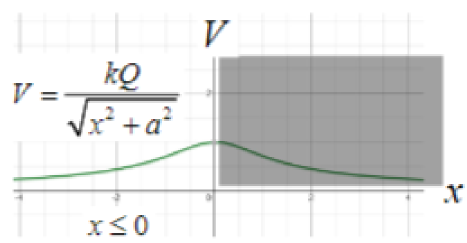
$$E_x = -\frac{dV}{dx} = -kQ\frac{d}{dx}\left(x^2+a^2\right)^{-1/2} = \cancel{-kQ\left(-\frac{1}{2}\right)\left(x^2+a^2\right)^{-3/2}(2x)}$$

$$= \boxed{\frac{kQx}{\left(x^2+a^2\right)^{3/2}}}$$ Same as Chapter-22 $x \geq 0$

Ring

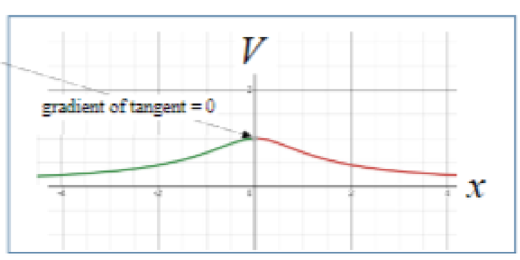




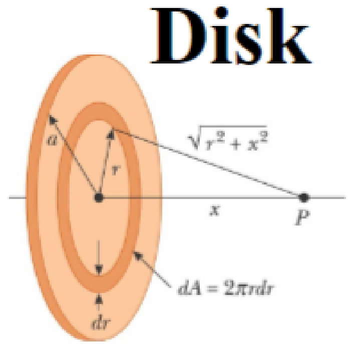


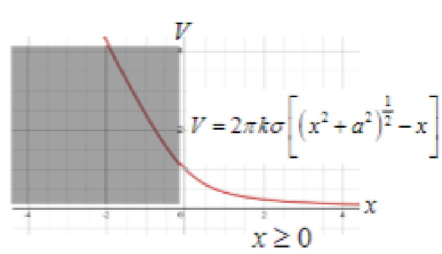
$$E_x = -\frac{dV}{dx} = \frac{kQx}{\left(x^2+a^2\right)^{3/2}} = 0$$

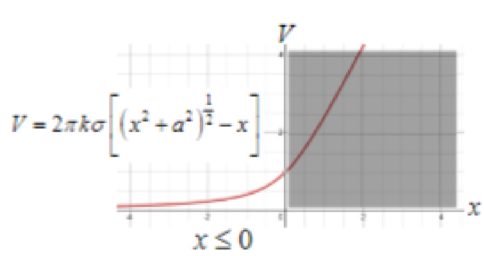
is valid for a ring when $x = 0$.



Disk

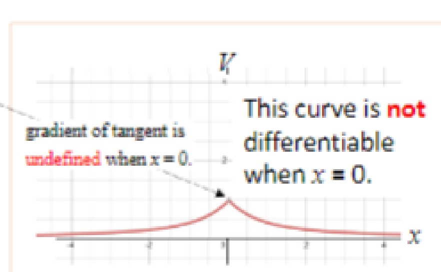






$$E_x = -\frac{dV}{dx} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

is invalid for a disk when $x = 0$.



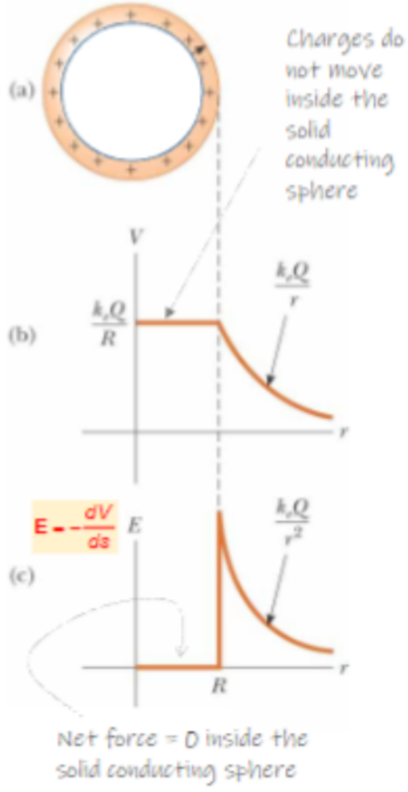
Potential Due to Charged Conductors

- In a charged conductor at **electrostatic equilibrium**:
  - The **electric field inside** is zero, as charges have redistributed to cancel any internal field.
  - All points on the conductor's surface are at the **same potential**:

$$\Delta V = 0 \quad \text{(between any two points on the surface)}$$

  - Since (E = 0) inside the conductor, the **potential remains constant** throughout the entire conductor.
- **Equipotential Surface**:
  - The surface of a conductor in electrostatic equilibrium forms an equipotential surface.

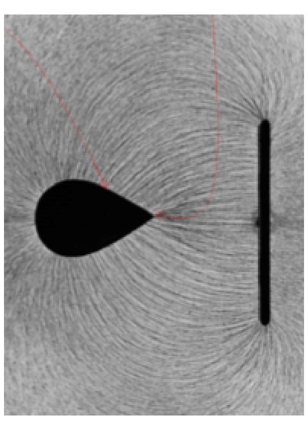
- Moving a charge along this surface requires no work.



Electric Potential and Electric Field Relationship

- The electric field at the surface of a charged conductor is **perpendicular to the surface**.
- The magnitude of E near the surface is highest at points with **small radius of curvature** (sharp points), where charge density is highest:

$$E = -\frac{dV}{ds}$$



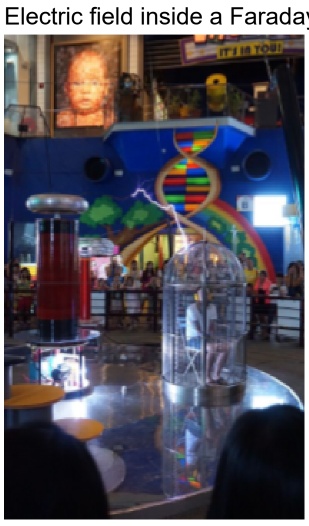
Corona Discharge

- At sharp points of a conductor, if the electric field exceeds a critical value, it can ionize nearby air, causing **corona discharge**.
- This phenomenon creates a glow and can discharge electrons into the surrounding air, creating additional ionization.

Electrostatic Shielding: The Faraday Cage

- **Faraday Cage**: A conductive enclosure that blocks external electric fields.
- Electric field inside a Faraday cage is zero, protecting the interior from electric effects, such as lightning strikes.



Practical Applications: Electrostatic Precipitator

- Utilizes electric discharge to **remove particulate matter** from gases.

- As gas flows through a duct with an electric field, particles become charged and move to the walls, where they can be collected.