NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2017-2018

MA1301 INTRODUCTORY MATHEMATICS

November 2017 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **SIX (6)** questions and comprises **TWENTY THREE (23)** printed pages.
2. Answer **ALL** questions on this booklet. For each question, write your working and answer in the space provided inside the booklet following that question.
3. This is a CLOSED BOOK examination. You are allowed to use ONE A4-sized, double-sided help sheets.
4. You may use non-programmable and non-graphing calculators. However, you should lay out systematically the various steps in the calculations.

Matriculation number

Question	1	2	3	4	5	6	Total
Marks							

Question 1. [20 marks]

1(a) Find the following integrals.

(i) $\int \frac{1}{x(x+2)}\, dx.$

(ii) $\int \frac{1}{e^x + 2}\, dx.$

Space for writing.

1(b) The curve for which $\frac{dy}{dx} = 4 + \frac{k}{x^2}$, where k is a constant, has a turning point at $(\frac{1}{2}, 4)$. Find

(i) the value of k,

(ii) the equation of the curve.

Question 2. [16 marks]

2(a) Use the substitution $y = v + 2x$ to solve the differential equation

$$\frac{dy}{dx} = 2 + \frac{1}{(2x - y)^2},$$

given that the solution curve passes through the origin.

Space for writing.

2(b) Find the particular solution of the differential equation

$$(2y-1)\frac{dy}{dx} - 2e^y = 0$$

for which $y = 0$ when $x = 2$.

Space for writing.

Question 3. [16 marks] Not tested

3(a) Suppose that $|z(1+i)| = \sqrt{32}$ and $\arg\left(\frac{1-i}{z}\right) = \frac{\pi}{6}$.

(i) Express z in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(ii) Find the smallest positive integer N such that z^N is a real number.

Space for writing.

3(b) Solve the simultaneous equations

$$3z + w = 15 + 6i$$

$$6z + w = \frac{150}{1 - 7i},$$

giving z and w in the form $a + bi$, where a and b are real. Hence, find the value of the real number x for which $(iz + w^*)(x + i)$ is a real number.

Space for writing.

Question 4. [20 marks]

Let $f(x) = 9 - 4x^2$ and $g(x) = 4x^2 + 1$. Let R be the region bounded by the curves $y = f(x)$ and $y = g(x)$.

(i) Sketch on a **single** diagram the graphs of $f(x)$ and $g(x)$.

(ii) Find the area of R.

(iii) Find the volume generated when R is rotated through $360°$ about the $x-$axis.

(iv) Find the volume generated when R is rotated through $360°$ about the $y-$axis.

Space for writing.

Space for writing.

Space for writing.

Question 5. [16 marks]
Two lines L_1 and L_2 have vector equations given respectively by

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{2i} + \mathbf{j} + \mathbf{k}) \quad \textbf{and} \quad \mathbf{r} = \mathbf{4i} + \mathbf{j} + \mathbf{10k} + \mu(\mathbf{i} + \mathbf{3k}).$$

(i) Show that L_1 and L_2 intersects, and find the point of the intersection.

(ii) Find the acute angle between L_1 and L_2.

(iii) Show that the point $A(3, 3, 7)$ does not lie on the line L_1, and determine the foot of perpendicular from A to L_1.

Space for writing.

Space for writing.

Space for writing.

Question 6. [12 marks]

A closed cylindrical can of radius r cm and height h cm is to be constructed to hold a volume of 288π cm^3. If the material for the curved side of the can costs 60 cents per cm^2 and the material for the top and base costs 40 cents per cm^2, find the minimum cost of the material required.

Space for writing.

END OF PAPER