

Semiconductor & PN Junction

Materials can be classified according to their electrical properties:

1. Conductors

2. Insulators

3. Semiconductors.

1. Conductors.

TL; DR \rightarrow > 4 valence. \rightarrow Insulators

$\rightarrow = 4$ valence \rightarrow Semiconductors.

$\rightarrow < 4$ valence \rightarrow Conductors.

- Atoms can have 1, 2, or 3 very loosely bound valence electrons.

- Valence electrons are electrons at the outermost orbit of the atom. Such electrons can easily break away and become free electrons.

- Hence a conductor has many free electrons available to support current flow when a voltage is applied.



2. Insulators.

- Materials that don't conduct electrical current under normal conditions.

- Usually compounds rather than elements.

- Atoms have 5 or more very tightly bound valence electrons which cannot break away easily from the atoms.

- Hence very few free electrons present to support current flow when voltage is applied.

3. Semiconductors.

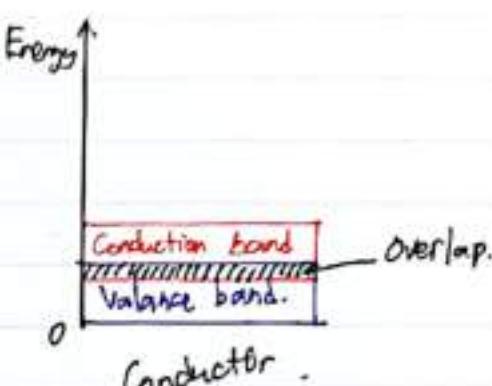
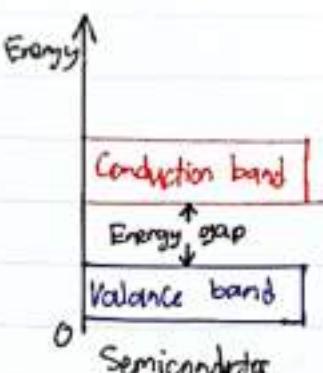
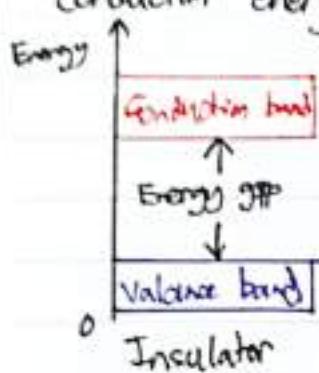
- Materials which electrical conductivity is between that of conductor & insulator.

- Intrinsic (pure) semiconductor is neither a good conductor nor a good insulator.

- They have 4 valence electrons which are moderately bound to the atom.

- Examples \rightarrow carbon, silicon, Germanium

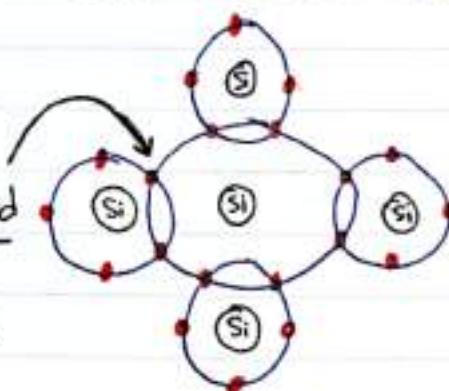
When a valence electron acquires enough additional energy, it is able to leave its valence shell and become a free electron. Such electrons are said to move from the valence energy band to the conduction energy band.



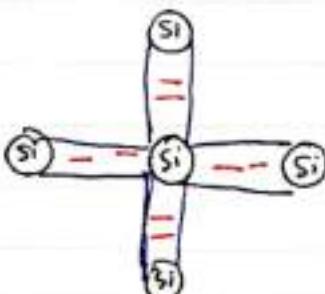
The conduction band and valence band in conductor overlaps resulting in large number of free electrons even without external energy.

Covalent bond.

- Pure (intrinsic) silicon has no impurities and is a crystalline material
- Its atoms are held together by covalent bonds (sharing of electrons)
- Center atom shares 1 electron with each of four surrounding atoms creating a covalent bond with each.
- The surrounding atoms are in turn bonded to other atoms, and so on.

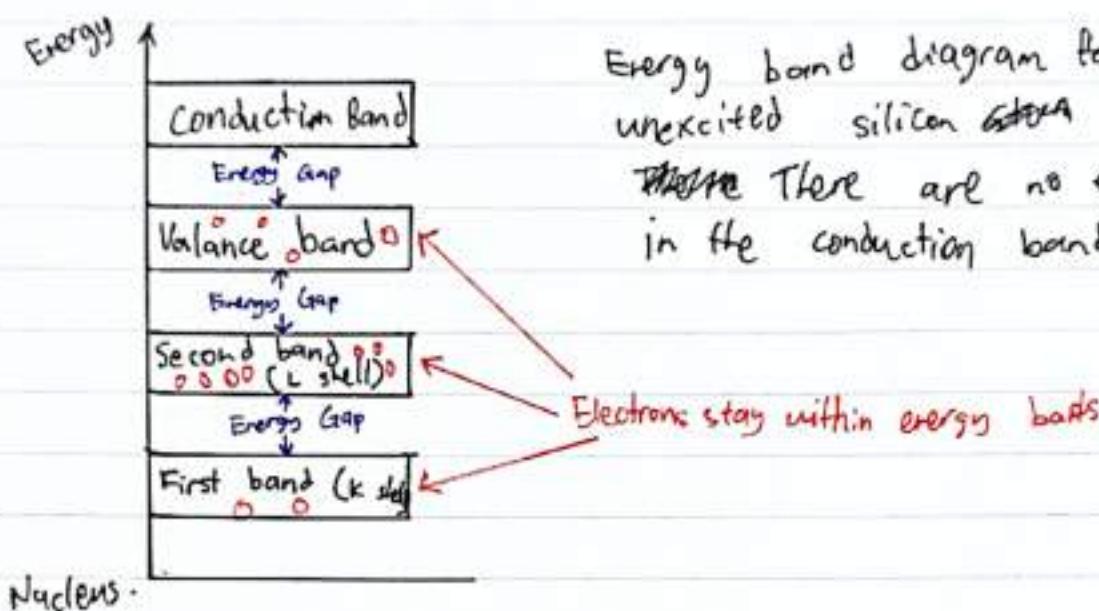


- Another way to represent the covalent bonds.
- Bonding Diagram. The red negative signs represent the shared valence electrons.



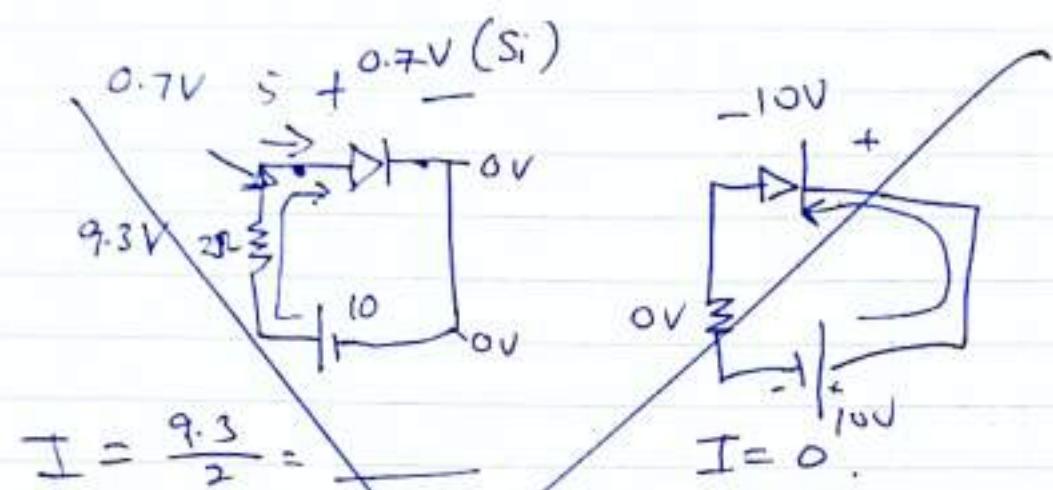
Semiconductor & PN Junction

- If an electron is unexcited, it stays ~~cation~~ within its prescribed atomic shell.
- Atomic shells can be looked upon as energy bands where electrons with similar energy levels reside.



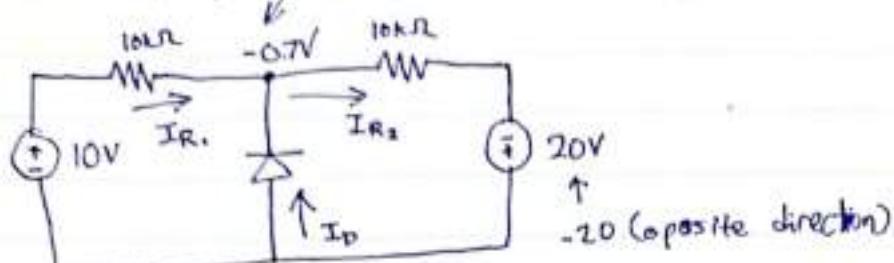
Energy band diagram for an unexcited silicon atom.
There are no electrons in the conduction band.

Electrons stay within energy bands



reverse, open
doped w trivalent
reduce free electron \rightarrow add trivalent.

\leftarrow forward biased \rightarrow voltage must be smaller than $-0.7V$.



$$I_{R_1} + I_D = I_{R_2}$$

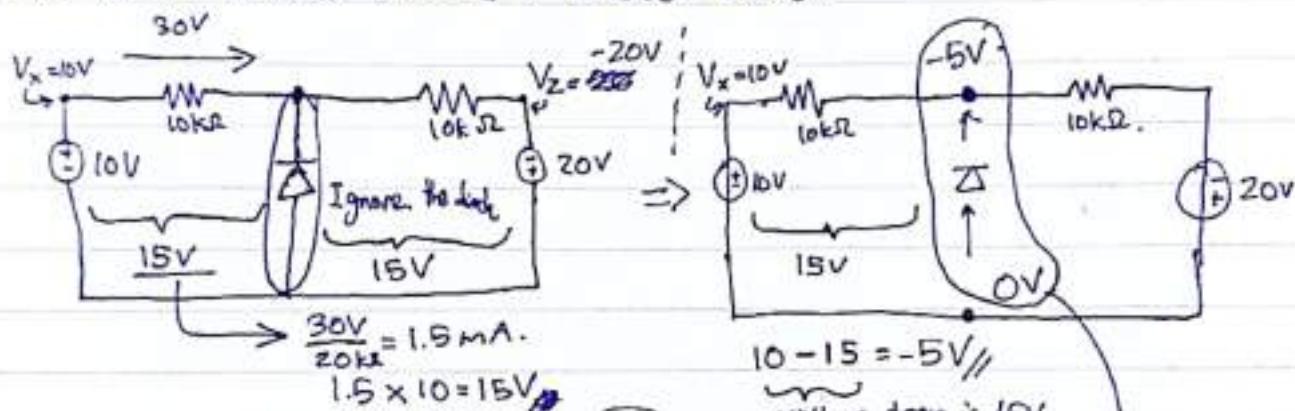
$$I_D = I_{R_2} - I_{R_1}$$

$$I_{R_1} = \frac{10V - (-0.7V)}{10k\Omega} = 1.07mA.$$

$$I_{R_2} = \frac{(-0.7V) - (-20V)}{10k\Omega} = 1.93mA$$

$$I_D = 1.93 - 1.07 = 0.86mA$$

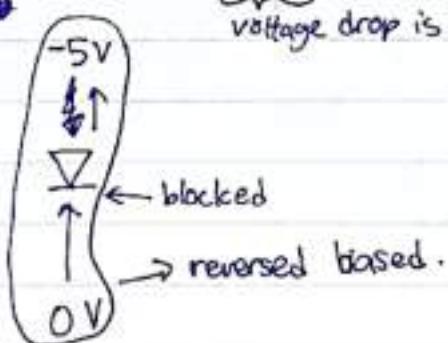
How to tell if it is forward / reversed biased.



$$10 - 15 = -5V$$

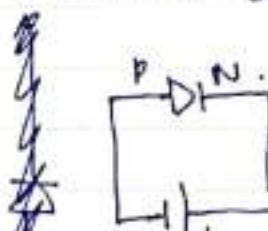
voltage drop is 15V

0 can go to -5 means forward.

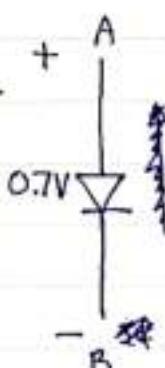


Forward \rightarrow current can flow

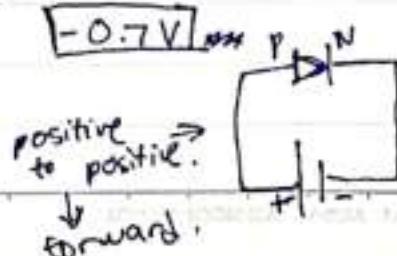
Reversed \rightarrow current cannot flow.



positive to negative
 \rightarrow reversed.

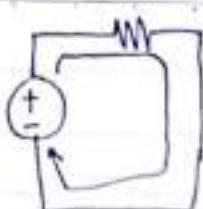


If B is 10V,
A must be 10.7V or more
 \therefore must have 0.7V difference.

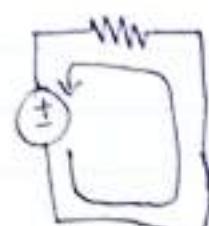


positive to positive?

forward.

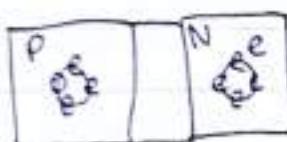


conventional current theory \rightarrow calculation use this positive-to-negative

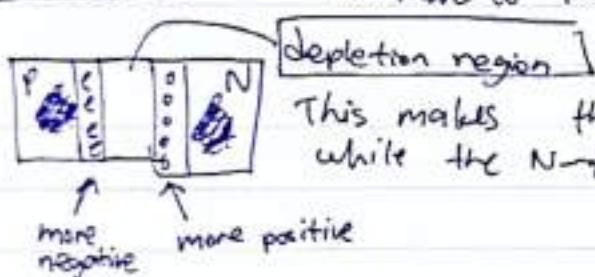


Electron flow \rightarrow IRL is this cuz current flow is due to electron flow \rightarrow negative to positive.

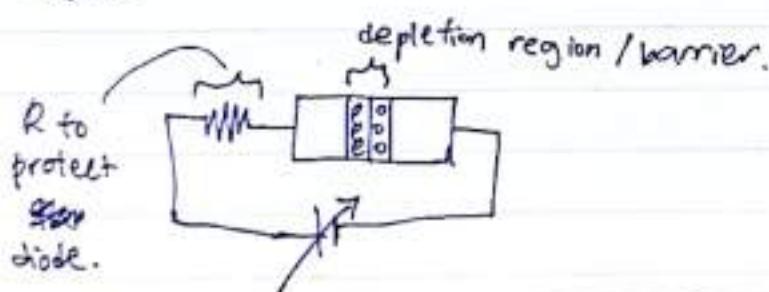
P-N Junction.



When they are met, all the electron from the N-type will move to the P-type.



This makes the P-type more negatively charged while the N-type is more positively charged.



Normal Diode.
(forward biased)

Zener (reversed) Diode (biased).
given voltage almost constant if used in reversed biased.

If zener diode in forward biased will act like normal diode.

Semiconductor & PN Junction.

Materials can be classified according to their electrical properties:

1. Conductor → - Valence shell have either 1, 2, or 3 electrons.

- Since electron number is very low or close to 0, thus they will happily give out their electrons, hence they are good at conducting current.

2. Insulator → - Valence shell have either 5, 6, 7 or 8 electrons.

- Since electron number are closer or are 8, thus they will not easily give out electron, hence they are bad at conducting current, as the flow of current is also called as the flow of electron.

3. Semiconductor → - Valence shell have exactly 4 electrons.

- They are in the middle, so they are not easily giving or taking electrons.

- It's neutrally charged.

- Example: Silicon, Germanium.

- The pure semiconductor are also known as intrinsic (pure) semiconductor.

The 2 types of Semiconductor:

- N-type semiconductor. → Extrinsic (impure) semiconductor

- Negative type (charged)

- Have 1 extra electron.

- Wants a hole.

- The atom with the extra electron is called an impurity atom, where it has 5 electrons, this is called "pentavalent atom".

- This makes it more negatively charged:

- ~~Majority~~ • Majority carrier are electrons

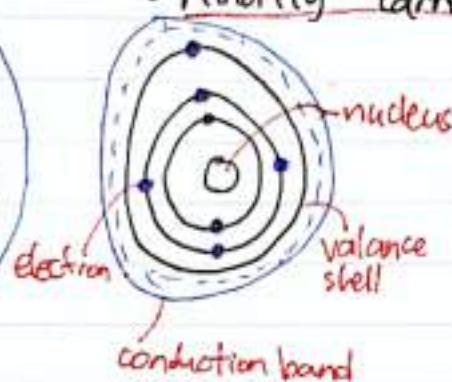
- Minority carrier are holes.

* Majority → i.e. there are 2 million N-type semiconductor, this will also mean that there will be 2 million extra electrons, this means that there are more electrons than holes, thus this is called as the majority carrier.

- P-type semiconductor \rightarrow - Extrinsic (impure) semiconductor.
- Positive type (charged)
- Have 1 extra hole / 1 less electron.
- wants an electron.
- The atom with 1 less electron is called an impurity atom, where it has 3 electrons, this is called "trivalent atom"
- This makes it more positively charged:
 - Majority carrier are holes
 - Minority carrier are electrons.



Insulator

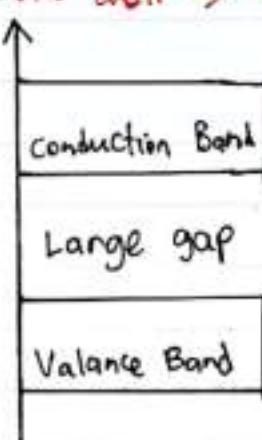
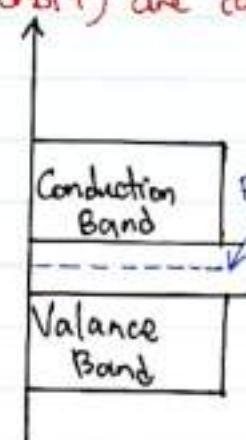
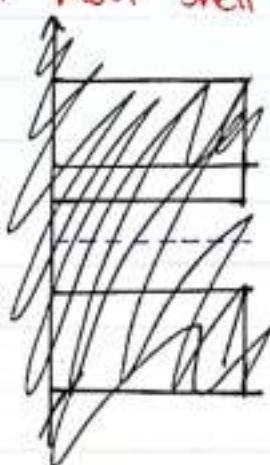
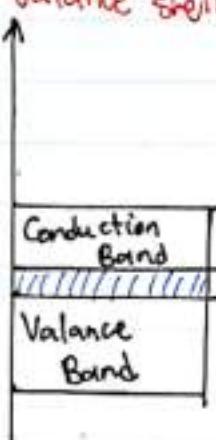


Conductor



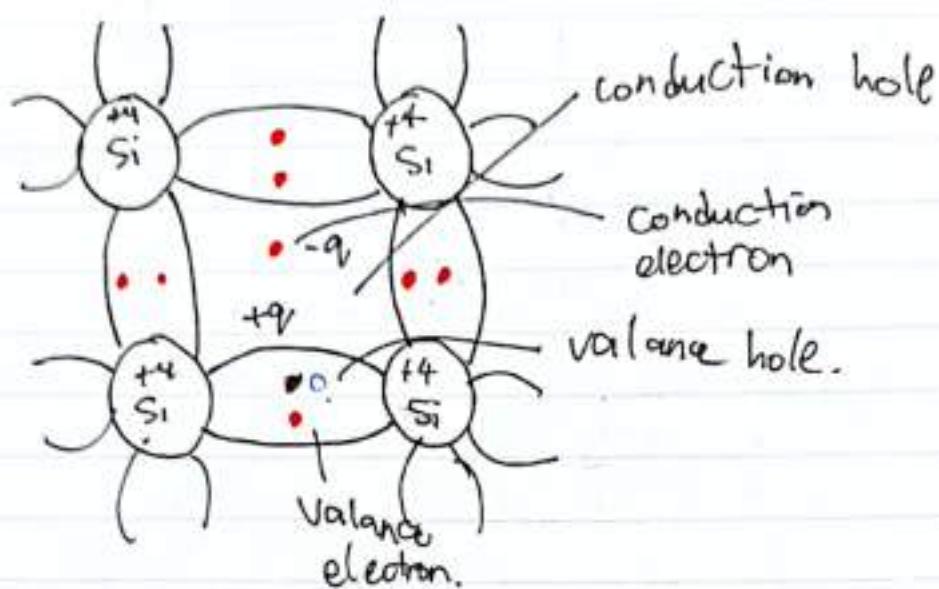
Semiconductor

Valence shell \rightarrow The outer most shell (orbit) are called valence shell

Insulator -
Large gapSemiconductor -
Small gapConductor -
Very close
(overlap).

Semiconductor & PN Junction.

Electron - Hole - Pair



When pure silicon receives heat or light energy from surroundings, some valence electron will gain sufficient energy to jump from the valence shell into the conduction band.

- Electron-hole-pair refers to the break-away free electrons and the hole left behind. The process is called ionisation.

- The hole is known as a plus (positive charge), as it is positively charged, it is waiting for an electron (which is negatively charged).

$$\text{electron} + \text{hole} = \text{electron-hole-pair}$$

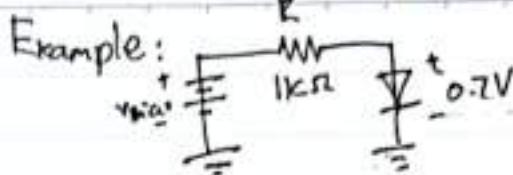
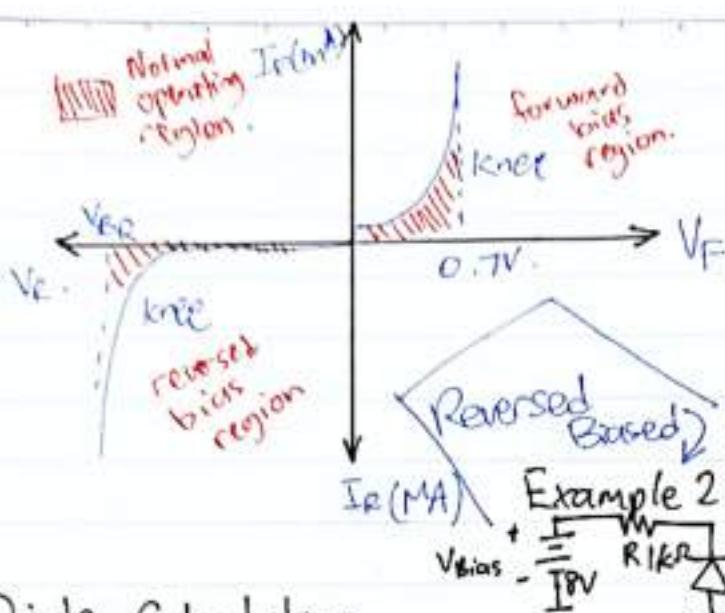
→ Direction of the electron moving.

← Direction of the hole moving.

- The hole is filled with the electron, and place of the original electron (that is now in the new hole) is turned into a hole, the process will happen repeatedly.

- The pure semiconductor consists of N-type and P-type semiconductor.

Forward Biased \rightarrow

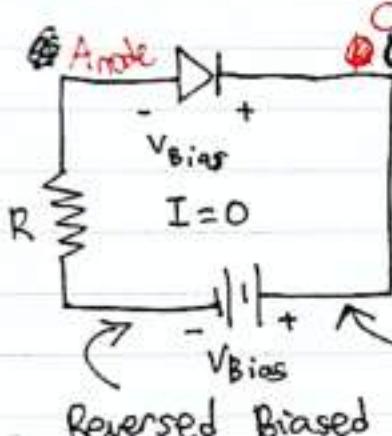


Find I_F and V_R .

$$I_F = \frac{V_{bias} - V_F}{R} = \frac{8 - 0.7}{1} = 7.3 \text{ mA}_{//}$$

$$V_R = I_F \times R = 7.3 \text{ mA} \times 1k\Omega = 7.3V_{//}$$

Diode Calculation.

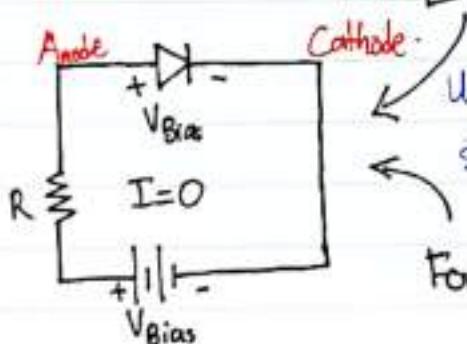


When diode is reversed biased, it acts as an open switch.

Anode terminal of the diode, connecting to p-region

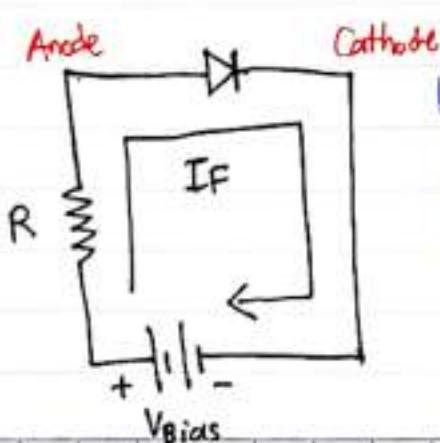
Cathode terminal of the diode, connecting to n-region

$$\begin{aligned} I_D &= 0A \\ V_R &= I_D \times R = 0V \\ V_{diode} &= V_{bias} - V_R = V_{bias} \end{aligned}$$



When $V_{bias} < 0.7V$, the diode acts as an open switch.

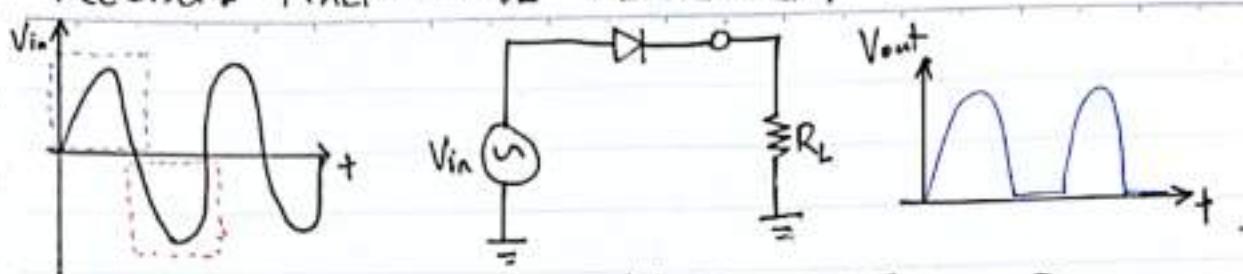
Forward Biased, but $V_{bias} < 0.7V$



When $V_{bias} \geq 0.7V$, the diode acts as a closed switch in series with a small voltage supply of 0.7V

$$\begin{aligned} V_{bias} &= I_F \times R + V_F \\ \therefore I_F &= \frac{V_{bias} - V_F}{R} \end{aligned}$$

Rectifier. HALF-WAVE RECTIFIER.

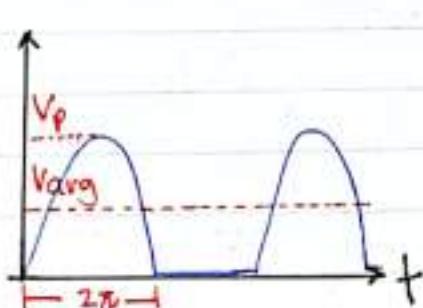


HALF-WAVE RECTIFIER.

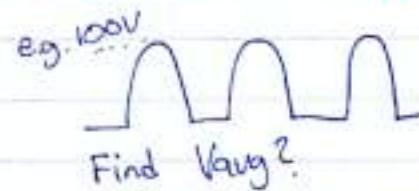
- The diode will convert input voltage (V_{in}) waveform into half wave output (V_{out})
 - Only the positive half-cycles appear across the load.
 - ~~Output~~ Output is a pulsating d.c. voltage.

When the input (V_{in}) goes positive, the diode is forward-biased and conducts current through the load resistor R_L . Thus, this will develop an output voltage V_{out} across it which has the same shape as the positive half-cycle of the input voltage.

When the input goes negative during the second half of this cycle, the diode is reverse-biased. Thus, ~~no~~ no current flows into the load R_L (i.e. $I=0A$), so the voltage difference across the load R_L is $0V$. $\rightarrow V_{RL} = I_{RL} \times R_L = 0 \times R_L = 0V$,



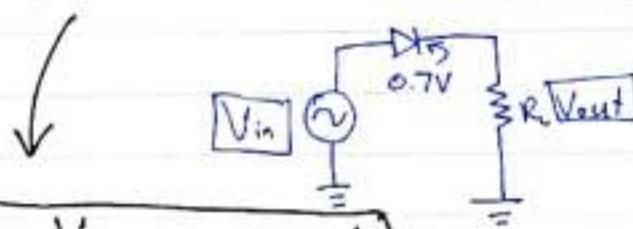
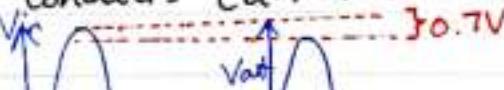
$$V_{avg} = \frac{V_p}{\pi}$$



$$V_{avg} = \frac{V_p}{\pi} = \frac{100}{\pi} = 31.83V$$

During the positive half cycle, the input voltage must overcome the barrier potential before the diode conducts current.

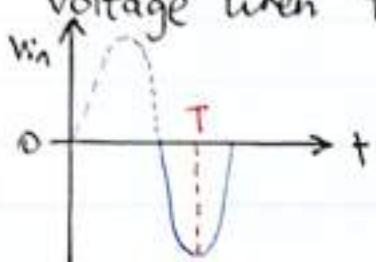
Thus, $V_{out(p)} = V_{in(p)} - 0.7V$



$$V_{out(p)} = V_{sec(p)} - 0.7V$$

Peak Inverse Voltage (PIV).

- PIV occurs at peak of each negative alteration of the input voltage when the diode is reverse-biased



$$\text{PIV} = V_{in(p)}$$

$$\text{PIV} = V_{sec(p)}$$

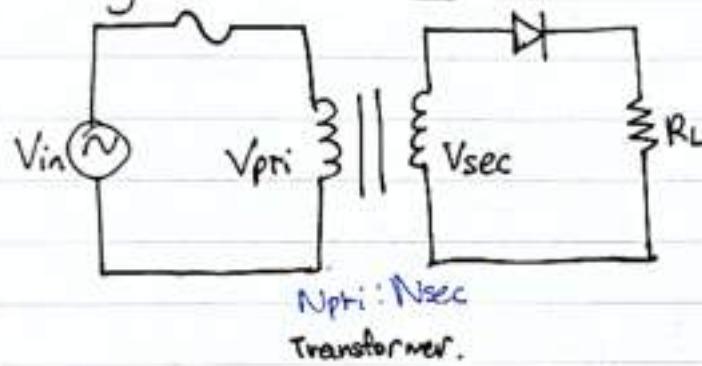
During the negative half cycle:
 $I=0$.

$$V_{RL} = IR_L = 0$$

$$V_{sec} V_k = V_{GND} = 0$$

$$\begin{aligned}\text{PIV} &= V_{KA(\max)} \\ &= V_{KA} - V_{A(\min)} \\ &= 0 - (-V_{in(p)}) \\ &= V_{in(p)}/\end{aligned}$$

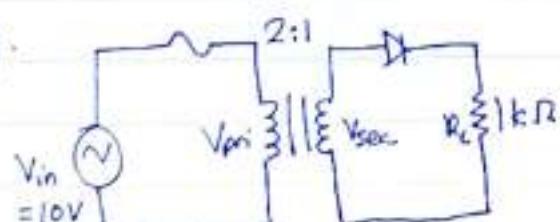
Using Transformers



A transformer is often used to couple the a.c. input voltage from the source to the rectifier. This has advantages:

- 1) It allows the source voltage to be stepped up or down as needed.
- 2). The a.c. source is electrically isolated from the rectifier for preventing a shock hazard in the secondary.

e.g.

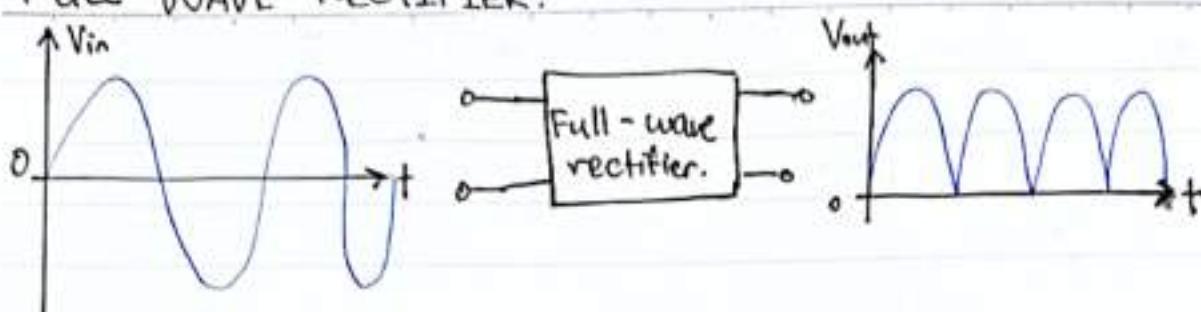


Find $V_{out(p)}$ when $V_{in} = 10V_{peak}$.

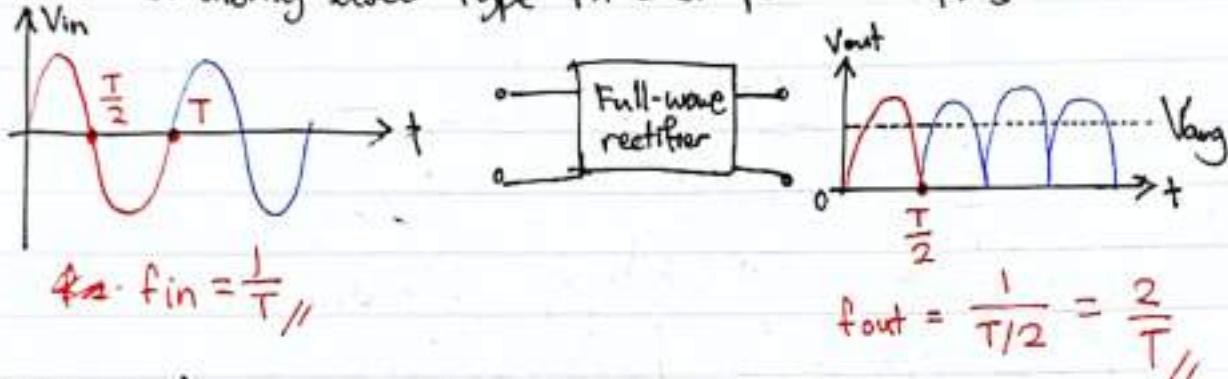
$$V_{sec(p)} = \left(\frac{N_{sec}}{N_{pri}} \right) \times V_{pri(p)} = \frac{1}{2} \times 10V = 5V$$

$$\begin{aligned}V_{out(p)} &= V_{sec(p)} - 0.7V = 5 - 0.7V = 4.3V \\ &\approx 4.3V/\end{aligned}$$

FULL-WAVE RECTIFIER.



Most commonly used type in d.c. power supplies.

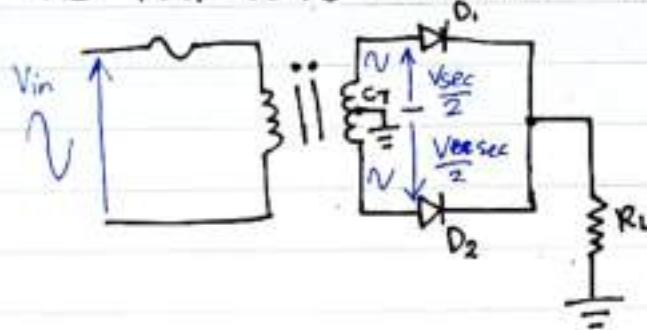


- Output frequency is twice that of input frequency.

- Average value for a full-wave rectifier sinusoidal voltage is twice that of half-wave.

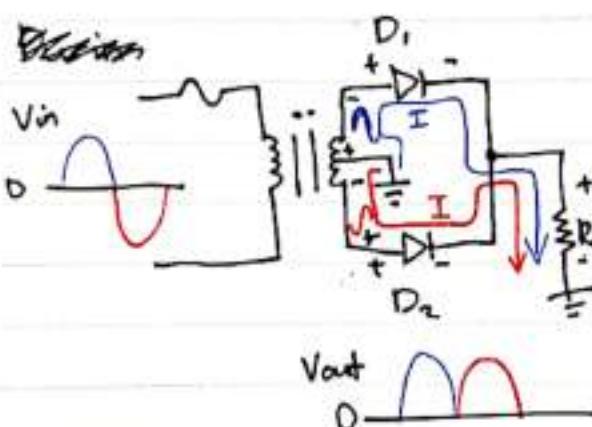
$$V_{\text{avg}} = \frac{2V_{\text{out(P)}}}{\pi}$$

The Full-wave centre-tapped transformer rectification.



This uses 2 diodes connected to the secondary of a centre-tapped transformer.

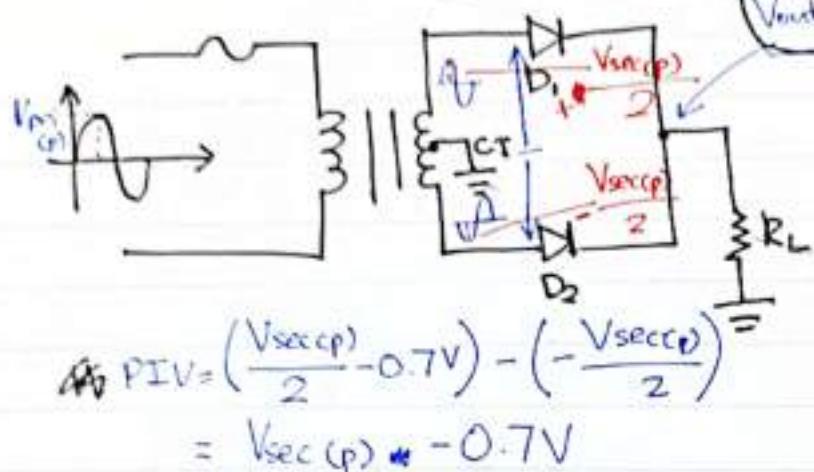
\Leftrightarrow $D_1 \rightarrow$ forward biased
 $D_2 \rightarrow$ Reverse biased.



Positive half cycle \rightarrow current path through D_1 and R_L .

Negative half cycle \rightarrow current path through D_2 and R_L

Peak Inverse Voltage (PIV)



$$V_{\text{out}(p)} = \frac{V_{\text{sec}(p)}}{2} - 0.7V$$

When D_2 is reversed biased, the PIV across D_2 is the p.d. between the cathode and anode

$$\text{As derived previously } \rightarrow V_{\text{out}(p)} = \frac{V_{\text{sec}(p)}}{2} - 0.7V$$

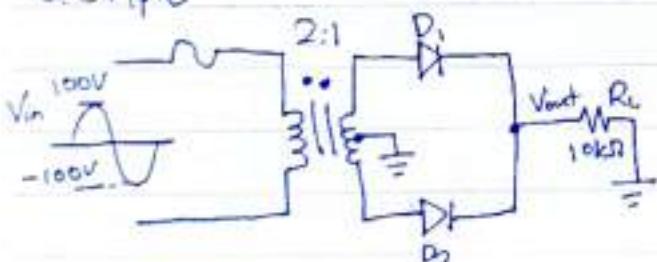
$$\frac{V_{\text{sec}(p)}}{2} = V_{\text{out}(p)} + 0.7V$$

$$V_{\text{sec}(p)} = 2V_{\text{out}(p)} + 1.4V$$

$$\text{Therefore } \rightarrow \text{PIV} = (2V_{\text{out}(p)} + 1.4V) - 0.7V$$

$$\boxed{\text{PIV} = 2V_{\text{out}(p)} + 0.7V}$$

example



Find a) secondary winding V_p .

b) $V_{\text{out}(p)}$

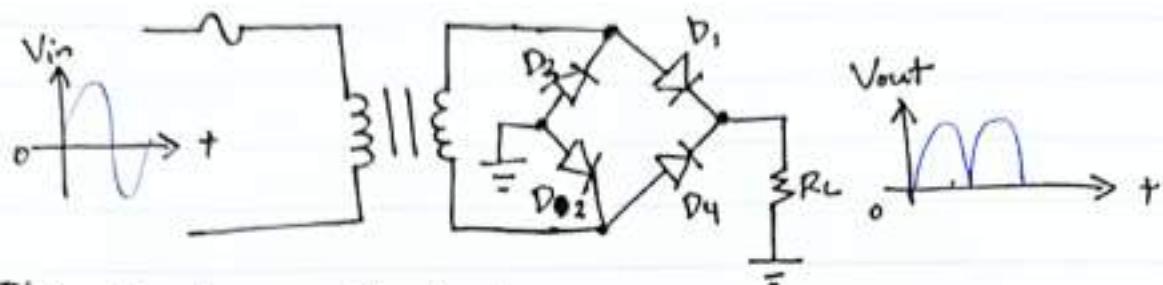
c) PIV

$$\begin{aligned} \text{a) } V_{\text{sec}(p)} &= \left(\frac{N_{\text{sec}}}{N_{\text{pri}}} \right) \times V_{\text{pri}(p)} \\ &= \frac{1}{2} \times 100V \\ &= 50V_{\text{II}} \end{aligned}$$

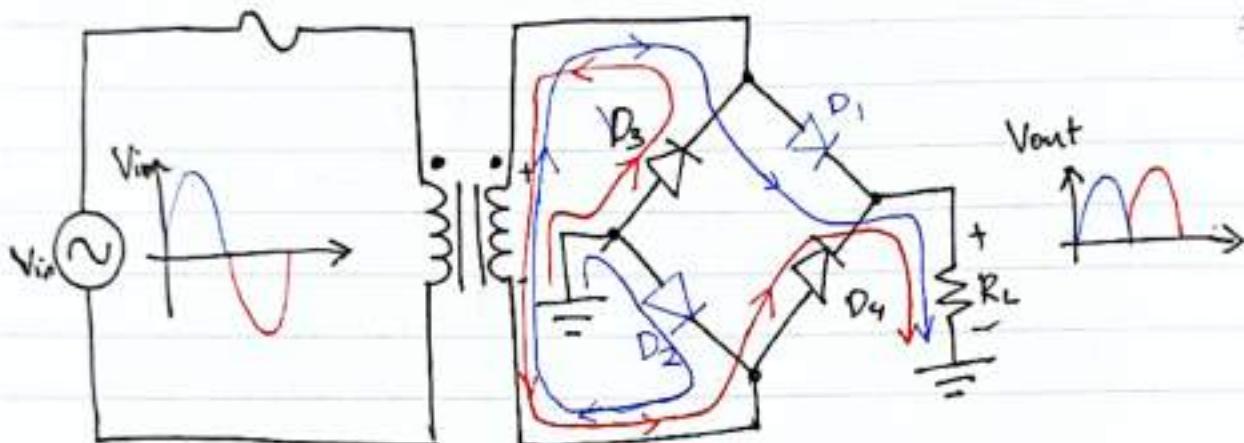
$$\begin{aligned} \text{b) } V_{\text{out}(p)} &= \frac{V_{\text{sec}(p)}}{2} - 0.7V \\ &= \frac{50}{2} - 0.7 \\ &= 24.3V_{\text{II}} \end{aligned}$$

$$\begin{aligned} \text{c) PIV} &= 2V_{\text{out}(p)} + 0.7V \\ &= 2(24.3) + 0.7 \\ &= 49.3V_{\text{II}} \end{aligned}$$

Full-Wave Bridge Rectifier



This circuit uses 4 diodes.



~~D₁ and D₂ → Forward biased
D₃ and D₄ → Reversed biased~~

Positive half cycle.

- A Voltage is developed across R_L in the path shown.
- D_1 and D_2 \rightarrow forward biased.
- D_3 and D_4 \rightarrow reversed biased.

Negative half cycle.

- D_3 and D_4 \rightarrow forward biased.
- D_1 and D_2 \rightarrow reversed biased.
- Conducts current in the same direction through R_L as during the positive half cycle.

Bridge Output Voltage

Since 2 diode are always in series with the R_L during both half-cycles (positive or negative half cycles). Consider each has a voltage drop of 0.7 V each.

$$V_{out} = V_{sec(p)} - 1.4V$$

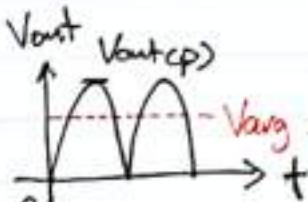
Bridge Output Voltage

- Since 2 diodes are always in series with R_L , during both positive and negative half cycles. We can consider each has a voltage drop of 0.7V. Thus,

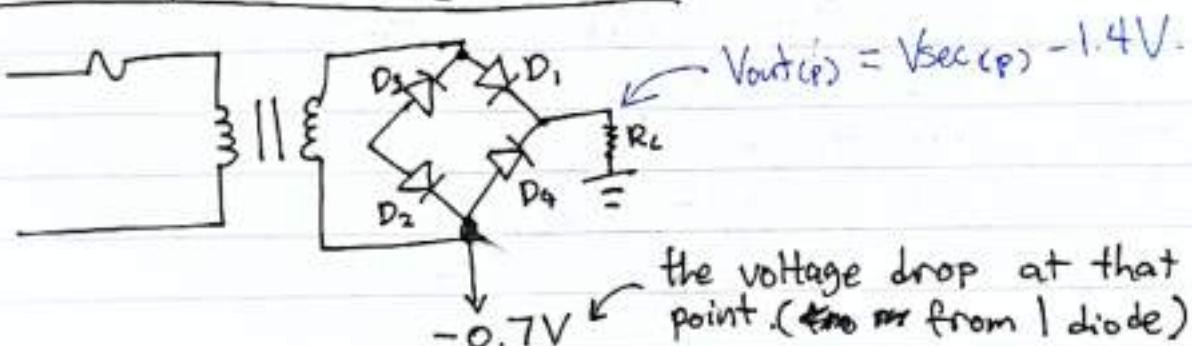
$$V_{out(p)} = V_{sec(p)} - 1.4V$$

- The average output voltage will hence be.

$$V_{out(\text{avg})} = \frac{2}{\pi} \times V_{out(p)}$$



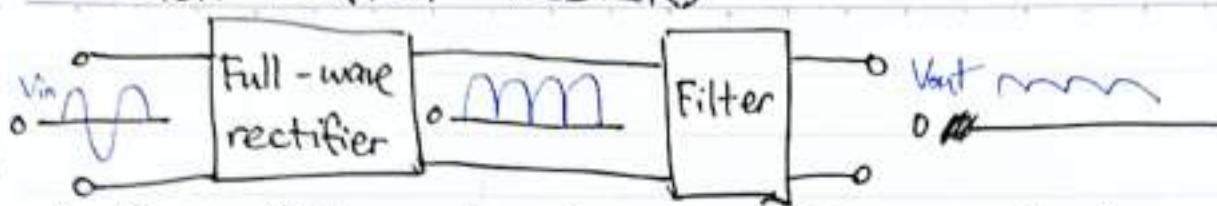
Peak Inverse Voltage (PIV)



$$PIV = V_{out(p)} - (-0.7V)$$

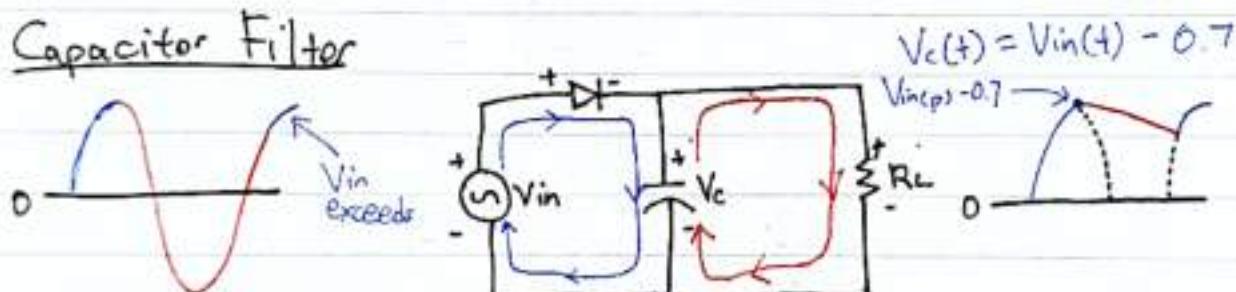
$$PIV = V_{out(p)} + 0.7V$$

POWER SUPPLY FILTERS

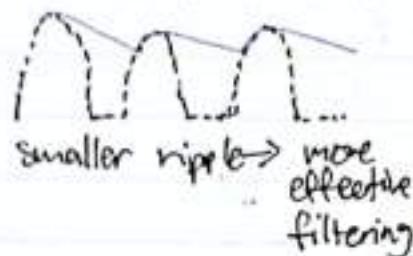
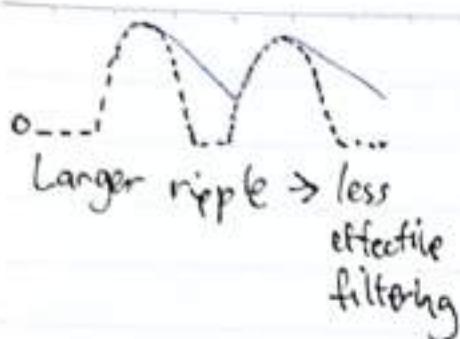


- A Power filter eliminates or reduces fluctuations in output voltage of a rectifier.
- It produces a near constant-level dc voltage.
- Filtering is necessary as electronic circuits need a constant source of dc voltage and current to provide power and biasing for proper operation.
- The small amount of fluctuation in the filter output is called ripple.

Capacitor Filter



- During the positive first-quarter cycle of the input, the diode is forward biased, allowing the capacitor to charge to within 0.7V of the input peak.
- When the input decreases below its peak, the capacitor retains its charge and diode becomes reverse-biased because the cathode is more positive than the anode.
- $V_c(t) = (V_{in(p)} - 0.7) e^{-\frac{t}{RL}}$
- Capacitor discharges only through the R_L at a rate determined by the $R_L C$ time constant (normally long).
- Larger time constant, slower discharge time.
- During the first quarter of the next cycle, the diode will again become forward-biased when the input voltage exceeds the capacitor voltage by approx. 0.7V.



e.g. ripple factor



$$V_{dc} = \frac{(8+6)}{2} = 7V$$

$$r = \frac{(8-6)}{2\sqrt{3}(7)} = 0.0825V$$

~~Thw~~

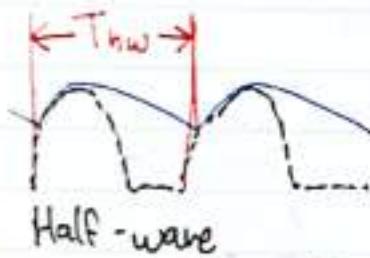


Half wave



Full wave

The full wave rectified wave has a smaller ripple \rightarrow for the same R_L and filter Capacitance.



Half-wave

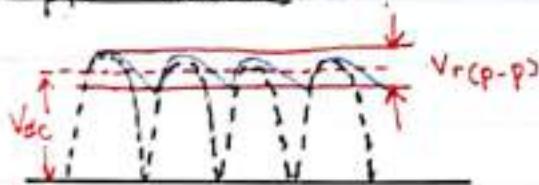
$$\text{eg. } f_{hw} = \frac{1}{Thw} = \frac{1}{20} = 50\text{Hz}$$



Full-wave

$$\text{eg. } f_{fw} = \frac{1}{Tfsw} = \frac{1}{Thw/2} = \frac{1}{20/2} = 100\text{Hz}$$

Ripple factor



- * The lower the ripple factor is, the better the filter.
- * The ripple can be lowered by increasing the value of the filter capacitor or increasing the R_L .

The ripple factor (r) is an indication of the effectiveness of the filter.

$$r = \frac{\sqrt{r}(\text{rms})}{V_{dc}}$$

rms of the ripple voltage

$V_r(\text{rms})$
↓
can be replaced to $V_r(\text{p-p})$

$$r = \frac{\sqrt{r}(\text{p-p})}{2\sqrt{3} V_{dc}}$$

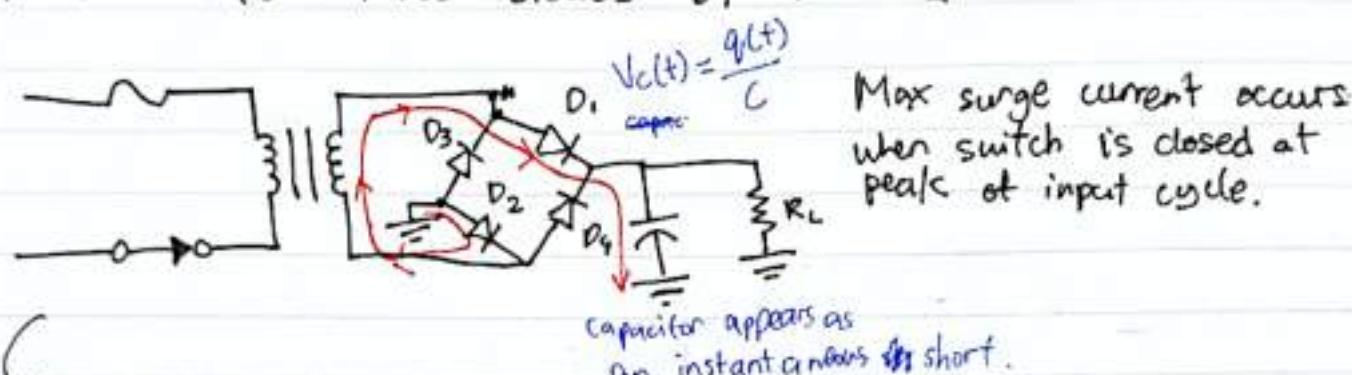
peak to peak
ripple voltage.

$r \downarrow$ if Filter capacitor \uparrow / $R_L \uparrow$

POWER SUPPLY FILTERS

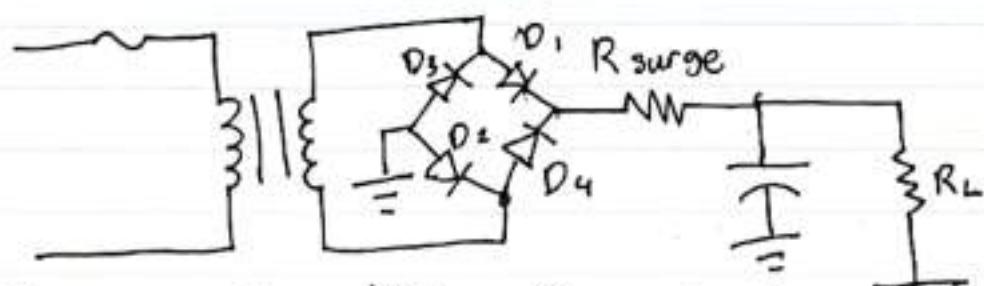
Surge Current in the Capacitor Filter

- Before the switch is closed, the filter capacitor is unchanged.
- At the instant the switch is closed, voltage is connected to the bridge and the unchanged capacitor appears as a short.
- This produces an initial surge of current I_{surge} through the 2 forward-biased diodes D_1 and D_2 .



→ Thus, a surge limiting resistor is sometimes used to protect the diodes.

- The value of ~~this~~ it must be smaller than the R_L .
- Diodes must have a maximum forward surge current rating that can hold momentary surge of current - specified as I_{FSM} .
- The minimum surge-limiting resistor : $R_{\text{surge}} = \frac{V_{\text{peak}} - L^{\text{AV}}}{I_{FSM}}$



- A series resistor ~~is~~ R_{surge} limits the surge current

VOLTAGE REGULATOR.

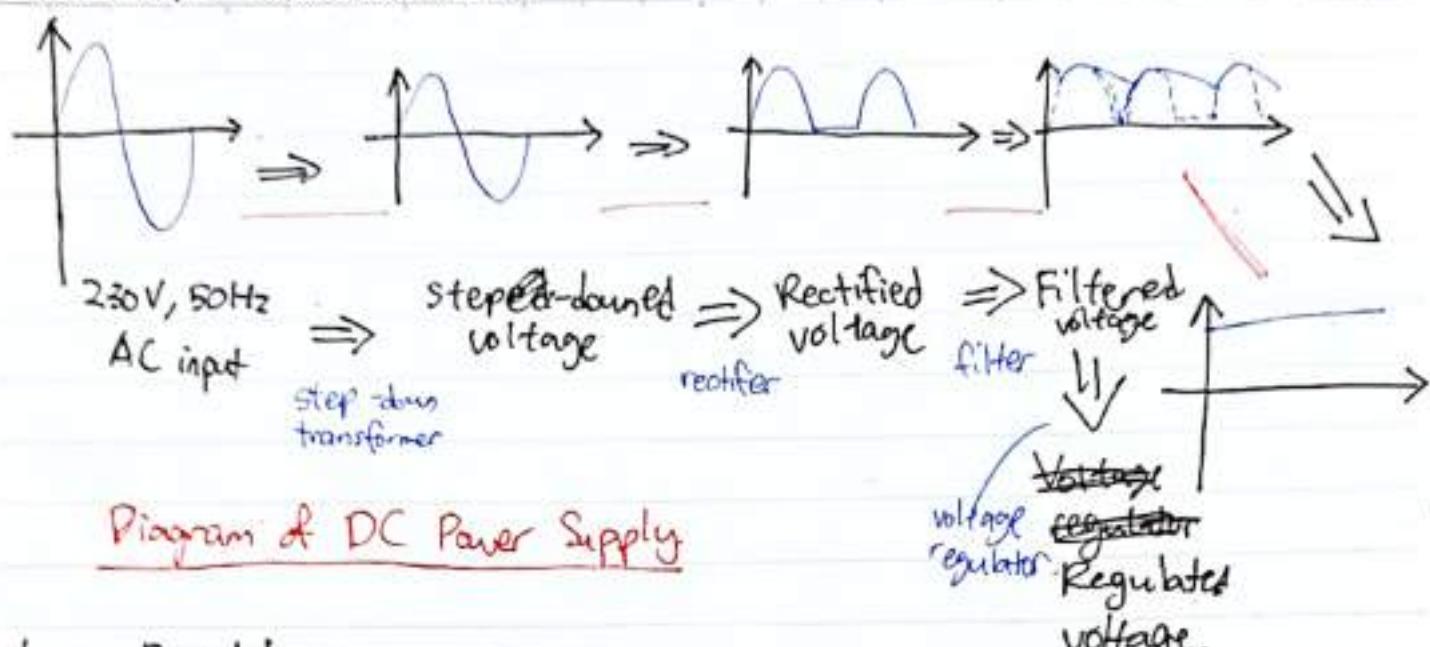


Diagram of DC Power Supply

Line Regulation.

Ability of a voltage regulator to keep its output voltage not to have significant changes even when the input voltage has some measurable changes.

When the dc input (line) changes, voltage regulator must maintain a nearly constant output voltage.

This is defined as the percentage change in the output voltage for a given change in the input (line) voltage.
Smaller the line regulation, the better the regulator.

$$\text{Line Regulation} = \frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \times 100\%$$

Load Regulation

When the amount of current through a load changes due to a varying R_L , the voltage regulator must maintain a near constant output voltage across the load.

Smaller the load regulation, the better the regulator.

e.g. $V_{NL} = 12V$

$V_{FL} = 11.9V$
load regulation =

$$\text{Load regulation} = \left(\frac{V_{NL} - V_{FL}}{V_{FL}} \right) \times 100\% \quad \begin{array}{l} NL = \text{no load} \\ FL = \text{full load} \end{array}$$

$$\frac{12 - 11.9}{11.9} \times 100\% = 0.840\% \quad \text{It can also be expressed as} \rightarrow \text{load regulation} = \frac{0.840\%}{10mA} = 0.084\%/\text{mA}$$

POWER SUPPLY FILTERS

78XX IC \rightarrow positive voltage output

79XX IC \rightarrow negative voltage output

\curvearrowleft first 2 digits ~~get~~ are specify the magnitude of the output voltage.

Three-terminal linear IC regulators are available to provide fixed-value output voltages & either positive / negative polarities.

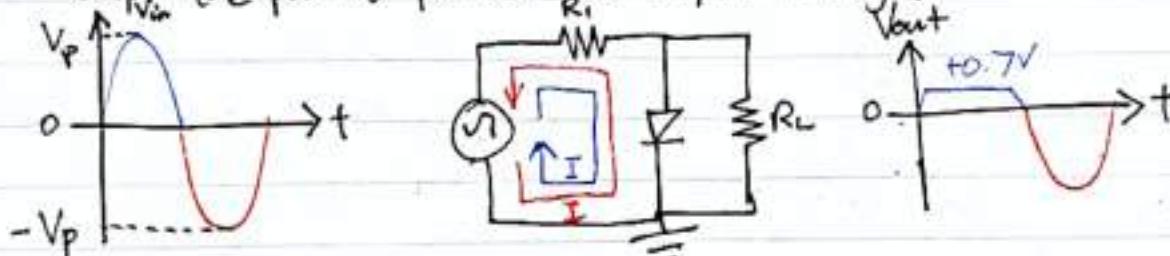
DIODE LIMITING CIRCUITS

- Diode Limiting Circuits (also called limiters or clippers) are used to clip off portions of signal voltages above or below certain levels.

- 3 Main Categories:

* Positive Diode Limiter

- Clip V_{in} the positive part of the input voltage.



Positive alteration \rightarrow diode is forward biased.

- According to the practical diode model, when silicon diode conducts, the forward voltage across the diode is clipped at 0.7V

- During Negative alteration \rightarrow diode is reverse biased.

- Thus it does not conduct.

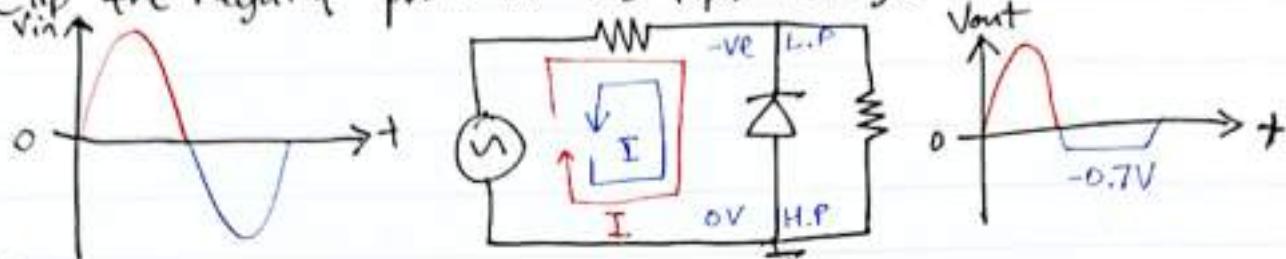
- When no current flows through the diode.

$$I_{R_1} = I_{R_L} = \frac{V_{in}}{R_1 + R_L} \quad \text{and} \quad V_{out} = I_{RL} \times R_L = \frac{R_L}{R_1 + R_L} \times V_{in}$$

DIODE LIMITING CIRCUITS.

★ Negative Diode Limiter.

- Clip the negative part of the input voltage.



- Negative alteration \rightarrow forward biased.

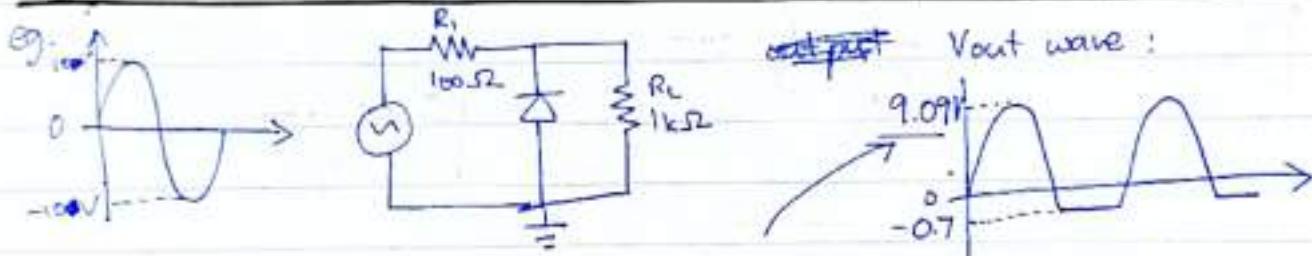
- When silicon diode conducts \rightarrow voltage across is clipped at $-0.7V$

- Positive alteration \rightarrow reverse biased

- Diode does not conduct

$$V_{out} = \left(\frac{R_L}{R_1 + R_L} \right) \times V_{in}$$

★ If R_1 is small compared to R_L then $V_{out} \approx V_{in}$



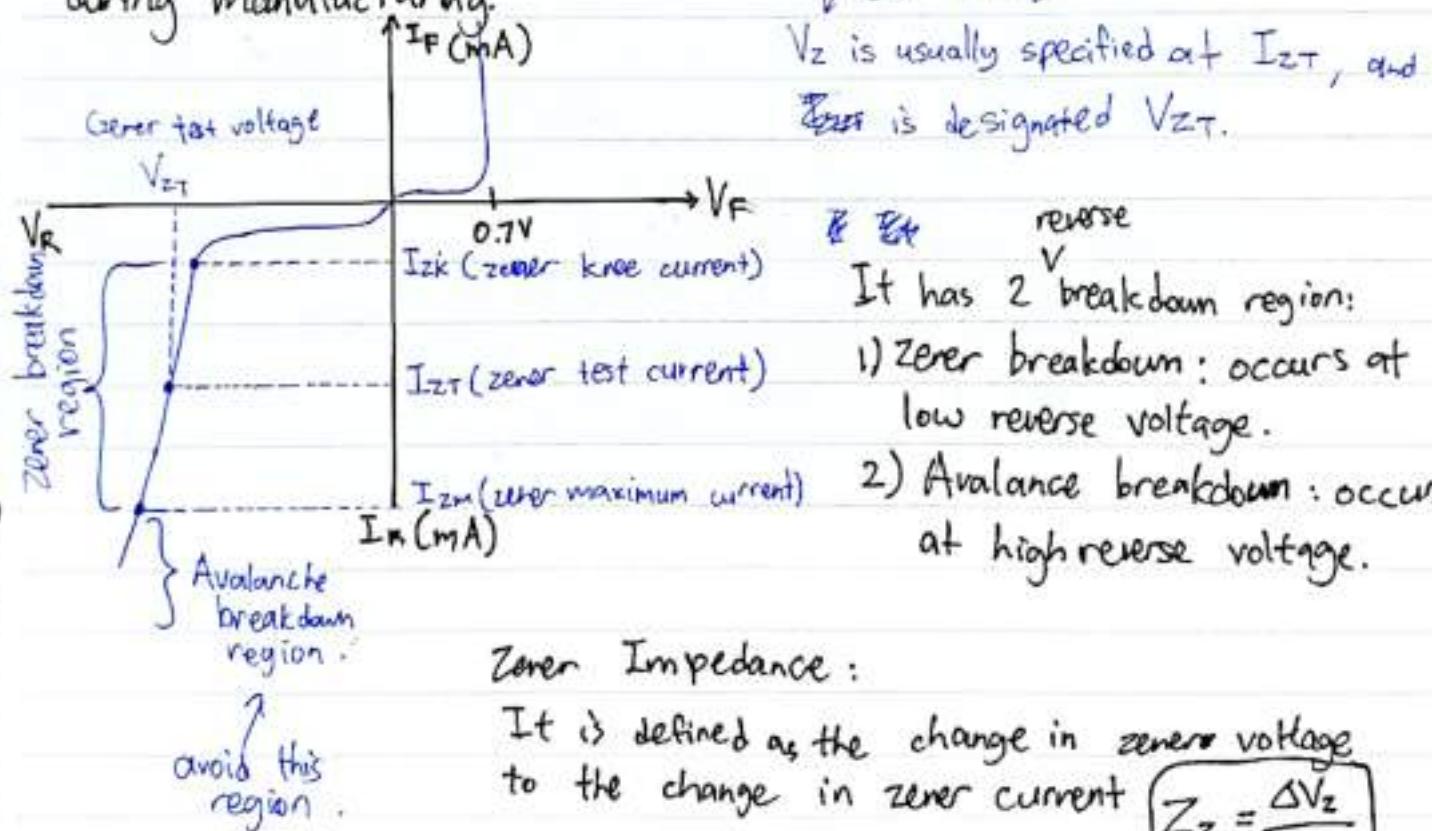
$$V_{out} = \frac{R_L}{R_1 + R_L} \times V_{in} = \frac{1}{0.1 + 1} \times 10 \text{ m} \approx 9.09V$$

ZENER DIODE.



- Unlike normal p-n junction diodes that are operated in a reverse-breakdown region may cause overheating and damage.

- Zener diode are designed to operate in the reverse-breakdown region.
- Zener diode operates like an ordinary diodes when forward biased.
- They are designed to breakdown in a controlled manner at a pre-determined voltage when reversed biased.
- Once breakdown takes place, voltage across the diode is almost constant, ~~this~~ this voltage is the V_z .
- This breakdown voltage is set by carefully controlling doping level during manufacturing.



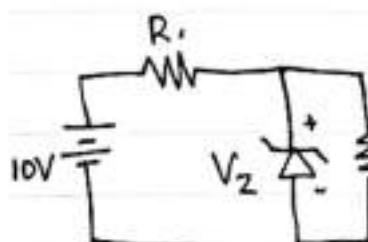
It has 2 breakdown regions:

- 1) Zener breakdown: occurs at low reverse voltage.
- 2) Avalanche breakdown: occurs at high reverse voltage.

Zener Impedance:

It is defined as the change in zener voltage to the change in zener current

$$Z_z = \frac{\Delta V_z}{\Delta I_z}$$



In the past Zener Diodes were widely used to regulate D.C. voltages, but IC voltage ~~regulators~~ regulators are now more commonly used for this purpose.

When $V_{bias} >$ the zener breakdown voltage, the zener diode breaks down and V_z is the zener breakdown voltage.

LED (Light Emitting Diode)



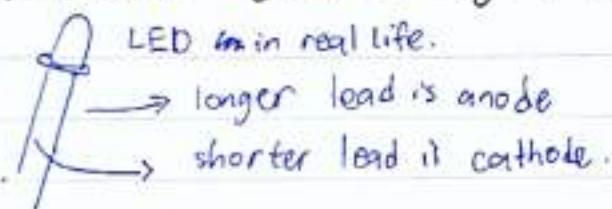
$\uparrow I \rightarrow \uparrow$ brightness

$\uparrow R \rightarrow \downarrow$ brightness (cuz $\downarrow I$)

Use Variable resistor to adjust brightness

LED.

- Special diode that emits light when sufficient forward current flows through it.
- When forward-biased, electrons cross the p-n junction and ~~are~~ recombine with holes, ^{from the V_n-type} into the p-type.
- This releases energy in the form of ~~a little heat~~ and mostly light.
- This process is called electroluminescence,
- Different impurities can be added during ~~the~~ doping process to obtain different coloured light, it ~~can~~ can also emit infrared



infrared is invisible to human eyes.

Gallium Arsenide (GaAs) \rightarrow Infrared light

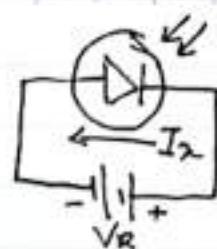
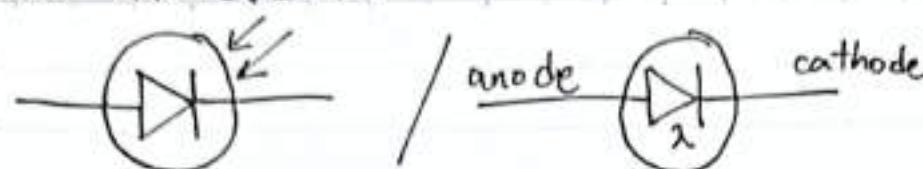
Gallium Arsenide Phosphide (GaAsP) \rightarrow red / yellow.

Gallium Phosphide (GaP) \rightarrow ~~other~~ Red / Green.

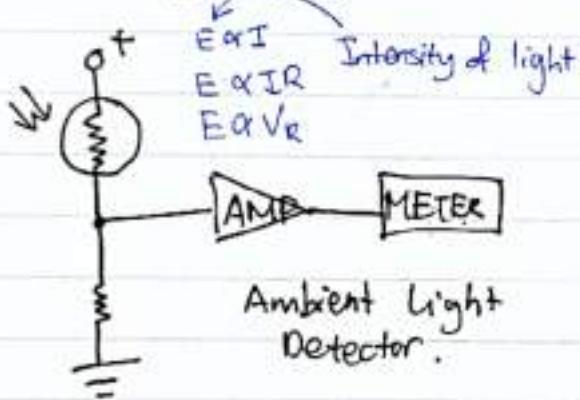
Si & Ge are not used as they produce more heat than light.

- Forward voltage across an LED is greater than that of ~~a diode~~ diode at 1.2V to 3.2V.
- Reverse breakdown is also much less compared to junction diode at around 3V to 10V.

LED PHOTODIODE



- When light is shined \rightarrow current \uparrow
- Work on reverse bias
- Amount of reverse current (I_d) depends on the amount of light irradiating the photo diode.
- Sensitivity of photodiode can change
- Intensity of light measured in $\frac{\text{mW}}{\text{cm}^2}$ or $\frac{\text{lumens}}{\text{m}^2}$ (lux)



- The photo diode can be biased with a constant voltage across it.

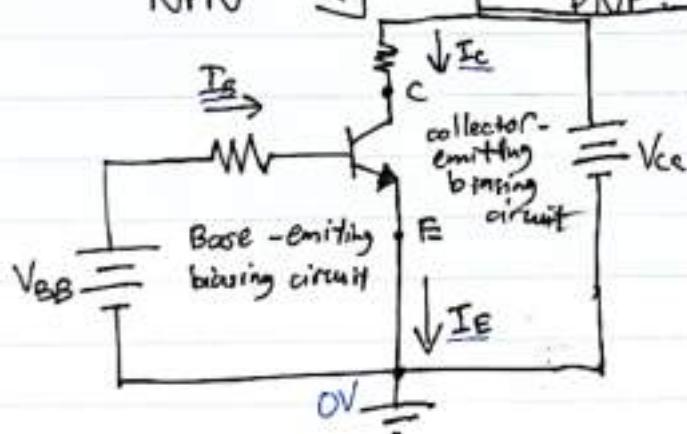
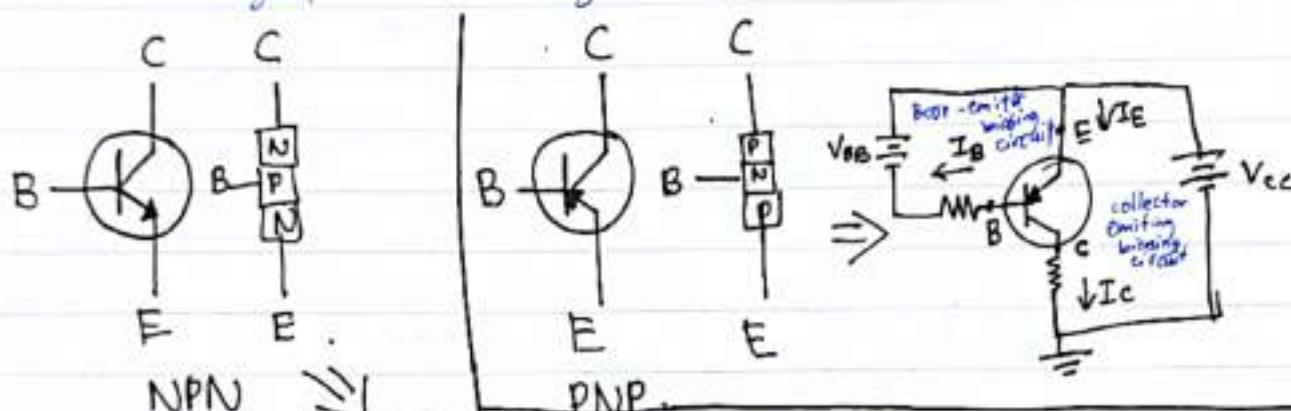
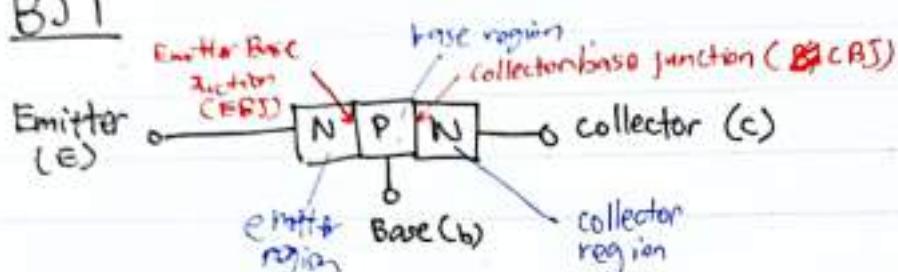
* It works in reverse-biased. At low light, the R is very high and only very ~~is~~ small/negligible current passes through.
At high light, its resistance drops exponentially and allows a reverse current to flow through it.

BIPOLAR JUNCTION TRANSISTOR (BJT)

Transistors

- Found in many household appliance \rightarrow first created in late 1940s.

BJT



$$I_E = I_B + I_C$$

$$V_{BE} = V_B - V_E$$

$$V_{CE} = V_C - V_E$$

$$V_E = 0$$

If $V_{BB} \leq 0.7V$

$$V_{BE} = V_{BB}, \\ I_B = 0.$$

I_B will have current if V_{BB} is greater than 0.7V

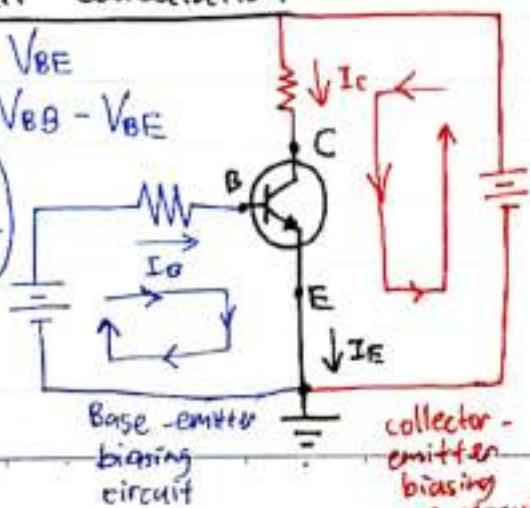
Basic Biasing Circuit Calculation

$$V_{BB} = I_R \times R_B + V_{BE}$$

$$I_{BB} \times I_B \times R_B = V_{BB} - V_{BE}$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B}$$

$$I_E = I_B + I_C$$



$$V_{CC} = I_C \times R_C + V_{CE}$$

$$I_C R_C = V_{CC} - V_{CE}$$

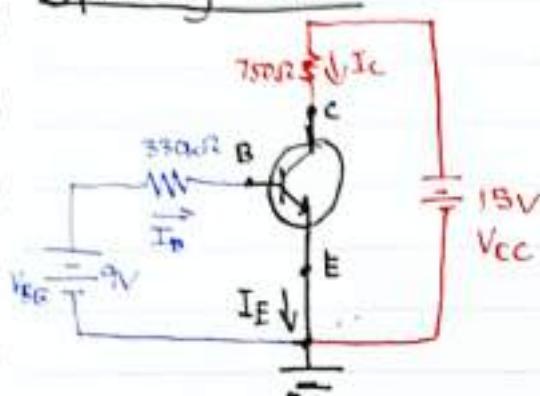
$$I_C = \frac{V_{CC} - V_{CE}}{R_C}$$

$$I_C = -\frac{1}{R_C} V_{CE} + \frac{V_{CC}}{R_C}$$

Load line equation

BIPOLAR JUNCTION TRANSISTOR (BJT)

Operating Point



$$V_{BE} = 9V > 0.7V$$

0.7V is the knee voltage of a p-n junction

B-E junction conduct

$$V_{FE} = 0.7V$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$9V = I_B \times 330k\Omega + 0.7V$$

$$I_B = \frac{9 - 0.7}{330k\Omega} = 25\text{mA}$$

Operating point or Q-point (Quiescent point) is the V_{CE} and I_C of the biasing circuit.

$$V_{CC} = I_C R_C + V_{CE}$$

$$I_C = -\frac{1}{R_C} V_{CE} + \frac{V_{CC}}{R_C}$$

$$I_C = -\frac{4}{3} V_{CE} + 20(\text{mA})$$

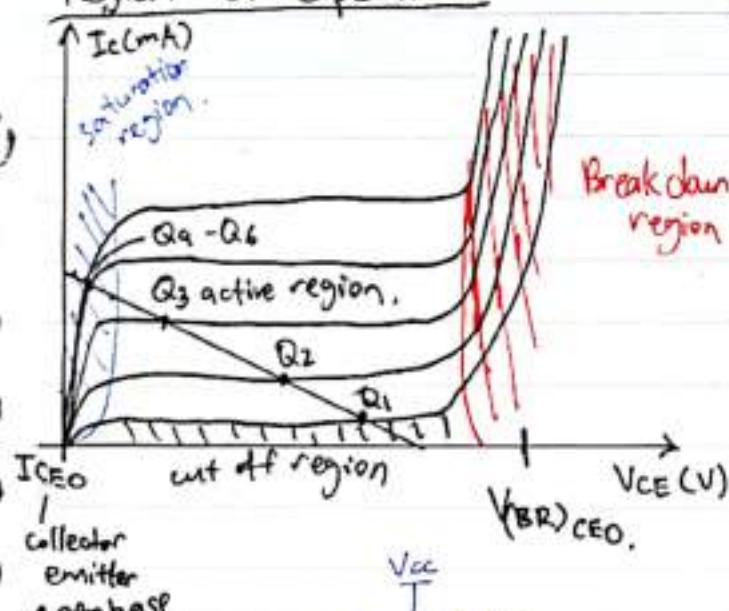
$$\left(0, \frac{V_{CC}}{R_C}\right) = \left(0, \frac{15V}{0.75k\Omega}\right) = (0, 20\text{mA})$$

y-intercept

$$(V_{CC}, 0) = (15V, 0)$$

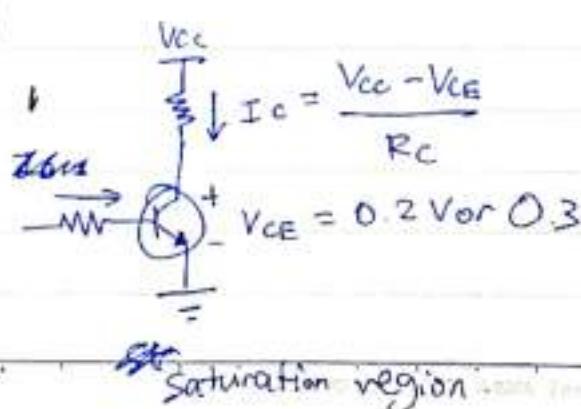
x-intercept

Region of Operation



$I_B = 0$ \rightarrow $V_{CE} \approx V_{CC}$.
Cut-off region

Region	I_B, I_C	$V_{CE} = V_{CC} - I_C R_C$
Q_1 cut-off	$I_B = 0$, $I_C \approx 0$	$V_{CE} \approx V_{CC} - 0 \cdot R_C$ $V_{CE} \approx V_{CC}$
Q_2, Q_3 active region	$I_C \propto I_B$	$0.2V < V_{CE} < V_{CC}$ or $0.3V$
$Q_4 - Q_6$ Saturation	$I_C \approx I_B$	$V_{CE} = 0.2V / 0.3V$



Saturation region

- When a transistor is biased in the cutoff region, the transistor operates as an open switch. $I_B = 0$; $I_C \approx 0$ and $V_{CE} \approx V_{CC}$
- When a transistor is biased in the active region, it operates as a current amplifier. $I_C = \beta I_B$; $V_{CE} = V_{CC} - I_C \cdot R_{CE}$ and $V_{CE(sat)} < V_{CE} < V_{CC}$, where $V_{CE(sat)} = 0.2V$
- When a transistor is biased in the saturation region, it operates as a closed switch. $I_C/I_B < \beta$; $I_C = (V_{CC} - V_{CE(sat)})/R_C$; $V_{CE} = V_{CE(sat)}$

$$\beta = \frac{\text{collector current}}{\text{Base current}} = \frac{I_C}{I_B}$$

When $I_B = I_{B(\min)}$

- In both active and saturation region.
- $I_C = \beta I_B$
- $I_C = I_{C(sat)}$
- $V_{CE} = V_{CE(sat)} = 0.2V.$

Thus,

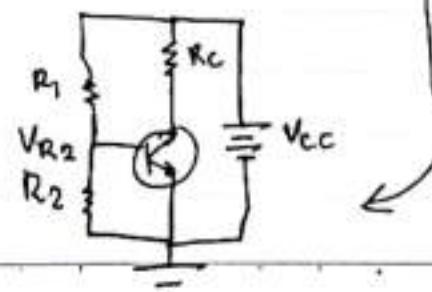
- $\beta I_{B(\min)} = I_{C(sat)}$ where $I_{B(\min)}$ is the minimum base current to bias the transistor to saturation.
- $I_{B(\min)} = \frac{I_{C(sat)}}{\beta}$

When $I_B > I_{B(\min)}$

- In saturation region only
- $I_C \neq \beta I_B$
- $I_C = I_{C(sat)}$
- $V_{CE} = V_{CE(sat)} = 0.2V$
- $I_{B(\min)} = \frac{I_{C(sat)}}{\beta}$

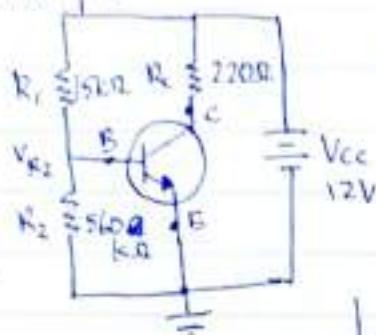
- To ensure deep saturation, set $I_B = 2I_{B(\min)}$

$$\frac{R_2}{R_1 + R_2} \times V_{CC} = V_{R_2}$$



BIPOLAR JUNCTION TRANSISTOR (BJT)

Example:



a) find I_B , I_C & V_{CE} when $\beta = 150$ and $V_{CE(sat)} = 0.2V$

$$\frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{0.56}{15 + 0.56} \times 12 \approx 0.43 \rightarrow < 0.7V$$

cut off region.

$\therefore I_B = 0$, $I_C \approx 0$, and $V_{CE} \approx 12V$

b) If R_2 is changed to $1.2k\Omega$

c) If $R_2 = 18k\Omega$.

$$\frac{18}{15+18} \times 12 = 6.55V > 0.7V$$

not off
and $V_{CE} = 0.7V$

$$I_{C(sat)} = \frac{12 - 0.2}{0.2} \approx 53.6mA$$

$$I_{C(min)} = \frac{12 - 0.2}{15} \times \frac{53.6}{150} \approx 0.36mA$$

$\approx 0.36mA$.

$$I_B = \frac{12 - 0.7}{15} - \frac{0.7}{1.2}$$

$\approx 0.7144mA$

$\therefore I_B \approx 0.71 > I_{B(min)} \approx 0.36$
saturation region.

$$V_{CE} = V_{CE(sat)} = 0.2V$$

$$I_C = I_{C(sat)} = \frac{12 - 0.2}{0.2} = 53.6mA$$

$I_C \neq \beta I_B$ in Saturation Region

$$\frac{1.2}{15+1.2} \times 12 \approx 0.9V \rightarrow > 0.7V$$

not off, and $V_{CE} = 0.7V$

$$I_{C(sat)} = \frac{V_{CC} - V_{CE(sat)}}{R_C} = \frac{12 - 0.2}{0.22} \approx 53.6mA$$

$$I_{B(min)} = \frac{I_{C(sat)}}{\beta} \approx \frac{53.6}{150} \approx 0.36mA$$

$$I_B = I_{R_1} - I_{R_2}$$

$$I_B = \frac{V_{CC} - V_{R_2}}{R_1} - \frac{V_{R_2}}{R_2} = \frac{12 - 0.7}{15} - \frac{0.7}{1.2} \approx 0.17mA$$

$\therefore I_B \approx 0.17mA < I_{B(min)} \approx 0.36mA$
active region.

$$I_C = \beta I_B \approx 150 \times 0.17 \approx 25.5mA$$

$$V_{CE} = V_{CC} - I_C R_C \approx 12 - 25.5 \times 0.22 = 6.39V$$

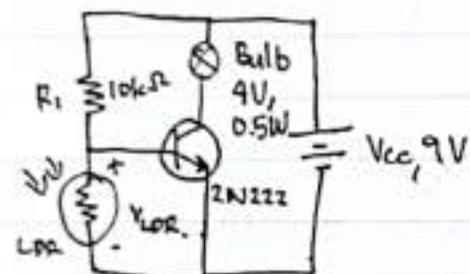
TRANSDUCER.

LDR Converts energy from one form to another



LDR

light intensity $\propto \frac{1}{R}$ (decreases R when light increases)
When connected with a BJT, can turn to an automatic lighting circuit



$$R_{LDR} = 5\text{k}\Omega \text{ at } 50\text{lux} \text{ to } 1\text{k}\Omega \text{ at } 500\text{lux}$$

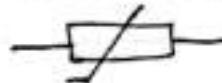
if when $\frac{R_{LDR}}{R_1 + R_{LDR}} \times V_{cc} \leq 0.7$, $I_B = 0$, transistor goes off, and bulb goes off.

if when $\frac{R_{LDR}}{R_1 + R_{LDR}} \times V_{cc} > 0.7$, $V_{BE} = 0.7\text{V}$, $I_B = \frac{V_{cc} - 0.7}{R_1} - \frac{0.7}{R_{LDR}}$

Saturation Region

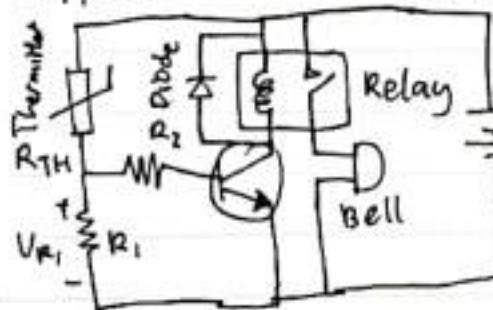
Light Intensity \uparrow , $R_{LDR} \downarrow$, $I_B \uparrow$, $I_c \uparrow$, Bulb Brightness \uparrow

Thermistor



- Semiconductor that varies its resistance with temperature
- Temp \uparrow , R \downarrow
- sensitivity at about $-4\% / {}^\circ\text{C}$ (-ve co.eff of temp).
- Range of -50°C to $+300^\circ\text{C}$

Application: Fire alarm.



• No fire, R_{TH} very high and $\frac{R_1}{R_{TH} + R_1} \times V_{cc} < 0.7\text{V}$
 $\therefore I_B = 0$ & $I_c = 0$.
 - Relay not energized, and stay in normal open state
 - Bell off.

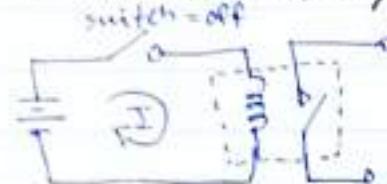
- When fire breaks out, R_{TH} drops and $\frac{R_1}{R_{TH} + R_1} \times V_{cc} > 0.7\text{V}$
 $\therefore I_B > 0$ and $I_c > 0$.
 - Relay energized and close the relay \rightarrow bell on.

TRANSDUCER

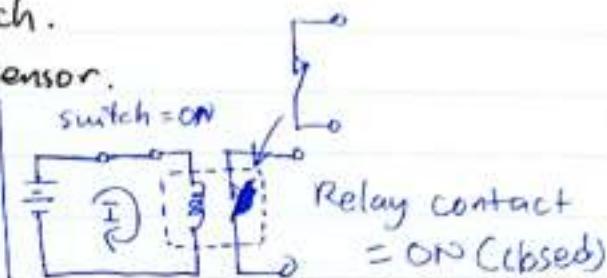
Relay

- electrically controlled switch.

- not a transducer, nor a sensor.

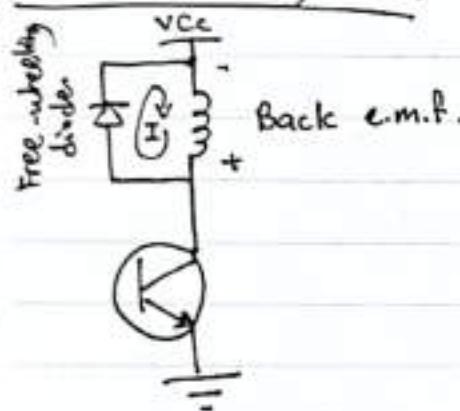


Relay contact
= OFF (open)



Relay contact
= ON (closed)

Free-wheeling diode



When the transistor switches off, there is a collapse in the magnetic field around the relay coil. This causes a back E.M.F. to be induced across the coil. The energy can now be returned to the circuit through it. This prevents the transistor from being damaged.

When no free-wheeling diode high current surge discharged through the transistor. This may damage the transistor.

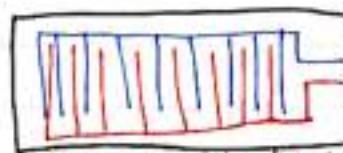
Moisture Sensor

- Operates on principle that water is a better conductor of electricity compared to air.

- Resistance ~~is~~ between the moisture sensor contacts changes in the following manner.

- High resistance (open circuit) when dry.

- Low resistance when wet.



Simple PCB Moisture Sensor

- ~~Advantages~~ Advantages of using Output Relay

- Separated high output power supplies can be used for demanding loads

- Separated AC supplies can be used for AC loads.

No. _____

Date _____

- ★ LDR $\rightarrow R \downarrow$ non-linearly with light intensity
- ★ Thermistor $\rightarrow R \downarrow$ exponentially with temperature
- ★ Moisture sensor \rightarrow resistance decreases with moisture level.
- ★ A BJT circuit can be used to detect the changes in the sensor resistance and change the collector current accordingly.
- ★ A relay can be controlled by a sensor circuit and can be used to power up a demanding load or an AC load with an insulated power source.
- ★ A free-wheeling diode connected to a relay and a transistor protects the transistor from the induced surge current when the transistor switches ON to OFF.

$$\Delta I_z = I_z - I_{z_T} \quad I_T = \frac{V_T - V_Z}{R}$$

$$\Delta V_Z = \Delta I_z \times Z_Z \quad I_L = I_T - I_Z$$

$$V_Z = \Delta V_Z + V_{Z_T} \quad R_L = \frac{V_Z}{I_L}$$

~~$I_C = \frac{V_{CC} - V_{BE}}{R_L}$~~

Automatic fire alarm circuit.

No fire $\rightarrow R_{TH}$ high, $V_B = \text{low}$.

$V_{BE} < 0.7V$, transistor = cut off.

$I_C = 0 \rightarrow$ relay coil not magnetized.

relay contact \rightarrow open.

Bell \rightarrow off

Fire break out $\rightarrow R_{TH} = \text{low}$, $V_B = \text{high}$

$V_{BE} = 0.7$, transistor = active. \rightarrow saturation.

I_C = flows, \rightarrow relay coil magnetized

relay contact \rightarrow closed.

Bell \rightarrow on.

Alt.

No fire $\rightarrow R_{TH}$ very high, $\frac{R_1}{R_{TH} + R_1} \times V_{CC} = V_R < 0.7V$

$I_B = 0 \wedge I_C = 0$.

Relay not energized, contact open.

Bell off.

Break out $\rightarrow R_{TH}$ very low, $\frac{R_1}{R_{TH} + R_1} \times V_{CC} = V_R > 0.7V$

$I_B > 0 \wedge I_C > 0$

Relay will be energized, contact closed.

Bell on.

Q point	Region	I_B, I_C	$V_{CE} = V_{CC} - I_C R_C$
Q_1	Cutoff	$I_B = 0, I_C \approx 0$	$V_{CE} \approx V_{CC}$
Q_2, Q_3	Active	$I_B \propto I_C$	$0.2/0.3 < V_{CE} < V_{CC}$
$Q_4 - Q_6$	Saturation	\times	$0.2/0.3 = V_{CE}$



$$P = IV.$$

$$P = I^2 R.$$

$$P = \frac{V^2}{R}.$$

Operational Amplifier Fundamentals

Characteristics of an op-amp:

- The need for an amplifier,
- amplification (gain),
- bandwidth,
- input & output impedances, and
- distortion.

Amplifiers

- When watching the TV, you increase the volume if you are unable to hear it.
- When the picture on the TV gets too dark, you increase the brightness.
- In both cases, you are taking a relatively weak signal and making it stronger (increasing its power).
- The process of increasing the amplitude or power of a signal is called amplification.
- A circuit used to perform this function is called an amplifier.
- The most common types of amplifiers:
 - * Voltage amplifier,
 - & Current amplifier,
 - * Power amplifier.

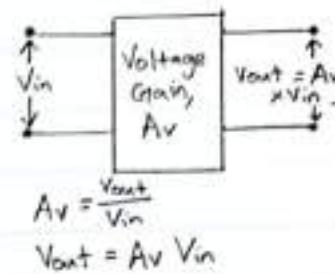
Ideal amplifier characteristics

- Infinite Voltage $\rightarrow A_v = \infty$
- Infinite input impedance $\rightarrow Z_{in} = \infty$
- zero output impedance $\rightarrow Z_{out} = 0$
- Infinite bandwidth, $\rightarrow BW = \infty$
- zero distortion (Input / Output relationship \leftrightarrow being linear) (V_o is proportional to V_i)
- Infinite Gain $\rightarrow A_v = \infty$

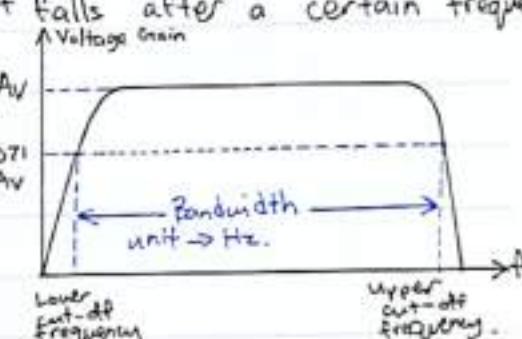
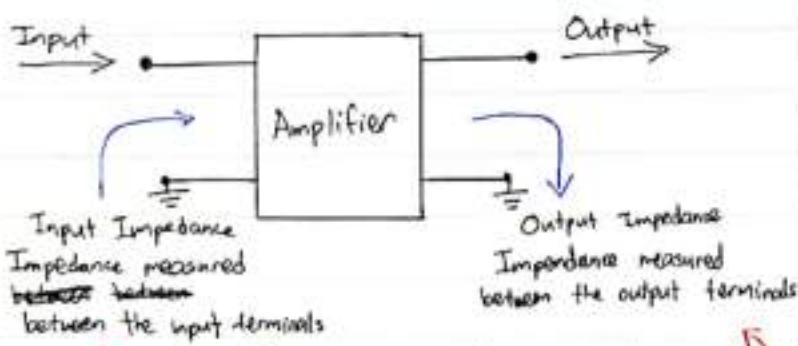
Effects

Gain

- Amplifiers are circuits that exhibit a property called gain.
- The gain of an amplifier is a multiplier that exists between the input and output.
- An amplifier has a $A_v = 100 \rightarrow V_{out}$ will be 100x larger than V_{in} .
- The gain of an amplifier is determined by the amplifier's circuit component value.

Band-width

- The gain of the amplifier is not uniform - it falls after a certain frequency.
- The range of frequencies over which a useful gain is available is called the bandwidth of the amplifier.
- Difference between upper and lower cut-off frequency.
- Freq_{in} should be higher than the lower cut-off freq. and lower than the upper cut-off freq.
- Bandwidth of an amplifier depends on the circuit component values and the type of active components used.

Impedance (Z_{in} & Z_{out} / Z_{in} & Z_{out})

Volt diff in the input divided by the input current

Impedance is not a signal, it is a resistance-like property.

- Impedance is resistance-like property used in AC circuits
- An AC signal comes with the parameters of frequency and phase.
- ∴ Impedance is also a function of frequency and phase.

Volt diff in the output, divided by the output current

Recall:

$$Z_R = R \Omega$$

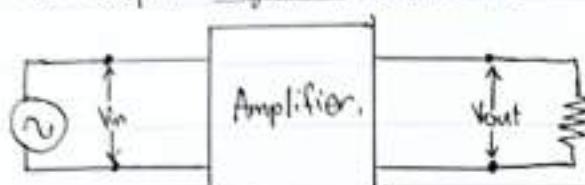
$$Z_C = \frac{1}{\omega C} \angle -90^\circ \Omega$$
~~$$Z_L = \omega L \angle 90^\circ \Omega$$~~

$$Z_L = \omega L \angle 90^\circ \Omega$$

Operational Amplifier Fundamentals

Distortion

- The input signal and the output signal must retain the same shape
- It has a linear input and output relationship
- If the shape of the output signal does not resemble the input signal, the output signal is distorted - this is undesirable.



✓ may have diff amplitude & phase → same shape = not distorted.

↙ Sine wave in → Sine wave out.

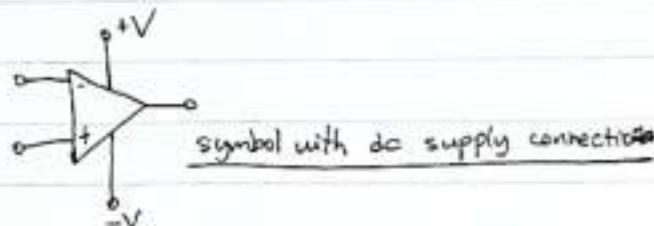
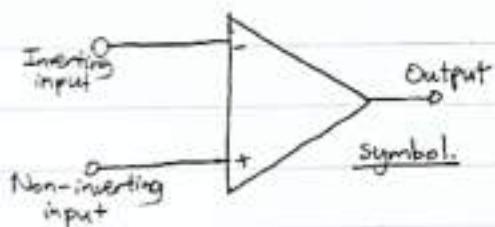


✗ not same shape = distorted

Op - Amps

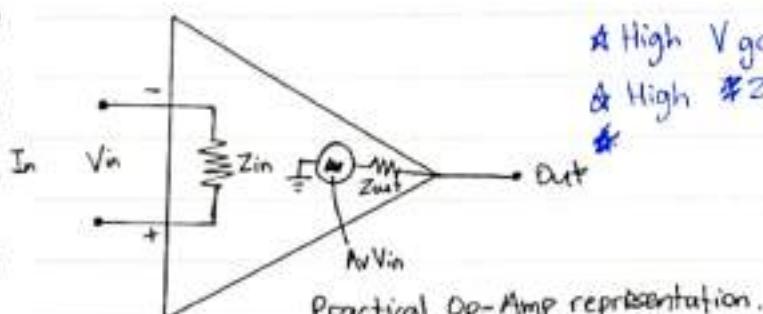
- Early op-amps are used to perform math operations - e.g. addition, subtraction, integration and differentiation - hence the term "operational".

Symbols and terminals



- It comes in DIP (Dual In-line package) and SMT (Surface-Mount Technology).

The practical Op - Amps



- * High V_{gain}
- * High Z_{in}
- * Low Z_{out}
- * Wide BW.

- p-p V_{out} usually limited to slightly less than the two supply voltages.
- ~~Iout~~ I_{out} also limited by the internal restrictions such as power dissipation and component rating
 - e.g. max power rating.

No.

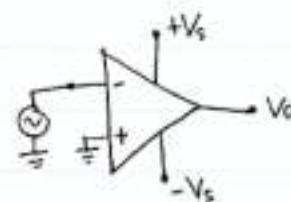
Date

Op-Amp Input Modes.

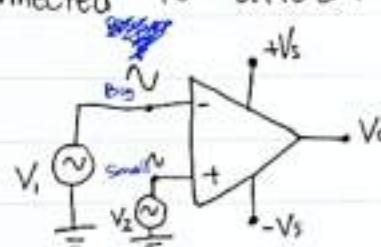
Modes of Signal Operation

Op-Amp circuits use one of the 3 basic input modes of operation:

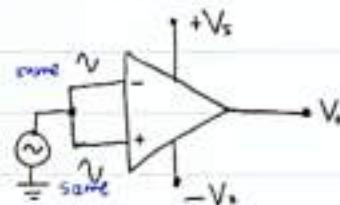
1. Single-ended input mode has one input connected to the input signal and the other connected to ground.



2. Differential input where the inputs are connected to different signal sources.



3. Common-mode input, where both inputs are connected to the same signal source.



Saturated Output Voltage

- The limit on the output voltage is set by the supply voltages, +V_s and -V_s.
- The upper limit is called the positive saturation voltage, +V_{sat} \approx +V_s - 1V.
- The lower limit is called the negative saturation voltage, -V_{sat} \approx -V_s + 2V.
- e.g. a supply voltage of $\pm 15V$, +V_{sat} \approx +14V and -V_{sat} \approx -13V.

Therefore, V_o is restricted to a symmetrical p-p swing of $\pm 13V$.

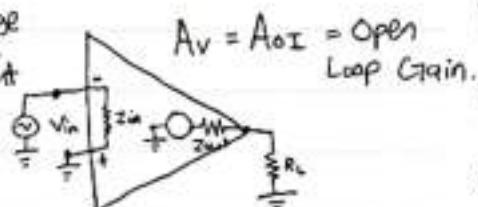
Op-Amp Open Loop Voltage Gain (A_{OI})

- The open-loop voltage gain of an op-amp is the internal voltage gain of the device and represents the ratio of output voltage to input voltage when output signal is not feedbacked to the inputs. i.e. when output pin is not connected to any of the input pins through any wire or components.

- The open-loop voltage gain is set entirely by the ~~internal design~~ internal design.

- Typical open-loop voltage gain is usually larger than 100,000.

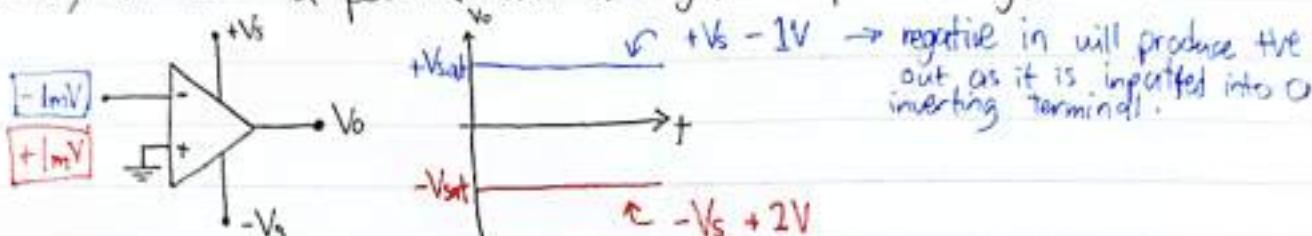
- Data sheets often refer to the open-loop voltage gain as the large-signal voltage gain.



Operational Amplifier Fundamentals

Disadvantages of Open Loop Op-Amp

- Extremely small input voltage drives the op-amp to saturation.
e.g. $V_{IN} = 1\text{mV}$ and $A_{OL} = 100,000$, then $V_{OUT} = V_{IN} A_{OL} = (1\text{mV})(100000) = 100\text{V}$,
- Since the output level of an op-amp can never reach 100V, it is driven deep into saturation and the output is limited to its maximum output levels, for both a positive and a negative input voltage of 1mV.



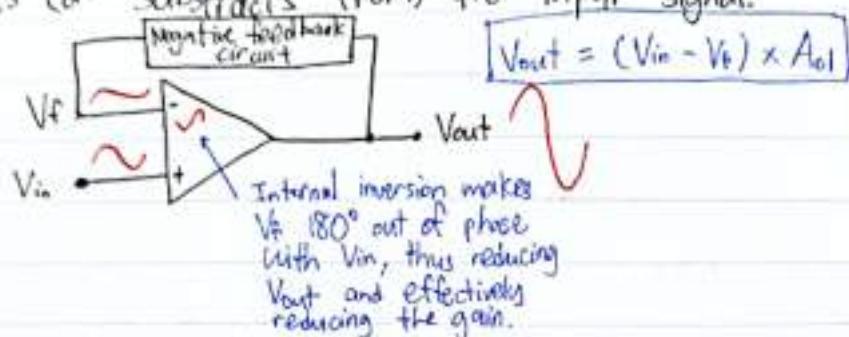
- When the voltage at the non-inverting terminal (+ve) is higher than the voltage at the inverting terminal (-ve), then the output voltage will be a positive value.
- When the voltage at the non-inverting terminal (+ve) is lower than the voltage at the inverting terminal (-ve), then the output voltage will be a negative value.
- When the input is applied to the inverting terminal of an op-amp, then the op-amp will have a negative gain (opposite).
- The extremely high open-loop gain of an op-amp creates an unstable situation.
 - Small input noise voltage amplified such that amplifier is driven out of the linear region.
 - Can also cause unwanted oscillations.
- Open-loop gain parameter of an op-amp can vary greatly from one device to another. It is not well-controlled parameter during ~~manufacturing~~ manufacturing.

Why Use Negative Feedback?

- The main disadvantages of an open-loop op-amp are caused by its extremely high open-loop gain.
- Negative feedback creates an effective reduction in gain by taking a portion of the output and applying it back to the inverting terminal.
- Closed-loop gain is usually much less than the open-loop gain and can be controlled by the external components connected to the op-amp.

Op-Amp with Negative Feedback

- Negative feedback is one of the most useful concepts in electronics, esp in op-amp applications.
- Negative feedback is the process whereby a portion of the output voltage of an amplifier is ~~return~~ returned to the input with a phase angle that opposes (or subtracts from) the input signal.



Since output is feedback to the inverting terminal, higher the output, higher the $|V_f|$ and lower the $(V_{in} - V_f)$ difference as a result, V_{out} will reduce and prevent from saturation.

- In addition to providing a controlled, stable voltage gain, ~~the~~ negative feedback also ~~not~~ provides for control of the input and output impedances and amplifier band width.

	Voltage Gain	Input Impedance	Output Impedance	Band width
Without Feedback	$A_v = A_{ol}$ is too high for linear amplifier applications.	Relatively high	Relatively low	Relatively narrow (because the gain is so high).
With negative feedback	A_{ol} A_{ol} is set to desired value by the feedback feedback circuit	Can be increased or reduced to a desired value depending on the type of circuit	Can be reduced to a desired value	Significantly wider

$$A_{cl} = A_{ol}$$

$$A_{cl} = A_{ol}$$

With negative feedback, the closed-loop voltage gain (A_{cl}) can be reduced and controlled so that the op-amp can function as a linear amplifier.

Non-Inverting Amplifiers.

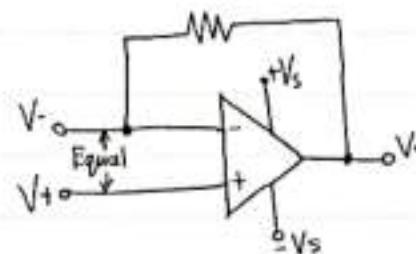
Introduction

- Non-inverting and inverting amplifier are ~~are~~ different the 2 basic config. of an operational amplifier. Negative feedback is applied in both configurations to provide linear operation and undistorted output.
- In a non-inverting amplifier, the ~~a~~ input signal is applied to the non-inverting terminal of an op-amp; whereas in a inverting amplifier, the input signal is applied to the inverting terminal through an input resistor.
- Another difference between a non-inverting amplifier and an inverting amplifier is that the output signal of a non-inverting amplifier is in phase with the input signal, but the output signal of an inverting amplifier is out of phase with the input signal.

Non-Inverting Op-Amp Amplifier.

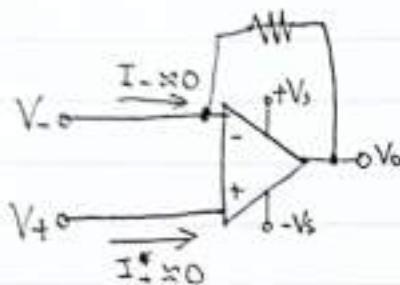
Input Voltages of Op-Amp with Negative Feedback, $V_+ \approx V_-$

- With negative feedback, the op-amp operates in the linear region. Therefore,
 - $V_{out} = (V_+ - V_-) \times A_v$
 - V_- and V_+ are the input voltage at the inverting and non-inverting terminals.
 - $-V_{sat} \leq V_{out} \leq +V_{sat}$
 - $\therefore A_v$ is very large
 - \therefore The difference $(V_+ - V_-)$ must be very small. Therefore
 - $V_+ \approx V_-$
 - This is an important approximation when analysing op-amp circuits with negative feedbacks.



Input Current to Op-Amp is Zero.

- The input Impedance of an op-amp is very high. It can be found in the range of $1M\Omega$ to $1T\Omega$. Therefore,
 - $Z_{in} \approx \infty$
 - $I_- \approx 0$ and $I_+ \approx 0$, ie, no input current flowing into the op-amp.
 - These are important approximations when analysing op-amp circuits.

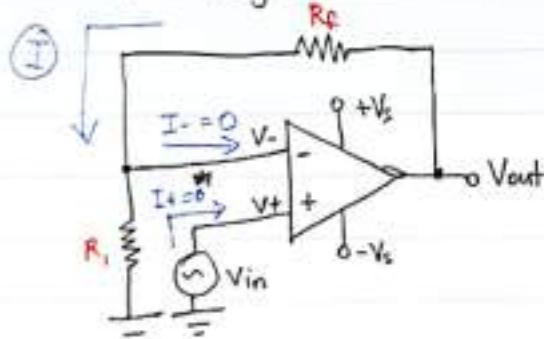


Non-Inverting Amplifier.

- Assume ideal op-Amp is used, since the input current to the ~~op-amp~~ ~~op-amp = 0~~, the same current that flows through R_f will also flow into R_i .

$$V_{out} = IR_i + IR_f = I(R_i + R_f)$$

- Due to negative feedback, $V_+ = V_-$ | - Since $V_+ = V_{in}$ and $V_- = IR$.



$$\text{Hence, } Acl = \frac{V_{out}}{V_{in}} = \frac{I(R_i + R_f)}{IR_i} = \frac{R_i + R_f}{R_i}$$

$$= 1 + \frac{R_f}{R_i}$$

- The closed loop voltage gain of a non-inverting amplifier is
- The voltage gain is positive. It indicates that the output signal is in phase with the input signal.

$$\cdot V_{out} = \left(1 + \frac{R_f}{R_i}\right) V_{in}$$

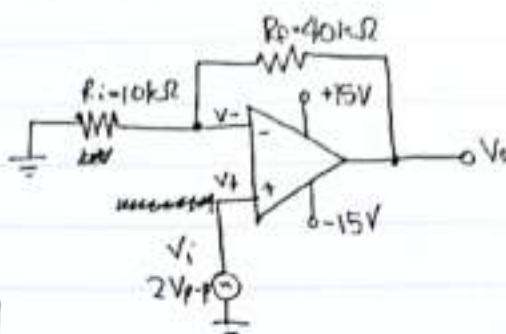
Where the Z is the impedance of the whole non-inverting amplifier.

$$\cdot Z_{in} = \frac{V_{in}}{I_i} = \frac{V_{in}}{I_+} \rightarrow \infty$$

(Assume ideal op-amp ~~used~~. Proof is not provided.)

Non-Inverting Amplifiers

- Calculate Voltage gain, A_v .
- Calculate the output voltage, $V_{o(p-p)}$
- Sketch input & output voltage wave, and show the phase relationship
- Sketch the output wave if the value of R_f is firstly changed to $90k\Omega$ and later changed to $190k\Omega$.



SOLUTIONS

a) A_v , Voltage Gain,

$$A_{cl} = 1 + \frac{R_f}{R_i} = 1 + \frac{40}{10} = 5 //$$

b) The output voltage,

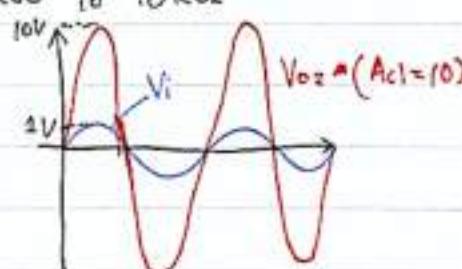
$$V_o = A_{cl} \times V_i = 5 \times 2 = 10V_{p-p}$$



c) If R_f change from $40k\Omega$ to $90k\Omega$

$$\text{The voltage gain, } A_{cl} = 1 + \frac{90}{10} = 10$$

$$\text{The output voltage, } V_o = 10 \times 2 = 20V_{p-p}$$



R_f changes from $90k\Omega$ to $190k\Omega$

The voltage gain,

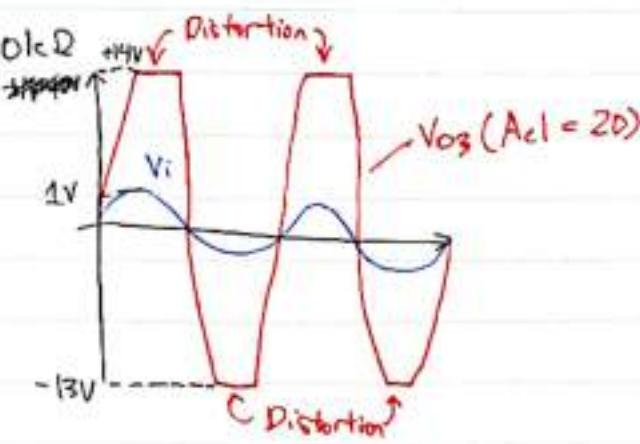
$$A_{cl} = 1 + \frac{190}{10} = 20$$

The output voltage,

$$V_o = 20 \times 2 = 40V_{p-p}$$

exceeded
the max
voltage

which is
max $30V_{p-p}$ thus it
will clip. (distorted)



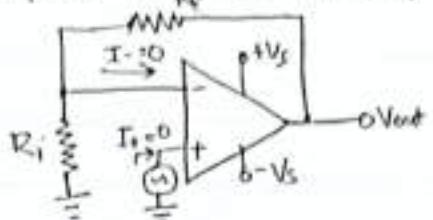
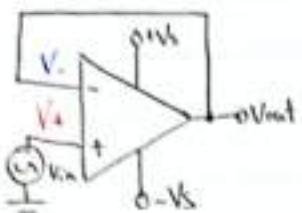
why $+14V$ & $-13V$? Because the output voltage is forced into saturation causing the output waveform to be distorted

$$+V_{sat} \approx +V_s - 1V \text{ and } -V_{sat} \approx -V_s + 2V$$

$$\text{where } V_s = \pm 15V$$

Voltage Follower

- This is a special case of non-inverting amplifier where $\rightarrow R_f = 0$, $R_i = \infty$



- Due to the extremely large voltage gain of an op-amp, the differential input voltage is zero when there is a -ve feedback loop. Therefore,

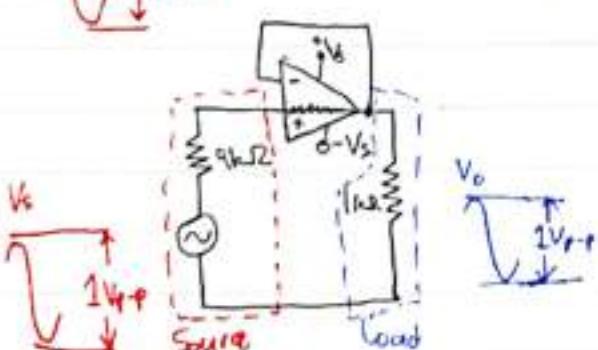
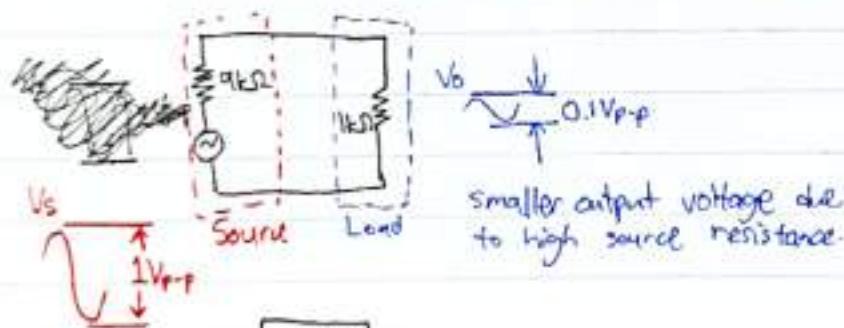
$$V_+ = V_-$$

- Since $V_{in} = V_+ = V_- = V_{out}$

- For Voltage follower, $V_{in} = V_{out}$

- Characteristics:
 - Extremely high input impedance
 - Extremely low output impedance
 - Capable of driving a relatively low resistive load due to its very small output impedance.
 - Often used as buffer circuit to match a high internal resistance signal source to a low resistive load.

- Voltage gain of voltage follower is $A_v = 1$



Inverting Amplifiers

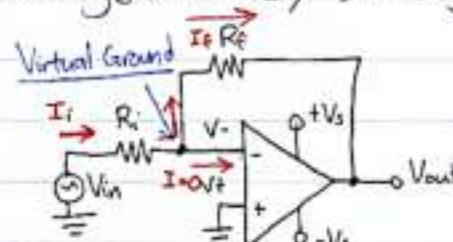
Introduction to Inverting Amplifiers

- Inverting amplifier and non-inverting amplifier are the two basic configurations of an op-amp. Negative feedback is applied in both configurations to provide linear operation and undistorted output.
- In a non-inverting amplifier, the input signal is applied to the non-inverting terminal of the op-amp; whereas in an inverting amplifier, the input signal is applied to the inverting terminal, through an input resistor. Another major difference between the 2 amplifiers is that the output signal of a non-inverting amplifier is in phase with the input signal but the output signal of an inverting amplifier is 180° out of phase with the input signal.
- Some variations of the inverting amplifier are used to perform math operations, such as summing(addition), scaling & averaging.

- Assume ideal op-amp is used.

- With negative feedback applied,

$$V_- = V_+$$



- Since V_+ is connected to ground, V_- will also be ~~zero~~ at ground potential.

- As V_- is not physically connected to the ground unlike V_+ , it is referred to as Virtual Ground.

$$\text{Current flowing through } R_i \text{ is: } I_i = \frac{V_{in} - 0V}{R_i} = \frac{V_{in}}{R_i}$$

$$\text{Current flowing through } R_f \text{ is: } I_f = \frac{0 - V_{out}}{R_f} = \frac{-V_{out}}{R_f}$$

- Since no current flows into the input terminals, I_i fully flows into the feedback resistor R_f . Thus, $I_i = I_f \Rightarrow \frac{V_{in}}{R_i} = \frac{-V_{out}}{R_f} \Rightarrow A_{in} = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_i}$

* The negative sign in the expression denotes that the output signal is 180° out-of-phase with the input signal. \rightarrow Output Voltage $\Rightarrow V_{out} = -\left(\frac{R_f}{R_i}\right)V_{in}$

$$\text{Input Impedance of the inverting amplifier is: } Z_{in} = \frac{V_{in}}{I_i} = R_i$$

- The output impedance of an inverting amplifier is

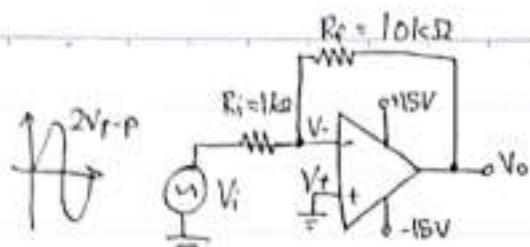
$$Z_{out} = \text{internal output impedance of op-amp} // R_f$$

* Proof is not provided.

No.

Date

Example.

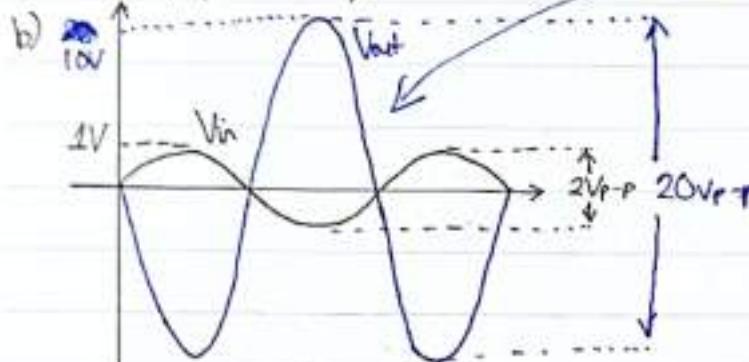
a) Find V_{out-p}

b) Sketch in 2 out waveforms, and

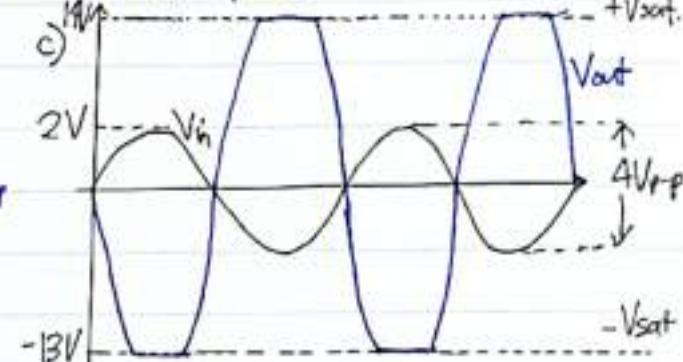
c) out waveform if the amplitude of the
first V_{in} is doubled.

$$a) V_{out} = -\left(\frac{R_f}{R_i}\right)V_{in} = -\left(\frac{10k\Omega}{1k\Omega}\right)2V_{p-p} = -20V_{p-p}$$

$$\therefore V_{out-p} = 20V_{p-p}$$



180° out-of-phase



$$+V_{sat} = 15 - 1 \\ = 14V$$

$$-V_{sat} = -15 + 2 \\ = -13V$$

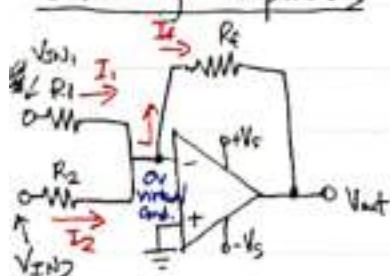
$$V_{out} = -\left(\frac{R_f}{R_i}\right)V_{in} = -\left(\frac{10k\Omega}{1k\Omega}\right)4V_{p-p} = -40V_{p-p}$$

{ will be clipped at $+V_{sat}$ / $-V_{sat}$ cuz more than the V_{sat} value.

Summing Amplifiers and Variations

- Variation of the inverting amplifier
- Has 2 or more inputs, and its output voltage is proportional to the -ve of the algebraic sum of it's V_{in} .
- Variations of the summing amplifier include the = averaging amplifier, and
 - scaling amplifier.

Summing Amplifiers



- Since no current flows into the op-amp input terminals, I_1 and I_2 are fully diverted into the feedback resistor R_f .

$$\therefore I_f = I_1 + I_2 = \frac{V_{IN1}}{R_1} + \frac{V_{IN2}}{R_2}$$

$$\therefore 0 - V_{out} = I_f R_f$$

$$V_{out} = -I_f R_f$$

$$V_{out} = -\left(\frac{V_{IN1}}{R_1} + \frac{V_{IN2}}{R_2}\right)R_f$$

$$V_{out} = -\left(\frac{R_f}{R_1}V_{IN1} + \frac{R_f}{R_2}V_{IN2}\right)$$

IP with Unity Gain

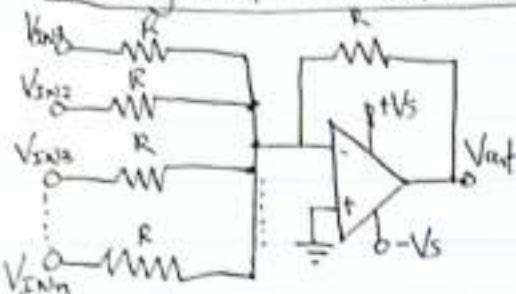
If all the resistors

all equal, $R_1 = R_2 = R_f$,

$$V_{out} = -(V_{IN1} + V_{IN2})$$

Inverting Amplifiers:

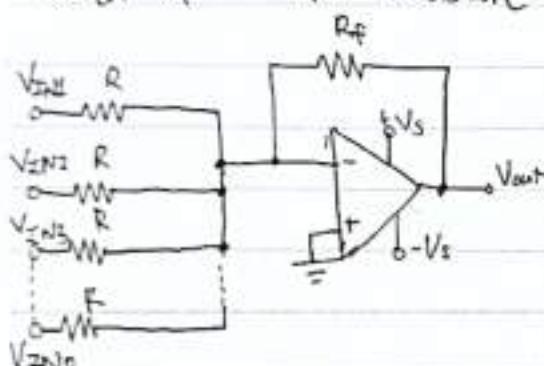
Summing Amplifiers with "n" Identical Input Resistors



$$V_{out} = -(V_{IN1} + V_{IN2} + V_{IN3} + \dots + V_{INn}) / R_f$$

Summing Amplifier with Gain Greater Than Unity

- When $R_f >$ Input resistance R , the amplifier has a gain of R_f/R .



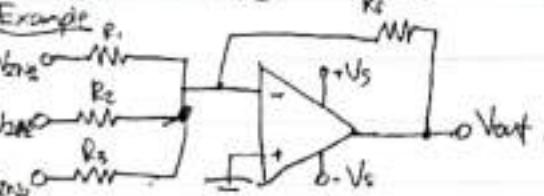
$$V_{out} = -\frac{R_f}{R} (V_{IN1} + V_{IN2} + V_{IN3} + \dots + V_{INn})$$

Multi-channel Scaling Amplifier

- By using different values of $R_1, R_2, R_3 \dots R_n$, the inputs of a summing amplifier can be subjected to different voltage gains.

$$- V_{out} = -\left(\frac{R_f}{R_1} V_{IN1} + \frac{R_f}{R_2} V_{IN2} + \frac{R_f}{R_3} V_{IN3} + \dots + \frac{R_f}{R_n} V_{INn}\right)$$

Example e.



RF = 1kΩ

$$V_{IN1} = 1V, V_{IN2} = 2V, V_{IN3} = 3V$$

$$R_1 = 15k\Omega, R_2 = 15k\Omega, R_3 = 15k\Omega, R_f = 15k\Omega$$

$$V_{out} = -\left(\frac{15}{15}(1) + \frac{15}{15}(2) + \frac{15}{15}(3)\right) = -6V$$

Only 2 → No V_{IN3} & R_3

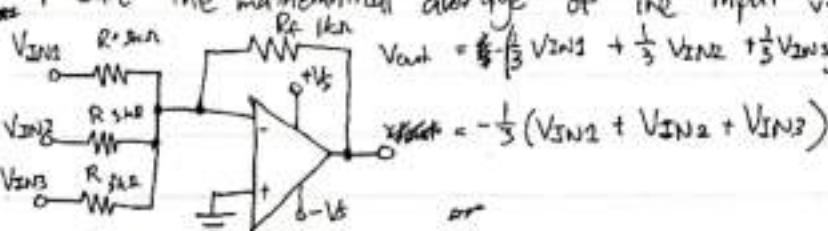
$$\rightarrow V_{IN1} = 0.2V, R_1 = 1k\Omega, R_f = 10k\Omega$$

$$V_{IN2} = 0.3V, R_2 = 1k\Omega$$

Averaging Amplifier

A. Summing amplifier can be made to

produce the mathematical average of the input voltages by having $R/R_p = n^{-1}$ input resistors.



$$V_{out} = -\left(\frac{1}{1}(0.2) + \frac{1}{1}(0.5)\right) = -7V$$

$$V_{out} = -\left(\frac{R_f}{R_1} V_{IN1} + \frac{R_f}{R_2} V_{IN2} + \frac{R_f}{R_3} V_{IN3} + \dots + \frac{R_f}{R_n} V_{INn}\right)$$

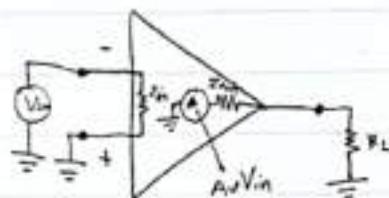
$$+ \dots + \frac{R_f}{R_n} V_{INn}$$

Comparators

Intro to comparators

- The open-loop voltage gain of an op-amp is extremely high. A very small voltage difference across the two input terminals will be able to saturate the output voltage to $\pm V_{sat}$.
- Whether the output voltage is $+V_{sat}$ or $-V_{sat}$ depends on which of the two input voltages is higher. An open-loop op-amp used for comparing two voltages is referred to as a comparator.
- Op-amps were originally designed for analog circuits. The response of an op-amp comparator is not very fast, so it is better to use a dedicated comparator IC for fast changing input signals. The internal circuit of a dedicated comparator is different from that of an op-amp.
- A dedicated comparator, such as LM311, or CM39, is designed to work with other digital devices. However this module mainly focuses on analog circuits. Thus we will ~~limit~~

Op-Amp in the Open-Loop Configuration



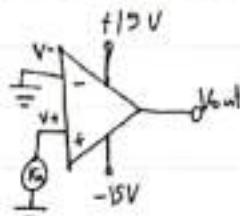
$A_v = A_{ol} = \text{Open-Loop Voltage Gain}$.

$$V_{out} = (V_+ - V_-) \times \frac{A_{ol}}{A_{ol} + 1}$$

Op-Amp Comparators

- 3 Major type of comparators:
1. Zero level comparator (or decoder),
 2. Non-zero level comparator
 3. Comparator with hysteresis.

Zero Level Comparators



- Very small diff. in voltage between the 2 inputs drives the amplifier saturation.

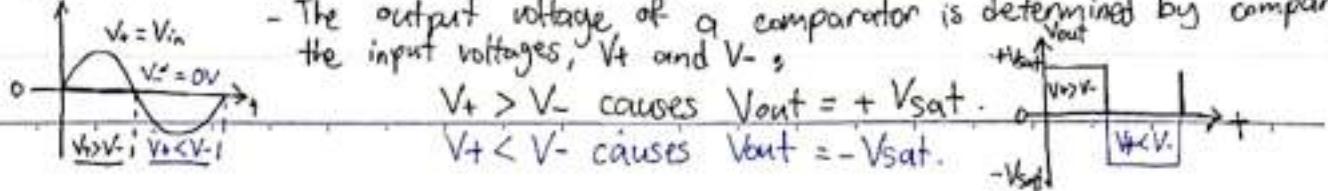
- E.g. If $A_{ol} = 100,000$ & Voltage diff. between the 2 inputs = 0.25mV

- The amplifier will be driven to saturation.

- The output voltage of a comparator is determined by comparing the input voltages, V_+ and V_- ,

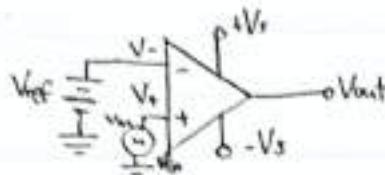
$$V_+ > V_- \text{ causes } V_{out} = +V_{sat}$$

$$V_+ < V_- \text{ causes } V_{out} = -V_{sat}$$



Comparators

Non-Zero Level Comparator

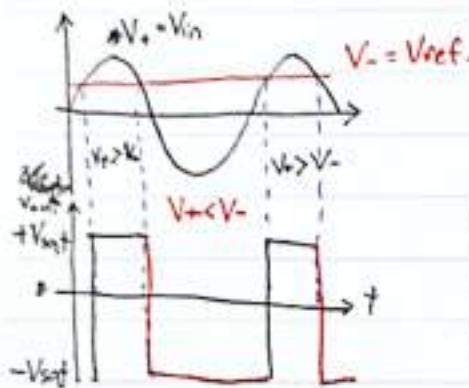
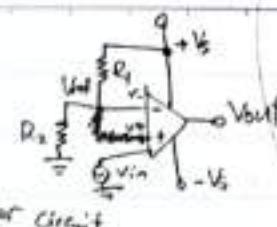


$$V_+ > V_- \Rightarrow V_{\text{out}} = +V_{\text{sat}}$$

$$V_+ < V_- \Rightarrow V_{\text{out}} = -V_{\text{sat}}$$

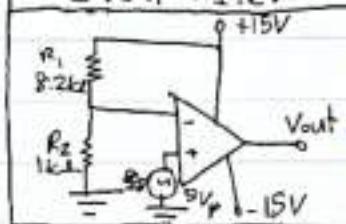
$V_{\text{ref}} \Rightarrow$ usually use a potential ~~meter~~.

$$V_{\text{ref}} = \left(\frac{R_2}{R_1 + R_2} \right) \times V_s$$

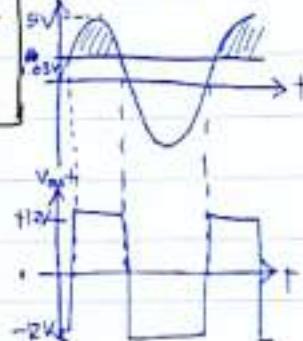


Example

5Vp sine wave \rightarrow input to a comparator. Sketch input & output wave. State amplitude & phase relationship $\pm V_{\text{sat}} = \pm 12V$



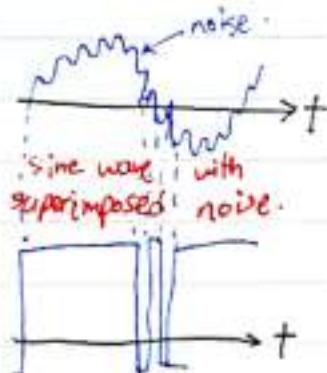
$$\text{Sol. } V_{\text{ref}} = \frac{R_2}{R_1 + R_2} \times V_s = \frac{1}{8.2+1} \times 5 = 1.63V$$



Hysteresis Comparator

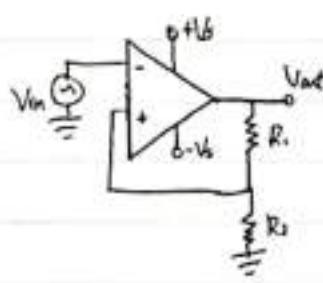
Effects of Input Noise on Comparator Operation

- In many practical situations, unwanted voltage fluctuation (noise) appears on the input line.
- The noise voltage becomes superimposed on the input voltage and can cause a comparator to erratically switch output states.



Reducing Noise Effect with Hysteresis

- To make the comparator less sensitive to noise applying positive feedback in an op-amp circuit known as hysteresis
- Hysteresis \rightarrow higher reference & reference level when $V_{\text{out}} = +V_{\text{sat}}$, and lower reference level when $V_{\text{out}} = -V_{\text{sat}}$.

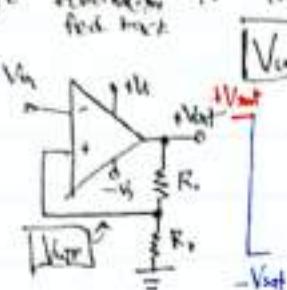
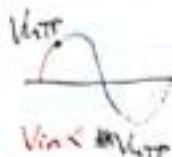


- The two reference voltages are ~~referenced to~~ referred to as the upper trigger point (UTP) and the lower trigger point (LTP).

No.

Date

- Assuming that the output voltage is at its positive maximum, $+V_{sat}$.
- The voltage fed back to the non-inverting input is V_{UTP} . $\rightarrow V_{UTP} = \frac{R_2}{R_1 + R_2} \times V_{sat}$.

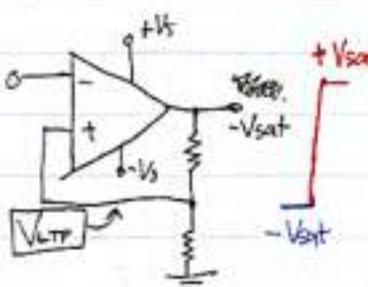


V_{UTP} - Output at the maximum positive voltage.

- Input exceeds V_{UTP} , output ~~saturates~~ switches from the maximum ~~pos~~ positive voltage to the maximum negative voltage.

- When the input voltage exceeds V_{UTP} , the output voltage drops to its negative maximum, $-V_{sat}$.

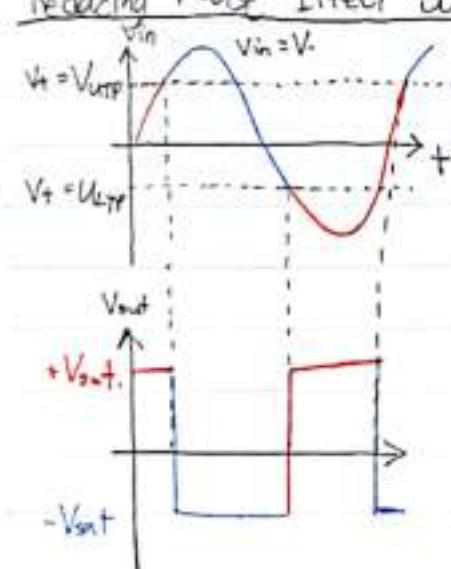
- The voltage fed back to the non-inverting input is V_{LTP} .



$-V_{LTP}$ - Output at the maximum negative voltage.

- Input goes below V_{LTP} , output switches from the ~~maximum neg~~ maximum negative voltage to the maximum ~~pos~~ positive voltage.

Reducing Noise Effect with hysteresis.



- To switch from the maximum negative voltage back to the maximum positive voltage, input voltage must now fall below V_{LTP} .

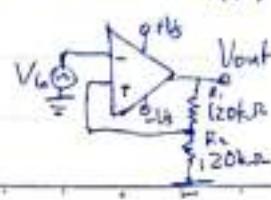
- Small amount of noise voltage therefore has no ~~effect~~ effect on the output.

- A comparator with hysteresis is also called a Schmitt trigger.

- The amount of hysteresis is defined by the difference of the 2 trigger levels

Example

Find V_{UTP} & V_{LTP} , Assume that $\pm V_{sat} = 8V$



$$V_{UTP} = \frac{R_2}{R_1 + R_2} \times V_{sat} = 0.5 \times 8 = 4V$$

$$V_{LTP} = \frac{R_2}{R_1 + R_2} \times -V_{sat} = 0.5 \times (-8) = -4V$$

$$V_{Hys} = V_{UTP} - V_{LTP}$$

Phasors & Complex Numbers

- When you throw a bowling ball, the ball is rotating at a constant angular velocity (angular speed in a specific direction).



- The angular movement at a point on the surface (circumference) of the ball is circular (relative to its axis of rotation).

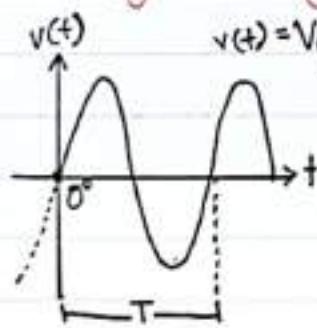
- When the movement against its time, it shows a sinusoidal wave.

- Feature of a sinusoidal wave : - Angular velocity, frequency, period, Direction of rotation, Peak value, amplitude

The direction of the rotation provides a reference to polarity of angle subtended at the center.

(or rotation axis) & so :

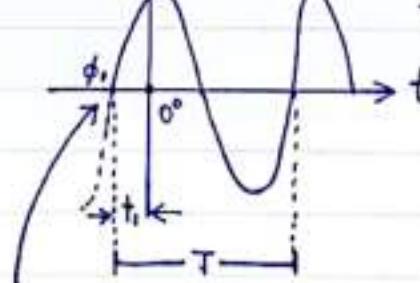
- A positive angle \rightarrow rotation is counter-clockwise.
- A negative angle \rightarrow rotation is clockwise.



$$v(t) = V_p \sin(2\pi ft + \phi_1)$$

$$t_1 = \frac{(\phi_1)}{2\pi} \times T$$

- The angle ϕ_1 has a positive value.



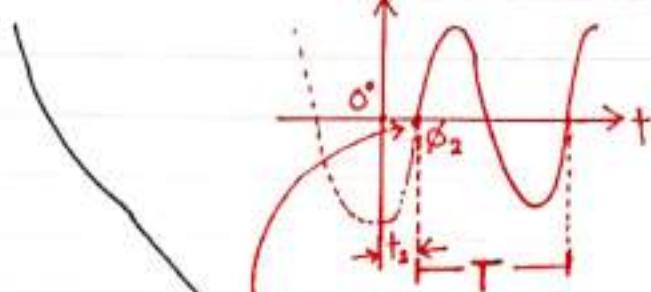
\rightarrow crosses earlier. \rightarrow positive slope crosses zero level at ϕ_1 .

A sine wave that

$$v(t) = V_p \sin(2\pi ft + \phi_2)$$

$$t_2 = \frac{(\phi_2)}{2\pi} \times T$$

- The angle ϕ_2 has a negative value.



\rightarrow crosses later \rightarrow positive slope crosses zero level at ϕ_2 .

AC voltage & AC current \rightarrow sinusoidal.

\hookrightarrow magnitude represented by either peak or can be rms values.

Calculating power by an AC supply. \rightarrow all AC voltage & current are referred using the rms values.

Singapore \rightarrow AC 230V \rightarrow rms value of 230V

power adaptor \rightarrow 100V - 240V ac \rightarrow you can input rms 100V - 240V rms to your adaptor.

Sinusoidal wave \rightarrow has magnitude & an angular position / direction
vector quantity

AC voltage & currents are called phasors.

A voltage phasor has the expression. $\rightarrow \vec{V} = C \angle \phi$ phase angle in degrees.
phasor for voltage ↑ magnitude in rms

- The magnitude C is always positive.

\rightarrow This is because only positive rms value are considered.

\rightarrow This is different from a vector that can show positive or negative magnitude.

- The phase angle ϕ can be zero, a positive or a negative value.

* The phase angle is only meaningful if there are 2 or more sine waves to be compared with each other.

~~$v(t) = 300 \sin(2\pi ft + 30^\circ)$~~

$$v(t) = 100 \sin(2\pi ft + 30^\circ) V$$

$$\text{The magnitude } C = \frac{100}{\sqrt{2}} = 70.7 V_{(\text{rms})}$$

$$\text{The phase angle } \phi = 30^\circ$$

$$\text{The phasor } \vec{V} = 70.7 \angle 30^\circ V_{//}$$

$$i(t) = \frac{300}{I_p} \sin(2000\pi t - 60^\circ) A$$

$$\text{The magnitude } C = \frac{300}{\sqrt{2}} = 212 \text{ mA}$$

$$\text{The phase angle} = -60^\circ$$

$$\text{The phasor } \vec{I} = 212 \angle -60^\circ \text{ mA}$$

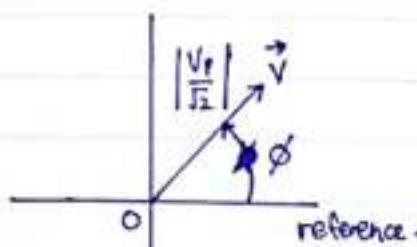
Phasors & Complex Numbers.

Sinusoidal expression is a function of time (i.e. in time domain).

$$v(t) = V_p \sin(2\pi ft + \phi) V$$

The phasor $\vec{V} = \frac{V_p}{\sqrt{2}} \angle \phi V$.

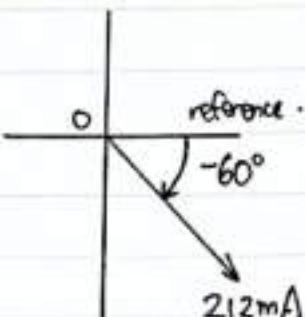
If the angle ϕ is positive.



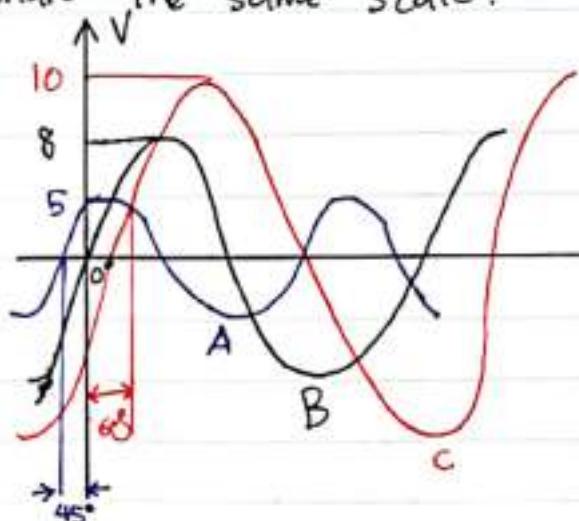
$$i(t) = 300 \sin(2000\pi t - 60^\circ) \text{ mA.}$$

The phasor $\vec{V} = 212 \angle -60^\circ \text{ mA.}$

If the angle were to be positive.



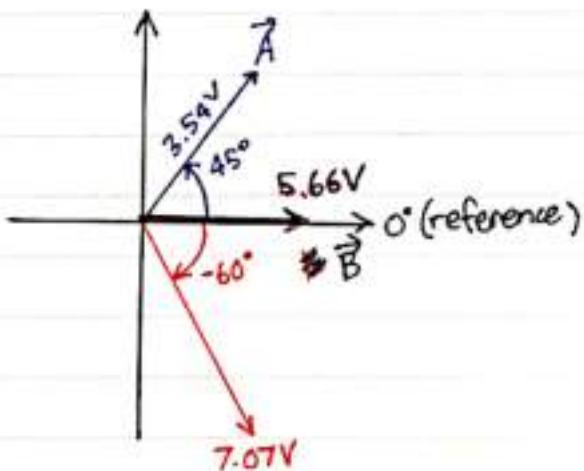
- When two or more sinusoidal functions are operating at the same frequency, their phasors can be sketched on the same phasor diagram.
- A phasor diagram may have a mix of phasors comprising voltages and currents.
- Voltages and currents have different units and don't therefore share the same scale.



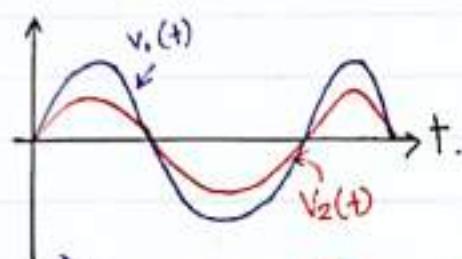
$$\vec{A} = \frac{5}{\sqrt{2}} \angle 45^\circ V = 3.54 \angle 45^\circ V$$

$$\vec{B} = \frac{5\sqrt{2}}{2} \angle 0^\circ V = 5.66 \angle 0^\circ V$$

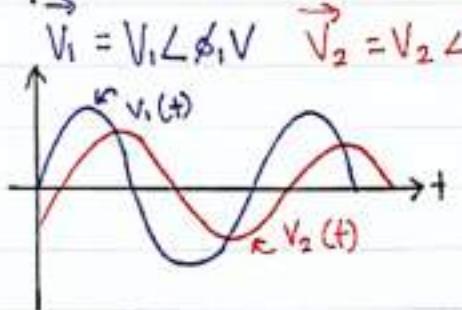
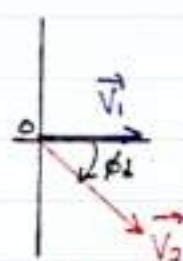
$$\vec{C} = \frac{\sqrt{15}}{2} \angle -60^\circ V = 7.07 \angle -60^\circ V$$



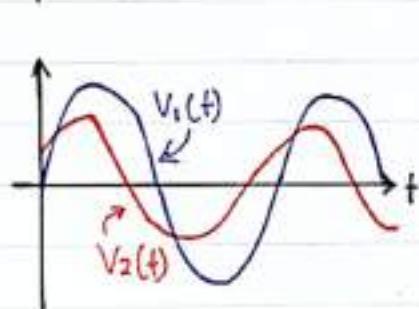
- If we are only representing one wave, we are usually not interested in whether it starts from the origin.
- However, if we are representing 2 or more waves, and they are not synchronized (out-of-phase), then their initial phase ~~is~~ is important.



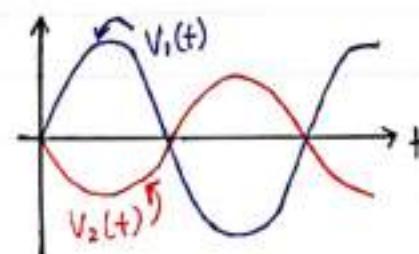
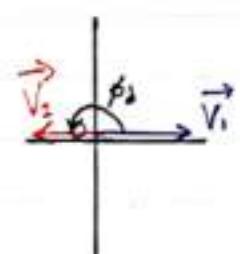
- Both waves are in phase.
 - No phase difference.
- $$\phi_1 = \phi_2$$



- Both waves are out-of-phase.
 - Phase difference $\rightarrow \phi_d = |\phi_1 - \phi_2|$
- $V_1(t)$ leads $V_2(t)$ by ϕ_d
 $V_2(t)$ lags $V_1(t)$ by ϕ_d



- Both waves are out-of-phase.
 - Phase difference $\rightarrow \phi_d = |\phi_1 - \phi_2|$
- $V_2(t)$ leads $V_1(t)$ by ϕ_d
 $V_1(t)$ lags $V_2(t)$ by ϕ_d



- When 2 waves are 180° out-of-phase / inverted of each other.
- Phase difference $\rightarrow \phi_d = |\phi_1 - \phi_2| = 180^\circ$

- Inappropriate to state which wave is leading / lagging the other.

Phasors & Complex Numbers.

Addition

$$V_1(t) = 20 \sin(2000t - 30^\circ) V, V_2(t) = 10 \sin(2000t + 45^\circ) V$$

$$V_1(t) + V_2(t) = ?$$

$\vec{V}_1(t) = \vec{V}_1 = \frac{20}{\sqrt{2}} \angle -30^\circ V = 14.14 \angle -30^\circ V, \vec{V}_2(t) = \vec{V}_2 = \frac{10}{\sqrt{2}} \angle 45^\circ V = 7.07 \angle 45^\circ V$

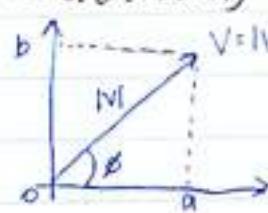
- At this juncture, we will learn about another form of phasor representation.

- Trigonometrically, a phasor can be decomposed (broken down) into 2 components.

Let phasor $\vec{V} = |V| \angle \phi$

* Horizontal Component $a = |V| \cos \phi$

* Vertical Component $b = |V| \sin \phi$



j Operator

- A phasor $\vec{V} = |V| \angle \phi$ is a complex number.

- To differentiate the vertical component from the horizontal, the ~~j~~ operator is used to represent the vertical axis.

- Mathematically,

* $j = \sqrt{-1}, j^2 = -1, \frac{1}{j} = -j$

- In phasor, j denotes an angle of 90° rotated counterclockwise from the horizontal axis.

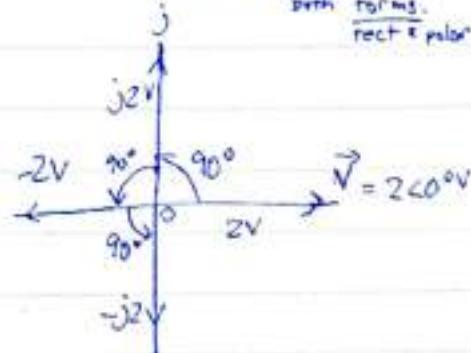
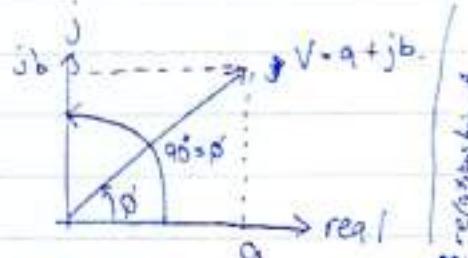
- To illustrate the meaning of j operator, let phasor $\vec{V} = 2 \angle 0^\circ V$ and is on the horizontal axis.

- Multiplying the magnitude 2 by j gives $j2 V$

* which is rotated 90° counterclockwise.

- A further 90° counterclockwise of rotation gives $j^2(2) V$ or $-2V$.

- Another 90° counterclockwise of rotation gives $j^3(2)V$ or $-j2V$.



~~$\vec{V} = a + jb$~~

- This is the rectangular form

- To convert from polar to rectangular.

$$\rightarrow a = |V| \cos \phi, jb = j|V| \sin \phi$$

- To convert from rectangular to polar

$$|V| = \sqrt{a^2 + b^2}, \phi = \tan^{-1} \left(\frac{b}{a} \right)$$

eg. $\vec{V}_1 = 14.14 \angle -30^\circ V$ find $a + jb$

$a = 14.14 \cos(-30^\circ) = 12.26 V$

$jb = j 14.14 \sin(-30^\circ) = -j7.07 V$

$\therefore 12.26 - j7.07 V$

Addition

$$\vec{V}_1 = a_1 + jb_1 \text{ and } \vec{V}_2 = a_2 + jb_2$$

$$\vec{V}_1 + \vec{V}_2 = (a_1 + a_2) + j(b_1 + b_2)$$

Using sine wave.

$$14.14 \angle -30^\circ \rightarrow 20 \sin(2000t - 30^\circ) V.$$

$$\vec{V}_1 = 14.14 \angle -30^\circ \rightarrow V_1(t) = 10 \sin(2000t - 30^\circ) V$$

$$\vec{V}_2 = 7.07 \angle 45^\circ \rightarrow V_2(t) = 10 \sin(2000t + 45^\circ) V$$

$$\vec{V}_2 = 7.07 \angle 45^\circ \rightarrow V_2(t) = 10 \sin(2000t + 45^\circ) V$$

$$V_{12}(t) = \underline{V_p} \sin(2000t + 45^\circ) V$$

Important: The frequency of a new waveform follows that of the ~~original~~ original functions it derived from (in this case the $2000t$).

Subtraction

$$\vec{V}_1 - \vec{V}_2 = (a_1 - a_2) + j(b_1 - b_2)$$

Multiplication

$$\vec{V}_1 \times \vec{V}_2 = (a_1 + jb_1) \times (a_2 + jb_2)$$

$$\text{rect.} = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1)$$

$$\vec{V}_1 \times \vec{V}_2 = (V_1 \angle \phi_1) \times (V_2 \angle \phi_2)$$

$$\text{Polar} = (|V_1| \times |V_2|) \angle (\phi_1 + \phi_2)$$

Division

$$\vec{V}_1 \div \vec{V}_2 = (V_1 \angle \phi_1) \div (V_2 \angle \phi_2)$$

$$= \left(\frac{|V_1|}{|V_2|}\right) \angle (\phi_1 - \phi_2)$$

$$\text{e.g. } \vec{V}_1 = 14.14 \angle -30^\circ \text{ and } \vec{V}_2 = 7.07 \angle 45^\circ V$$

$$\vec{V}_1 + \vec{V}_2 = ?$$

(1) convert to rectangular form \rightarrow

$$\vec{V}_1 = 12.25 - j7.07 \quad \vec{V}_2 = 5 + j5V$$

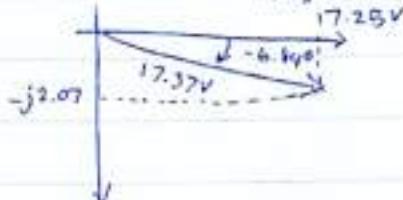
(2) adding:

$$\vec{V}_1 + \vec{V}_2 = (12.25 + 5) + j(-7.07 + 5)V = 17.25 - j2.07V$$

(3) convert back to polar.

$$|V| = \sqrt{17.25^2 + 2.07^2} = 17.37V$$

$$\phi' = \tan^{-1}\left(\frac{-2.07}{17.25}\right) = -6.84^\circ \therefore 17.37 \angle -6.84^\circ V$$



$$\text{e.g. } \vec{V}_1 = 18 + j5V \text{ and } \vec{V}_2 = 6 - j3V$$

$$\vec{V}_1 - \vec{V}_2 = (18 - 6) + j(5 - (-3))V$$

$$= 12 + j8V$$

$$\text{e.g. } \vec{V}_1 = 15 + j5V \text{ and } \vec{I}_2 = 6 + j3A$$

$$\vec{V}_1 \times \vec{I}_2 = ((15 \times 6) - (5 \times 3)) + j((5 \times 6) + (15 \times 3))VA$$

$$= 75 + j75VA$$

$$\vec{V}_1 = 15.81 \angle 18.43^\circ V \text{ and } \vec{I}_2 = 6.7 \angle 26.57^\circ A$$

$$\vec{V}_1 \times \vec{I}_2 = (15.81 \times 6.7) \angle (18.43^\circ + 26.57^\circ)VA$$

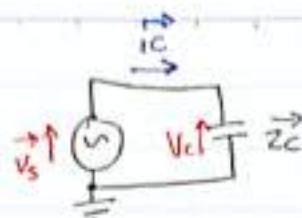
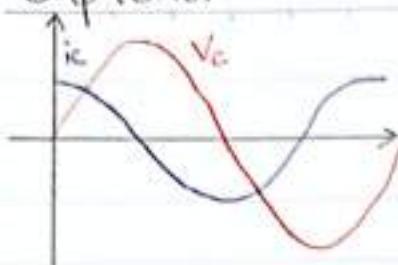
$$= 106 \angle 45VA$$

$$\text{e.g. } \vec{V}_1 = 15.81 \angle 18.43^\circ V \text{ and } \vec{I}_1 = 6.7 \angle 26.57^\circ A$$

$$\vec{V}_1 = \frac{15.81}{6.7} \angle (18.43 - 26.57)$$

$$= 2.36 \angle -8.14^\circ S2A$$

Capacitor



* Resistive ac circuit

- A sinusoidal voltage always produces a sinusoidal current provided the signal is not distorted.
- Operated at same frequency
- Current and voltage waves are moving up and down simultaneously and crossing their respective zero level at the same instant.
- The current & voltage are said to be in phase.

→ To show, let $V_s(t) = V_p \sin(2\pi ft) V$

The current flowing $\rightarrow i_R(t) = I_p \sin(2\pi ft) A$.

The voltage developed across the resistor $\rightarrow V_R(t) = V_p \sin(2\pi ft) V$

$$\therefore \text{The ratio} = \frac{V_R}{I_R} = \frac{V_p \sin(2\pi ft) V}{I_p A} = \frac{V_p V}{I_p A} = \frac{|V_p| \sqrt{2} V}{|I_p| \sqrt{2} A} = \frac{|V_p| V}{|I_p| A}$$

where $|V_p|$ & $|I_p|$ are rms

- From Ohm's law, the ratio gives resistance in ohm.

- Converting to phasor domain $\rightarrow \frac{\vec{V}_R}{\vec{I}_R} = \frac{\vec{V}_p}{\vec{I}_p} = \frac{|V_p|}{|I_p|} \angle 0^\circ$

- In a resistive ac circuit, the ratio is $R \rightarrow \frac{\vec{V}_R}{\vec{I}_R} = \frac{|V_p|}{|I_p|} \angle 0^\circ$

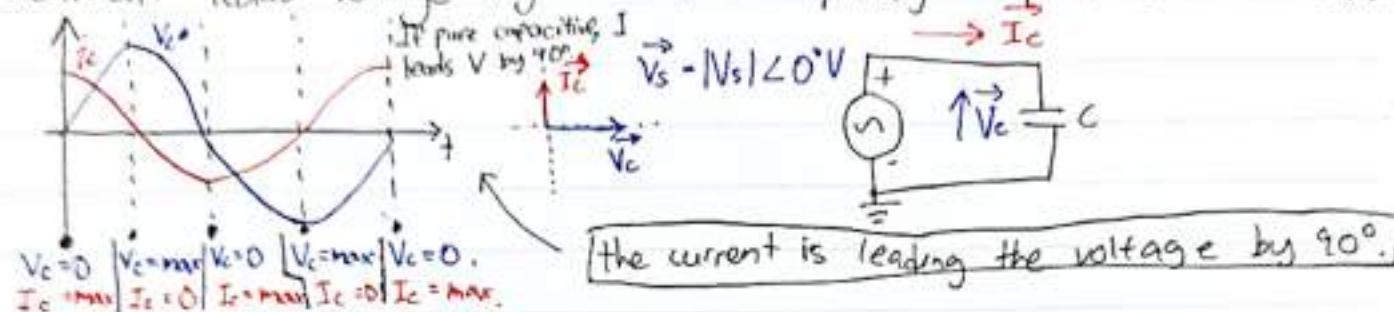
* Capacitive ac circuit

- A capacitance doesn't allow current to pass through it.
- In a dc circuit, during the charging or the discharging process do we observe any current flow in the circuit.
- It is precisely the ever-changing voltage across the capacitor that generates the current in a capacitive ac circuit.
- It's the flow of charges that eventually accumulate on the plate surfaces of the capacitor during the charging process.
 - The discharging process reverses the polarity of the charges on the plates
- The alternations of charges on the capacitor constitute the current flow in the circuit.
- No current actually flows through the capacitor, regardless of the circuit.

X

VI phase relationship in a capacitive ac circuit

- A capacitor behaves differently from a resistor in an ac circuit.
- Current leads voltage by 90° in a purely capacitive ac circuit.



- In a capacitive ac circuit, current & voltage waves are sinusoidal that cross their zero level at different ~~times~~ instants.
- ~~For capacitor~~ Why? $\rightarrow Q = CV$ or $i_c(t) \Delta t = C \Delta V$ where ΔV is change of V over time Δt .

$$\rightarrow \text{Thus in ac, current } i_c(t) = C \frac{dV}{dt} A_p$$

$$\rightarrow \text{The supply voltage is } V_s(t) \text{ where } V_s(t) = V_p \sin(2\pi ft) V$$

$$\rightarrow \text{Sub in for } V_c(t) \text{ in the } i_c(t) \text{ circuit} \rightarrow i_c(t) = C \frac{d}{dt} V_p \sin(2\pi ft) A_p$$

- The current after differentiation is.

$$\left. \begin{aligned} i_c(t) &= (2\pi ft) V_p \sin(2\pi ft) A_p \\ i_c(t) &= (2\pi f) V_p \sin(2\pi ft + 90^\circ) A_p \end{aligned} \right\} \text{same.}$$

- Compared with a standard current expression.

$$i_c(t) = I_p \sin(2\pi ft + \phi) A_p$$

$$\rightarrow I_p = (2\pi fC) V_p, \text{ Phase angle } \phi = 90^\circ$$

- The voltage across the capacitor is $V_c(t)$

$$V_c(t) = V_p \sin(2\pi ft) V$$

- The ratio of voltage to current.

$$\frac{V_c}{I_c} = \frac{V_p \sin(2\pi ft) V}{I_p \cos(2\pi ft) A_p} = \frac{V_p \sin(2\pi ft) V}{I_p \sin(2\pi ft + 90^\circ) A_p}$$

- Convert to phasor domain.

$$\frac{\vec{V}_c}{I_c} = \frac{\vec{V}_c}{\vec{I}_c} = \frac{|V_c| \angle 0^\circ V}{|I_c| \angle 90^\circ A} = \frac{|V_c|}{|I_c|} \angle (0^\circ - 90^\circ) \Omega = \frac{|V_c|}{|I_c|} \angle -90^\circ \Omega$$

$|V_c| \& |I_c| \rightarrow \text{rms value.}$

* It's magnitude is called

reactance X_c

$$X_c = \frac{|V_c|}{|I_c|} \Omega$$

* It's impedance angle ϕ_c

$$\phi_c = -90^\circ$$

* \rightarrow Mathematically, reactance

X_c is expressed in

terms of Capacitance C

and operating freq. f

$$\rightarrow X_c = \frac{1}{2\pi fC} \Omega$$

use ohm's law.

* The ratio ~~express~~ does express the opposition to current flow in an ac circuit.

* In a capacitive ac circuit, the ratio is termed as impedance Z_c

$$\frac{V_c}{I_c} = Z_c = \frac{|V_c|}{|I_c|} \angle -90^\circ \Omega$$

Capacitor

e.g. Find the impedance of a 22nF capacitor connected to a 10V , 2000Hz

$$\text{Capacitance } C = 22\text{nF} = 22 \times 10^{-9}\text{F.}$$

$$\text{Reactance } X_C = \frac{1}{2\pi \times 2000 \times 22 \times 10^{-9}} \Omega = 3617\Omega$$

$$\text{Impedance } Z_C = 3617 \angle -90^\circ \Omega \text{ (polar), or } -j3617 \Omega \text{ (rect)}$$

* Find the current flowing in a 3.3mF capacitor connected to a 5V , 50Hz

$$\text{Capacitance } C = 3.3\text{mF} = 3.3 \times 10^{-6}\text{F.}$$

$$\text{Impedance } Z_C = \frac{1}{2\pi \times 5000 \times 3.3 \times 10^{-6}} \angle -90^\circ \Omega = 9.646 \angle -90^\circ \Omega$$

$$\text{Current } = \frac{\vec{V}_s}{Z_s} = \vec{I}_C = \frac{5 \angle 0^\circ \text{V}}{9.646 \angle -90^\circ \Omega} = 518 \angle 90^\circ \text{mA} / j518 \text{mA}$$

* Find Capacitance when a current of 8mA is flowing connected to a 8V , 1.5kHz

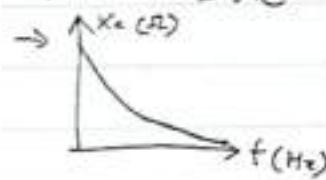
$$\text{Impedance } Z_C = \frac{8 \angle 0^\circ \text{V}}{8 \angle 90^\circ \text{mA}} = 1 \angle -90^\circ \Omega$$

$$\text{Reactance } X_C = \frac{1}{2\pi \times 1.5 \times 10^3 \times C} \Omega = 1000 \Omega$$

$$C = 106 \text{nF}$$

Reactance against operating frequency

If $X_C = \frac{1}{2\pi f C} \Omega \rightarrow$ Then for given C , X_C is ~~proportional~~^{As} inversely proportional to f ,



- This characteristic curve for reactance has applications like
- * Filters \rightarrow allows ac signals to pass through a circuit within a frequency range \rightarrow called ~~ac~~ band pass or band stop.
- * ~~Hi-Fi~~ sound fx \rightarrow capacitor passes high frequency energy to tweeter Hi-Fi
- * AC coupling \rightarrow allows only ac signals to cross between circuits.

Resistor in an ac circuit

$$\vec{Z} = \frac{V}{I} \angle 0^\circ \Omega = R \Omega. \quad \text{Resistance in series} = R_T = R_1 + R_2 + R_3 + \dots + R_n$$

$$\rightarrow \vec{Z} = R \Omega \quad \text{When expressed in impedance } (\vec{Z}_T) = \vec{Z}_T = R_T = R_1 + R_2 + R_3 + \dots + R_n$$

$$\text{If parallel} = \frac{1}{\vec{Z}_T} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

~~Capacitance in series $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$~~

use

V

- Admittance (\vec{Y}_T) for a parallel circuit = $\frac{1}{\vec{Z}_T}$, $\vec{Y}_1 = \frac{1}{R_1}$, $\vec{Y}_2 = \frac{1}{R_2}$, $\vec{Y}_3 = \frac{1}{R_3}$... $\vec{Y}_n = \frac{1}{R_n}$

- Expressed in admittance (\vec{Y}_T) = $\vec{Y}_1 + \vec{Y}_2 + \dots + \vec{Y}_n$

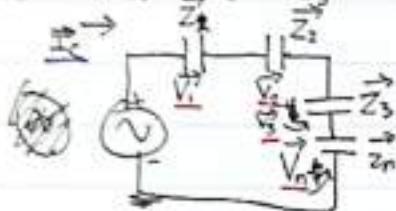
- Likewise, in terms of conductance (G)

$$\vec{Y}_T = G_T = G_1 + G_2 + \dots + G_n$$

$$\text{Admittance : } \vec{Y} = \frac{1}{V} \angle 0^\circ = GS \quad (\text{Siemens})$$

Capacitor in series connections

$$C_T = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n} \leftarrow \text{in both ac & dc.}$$



By converting the capacitance to their impedance.
→ Impedance value = \vec{Z}_T

$$\vec{Z}_T = \frac{1}{C_T} (V_s) = \frac{1}{C_T} (V_1 + V_2 + V_3 + \dots + V_n)$$

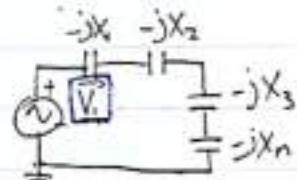
$$- \vec{Z}_T = \frac{\vec{V}_s}{I_c} = | \frac{V_s}{I_c} | \angle -90^\circ \Omega = -jX_T \Omega.$$

$$\vec{V}_1 = \vec{V}_s \times \frac{\vec{Z}_1}{\vec{Z}_T}$$

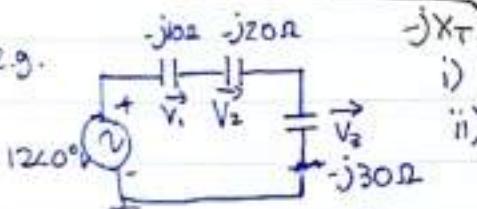
~~Series connection~~

- Total impedance (\vec{Z}_T) in terms of series component

$$\vec{Z}_T = -jX_1 - jX_2 - jX_3 - \dots - jX_n.$$



e.g.



$$\text{i) Find } (\vec{Z}_T)$$

$$\text{ii) Find } \vec{V}_1, \vec{V}_2, \vec{V}_3$$

$$\begin{aligned} \text{i) } \vec{Z}_T &= -jX_1 - jX_2 - jX_3 \\ &= (-j10) + (-j20) + (-j30) \\ &= -j60 \Omega \end{aligned}$$

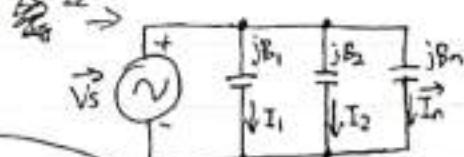
$$\text{ii) } \vec{V}_1 = 12 \angle 0^\circ V \times \frac{-j10}{-j60} = 2 \angle 0^\circ V_{II}$$

$$\vec{V}_2 = 12 \angle 0^\circ V \times \frac{-j20}{-j60} = 4 \angle 0^\circ V_{II}$$

$$\vec{V}_3 = 12 \angle 0^\circ V \times \frac{-j30}{-j60} = 6 \angle 0^\circ V_{II}$$

Hence (\vec{Y}_T) in terms of parallel = $\vec{Y}_T = jB_1 + jB_2 + \dots + jB_n$.

$$\vec{I}_T$$



Capacitance in parallel connections

$$C_T = C_1 + C_2 + C_3 + \dots + C_n$$

$$\vec{I}_c = \vec{V}_s \left[\frac{\vec{I}_1}{\vec{Z}_1}, \frac{\vec{I}_2}{\vec{Z}_2}, \frac{\vec{I}_3}{\vec{Z}_3}, \dots, \frac{\vec{I}_n}{\vec{Z}_n} \right] \vec{Z}_T = \frac{1}{V_s} (\vec{I}_c) = \frac{1}{V_s} (\vec{I}_1 + \vec{I}_2 + \vec{I}_3 + \dots + \vec{I}_n)$$

$$\frac{1}{\vec{Z}_T} = \frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3} + \dots + \frac{1}{\vec{Z}_n}$$

$$\frac{1}{\vec{Z}_T} = \vec{Y}_T$$

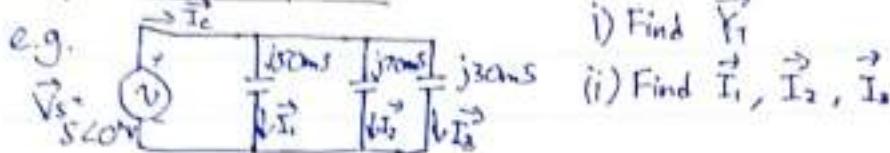
$$\vec{Y}_T = \vec{Y}_1 + \vec{Y}_2 + \vec{Y}_3 + \dots + \vec{Y}_n$$

$$\text{Chris's Law} \rightarrow \vec{V}_T = \frac{\vec{I}_c}{V_s} = \frac{\vec{I}_c}{\vec{V}_s} \angle 90^\circ \Omega = jB_T S.$$

B_T = total susceptance

Capacitors

Capacitors in parallel



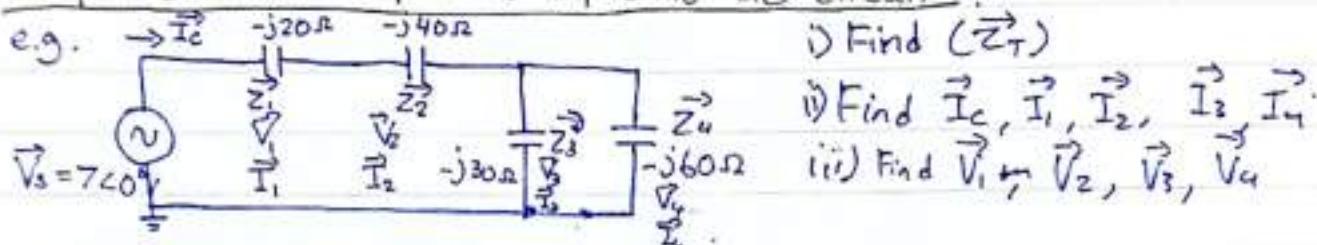
$$\text{i)} \vec{Y}_T = j50mS + j70mS + j30mS = j150mS,$$

$$\text{ii)} \vec{I}_1 = \vec{V}_s \times \vec{Y}_1 = 5\angle 0^\circ \times 150 \angle 90^\circ mS = 750 \angle 90^\circ mA,$$

$$\vec{I}_2 = \vec{V}_s \times \vec{Y}_2 = 5\angle 0^\circ \times 70 \angle 90^\circ mS = 350 \angle 90^\circ mA,$$

$$\vec{I}_3 = \vec{V}_s \times \vec{Y}_3 = 5\angle 0^\circ \times 30 \angle 90^\circ mS = 150 \angle 90^\circ mA,$$

V-I phase relationship in a capacitive ac circuit



$$\text{i)} \vec{Z}_1 + \vec{Z}_2 = -j20 + -j40 = -j60 \Omega$$

$$\vec{Z}_3 + \vec{Z}_4 = \frac{1}{-j10} + \frac{1}{-j60} = \frac{1}{Z_{34}}, Z_{34} = -j20 \Omega$$

$$\vec{Z}_T = -j60 + -j20 = -j80 \Omega,$$

$$\text{ii)} \vec{I}_c = \frac{\vec{V}_s}{\vec{Z}_T} = \frac{7\angle 0^\circ V}{-j80 \Omega} = 0.0875 \angle 90^\circ A = 8.75 \angle 90^\circ mA,$$

$$\vec{I}_1 = \vec{I}_2 = 8.75 \angle 90^\circ mA,$$

$$\vec{I}_3 = 8.75 \angle 90^\circ \times \frac{-j20}{-j60 \times -j30} = 58.33 \angle 90^\circ mA,$$

$$\vec{I}_4 = 8.75 \angle 90^\circ \times \frac{-j30}{-j30 + j60} = 29.17 \angle 90^\circ mA,$$

$$\text{iii)} \vec{V}_1 = V_s \times \frac{\vec{Z}_1}{\vec{Z}_T} = 7\angle 0^\circ \times \frac{-j20}{-j80} = 1.75 \angle 0^\circ V,$$

$$\vec{V}_2 = V_s \times \frac{\vec{Z}_2}{\vec{Z}_T} = 7\angle 0^\circ \times \frac{-j40}{-j80} = 3.5 \angle 0^\circ V,$$

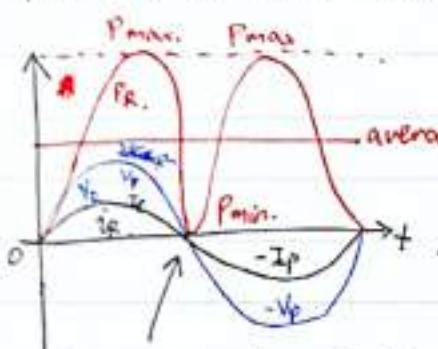
$$\vec{V}_3 = \vec{V}_4 = V_s \times \frac{\vec{Z}_{34}}{\vec{Z}_T} = 7\angle 0^\circ \times \frac{-j20}{-j80} = 1.75 \angle 0^\circ V,$$

Types of AC capacitive circuit:

1) Instantaneous power, 2) True Power, 3) Reactive power.

1) Instantaneous power.

- In a resistive circuit, voltage (V_R) current (I_R) vary from zero to their respective positive peaks (V_p and I_p) and negative peaks ($-V_p$ and $-I_p$).
- At $V_p \& I_p$, power $\rightarrow P_{max}$.
- At $-V_p \& -I_p$, power $\rightarrow P_{min}$.



Instantaneous power (P_i) = 0, when either $V_R / I_R = 0$ when voltage & current are at zero point.

$$\begin{aligned} - P_{avg} &= \frac{1}{T} \int_0^T V_R i_R dt \Rightarrow \frac{V_p I_p}{2} = \frac{V_p}{\sqrt{2}} \frac{I_p}{\sqrt{2}} = V_{rms} I_{rms} \rightarrow (V_p \times I_p) = P_{max} \\ - P_{avg} &= \frac{V_p I_p}{2} = \frac{P_{max}}{2} / V_{rms} \times I_{rms} \end{aligned}$$

2) True power

- P_{avg} can also be called the real and true power.

- Symbol of P_{true} .

- Unit of W

- Only resistors and components that contain resistance dissipate real power.

Apparent Power.

$$\begin{aligned} - V_{s(rms)} \times I_{s(rms)} &= \text{apparent power.} \\ - \text{Symbol } S &= V_s \times I_s \\ - \text{Unit in volt - ampere or VA.} \end{aligned} \quad \left. \begin{array}{l} \text{When ac receiving power,} \\ \text{it draws current.} \end{array} \right\}$$

Power factor.

- Apparent power \rightarrow used to provide power in ac circuit.

- The useful power \rightarrow true power.

- Power factor (pf) is defined as the ratio of true power (P) to apparent power (S) = $pf = \frac{P}{S}$ \rightarrow no unit. \rightarrow value of range $0 \leq pf \leq 1$.

- In a resistive ac circuit, $pf = 1 \rightarrow$ as the entire power entire apparent power is converted to true power ($S = P$), $pf = \frac{P}{P} = 1$,

Capacitors

$$P_{avg} = \frac{1}{T} \int_0^T V_c i_c dt = 0W.$$

- During positive half of the cycle \rightarrow power delivered to capacitor.
- During negative half of the cycle \rightarrow same amount of power is returned to the system.
- Hence the net power delivered to the capacitor over a cycle = 0.

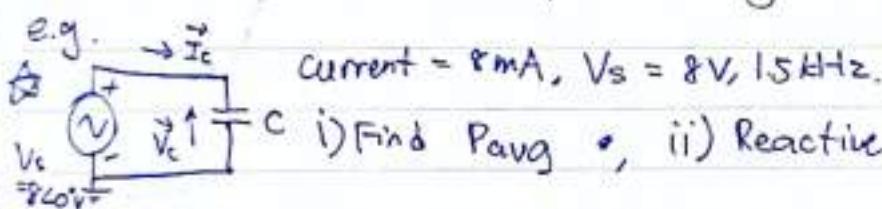
Reactive power

- The capacitor, like a resistor, still encounters the $V + I$ in an ac circuit.
- $V_{rms} \times I_{rms}$ \rightarrow power by the capacitor over a cycle.
- This is reactive power. \rightarrow symbol Q (or P_r)
 \rightarrow unit is volt-ampere-reactive (VAR)

cannot use W as it's not true/real power.

- Indication of the power shifted to and fro between the sources and the capacitor over a cycle.

\therefore Reactive power in a capacitive circuit \rightarrow $Q = V_{rms} I_{rms}$



i) Find P_{avg} , ii) Reactive power.

$$i) P_{avg} = 0W,$$

$$ii) Q = V_{rms} \times I_{rms} = 8V \times (8 \times 10^{-3})A = 64m\text{VAR}_{//}$$

reactance.

* Capacitor = 10Ω , $V_s = 5V$, 5kHz.

Find Reactive power

$$Q = V_{rms} I_{rms} = \frac{V_{rms}^2}{X_C} = \frac{5^2}{10} = 2.5 \text{ VAR}_{//}$$

* Capacitor = 22nF , $V_s = 10V$, 2000Hz

i) Find true power ii) reactive power.

i) True power (P) = 0W,

ii) $X_C = \frac{1}{2\pi \times 2000 \times (22 \times 10^{-9})} \approx 361752$

$$Q = V_{rms} I_{rms} = \frac{V_{rms}^2}{X_C} = \frac{10V^2}{361752} = 0.0276 \text{ VAR} = 27.6m\text{VAR}_{//}$$

* In purely capacitive ac circuit, there is no power dissipation.

* $\text{pf} = \frac{Q}{S} = 0 \rightarrow$ power factor.

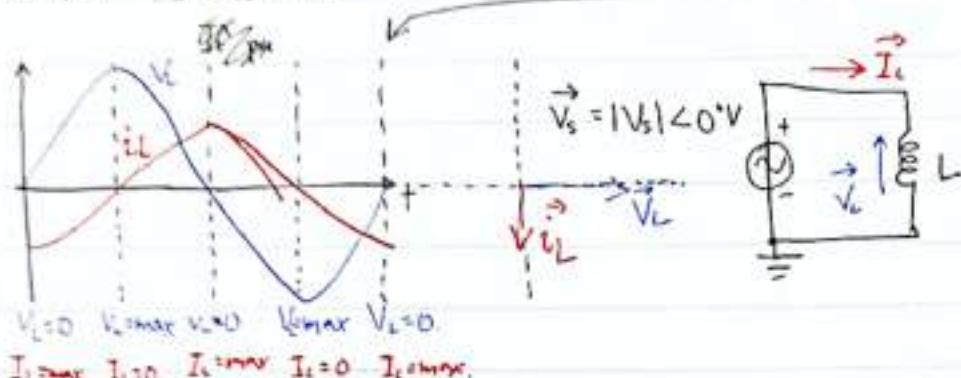
* The entire apparent power is converted to reactive power.

$$P = 0W$$

$$S = Q$$

Inductors

- In a purely inductive circuit, \vec{V}_L leads \vec{I}_L by 90°
- $V_L = L I_p \cos(\phi)$
- Inductive impedance is given by: $Z_L = X_L \angle 90^\circ = jX_L$
where $X_L = 2\pi f L$



- Though it's not a resistance, the ratio does express the opposition to current flow in an ac circuit. \rightarrow the inductive ac circuit, the ratio is termed impedance, Z_L .
- Its magnitude is ~~equal~~ the reactance $X_L \rightarrow \frac{|V_L|}{|I_L|} \Omega$; impedance angle $\phi_L = 90^\circ$

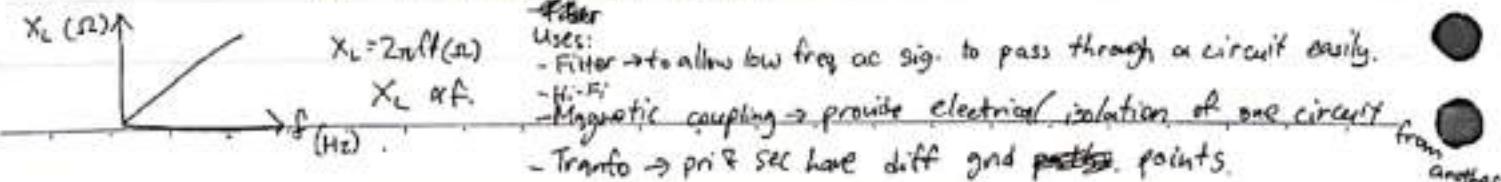
$$\therefore X_L = 2\pi f L \Omega$$

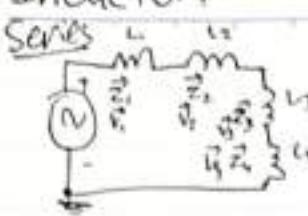
e.g.
Find impedance?
 $L = 660 \text{ mH} = 660 \times 10^{-3} \text{ H}$.
 $\therefore \text{Impedance} = 4.15 \angle 90^\circ \Omega$

$$\text{Reactance } X_L = 2\pi \times 3000 \times 660 \times 10^{-3} \Omega = 4.15 \Omega \quad j4.15 \Omega$$

Find I_L ?
 $Z_L = 2\pi \times 10000 \times 47 \times 10^{-3} \angle 90^\circ \Omega = 2953 \angle 90^\circ \Omega$
 $\vec{I}_L = \frac{\vec{V}_s}{Z_L} = \frac{6 \angle 0^\circ}{2953 \angle 90^\circ}$
 $= 2.03 \angle -90^\circ \text{ mA}$
or $j2.03 \text{ mA}$

Find L ?
 $I_L = 5 \text{ mA}, Z_L = \frac{V_s}{I_L} = \frac{10 \angle 0^\circ}{5 \angle -90^\circ} = 2 \angle 90^\circ \text{ k}\Omega$
 $X_L = 2\pi \times 6 \times 10^{-3} \times L = 2 \text{ k}\Omega$
 $\Rightarrow L = \frac{2 \times 10^3}{2\pi \times 6 \times 10^{-3}} \text{ H} = 0.053 \text{ H} = 53 \text{ mH}$



Inductors

$$Z_T = L_1 + L_2 + L_3 + L_4 + \dots + L_n$$

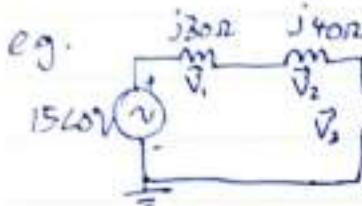
$$Z_T = \frac{1}{I_s} (\vec{V}_s) \quad (V_s = V_1 + V_2 + V_3 + \dots + V_n)$$

$$\vec{Z}_T = Z_1 + Z_2 + Z_3 + Z_4 + \dots + Z_n$$

$$\vec{Z}_T = \frac{\vec{V}_s}{I_s} = |V_s| \angle 90^\circ \Omega \quad [jX + \Omega]$$

$$\vec{Z}_T = jX_1 + jX_2 + \dots + jX_n$$

$$= Z_1 \quad = Z_2 \quad = Z_n$$



$$i) \text{Find } \vec{Z}_T$$

$$ii) \text{Find } \vec{V}_1, \vec{V}_2, \vec{V}_3$$

$$iii) \vec{V}_T = j30 + j40 + j50$$

$$= j120 \Omega //$$

$$i) \vec{V}_1 = V_s \times \frac{\vec{Z}_1}{\vec{Z}_T} = 15 \angle 0^\circ \times \frac{j30}{j120} = 3.75 \angle 0^\circ$$

$$\vec{V}_2 = V_s \times \frac{\vec{Z}_2}{\vec{Z}_T} = 15 \angle 0^\circ \times \frac{j40}{j120} = 5 \angle 0^\circ \angle 0^\circ$$

$$\vec{V}_3 = V_s \times \frac{\vec{Z}_3}{\vec{Z}_T} = 15 \angle 0^\circ \times \frac{j50}{j120} = 6.25 \angle 0^\circ \angle 0^\circ$$

Parallel

$$L_T = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}$$

$$\frac{1}{Z_T} = \frac{1}{V_s} (\vec{I}_s) \quad (\vec{I}_s = I_1 + I_2 + I_3 + \dots + I_n)$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots + \frac{1}{Z_n}$$

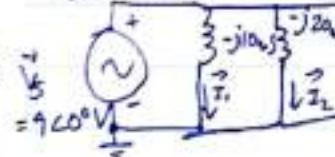
$$\text{Admittance. } Y_T = \frac{1}{Z_T}$$

$$\vec{Y}_T = \vec{Y}_1 + \vec{Y}_2 + \vec{Y}_3 + \dots + \vec{Y}_n$$

$$\vec{Y}_T = \frac{\vec{I}_s}{V_s} = \left| \frac{I_s}{V_s} \right| \angle -90^\circ S = -jB_T S. \quad \text{Susceptance } \rightarrow B.$$

$$\text{Hence. } \vec{Y}_T = -jB_T = -jB_1 + -jB_2 + -jB_3 + \dots + -jB_n$$

$$\text{e.g. } \rightarrow \vec{I}_s \quad \text{Find i) } \vec{Y}_T; \text{ ii) } \vec{I}_1, \vec{I}_2, \vec{I}_3, \vec{I}_4$$



$$i) \vec{Y}_T = -j10 + j20 - j50 = -j80 \text{ mS} //$$

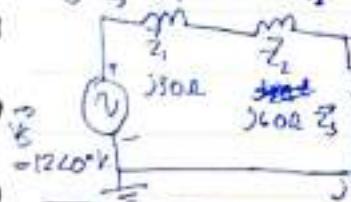
$$ii) \vec{I}_1 = V_s \times \vec{Y}_1 = (9 \angle 0^\circ) \times (10 \angle -90^\circ \text{ mS}) = 90 \angle -90^\circ \text{ mA},$$

$$\vec{I}_2 = V_s \times \vec{Y}_2 = (9 \angle 0^\circ) \times (20 \angle -90^\circ \text{ mS}) = 180 \angle -90^\circ \text{ mA},$$

$$\vec{I}_3 = V_s \times \vec{Y}_3 = (9 \angle 0^\circ) \times (50 \angle -90^\circ \text{ mS}) = 450 \angle -90^\circ \text{ mA},$$

$$\text{e.g. } \vec{I}_s, \vec{V}_1, \vec{I}_1, \vec{V}_2, \vec{I}_2, \vec{V}_3, \vec{I}_3, \vec{V}_4, \vec{I}_4$$

$$i) \text{Find } Z_T \quad ii) \vec{I}_s, \vec{I}_1, \vec{I}_2, \vec{I}_3, \vec{I}_4, \vec{V}_1, \vec{V}_2, \vec{V}_3, \vec{V}_4.$$



$$i) Z_{T2} = j30 + j60 = j90.$$

$$Z_{T34} = \frac{1}{j20} + \frac{1}{j30} = -j0.08333; Z_{T4} = \frac{1}{j30} = \frac{1}{-j0.08333} = j12.02 \Omega$$

$$Z_T = j90 + j12 = j102 \Omega //$$

$$iii) \vec{V}_1 = V_s \times \frac{\vec{Z}_1}{\vec{Z}_T} = (12 \angle 0^\circ) \times \frac{j30}{j102} = 3.53 \angle 0^\circ \text{ V} //,$$

$$\vec{V}_2 = V_s \times \frac{\vec{Z}_2}{\vec{Z}_T} = (12 \angle 0^\circ) \times \frac{j60}{j102} = 7.06 \angle 0^\circ \text{ V} //,$$

$$\vec{V}_3 = \vec{V}_4 = V_s \times \frac{\vec{Z}_3}{\vec{Z}_T} = (12 \angle 0^\circ) \times \frac{j12}{j102} = 1.41 \angle 0^\circ \text{ V} //,$$

$$i) \vec{I}_s = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 + \vec{I}_4 = 117.6 \angle 90^\circ + 117.6 \angle 90^\circ + 70.6 \angle 90^\circ + 1.41 \angle 0^\circ \text{ mA} //,$$

$$ii) \vec{I}_1 = \vec{I}_2 = \vec{I}_3 = \vec{I}_4 = 117.6 \angle 90^\circ \text{ mA} //,$$

$$iii) \vec{I}_1 = \vec{I}_2 = \vec{I}_3 = \vec{I}_4 = 117.6 \angle 90^\circ \times \frac{j30}{j102} = 35.83 \angle 90^\circ \text{ mA} //,$$

$$iv) \vec{I}_1 = \vec{I}_2 = \vec{I}_3 = \vec{I}_4 = 117.6 \angle 90^\circ \times \frac{j60}{j102} = 70.6 \angle 90^\circ \text{ mA} //,$$

$$v) \vec{I}_1 = \vec{I}_2 = \vec{I}_3 = \vec{I}_4 = 117.6 \angle 90^\circ \times \frac{j12}{j102} = 1.41 \angle 0^\circ \text{ mA} //,$$

$$vi) \vec{I}_1 = \vec{I}_2 = \vec{I}_3 = \vec{I}_4 = 117.6 \angle 90^\circ - 70.6 \angle 90^\circ = 47 \angle 90^\circ \text{ mA} //,$$

$$vii) \vec{I}_1 = \vec{I}_2 = \vec{I}_3 = \vec{I}_4 = 117.6 \angle 90^\circ - 1.41 \angle 0^\circ = 117.59 \angle 90^\circ \text{ mA} //,$$

RC Circuits.

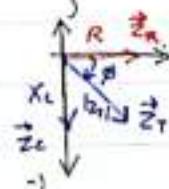
$$V_s = |V_s| \angle \phi V$$

$$\begin{aligned} Z_R &= R \\ Z_C &= -jX_C \\ Z_T &= Z_R + Z_C \\ \therefore Z_T &= R - jX_C \\ Z_T &= |Z_T| \angle \phi \\ |Z_T| &= \sqrt{R^2 + X_C^2} \\ \phi' &= \tan^{-1}\left(\frac{-X_C}{R}\right) = -\tan^{-1}\left(\frac{|X_C|}{R}\right) \\ \text{As } \phi' \rightarrow \text{negative value} \\ \delta \geq \phi' \geq -90^\circ \end{aligned}$$

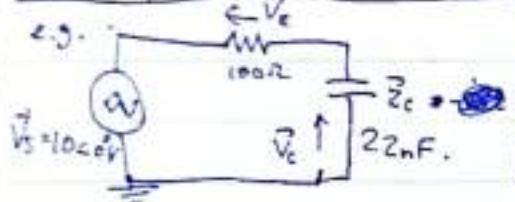
e.g. Find Z_T ?

$$\begin{aligned} V_1 &= 100V \\ Z &= 2\pi f C \\ \text{Reactance } X_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 4 \times 10^{-6}} \\ &= 20\Omega \\ \text{Capacitive Impedance } Z_C &= -jX_C = -j20\Omega \\ Z_T &= R + Z_C = 100 - j20\Omega \end{aligned}$$

$$\begin{aligned} Z_R &= R \angle 0^\circ \Omega \\ Z_C &= X_C \angle -90^\circ \Omega \\ \therefore Z_T &= \sqrt{R^2 + X_C^2} \angle \phi \Omega \end{aligned}$$



$$V_R = \frac{R}{Z_T} V_s ; V_C = \frac{Z_C}{Z_T} V_s$$



Determine V_C a) $f = 1000$ Hz

$$\begin{aligned} b) &= 500 \text{ kHz} \\ b) X_C &= \frac{1}{2\pi \times 500 \times 10^3 \times 22 \times 10^{-9}} \\ &= 14.5 \Omega \end{aligned}$$

$$Z_T = (100 - j14.5) \Omega$$

$$V_C = \frac{X_C}{|Z_T|} = \frac{14.5}{\sqrt{100^2 + 14.5^2}} \times 10V$$

$$V_C \approx 14.3V \text{ or } 14.3\% \text{ of } V_s$$

$$\begin{aligned} R &= |Z_T| \cos \phi \\ X_C &= |Z_T| \sin \phi \end{aligned}$$

e.g. $Z_T = 100 \angle -11.3^\circ \Omega$

$$\begin{aligned} a) X_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 22 \times 10^{-9}} \\ &= 7234 \Omega \end{aligned}$$

$$Z_T = (100 - j7234) \Omega$$

$$VDR \Rightarrow V_C = \frac{X_C}{|Z_T|} V_s$$

$$= \frac{7234}{\sqrt{100^2 + 7234^2}} (10V)$$

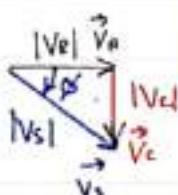
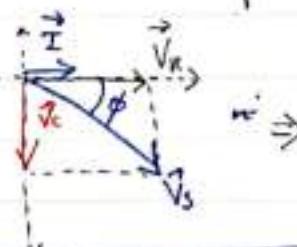
$$V_C \approx 10V = V_s$$

$$\begin{aligned} I &= |I| \angle 0^\circ A \\ V_R &= |V_R| \angle 0^\circ V \\ Z_C, V_C &= |V_C| \angle -90^\circ V \end{aligned}$$

$$\begin{aligned} |V_s| &= \sqrt{|V_R|^2 + |V_C|^2} \\ \phi &= -\tan^{-1}\left(\frac{|V_C|}{|V_R|}\right) \end{aligned}$$

The phasors

$$V_s = |V_s| \angle \phi V \quad \text{Source voltage.}$$



$$\begin{aligned} |V_R| &= |V_s| \cos \phi \\ |V_C| &= |V_s| \sin \phi \end{aligned}$$

e.g. $I = 0.2 \angle 0^\circ A$

$$\begin{aligned} V_R &= I \times R = (0.2 \angle 0^\circ A)(100) \\ &= 20 \angle 0^\circ V \end{aligned}$$

$$\begin{aligned} V_C &= I \cdot (X_C \angle -90^\circ) \\ &= (0.2 \angle 0^\circ A)(20 \angle -90^\circ \Omega) \\ &= 4(0^\circ \angle 90^\circ) = 4 \angle -90^\circ V \end{aligned}$$

$$V_s = |V_s| \neq \phi V$$

$$\begin{aligned} |V_s| &= \sqrt{|V_R|^2 + |V_C|^2} = \sqrt{20^2 + 4^2} \\ &= 20.4V \end{aligned}$$

$$\phi = \tan^{-1}\left(\frac{4}{20}\right) = -11.3^\circ$$

$$\therefore V_s = 20.4 \angle -11.3^\circ V$$

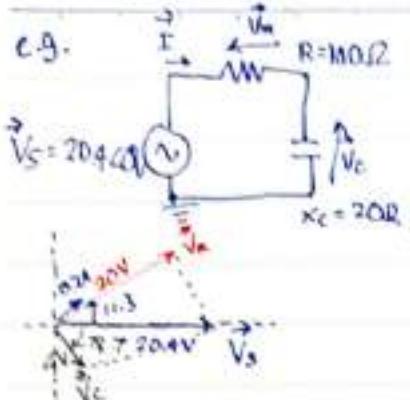
$$\text{Impt} \rightarrow (X_C < -90^\circ) \quad \vec{V}_c = \vec{I} (X_C < -90^\circ)$$

No. PEEE-15.

\Rightarrow Series RC has a leading pf.

Date 16.2.22

RC circuit



Find \vec{I} , \vec{V}_R , \vec{V}_C & sketch the phasor diagram (with V_s)

$$Z_T = |Z_T| \angle \phi \text{ rad} ; \quad |Z_T| = \sqrt{|R|^2 + |X_C|^2} = \sqrt{100^2 + 20^2} = 102 \Omega$$

$$\phi = -\tan^{-1}\left(\frac{X_C}{R}\right) = -\tan^{-1}\left(\frac{20}{100}\right) = -11.3^\circ \therefore Z_T = 102 \angle -11.3^\circ \Omega.$$

$$I = \frac{V_s}{Z_T} = \frac{20\angle 45^\circ}{102} \angle b - (-11.3^\circ) = 0.2 \angle 11.3^\circ A_{\parallel}$$

$$V_R = I \times R = (0.2 \angle 11.3^\circ) \times (100 \Omega) = 20 \angle 11.3^\circ V_{\parallel}$$

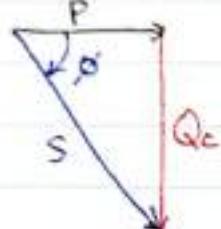
$$V_C = I \times (X_C \angle -90^\circ) = (0.2 \angle 11.3^\circ) \times (20 \angle -90^\circ) = 4 \angle -78.7^\circ$$

$$= I^2 |Z_T|$$

3 types of power: 1. Apparent power $\rightarrow S = |I| \times |V_s| \rightarrow$ unit VA

2. Reactive power $\rightarrow Q_c = |I|^2 \times X_c \rightarrow$ unit VAR.

3. Real power $\rightarrow P = I^2 R \rightarrow$ unit Watt.



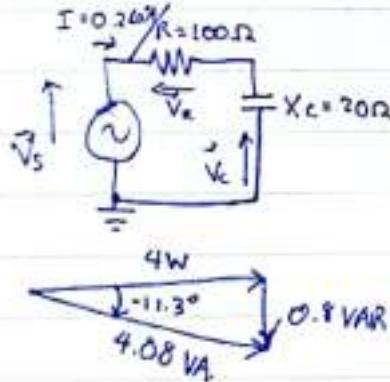
$$P = S \cos \phi \quad Q_c = S \sin \phi \quad S = \sqrt{P^2 + Q_c^2}$$

\hookrightarrow It is also the avg power over a cycle

$$\text{pf} = \frac{P}{S} = \frac{S \cos \phi}{S} = \cos \phi \rightarrow \text{no unit} \rightarrow \text{it is leading.}$$

$$\phi = -\tan^{-1}\left(\frac{Q_c}{P}\right)$$

e.g. Sketch the power triangle and find pf.



$$P = I^2 R = 0.2^2 \times 100 = 4 W$$

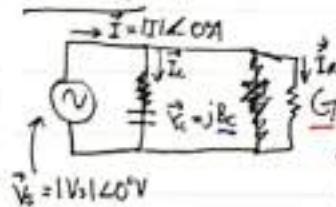
$$Q_c = I^2 X_C = 0.2^2 \times 20 = 0.8 \text{ VAR}$$

$$S = I^2 |Z_T| = (0.2^2) (\sqrt{100^2 + 20^2}) = 4.08 \text{ VA}$$

$$\phi = -\tan^{-1}\left(\frac{Q_c}{P}\right) = -\tan^{-1}\left(\frac{0.8}{4}\right) = -11.3^\circ$$

$$\text{pf} = \cos(-11.3^\circ) = 0.981 \text{ (leading)}$$

Parallel.



Susceptance $B_C = \frac{1}{X_C} \rightarrow$ unit is SIEMEN.

Conductance $G = \frac{1}{R} \rightarrow$ unit is SIEMEN

$$Y_T = |Y_T| \angle \phi \leftarrow \text{positive value} \quad 0^\circ \leq \phi \leq 90^\circ$$

$$\sqrt{G^2 + B_C^2} \quad \tan^{-1}\left(\frac{B_C}{G}\right)$$

$$I_C = Y_C V_s$$

$$\text{Admittance of } R \rightarrow \vec{Y}_R = G \angle 0^\circ S$$

$$\text{Admittance of } C \rightarrow \vec{Y}_C = B_C \angle 90^\circ S$$

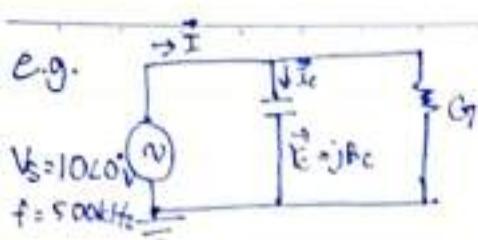
$$\text{Total admittance} \rightarrow \vec{Y}_T = \sqrt{G^2 + B_C^2} \angle \phi S$$



$$G = |Y_T| \cos \phi$$

$$B_C = |Y_T| \sin \phi$$



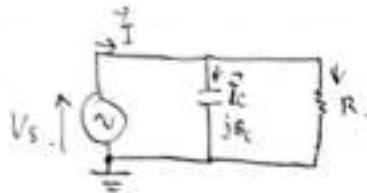


$$R = 100 \Omega, C = 22 \text{nF} \rightarrow \text{find } \vec{I}_c$$

$$\omega_c = 2\pi f C = 2\pi (500,000) \times (22 \times 10^{-9} \text{S}) = 69.1 \text{mS}$$

~~∴ $\vec{I}_c = \vec{V}_c \times \vec{G} = 69.1 \text{mS} \times 10 \angle 0^\circ \text{V} = 691 \angle 90^\circ \text{mA}$~~

$$\vec{I}_c = \vec{V}_c \vec{V}_s = (69.1 \text{mS}) \times (10 \angle 0^\circ \text{V}) = 691 \angle 90^\circ \text{mA}$$



$$\vec{V}_s = |V_s| \angle 0^\circ \text{V}$$

$$\vec{I} = |I| \angle 0^\circ \text{A}$$

$$\therefore \vec{I}_R / G = |I_R| \angle 0^\circ \text{A}$$

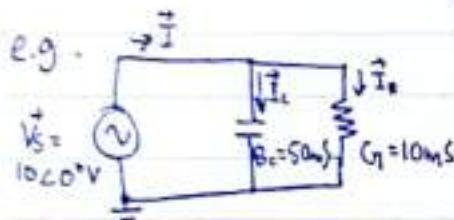
$$\vec{I}_c / j\omega_c = |I_c| \angle -90^\circ \text{A}$$

$$|I| = \sqrt{|I_R|^2 + |I_c|^2}$$

$$\phi = \tan^{-1} \left(\frac{|I_c|}{|I_R|} \right) = \tan^{-1} \left(\frac{\omega_c}{G} \right)$$

$$|I_R| = |I| \cos \phi$$

$$|I_c| = |I| \sin \phi$$



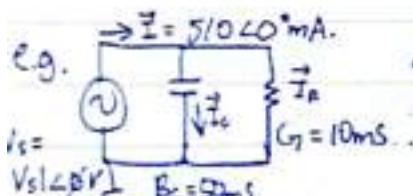
Find $\vec{I}_R, \vec{I}_c, \vec{I}$

$$\vec{I}_R = \vec{V}_s \times G = (10 \angle 0^\circ) \times (10) = 100 \angle 0^\circ \text{mA}_{\parallel}$$

$$\vec{I}_c = \vec{V}_s \times (B_c \angle 90^\circ) = (10 \angle 0^\circ) \times (50 \angle 90^\circ \text{mS}) = 500 \angle 90^\circ \text{mA}_{\parallel}$$

$$|I| = \sqrt{|I_R|^2 + |I_c|^2} = \sqrt{100^2 + 500^2} = 510 \text{mA}$$

$$\phi = \tan^{-1} \left(\frac{|I_c|}{|I_R|} \right) = \tan^{-1} \left(\frac{500}{100} \right) = 78.7^\circ \quad \therefore \vec{I} = 510 \angle 78.7^\circ \text{mA}_{\parallel}$$



$$\therefore \vec{V}_T = |V_T| \angle \phi \text{V}$$

$$|V_T| = \sqrt{|G|^2 + |B_c|^2} = \sqrt{10^2 + 50^2 \text{mS}} = 51 \text{mS} \quad \therefore \vec{V}_T = 51 \angle 78.7^\circ \text{mS}$$

$$\phi = \tan^{-1} \left(\frac{B_c}{G} \right) = \tan^{-1} \left(\frac{50}{10} \right) = 78.7^\circ$$

$$\vec{V}_s = \frac{\vec{I}}{V_T} = \frac{510}{51} \angle (0^\circ - 78.7^\circ) \text{V} = 10 \angle -78.7^\circ \text{V}_{\parallel}$$

$$\vec{I}_R = \vec{V}_s \times G = (10 \angle -78.7^\circ) \times (10) = 100 \angle -78.7 \text{mA}_{\parallel}$$

$$\vec{I}_c = \vec{V}_s \times (B_c \angle 90^\circ) = (10 \angle -78.7^\circ) \times (50 \angle 90^\circ \text{mS})$$

$$= 500 \angle 11.3 \text{mA}_{\parallel}$$

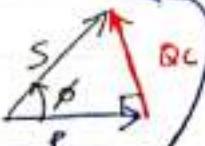
also absorbed in circuit.

In parallel \rightarrow real power is dissipated as heat in Resistor.

\rightarrow real power ~~losses~~ also known as active power / true power.

$$\begin{aligned} P &= S \cos \phi \\ Q_c &= S \sin \phi \end{aligned}$$

$$S = \sqrt{P^2 + Q_c^2}$$



$$S = V_s^2 / Y_T / R$$

$$P = V_s^2 R$$

$$Q_c = V_s^2 B_c$$

$$I = V_s / Y_T$$

$$I_c = V_s B_c$$

$$I_R = V_s G$$

$$R \rightarrow G, |Y_T| = \sqrt{G^2 + B_c^2}$$

$$PF = \frac{P}{S} = \frac{G}{Y_T} = \frac{S \cos \phi}{S} = \cos \phi$$

RL Circuitseries

$$\vec{Z}_T = R + jX_L$$

$$R + jX_L \angle 90^\circ$$

$$PF = \frac{P}{S} = \frac{R}{R+jX_L} = \cos\phi$$

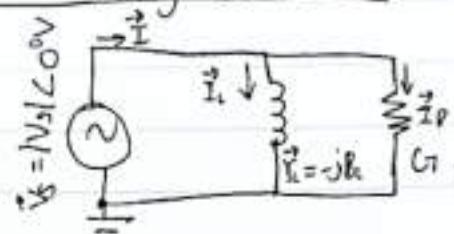
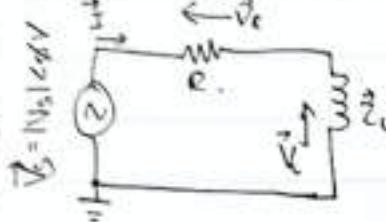
parallel

$$\vec{Y}_T = G - jB_L$$

$$= G - B_L \angle -90^\circ$$

power factor is always lagging. \leftarrow since I is lagging V_s by ϕ

POWER \rightarrow same like RC. \leftarrow just change C to L \leftarrow X_c become X_L.

RLC CircuitSeries:

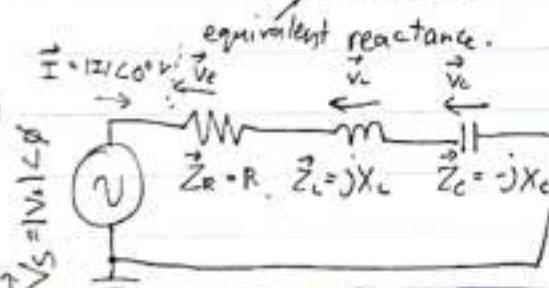
$$\vec{Z}_T = R + (X_L \angle 90^\circ) - (X_C \angle 90^\circ)$$

$$= R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$= X = X_L - X_C$$

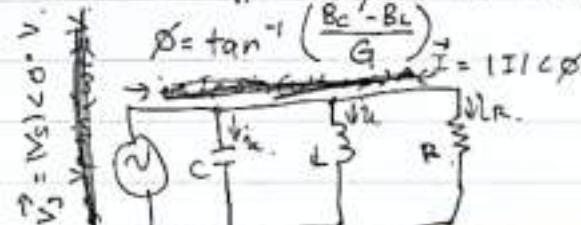
equivalent reactance.

parallel

$$\vec{Y}_T = G - jB_L + jB_C$$

$$\vec{Z}_T = \frac{1}{\vec{Y}_T} = \frac{1}{G-j(B_C-B_L)} = \vec{Z}_T \angle -\phi$$

$$\phi = \tan^{-1} \left(\frac{B_C - B_L}{G} \right)$$



(leading)

equivalent susceptance.

$$B = B_C - B_L$$

$$\vec{Z}_T = 1Z_1 \angle \phi$$

$$|Z_T| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

X_L > X_C \rightarrow $\phi + 90^\circ$ X_L < X_C \rightarrow $\phi - 90^\circ$

lagging

$$\vec{Y}_T = f |Y_T| \angle \phi$$

$$|Y_T| = \sqrt{G^2 + (B_C - B_L)^2}$$

$$\phi = \tan^{-1} \left(\frac{B_C - B_L}{G} \right)$$

$$B_C > B_L \rightarrow \phi + 90^\circ$$

$$B_C < B_L \rightarrow \phi - 90^\circ$$

lagging

$$\vec{Y}_R = G \angle 0^\circ S$$

$$\vec{Y}_L = B_L \angle -90^\circ S$$

$$\vec{Y}_C = B_C \angle 90^\circ S$$

$$S = \sqrt{P^2 + (Q_C - Q_L)^2}$$

$$P = S \cos \phi$$

$$Q_L = S \sin \phi$$

$$\vec{Z}_R = R \angle 0^\circ \Omega$$

$$\vec{Z}_L = X_L \angle 90^\circ \Omega$$

$$\vec{Z}_C = X_C \angle -90^\circ \Omega$$

find pf

leading

$$Q_L > Q_C \rightarrow S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$P = S \cos \phi$$

$$Q = S \sin \phi$$

$$Q_L > Q_C \rightarrow S = \sqrt{P^2 + (Q_C - Q_L)^2}$$

$$P = S \cos \phi$$

$$Q = S \sin \phi$$

(C-L) when C > L.

(L-C) when L > C.

(C-L)

The Power \rightarrow same as RC & RLC \leftarrow just change C or L to ~~(C-L)~~ (L-C)