How does MixSIAR treat source data?

A key advance of Bayesian mixing models is that they account for uncertainty not only in the mixture data, but also in the sources (Moore and Semmens 2008). In both cases, the fitted source parameters μ^s_{jk} and Σ^s_k are used to construct the mixture likelihood for consumer i and tracer j:

$$Y_{ij} \sim MVN\left(\sum_k p_k \mu^s_{jk}, \Sigma\right),$$

with the source covariance, Σ^s_k, informing the mixture covariance (exactly how depends on which error structure option you've chosen).

The best way to fit the source parameters μ^s_{jk} and Σ^s_k depends on the type of source data you have.

Raw source data:

Best case is you have raw source data - then there is information to fit the covariance between tracers/isotopes. The data for source k, tracer j, Y^s_{jk}, are fit hierarchically:

$$Y^s_{jk} \sim MVN\left(\mu^s_{jk}, \Sigma^s_k\right),$$

where the priors for the source means, μ^s_{jk}, are:

$$\mu^s_{jk} \sim \mathcal{N}(0, .001)$$

The source covariance matrix, Σ^s_k, is constructed as in Hopkins & Ferguson (2012):

Uninformative priors on source precisions:

$$\tau_{jk} \sim gamma(.001, .001);$$

Uninformative priors on source correlations:

$$\rho_{ij} \sim \mathcal{U}(-1, 1),$$

$$\rho_{ji} = \rho_{ij},$$

$$\rho_{ii} = 1$$

Summary statistics only (no covariance):

If you don't have raw source data and only provide summary statistics (mean, variance, and sample size), we cannot fit the above model with covariance. Instead, we fit the source parameters μ^s_{jk} and Σ^s_k as in Ward et al. (2010):

$$\mu_{jk} \sim \mathcal{N}\left(m_{jk}, n_k/s^2_{jk}\right),$$

$$tmp.X_{jk} \sim \chi^2(n_k),$$

$$\tau_{jk} = \frac{tmp.X_{jk}}{s^2_{jk}\left(n_k - 1\right)},$$

$$\Sigma^s_k = diag\left(\frac{1}{\tau_{jk}}\right),$$

where:
m_{jk} = tracer j sample mean for source k (data),

s_{jk}^2 = tracer j sample variance for source k (data),
n_k = source k sample size (data),
μ_{jk} = tracer j mean for source k (parameter),
τ_{jk} = tracer j precision for source k (parameter),
Σ_k^s = source k covariance matrix (calculated from τ_{jk} terms).

Then, μ_{jk}^s and Σ_k^s are used in the mixture likelihood as before.