

¹ The utility of spatial model-based estimators of
² unobserved bycatch: future or folly?

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¹²

¹³ **Abstract**

¹⁴ Quantifying effects of fishing on non-targeted (bycatch) species is an important management and conservation
¹⁵ issue. Bycatch estimates are typically calculated using data collected by on-board observers, but observer
¹⁶ programs are costly and therefore often only cover a small percentage of the fishery. The challenge is then to
¹⁷ estimate bycatch for the unobserved fishing activity. The status quo for most fisheries is to assume the ratio of
¹⁸ bycatch-to-effort is constant and multiply this ratio times the effort in the unobserved activity (ratio estimator).

¹⁹ We used a dataset with 100% observer coverage, 35,440 hauls from the U.S. West Coast groundfish trawl
²⁰ fishery, to evaluate the ratio estimator against methods that utilize fine-scale spatial information: generalized
²¹ additive models (GAMs) and random forests. Applied to 15 species representing a range of bycatch rates,
²² including spatial locations improved model predictive ability, whereas including effort-associated covariates
²³ generally did not. Random forests performed best for all species (lower root mean square error), but were
²⁴ slightly biased (overpredicting total bycatch). Thus, the choice of bycatch estimation method involves a
²⁵ tradeoff between bias and precision, and which method is optimal may depend on the species bycatch rate
²⁶ and how the estimates are to be used.

²⁷ **Keywords**

²⁸ bycatch estimation, fishing effort, ratio estimator, spatial model, GAM (generalized additive model), random
²⁹ forest, bias-variance tradeoff, U.S. West Coast groundfish fishery

³⁰ **Introduction**

³¹ The incidental bycatch of non-targeted species by fisheries in the US and around the world has been highlighted
³² as an issue of both conservation concern and fisheries inefficiency (Harrington *et al.*, 2005), and reducing or
³³ eliminating bycatch and incidental mortality is a goal of many fisheries around the world. There are several
³⁴ reasons why a species might be considered bycatch or discarded: the species may be of little or no commercial
³⁵ value, the species may be protected (e.g. marine mammals, turtles, birds), the species may be permitted to be
³⁶ caught but in a different fishery, or the quota for the targeted species in a given time period may be exceeded.
³⁷ In addition to making fisheries more efficient, reducing bycatch can have positive socioeconomic benefits to
³⁸ fishers. Two such examples include: fisheries remaining open longer (before bycatch quotas are met), and
³⁹ valuable stocks rebuilding more quickly due to reductions in take of overfished bycatch species (NMFS, 2016).

⁴⁰ Quantifying the amount of bycatch or discards for a given fishery can be challenging. One of the most reliable
⁴¹ sources of information is the use of onboard scientific observers. Because observer programs are typically
⁴² expensive, few fisheries around the world are able to maintain 100% observer coverage. Instead, a subset of
⁴³ fishing activities is typically monitored (trips, vessels). Assuming these observed units are representative
⁴⁴ of unobserved fishing, ratio estimators can be used to expand the observed bycatch ratio (i.e. the ratio of
⁴⁵ bycatch-to-effort) to the remainder of the fishery. In situations where bycatch rates are assumed to vary by
⁴⁶ strata (e.g. by season, depth, or latitude), the ratio estimator can be applied separately to each stratum and
⁴⁷ then summed to generate a total index of bycatch, $\sum_{s=1}^S \frac{d_s}{r_s} R_s$, where d_s is the observed bycatch or discards
⁴⁸ for stratum s , r_s is the retained observed catch for stratum s and R_s is the total landed catch in stratum
⁴⁹ s (Cochran, 1963). Importantly, ratio estimators do not incorporate a formal underlying statistical model
⁵⁰ (i.e. are free of any assumptions regarding data structure), and are thus sample-based estimators, rather than
⁵¹ model-based estimators (McCracken, 2000). These stratified ratio estimators have been widely used around
⁵² the world and applied to estimates of both discards (Anderson and Clark, 2003; Amandè *et al.*, 2010b) and
⁵³ protected species (Rogan and Mackey, 2007).

⁵⁴ Despite their widespread use, there are a number of potential issues in applying ratio estimators to enumerate
⁵⁵ fleetwide bycatch. First, using observed catches of target species or any other measure of effort implicitly
⁵⁶ makes an assumption about a linear relationship between non-target and target catches (Fonteneau and

57 Richard, 2003). This may be unrealistic, particularly as the distribution of catches of non-target species is
58 often zero-inflated, or has a small number of observations containing extremely high values (Ortiz and Arocha,
59 2004; Rochet and Trenkel, 2005). Second, for species with low bycatch rates in fisheries with low observer
60 coverage (i.e. rare-event bycatch), it is common for zero bycatch events to be observed in a given year (ratio
61 estimator = 0), and when bycatch events are observed, the ratio estimator often delivers implausibly high
62 estimates (McCracken, 2004; Martin *et al.*, 2015). Third, the boundaries of strata used in a ratio estimator
63 can be somewhat arbitrary whenever post-stratified boundaries are used (as is common in multispecies
64 sampling designs). A fourth and related point is that within each stratum, bycatch rates are assumed to be
65 uniform, while in reality they may vary by season, depth, or other factors.

66 One of the biggest questions related to bycatch estimation is whether model-based estimators that incorporate
67 explicit spatial information (beyond any implicit spatial information incorporated by strata) offer any
68 advantage over the widely used stratified ratio estimator. Like fishery independent catch per unit effort
69 (CPUE) data, fishery dependent bycatch patterns are spatially correlated (Lewison *et al.*, 2009). Accounting
70 for spatial correlation in model-based estimators has been extensively summarized in the geostatistical
71 literature (Grondona and Cressie, 1991; e.g. Brus and de Gruijter, 1997). Similar comparisons have recently
72 been applied to index standardization of fisheries survey data (Maunder and Punt, 2004). In the majority
73 of cases, spatially explicit model-based estimators have increased precision relative to simpler estimators
74 that assign observations to strata (Thorson and Ward, 2013; Shelton *et al.*, 2014; Thorson *et al.*, 2015).

75 There are a number of additional advantages of spatial models, including the ability to better quantify shifts
76 in distribution (Thorson *et al.*, 2016), and improved ability to identify fine scale hotspots of high bycatch
77 (Cosandey-Godin *et al.*, 2014; Ward *et al.*, 2015). While the majority of these recent analyses of fishery
78 dependent data have relied on parametric methods (delta-GLMM models; Thorson and Ward, 2013), semi-
79 or non-parametric models such as generalized additive models (GAMs) or random forests (RFs) have also
80 been used to include spatial variation (Winker *et al.*, 2013; Li *et al.*, 2015; Thorson *et al.*, 2015).

81 While recent work has included fishing location information in spatial model-based estimates of bycatch
82 (Orphanides, 2009), there is little guidance on how to model spatiotemporal variation, and how different
83 spatial modeling approaches compare in their bias or precision against the traditional ratio estimator. To
84 evaluate these different bycatch estimators, we developed a simulation study from observer data collected
85 from the West Coast Groundfish Observer Program (WCGOP) at the Northwest Fisheries Science Center.

86 While observers have been monitoring a portion of trips in the groundfish fishery since 2002, since 2011
87 regulations require an observer on every groundfish trip (100% coverage). Thus, years with 100% coverage
88 can be subsampled to generate smaller datasets that can be used to expand estimates to the fleet total,

and the relative performance of different methods can be compared because the true bycatch is known. We begin by using the entirety of the dataset to test the ratio estimator assumption of a linear relationship between bycatch and available metrics of fishing effort. Next, we use randomly generated subsamples of the observer data to evaluate (1) the relative performance of spatial model-based bycatch estimates against the conventional stratified ratio estimator, and (2) the sensitivity of model performance to varying levels of observer coverage.

Methods

Fisheries observer data

To evaluate the performance of ratio estimators versus spatial model-based estimates of fleet-wide bycatch, we used a dataset from the United States with 100% observer coverage, the West Coast Groundfish Observer Program (WCGOP) of the Northwest Fisheries Science Center (NWFSC, Bellman *et al.*, 2010). The WCGOP monitors commercial bottom trawls on the west coast of the USA, which primarily target groundfish such as Dover sole (*Microstomus pacificus*), thornyheads (*Sebastolobus spp.*), sablefish (*Anoplopoma fimbria*), and rockfish (*Sebastodes spp.*). The fishery moved to an individual fishing quota (IFQ) system with 100% observer coverage in 2011, and we used the 35,440 post-IFQ hauls (4,007 trips) observed from 2011-2015 in the area north of Cape Falcon, Oregon (45.77° N, Fig. 1). In 2015, a small portion of the fleet began experimenting with the use of electronic monitoring equipment in lieu of an observer. We excluded any such trips from our analysis. Observers recorded haul duration, location, date, time, depth, gear type, and at-sea catch including at-sea discarded bycatch (for details see NWFSC, 2016). Because fishermen are permitted to land a low quota of valuable non-target species under IFQ management, we only considered 15 species that are exclusively discarded and cover wide ranges of bycatch rates and levels of management concern (Table 1). Species such as Dungeness crab or Pacific halibut are of high value, but as each are permitted in other fisheries, they are considered bycatch in the groundfish fishery.

Relationship between bycatch and effort

While the stratified ratio estimator typically involves multiplying the bycatch-to-target catch ratio by the total target catch within strata, it is certainly possible to replace target catch with other metrics of effort, such as haul duration. This may be advantageous if a linear relationship exists between bycatch and haul duration, but not between bycatch and target catch. To investigate whether there was a linear relationship between bycatch and available metrics of fishing effort, retained catch of target species (kg) and haul duration

118 (minutes), we fit log-log linear models for each species:

$$\log(\text{Bycatch}) = \alpha + \beta \log(\text{Effort}) + \epsilon$$

119

$$\epsilon \sim \mathcal{N}(0, \sigma^2)$$

120 The slope term, β , of a log-log linear model is the exponent of an assumed power law, i.e:

$$\text{Bycatch} = e^\alpha \text{ Effort}^\beta e^\epsilon$$

121 Thus, if a linear relationship between bycatch and fishing effort exists, the power law exponent should equal
122 one ($\beta = 1$). Exponents greater than one ($\beta > 1$) imply positive concavity and exponents less than one
123 ($\beta < 1$) imply negative concavity, while $\beta = 0$ if no relationship exists.

124 **Simulation design**

125 We compared the performance of the stratified ratio estimator with two spatial modeling frameworks: GAM
126 and RF. All analyses were conducted using R v3.4.4 (R Core Team, 2018). We designed our data sub-sampling
127 experiment to calculate predictive performance by cross-validation. We generated 200 ‘training’ datasets with
128 reduced observer coverage (e.g. 20%, 40%), by sampling trips (collections of hauls) without replacement from
129 the complete dataset. We used trip as the cross-validation sample unit because this mirrors sampling schemes
130 in observer programs with less than 100% coverage (i.e. observers are placed on vessels on a trip-by-trip basis,
131 and then observe all hauls within the trip). These training datasets were generated once for all species, so
132 that models were evaluated against the same simulated datasets. Hauls from unobserved trips were then
133 used as the ‘test’ dataset to evaluate predictions. This repeated training/test split procedure is also known
134 as “leave-group-out cross-validation” or “Monte Carlo cross-validation,” and a set of 200 train/test splits is
135 recommended as a good sample size (Kuhn and Johnson, 2013).

136 **Status quo: ratio estimator**

137 We implemented the stratified ratio estimator as described in the Introduction and Bellman *et al.* (2010),
138 where observed estimates of bycatch in each strata are expanded based on the ratio of observed to total effort
139 (total target catch or haul duration) and total estimates are generated as sums over strata (Cochran, 1963).
140 An important note from a modeling perspective is that the ratio estimator is stratified by year (5 levels:
141 2011, 2012, 2013, 2014, 2015), season (two levels: summer, winter), depth (three levels: 0-125, 126-250, >

¹⁴² 250 fathoms), and bimonthly period (six levels: Jan-Feb, Mar-Apr, . . . , Nov-Dec). Any stratum with zero
¹⁴³ sampled bycatch is expanded to predict zero total bycatch in that stratum.

¹⁴⁴ **Spatial framework #1: generalized additive models (GAMs)**

¹⁴⁵ We fit GAMs using two alternative methods of accounting for zeros. Our first approach, the “GAM-Delta”
¹⁴⁶ model, partitioned the data into separate presence/absence (‘binomial’) and ‘positive’ components (a delta, or
¹⁴⁷ hurdle, model as in Pennington, 1983; Maunder and Punt, 2004). The GAM-Delta model estimates of total
¹⁴⁸ density were then calculated by multiplying the binomial and positive components (as in Lo *et al.*, 1992).
¹⁴⁹ The second approach, the “GAM-Tweedie” model, treats zero inflated catch data as arising from a Tweedie
¹⁵⁰ distribution with power parameter $1 < p < 2$, which is a compound Poisson process where catch is modeled
¹⁵¹ as the sum of N independent gamma random variables, with N following a Poisson distribution (Tweedie,
¹⁵² 1984). Assuming a Tweedie distribution is reasonable, as the haul catch (weight) can be thought of as a sum
¹⁵³ of the weight of N fish, where the weight of each fish is gamma-distributed (Candy, 2004). Importantly, this
¹⁵⁴ allows for hauls with zero catch, since N can be zero. We estimated the Tweedie power parameter, p , for each
¹⁵⁵ species outside the model using maximum likelihood, and then fit GAMs using these fixed, species-specific p
¹⁵⁶ values.

¹⁵⁷ We fit both the GAM-Delta and GAM-Tweedie models using the ‘mgcv’ library (v1.8-17, Wood, 2006) and
¹⁵⁸ the same covariates as the ratio estimator, adding a 2D thin plate regression spline on location:

$$\text{bycatch} \sim \text{year} + \text{season} + \text{bimonth} + \text{bimonth}^2 + \text{depth interval} + \text{s(lat, long, } k = 50\text{)}$$

¹⁵⁹ Tensor product splines were also considered for the 2D spline, since they are designed for cases where the scale
¹⁶⁰ differs in the two dimensions (as in our case, along- vs. cross-shore distance). We used thin plate regression
¹⁶¹ splines instead, however, because they had better predictive performance in preliminary testing. We used
¹⁶² the same factor covariates as the ratio estimator (fixed effects of year, season, bimonthly period, and depth
¹⁶³ interval) for two reasons. First, this offered a more direct comparison between the ratio estimator and GAMs.
¹⁶⁴ Second, this analysis aims to inform the process of producing yearly bycatch estimates for dozens of species
¹⁶⁵ in a highly multispecies trawl fishery, where lengthy model selection is impractical given current logistical
¹⁶⁶ constraints (Bellman *et al.*, 2010).

¹⁶⁷ We fit four variations of each GAM model to determine the effect of including location and effort on predictive
¹⁶⁸ performance: no effort or location, location only, effort only, and both location and effort.

169 **Spatial framework #2: random forests (RFs)**

170 Similar to the GAMs, we fit two random forest models. “RF-Delta” considered the binomial and positive
171 data independently and multiplied them together to calculate total bycatch density. “RF-Total” treated the
172 binomial and positive data as occurring from the same process in a single model.

173 We used ‘randomForest’ (v4.6-12, Liaw and Wiener, 2002) to fit the RFs, and we used the same covariates as
174 the ratio estimator and GAMs, plus linear and quadratic terms for location:

$$\text{bycatch} \sim \text{year} + \text{season} + \text{bimonth} + \text{bimonth}^2 + \text{depth interval} + \text{lat} + \text{lat}^2 + \text{lon} + \text{lon}^2$$

175 Since RFs are claimed to not overfit data (Breiman, 2001) and suffer less from incorporating numerous, possibly
176 correlated and uninformative covariates (Biau and Scornet, 2016), we fit a third RF model using all available
177 covariates without stratification. We expected this “RF-All” model to outperform the RF-Delta and RF-Total
178 models because, presumably, information is lost by not including covariates (haul number in trip, gear, time
179 of day) and stratifying depth (to depth interval), date (to season and bimonthly period), and location (areas
180 by latitude). We included day-of-year and hour-of-day as periodic functions (i.e. $\text{sinhour} = \sin\left[\frac{2\pi\text{hour}}{24}\right]$):

$$\text{bycatch} \sim \text{year} + \text{depth} + \text{haul number} + \text{gear} + \text{cosday} + \text{sinday} + \text{coshour} + \text{sinhour} + \text{lat} + \text{lat}^2 + \text{lon} + \text{lon}^2$$

181 **Model evaluation**

182 For each simulated dataset, we calculated model performance as root mean square error (RMSE) using the
183 predicted and observed bycatch. RMSE was calculated by year, and also averaged across years. As RMSE
184 can be expressed as the sum of variance and squared bias, we also generated estimates of the bias from each
185 prediction, in order to better understand the relative contributions to total RMSE (in other words, why some
186 models do better than others).

187 **Results**

188 **Weak relationship between effort and bycatch**

189 For nearly all of the 15 species included in our analysis, we found that relationships between bycatch and
190 effort (both target catch and haul duration) were either weak or nonlinear, as most power law exponents

191 from the log-log regression were much less than 1 ($25/30 < 0.5$, Figs. 2, 3, and S1). In only a few cases were
192 the estimated coefficients close to 1.0 (the relationship assumed when effort is included as an offset).

193 **Model comparison: RF had lower error but slight bias**

194 Compared to the ratio estimator, we found that the RF-Total model (not applying a hurdle or delta model)
195 produced estimates of total bycatch that had lower RMSE (26% lower averaged across species, Fig. 4). For
196 most species and years, median bycatch estimates from the ratio estimator and RF-Total were close to each
197 other and the true, observed bycatch, but the RF-Total model was more precise (Fig. 5). However, RF-Total
198 had higher bias compared to the ratio method (median percent error across all species and years: RF = 0.068,
199 Ratio = -0.011, Fig. 6). The GAM-Tweedie model appeared to have convergence issues for some simulations
200 in one-fifth of the species (Black skate, California slickhead, and Grenadier), but for the simulations that did
201 converge, it performed similarly to the ratio estimator (Fig. 4).

202 Though delta models have been widely used in the index standardization of fisheries data (Maunder and
203 Punt, 2004), both GAM and RF models with an aggregated response consistently outperformed delta models
204 (Fig. S2).

205 **Effect of including fishing effort and spatial locations**

206 We found minimal gain in predictive performance when fishing effort was included as a covariate. In all
207 models compared, any effect of effort was smaller than the effect of including spatial locations (Fig. S3). An
208 important difference between the GAM and RF models was that for many species, adding spatial locations
209 to GAMs led to worse predictions, while adding location information to the RF models either improved
210 predictions (especially for RF-Delta) or had no effect.

211 **Influence of data richness on model performance**

212 As expected, model performance improved for higher observer coverage (20% vs. 40%, Fig. S4). Averaged
213 across species, RF had markedly lower median RMSE than the ratio estimator. In fact, the RF models based
214 on 20% observer coverage (0.155 median RMSE) outperformed the ratio estimator based on 40% observer
215 coverage (0.180 median RMSE). Similarly, the performance advantage (indicated by lower RMSE) of RF over
216 the ratio estimator was most pronounced for species with low bycatch rates, and decreased for species with
217 higher bycatch rates (Fig. 7).

218 **Discussion**

219 In terms of the relative performance across models, our results are consistent with previous studies showing
220 that non-parametric methods such as random forests offer improved predictive capabilities over GAMs and
221 delta-GLMM models (Marmion *et al.*, 2009; Knudby *et al.*, 2010; Rooper *et al.*, 2017). Including the spatial
222 location of fishing offered a considerable improvement in RMSE for many species, particularly in the RF-Delta
223 modeling framework (Fig. S3). However, once spatial information was included, the addition of effort had
224 a minimal effect in reducing RMSE. This result is not surprising, given the weak relationships between
225 bycatch and effort revealed by our log-log analyses (Figs. 2, 3, and S1). We found decreases in RMSE for
226 all species and models as observer coverage increased from 20% to 40% (Fig. S4). The improvement in
227 predictive capabilities with increasing observer coverage is consistent with previous simulation experiments
228 using different fisheries (Babcock *et al.*, 2003; Amandè *et al.*, 2010a).

229 For an observer program tasked with producing yearly bycatch estimates for many species, the ideal bycatch
230 estimation model is simple, converges rapidly, performs well on average, and never performs much worse than
231 a default option like a ratio estimator. Therefore, the fact that RF had equal or lower prediction error than
232 the ratio estimator for all species and scenarios is an important finding. The desire for one simple model
233 also informed our selection of candidate models; we did not test an exhaustive list of modeling options for
234 spatiotemporal bycatch data, but a subset of models that analysts are familiar with and can apply quickly.
235 We assumed that each species in our simulations were affected by the same set of covariates; ideally, a single
236 best model could be developed for each species in a given fishery, with unique covariates. We also restricted
237 covariates in our analysis to the same information that is typically used in the ratio estimator, even though
238 some covariates (e.g. depth, date of year) could be treated as continuous rather than discrete factor variables.
239 However, including all available covariates without stratification in a RF model, RF-All, actually performed
240 worse than the model with fewer, stratified covariates, RF-Total (Fig. S2). While RF are touted as robust
241 to overfitting and the inclusion of noninformative covariates (Breiman, 2001; Biau and Scornet, 2016), one
242 possible explanation for this result is that RF-All did overfit the data.

243 The second important finding from our simulations with practical implications for management is that the
244 choice of one estimator over another is accompanied by an implicit tradeoff between bias and variance. While
245 RF had equal or lower prediction error than the ratio estimator for all species, RF was slightly biased high
246 (overestimating true bycatch, Fig. 6). On the other hand, RF estimates were much less variable than the ratio
247 estimator. This bias-variance tradeoff was apparent for all species in our simulations (Fig. 8), but depended
248 on the species' bycatch rate (Fig. 7). For commonly-caught species like Sandpaper skate or Brown cat shark,

where RF and the ratio estimator had similar RMSE, RF offered slight reductions in uncertainty but had large increases in bias. For rarely-caught species, like California slickhead or Dungeness crab, RF exchanged large reductions in uncertainty for modest increases in bias. The recommendation of one methodology over another largely depends on what the bycatch estimates will be used for. Stock assessment scientists, for example, may be largely interested in unbiased but imprecise estimates, such as the ratio estimator, which can then be fitted and smoothed statistically during model fitting. On the other hand, scientists or policy makers who are more concerned about relative changes in bycatch over time may prefer more precise estimators (such as RF) that are more robust to noise arising from sampling less than 100% of the fishery. We recommend further research regarding circumstances when it is important to minimize bias versus imprecision when processing data for inclusion in a second-stage model (Szpiro and Paciorek, 2013).

The bias of a RF model is roughly equal to the bias of the individual regression trees it comprises, so it should not be expected to produce unbiased estimates (Breiman, 1999; Kuhn and Johnson, 2013; Xu, 2013). RF bias depends on the response variable distribution—RF will be unbiased for a uniform response, and we can expect positive bias for typical fisheries catch distributions (positive, right-skewed). Why? Consider how each individual tree in a RF generates predictions for the tails of a distribution. Terminal nodes for extreme values use the mean of the training data in those nodes, so trees tend to overpredict in the lower tail and underpredict in the upper tail. Because bycatch is right-skewed, there are more observations in the lower tail, and therefore more overprediction than underprediction. Several bias correction methods have been proposed, and we tested two: 1) Cubist, which fits a linear model in terminal nodes instead of using the data mean (Quinlan, 1992, 1993), and 2) Xu (2013), which fits a second RF model to the residuals of the original RF. Unfortunately, Cubist reduced but did not eliminate bias, and Xu (2013) performed poorly (e.g. for Dungeness crab, Cubist reduced median percent error from 0.055 to 0.043, Fig. S5).

Based on the results from our simulation study, there are several potential avenues of future research that will help to advance the inclusion of spatial information into bycatch estimation. First, additional work could be done to improve variance estimation for non-parametric methods such as RF. Resampling or bootstrapped estimates could be generated for fisheries with less than 100% observer coverage, and variance estimates could be compared to analytic estimates via the ratio estimator (Cochran, 1963). Second, it may be useful to perform a more detailed comparison between the models used here, and the spatiotemporal delta-GLMM models that have been widely used for fisheries survey data (Thorson *et al.*, 2015). Similarly, multispecies spatiotemporal models may improve predictions of local density by sharing information about underlying spatial patterns (Latimer *et al.*, 2009; Warton *et al.*, 2015; Ovaskainen *et al.*, 2016; Thorson and Barnett, 2017; Thorson *et al.*, 2017). Additionally, advice on the number and distribution of knots or random effects

281 in spatiotemporal models would be useful for analysts interested in applying these models.

282 **Supplementary material**

283 The following supplementary material is available online:

284 Table S1: Annual bycatch (mt) and bycatch rate (percent of hauls) for species selected from the U.S. WCGOP
285 dataset.

286 Figure S1: Estimated relationships between fishing effort (haul duration in hours) and bycatch (kg) for 15
287 species analyzed in the West Coast groundfish trawl fishery.

288 Figure S2: Predictive performance of the ratio estimator (status quo) and two spatial modeling frameworks:
289 generalized additive model (GAM) and random forests (RF).

290 Figure S3: Change in predictive performance (normalized RMSE) when adding fishing effort and spatial
291 location as covariates in each model.

292 Figure S4: Predictive performance (normalized RMSE) for different levels of simulated observer coverage.

293 Figure S5: Performance of RF bias correction methods (percent error, PE, averaged across years 2011–2015).

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Table 1: Total bycatch (mt) and bycatch rate (percent of hauls) for species selected from the U.S. West Coast Groundfish Observer Program (WCGOP) dataset. All selected species are exclusively discarded. The summarized data are 35,440 post-IFQ hauls (4,007 trips) observed from 2011-2015 in the area north of Cape Falcon, Oregon (45.77° N).

Species	Catch (mt)	% Hauls
Big skate	185.4	12.9
Black skate	72.0	15.2
Brown cat shark	113.4	45.1
California slickhead	32.0	9.2
Dungeness crab	547.9	29.4
Grenadier	452.9	28.8
Octopus	16.9	13.9
Pacific hake	727.9	56.7
Pacific halibut	306.8	31.0
Rosethorn rockfish	3.2	4.2
Sandpaper skate	162.1	50.6
Slender sole	160.5	26.4
Spiny dogfish shark	1216.5	43.3
Spotted ratfish	295.1	42.7
Tanner crab	494.8	39.9

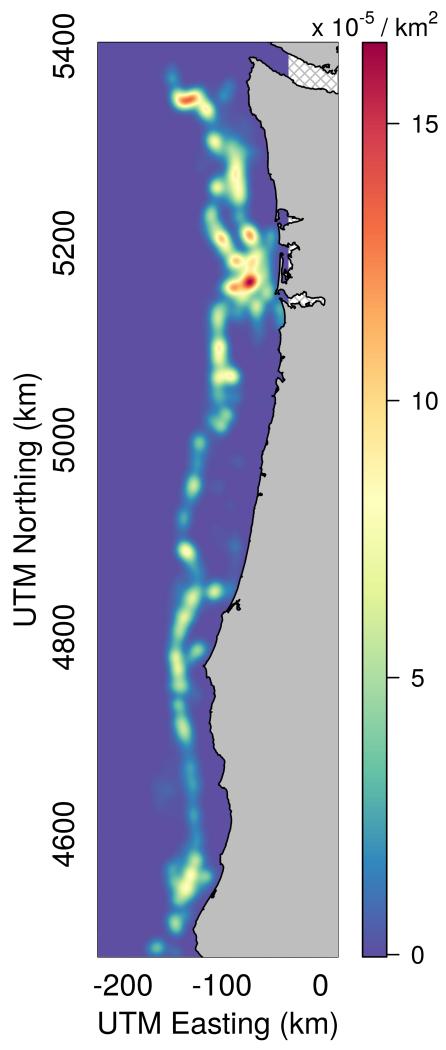


Figure 1: Fishing effort density in the West Coast groundfish trawl fishery from 2011 to 2015 in the area north of Cape Falcon, Oregon (45.77° N). The West Coast Groundfish Observer Program monitored and collected data from 35,440 hauls from all (100%) of the 4,007 trips. Fishing effort was smoothed using a bivariate kernel density estimate ('bkde2D' function in R package 'KernSmooth') to ensure that fishing locations were anonymized.

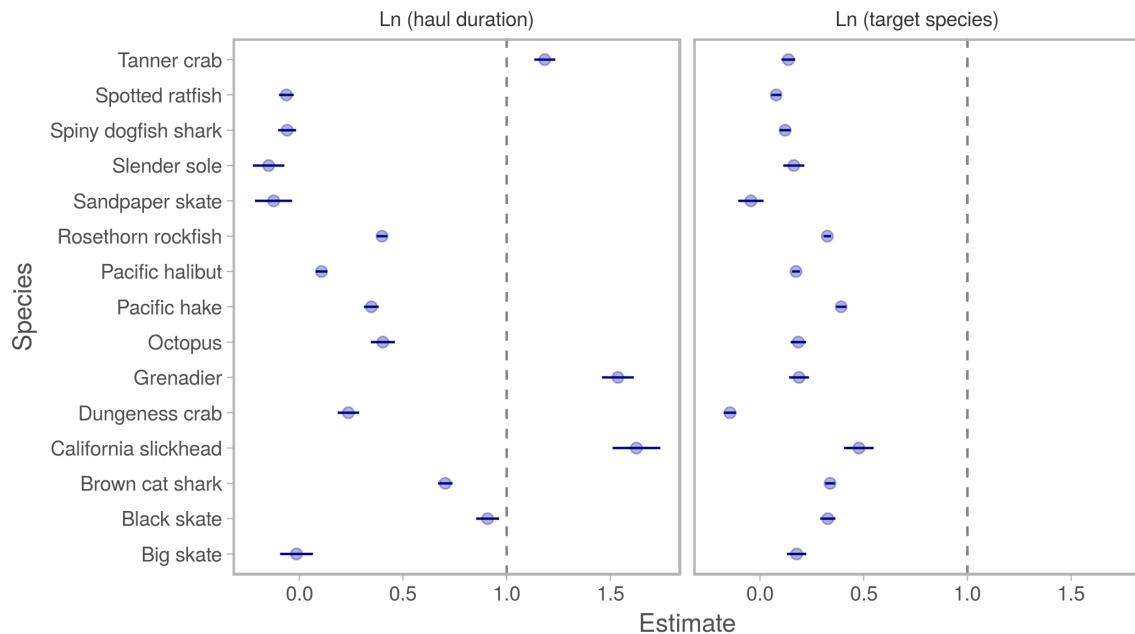


Figure 2: Estimated relationships between fishing effort, defined as haul duration (hours, left panel) or catch of target species (kg, right panel), and bycatch for 15 species analyzed in the West Coast groundfish trawl fishery. The slope terms, β , of log-log linear models are exponents of an assumed power law fit to each species, $\text{Bycatch} = \alpha \text{Effort}^\beta$, with 95% CIs shown for each estimate. Most β are much less than 1 (left of dashed line), indicating the relationship between bycatch and effort is either weak or less-than-linear. Data ($n = 35,440$) consist of observed hauls from the West Coast Groundfish Observer Program recorded from 2011 to 2015 in the area north of Cape Falcon, Oregon (45.77° N).

Effort = Log(Target Catch)

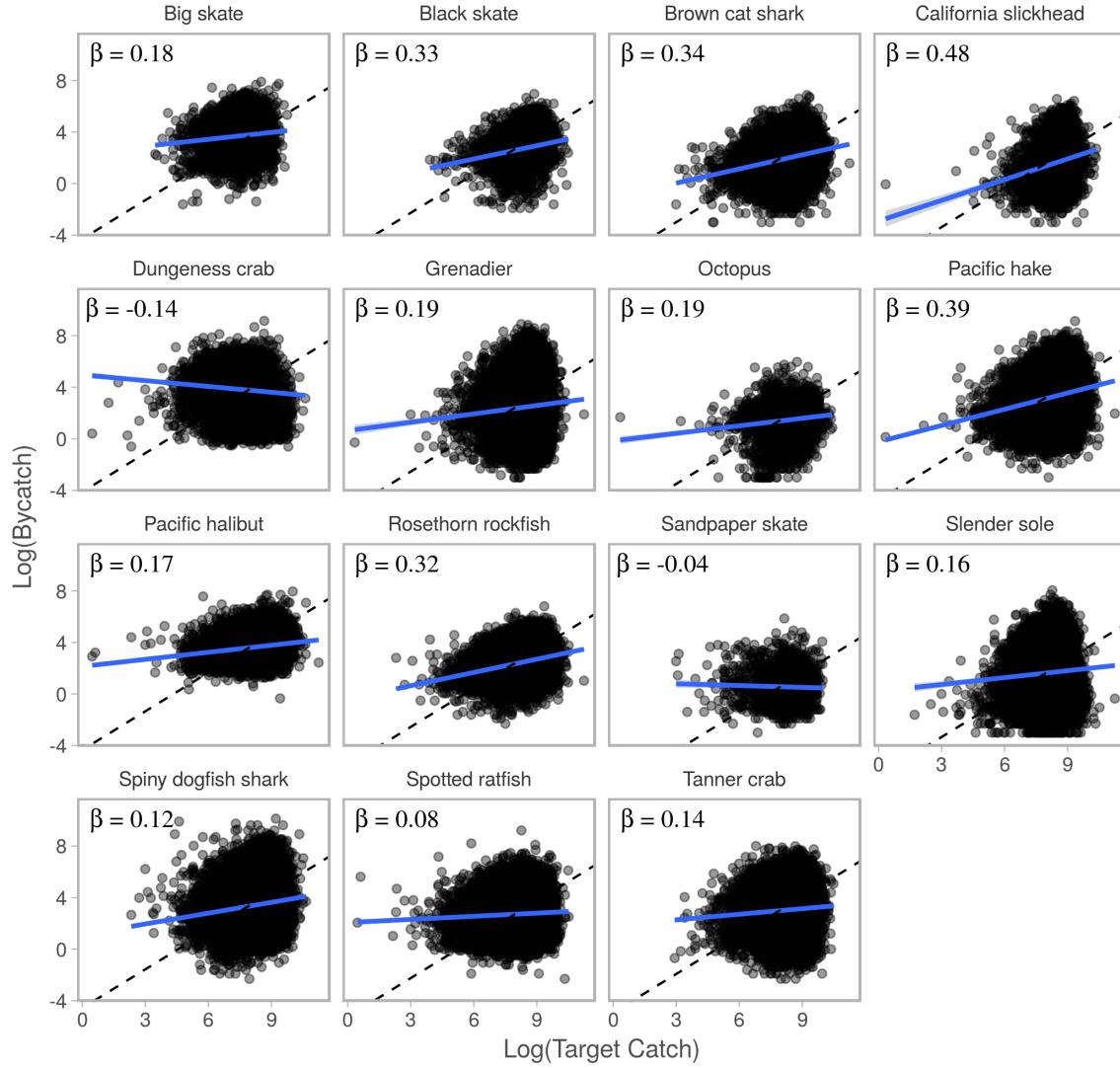


Figure 3: Relationship between fishing effort (catch of target species in kg) and bycatch (kg) of 15 selected species in the West Coast groundfish trawl fishery. The slope terms, β , of log-log linear models are exponents of an assumed power law fit to each species, $\text{Bycatch} = \alpha \text{Effort}^\beta$. All β are much less than 1, indicating the relationship between Bycatch and Effort is either weak or less-than-linear. Each data point ($n = 35,440$) is an observed haul from the West Coast Groundfish Observer Program recorded from 2011 to 2015 in the area north of Cape Falcon, Oregon (45.77° N).

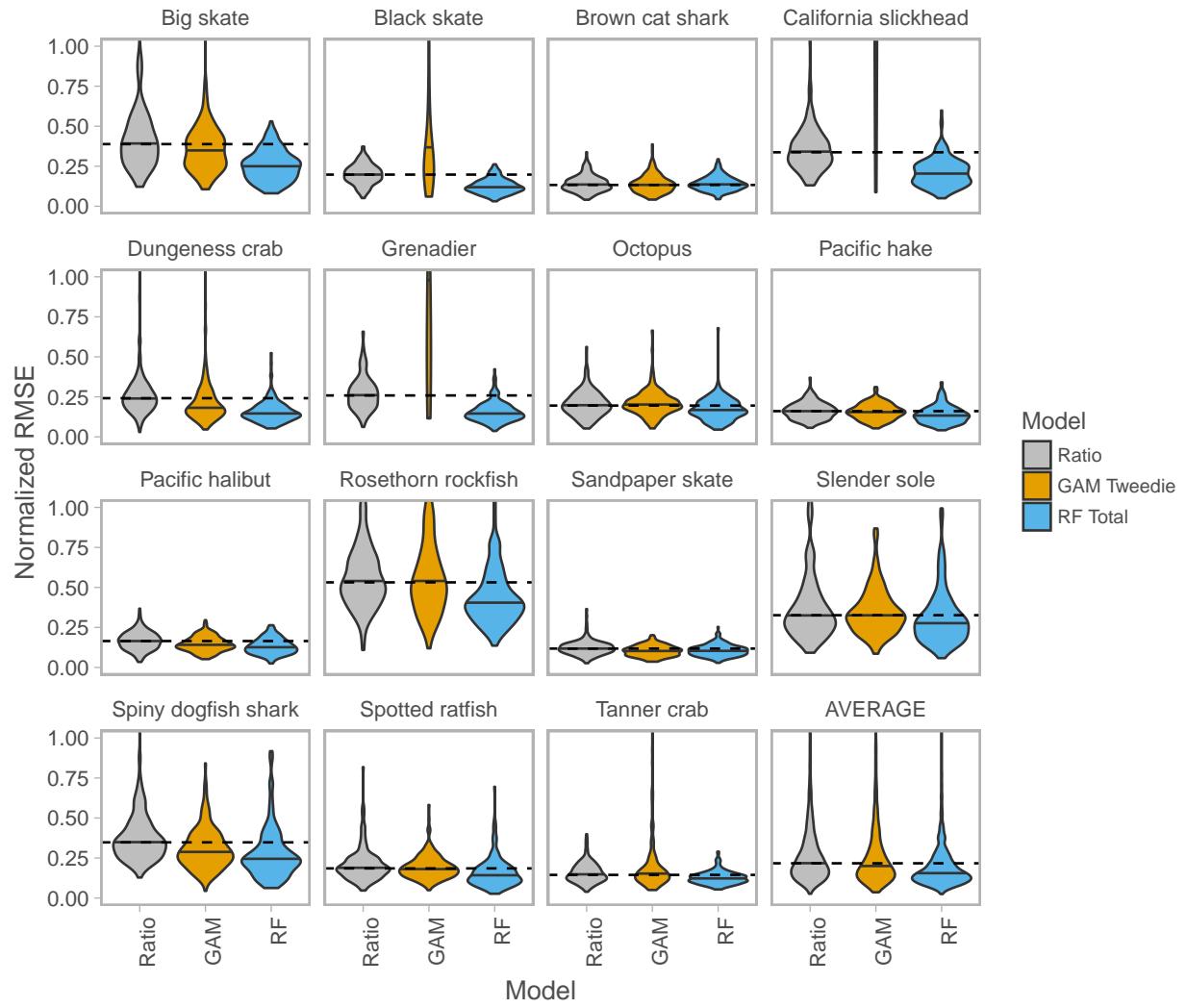


Figure 4: Predictive performance of the ratio estimator (status quo) and two spatial modeling frameworks: generalized additive model (GAM) and random forest (RF). We fit each model to 200 'training' datasets simulated with 20% observer coverage, then predicted bycatch in unobserved hauls to calculate annual estimates of fleet-wide bycatch. We calculated model performance (RMSE) using the true, observed bycatch. For each species, the dashed line indicates the median RMSE for the ratio estimator, and solid lines indicate median RMSE for each model. The GAM-Tweedie had convergence issues for 3/15 species. RF-Total outperformed the ratio estimator for all species, and on average had 26% lower RMSE (RF = 0.16, Ratio = 0.22).

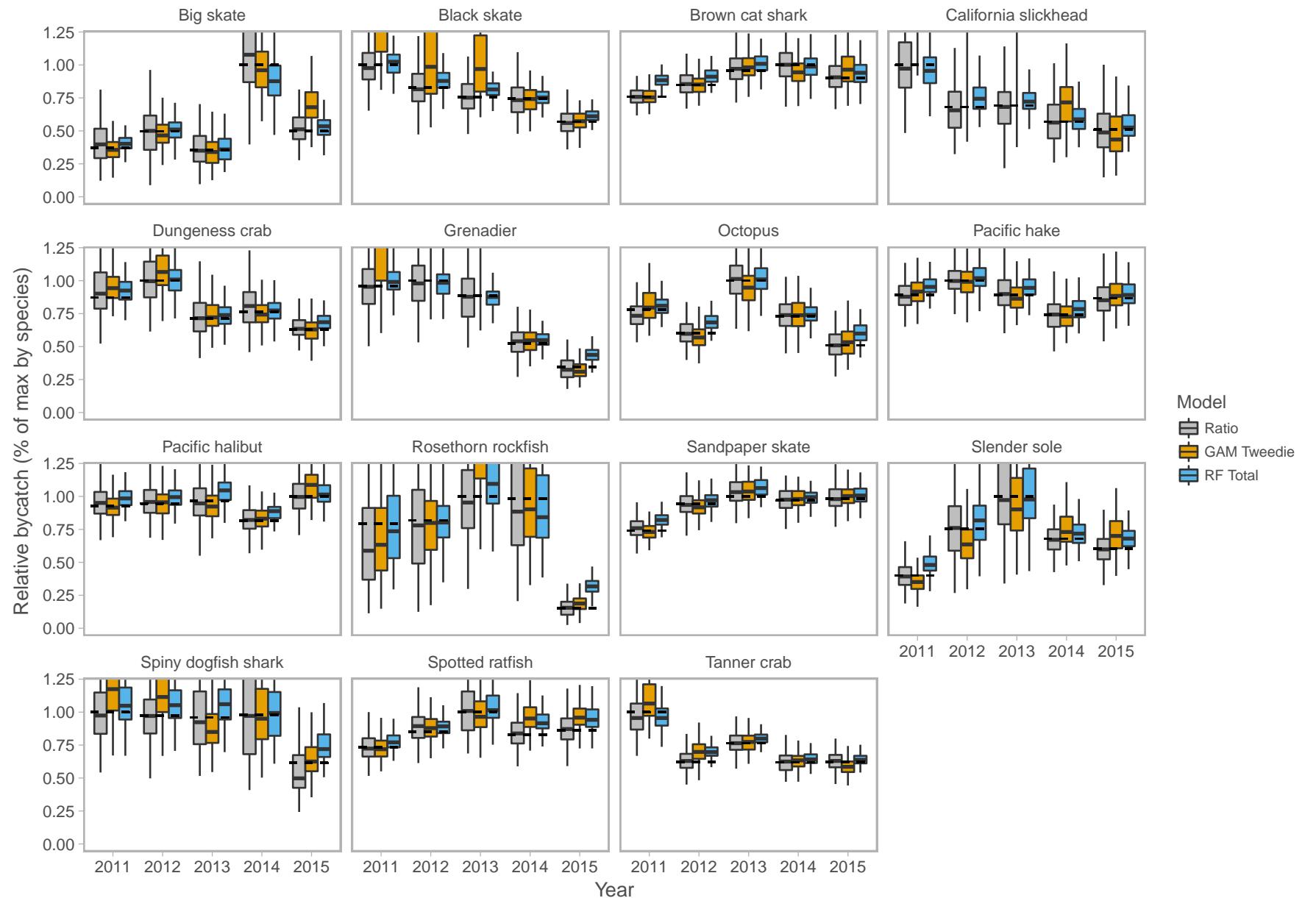


Figure 5: Predictive performance of the ratio estimator (status quo) and two spatial modeling frameworks: generalized additive model (GAM) and random forests (RF). We fit each model to 200 'training' datasets simulated with 20% observer coverage, then predicted bycatch in unobserved hauls to calculate annual estimates of fleet-wide bycatch. For each species and year, the dashed lines indicate the true observed bycatch.

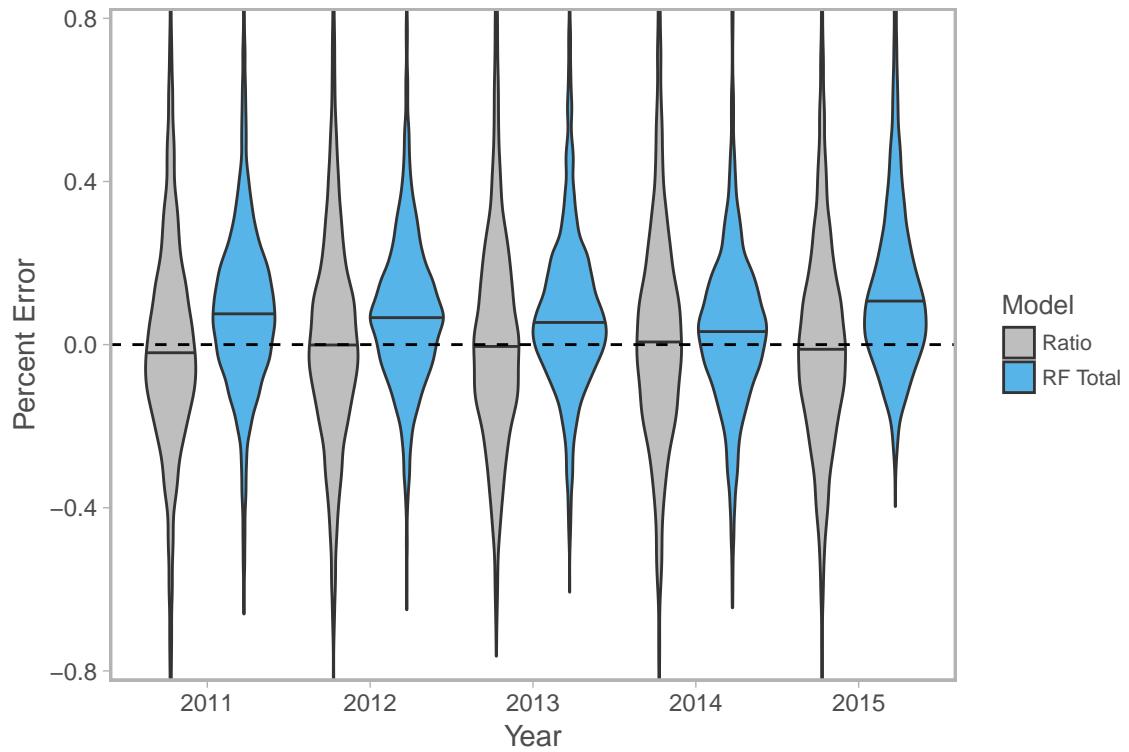


Figure 6: Percent error of annual bycatch predictions using the ratio estimator (status quo) and random forests (RF), averaged across 15 species in the West Coast groundfish trawl fishery. Averaged across species, RF-Total was more precise than the ratio estimator (median absolute percent error: RF = 0.118, Ratio = 0.155), but with slight positive bias (median percent error = 0.068). Median percent error (bias) of the ratio estimator was very slightly negative (-0.011). We fit each model to 200 'training' datasets simulated with 20% observer coverage, then predicted bycatch in unobserved hauls to calculate annual estimates of fleet-wide bycatch. Percent error was calculated using the true, observed bycatch.

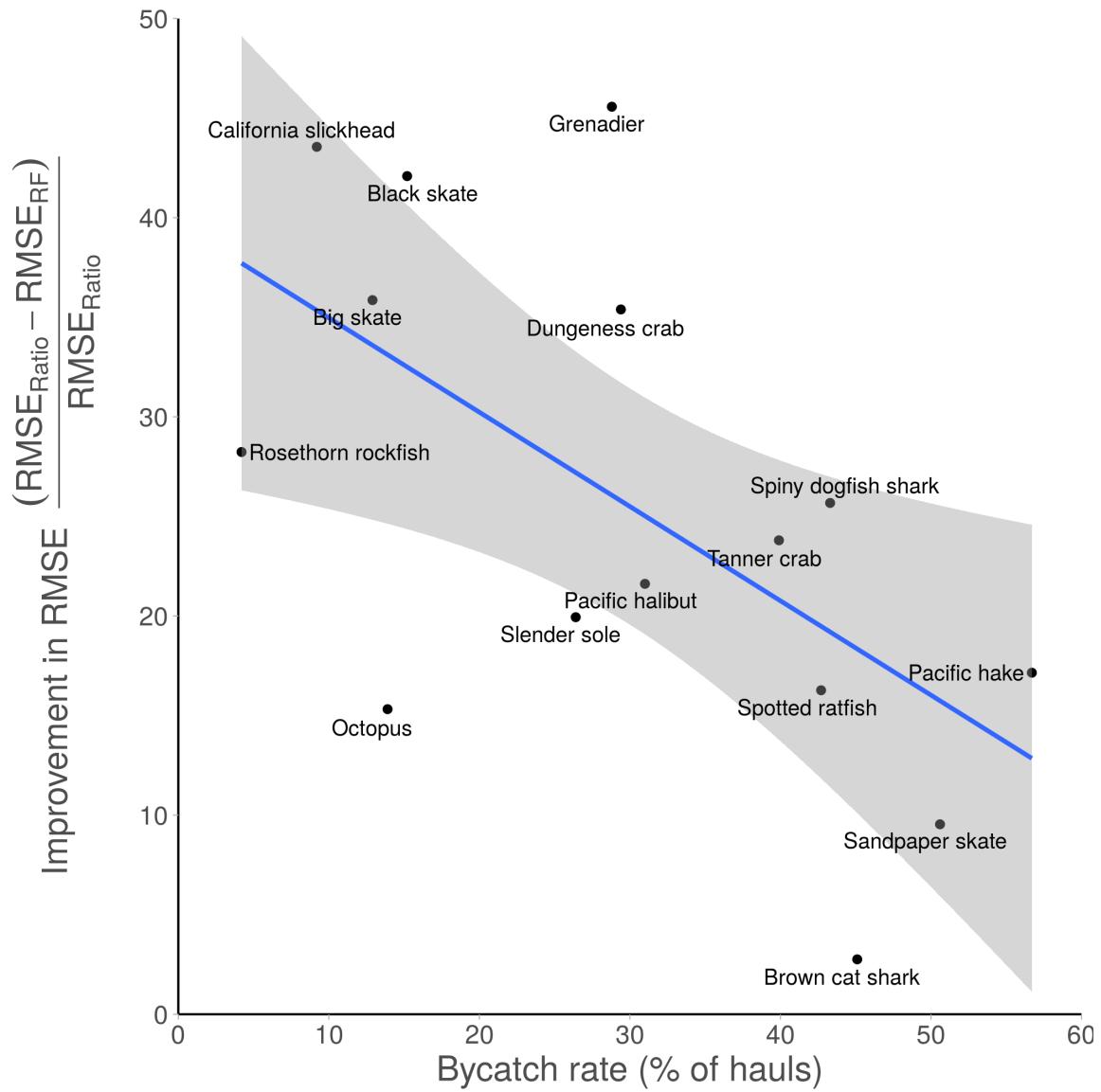


Figure 7: RF reduction in prediction error compared to the ratio estimator, as a function of bycatch rate for 15 species in the U.S. West Coast groundfish trawl fishery. RF improved on the ratio estimator for all species (26% lower RMSE on average), but this improvement was greater for species with lower bycatch rates (e.g. Rosethorn rockfish, California slickhead, Big skate, Black skate, Dungeness crab, Grenadier).

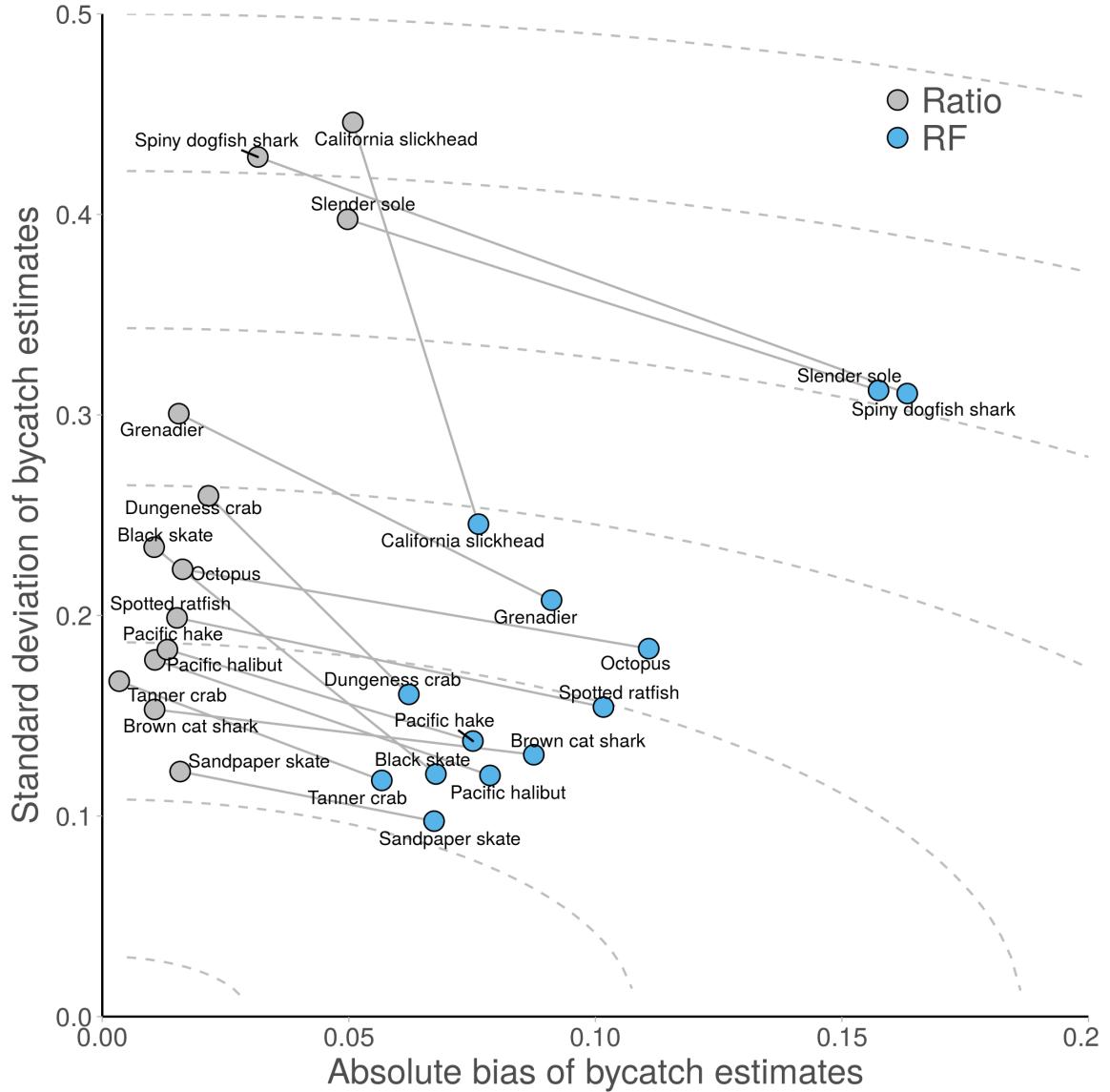


Figure 8: Bias-variance trade-off between the ratio estimator and RF. RF achieves more accurate predictions (lower RMSE) by allowing some bias but greatly reducing the variance of its estimates. The ratio estimator has very low bias but much higher variance (i.e. it underfits the data and is more sensitive to which hauls are observed). Dashed grey lines indicate iso-RMSE curves. Species with lines that are nearly parallel to the iso-RMSE curves (e.g. Octopus, Brown cat shark) indicate that RF and the ratio estimator perform similarly (same RMSE). Species with lines that cross iso-RMSE curves (e.g. Dungeness crab, California slickhead, Spiny dogfish shark) indicate RF greatly improves on the ratio estimator (lower RMSE). RF has lower RMSE for species with lower bycatch rates (Fig. 7).

¹ Supplementary material for “The utility of spatial
² model-based estimators of unobserved bycatch: future
³ or folly?”

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⁸ **List of Tables**

⁹ S1 Annual bycatch (mt) and bycatch rate (percent of hauls) for species selected from the U.S. WCGOP dataset.	¹⁰	2
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¹¹ **List of Figures**

¹² S1 Estimated relationships between fishing effort (haul duration in hours) and bycatch (kg) for 15 species analyzed in the West Coast groundfish trawl fishery.	¹³	3
¹⁴ S2 Predictive performance of the ratio estimator (status quo) and two spatial modeling frameworks: generalized additive model (GAM) and random forests (RF).	¹⁵	4
¹⁶ S3 Change in predictive performance (normalized RMSE) when adding fishing effort and spatial location as covariates in each model.	¹⁷	5
¹⁸ S4 Predictive performance (normalized RMSE) for different levels of simulated observer coverage.	¹⁹	6
S5 Performance of RF bias correction methods (percent error, PE, averaged across years 2011-2015).	7

Table S1: Annual bycatch (mt) and bycatch rate (percent of hauls) for species selected from the U.S. West Coast Groundfish Observer Program (WCGOP) dataset. All selected species are exclusively discarded. The summarized data are 35,440 post-IFQ hauls (4,007 trips) observed from 2011-2015 in the area north of Cape Falcon, Oregon (45.77° N).

Species	2011		2012		2013		2014		2015	
	Catch (mt)	% Hauls								
Big skate	25.2	10.2	33.9	10.8	24.1	9.1	68.2	17.9	34.0	18.5
Black skate	18.5	17.3	15.3	14.4	14.0	15.2	13.7	15.3	10.5	13.3
Brown cat shark	19.3	45.6	21.5	43.5	24.3	45.4	25.4	45.4	22.9	45.8
California slickhead	9.3	12.3	6.3	8.1	6.4	9.0	5.3	9.3	4.7	6.7
Dungeness crab	120.1	27.6	137.8	32.7	98.2	25.3	105.0	31.9	86.8	30.7
Grenadier	116.8	34.0	121.9	29.8	108.1	29.8	64.0	26.0	42.0	22.5
Octopus	3.7	15.9	2.8	13.2	4.7	15.4	3.4	13.2	2.4	10.9
Pacific hake	147.6	55.1	165.8	58.2	148.0	54.2	122.7	56.2	143.8	60.7
Pacific halibut	61.0	29.3	62.3	30.3	63.7	27.1	53.8	33.9	65.9	36.2
Rosethorn rockfish	0.7	3.3	0.7	4.5	0.9	5.9	0.8	4.2	0.1	2.5
Sandpaper skate	25.9	44.9	33.0	48.4	35.0	51.8	33.9	53.9	34.3	55.4
Slender sole	18.7	20.7	35.2	23.6	46.7	26.9	31.7	31.3	28.2	31.2
Spiny dogfish shark	268.7	42.5	261.4	46.5	258.0	39.2	262.9	46.9	165.5	42.2
Spotted ratfish	50.7	37.5	58.7	42.3	69.0	41.9	57.3	44.4	59.4	48.8
Tanner crab	136.3	46.3	85.1	38.6	104.2	39.7	84.3	39.4	84.9	34.4

²⁰ **Figure S1**

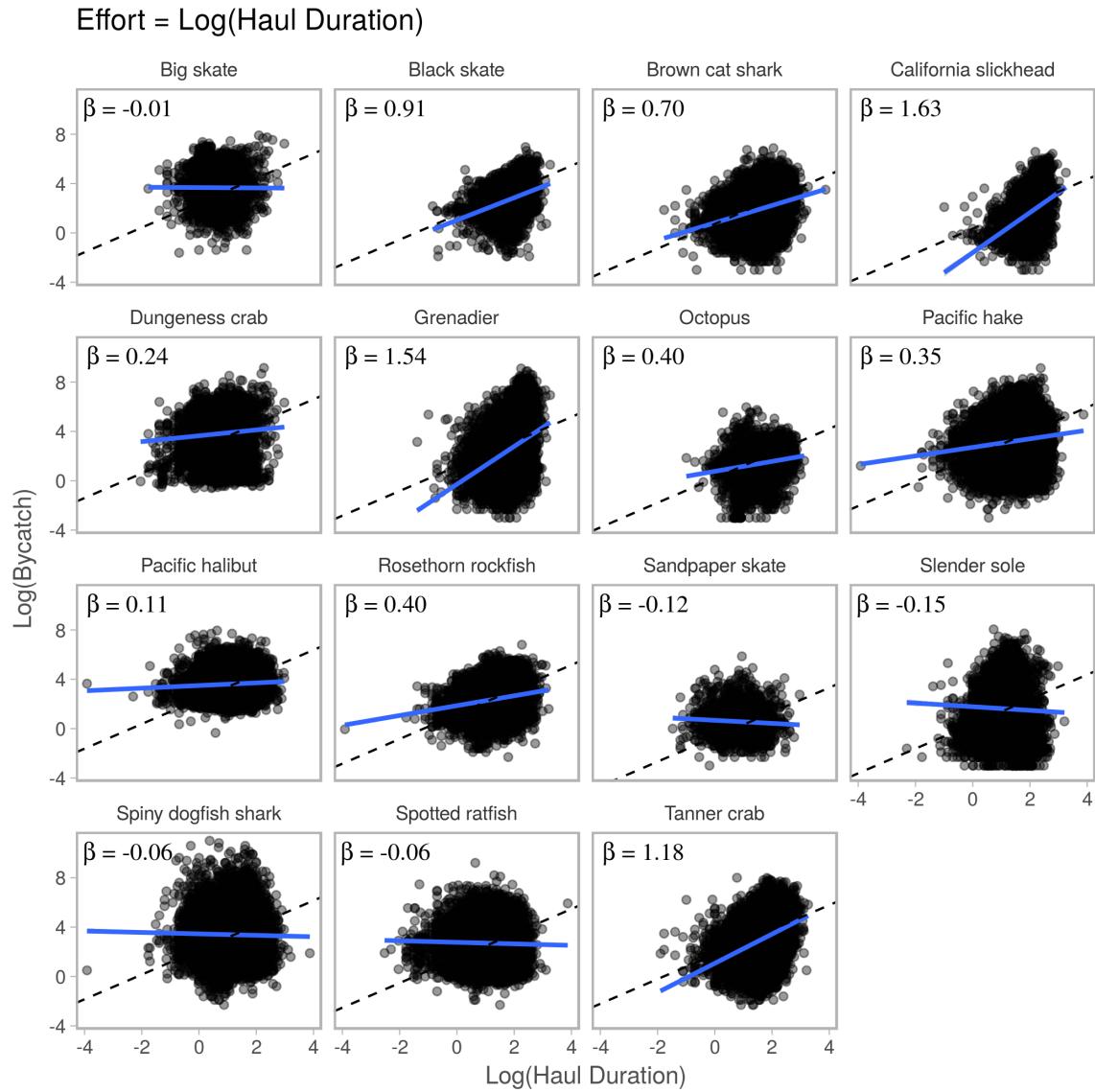


Figure S1: Estimated relationships between fishing effort (haul duration in hours) and bycatch (kg) for 15 species analyzed in the West Coast groundfish trawl fishery. The slope terms, β , of log-log linear models are exponents of an assumed power law fit to each species, $\text{Bycatch} = \alpha \text{Effort}^\beta$. Most β are much less than 1, indicating the relationship between bycatch and effort is either weak or not linear. Data ($n = 35,440$) consist of observed hauls from the West Coast Groundfish Observer Program recorded from 2011 to 2015 in the area north of Cape Falcon, Oregon (45.77° N).

21 **Figure S2**

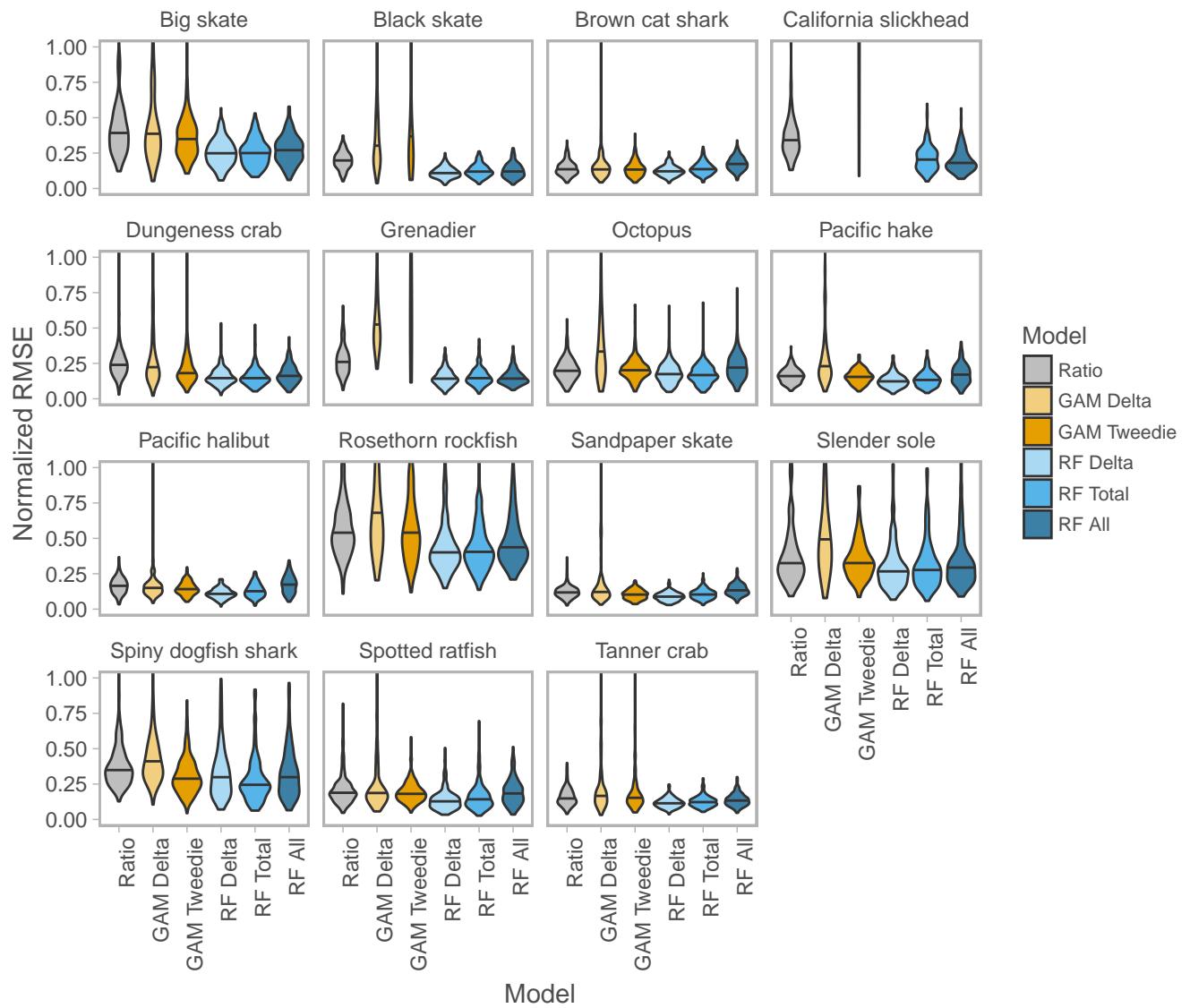


Figure S2: Predictive performance of the ratio estimator (status quo) and two spatial modeling frameworks: generalized additive model (GAM) and random forests (RF). We fit each model to 200 'training' datasets simulated with 20% observer coverage, then predicted bycatch in unobserved hauls to calculate annual estimates of fleet-wide bycatch. For each species, the dashed line indicates the median RMSE for the ratio estimator, and solid lines indicate median RMSE for each model. For both GAMs and RFs, the non-delta models outperformed the delta models.

22 **Figure S3**

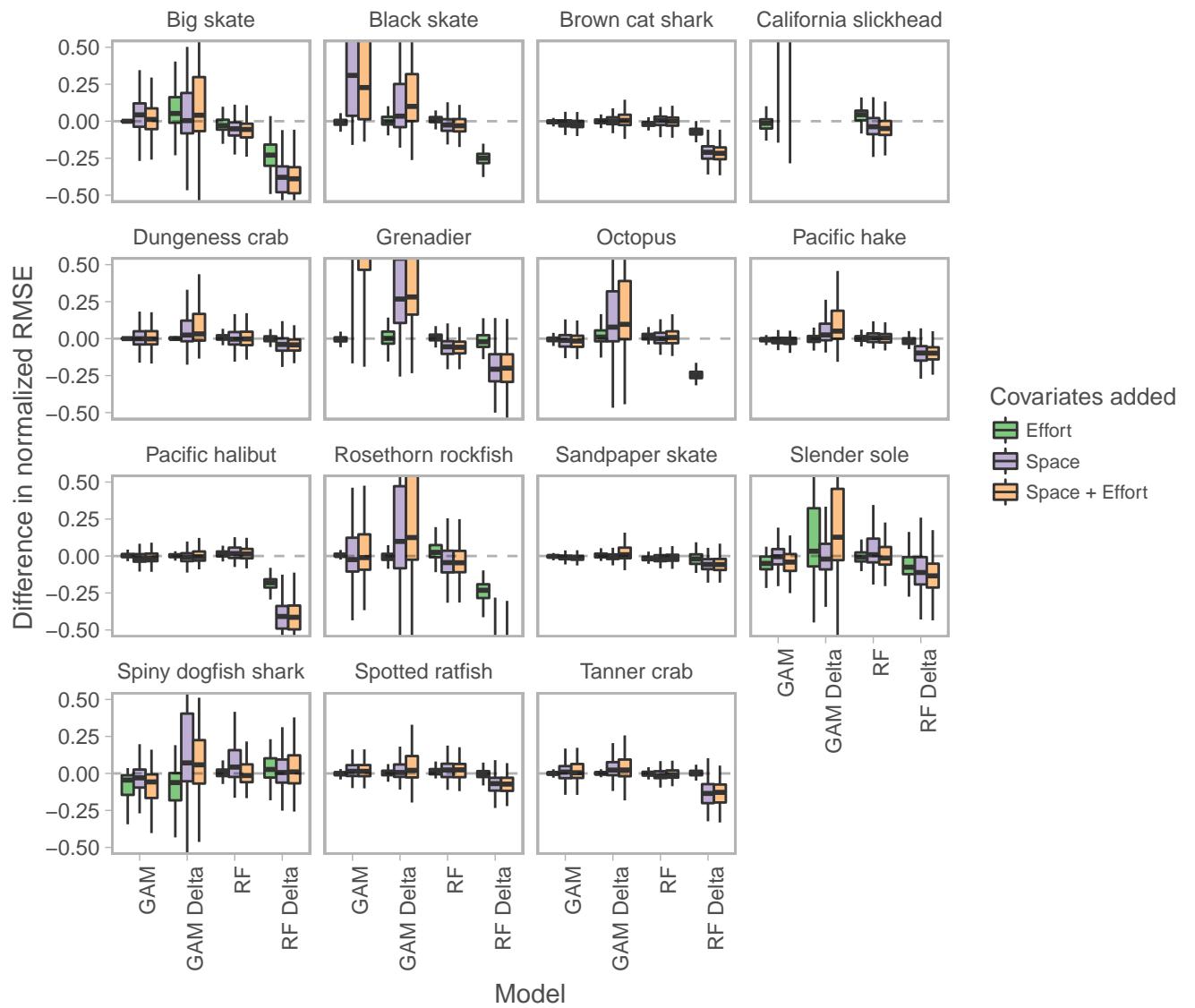


Figure S3: Change in predictive performance (normalized RMSE) when adding fishing effort and spatial location as covariates in each model. For many species, adding space to the GAM-Delta and GAM-Tweedie models led to worse predictions (positive change in RMSE, above dashed line). On the other hand, adding space to the RF-Delta model consistently improved predictions (negative change in RMSE, below dashed line). For RF-Total, including space had either slightly improved predictions or had no effect. Adding effort had little effect for nearly all species and models, and never had a larger effect than adding space.

²³ **Figure S4**

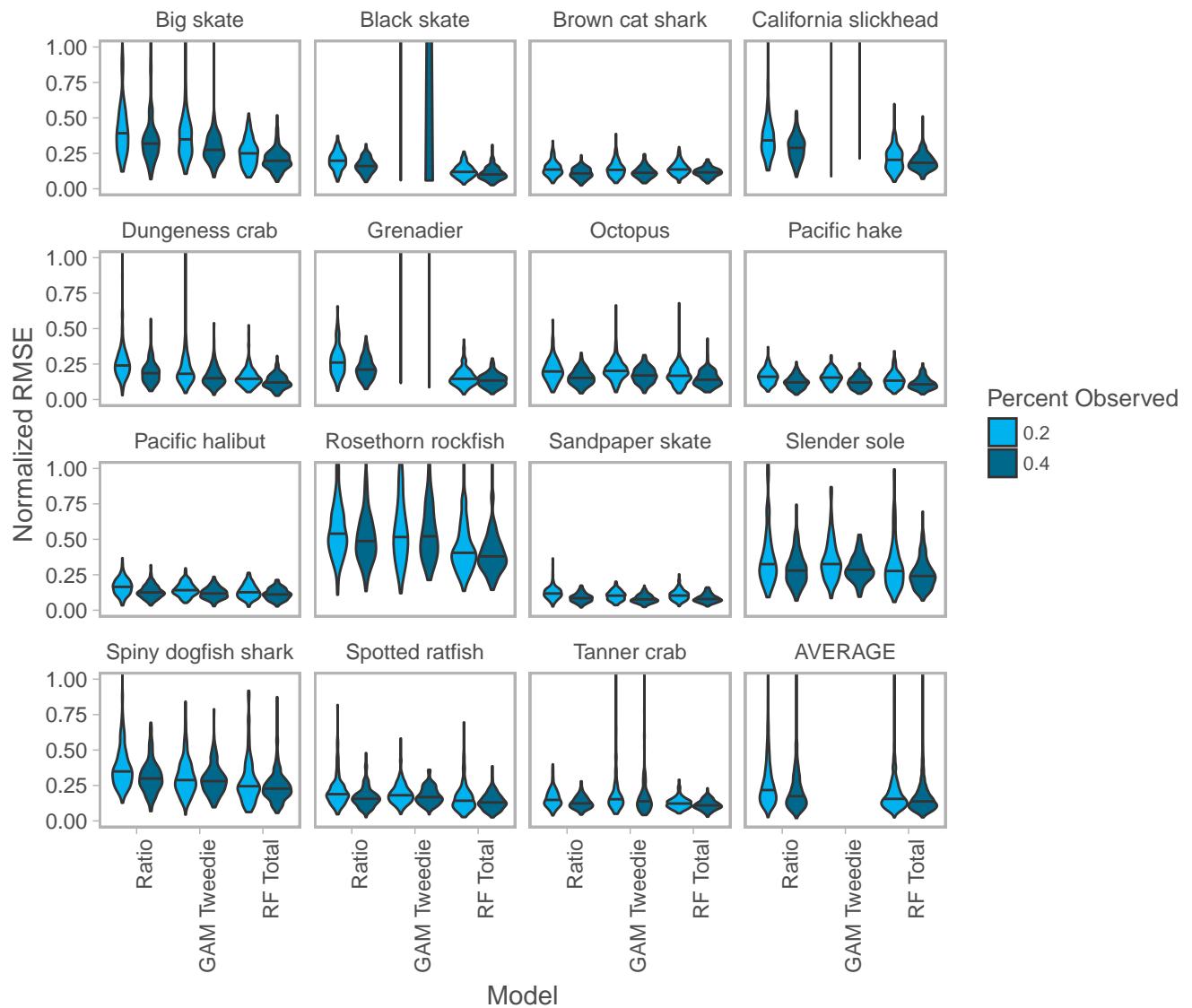


Figure S4: Predictive performance (normalized RMSE) for different levels of simulated observer coverage. Averaged across species, RF-Total had lower median RMSE than the ratio estimator, even at half the observer coverage (RF-Total at 20%: 0.155, Ratio at 40%: 0.180). GAM-Tweedie failed to converge for 3/15 species.

24 Figure S5

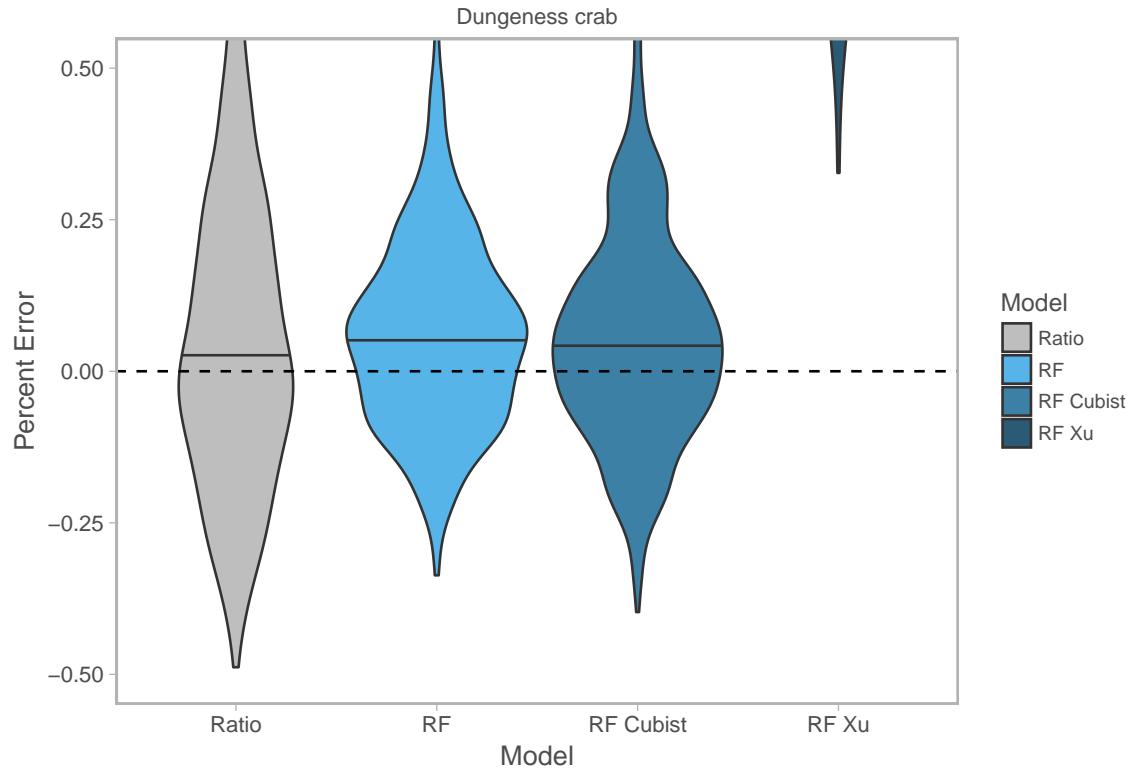


Figure S5: Performance of RF bias correction methods (percent error, PE, averaged across years 2011-2015). The ratio estimator is unbiased (median PE = 0.002). RF is positively biased (median PE = 0.055) and Cubist is less positively biased (median PE = 0.043). Cubist reduces bias by fitting a linear model in regression tree terminal nodes instead of using the data mean (Quinlan 1992, Quinlan 1993). The second method, Xu (2013), fits a second RF model to the residuals of the original RF, but this method performed poorly (median PE = 1.107, off chart).