

## URBAN POPULATION DYNAMICS

1. Using the information given to us in the matrix, it is easy to see that the data in the matrix is parallel to a human society in a practical sense. The population in 2000 is as follows:

Age Group	Population Distribution
0-9	210,000
10-19	190,000
20-29	180,000
30-39	210,000
40-49	200,000
50-59	170,000
60-69	120,000
70-79	90,000
80+	50,000

All this information is given through the vector  $x(0)$ . The Leslie matrix also contains a lot of information: it explains the survival rate of an age group to advance to the next age group. The top row of the Leslie matrix describes fecundity – per capita average number of female off springs reaching the age group 0-9 from mothers of the current age group they represent.

In this particular example, the population appears to have a large population of children 0-9 and 30-39 year-old adults. Social factors that influence the downward trend of the distribution would be because of time. The real-life model shows that the population of the generation group shrinks the later they live for. This is only natural due to death. The Leslie Matrix shows that most children are born from mothers of the 10-19 year old column, with 20-29 and 30-39 following closely behind. The number falls off for further columns due to woman's inability to produce children at a later age. Lastly, the Leslie Matrix shows the percentage of people who survive from one age group to the next. Because children are more prone to put themselves in danger, they have 70% chance of surviving until age 10 in this particular population. It remains fairly steady later in life, peaking at 20-29 and 30-39. Later in life (closer to 60-69 and 70-79) the numbers begin shrinking again, with only 77% of 60-69 year olds living until 70, and only 40% of 70-79 year olds living until 80. This is again due to the fragility of the human condition at this age. We are more prone to diseases and health problems the older we get, increasing the chances of death.

2. Using power\_method.java, one can determine the population distribution in years 2010, 2020, 2030, 2040, 2050, etc.

**2010: Loop 1**

0-9	635,000
10-19	147,000
20-29	161,500
30-39	162,000
40-49	189,000
50-59	176,000
60-69	138,000
70-79	92,000
80+	36,000

**2020: Loop 2**

0-9	518,750
10-19	444,500
20-29	124,950
30-39	145,350
40-49	145,800
50-59	166,320
60-69	140,800
70-79	104,720
80+	36,960

**2030: Loop 3**

0-9	816,240
10-19	363,124
20-29	377,825
30-39	112,455
40-49	130,815
50-59	128,304
60-69	133,056
70-79	108,416
80+	41,888

**2040: Loop 4**

0-9	965,648
10-19	571,367
20-29	308,656
30-39	340,042
40-49	101,209
50-59	115,117
60-69	102,643
70-79	102,453
80+	43,366

**2050: Loop 5**

0-9	1,341,322
10-19	675,953
20-29	485,662
30-39	277,790
40-49	306,038
50-59	89,064
60-69	92,093
70-79	79,035
80+	40,981

**Population Totals**

<b>2010</b>	1,736,500
<b>2020</b>	1,828,150
<b>2030</b>	2,212,123
<b>2040</b>	2,650,501
<b>2050</b>	3,387,938

### Percent Increase in Total Population

Year Range	Fractional Change:
2000-2010	22.2887323943662%
2010-2020	5.277857759861791%
2020-2030	21.00336405655991%
2030-2040	19.81707165469551%
2040-2050	7.822551283700705

3. The dominant eigenvalue for this situation is estimated to be 1.2886562339310283. This is the population's growth rate at the stable age distribution, which is given by the eigenvalue's corresponding eigenvector:

72.77316642479103  
39.53049320372915  
26.07438534687458  
18.210401031935042  
12.718179214285893  
8.685014213938615  
5.391671718341888  
3.221640584827573  
1.0

The population becomes stable in the long run since the values converge ( $\|A^k\|$ ) under the tolerance after 86 iterations. In every iteration, we get closer and closer to the eigenvalue and eigenvector. This is due to the decreasing difference between  $k+1$  and  $k$  in the equation  $n(k+1) = A n(k)$ . As the value gets smaller and smaller, the difference between each iteration lessens, eventually falling beneath an error threshold.

4. In 2020, we cut the second age group's birth rate in half. The year 2020 corresponds with x(2):

5.1875  
 4.445  
 1.2495  
 1.4535  
 1.4580000000000002  
 1.6632  
 1.4080000000000001  
 1.0472000000000001  
 0.3696

The birth rate of the second age group is 1.2 (This is the second column of the first row). Cut in half, it is merely .6. Ran through our program this changes the following

**2030:**

Age Groups	Pre-Change	Estimated Population
0-9	816,240	549,540.0000000001
10-19	363,124	363,124.9999999996
20-29	377,825	377,825.0000000003
30-39	112,455	112,455.0000000002
40-49	130,815	130,815.0000000001
50-59	128,304	128,304
60-69	133,056	133,056.0000000002
70-79	108,416	10,841.6
80+	41,888	41,888

**2040:**

Age Groups	Pre-Change	Estimated Population
0-9	965,648	747,773.5000000001
10-19	571,367	384,678.0000000003
20-29	308,656	308,656.2499999994
30-39	340,042	340,042.5000000003
40-49	101,209	101,209.5000000002
50-59	115,117	115,117.2
60-69	102,643	102,643.2
70-79	102,453	102,453.1200000002
80+	43,366	43,366.4000000005

**2050:**

Age Groups	Pre-Change	Estimation Population
0-9	1,341,322	886,487.8749999999
10-19	675,953	523,441.4500000001
20-29	485,662	326,976.3
30-39	277,790	277,790.62499999996
40-49	306,038	306,038.25000000002
50-59	89,064	89,064.36000000002
60-69	92,093	92,093.76000000001
70-79	79,035	79,035.264
80+	40,981	40,981.24800000001

The estimated eigenvalue is 1.1679027367203167

The estimated eigenvector is:

33.12287674923348  
19.85269234813984  
14.448796090175751  
11.134417338052286  
8.580316912747959  
6.465160707465589  
4.428561046281154  
2.9197568412193515  
1.0

In the long run, this will not affect age groups who were born past 2020. As time goes on, there is a huge deficit between the two situations. As previously mentioned, the eigenvector represents the growth rate at the stable age distribution. The new matrix's eigenvector is smaller. In the long run, the population will grow slower than the old matrix.