Algebraic Manipulations

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§1 Factorizations

Theorem 1.1 (Difference of Squares)

Example 1.2 (Kinematic Equations)
For motion with constant acceleration a, we have

$$\begin{aligned} a(t) &= a, \\ v(t) &= v_0 + at, \\ x(t) &= x_0 + v_0 t + \frac{1}{2}at^2, \end{aligned}$$

where x_0 and v_0 are the intial position and velocity at $t = 0$. Show that if an object has a displacement d with constant acceleration a, then the intial and final velocities satisfy

$$v_{\text{f}}^2 = v_{\text{i}}^2 + 2ad.$$

Solution. Note that we can write

$$\begin{aligned} v_{\text{f}}^2 &= v_{\text{i}}^2 + 2ad \\ \implies v_{\text{f}}^2 - v_{\text{i}}^2 &= 2ad \\ \implies (v_{\text{f}} + v_{\text{i}})(v_{\text{f}} - v_{\text{i}}) &= 2ad, \end{aligned}$$

where we get the last line by the difference of squares theorem shown above. But since v_{f} is equal to $v_{\text{i}} + at$, we have

$$(2v_{\text{i}} + at)(v_{\text{i}} + at - v_{\text{i}}) = 2ad$$

Dividing both sides by $2a$, we get

$$v_{\text{i}}t + \frac{1}{2}at^2 = d.$$

But this is just the third kinematic equation listed above (where we set $x(t) - x_0 = d$), so we are done. $\square$

§1.1 Partial Fractions

§2 Substitutions

§3 Symmetry

§4 Problems