

Exercise 4 : Task 2

4.2 a)

$$\delta(t-t_0) = \begin{cases} \infty & , \quad t=t_0 \\ 0 & , \quad t \neq t_0 \end{cases}$$

(b)

$$\delta(x-x_0) = \begin{cases} 1 & , \quad x=x_0 \\ 0 & , \quad x \neq x_0 \end{cases}$$

(c)

$$\delta(a-t) = \delta(-(t-a))$$

$$\delta(t-a) \neq \delta(-(t-a))$$

These two functions will always be mirrored across $x=0$. $\delta(t-a)$ has an impulse at $t=a$, $\delta(a-t)$ has an impulse at $t=-a$



4.3

→ Repeat with $f(t) = A$ for $0 \leq t \leq T$

(a)

$$f(t-t_0) = \begin{cases} A & , \quad t_0 \leq t < T+t_0 \\ 0 & , \quad \text{else} \end{cases}$$

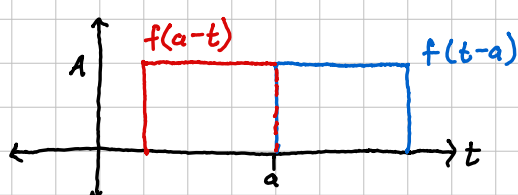
(b)

$$f(x-x_0) = \begin{cases} A & , \quad x_0 \leq x < T+x_0 \\ 0 & , \quad \text{else} \end{cases}$$

(c)

$$f(a-t) = f(-(t-a)) = \begin{cases} A & a \geq t > a-T \\ 0 & \text{else} \end{cases}$$

$$f(t-a) = \begin{cases} A & a \leq t < T+a \\ 0 & \text{else} \end{cases}$$



$$\begin{aligned} a \geq t > a-T \\ a-T \leq t \leq a \end{aligned}$$

These two functions are mirrored across the line $x=a$.

$$\begin{aligned} 0 \leq a-t < T \\ -a \leq -t < T-a \\ a \geq t > a-T \\ a-T < t \leq a \end{aligned}$$

4.11

$$\mathcal{F}\{(f*h)(t)\} =$$

$$\int_{-\infty}^{\infty} (f*h)(t) e^{-j2\pi\mu t} dt =$$

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau \right] e^{-j2\pi\mu t} dt =$$

$$\int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau) e^{-j2\pi\mu t} dt \right] d\tau =$$

$$\int_{-\infty}^{\infty} f(\tau) H(\mu) e^{j2\pi\mu\tau} d\tau =$$

$$H(\mu) \int_{-\infty}^{\infty} f(\tau) e^{-j2\pi\mu\tau} d\tau =$$

$$H(\mu) F(\mu) =$$

$$(F \cdot H)(\mu)$$

$$\mathcal{F}^{-1}\{(F \cdot H)(\mu)\} =$$

$$\int_{-\infty}^{\infty} (F \cdot H)(\mu) e^{j2\pi\mu t} d\mu =$$

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} F(\nu) H(\mu-\nu) d\nu \right] e^{j2\pi\mu t} d\mu =$$

$$\int_{-\infty}^{\infty} F(\nu) \left[\int_{-\infty}^{\infty} H(\mu-\nu) e^{j2\pi\mu t} d\mu \right] d\nu =$$

$$\int_{-\infty}^{\infty} F(\nu) h(t) e^{j2\pi\nu t} d\nu =$$

$$h(t) \int_{-\infty}^{\infty} F(\nu) e^{j2\pi\nu t} d\nu =$$

$$h(t) f(t) =$$

$$(f \cdot h)(t)$$