Exercise 4 : Task 5

$$f_0[n] = e^{-n} \qquad n = 0, 1, \dots N-1$$

$$f_1[n] = \begin{cases} e^{-n} & n = 0, 1, \dots N-1 \\ 0 & n = N, N+1, \dots 2N-1 \end{cases}$$

$$f[n] = \begin{cases} e^{-n} & n = 0, 1, \dots N-1 \\ 0 & \text{for all other } n \in \mathbb{Z} \end{cases}$$

1) $F_0[k] = \mathcal{F}\{f_0[n]\}$

$$F_0[k] = \sum_{n=0}^{N-1} f_0[n] \, e^{-j2\pi n \frac{k}{N\Delta T}} \qquad , \quad \Delta T = 1$$

$$F_0[k] = \sum_{n=0}^{N-1} e^{-n} \, e^{-j2\pi n \frac{k}{N}}$$

$$F_0[k] = \frac{1 - e^{-N} e^{-j2\pi k}}{1 - e^{-1} e^{-j2\pi k/N}} \qquad , \quad e^{-j2\pi m} = 1$$

$$F_0[k] = \frac{1 - e^{-N}}{1 - e^{-1} e^{-j2\pi k/N}}$$

2) $F_1[k] = \mathcal{F}\{f_1[n]\}$

$$F_1[k] = \sum_{n=0}^{2N-1} f_1[n] \, e^{-j2\pi n \frac{k}{2N\Delta T}} \qquad , \quad \Delta T = 1$$

$$F_1[k] = \sum_{n=0}^{N-1} e^{-n} \, e^{-j\pi n k/N}$$

$$F_1[k] = \frac{1 - e^{-N} e^{-j\pi k}}{1 - e^{-1} e^{-j\pi k/N}} \qquad e^{-j\pi k} = (-1)^k$$

$$F_1[k] = \frac{1 - (-1)^k e^{-N}}{1 - e^{-1} e^{-j\pi k/N}}$$

3) for $n = 0, 1, \dots N-1$, $\quad F_0[n] = F_1[2n]$

4) $F(e^{j\omega}) = \sum_{n \in \mathbb{Z}} f[n] \, e^{-j\omega n}$

$$F(e^{j\omega}) = \sum_{n=0}^{N-1} e^{-n} \, e^{-j\omega n}$$

$$F(e^{j\omega}) = \frac{1 - e^{-N} e^{-j\omega N}}{1 - e^{-1} e^{-j\omega}}$$

5) $F_0[k]$ and, $F[k]$ represent the frequency components of the same function, but $F_1[k]$ is sampled twice as densely in the frequency space. Both $F_0[k]$ and $F_1[k]$ are discretized versions of $F(e^{j\omega})$ within the frequency domain.

6) Zero padding in the time domain increases the frequency domain resolution of the DFT.