Optimal window for emphasizing a selected time-frequency pattern

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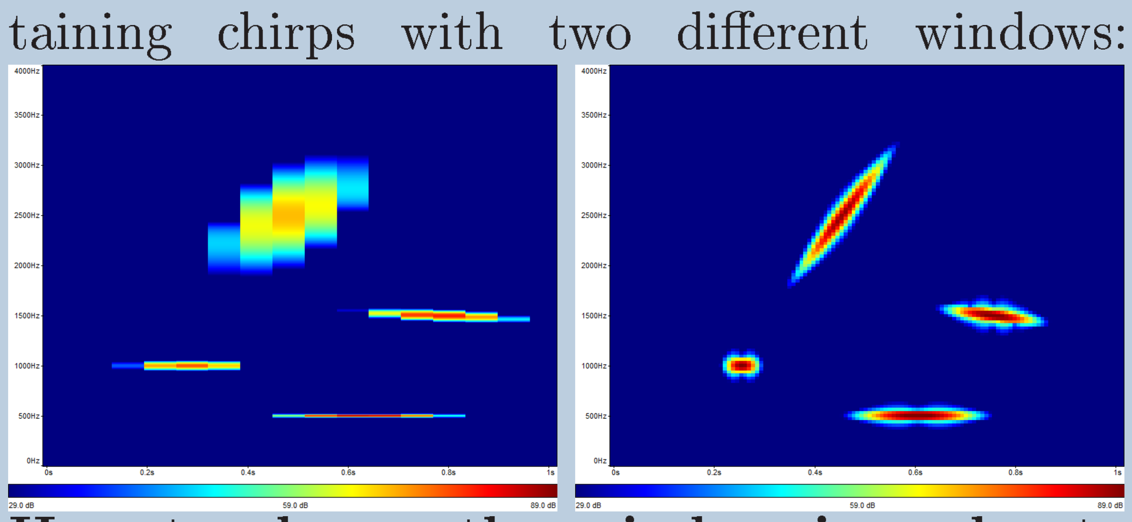
Problem

The uncertainty principle prevents Time-frequency representations to be accurate in both time and frequency.
Example: spectrogram of a signal containing chirps with two different windows:



How to choose the window in order to maximally concentrate a selected TF component?

The Gabor transform

Let $\psi \in \mathbb{C}^N$ a vector of length N, and denote by ψ_z, with $z=(\tau,\xi)$ a time-frequency shifted copy of ψ with time and frequency shifts τ and ξ:

$$\psi_z(t) = e^{\frac{2i\pi}{N}\xi t}\psi(t-\tau)\ .$$

The ψ_z will be termed Gabor atoms. In the discrete setting, τ, ξ belong to a lattice of discrete values: for $n, m \in [0, 1, 2, \cdots N-1]$, $\tau = an$, $\xi = bm$. The Gabor transform $\mathcal{V}_\psi s$ of a signal s is given by the projection on the atoms:

$$\mathcal{V}_\psi s(z) = \langle s, \psi_z \rangle. \qquad (1)$$

Concentration measures [1, 2]

Classical families of concentration measures are provided by L^p-(quasi)-norms, for $p \geq 0$

$$I_\psi(p) = \|\mathcal{V}_\psi s\|_p^p = \sum_z |\langle s, \psi_z\rangle|^p, \qquad (2)$$

which are closely related to Rényi entropies, and the Shannon entropy measure:

$$S(\mathcal{V}_\psi s) = -\sum_z |\mathcal{V}_\psi s(z)|^2 \log(|\mathcal{V}_\psi s(z)|^2). \qquad (3)$$

For $p < 2$, maximizing the L^p-norm (under unit L^2 norm constraint) favors spreading while for $p > 2$ this increases sparsity. The Shannon entropy is the derivative of I_ψ at $p = 2$. Small values for the Shannon entropy is a sign of sparsity.

References

[1] R. G. Baraniuk, P. Flandrin, A. J. E. M. Janssen, O. J. J. Michel, Measuring Time-Frequency Information Content Using the Rényi Entropies, IEEE Trans. info. th., 47, 4, 2001.

[2] F. Jaillet and B. Torresani. Time-frequency jigsaw puzzle: adaptive and multilayered Gabor expansions. International Journal for Wavelets and Multiresolution Information Processing, 5(2):293âĂŞ316, 2007.

[3] H. G. Feichtinger, D. Onchis-Moaca, B. Ricaud, B. Torrésani, C. Wiesmeyr *A method for optimizing the ambiguity function concentration*, EUSIPCO12, Bucarest (2012)

Acknowledgements

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`http://unlocx.math.uni-bremen.de`.

Optimization principle

Maximize the TF concentration via a variational approach, adapted from [3] and using a gradient method (faster convergence).

1). Make a first Gabor transform of the signal s and choose a region of interest in the TF plane.

2). Make a synthesis of the selected region by inverse Gabor transform. This gives a time signal y containing only the TF pattern of interest.

3). Optimize the Gabor window in order to maximize the L^p-norm, $p > 2$. The problem to solve reads

$$\psi_{\mathrm{opt}} = \underset{\psi:\|\psi\|=1}{\arg\max}\ \Gamma(\psi), \qquad \Gamma(\psi) = \sum_z |\mathcal{V}_\psi y(z)|^p\,, \quad p > 2. \qquad (4)$$

When a global maximum exists (depending on the shape of $|\mathcal{V}_\psi y|$), a gradient ascent method can be used. For this, it is needed to calculate the functional derivative Γ' of Γ with respect to the complex-valued function ψ. This reads:

$$\Gamma'(\psi) = \sum_z |\mathcal{V}_y\psi(z)|^{p-2}\ \mathcal{V}_y\psi(z)\ y_z. \qquad (5)$$

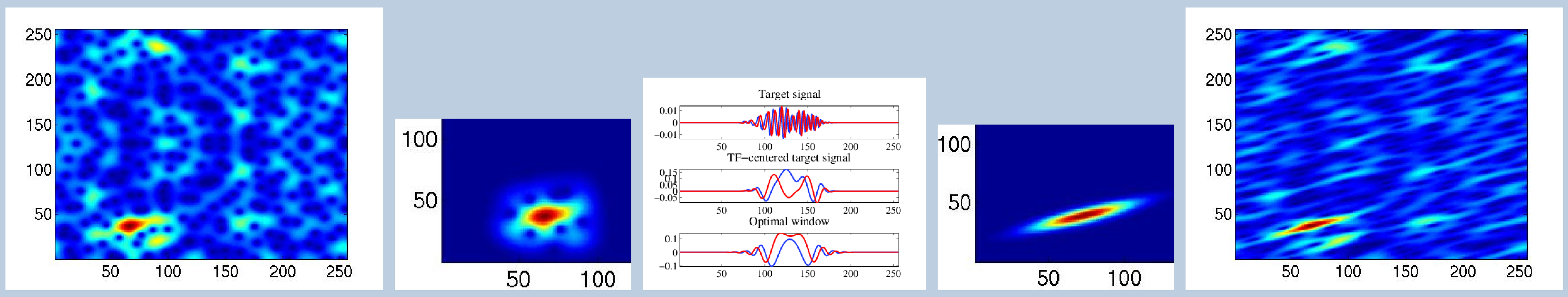
This is a Gabor multiplier with weight: $|\mathcal{V}_y\psi(z)|^{p-2}$ and window y. The optimization algorithm is as follows: Take an intial ψ_0 and a step length $\lambda > 0$, then at iteration k:

1. Compute: $\psi_{k+1} = \psi_k + \lambda\Gamma'(\psi_k)$,
2. Normalize ψ_{k+1} such that $\|\psi_{k+1}\|_2 = 1$,
3. Loop to step 1 until convergence is reached, i.e.: $\|\psi_{k+1} - \psi_k\| < \varepsilon$.

4). Use the optimal window for the Gabor transform of s.

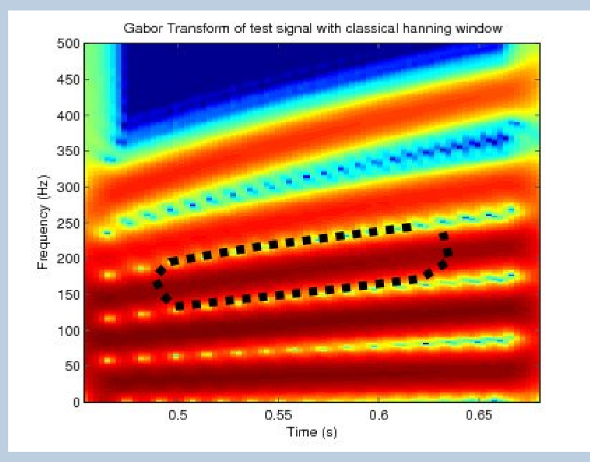
Results

Example 1. Small chirp signal embedded in white noise.

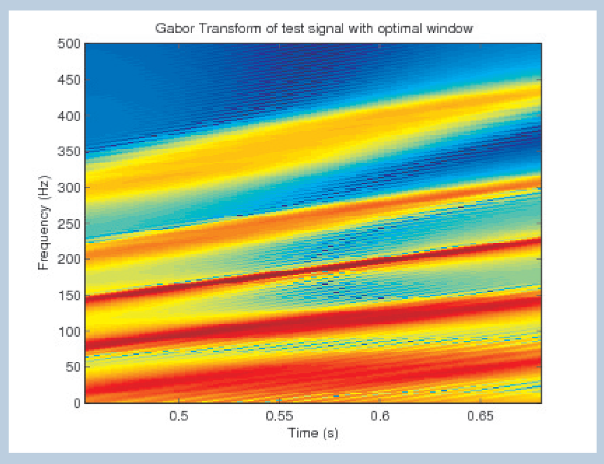


From left to right: 1) Spectrogram of the signal with a Gaussian window, 2) TF component extracted, 3) Graphs in the time domain. 4) Spectrogram of the optimal window with itself (Ambiguity function). 5) Spectrogram of the signal with the optimal window.

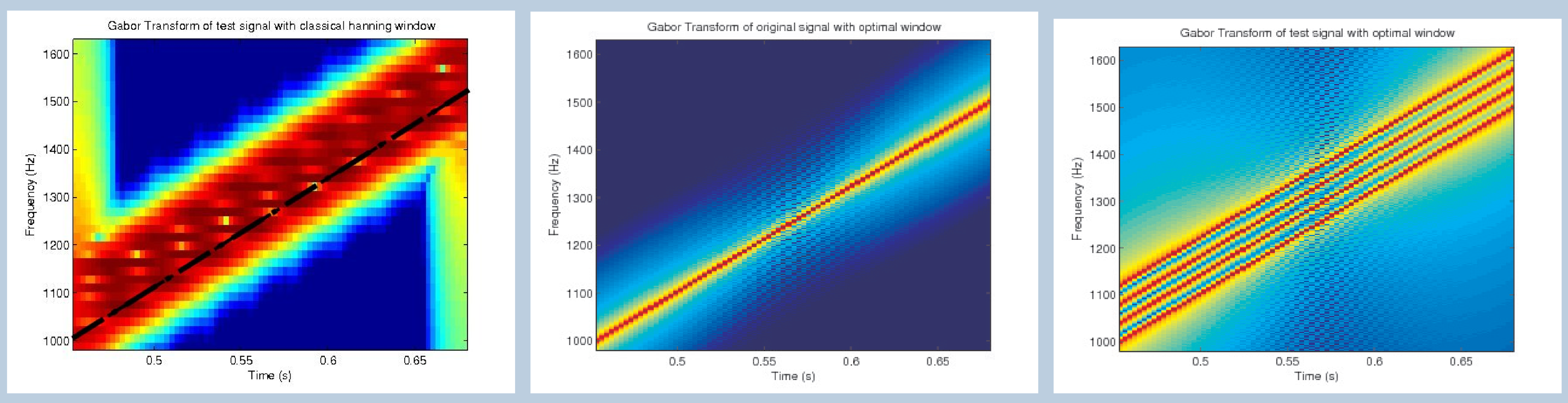
Example 2. Evolution of a motor noise when accelerating.



Left: with Hanning window and selection, right: with optimal window. The optimal window is obtained by optimizing on the middle ridge of the signal.



Example 3. 4 chirps with very close TF locations. User defined optimization: the user draws a curve in the TF plane which makes a synthetic chirp. The curve is supposed to match the pattern.



Left: Spectrogram w. Hanning window + user defined line, middle: Ambiguity function of the optimal window, right: spectrogram w. optimal window.

Advantages: Fast convergence for simple TF shapes. Require the computation of a Gabor multiplier at each step: fast computation when lattice parameters a, b are large, this is the case in practice.

Perspectives

- Add new constraints to control the shape of the window. Ex: the TF shape may be too elongated in one direction.
- Study convergence issues. Numerical tests shown that the algo. failed to converge for some shapes.
- Take into account the concentration of the dual window for an efficient synthesis.