

CONCOURS D'ENTREE EN 1ère ANNEE – SESSION AVRIL 2023

EPREUVE DE MATHEMATIQUES

Duration 3h00 - Coefficient 4

EXERCICE 1 : 3 Points

In an assembly of 250 persons, there are 120 men with black trousers, 85 men with tie of which 50 are in black trousseurs. A person is chosen at random from the assembly for an interview.

- 1) What is the probability that he is in a black trouser? 0.5pt**
- 2) What is the probability that he is in a black trouser and has a tie? 0.5pt**
- 3) What is the probability that he is in black trousers or has a tie? 1pt**
- 4) What is the probability that he wears neither a black trouser nor a tie? 1pt**

EXERCICE 2 : 5 Points

The municipal authority of a town in Cameroon envisage to construct low cost houses for municipal councilors. In order to fix the lodging price, the council accountant reveals the salaries X_i and rent proposition Y_i of a sample of eight agents. The results expressed in thousands of francs CFA are shown on the table below.

X_i	50	100	60	120	120	100	150	160
Y_i	15	20	15	30	25	25	40	35

The table above defines a statistical series with double character (X, Y)

Where X is the salary and Y the rent proposition.

- 1) a) Plot a scatter diagram representing the clouds of the series. **1pt**
b) Determine the coordinate of the mean of the series. **1pt**
- 2) a) Calculate the coefficient of linear correlation of the series. **1pt**
b) Can the salaries permit to explain the rent proposition? **0.5pt**
- 3) Use the least square method to determine the linear regression line of Y on X **1.5pt**

EXERCICE 3 : 4 Points

- 1) Write in a simpler form $c = \ln \frac{4}{9} + \frac{1}{2} \ln 36 + \frac{2}{3} \ln \frac{27}{8}$. **0.5pt**
- 2) Express in the form $X + Yi$, the complex numbers: **1.5pt**

$$Z_1 = 4(-2 + 3i) + 3(-5 - 8i) \quad \text{and} \quad Z_2 = \left(\frac{1+i}{2-i} \right)^2 + \frac{3+6i}{3-4i}$$

- 3) Solve the equation: $z^3 + (1+i)z^2 + (i-1)z - i = 0$.
- a) Given that one of the roots is purely imaginary. **0.5pt**
- b) Determine real numbers a and b such that :
- $$z^3 + (1+i)z^2 + (i-1)z - i = (z - ai)(z^2 + bz + c). \quad \textbf{0.5pt}$$
- c) Deduce all the roots of the equation. **1pt**

EXERCICE 4 : 8 Points

Consider the function of real values f defined on \mathbb{R} by: $f(x) =$

$$\begin{cases} 2 - e^x & \text{if } x < 0 \\ 1 + \ln(1 + x) & \text{if } x \geq 0 \end{cases}$$

And (C_f) is the curve representing f in an orthonormal reference frame

- 1) Determine the domain of definition of f 0.5pt**
- 2) Study the continuity and the differentiability f at 0 1pt**
- 3) Write the equation of the tangent to (C_f) at the point $x=0$ 0.5pt**
- 4) Calculate the limits of f at $x=-\infty$ and at $x = +\infty$ 0.5pt**
- 5) Study the infinite branches of the curve (C_f) . 0.5pt**
- 6) Study the variations and draw a table of variation of f 1pt**
- 7) Sketch (C_f) after sketching the tangent to (C_f) at the point $x = 0$ 1.5pt**
- 8) Using integration by parts, evaluate $\int_0^1 \ln(1 + x) dx$ 0.5pt**
- 9) Given that K is the area of the domain limited by (C_f) , the $x - axis$ and the lines with equations $x = -\ln 2$ and $x = 1$
 Write K in an integrable form and calculate K . 1pt**
- 10) Show that the restriction of g of f in the interval $[0; +\infty[$ is a bijection of $[0; +\infty[$ in an interval J and determine J . 0.5pt**
- 11) Show that the equation $g(x) = x$ has a unique solution α in the interval $1 \leq \alpha \leq 3$ 0.5pt**