<u>CONCOURS D'ENTREE EN 1ère ANNEE – SESSION D'AOUT 2019</u>

<u>EPREUVE DE MATHEMATIQUES</u>

<u>Duration 3h00 - Coefficient 4</u>

<u>EXERCISE 1</u>: **3.5 Points**

Consider in the set ₵ of complex numbers, the system of equations:

$$\begin{cases} iz + (1-i)z' = 0 & (i) \\ z - iz' = -i & (ii) \end{cases}$$

1- Solve the system of equation. **1pt**
2- Write each solution in exponential form. **1.5pt**
3- Give the cube roots of the complex number, $u = -i$ **1pt**

<u>EXERCISE 2 : 5.5 Points</u>

Consider the function f defined on IR by : $f(x) = e^{-x}\ln(1+e^{x})$ and (C) its curve represented in an orthonormal frame.

1- Study the infinite branches of f. **1pt**

2- Consider the function g defined on the interval $]-1;\ +\infty[$ by :

$$g(t) = \frac{t}{1+t} - \ln(1+t)$$

Study the variations of g and deduce the sign of g according to the values of t. **1pt**

3-

3.1. Calculate the derivative of f and express $f'(x)$ as a function of $g(e^{x})$. **0.75pt**

3.2. Deduce the sense of variation of the function f then draw a table of variations. **0.5pt**

3.3. Sketch the curve (C) and its asymptotes. **0.75pt**

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4- Let F be the function defined on IR by: $F(x) = \int_0^x f(t)dt$

4.1 Study the sense of variation of F. **0.5pt**

4.2 Use integration by parts to evaluate F. **1pt**

EXERCISE 3 : 5.5 Points

Let n be a natural number greater than or equal to 2.

Given n bags of coins, $S_1.S_2, S_3, \dots, S_n$. At the beginning, the bag S_1 has two black coins and 1 white coin and each of the other bags contain one black coin and one white coin We are required to study the successive withdrawal of a coin from these bags, carried out as follows:

First stage: We randomly draw a coin from S_1

Second stage: We place this coin in S_2 and withdraw a coin in S_2

Third Stage: After placing the coin in S_3, the coin drawn fromS_2, we randomly draw a coin from S_3 and so on.

For all natural number k such that $1 \leq k \leq n$, we note E_k the event << the coin drawn from S_k is white >>

1- Determine the probability E_1 denoted $p(E_1)$, and the conditional probabilities : $p({}^{E_2}/_{E_1})$, and $p({}^{E_2}/_{\bar{E}_1})$. Deduce the probability of the event E_2 denoted $p_r(E_2)$. **1.5pt**

2- For all natural number k such that : $1 \leq k \leq n$, the probability of the event E_k and denoted p_k. Justify the relation $p_{k+1} = \frac{1}{3}p_k + \frac{1}{3}$ **1pt**

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3- Consider the sequence (v_k) defined for all natural number k by $v_k = p_k - \frac{1}{2}$. Show that (v_k) is a geometric sequence. Stating its first term and common ratio. **1.5pt**

4- State the general term of the sequence (p_k) and calculate its limits. **0.5pt**

5- Given that $n = 10$. Determine for what values of k we have $0{,}4999 < p_k < 0{,}5000$. **1pt**

EXERCISE 4 : 5.5 Points

1- A group of enterprises has three warehouses. In the first, the average salary in thousand of francs is 100, in the second it is 120 and in the third it is 90. Given that the average salary of the group is104, there are 10 salaries in the first warehouse and 20 salaries in the second, what is the effective number of salaries in the third? **1.5pt**

2- This group of enterprise has 200 sale points distributed in many countries. Each sale point states its capital expressed in million of francs for the month of February 2019. The figures are shown on the table below:

77	129	56	67	78	176	75	101	68	96
89	46	178	125	97	26	50	29	151	141
120	77	33	75	17	113	97	80	144	109
96	60	152	20	84	123	105	57	102	100
61	118	140	95	84	152	101	105	63	95
115	3	104	107	47	83	75	50	119	137
2	46	80	99	120	159	110	108	138	143
93	94	124	17	110	115	139	55	146	72
31	121	109	128	64	136	103	112	149	114
136	144	96	107	86	62	100	105	78	46
50	137	71	74	113	141	42	135	89	38
114	136	115	28	71	114	123	87	124	94
127	34	116	111	107	93	29	113	99	115
113	122	137	86	59	107	78	189	5	114
121	116	68	120	81	124	124	67	93	107
125	123	134	140	49	96	101	99	102	72
79	48	90	128	142	86	72	117	28	16
81	106	125	79	35	82	124	50	77	149
98	103	103	101	97	96	193	152	127	107
109	127	76	97	62	105	50	80	108	62

2.1. Draw a group frequency distribution table from these results. You may use the following intervals:
$[0,30[$; $[30,50[$; $[50,70[$; $[70,90[$; $[90,100[$; $[100,110[$;
$[110,120[$; $[120,130[$; $[130,150[$; $[150,200[$ **2.5pts**

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2.2. Determine the frequency and number of the sale points with the following capital:

i) Less than 105 million of francs; **0.5pt**

ii) Greater than 60 million of francs; **0.5pt**

iii) Between 35 and 150 million of francs. **0.5pt**

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