EXERCISE I (5 Mark

Given that the sequence u_n $\begin{cases} u_0 = a \\ u_{n+1} = u_n(2-u_n) \end{cases}$ Where a is a given real number with 0 < *a* < 1.

1. Suppose that $a = \frac{1}{8}$

a. Calculate u_1 and u_2. **0.5mk**

b. Draw in an orthogonal reference frame the curve P, representing the function f: $f(x) = x(2-x)$ and the straight line (*y* = *x*). **0.75mk**

c. Use d and p to construct on the x-axis the point A_1, A_2, A_3 with abscissa, respectively u_1, u_2, u_3. **1mk**

2. We suppose in this question that a is any, number (0 < *a* < 1).

a. Prove, by induction that $0 < u_n < 1$. **0.5mk**

b. Show that, (u_n)is increasing . **0.5mk**

c. Deduce the convergence of the sequence? **0.25mk**

3. Given once more that $a = \frac{1}{8}$ and consider the sequence $v_n = 1 - u_n$.

a. Express v_{n+1} as a function of v_n **0.5mk**

b. deduce the expression of v_n as a function of *n*. **0.5mk**

c. Determiner the limit of v_n and that of u_n. **0.5mk**

EXECISE II (5 Marks)

The table below describes the mean number y of object that a worker who just stated work on a channel produce in one day, let x^{th} day that he worked on this channel .

x_i	1	3	5	7	9
y_i	27	41	46	48	49

A. Here we are going to use a calculator to calculate the statistical functions (the details of the calculations are not necessary)

1. The plane p is considered as an orthogonal reference frame with graphical unit 1cm for a day on the x-axis and 1cm for a day on the y axis draw a scatter diagram representing the associated points (x_i, y_i). **1mk**

2. Determine the coordinate of the mean point G and plot this point on the same graph

3. a. Determine the value to the nearest 10^{-2} the coefficient of the linear correlation (x_i, y_i). **1mk**

b. Give an equation of the linear regression line of y on x by the least square method. Represent the straight line d on your graph. **1 mk**

c. What day, the worker will produce 83 objects **1mk**

EXERCISE III (5 Marks)

Given that i is a complex number with modulus 1 and argument $\frac{\pi}{2}$. The complex plane is taken as and orthogonal reference $(O; \vec{u}, \vec{v})$ with graphical unit 1 cm.

1. Solve in the set of complex number the equation: $z^2 + 3z + 3 = 0$.Giving your solution in algebraic form **1.5mk**

2. Given that the complex number: $z_1 = -\frac{3}{2} + \frac{\sqrt{3}}{2}i$ and $z_2 = \overline{z_1}$.

a. Express z_1 in trigonometric form. **0.5mk**

b. Construct with precision in the frame $(O\,;\vec{u},\vec{v})$ the point A and B with coordinate respectively z_1 and z_2. Leave visible the treat of your construction. **1mk**

3. Given that D is point represented by $z_3 = \frac{7}{2} - \frac{\sqrt{3}}{2}i$ and K the point represented by $z_4 = 1$.

a. Show that the point A, B and D belong to a circle C with center K**.** **1mk**

b. show that the point K is the midpoint of the segment $[AD]$. **1mk**

EXERCISE IV (5 Marks)

Two breeders produce a race of decorating fish that can only take its final color after the age of three months:

- .For the fry of the first breed, between the age of two months and the age of three months 10% die, 75% becomes red and the remaining 15% become grey

- For the fry of the second breed ,between the age of two months and the age of tree months, 5% die, 65% becomes red and the remaining 30% becomes grey. A pet shop buys the fry at the age of two months 60% from the first breeder and 40% from the second.

1. A child buys a fish the next day he arrived the pet shop that is at the age of two months

a. Show that the probability that the fish is alive one month later is 0.92 **0.75mk**

b. Determine the probability that one month later the fish is red **0.75mk**

c. Knowing that the fish becomes grey at the age of three months what is the probability that it come from the first breeder. **0.75mk**

2. A person chooses at random an independently 5 fries of 2 month old, what is the probability that one month later only three are alive. Give your answer to the nearest 10^{-2}. **0.75mk**

3. The pet shop decide to keep the fries until the age of three months so that they will be sold with their final color the shop gains 1 euro if the fish is red, 0.25 euro I it is grey and loses 0.1 euro if it did not live **0.75mk**

Let X be the random variable equal to the algebraic gain of the pet shop per fish bought

Determine the probability law of X, determine the expectation of X rounded to the nearest centime.. **1.25mk**