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Talk Notes

Slide 1: Hello. Today, I would like to talk about wedge product matrices and their applications.

Slide 2:

- The main topic my thesis addresses is good old-fashioned linear algebra, which sees widespread use in many different fields of mathematics.

- In my thesis, I define wedge product matrices which generalise the determinant of a square matrix with entries in a commutative ring. Then, I investigate a variety of applications of wedge product matrices.

- In this talk, I will focus on two particular applications — quasideterminants and a matrix orbit of principal congruence subgroups ...
- Quasideterminants appear in the field of Lie algebras, the matrix orbit appears in the context of number theory. STATE THIS CLEARLY!

Slide 3:

- So, what are wedge product matrices? Let R be a commutative ring, ...

- The k^{th} wedge product matrix of A, denoted by $\Lambda^k(A)$, is given by the following equation in red. The equality takes place in the free R-module $\bigwedge^k(R^n)$. Let me explain what this means ...

- If you take the wedge product of k columns of A as selected by the set J, you obtain an element of $\bigwedge^k(R^n)$. By rewriting this with the standard basis, the resulting coefficients form a column of $\Lambda^k(A)$ ...

Slide 4:

- So, what are the properties satisfied by wedge product matrices? The first property states that wedge product matrices are multiplicative ...

- The second equation states that the elements of $\Lambda^k(A)$ are the determinants of $k \times k$ minors of A ...

- As a particular case of wedge product matrices, if we set $k = n$ then ...

- Finally, the determinant of $\Lambda^k(A)$ is ...

Slide 5:

- Adjugate matrices as defined in my thesis are closely related to wedge product matrices and are derived from the Laplace expansion of the determinant ...

- In the summation defining Laplace expansion, if we collect all the terms except for the $\Lambda^k(A)$ term ...

- The signs s_L and s_H are numbers, either -1 or 1, which depend on ...

- By definition of $\Upsilon^{n-k}(A)$, the following equation involving $\Lambda^k(A)$ is satisfied ...

- Adjugate matrices share similar properties to their wedge product counterparts ...

Slide 6:

- As the first application of wedge product matrices, we will look at quasideterminants, which are ...

- In his book "Yangians and classical Lie algebras", Alexander Molev uses quasideterminants to ...

- Molev also proves that in a commutative ring, ...

Slide 7:

- In order to understand what a quasideterminant is, it is best to peruse an example ...

- In my thesis, I generalise the definition of a quasideterminant, by allowing the selection of more than one row or one column ...

- The equations below are the definitions of the quasideterminant and the general quasideterminant. Note that quasideterminants only exist provided that ...

- In a commutative ring, Molev proved that ...

- In my thesis, I generalise this particular result where...

Slide 8:

- Here are some properties satisfied by quasideterminants in a commutative ring ...

- The second equation below states that the determinant of a general quasideterminant can be computed by the original quasideterminants defined by Gel'fand and Retakh ...

- I would like to point out that these equations only make sense ...

Slide 9:

- As our final application of wedge product matrices, we will investigate a particular matrix orbit of principal congruence subgroups.

- The setting for this is ...

- Bump and Hoffstein compute Fourier expansions of such minimal parabolic Eisenstein series ...

- Bump and Hoffstein work in the ring of ...

- $\Gamma(3)$ is defined as ...

- The subgroup $\Gamma_\infty(3)$ of $\Gamma(3)$ is defined by ...

Slide 10:

- Let A be an element of $\Gamma(3)$. In order to investigate the coset $\Gamma_\infty(3) \cdot A$, ...

- To see why Λ^1 and Λ^2 invariants are preserved in the matrix orbit $\Gamma_\infty(3) \cdot A$, ...

- We collect the Λ^1 and Λ^2 invariants in a six-tuple ...

Slide 11:

- The invariant set of A satisfies four important conditions, which are called ...

- With regards to condition (I3), the gcd makes sense because ...

- The main theorem regarding invariant sets is given below ...

Slide 12:

- Before we proceed, I will connect my approach to the invariants of a $\Gamma_\infty(3)$ orbit with that of Bump and Hoffstein ...

- The invariants of A, in Bump and Hoffstein's paper, is ...

- To see how the Λ^2 invariants are related to the bottom row of ${}^\iota A$, ...

- Thus, the invariants as defined in my thesis are ...

Slide 13:

- Finally, I would like to talk about a particular theorem proved in my thesis ...

- In my thesis, I proved that one can construct a matrix representative X such that ...

- The case where $A_1 = B_1 = 0$ is ...

- The approach I took to constructing the representative hinges on multiplying on the ring by these three block matrices, which is inspired by Robert Steinberg in his paper ...

- For instance, if you have a matrix in $GL_3(Z)$ and then multiply on the right by ...

- This works in the more general ring of Eisenstein integers because ...

Slide 14: This concludes the talk. Thank you for listening.