Table of Basic Integrals

Basic Forms

$\int x^n dx = \frac{1}{n+1}x^{n+1}, \ \ n \neq -1$

$\int \frac{1}{x} dx = \ln|x|$

$\int u dv = uv - \int v du$

$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b|$

Integrals of Rational Functions

$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a}$

$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1$

$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$

$\int \frac{1}{1+x^2} dx = \tan^{-1} x$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$\int \frac{x}{a^2+x^2} dx = \frac{1}{2}\ln|a^2+x^2|$

$\int \frac{x^2}{a^2+x^2} dx = x - a\tan^{-1}\frac{x}{a}$

$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2|$

$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$

$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, \ a \neq b$

$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|a+x|$

$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$

Integrals with Roots

$\int \sqrt{x-a}\, dx = \frac{2}{3}(x-a)^{3/2}$

$\int \frac{1}{\sqrt{x \pm a}}\, dx = 2\sqrt{x \pm a}$

$\int \frac{1}{\sqrt{a-x}}\, dx = -2\sqrt{a-x}$

$\int x\sqrt{x-a}\, dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, \text{ or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, \text{ or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$

$\int \sqrt{ax+b}\, dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b}$

$\int (ax+b)^{3/2}\, dx = \frac{2}{5a}(ax+b)^{5/2}$

$\int \frac{x}{\sqrt{x \pm a}}\, dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a}$

$\int \sqrt{\frac{x}{a-x}}\, dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a}$

$\int \sqrt{\frac{x}{a+x}}\, dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right]$

$\int x\sqrt{ax+b}\, dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b}$

$\int \sqrt{x(ax+b)}\, dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right]$

$\int \sqrt{x^3(ax+b)}\, dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|$

$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right|$

$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

$\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2}$

$\int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

$\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\frac{x}{a}$

$\int \frac{x}{\sqrt{x^2 \pm a^2}}\, dx = \sqrt{x^2 \pm a^2}$

$\int \frac{x}{\sqrt{a^2-x^2}}\, dx = -\sqrt{a^2-x^2}$

$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\, dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right|$

$\int \sqrt{ax^2+bx+c}\, dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx^+c)}\right|$

$\int x\sqrt{ax^2+bx+c}\, dx = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c}\left(-3b^2+2abx+8a(c+ax^2)\right) + 3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right)$

$\int \frac{1}{\sqrt{ax^2+bx+c}}\, dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$

$\int \frac{x}{\sqrt{ax^2+bx+c}}\, dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$

$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$

Integrals with Logarithms

$\int \ln ax\, dx = x\ln ax - x$

$\int x\ln x\, dx = \frac{1}{2}x^2\ln x - \frac{x^2}{4}$

$\int x^2\ln x\, dx = \frac{1}{3}x^3\ln x - \frac{x^3}{9}$

$\int x^n\ln x\, dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right), \quad n \neq -1$

$\int \frac{\ln ax}{x}\, dx = \frac{1}{2}(\ln ax)^2$

$\int \frac{\ln x}{x^2}\, dx = -\frac{1}{x} - \frac{\ln x}{x}$

$\int \ln(ax+b)\, dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0$

$\int \ln(x^2+a^2)\, dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x$

$\int \ln(x^2-a^2)\, dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x$

$\int \ln\left(ax^2+bx+c\right)\, dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2+bx+c\right)$

$\int x\ln(ax+b)\, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b)$

$\int x\ln\left(a^2-b^2x^2\right)\, dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right)$

$\int (\ln x)^2\, dx = 2x - 2x\ln x + x(\ln x)^2$

$\int (\ln x)^3\, dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x$

$\int x(\ln x)^2\, dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x$

$\int x^2(\ln x)^2\, dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x$

Integrals with Exponentials

$\int e^{ax}\, dx = \frac{1}{a}e^{ax}$

$\int \sqrt{x}e^{ax}\, dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\text{erf}\left(i\sqrt{ax}\right), \text{ where erf}(x) =$

$\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$

$\int xe^x\ dx = (x-1)e^x$

$\int xe^{ax}\ dx = \left(\frac{x}{a}-\frac{1}{a^2}\right)e^{ax}$

$\int x^2e^x\ dx = \left(x^2-2x+2\right)e^x$

$\int x^2e^{ax}\ dx = \left(\frac{x^2}{a}-\frac{2x}{a^2}+\frac{2}{a^3}\right)e^{ax}$

$\int x^3e^x\ dx = \left(x^3-3x^2+6x-6\right)e^x$

$\int x^ne^{ax}\ dx = \dfrac{x^ne^{ax}}{a} - \dfrac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x$

$\int x^ne^{ax}\ dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n,-ax]$, where $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t$

$\int e^{ax^2}\ dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right)$

$\int e^{-ax^2}\ dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right)$

$\int xe^{-ax^2}\ dx = -\dfrac{1}{2a}e^{-ax^2}$

$\int x^2e^{-ax^2}\ dx = \dfrac{1}{4}\sqrt{\dfrac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \dfrac{x}{2a}e^{-ax^2}$

Integrals with Trigonometric Functions

$\int \sin ax\ dx = -\frac{1}{a}\cos ax$

$\int \sin^2 ax\ dx = \frac{x}{2}-\frac{\sin 2ax}{4a}$

$\int \sin^3 ax\ dx = -\frac{3\cos ax}{4a}+\frac{\cos 3ax}{12a}$

$\int \sin^n ax\ dx = -\frac{1}{a}\cos ax\ \ {}_2F_1\left[\frac12,\frac{1-n}{2},\frac32,\cos^2 ax\right]$

$\int \cos ax\ dx = \frac{1}{a}\sin ax$

$\int \cos^2 ax\ dx = \frac{x}{2}+\frac{\sin 2ax}{4a}$

$\int \cos^3 axdx = \frac{3\sin ax}{4a}+\frac{\sin 3ax}{12a}$

$\int \cos^p axdx = -\frac{1}{a(1+p)}\cos^{1+p}ax\times{}_2F_1\left[\frac{1+p}{2},\frac12,\frac{3+p}{2},\cos^2 ax\right]$

$\int \cos x\sin x\ \ dx = \frac12\sin^2 x + c_1 = -\frac12\cos^2 x + c_2 = -\frac14\cos 2x + c_3$

$\int \cos ax\sin bx\ dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a\neq b$

$\int \sin^2 ax\cos bx\ dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$

$\int \sin^2 x\cos x\ dx = \frac13\sin^3 x$

$\int \cos^2 ax\sin bx\ dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$

$\int \cos^2 ax\sin ax\ dx = -\frac{1}{3a}\cos^3 ax$

$\int \sin^2 ax\cos^2 bxdx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$

$\int \sin^2 ax\cos^2 ax\ dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$

$\int \tan ax\ dx = -\frac1a\ln\cos ax$

$\int \tan^2 ax\ dx = -x + \frac1a\tan ax$

$\int \tan^n ax\ dx = \frac{\tan^{n+1}ax}{a(1+n)}\times{}_2F_1\left(\frac{n+1}{2},1,\frac{n+3}{2},-\tan^2 ax\right)$

$\int \tan^3 axdx = \frac1a\ln\cos ax + \frac{1}{2a}\sec^2 ax$

$\int \sec x\ dx = \ln|\sec x+\tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right)$

$\int \sec^2 ax\ dx = \frac1a\tan ax$

$\int \sec^3 x\ dx = \frac12\sec x\tan x + \frac12\ln|\sec x+\tan x|$

$\int \sec x\tan x\ dx = \sec x$

$\int \sec^2 x\tan x\ dx = \frac12\sec^2 x$

$\int \sec^n x\tan x\ dx = \frac1n\sec^n x, n\neq 0$

$\int \csc x\ dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C$

$\int \csc^2 ax\ dx = -\frac1a\cot ax$

$\int \csc^3 x\ dx = -\frac12\cot x\csc x + \frac12\ln|\csc x-\cot x|$

$\int \csc^n x\cot x\ dx = -\frac1n\csc^n x, n\neq 0$

$\int \sec x\csc x\ dx = \ln|\tan x|$

Products of Trigonometric Functions and Monomials

$\int x\cos x\ dx = \cos x + x\sin x$

$\int x\cos ax\ dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$

$\int x^2\cos x\ dx = 2x\cos x + \left(x^2-2\right)\sin x$

$\int x^2\cos ax\ dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax$

$\int x^n\cos xdx = -\frac12(i)^{n+1}\left[\Gamma(n+1,-ix)+(-1)^n\Gamma(n+1,ix)\right]$

$\int x^n\cos ax\ dx = \frac12(ia)^{1-n}\left[(-1)^n\Gamma(n+1,-iax)-\Gamma(n+1,ixa\right.$

$\int x\sin x\ dx = -x\cos x + \sin x$

$\int x\sin ax\ dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}$

$\int x^2\sin x\ dx = \left(2-x^2\right)\cos x + 2x\sin x$

$\int x^2\sin ax\ dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2}$

$\int x^n\sin x\,dx = -\frac12(i)^n\left[\Gamma(n+1,-ix)-(-1)^n\Gamma(n+1,-ix)\right]$

$\int x\cos^2 x\ dx = \frac{x^2}{4}+\frac18\cos 2x + \frac14 x\sin 2x$

$\int x\sin^2 x\ dx = \frac{x^2}{4}-\frac18\cos 2x - \frac14 x\sin 2x$

$\int x\tan^2 x\ dx = -\frac{x^2}{2} + \ln\cos x + x\tan x$

$\int x\sec^2 x\ dx = \ln\cos x + x\tan x$

Products of Trigonometric Functions and Exponentials

$\int e^x\sin x\ dx = \frac12 e^x(\sin x - \cos x)$

$\int e^{bx}\sin ax\ dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax)$

$\int e^x\cos x\ dx = \frac12 e^x(\sin x + \cos x)$

$\int e^{bx}\cos ax\ dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax)$

$\int xe^x \sin x \; dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x)$

$\int xe^x \cos x \; dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x)$

Integrals of Hyperbolic Functions

$\int \cosh ax \; dx = \frac{1}{a}\sinh ax$

$$\int e^{ax}\cosh bx \, dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases}$$

$\int \sinh ax \; dx = \frac{1}{a}\cosh ax$

$$\int e^{ax}\sinh bx \, dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases}$$

$\int \tanh ax \, dx = \frac{1}{a}\ln\cosh ax$

$$\int e^{ax}\tanh bx \, dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1+\dfrac{a}{2b}, 1, 2+\dfrac{a}{2b}, -e^{2bx}\right] \\ \qquad -\dfrac{1}{a}e^{ax}{}_2F_1\left[1, \dfrac{a}{2b}, 1+\dfrac{a}{2b}, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases}$$

$\int \cos ax \cosh bx \, dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right]$

$\int \cos ax \sinh bx \, dx = \frac{1}{a^2+b^2}\left[b\cos ax\cosh bx + a\sin ax\sinh bx\right]$

$\int \sin ax \cosh bx \, dx = \frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx + b\sin ax\sinh bx\right]$

$\int \sin ax \sinh bx \, dx = \frac{1}{a^2+b^2}\left[b\cosh bx\sin ax - a\cos ax\sinh bx\right]$

$\int \sinh ax \cosh ax dx = \frac{1}{4a}\left[-2ax + \sinh 2ax\right]$

$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2-a^2}\left[b\cosh bx\sinh ax - a\cosh ax\sinh bx\right]$