Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \quad (1)$$

$$\int \frac{1}{x}dx = \ln|x| \quad (2)$$

$$\int udv = uv - \int vdu \quad (3)$$

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| \quad (4)$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2}dx = -\frac{1}{x+a} \quad (5)$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \quad (6)$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \quad (7)$$

$$\int \frac{1}{1+x^2}dx = \tan^{-1}x \quad (8)$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \quad (9)$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2| \quad (10)$$

$$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a} \quad (11)$$

$$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \quad (12)$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \quad (13)$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a \neq b \quad (14)$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x| \quad (15)$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \quad (16)$$

Integrals with Roots

$$\int \sqrt{x-a}dx = \frac{2}{3}(x-a)^{3/2} \quad (17)$$

$$\int \frac{1}{\sqrt{x\pm a}}dx = 2\sqrt{x\pm a} \quad (18)$$

$$\int \frac{1}{\sqrt{a-x}}dx = -2\sqrt{a-x} \quad (19)$$

$$\int x\sqrt{x-a}dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \quad (20)$$

$$\int \sqrt{ax+b}dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \quad (21)$$

$$\int (ax+b)^{3/2}dx = \frac{2}{5a}(ax+b)^{5/2} \quad (22)$$

$$\int \frac{x}{\sqrt{x\pm a}}dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} \quad (23)$$

$$\int \sqrt{\frac{x}{a-x}}dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \quad (24)$$

$$\int \sqrt{\frac{x}{a+x}}dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right] \quad (25)$$

$$\int x\sqrt{ax+b}dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \quad (26)$$

$$\int \sqrt{x(ax+b)}dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right] \quad (27)$$

$$\int \sqrt{x^3(ax+b)}dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right| \quad (28)$$

$$\int \sqrt{x^2\pm a^2}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \quad (29)$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{30}$$

$$\int x\sqrt{x^2\pm a^2}dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{36}$$

$$\int \sqrt{ax^2+bx+c}dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{37}$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c} \times\left(-3b^2+2abx+8a(c+ax^2)\right) +3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln axdx = x\ln ax - x \tag{42}$$

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}(\ln ax)^2 \tag{43}$$

$$\int \ln(ax+b)dx = \left(x+\frac{b}{a}\right)\ln(ax+b) - x, a\neq 0 \tag{44}$$

$$\int \ln(x^2+a^2)\,\mathrm{dx} = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{45}$$

$$\int \ln(x^2-a^2)\,\mathrm{dx} = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{46}$$

$$\int \ln\left(ax^2+bx+c\right)dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right) \tag{47}$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2-\frac{b^2}{a^2}\right)\ln(ax+b) \tag{48}$$

$$\int x\ln\left(a^2-b^2x^2\right)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2-\frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{49}$$

Integrals with Exponentials

$$\int e^{ax}dx = \frac{1}{a}e^{ax} \tag{50}$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}\left(i\sqrt{ax}\right), \quad \text{where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{51}$$

$$\int xe^{x}dx = (x-1)e^{x} \tag{52}$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{53}$$

$$\int x^2e^{x}dx = \left(x^2 - 2x + 2\right)e^{x} \tag{54}$$

$$\int x^2e^{ax}dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{55}$$

$$\int x^3e^{x}dx = \left(x^3 - 3x^2 + 6x - 6\right)e^{x} \tag{56}$$

$$\int x^ne^{ax}\,\mathrm{d}x = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{57}$$

$$\int x^ne^{ax}\,\mathrm{d}x = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n,-ax], \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{58}$$

$$\int e^{ax^2}\,\mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}\left(ix\sqrt{a}\right) \tag{59}$$

$$\int e^{-ax^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}\left(x\sqrt{a}\right) \tag{60}$$

$$\int xe^{-ax^2}\,\mathrm{dx} = -\frac{1}{2a}e^{-ax^2} \tag{61}$$

$$\int x^2e^{-ax^2}\,\mathrm{dx} = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\operatorname{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{62}$$

Integrals with Trigonometric Functions

$$\int \sin axdx = -\frac{1}{a}\cos ax \tag{63}$$

$$\int \sin^2 axdx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{64}$$

$$\int \sin^n axdx = -\frac{1}{a}\cos ax\ {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \tag{65}$$

$$\int \sin^3 axdx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{66}$$

$$\int \cos axdx = \frac{1}{a}\sin ax \tag{67}$$

$$\int \cos^2 axdx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{68}$$

$$\int \cos^p axdx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \tag{69}$$

$$\int \cos^3 axdx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{70}$$

$$\int \cos ax \sin bxdx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{71}$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \quad (72)$$

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x \quad (73)$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \quad (74)$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a}\cos^3 ax \quad (75)$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \quad (76)$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \quad (77)$$

$$\int \tan ax dx = -\frac{1}{a}\ln \cos ax \quad (78)$$

$$\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax \quad (79)$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \quad (80)$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln \cos ax + \frac{1}{2a}\sec^2 ax \quad (81)$$

$$\int \sec x dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \quad (82)$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax \quad (83)$$

$$\int \sec^3 x \, \mathrm{dx} = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| \quad (84)$$

$$\int \sec x \tan x dx = \sec x \quad (85)$$

$$\int \sec^2 x \tan x dx = \frac{1}{2}\sec^2 x \quad (86)$$

$$\int \sec^n x \tan x dx = \frac{1}{n}\sec^n x, n \neq 0 \quad (87)$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \quad (88)$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax \quad (89)$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| \quad (90)$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x, n \neq 0 \quad (91)$$

$$\int \sec x \csc x dx = \ln|\tan x| \quad (92)$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x dx = \cos x + x \sin x \quad (93)$$

$$\int x \cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \quad (94)$$

$$\int x^2 \cos x dx = 2x \cos x + \left(x^2 - 2\right)\sin x \quad (95)$$

$$\int x^2 \cos ax dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax \quad (96)$$

$$\int x^n cosx dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)\right] \quad (97)$$

$$\int x^n cosax dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] \quad (98)$$

$$\int x \sin x dx = -x \cos x + \sin x \quad (99)$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (100)$$

$$\int x^2 \sin x dx = \left(2 - x^2\right) \cos x + 2x \sin x \quad (101)$$

$$\int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (102)$$

$$\int x^n \sin x dx = -\frac{1}{2}(i)^n \left[\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, ix)\right] \quad (103)$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (104)$$

$$\int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (105)$$

$$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (106)$$

$$\int e^{bx} \cos ax dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (107)$$

$$\int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (108)$$

$$\int x e^x \cos x dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (109)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a} \sinh ax \quad (110)$$

$$\int e^{ax} \cosh bx dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2}[a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (111)$$

$$\int \sinh ax dx = \frac{1}{a} \cosh ax \quad (112)$$

$$\int e^{ax} \sinh bx dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2}[-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (113)$$

$$\int e^{ax} \tanh bx dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1\left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx}\right] \\ \quad -\frac{1}{a} e^{ax} {}_2F_1\left[\frac{a}{2b}, 1, 1E, -e^{2bx}\right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (114)$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \quad (115)$$

$$\int \cos ax \cosh bx dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \quad (116)$$

$$\int \cos ax \sinh bx dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \quad (117)$$

$$\int \sin ax \cosh bx dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \quad (118)$$

$$\int \sin ax \sinh bx dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \quad (119)$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (120)$$

$$\int \sinh ax \cosh bx dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \quad (121)$$