The Kaluza-Klein example from section 2.5 of hep-th/0701238

This example shows how to use `@split_index` in a somewhat more complicated setting.

```
{\mu,\nu,\rho,\sigma,\kappa,\lambda,\eta,\chi#}::Indices(full, position=independent).
{m,n,p,q,r,s,t,u,v,m#}::Indices(subspace, position=independent, parent=full).
```

Assigning list property Indices to $\{\mu, \nu, \rho, \sigma, \kappa, \lambda, \eta, \chi\#\}$.
Assigning list property Indices to $\{m, n, p, q, r, s, t, u, v, m\#\}$.
Note the appearance of `parent=full`. This indicates that the indices in the second set span a subspace of the indices in the first set. Also note that we have declared the indices as `position=independent`, to prevent cadabra from automatically lowering or raising them when canonicalising (it does not really help us here).

The remaining declarations are standard,

```
\partial{#}::PartialDerivative.
g_{\mu\nu}::Metric.
g^{\mu\nu}::InverseMetric.
g_{\mu? \nu?}::Symmetric.
g^{\mu? \nu?}::Symmetric.
h_{m n}::Metric.
h^{m n}::InverseMetric.
\delta^{\mu?}_{\nu?}::KroneckerDelta.
\delta_{\mu?}^{\nu?}::KroneckerDelta.
F_{m n}::AntiSymmetric.
```

Assigning property PartialDerivative to $\partial\#$.
Assigning property Metric to $g_{\mu\nu}$.
Assigning property InverseMetric to $g^{\mu\nu}$.
Assigning property Symmetric to $g_{\mu?\nu?}$.
Assigning property Symmetric to $g^{\mu?\nu?}$.
Assigning property Metric to h_{mn}.
Assigning property InverseMetric to h^{mn}.
Assigning property KroneckerDelta to $\delta^{\mu?}{}_{\nu?}$.
Assigning property KroneckerDelta to $\delta_{\mu?}{}^{\nu?}$.
Assigning property AntiSymmetric to F_{mn}.
We will want to expand the Riemann tensor in terms of the metric. The following two substitution rules do the conversion from Riemann tensor to Christoffel symbol and from Christoffel symbol to metric (a library with common substitution rules like these is in preparation).

```
RtoG:= R^{\lambda?}_{\mu?\nu?\kappa?} ->
 - \partial_{\kappa?}{ \Gamma^{\lambda?}_{\mu?\nu?} }
 + \partial_{\nu?}{ \Gamma^{\lambda?}_{\mu?\kappa?} }
 - \Gamma^{\eta}_{\mu?\nu?} \Gamma^{\lambda?}_{\kappa?\eta}
 + \Gamma^{\eta}_{\mu?\kappa?} \Gamma^{\lambda?}_{\nu?\eta};

Gtog:= \Gamma^{\lambda?}_{\mu?\nu?} ->
  (1/2) * g^{\lambda?\kappa} (
      \partial_{\nu?}{ g_{\kappa\mu?} } + \partial_{\mu?}{ g_{\kappa\nu?} } - \partial_{\kap
```

$$RtoG \ := R^{\lambda?}{}_{\mu?\nu?\kappa?} \rightarrow (-\partial_{\kappa?}\Gamma^{\lambda?}{}_{\mu?\nu?} + \partial_{\nu?}\Gamma^{\lambda?}{}_{\mu?\kappa?} - \Gamma^{\eta}{}_{\mu?\nu?}\Gamma^{\lambda?}{}_{\kappa?\eta} + \Gamma^{\eta}{}_{\mu?\kappa?}\Gamma^{\lambda?}{}_{\nu?\eta});$$

$$Gtog \ := \Gamma^{\lambda?}{}_{\mu?\nu?} \rightarrow \frac{1}{2} g^{\lambda?\kappa}(\partial_{\nu?}g_{\kappa\mu?} + \partial_{\mu?}g_{\kappa\nu?} - \partial_{\kappa}g_{\mu?\nu?});$$

Now input the R_{m4n4} component and do the substitution. After each substitution, we distribute products over sums. We also apply the product rule to distribute derivatives over factors in a product.

```
todo:= g_{m1 m} R^{m1}_{4 n 4} + g_{4 m} R^{4}_{4 n 4};
@substitute!(%)( @(RtoG) ):
@substitute!(%)( @(Gtog) ):
@distribute!(%):
@prodrule!(%):
@distribute!(%):
@prodsort!(%);
```

$$todo \ := g_{m1m}R^{m1}{}_{4n4} + g_{4m}R^{4}{}_{4n4};$$

$$
\begin{aligned}
todo\ :=\ & -\frac{1}{2}\,\partial_{4}g^{m1\kappa}\,\partial_{n}g_{\kappa 4}\,g_{m1m} - \frac{1}{2}\,\partial_{4n}g_{\kappa 4}\,g_{m1m}g^{m1\kappa} - \frac{1}{2}\,\partial_{4}g_{\kappa n}\,\partial_{4}g^{m1\kappa}\,g_{m1m} - \frac{1}{2}\,\partial_{44}g_{\kappa n}\,g_{m1m}g^{m1\kappa}\\
& + \frac{1}{2}\,\partial_{4}g^{m1\kappa}\,\partial_{\kappa}g_{4n}\,g_{m1m} + \frac{1}{2}\,\partial_{4\kappa}g_{4n}\,g_{m1m}g^{m1\kappa} + \frac{1}{2}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g^{m1\kappa}\,g_{m1m} + \frac{1}{2}\,\partial_{n4}g_{\kappa 4}\,g_{m1m}g^{m1\kappa}\\
& + \frac{1}{2}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g^{m1\kappa}\,g_{m1m} + \frac{1}{2}\,\partial_{n4}g_{\kappa 4}\,g_{m1m}g^{m1\kappa} - \frac{1}{2}\,\partial_{\kappa}g_{44}\,\partial_{n}g^{m1\kappa}\,g_{m1m} - \frac{1}{2}\,\partial_{n\kappa}g_{44}\,g_{m1m}g^{m1\kappa}\\
& - \frac{1}{4}\,\partial_{\eta}g_{\mu 4}\,\partial_{n}g_{\kappa 4}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} - \frac{1}{4}\,\partial_{4}g_{\mu\eta}\,\partial_{n}g_{\kappa 4}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} + \frac{1}{4}\,\partial_{\mu}g_{4\eta}\,\partial_{n}g_{\kappa 4}\,g^{\eta\kappa}g_{m1m}g^{m1\mu}\\
& - \frac{1}{4}\,\partial_{4}g_{\kappa n}\,\partial_{\eta}g_{\mu 4}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} - \frac{1}{4}\,\partial_{4}g_{\mu\eta}\,\partial_{4}g_{\kappa n}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} + \frac{1}{4}\,\partial_{4}g_{\kappa n}\,\partial_{\mu}g_{4\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu}\\
& + \frac{1}{4}\,\partial_{\kappa}g_{4n}\,\partial_{\eta}g_{\mu 4}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} + \frac{1}{4}\,\partial_{4}g_{\mu\eta}\,\partial_{\kappa}g_{4n}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} - \frac{1}{4}\,\partial_{\mu}g_{4\eta}\,\partial_{\kappa}g_{4n}\,g^{\eta\kappa}g_{m1m}g^{m1\mu}\\
& + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\eta}g_{\mu n}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g_{\mu\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} - \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\mu}g_{n\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu}\\
& + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\eta}g_{\mu n}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g_{\mu\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} - \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\mu}g_{n\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu}\\
& - \frac{1}{4}\,\partial_{\kappa}g_{44}\,\partial_{\eta}g_{\mu n}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} - \frac{1}{4}\,\partial_{\kappa}g_{44}\,\partial_{n}g_{\mu\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu} + \frac{1}{4}\,\partial_{\kappa}g_{44}\,\partial_{\mu}g_{n\eta}\,g^{\eta\kappa}g_{m1m}g^{m1\mu}\\
& - \frac{1}{2}\,\partial_{4}g^{4\kappa}\,\partial_{n}g_{\kappa 4}\,g_{4m} - \frac{1}{2}\,\partial_{4n}g_{\kappa 4}\,g^{4\kappa}g_{4m} - \frac{1}{2}\,\partial_{4}g^{4\kappa}\,\partial_{4}g_{\kappa n}\,g_{4m} - \frac{1}{2}\,\partial_{44}g_{\kappa n}\,g^{4\kappa}g_{4m}\\
& + \frac{1}{2}\,\partial_{4}g^{4\kappa}\,\partial_{\kappa}g_{4n}\,g_{4m} + \frac{1}{2}\,\partial_{4\kappa}g_{4n}\,g^{4\kappa}g_{4m} + \frac{1}{2}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g^{4\kappa}\,g_{4m} + \frac{1}{2}\,\partial_{n4}g_{\kappa 4}\,g^{4\kappa}g_{4m}\\
& + \frac{1}{2}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g^{4\kappa}\,g_{4m} + \frac{1}{2}\,\partial_{n4}g_{\kappa 4}\,g^{4\kappa}g_{4m} - \frac{1}{2}\,\partial_{\kappa}g_{44}\,\partial_{n}g^{4\kappa}\,g_{4m} - \frac{1}{2}\,\partial_{n\kappa}g_{44}\,g^{4\kappa}g_{4m}\\
& - \frac{1}{4}\,\partial_{\eta}g_{\mu 4}\,\partial_{n}g_{\kappa 4}\,g^{4\mu}g_{4m}g^{\eta\kappa} - \frac{1}{4}\,\partial_{4}g_{\mu\eta}\,\partial_{n}g_{\kappa 4}\,g^{4\mu}g_{4m}g^{\eta\kappa} + \frac{1}{4}\,\partial_{\mu}g_{4\eta}\,\partial_{n}g_{\kappa 4}\,g^{4\mu}g_{4m}g^{\eta\kappa}\\
& - \frac{1}{4}\,\partial_{4}g_{\kappa n}\,\partial_{\eta}g_{\mu 4}\,g^{4\mu}g_{4m}g^{\eta\kappa} - \frac{1}{4}\,\partial_{4}g_{\mu\eta}\,\partial_{4}g_{\kappa n}\,g^{4\mu}g_{4m}g^{\eta\kappa} + \frac{1}{4}\,\partial_{4}g_{\kappa n}\,\partial_{\mu}g_{4\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa}\\
& + \frac{1}{4}\,\partial_{\kappa}g_{4n}\,\partial_{\eta}g_{\mu 4}\,g^{4\mu}g_{4m}g^{\eta\kappa} + \frac{1}{4}\,\partial_{4}g_{\mu\eta}\,\partial_{\kappa}g_{4n}\,g^{4\mu}g_{4m}g^{\eta\kappa} - \frac{1}{4}\,\partial_{\mu}g_{4\eta}\,\partial_{\kappa}g_{4n}\,g^{4\mu}g_{4m}g^{\eta\kappa}\\
& + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\eta}g_{\mu n}\,g^{4\mu}g_{4m}g^{\eta\kappa} + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g_{\mu\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa} - \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\mu}g_{n\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa}\\
& + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\eta}g_{\mu n}\,g^{4\mu}g_{4m}g^{\eta\kappa} + \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{n}g_{\mu\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa} - \frac{1}{4}\,\partial_{4}g_{\kappa 4}\,\partial_{\mu}g_{n\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa}\\
& - \frac{1}{4}\,\partial_{\kappa}g_{44}\,\partial_{\eta}g_{\mu n}\,g^{4\mu}g_{4m}g^{\eta\kappa} - \frac{1}{4}\,\partial_{\kappa}g_{44}\,\partial_{n}g_{\mu\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa} + \frac{1}{4}\,\partial_{\kappa}g_{44}\,\partial_{\mu}g_{n\eta}\,g^{4\mu}g_{4m}g^{\eta\kappa};
\end{aligned}
$$

We now split the μ index into a m part and the remaining 4 direction (the `!!` version of the command makes it apply until the result no longer changes). After that, we remove x^4 derivatives of the gauge field and write the expression in canonical form,

```
@split_index!!(%){\mu,m1,4}:
@canonicalise!(%):
@substitute!(%)( \partial_{4}{A??} -> 0 ):
@substitute!(%)( \partial_{4 m?}{A??} -> 0 );
```

$$
\begin{aligned}
todo \; := & -\frac{1}{2}\,\partial_{m1}g_{44}\,\partial_{n}g^{m1p}\,g_{mp} - \frac{1}{2}\,\partial_{nm1}g_{44}\,g_{mp}g^{m1p} - \frac{1}{4}\,\partial_{n}g_{4m1}\,\partial_{p}g_{4q}\,g^{m1p}g_{mr}g^{qr} \\
& - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{n}g_{4p}\,g^{m1p}g_{mq}g^{4q} - \frac{1}{4}\,\partial_{m1}g_{4p}\,\partial_{n}g_{44}\,g^{4m1}g_{mq}g^{pq} \\
& - \frac{1}{4}\,\partial_{n}g_{44}\,\partial_{m1}g_{44}\,g^{4m1}g_{mp}g^{4p} + \frac{1}{4}\,\partial_{m1}g_{4p}\,\partial_{n}g_{4q}\,g^{m1r}g_{mr}g^{pq} + \frac{1}{4}\,\partial_{m1}g_{4p}\,\partial_{n}g_{44}\,g^{4p}g_{mq}g^{m1q} \\
& + \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{n}g_{4p}\,g^{4p}g_{mq}g^{m1q} + \frac{1}{4}\,\partial_{n}g_{44}\,\partial_{m1}g_{44}\,g^{44}g_{mp}g^{m1p} + \frac{1}{4}\,\partial_{m1}g_{4n}\,\partial_{p}g_{4q}\,g^{m1p}g_{mr}g^{qr} \\
& + \frac{1}{4}\,\partial_{m1}g_{4n}\,\partial_{p}g_{44}\,g^{m1p}g_{mq}g^{4q} - \frac{1}{4}\,\partial_{m1}g_{4p}\,\partial_{q}g_{4n}\,g^{m1r}g_{mr}g^{pq} - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{p}g_{4n}\,g^{4p}g_{mq}g^{m1q} \\
& - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{p}g_{nq}\,g^{m1p}g_{mr}g^{qr} - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{p}g_{4n}\,g^{m1p}g_{mq}g^{4q} - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{n}g_{pq}\,g^{m1p}g_{mr}g^{qr} \\
& - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{n}g_{4p}\,g^{m1p}g_{mq}g^{4q} - \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{n}g_{4p}\,g^{4m1}g_{mq}g^{pq} - \frac{1}{4}\,\partial_{n}g_{44}\,\partial_{m1}g_{44}\,g^{4m1}g_{mp}g^{4p} \\
& + \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{p}g_{nq}\,g^{m1q}g_{mr}g^{pr} + \frac{1}{4}\,\partial_{m1}g_{44}\,\partial_{p}g_{4n}\,g^{4m1}g_{mq}g^{pq} - \frac{1}{2}\,\partial_{p}g_{44}\,\partial_{n}g^{4p}\,g_{4m} \\
& - \frac{1}{2}\,\partial_{np}g_{44}\,g^{4p}g_{4m} - \frac{1}{4}\,\partial_{n}g_{4p}\,\partial_{q}g_{4r}\,g^{4r}g_{4m}g^{pq} - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{n}g_{4q}\,g^{44}g_{4m}g^{pq} \\
& - \frac{1}{4}\,\partial_{p}g_{4q}\,\partial_{n}g_{44}\,g^{4q}g_{4m}g^{4p} - \frac{1}{4}\,\partial_{n}g_{44}\,\partial_{p}g_{44}\,g^{44}g_{4m}g^{4p} + \frac{1}{4}\,\partial_{p}g_{4q}\,\partial_{n}g_{4r}\,g^{4p}g_{4m}g^{qr} \\
& + \frac{1}{4}\,\partial_{p}g_{4q}\,\partial_{n}g_{44}\,g^{4p}g_{4m}g^{4q} + \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{n}g_{4q}\,g^{4p}g_{4m}g^{4q} + \frac{1}{4}\,\partial_{n}g_{44}\,\partial_{p}g_{44}\,g^{4p}g_{4m}g^{44} \\
& + \frac{1}{4}\,\partial_{p}g_{4n}\,\partial_{q}g_{4r}\,g^{4r}g_{4m}g^{pq} + \frac{1}{4}\,\partial_{p}g_{4n}\,\partial_{q}g_{44}\,g^{44}g_{4m}g^{pq} - \frac{1}{4}\,\partial_{p}g_{4q}\,\partial_{r}g_{4n}\,g^{4p}g_{4m}g^{qr} \\
& - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{q}g_{4n}\,g^{4p}g_{4m}g^{4q} - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{q}g_{nr}\,g^{4r}g_{4m}g^{pq} - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{q}g_{4n}\,g^{44}g_{4m}g^{pq} \\
& - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{n}g_{qr}\,g^{4q}g_{4m}g^{pr} - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{n}g_{4q}\,g^{44}g_{4m}g^{pq} - \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{n}g_{4q}\,g^{4q}g_{4m}g^{4p} \\
& - \frac{1}{4}\,\partial_{n}g_{44}\,\partial_{p}g_{44}\,g^{44}g_{4m}g^{4p} + \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{q}g_{nr}\,g^{4q}g_{4m}g^{pr} + \frac{1}{4}\,\partial_{p}g_{44}\,\partial_{q}g_{4n}\,g^{4q}g_{4m}g^{4p};
\end{aligned}
$$

In the next step, we insert the metric ansatz and simplify the result as much as possible.

```
@substitute!(%)( g_{4 4} -> \phi ):
@substitute!(%)( g_{4 m} -> \phi A_{m} ):
@substitute!(%)( g_{m n} -> \phi**{-1} h_{m n} + \phi A_{m} A_{n} ):
@substitute!(%)( g^{4 4} -> \phi**{-1} + \phi A_{m} h^{m n} A_{n} ):
@substitute!(%)( g^{4 m} -> - \phi h^{m n} A_{n} ):
@substitute!(%)( g^{m n} -> \phi h^{m n} ):
@distribute!(%):
@prodrule!(%):
@distribute!(%):
@prodrule!(%):
@canonicalise!(%):
@substitute!!(%)( h_{m1 m2} h^{m3 m2} -> \delta_{m1}^{m3} ):
@eliminate_kr!(%):
@collect_factors!(%):
@prodsort!(%):
@collect_terms!(%);
```

$$
\begin{aligned}
todo \; := & -\frac{1}{2}\,\partial_m\phi\,\partial_n\phi\,\phi^{-1} - \frac{1}{2}\,\partial_{m1}\phi\,\partial_n h^{m1p}\,h_{mp} - \frac{1}{4}\,A_m A_p \partial_{m1}\phi\,\partial_n\phi\,\phi h^{m1p} \\
& - \frac{1}{2}\,A_m A_p \partial_{m1}\phi\,\partial_n h^{m1p}\,\phi^2 - \frac{1}{2}\,\partial_{nm}\phi\, - \frac{1}{2}\,A_m A_p \partial_{nm1}\phi\,\phi^2 h^{m1p} \\
& - \frac{1}{4}\,A_{m1}\partial_p A_m\,\partial_n\phi\,\phi^2 h^{m1p} - \frac{1}{4}\,A_m \partial_n A_{m1}\,\partial_p\phi\,\phi^2 h^{m1p} - \frac{1}{4}\,\partial_n A_{m1}\,\partial_p A_m\,\phi^3 h^{m1p} \\
& - \frac{1}{4}\,A_m A_q A_r \partial_n A_{m1}\,\partial_p\phi\,\phi^4 h^{m1p} h^{qr} - \frac{1}{4}\,A_m A_r \partial_n A_{m1}\,\partial_p A_q\,\phi^5 h^{m1p} h^{qr} \\
& + \frac{1}{2}\,A_r \partial_n A_p\,\partial_{m1}\phi\,\phi^2 h_{mq} h^{m1p} h^{qr} + \frac{1}{4}\,A_m A_q A_r \partial_n A_p\,\partial_{m1}\phi\,\phi^4 h^{m1p} h^{qr} \\
& + \frac{1}{4}\,A_q \partial_{m1} A_m\,\partial_n\phi\,\phi^2 h^{m1q} + \frac{1}{4}\,A_p \partial_n A_q\,\partial_m\phi\,\phi^2 h^{pq} + \frac{1}{4}\,A_q \partial_m A_p\,\partial_n\phi\,\phi^2 h^{pq} \\
& + \frac{1}{4}\,\partial_m A_q\,\partial_n A_{m1}\,\phi^3 h^{m1q} + \frac{1}{4}\,A_m A_r \partial_n A_{m1}\,\partial_p A_q\,\phi^5 h^{m1q} h^{pr} - \frac{1}{4}\,A_r \partial_m A_p\,\partial_n\phi\,\phi^2 h^{pr} \\
& - \frac{1}{4}\,A_r \partial_n A_p\,\partial_m\phi\,\phi^2 h^{pr} + \frac{1}{4}\,A_m \partial_{m1} A_n\,\partial_p\phi\,\phi^2 h^{m1p} + \frac{1}{4}\,\partial_{m1} A_n\,\partial_p A_m\,\phi^3 h^{m1p} \\
& + \frac{1}{4}\,A_m A_r \partial_{m1} A_n\,\partial_p A_q\,\phi^5 h^{m1p} h^{qr} - \frac{1}{4}\,A_r \partial_{m1} A_n\,\partial_p\phi\,\phi^2 h_{mq} h^{m1p} h^{qr} - \frac{1}{4}\,A_p \partial_q A_n\,\partial_m\phi\,\phi^2 h^{pq} \\
& - \frac{1}{4}\,A_n \partial_m A_p\,\partial_q\phi\,\phi^2 h^{pq} - \frac{1}{4}\,\partial_m A_q\,\partial_{m1} A_n\,\phi^3 h^{m1q} - \frac{1}{4}\,A_m A_n A_r \partial_{m1} A_p\,\partial_q\phi\,\phi^4 h^{m1r} h^{pq} \\
& - \frac{1}{4}\,A_m A_r \partial_{m1} A_n\,\partial_p A_q\,\phi^5 h^{m1q} h^{pr} + \frac{1}{4}\,A_r \partial_p A_n\,\partial_m\phi\,\phi^2 h^{pr} + \frac{1}{4}\,\partial_{m1}\phi\,\partial_p\phi\,\phi^{-1} h_{nm} h^{m1p} \\
& - \frac{1}{4}\,\partial_{m1}\phi\,\partial_p h_{nm}\,h^{m1p} + \frac{1}{4}\,A_m A_r \partial_{m1}\phi\,\partial_p\phi\,\phi h_{nq} h^{m1p} h^{qr} - \frac{1}{4}\,A_m A_r \partial_{m1}\phi\,\partial_p h_{nq}\,\phi^2 h^{m1p} h^{qr} \\
& - \frac{1}{4}\,A_m \partial_p A_n\,\partial_{m1}\phi\,\phi^2 h^{m1p} + \frac{1}{4}\,A_r \partial_p A_n\,\partial_{m1}\phi\,\phi^2 h_{mq} h^{m1p} h^{qr} + \frac{1}{4}\,\partial_{m1}\phi\,\partial_n\phi\,\phi^{-1} h_{pm} h^{m1p} \\
& - \frac{1}{4}\,\partial_{m1}\phi\,\partial_n h_{pm}\,h^{m1p} + \frac{1}{4}\,A_m A_r \partial_{m1}\phi\,\partial_n\phi\,\phi h_{pq} h^{m1p} h^{qr} - \frac{1}{4}\,A_m A_r \partial_{m1}\phi\,\partial_n h_{pq}\,\phi^2 h^{m1p} h^{qr} \\
& - \frac{1}{4}\,A_m \partial_n A_p\,\partial_{m1}\phi\,\phi^2 h^{m1p} - \frac{1}{4}\,A_p \partial_n A_m\,\partial_{m1}\phi\,\phi^2 h^{m1p} + \frac{1}{4}\,A_q \partial_n A_m\,\partial_{m1}\phi\,\phi^2 h^{m1q} \\
& + \frac{1}{4}\,\partial_{m1}\phi\,\partial_m h_{nq}\,h^{m1q} - \frac{1}{4}\,A_m A_r \partial_n\phi\,\partial_p\phi\,\phi h^{pr} + \frac{1}{4}\,A_m A_r \partial_{m1}\phi\,\partial_p h_{nq}\,\phi^2 h^{m1q} h^{pr} \\
& + \frac{1}{4}\,A_n \partial_m A_q\,\partial_{m1}\phi\,\phi^2 h^{m1q} + \frac{1}{4}\,A_m A_n A_r \partial_p A_q\,\partial_{m1}\phi\,\phi^4 h^{m1q} h^{pr} + \frac{1}{2}\,A_m A_q \partial_p\phi\,\partial_n h^{pq}\,\phi^2 \\
& + \frac{1}{2}\,A_m A_q \partial_{np}\phi\,\phi^2 h^{pq} + \frac{1}{4}\,A_m A_r A_s \partial_n A_p\,\partial_q\phi\,\phi^4 h^{pq} h^{rs} + \frac{1}{4}\,A_m A_s \partial_n A_p\,\partial_q A_r\,\phi^5 h^{pq} h^{rs} \\
& - \frac{1}{4}\,A_m A_r A_s \partial_n A_q\,\partial_p\phi\,\phi^4 h^{pq} h^{rs} + \frac{1}{4}\,A_m A_s \partial_n\phi\,\partial_p\phi\,\phi h^{ps} - \frac{1}{4}\,A_m A_s \partial_n A_p\,\partial_q A_r\,\phi^5 h^{pr} h^{qs} \\
& - \frac{1}{4}\,A_m A_s \partial_p A_n\,\partial_q A_r\,\phi^5 h^{pq} h^{rs} + \frac{1}{4}\,A_m \partial_p A_n\,\partial_q\phi\,\phi^2 h^{pq} + \frac{1}{4}\,A_m A_n A_s \partial_p A_q\,\partial_r\phi\,\phi^4 h^{ps} h^{qr} \\
& + \frac{1}{4}\,A_m A_s \partial_p A_n\,\partial_q A_r\,\phi^5 h^{pr} h^{qs} - \frac{1}{4}\,A_m A_s \partial_p\phi\,\partial_q\phi\,\phi h_{nr} h^{pq} h^{rs} \\
& + \frac{1}{4}\,A_m A_s \partial_p\phi\,\partial_q h_{nr}\,\phi^2 h^{pq} h^{rs} - \frac{1}{4}\,A_m \partial_q A_n\,\partial_p\phi\,\phi^2 h^{pq} + \frac{1}{4}\,A_m A_s \partial_p\phi\,\partial_n h_{qr}\,\phi^2 h^{pq} h^{rs} \\
& - \frac{1}{4}\,A_m A_s \partial_p\phi\,\partial_q h_{nr}\,\phi^2 h^{pr} h^{qs} - \frac{1}{4}\,A_m A_n A_s \partial_q A_r\,\partial_p\phi\,\phi^4 h^{pr} h^{qs};
\end{aligned}
$$

Some derivatives have to be rewritten to a canonical form,

```
@substitute!(%)( \partial_{p}{h^{n m}} h_{q m} -> - \partial_{p}{h_{q m}} h^{n m} ):
@canonicalise!(%):
@substitute!(%)( h_{m1 m2} h^{m3 m2} -> \delta_{m1}^{m3} ):
@eliminate_kr!(%);
```

$$
\begin{aligned}
todo\ :=\ & -\frac{1}{2}\,\partial_m\phi\,\partial_n\phi\,\phi^{-1}+\frac{1}{2}\,\partial_{m1}\phi\,\partial_n h_{mp}\,h^{m1p}-\frac{1}{4}\,A_m A_{m1}\partial_n\phi\,\partial_p\phi\,\phi h^{m1p}\\
&-\frac{1}{2}\,A_m A_{m1}\partial_p\phi\,\partial_n h^{m1p}\,\phi^2-\frac{1}{2}\,\partial_{mn}\phi\,-\frac{1}{2}\,A_m A_{m1}\partial_{np}\phi\,\phi^2 h^{m1p}\\
&-\frac{1}{4}\,A_{m1}\partial_p A_m\,\partial_n\phi\,\phi^2 h^{m1p}-\frac{1}{4}\,A_m\partial_n A_{m1}\,\partial_p\phi\,\phi^2 h^{m1p}-\frac{1}{4}\,\partial_n A_{m1}\,\partial_p A_m\,\phi^3 h^{m1p}\\
&-\frac{1}{4}\,A_m A_{m1} A_p\partial_n A_q\,\partial_r\phi\,\phi^4 h^{m1p}h^{qr}-\frac{1}{4}\,A_m A_{m1}\partial_n A_p\,\partial_q A_r\,\phi^5 h^{m1r}h^{pq}\\
&+\frac{1}{2}\,A_m\partial_n A_p\,\partial_q\phi\,\phi^2 h^{pq}+\frac{1}{4}\,A_m A_{m1} A_p\partial_n A_q\,\partial_r\phi\,\phi^4 h^{m1p}h^{qr}+\frac{1}{4}\,A_{m1}\partial_q A_m\,\partial_n\phi\,\phi^2 h^{m1q}\\
&+\frac{1}{4}\,A_p\partial_n A_q\,\partial_m\phi\,\phi^2 h^{pq}+\frac{1}{4}\,A_p\partial_m A_q\,\partial_n\phi\,\phi^2 h^{pq}+\frac{1}{4}\,\partial_m A_{m1}\,\partial_n A_q\,\phi^3 h^{m1q}\\
&+\frac{1}{4}\,A_m A_{m1}\partial_n A_p\,\partial_q A_r\,\phi^5 h^{m1q}h^{pr}-\frac{1}{4}\,A_p\partial_m A_r\,\partial_n\phi\,\phi^2 h^{pr}-\frac{1}{4}\,A_p\partial_n A_r\,\partial_m\phi\,\phi^2 h^{pr}\\
&+\frac{1}{4}\,A_m\partial_{m1}A_n\,\partial_p\phi\,\phi^2 h^{m1p}+\frac{1}{4}\,\partial_{m1}A_m\,\partial_p A_n\,\phi^3 h^{m1p}+\frac{1}{4}\,A_m A_{m1}\partial_p A_n\,\partial_q A_r\,\phi^5 h^{m1r}h^{pq}\\
&-\frac{1}{4}\,A_m\partial_p A_n\,\partial_q\phi\,\phi^2 h^{pq}-\frac{1}{4}\,A_p\partial_q A_n\,\partial_m\phi\,\phi^2 h^{pq}-\frac{1}{4}\,A_n\partial_m A_p\,\partial_q\phi\,\phi^2 h^{pq}\\
&-\frac{1}{4}\,\partial_m A_{m1}\,\partial_q A_n\,\phi^3 h^{m1q}-\frac{1}{4}\,A_m A_n A_{m1}\partial_p A_q\,\partial_r\phi\,\phi^4 h^{m1p}h^{qr}\\
&-\frac{1}{4}\,A_m A_{m1}\partial_p A_n\,\partial_q A_r\,\phi^5 h^{m1q}h^{pr}+\frac{1}{4}\,A_p\partial_r A_n\,\partial_m\phi\,\phi^2 h^{pr}+\frac{1}{4}\,\partial_{m1}\phi\,\partial_p\phi\,\phi^{-1}h_{mn}h^{m1p}\\
&-\frac{1}{4}\,\partial_{m1}\phi\,\partial_p h_{mn}\,h^{m1p}+\frac{1}{4}\,A_m A_n\partial_p\phi\,\partial_q\phi\,\phi h^{pq}-\frac{1}{4}\,A_m A_{m1}\partial_p\phi\,\partial_q h_{nr}\,\phi^2 h^{m1r}h^{pq}\\
&-\frac{1}{4}\,A_m\partial_{m1}A_n\,\partial_p\phi\,\phi^2 h^{m1p}+\frac{1}{4}\,A_m\partial_p A_n\,\partial_q\phi\,\phi^2 h^{pq}+\frac{1}{4}\,\partial_n\phi\,\partial_m\phi\,\phi^{-1}\\
&-\frac{1}{4}\,\partial_{m1}\phi\,\partial_n h_{mp}\,h^{m1p}+\frac{1}{4}\,A_m A_{m1}\partial_n\phi\,\partial_q\phi\,\phi h^{m1q}-\frac{1}{4}\,A_m A_{m1}\partial_p\phi\,\partial_n h_{qr}\,\phi^2 h^{m1q}h^{pr}\\
&-\frac{1}{4}\,A_m\partial_n A_{m1}\,\partial_p\phi\,\phi^2 h^{m1p}-\frac{1}{4}\,A_{m1}\partial_n A_m\,\partial_p\phi\,\phi^2 h^{m1p}+\frac{1}{4}\,A_{m1}\partial_n A_m\,\partial_q\phi\,\phi^2 h^{m1q}\\
&+\frac{1}{4}\,\partial_{m1}\phi\,\partial_m h_{nq}\,h^{m1q}-\frac{1}{4}\,A_m A_p\partial_n\phi\,\partial_r\phi\,\phi h^{pr}+\frac{1}{4}\,A_m A_{m1}\partial_p\phi\,\partial_q h_{nr}\,\phi^2 h^{m1q}h^{pr}\\
&+\frac{1}{4}\,A_n\partial_m A_{m1}\,\partial_q\phi\,\phi^2 h^{m1q}+\frac{1}{4}\,A_m A_n A_{m1}\partial_p A_q\,\partial_r\phi\,\phi^4 h^{m1p}h^{qr}\\
&+\frac{1}{2}\,A_m A_p\partial_q\phi\,\partial_n h^{pq}\,\phi^2+\frac{1}{2}\,A_m A_p\partial_{nq}\phi\,\phi^2 h^{pq}+\frac{1}{4}\,A_m A_p A_q\partial_n A_r\,\partial_s\phi\,\phi^4 h^{pq}h^{rs}\\
&+\frac{1}{4}\,A_m A_p\partial_n A_q\,\partial_r A_s\,\phi^5 h^{ps}h^{qr}-\frac{1}{4}\,A_m A_p A_q\partial_n A_r\,\partial_s\phi\,\phi^4 h^{pq}h^{rs}+\frac{1}{4}\,A_m A_p\partial_n\phi\,\partial_s\phi\,\phi h^{ps}\\
&-\frac{1}{4}\,A_m A_p\partial_n A_q\,\partial_r A_s\,\phi^5 h^{pr}h^{qs}-\frac{1}{4}\,A_m A_p\partial_q A_n\,\partial_r A_s\,\phi^5 h^{ps}h^{qr}+\frac{1}{4}\,A_m\partial_p A_n\,\partial_q\phi\,\phi^2 h^{pq}\\
&+\frac{1}{4}\,A_m A_n A_p\partial_q A_r\,\partial_s\phi\,\phi^4 h^{pq}h^{rs}+\frac{1}{4}\,A_m A_p\partial_q A_n\,\partial_r A_s\,\phi^5 h^{pr}h^{qs}-\frac{1}{4}\,A_m A_n\partial_q\phi\,\partial_r\phi\,\phi h^{qr}\\
&+\frac{1}{4}\,A_m A_p\partial_q\phi\,\partial_r h_{ns}\,\phi^2 h^{ps}h^{qr}-\frac{1}{4}\,A_m\partial_p A_n\,\partial_q\phi\,\phi^2 h^{pq}+\frac{1}{4}\,A_m A_p\partial_q\phi\,\partial_n h_{rs}\,\phi^2 h^{pr}h^{qs}\\
&-\frac{1}{4}\,A_m A_p\partial_q\phi\,\partial_r h_{ns}\,\phi^2 h^{pr}h^{qs}-\frac{1}{4}\,A_m A_n A_p\partial_q A_r\,\partial_s\phi\,\phi^4 h^{pq}h^{rs};
\end{aligned}
$$

Finally, we replace the derivative of the gauge field with the field strength,

```
@substitute!(%)( \partial_{n}{A_{m}} -> 1/2*\partial_{n}{A_{m}} + 1/2*F_{n m}
                                                  + 1/2*\partial_{m}{A_{n}} ):
@distribute!(%):
@prodsort!(%):
@canonicalise!(%):
@rename_dummies!(%):
@collect_terms!(%);
```

$$
\begin{aligned}
todo \;:=& -\frac{1}{4}\,\partial_{m}\phi\,\partial_{n}\phi\,\phi^{-1} + \frac{1}{4}\,\partial_{p}\phi\,\partial_{n}h_{mq}\,h^{pq} - \frac{1}{2}\,\partial_{mn}\phi\, + \frac{1}{4}\,F_{mp}F_{nq}\phi^{3}h^{pq} \\
&+ \frac{1}{4}\,\partial_{p}\phi\,\partial_{q}\phi\,\phi^{-1}h_{mn}h^{pq} - \frac{1}{4}\,\partial_{p}\phi\,\partial_{q}h_{mn}\,h^{pq} + \frac{1}{4}\,\partial_{p}\phi\,\partial_{m}h_{nq}\,h^{pq};
\end{aligned}
$$