Some simple gamma matrix algebra.

```
::PostDefaultRules( @@prodsort!(%), @@eliminate_kr!(%),
                                    @@canonicalise!(%), @@collect_terms!(%) ).
```

```
Assigning property PostDefaultRules to .
```

```
{s,r,l,k,m,n}::Indices(vector).
{s,r,l,k,m,n}::Integer(0..d-1).
\Gamma_{#}::GammaMatrix(metric=\delta).
\delta_{m n}::KroneckerDelta.
```

```
Assigning property Indices to s, r, l, k, m, n.
Assigning property Integer to s, r, l, k, m, n.
Assigning property GammaMatrix to \Gamma.
Assigning property KroneckerDelta to \delta.
```

The expression which we want to simplify:

```
\Gamma_{s r} \Gamma_{r l} \Gamma_{k m} \Gamma_{m s};
```

$$1 := (-1)\,\Gamma_{mr}\Gamma_{lm}\Gamma_{ks}\Gamma_{rs};$$

By joining all adjacent gamma matrices repeatedly (using the `!!` form of the command), we end up with

```
@join!!(%){expand};
```

$$1 := (-1)\,(2\,\Gamma_{lr} - \Gamma_{lr}d + \delta_{lr}d - \delta_{lr})(2\,\Gamma_{kr} - \Gamma_{kr}d + \delta_{kr} - \delta_{kr}d);$$

After distributing the result and joining once more, we collect factors and get the desired result:

```
@distribute!(%):
@canonicalise!(%):
@join!(%){expand}:
@distribute!(%):
@factorise!(%){d}:
@collect_factors!(%);
```

$$1 := \Gamma_{kl}(12 - 18\,d + 8\,d^2 - d^3) + \delta_{kl}(-3 + 6\,d - 4\,d^2 + d^3);$$