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04/03/2019

SFWRENG 4X03 Lab 4

1a) <u>netbp.m code:</u>

```
function netbp(points , labels , neurons , learning_rate , niter
, file)
    %NETBP Uses backpropagation to train a network
    % Initialize weights and biases
    rng(5000);
    W2 = 0.5*randn(neurons(1),2); W3 =
0.5*randn(neurons(2),neurons(1)); W4 =
0.5*randn(neurons(3),neurons(2));
    b2 = 0.5*randn(neurons(1),1); b3 = 0.5*randn(neurons(2),1);
b4 = 0.5*randn(neurons(3),1);
    % Forward and Back propagate

    savecost = zeros(niter,1); % value of cost function at each
iteration
    for counter = 1:niter
        k = randi(length(points)); % choose a training point at
random
        x = points(:,k);            %%for every point
        % Forward pass
        a2 = activate(x,W2,b2);
        a3 = activate(a2,W3,b3);
        a4 = activate(a3,W4,b4);
        % Backward pass
        delta4 = a4.*(1-a4).*(a4-labels(:,k));
        delta3 = a3.*(1-a3).*(W4'*delta4);
        delta2 = a2.*(1-a2).*(W3'*delta3);
        % Gradient step
        W2 = W2 - learning_rate*delta2*x';
        W3 = W3 - learning_rate*delta3*a2';
        W4 = W4 - learning_rate*delta4*a3';
        b2 = b2 - learning_rate*delta2;
        b3 = b3 - learning_rate*delta3;
        b4 = b4 - learning_rate*delta4;
        % Monitor progress
        newcost = cost(W2,W3,W4,b2,b3,b4); % display cost to
screen
        savecost(counter) = newcost;
    end
    % Show decay of cost function
    save costvec
    semilogy((1:1e4:niter),savecost(1:1e4:niter));

    function costval = cost(W2,W3,W4,b2,b3,b4)
```

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```
        costvec = zeros(length(points),1);
            for i = 1:length(points)        %%iterate for every
point
                x = points(:,i);            %% for every point
                a2 = activate(x,W2,b2);
                a3 = activate(a2,W3,b3);
                a4 = activate(a3,W4,b4);
                costvec(i) = norm(labels(:,i) - a4,2);
            end
        costval = norm(costvec,2)^2;
    end % of nested function
    save(file ,
'W2','W3','W4','b2','b3','b4','savecost','learning_rate');
end
```

1b) <u>classifypoints.m code:</u>

```
function category = classifypoints(file,points)

    load(file); %%load all variables
    category = zeros(1,length(points)); %% initialize return var


    for i = 1:length(points)
        x = points(:,i);                %%from netbp
        a2 = activate(x,W2,b2);
        a3 = activate(a2,W3,b3);
        output_vec = activate(a3,W4,b4);       %%past through
sigmoid activate function layer times


        if output_vec(1,1) >= output_vec(2,1)
            category(i) = 1;
        end
    end

end
```

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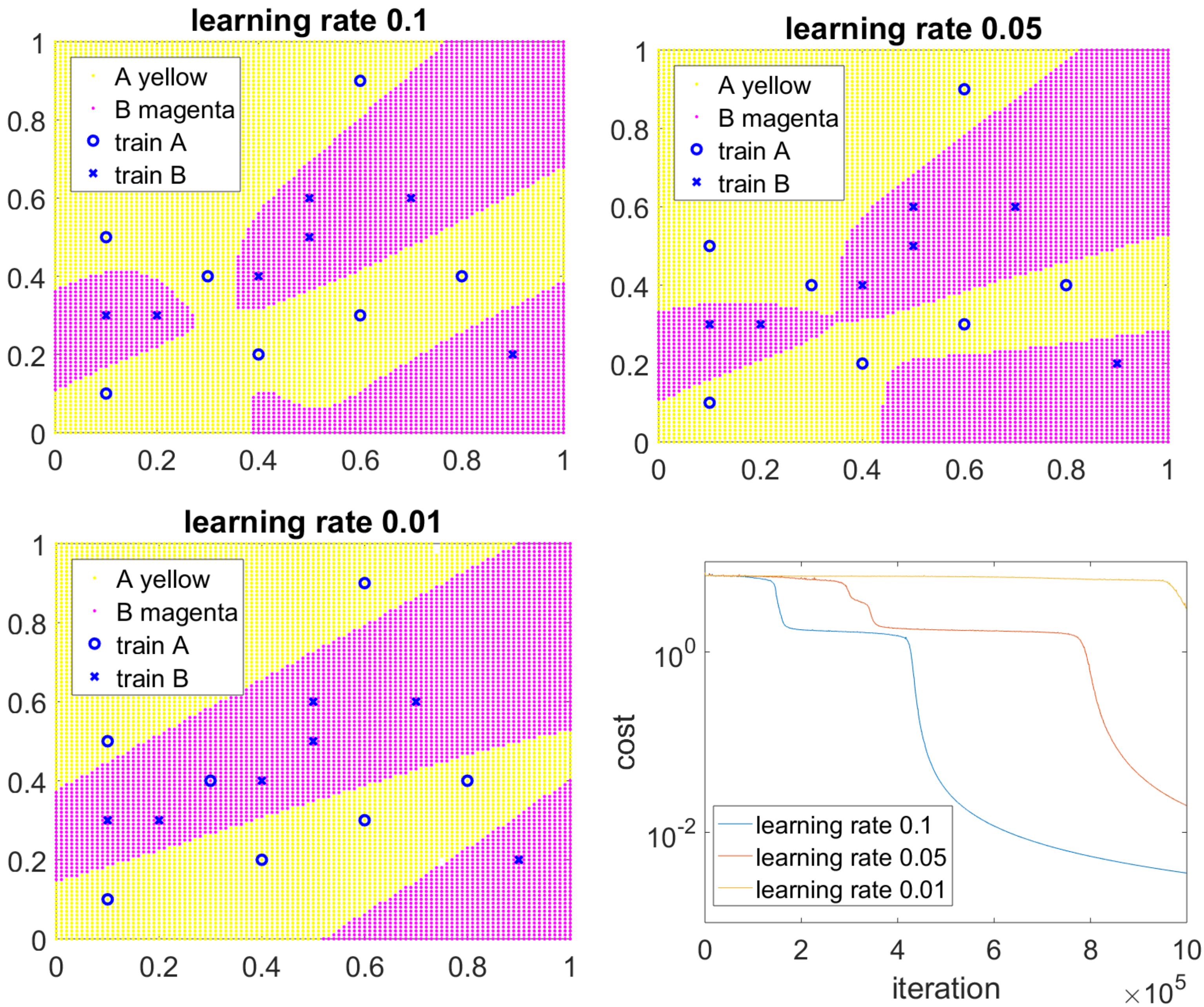
1c) best results at 4 layers,

neurons = [10, 16, 2], learning_rates = [0.1, 0.05, 0.01]



Across multiple learning rates, this neuron configuration can produce similar plots.

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2.

<u>bisection:</u>

f = @(x) x-exp(2-sqrt(x)): (1 root)

my root = 1.877321666106582e+00

fzero(f,1) = 1.877321666687556e+00

absolute error = 5.809739356266164e-10

My method is accurate to the 10th significant digit

f = @(x) x*sin(x^2) -1: (multiple roots in region, make sure both methods refer to same root)

my root = 4.368127214409469e+00

fzero(f,4.5) = 4.368127214401134e+00

absolute error = 8.335554468885675e-12

My method is accurate to the 12th significant digit

<u>Newtons:</u>

f = @(x) x^3 -2*x -5: (1 root)

my root = 2.094551481543604e+00

fzero(f,4.5) = 2.094551481542327e+00

absolute error = 1.277644656738630e-12

My method is accurate to the 12th significant digit

f = @(x) x*sin(x^2) -1: (multiple roots, make sure both methods refer to same root)

my root = 3.930741781510152e+00

fzero(f,4) = 3.930741781510152e+00

absolute error =0

My method is as accurate as the fzero method.

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Bisection.m code:

```
function root = bisection(f,a,b)

    if f(a)*f(b)<0 %%guess is proper
        mid = (a+b)/2;
        if abs(f(mid)) < 1e-9  %% close to 0 with error 
tolerance
            root = mid;
        elseif f(a)*f(mid)<0 %%root in a-mid interval
            root = bisection(a,mid,f);

        else            %% root in mid-b interval
            root = bisection(mid,b,f);
        end

    else
        disp('cannot compute with this guess')

    end
end
```

Newtons.m code:

```
function root = newtons(f,x0)
                         %%using syms package
    %---------------initialize--------%
    df = matlabFunction(diff(sym(f))) ;     %%compute derivative
    y0 = f(x0);

    root = lineRoot(x0,y0,df);  %%calculate root of line
    yRoot = f(root);       %%value at root

    %-------------loop---------%
    while abs(yRoot) > 1e-9   %%while not precise enough
        x0 = root;        %update x0
        y0 = f(x0);
        root = lineRoot(x0,y0,df);
        yRoot = f(root);

    end
    %%--------------------functions---------------%%
    function root = lineRoot(x0,y0,df)   %%computes next root
        slope =  df(x0);
        root = x0 - y0/slope;
    end

end
```

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3a)

The method does not finish executing under these circumstances. The line created at x0 = 1 is y = -2x -2 so the line's root is x = -1. This new guess then creates a line y = -2x+2, which has a root at x = 1. This brings us back to our initial guess, so the loop will never terminate.

b)

The method computes the root to be x = 1.600485180440241e+00. The outcome with the guess $x = 1 + 10^{-10}$ is different then the first outcome because the lines created do not form an endless loop of repeating roots.

4)

When x0 = 0 the method returns root = Inf. This happens because at x = 0 the slope of the function is 0 so the line created does not have a root. The calculation of the root divides by 0 so this causes the root to be x = Inf. The method finishes because it evaluates the value of the function at x = inf. The value y = NaN is then compared to the tolerance where it is "smaller" so the loop finishes.

The result I obtain when computing fsolve(f,1) = 3.129879311117518e+00, and the result I obtain when computing my method is x = 9.424744704881126e+00. This does not mean my method is incorrect, both methods are just referring to different roots of the function.

5a)

Replacing the derivative with a constant d, will result in the following behavior:

Guess will only move forward if: y0 and d have **different** signs

Guess will only move backward if: y0 and d have the **same** signs

This means that the condition for local convergence is that the guess must be moving in the direction of the root for every iteration.

b)

if $x_{k+1} = x_k - \frac{f(x_k)}{d}$ and the rate of convergence, $\mu = \lim_{k\to\infty} \frac{|x_{k+1} - L|}{|x_k - L|^q}$ where L is the value we converge to, the rate of convergence should be:

$$\mu = \lim_{k\to\infty} \frac{\left| x_k - \frac{f(x_k)}{d} - L \right|}{|x_k - L|^q}$$

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c)

The method will converge quadratically when $\lim_{k\to\infty} \frac{\left|x_k - \frac{f(x_k)}{d} - L\right|}{|x_k - L|^2} \leq M$ where $0 < M < \infty$

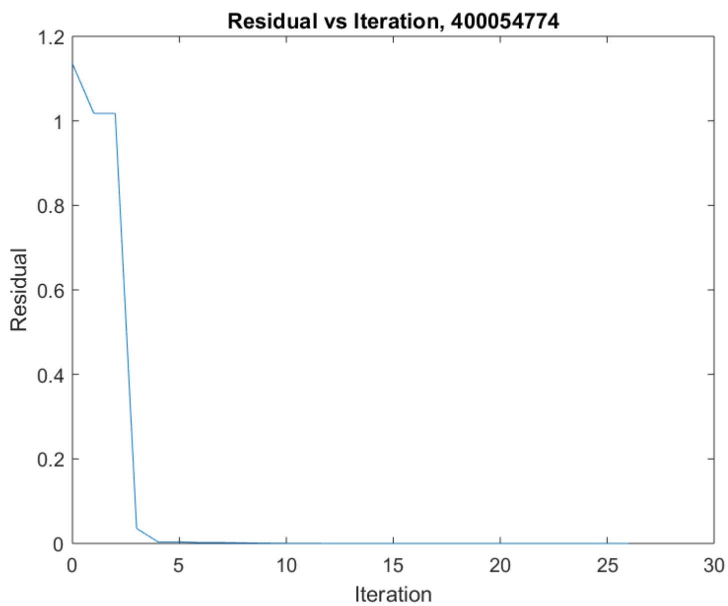
Therefore, it will converge quadratically when:

$$d \leq \frac{|-f(x_k)|}{|M(x_k - L)^2 - x_k + L|}$$

6

Yes, Newtons method will have difficulty solving this. The curves both have asymptotes so they never actually cross each other or the x-axis, therefore they do not have roots.

The output of fsolve states that the equation is solved however this does not mean it converges to a root. The residual becomes small but NOT zero.



I can conclude that this plot matches up with my statement about the residual.

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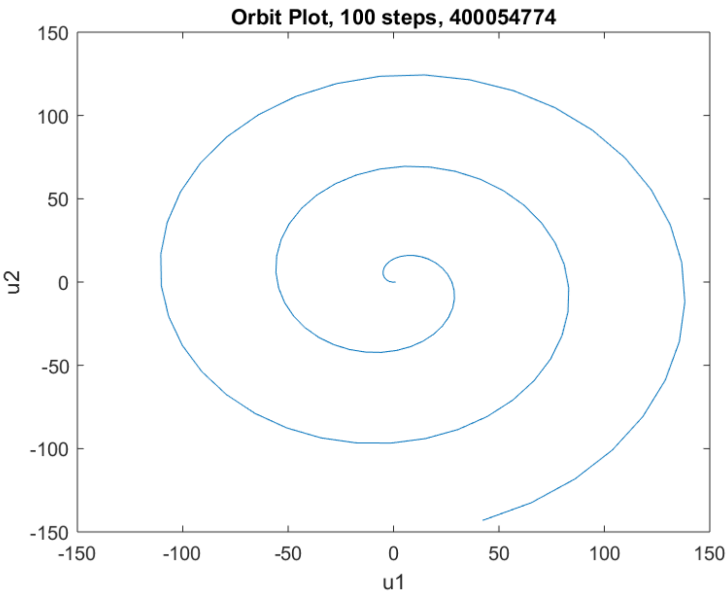
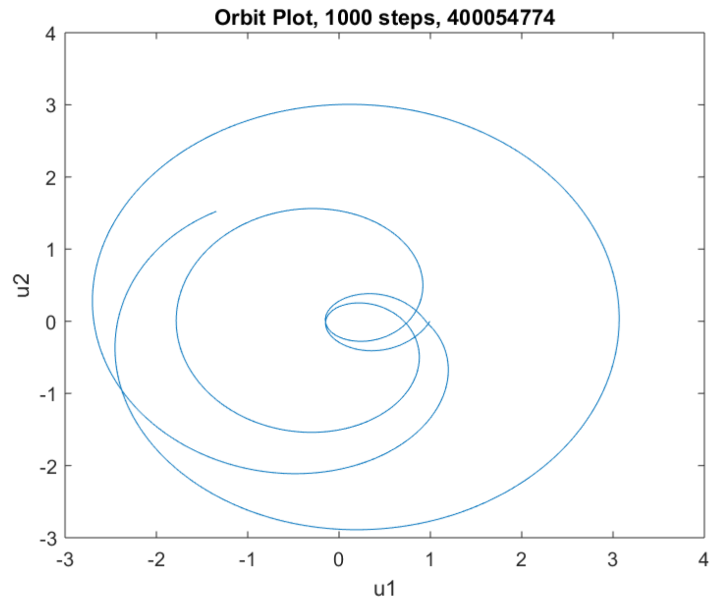
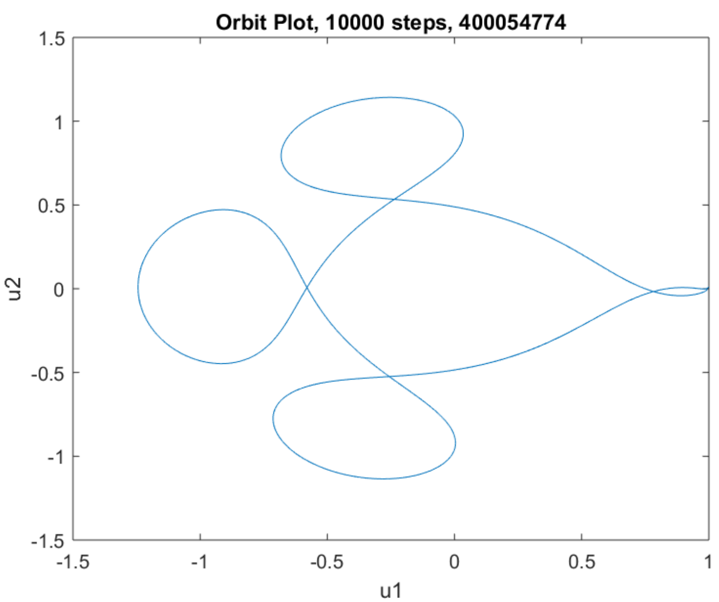
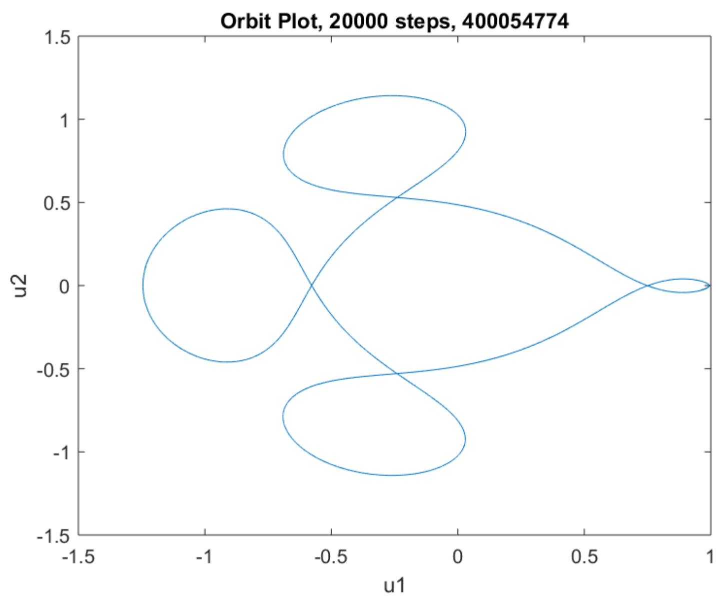
7.









The data from the plots determine that it takes around 10000 uniform steps for the orbit to appear qualitatively correct.

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8. A minor change of 10^-10 is enough to make a difference on each plot.