Never a Dull Deal: Faith, Hope and Probability in Bridge

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Author of *Bridge, Probability and Information*

NEVER A DULL DEAL:
FAITH, HOPE AND PROBABILITY IN BRIDGE

ROBERT F. MACKINNON

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Dedicated to the memory of my dear, gentle wife, Junko, who understood the basic principles of bridge, but who never wanted to play the game, except for that one time.

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FOREWORD

This book is about making choices. It is largely composed of material from blogs that have appeared over the years as afterthoughts to the ideas set down in my previous book, *Bridge, Probability and Information* (2010). The hope is that this second effort contains something new, amusing and useful for the reader when resting away from the table. I have the suspicion that like many senior citizens I have made the false assumption that what is worth saying once is worth repeating several times. There is this advantage: if the reader should nod off at some point, he need not flick back to see if he missed something important. He may have, but he can just read on and probably he will find the essentials repeated later on for his convenience. We begin with a glimpse into the environment in which we play our games.

A Brief Bridge Sermon

Here in Victoria, British Columbia, every week men and women of many faiths and races gather together in a church hall to play bridge in the spirit of fair and friendly competition under a policy of intolerance to rude and demeaning behavior.

As we gather to embark upon our game of bridge, we strive to keep foremost in our thoughts these three fundamentals:

- Faith, that our bidding system can get us to the right contracts;
- Hope, that our partner is going to have one of his better days;
- Probability, that the cards will sit where we want them to sit.

And the greatest of these is Probability for it provides us with our best advantage during the play and our best excuse when we are called to account during the post mortem. Probability knows not seasons. A player who hath Arithmetic but hath not Probability steers by the moon without benefit of the stars.

As for Charity — we look neither to give nor to receive undeservedly, although the law dictates we humbly and gratefully accept all gifts unwittingly given. Blessed is he who is parsimonious by nature, for a man may hold great cards, but he who giveth away tricks will not profit thereby. He is like unto a caravan bereft of camels, cast into the wilderness without teammates, without partners, without masterpoints.

Today's Lesson — Evolution

My friends, evolution advances in mysterious ways, and not always to the end one would wish. Recall the nineteenth century missionaries who sailed to Hawaii and inadvertently promoted the expansion of the American textile industry by persuading the natives to wear clothes even though the weather did not require them and the natives hadn't the wherewithal to pay for them. The unforeseen consequences of their invasion are apparent to this day: the multicolored Aloha shirt celebrating the overabundance of nature and the multilingual Hula dance where the hands tell a story and the hips deliver the message.

In the beginning days of bridge, each partner naively bid what he or she had going up the ladder until they reached the right contract at the right level. There they rested. This was not always easily accomplished, but at least declarer could blame only himself if he couldn't manage taking the number of tricks he himself had committed to. Naturally, bidding was on the cautious side. Overtricks were taken as a sign of good declarer play or of poor bidding. Let's look at an example hand and how bidding has changed.

First, in the early days just after WWII, when Tim, after surviving time sweating in the jungles, returned to the bridge table with his old chum, Sid, who was introducing him to duplicate.

Tim	**Sid**
♠ A Q 6 4 2	♠ 10 5
♥ 9	♥ A K Q J 8 4
♦ A 7 5 4	♦ K J 3 2
♣ K Q 9	♣ 3

1♠	2♥
3♦	4♦
4NT	5♥
5NT	pass

Their bidding was entirely natural. A club was led to the ♣A and a spade returned to North's ♠K. Exactly eleven tricks were taken. No problem.

Tim: Pinpoint bidding, pal. We beat every one of those suckers who stopped in 4NT.

Sid: Well… actually, Tim, anyone who makes eleven tricks in 3NT, 4NT or 5NT scores the same. So under the new scoring it's a tie for top.

Tim: You're kidding! You're telling me there is no advantage any more to reaching the right contract? Man, that really cheapens the game.

Sid: I know, I know, but it was done to appeal to the masses. Maybe in a few years when everybody gets to be a Life Master they'll toughen up the scoring rules.

Tim stayed in the army and got to participate in the Korean War, spending two years as a PoW. One of his Red Cross care packages was lined with a stained New York Times containing a bridge column devoted to the Blackwood Convention. Anxious to demonstrate his newfound toy, as soon as he returned home, Tim invited Sid, who now owned his own appliance repair shop, to another game of duplicate, during which a very similar hand arose. This time upon discovering Sid had an ace and two kings, Tim bid slam — because he found he couldn't stop in 5NT.

Tim	**Sid**
♠ A Q 6 4 2	♠ 10 5
♥ 9	♥ A K Q J 8 4
♦ A 7 5 4	♦ K J 3 2
♣ K Q 9	♣ 3

1♠	2♥
3♦	4♦
4NT	5♥
5NT	6♥
6NT	pass

A club was led to the ace and a low spade returned. Tim won the ♠A, ran the hearts pitching four spades and a diamond, returned to hand with the ♦A and eventually took the winning diamond finesse for twelve tricks and a top.

Tim: Phew! If they'd led a heart I might have taken the wrong finesse. This Blackwood is a good idea, getting us to 6NT ahead of everyone in 6♦.

Sid: Errr.. well no one will be in 6♦. Blackwood is supposed to keep you out of bad slams, not get you into them. How many points did you have? I had 14.

Tim: Points? What are points?

The years sped by and before they realized what was happening, Tim and Sid were greybeard Life Masters playing a 2/1 system that was designed to keep the bidding safely below 4NT. That required making bids that were forcing but not leading in any particular direction. The minor suits had become largely vestigial.

Tim	Sid
♠ A Q 6 4 2	♠ 10 5
♥ 9	♥ A K Q J 8 4
♦ A 7 5 4	♦ K J 3 2
♣ K Q 9	♣ 3

1♠	2♥
2NT	3NT
pass	

Tim: Tied for top, old chum — many will be in 4♥. Nice bid.
Sid: You know what they say, better to pass than to keep bidding the same suit.

The first lesson of evolution is this: as time goes by, entropy increases. This means that fewer bids must cover more ground, thus losing definition. The trend follows the second law of thermodynamics, which also predicts a further increase in the number of chaotic preempts and meaningless overcalls. So, although we must accept the laws of nature as scientific facts, that doesn't mean we approve of them. Here endeth the lesson.

After the Lesson

Here is a deal from a recent game wherein the novices earned a top against us with enterprising bidding. Was I guilty?

Dealer East
Neither vul.

	♠ Q 6 4 2	
	♥ A K 8	
	♦ 9 5 3	
	♣ 5 4 3	
♠ A 8 7	N / W E / S	♠ K J 10 9 5 3
♥ J 3 2		♥ 10 4
♦ J 7 6 4		♦ K 10
♣ A Q 7		♣ 9 6 2
	♠ —	
	♥ Q 9 7 6 5	
	♦ A Q 8 2	
	♣ K J 10 8	

Bob	Janice	John	Ajit
		2♠	3♥
3♠	4♥	all pass	

I was on lead against a pair who had played together for just a handful of games while Ajit's wife was out of town. Ajit, a retired chemist, had been taking lessons from an expert, so felt no reservations about entering the auction at the three-level on a bad suit. After a rather risky raise of partner's weak two I felt no temptation to sacrifice holding defensive values in four suits in a nine-loser hand. My opening lead was a trump.

After my passive lead, I somehow ended up on lead at Trick 12 holding the ♦7 under Ajit's ♦8, his tenth trick, giving us a clear bottom.

'You should have led a spade,' said John, 'after which repeated spade leads would have run declarer out of trumps.'

'You should have held the ♦8,' I replied. As with many a hasty analysis after a bad result, neither of us was correct. The double dummy solution after a spade lead is for declarer to resist the urge of entering dummy in trumps in order to finesse in clubs. He needs the ♦K onside, so he should work on clubs immediately, leading the ♣10 from his hand, planning to follow this up with a play of the ♣J once he sees the count from his RHO. (The more advantageous of the two most likely distributions of sides is ♠ 3-6 ♥ 3-2 ♦ 4-2 ♣ 3-3.) John was right in one respect — would Ajit have found the endplay after a spade lead? No one knows for sure, but that lead does nothing to disrupt declarer's timing.

INTRODUCTION

'Many a tear has to fall, but it's all in the game'
— Carl Sigman (1909 - 2000)

Most play bridge for the fun it provides, and the most fun comes from the quick thrill of a lucky play. That's a dangerous approach in the long run, so don't say I didn't warn you, but, hey, missed opportunities are just as bad, even though you don't feel them as acutely at the table. To pursue immediate pleasure or to avoid future pain? As with many dilemmas of a philosophical nature, it's your choice, no matter what Epicurus (341-270 BC) may have claimed. There is a third way: play with equanimity, free from anxiety, and always go with the percentages.

Parents set down rules for their kids: 'Brush your teeth after meals', 'Go to bed at nine', etc. Parents don't say to a four year old, 'Do what you think best' (although the trend appears to be in that direction). Kids respect the rules, they don't know any better, and to be fair, the world might be a better place if everyone was home in bed at nine o'clock, but as time goes by kids learn from experience to make exceptions. A distraught mother may admonish a daughter not to scream in public, but if the daughter grows up to be an opera singer, the rule goes by the board. When the prima donna is playing Tosca about to jump off the Papal parapet, her mother may urge, 'Scream as loud as you can, dear, the audience will love it.' Whether it's the opera house or the supermarket makes all the difference.

So it is when beginners are taught bridge. They are not told directly to do what they think best; rather they are taught rules, rules which will stand them in good stead in most situations, but rules that should be broken as the circumstances dictate. Beginner's rules are for beginners. Take the finesse, for example. Students are shown how declarer can create an extra trick by taking a finesse and are given numerous examples how this works. They are taught, 'Take your finesses and don't fist-pump when the desperate ones succeed.' Students are not told how to avoid a finesse by employing a strip-and-endplay, because that is a topic for the master class, and it may be hard to spot the possibility in any case. It's easier for the average player to keep on finessing. Only advanced players can follow the golden rule: choose the path most likely to lead to success.

In order to judge whether a given bid or play is likely to be successful, you need a working knowledge of the probabilities of the success of various options. At matchpoints especially you should not make a play that is against the odds. In many cases it is better to play for a plus rather than hope that a finesse will work when you feel it won't. How can you expect to win by playing against the odds? Before you can think in these terms, you have to learn how to estimate the odds at the time of decision, which may be when the dummy first appears, or near the end when more knowledge has been gathered.

The knowledge you seek is to what extent the current deal departs from normality. You have to gauge the state of affairs and decide what is most probable. Sometimes there is a rule that covers the situation and sometimes you may decide instinctively guided by previous experience on other hands of a similar nature, but often you have the clues available to make a decision based on the current probabilities. How to do this is the key problem.

Maximize the Gain or Minimize the Loss?

Don't mess with Mr. In-Between
— Johnny Mercer (1909-1976)

The Golden Rule at matchpoints is: always take the action that is most likely to succeed, using the *a posteriori* odds applicable at the time of decision. There are two fundamental approaches to decision-making that need to be considered: minimize the loss if you guess wrong, or maximize the gain if you guess right. It is not difficult to treat the problem theoretically where there are just two alternatives available. So we might think of the bidding outcomes as a game or a part-score, or a slam and a game, or a grand slam and a small slam, or we may apply the theory to a choice of card plays as well.

Differences due to system are the most obvious source of differences between the majority of players and the minority. These are most strongly felt in the bidding of slams. In a mixed field many players are content to end up in 3NT rather than in a minor suit slam simply because the field will not be confident enough in their methods to attempt the higher scoring contract. This approach feeds upon itself, as even superior players will play down to the field. Slams are becoming rare, whereas previously slam bidding was considered to be the keystone to good bidding practices. Now, failing to bid a cold slam may result in only a small loss as the vast majority will be stuck in the same boat. On the following hand I am ashamed to relate that I fell into the trap of bidding down to the level of the field.

Bob	**John**
♠ A Q 5	♠ K 10 8 7 6
♥ 10	♥ K 8 4 3 2
♦ A J 10	♦ 3
♣ A Q 10 8 5 2	♣ K 4

1♣*	1♠
2♦	3♣
3♦	3♠
4♣	4♠
pass	

The initial response to the Precision 1♣ showed a game-forcing hand with five or more spades. Using a series of asking bids I was able to discover that partner held at least five spades to the king, three high-card controls, and second-round control of the clubs. By bidding 5♣ I could find out whether the club control was a singleton or the ♣K. If John held a singleton club I would stop in 5♠ with work to be done — the field rated to be in game with 17 HCP opposite 9 HCP. If he held the ♣K, I would bid 6♠ with good chances of making five spade tricks, six club tricks and the ♦A. Playing to minimize my loss if I were wrong, I stopped abruptly in game when it would have been better (although not optimal) to go directly to 6♠ over 4♣ because the odds were greatly in favor of finding the ♣K opposite. Out of a field of eleven pairs, ten stopped in game so ostensibly I was not punished for my bad bidding. Nonetheless it was a mistake to defy the Golden Rule by rejecting an action that was more likely to be right than wrong.

Let's consider the scores you would receive by pushing on to slam regardless of what the field is doing. Assume eleven tables with eight pairs in game, two in slam. Here are the splits in matchpoints resulting from the decision on whether to bid slam or stay in game.

Bid slam and it makes	9	Bid game and slam makes	4
Bid slam and it doesn't make	1	Bid game and slam doesn't make	6

With 10 matchpoints available there is greater variability when you go against the majority and bid the slam. There is less variability when choosing to bid with the majority, even if they are wrong. Say PM represents the probability that the slam makes, M represents the number bidding slam and N, the number resting in game. The expected score for bidding slam is (M+N) x PM/2 and the expected score for bidding game is (M+N) x (1-PM)/2. If PM>0.5, the expected score for bidding slam will be greater than that for not bidding it, regardless of how the field has split. This is the basis of the Golden Rule.

A probability of 0.5 represents a state of maximum uncertainty. It follows that if one has some reason to suspect slam will make one should bid it. Trust your instincts, especially when they are right. Clearly, I was wrong not to bid 6♠. I might excuse myself by saying that near the end of a successful run I was happy to minimize my potential loss knowing I would have lots of company in game. In the long run this thinking is bad. How would my partner have felt if we had come in second overall by a couple of matchpoints? Not good.

A Matchpoint Anti-Finesse

Very often the decision reduces to fishing for a queen. Many feel they must take all the tricks available in a common contract, so will finesse at every opportunity. Play may degrade into a frenzy of finessing, declarers being unwilling to forego the extra trick obtained when the finesse happens to succeed. They are playing to maximize the number of tricks taken, and if the finesse fails it won't cost that much with most declarers playing in the same manner. However, the Golden Rule tells us one shouldn't take a finesse that is more likely to fail than not. The following hand recently played at the local club represents a situation where declarer does best by taking an anti-finesse.

West		East
♠ K Q	N	♠ A 8 4 3
♥ K 5 4 3	W E	♥ 8 7
♦ J 8	S	♦ A 10 5 4 3
♣ A K J 10 3		♣ 5 2

West overcalled the opening bid of 1♥ with a call of 1NT, not everyone's choice. Partner evoked Stayman then left him in 2NT. The opening lead was the ♥10 and questions were raised at the table as to why East didn't raise to 3NT. However, it appears his caution was justified as eight tricks looked to be the limit as the cards lay. South took his ♥A and continued a low heart to the ♥K, LHO following with the ♥9. At this point South had three heart tricks to take if and when he got in again.

Sometimes when the dummy first appears declarer realizes he is in a minority and is pretty sure that this is either a very good contract or a very bad one. In the above situation he cannot be sure what the field will be doing, so his major focus should be to play the hand safely while trying to arrange for an overtrick.

One of the main advantages for declarer is that upon seeing the dummy he immediately knows the division of sides. When the division of sides is 7-7-6-6 it often pays declarer to go passive and give up the obvious losers early rather than trying to create an additional winner by force. Sometimes pressure can be applied in this manner. The active approach is to overtake the ♠Q in dummy in order to take the club finesse. If it wins, continuing clubs will create nine tricks provided the RHO holds ♣Qx(x). If the finesse loses, there is still an excellent chance for taking eight tricks, via two spades, one heart, one diamond and four clubs.

The question to ask is whether or not the club finesse is likely to succeed. By overtaking with the ♠A the number of spade tricks is reduced from three to two, so declarer has to make an extra trick in clubs to make up for the loss. What are the chances the finesse will succeed? For his first-seat opening bid South needs the ♦K and at least one minor-suit queen. With 15 HCP he might have opened 1NT. North has 11 vacant places to South's 7, so the chances of South holding the ♣Q are less than 50%. That indicates declarer should avoid the club finesse.

What is the alternative plan? Declarer can cash the ♠KQ and play the ♣J hoping that North must take the ♣Q. If so, declarer has nine tricks easily. North must duck the ♣J if he holds four to the queen in order to destroy the communication with dummy and hold declarer to eight tricks. But some might win at the first opportunity and exit 'safely'. That is an edge that can be exploited. On the other hand, if South has the ♣Q, declarer is held to eight tricks immediately. It would be a cause for general merriment at the table if South held a singleton ♣Q, but nonetheless eight tricks would still be taken with declarer's communications still intact.

What was the situation at the table? Not surprisingly, North's shape was one of the two most likely candidates, 4=2=3=4, and he held the ♣Q as expected in that situation. This is exactly what declarer might have anticipated at Trick 2 by considering the most likely distributions. By not taking a losing finesse West might still have scored nine tricks on the extra chance of a defensive miscue. It is hard to guess in general how many matchpoints the overtrick would be worth, but we do have the results for this occasion.

The fourteen tables in play produced eight different contracts and nine different scores. Only three pairs played in 2NT, two making 120, one making 150. Making nine tricks in 2NT instead of eight would have added 5 matchpoints to the score, raising the percentage from 42% to 80%. That shows one needn't bid a close game to be successful at matchpoints, even if you would have made it if you had bid it. Two pairs were in 3NT, but both declarers failed, as they should have done, for a shared bottom; the highest East-West scores were attained by defending against a vulnerable 2♥ doubled.

Mr. In-Between

When you come to a fork in the road, take it.
— Yogi Berra (1925-2015)

There are those who oppose full application of the Golden Rule. They advocate being conservative in the bidding. Their idea is that by going along with the field they avoid getting a bad board through unforeseen circumstances. They prefer to proceed according to the judgment of the field rather than the lie of the cards which, being randomly dealt, are fickle. Change a six-spot to a five and you may be in big trouble without knowing it.

Conservatives argue that in a mixed field there are players who will present you with matchpoints through their misplays. So, for example, if you bid game and make twelve tricks, that might be an above average score because some have misplayed slam or ended up in the wrong contract. Of course the slam may have been cold, but maybe it wasn't. That is their thinking. However, you are not competing against these bad players for first place, you are competing against players who are at least as good as you are, in which case you need to play a sharp game and not pass up opportunities for a top score. Beware of missing opportunities. Who said that? Franz Liszt, the great lover turned Franciscan. (Reformed sinners are wise but they tend to lose their charm.)

If you are facing a bad pair you may bid boldly, as Rixi Markus suggested, anticipating a bad defense that might increase the chances of success. As in the end you are competing against the good players who hold your cards at the other tables, you can't afford to play for averages against a bad pair. Nonetheless, if the cards tell you that making slam is against the odds, you shouldn't bid it, in part because a bad defense to game may give you a good score through undeserved overtricks. That is, you may win on the play rather than on the bidding. On the other hand, bidding a good slam other good players may miss is a way to gain an advantage over them, an opportunity that shouldn't be missed just because of what the bad players may be doing. That is not safety.

A Girl Named Florida

What's in a name?
— William Shakespeare (1564-1616)

It is puzzling that so many players stick with the *a priori* odds rather than use the *a posteriori* odds that apply after information has been made available through the bidding and the play. In essence they are ignoring Bayes' Theorem. This was evident in a recent exchange of correspondence on bridgewinners.com, initiated on October 18, 2016 by Mike Wolf, concerning a possible application of Restricted Choice to discards in a potential squeeze position. One correspondent referred to a possible split in a key suit as Kxxx opposite Jx. The typographical convenience of using x's is inappropriate, because at the time of decision all the spot cards had been played. The question raised was this: is the jack to be considered a spot card that could have been played at random any time without damage? Although *a priori* all the spot cards and the jack were randomly dealt, it is better in theory to give the spot cards names (a, b, c, d) as a reminder that once the cards are given an identity during the play it is possible the odds have changed. It is reasonable to assume the jack would not be played at an early stage even though it would not be costly on a double dummy basis, so the conditions are not the same as when the cards were dealt. It is not correct to apply the *a priori* odds at a later stage, because the more extreme splits have been eliminated. The implications of this, which are relevant to the application of Restricted Choice, are discussed in a book that is not about bridge.

Talk sense to a fool and he calls you foolish
— Euripides (480-406 BC)

In 2008, Leonard Mlodinow published *The Drunkard's Walk*, a popular treatment on the mysteries of probability applied to real-life problems. No sooner had I finished reading it, than I received a call from a very nice lady who had decided to take up bridge upon retiring from a job in which she had dealt constantly with statistical data. Prompted by her love of numbers, she was drawn to my book. I congratulated her on her choices and wished her years of happy entertainment without mentioning the frustrations that go along with the game. However, she had called not to praise my book, but to correct it. She was familiar with the Monty Hall Problem, and was convinced that my treatment was wrong. I apologized for my inadequate explanation of the solution, but happily could refer her to *The Drunkard's Walk,* for a fuller treatment of the problem and its resolution, and thus for an independent confirmation of the validity of my approach. I hope she followed my advice, recovered my book from the trash bin, and corrected her long-held views.

The Monty Hall Problem is one often used by bridge writers to illustrate the application of conditional probability to card play, in particular, through the Principle of Restricted Choice. Later in this book, you will find many pages devoted to discussing variations of it. However, Mlodinow has provided us with another illustrative example that demonstrates directly the difficulty many encounter with the concept of probability linked to a state of partial knowledge. He calls it 'The Girl Named Florida Problem.'

Suppose that a couple have produced two naturally conceived children. What are the chances that they are both girls, assuming that at the time of conception a boy is as likely to result as a girl? The event is mathematically equivalent to tossing a coin, and the correct answer is 1 in 4. Next we ask, if one child is known to be a girl what are the chances the other is also a girl? Without going into the details, just accept for now that the chances of there being two girls is 1 in 3.

Next we ask, what are the chances of the couple having two girls given that one of their children is a girl called Florida? There are those who would argue that whether the girl was named Florida or Jane or Sarah should make no difference to the chances that their other child is a boy. Although the name Florida is unusual, there is no causal effect at work. However, it turns out that the naming of one child adds information and changes the probability for a second girl from 1 in 3 to 1 in 2.

The key point of the problem is that the number of children is specified beforehand, just as the number of cards in a suit is limited. You will encounter Miss Florida again later in this book, at which time I'll try to explain her problem in detail, and relate the story to the Principle of Restricted Choice.

Chapter 1

THE SHIFTING SANDS OF PROBABILITY

As I take one step and then another, I wonder what my chances are.
— Hayes Carll (b. 1976)

During elections we hear repeatedly about the results of opinion polls and are at the same time constantly warned about their unreliability. It is scary to think how often the chance of a disaster is within the margin of error, even in real life. The statistics change with time, but they generally prove to be an accurate gauge of how things are going to turn out in the end. Well, the same applies to bridge probabilities — they may change during the play of the cards, but they are usually a good guide to what will happen, barring the occasional nasty surprise.

In the world of bridge we have probabilities fed to us by the writers and commentators. I had always accepted these percentages at face values, but once I had the time to look more deeply into the whole concept of probability, I discovered the assumptions that lie hidden behind the figures and that are all too often ignored in the discussion, something that results in misleading conclusions. The idea that 'probabilities never change' was one of the first assertions that fell by the wayside, as of course this is contrary to common sense. The correct way of thinking is, 'probabilities change according to what information becomes available.' If they didn't, we'd be living in a strange world where ignorance would surely be bliss. The main theme of this first section of the book is that probabilities change with circumstances.

Some people are reluctant to admit that bridge is a game of guesses, but that is what it is, because it is played in an atmosphere of uncertainty. That's why on occasion weaker players can beat experts — sometimes the 'wrong' action turns out to be the winning one on a particular deal. In the long run, of course, it is a different matter. However, 'guess' is not a dirty word. Our lives are governed by chance. Probability is a way of organizing our guesses and assigning them proportions. Of course, at the end of the deal when all the cards have been revealed, we may realize that there were clues along the way that should have pointed us in the right direction. The skill of an expert is that he adapts his approach according to what has been revealed as the bidding and play proceed.

There are very successful players who haven't mastered Probability Theory, although they certainly apply it in a practical way based on experience. Multiple world champion Sabine Auken in her great memoir, *I Love This Game*, has described how her curiosity was piqued by probability only at a later stage of her career. The question arises, if that is so then why should we ordinary players be interested? If one is to use probability successfully then it is necessary to understand how hidden assumptions come into play so as not to be distracted unduly by the numbers lifted from textbooks. Probability should be a way of expressing common sense in numbers. Of course, once one has mastered the basics, there are many applications to be found away from the bridge table too.

One foundation of analysis is the table of probabilities of suit distributions to be found in *The Official Encyclopedia of Bridge* and elsewhere. Expert players are expected to know these backwards and forwards. If they don't play according to these numbers, they are often criticized by the pundits, but maybe the expert has his reasons. Well, I am nothing if not critical, and in many situations there can be arguments both ways. Let's begin by looking at how the experts played an eight-card suit missing the ace and jack when the bridge championship of the world was at stake in 2008.

Five Missing the Ace-Jack

The final of the women's 2008 World Bridge Games Championship was won by England over China by the slim margin of 1 IMP over 96 boards. The purist might say that the difference was a vital overtrick, implying that missing an overtrick can be a critical play even in such a long match. Well, yes, but the slender difference could also be more than made up by avoiding many of the simple errors one observes in the bidding and play. In that regard England was the steadier team throughout and deserved their win, although they almost blew it in the last 16-board segment. To me the lesson was this: bid aggressively, avoid major errors in the play of the hand and cooperate with your partner, and your team will be hard to beat. There are too many much larger factors that affect the results to be worried about overtricks.

One of the sources of percentages in play are those related to how a declarer should play combinations of cards in a given suit. An abundant source of this type of information is *The Dictionary of Suit Play Combinations*, a great reference book written by J. M. Roudinesco. If that is not enough, there is the computer program, *Suit Play*, created and made available on the Internet by Jeroen Warmerdam of the Netherlands. Let's look at the recommended play in eight-card suit combinations where the ace and jack are missing. If the Chinese women had got one of these correct they would have gained enough IMPs to win their final match handily. By examining the deals in detail, we can get a deeper understanding of how probability analysis should be applied under changing circumstances.

Going with the Odds

The first combination we shall ponder is:

Q 10

K 9 7 6 5 2

The textbook line is to play low to the queen and, regardless of whether that wins or loses, to run the ten on the next round. The chances of this providing four tricks is about 96%, while about 46% of the time five tricks will result. These figures are derived from considering the suit combinations in isolation from the full deal. As this is based on the prior expectations of how the cards were dealt, the approach yields a valid approximation when very little is known about the defenders' hands. If they have passed throughout, you may assume that the suits are likely to split more evenly than would be expected *a priori*, but that may not greatly affect the calculation of the odds. Here is the full deal from the final where the Chinese declarer went against the odds and lost 6 IMPs as a result.

North:
♠ A 9 4
♥ Q 10
♦ 8 6 2
♣ Q J 9 7 4

West:
♠ 10 8 3 2
♥ 8 3
♦ A 9 7 5
♣ K 3 2

East:
♠ Q 7 6 5
♥ A J 4
♦ 4 3
♣ A 10 8 6

South:
♠ K J
♥ K 9 7 6 5 2
♦ K Q J 10
♣ 5

Brock	Wang	Smith	Sun
		pass	1♥
pass	1NT*	pass	2♦
pass	2♥	pass	3♥
all pass			

Sun held a good hand in the context of a Precision opening bid, only five losers, so she raised herself to the three-level where others holding the hand were content to stay in 2♥. The lack of aces is a defect not overcome by the distribution, and perhaps the diamond suit is overly rich with honor cards while the heart suit is rather sparse in that regard. In the Open final, where Italy faced England, neither South was allowed to play in 2♥ as West balanced and East-West played in a contract of 2♠, going down. At the other table in the women's final, England's Nevena Senior played undisturbed in 2♥ making an overtrick. So Sun did the right thing in theory as her 3♥ contract appears to be solid and prevents the opposition from playing their optimum contract of 2♠.

The opening lead was an innocuous ♠2 won by the ♠K when Nicola Smith played her ♠Q. The play in the trump suit was now front-and-center, and if Sun had gone with the percentages, China would have won the championship. A point well made by Linda Lee in her blog of October 19, 2008 was that there was some urgency in the trump play as the lack of controls for the declaring side made it a race to the finish line, with declarer hoping to prevent the opponents establishing their setting trick before she made sure of her own nine.

Sun didn't feel the urgency even though the defenders held minor-suit aces that provided them with transportation. This wouldn't have mattered if Sun had played the trump suit optimally according to the *a priori* odds — that is, low to the ♥Q, planning to run the ♥10 next. The ♠A was still there as a safe entry to dummy to allow this sequence. Unfortunately for China, Sun chose to play a heart to the ♥10, losing to the ♥J. Nicola Smith had defended well throughout the final and here she was quick to take the opportunity of obtaining a ruff in diamonds. She switched to the ♦4 and Sally Brock took her ace and returned a diamond immediately just in case that ♦4 was a singleton. Not so, but it didn't matter; on winning the ♥A, Smith underled her ♣A and got the ruff that set the contract and brought 6 IMPs England's way, a critical swing late in the match.

When two high honors are missing, it is a great temptation to finesse against the lower honor first. The motivation is that this guards against one defender holding AJx in front of the queen. Another reason for adopting this approach would be that declarer places the ace behind the Q10, in which case the odds usually favor the honors being split. We shall examine this argument in the next segment.

Placing an Ace

The next deal was played early in the final matches of the Open, Women's and Bronze Medal series. It involved declarer play in a suit with the following construction:

♣ K 9 7 6 5

♣ Q 10 3

The textbook play for four tricks with the suit taken in isolation is to lead low to the ♣Q and, whatever happens on that trick, to pass the ♣10 next. That assumes declarer has no information on how the cards in clubs may be distributed. The probability of making four tricks is 54%, but the bidding may change the *a priori* odds. Let's look at the whole deal and how it was played in the Open Series where Italy faced England.

Dealer East North-South vul.	♠ A 6 5 ♥ K ♦ K 5 3 2 ♣ K 9 7 6 5	
♠ 9 4 ♥ Q J 8 7 5 3 ♦ 8 7 ♣ A 8 4	N W E S	♠ K Q J 7 3 ♥ 10 6 4 ♦ Q 9 6 ♣ J 2
	♠ 10 8 2 ♥ A 9 2 ♦ A J 10 4 ♣ Q 10 3	

Townsend	**Lauria**	**Gold**	**Versace**
		pass	1♦
1♥	2♣	dbl	3♣
pass	3♦	pass	3♥
dbl	5♣	all pass	

Gold led the ♠K and Lauria held up. Gold switched to the ♥4, which Lauria won to run the ♣9 around to Townsend's ♣A. The heart return did no damage, as a losing spade could be discarded on the ♥A. Obviously, Lauria's play in the club suit was predicated by the bidding and East's opening lead from a high honor sequence, which placed the ♣A in the West hand. He assumed East held the ♣J. He could also have taken the necessary finesse in diamonds at Trick 3 and led the ♣3 from dummy, guarding against a doubleton ♣AJ in the West hand.

In his book, *Playing with the Bridge Legends*, Barnet Shenkin makes the point that when amateur analysts (like myself) criticize experts, they are usually wrong — even with the help of Deep Finesse. I am willing to concede the point. Who am I to question the great Lauria? Nonetheless, no player is perfect and we amateurs mustn't give up on trying to understand the mental processes of the experts. How else are we to improve?

Child's Play?

The term 'Chinese finesse' is a derogatory one. It refers to leading an unsupported honor from the closed hand, and hoping the defenders err by not covering. The declarer runs the unsupported honor and makes an undeserved winner. I now propose that the term 'Chinese Drop' be applied to a play that also appears to be ridiculous, except when it works. The Chinese women could have won if they had employed a play that on the surface appears to be one that only a novice would make: low to the queen, then low to the king.

When it comes to the replay of the same deal in the Women's final, I cannot get rid of the feeling that something was definitely wrong in declarer's approach of making the standard percentage play in the club suit. Let's see if you agree.

Dhondy	Wang	Senior	Liu
		pass	pass
2♦*	dbl	2♥	3NT
all pass			

Heather Dhondy's lead was the ♥7, won by Liu Yi Qian in dummy. She led the ♣5 to her ♣Q and Dhondy ducked that since the ♣A was her only entry. Liu led a second club from her hand, Dhondy following with the last outstanding low club, the ♣8, and here Liu missed the seemingly ridiculous but winning play of going up with the ♣K to drop Senior's now bare ♣J. Let's see the most probable possibilities when Dhondy followed to the second club play:

	Dhondy	Senior
Situation #1	8 4	A J 2
Situation #2	J 8 4	A 2
Situation #3	A 8 4	J 2
Situation #4	A J 8 4	2

In Situation #1 there is no winning play, so we can rule that out, as we must play for success. Situations #2 and #3 are equally likely on the deal, but isn't it more likely on the bidding and play so far that Situation #3 holds? Without an entry in clubs, West might have led a spade, hoping to hit her partner's suit, whereas with the ♣A she would surely try to set up her own suit.

In my experience, when an opponent enters the bidding with a flimsy suit and leads that suit against 3NT, it is more likely that the bidder holds a top honor in my long suit. So, here when West is missing the ♥AK, I think it is more likely that she holds the ♣A. With regard to the dealing of the cards alone, of course, that conclusion would have no mathematical foundation.

One of the most useful pieces of information available to declarer is the division of sides. It is also the most neglected. When the dummy appears, a declarer knows how many cards are missing in each suit. One can link this to the probability that a particular distribution exists in one hand or the other. Here the question is whether Situation #4 is a live possibility, and, if so, how to take that possibility into account. That would give West 6 hearts and 4 clubs. Let's look at the possible divisions of sides when the opponents hold 7 spades, 9 hearts, 5 diamonds, and 5 clubs, the hearts are split 6-3 and West holds at least 2 clubs.

	I	II	III	IV	V
	♠ 3 – 4	♠ 2 – 5	♠ 2 – 5	♠ 3 – 4	♠ 2 – 5
	♥ 6 – 3	♥ 6 – 3	♥ 6 – 3	♥ 6 – 3	♥ 6 – 3
	♦ 2 – 3	♦ 2 – 3	♦ 3 – 2	♦ 1 – 4	♦ 1 – 4
	♣ 2 – 3	♣ 3 – 2	♣ 2 – 3	♣ 3 – 2	♣ 4 – 1
Weights:	100	60	60	50	15

The probability weights given along the bottom are easily calculated from the assumed splits. (The weights are the relative numbers of card combinations for each condition, starting with 100 for the greatest.) Condition I is by far the most likely situation, but we see that this is a losing configuration corresponding to Situation I. The same is true of Condition III. That leaves Conditions II and IV as the most likely winning position, corresponding to Situations #2 and #3. Condition V, representing Situation #4, is much less likely to have been dealt.

How the Unplayed Suits Contribute to Probabilities

Probabilities are derived from ratios of the number of suit combinations. In the case at hand there remain two honors missing, two possible splits in clubs, and three distributions of sides. We can add together the number of suit combinations in the unplayed suits, diamonds and spades, to obtain an estimate of the probabilities of success of the two possible winning plays, namely, running the ♣10 and going up with the ♣K.

Play	**II**	**IV**	**V**	**Total**	**Percentage**
Run the 10	210	175	175	490	56%
Play the K	210	175	0	385	44%

A calculation of the ratio of suit combinations yields the result that the probability of success for running the ♣10 is 56%, very close to the *a priori* probability. An assumption that underlies both calculations is that we are not at liberty to assume that the ♣A is more likely to have been dealt to the West hand. Another way of thinking of this assumption is that we are maximally uncertain as to the location of the ♣A. However, as noted above, I would be surprised if the

♣A were not held by the West player for the reasons stated. If the probability of West's having the ♣A were 56%, the two lines of play would have the same probability of success. Anything greater than 56%, and going up with the ♣K becomes the preferred play. I would estimate that given West's line of defense, she would hold the ♣A more than 3 times out of 5 (60%), and that is my basis for stating that the Chinese Drop represents the best chance at this stage where all the low clubs have been played. Yes, here the Chinese Drop is not as dumb a play as one might expect. I could argue that it is even the percentage play, given the information that is available at the time of decision.

Luck lies not with the player but in the placement of the cards.
Probability Theory, properly applied, is the best tool for extracting it.

The Maximally Likely Distribution of Sides

You might criticize this analysis as being rather superficial. It is certainly incomplete, as we have considered only one possible division of sides. That being said, I am somewhat amazed by how often the maximally likely division of sides turns out to be a reflection of reality. The longer the defenders follow with low cards to declarer's plays, the more likely that condition becomes. Those plays have eliminated the possibility of extreme splits.

When a declarer starts playing his suits, he should have a plan in mind. Taking into account the most likely division of sides makes sense. A simplification based solely on the maximum likelihood division is not rigorous, but it may focus the mind in a beneficial way. If the problem faced is a complex one, simplification may be the best approach. It is a start. By solving a simple problem, you hope that the more complex problem has the same solution. Probability is on your side. If you can think more deeply, and consider more possible distributions, great.

What you can do in many situations is gather more information before making a critical decision. Every bit of information helps, and the probabilities change accordingly. When Liu played the second round of clubs from her hand and West followed with a second low card, she gained information. The most likely distribution of sides could be eliminated as it would not leave her a winning choice. She should then have focused on the second most likely condition, specifically, the clubs splitting 3-2.

Probability and Information from a Surprise Action

The greater the surprise, the more information the action transmits. Let's suppose that on Board 2 Dhondy and Senior had not entered the auction. In this day and age, that would have been a surprise since when the dummy came down

Liu could see East-West held seventeen major-suit cards and 16 HCP between them. There are six equally most likely distributions of sides for a 7-9-5-5 division, namely,

I	II	III	IV	V	VI
♠ 3 – 4	♠ 3 – 4	♠ 4 – 3	♠ 4 – 3	♠ 4 – 3	♠ 3 – 4
♥ 5 – 4	♥ 5 – 4	♥ 4 – 5	♥ 4 – 5	♥ 5 – 4	♥ 4 – 5
♦ 3 – 2	♦ 2 – 3	♦ 3 – 2	♦ 2 – 3	♦ 2 – 3	♦ 3 – 2
♣ 2 – 3	♣ 3 – 2	♣ 2 – 3	♣ 3 – 2	♣ 2 – 3	♣ 3 – 2

Based on the inaction of this normally active pair, we might downgrade Conditions V and VI in each of which one player holds 5-4 in the majors. Next, consider the opening lead. If it is a heart, this is most consistent with Conditions I and II, as a player is most likely to lead from her longest suit. Note that a 3-2 split in clubs is as likely as a 2-3 split.

But suppose the opening lead was the ♦5 from the player on the left. That would be a surprise, and surprises greatly affect the probabilities. None of the above six distributions makes sense with this lead. Declarer might then consider other options, and in particular, that the lead was a singleton. She would then consider a distribution of sides based on that assumption and work from there. Here are some main candidates together with their probability weights relative to the above set.

	VII	VIII	IX	X	XI
	♠ 4 – 3	♠ 4 – 3	♠ 5 – 2	♠ 5 – 2	♠ 4 – 3
	♥ 5 – 4	♥ 6 – 3	♥ 4 – 5	♥ 5 – 4	♥ 4 – 5
	♦ 1 – 4	♦ 1 – 4	♦ 1 – 4	♦ 1 – 4	♦ 1 – 4
	♣ 3 – 2	♣ 2 – 3	♣ 3 – 2	♣ 2 – 3	♣ 4 – 1
Weights	50	33	30	30	25

The relative weights are based on the number of card combinations available on a random deal. However, the bidding (or lack of it) must also be taken into account. As the opening leader has passed throughout, more weight must be given to Condition XI. It makes sense that with a 4-4-4-1 shape a player would tend to remain silent throughout the auction whereas with 5-4+ in the majors, she would be inclined to make some noise, so Condition X belongs at the end of the line and Condition XI moves to the top.

As I pointed out at the start of the discussion, nothing is certain, and that is why we must revert to probabilities. There is no guarantee that the surprise diamond lead is a singleton. It is possible that the opening leader decided to make a 'safe' lead from three small diamonds rather than from honors or tenaces in the majors. She may even be attempting to make a deceptive lead. The general rule is that the best deceptions come from actions that appear to be normal. Some players will lead from worthless doubletons for no other reason than that they hope declarer gets it wrong. That's unusual, which is the reason why the technique is sometimes effective, so such occasions are rare. Based on likelihood considerations, what appears to be normal should be assumed to be normal. To be constantly suspicious of a normal action is to present yourself with one more way to lose. Be resigned to being deceived occasionally, and move on. That is the way to keep on the right side of probability; besides which, it's easier on the nerves.

Sound Advice: If the opponents took a 'wrong' view that worked out well, you have to look at it as their having given your side a chance for a good score that didn't quite pay off this time. Be patient. In the long run you will benefit from their poor decisions.

CHAPTER 2

NEVER A DULL DEAL

Little things mean a lot
— Edith Lindeman (1898-1984)

Probability is related to information, so it changes with the play of each card. Different information sets lead to different probabilities. 'Going with the odds' entails 'going with the information flow'. Here is a simple example. Suppose South is a declarer looking for the ♥Q with five hearts missing and has a two-way finesse available. On the basis of the deal alone, there is a 50-50 chance that Her Majesty lies in either of the defenders' hands. Now suppose the lead is the ♠2, from which declarer deduces that the spades are split four on the left and three on the right. This information leads to the conclusion that the ♥Q is more likely to have been dealt on the right in the ratio of the vacant places, 10:9, so it is correct to finesse through East. Next, suppose that at the other table North has become declarer in the same contract through a transfer sequence. The lead is the ♦2. By the same reasoning, North should play West for the ♥Q. He also is correct, but only one declarer is going to be successful. The information about one suit affects the probabilities in the other suits, but because the information is different, the deduction is also different.

Both the *a priori* odds and the vacant place calculation are based on the assumption that the unknown cards can be placed randomly between the two defenders. That is an assumption of a condition of maximum uncertainty with regard to the placement of the unknown cards. It loses validity if there are indications one way or the other. Let's continue to follow the first declarer, who doesn't immediately take the heart finesse through East, a decision which he thinks is favored in the ratio of 10:9. Instead he plays on diamonds and finds they are split 4-3 the other way from the spades. Now the vacant places are even at 6 and 6 between West and East. Due to this additional information, the chances of either defender holding the ♥Q have returned to their *a priori* value of 50-50. One feels that the more information one obtains, the more accurate will be the estimate of probability and the better the chances of making the right decision.

If the contract were 6NT, declarer might postpone the heart finesse until some more information was obtained from the club suit. If he can find out how the clubs split, knowing the pointed suits split evenly, he will be able to deduce the heart split and can play accordingly. The answer will not be a 50-50 decision any more, as one player will be known on the basis of the information available to hold three hearts, and the other, two. However, there are still no guarantees — a 60-40 finesse will still lose 40% of the time. We have been taught to assure the contract against bad breaks at team play and to forget about overtricks. As a consequence we are discouraged from making an exploratory move if that might lead to the defeat of the contract on a bad split. That in turn results in an over-reliance on the *a priori* odds which, as I noted, are based on a condition of maximum uncertainty.

At matchpoints, by contrast, risking the contract for the sake of an overtrick can be the correct play. Exploratory moves for the purpose of gathering information make good sense. Let's look at my misplay in a recent local Regional. It was a typical inexpert matchpoint game in which success often depends on the opponents not defending to best advantage. The deal was a routine 3NT across the whole field, so the play was all about overtricks. Some find such contracts dull, but I find they often contain elements of great interest. Besides which, our mistakes are stepping stones to improvement. (Sure they are.) First and foremost we must prepare ourselves to adapt to changing circumstances.

West		East
♠ K 7	N	♠ J 10 9 4
♥ A 7 2	W E	♥ K J 10 3
♦ J 8 7 3	S	♦ A Q
♣ A K 10 8		♣ Q 4 3

We arrived in 3NT after an uninterrupted Stayman auction that was probably duplicated at most tables. As declarer, I covered the lead of the ♠3 with dummy's ♠9 and won the trick while South, using standard carding, followed with the ♠2. How would you play this hand at teams, where the primary objective is to make the contract? After you answer that, the next question is: how would you play the hand at matchpoints where the objective is to win as many overtricks as possible?

The simplest way to assure nine tricks is to play the ♦Q at Trick 2, establishing two tricks in the suit. A spade continuation will set up the ninth trick, so the defenders' best return is a passive diamond to the now bare ace. No problem, as a third heart trick is assured by running the ♥J from dummy. At most declarer loses two spades and two tricks in the red suits. Giving up on the diamond finesse in this manner is playing the hand with extreme short-sighted pessimism, since declarer has a double stopper in every suit. There must be a reasonable limit to our fear of loss. However, if we play that way, 3NT does indeed become a very dull deal. However, as with our lives, there are no dull deals but a lack of initiative makes them so.

At matchpoints there is a chance for two overtricks in a common contract, so declarer must go all out to obtain them if they are available. We want to find North with the ♦K and South with the ♥Q. The opening spade lead gives us immediate odds of 10:9 for the second condition. With this in mind I led the ♥J from dummy, hoping for a cover; South played low, and I let the jack ride, losing to the queen. Unlucky? Or should I have followed Zia's advice that if they don't cover they don't have it? More on this later. Next North cashed his two spade winners and led a low diamond. Should I finesse or go up with ♦A and rely on bringing in four club tricks?

The *a priori* odds that clubs will split evenly is just 36%, so it would appear at first glance that the diamond finesse is better. However, having the ♣10 gives an *a priori* chance for four club tricks of about 60%. Judging the exact probabilities at this late stage requires more information on the expectation of the splits in the other suits. If North were expected to hold four clubs, then the line of trying for four club tricks would lose its luster.

My thinking at the time turned away from card combinations to psychology and motivation. I viewed North's strategy with suspicion. Why did he cash his spade winners and instead of exiting passively with a heart or even a spade? It would appear that I am committed to the diamond finesse regardless. It didn't occur to me at the time that the defender had no devious plan in mind and was merely happy to be ahead of the game by taking a trick in hearts to which he was not entitled. In a confused state of mind I decided to take the diamond finesse on the grounds that, after all, every declarer in the field would be taking that finesse. Wrong! Not only on the play itself, but, more importantly, with the thinking behind it. We should always keep in mind that a defender is working with different information and can't see the cards we hold, and vice versa. Here is the full deal.

	♠ A Q 5 3 ♥ Q 6 4 ♦ 10 5 2 ♣ J 9 6	
♠ K 7 ♥ A 7 2 ♦ J 8 7 3 ♣ A K 10 8	N W E S	♠ J 10 9 4 ♥ K J 10 3 ♦ A Q ♣ Q 4 3
	♠ 8 6 2 ♥ 9 8 5 ♦ K 9 6 4 ♣ 7 5 2	

This deal provides a fine example of 'Do as I say, not as I do.' I indulged in the kind of fragmented thinking that goes along with the acceptance of arguments based on the *a priori* odds in each suit taken without reference to the deal as a whole. The first step in the process after the opening lead is to examine dummy and set a realistic target for the number of tricks we hope to take. The lead gives us two tricks in spades, there are three tricks in clubs, and three top tricks in the red suits. There are two finesses to be taken in the red suits, either of which will set up our ninth trick even if it loses. If we get hearts right, the suit might even play for four tricks, as might clubs. So perhaps a realistic result is ten tricks. Of course, as this is matchpoints, declarer should strive for one trick more. The question arises of how we might negotiate to the desired ending without unduly risking what appears to be the normal result of making one overtrick.

Let me digress a moment to talk about the Deep Finesse effect. Deep Finesse is the double-dummy analysis software that is often used to generate the expected 'results' that appear on tournament hand records sheets. The program is a wonderful tool as it provides us with the optimal results obtainable on the given lie of the cards. However, it operates knowing the position of all the cards and does not deal with probabilities. To obtain this perfect result, usually declarer has to take any finesse that works, however unlikely a play this might be. The widespread availability of this analysis acts as an encouragement to take every finesse in sight and hope it succeeds. As with references to the *a priori* odds, this treatment fragments one's thinking into a suit-by-suit approach. It freezes the approach to be taken. In the above deal we focus our attention on the obvious finesses in the red suits, hoping first and foremost to take the heart finesse in the right direction.

If a declarer makes the optimal number of tricks as determined by Deep Finesse, he is entitled to feel he has done his part well enough, but to ensure a good score he may have to exceed the optimum. This usually comes about because the defense has erred, so a secondary aim of a declarer is to play the hand in such a way that the defenders won't find the best defense. On this deal, declarer aims to avoid the diamond finesse if possible. Why? Well, let's not forget the club suit. As noted above the club suit is expected to provide four tricks about 60% of the time, which is a better percentage than the diamond finesse taken in isolation, so, although the doubleton ♦AQ stands out in the dummy like a sore thumb, declarer wants to blind North to its importance, when he doesn't hold the ♦K, until it is too late. With a bit of luck in the club suit, the fourth club will be a winner, and the diamond finesse can be avoided. So timing becomes important. If the diamond finesse can't be avoided, then reluctantly we have to take our chances on it along with everyone else.

If declarer plays a spade at Trick 2 to establish a second spade trick, North should become aware of the danger and lead a diamond before taking his second and last spade trick. It has to be better tactics to take the heart finesse immediately at Trick 2 and hope for a defensive error. The question arises as to what is the best direction for that move. Preliminary evidence suggests that finessing through South has a slightly greater chance of success (10:9 odds), but it appears 'more natural' to finesse through North. (My partner certainly thought so.) More importantly, South can't lead a diamond. If South wins the ♥Q, she will undoubtedly return a spade, her partner's suit. If North then cashes a second spade before returning a diamond, good timing will have been achieved. If he doesn't take the second spade winner before playing a diamond, declarer resorts to the diamond finesse as a matter of necessity, losing nothing in the process. The question arises as to how much is risked by taking the heart finesse 'the wrong way'? Is it a good investment? To answer that we have to look more closely at the possible distributions involved.

The Distribution of Sides

If we accept the deduction that the spades are split 4-3, the most common distributions of the North-South cards are as follows:

	I	II	III	IV	V	VI
	N S	N S	N S	N S	N S	N S
	♠ 4 – 3	♠ 4 – 3	♠ 4 – 3	♠ 4 – 3	♠ 4 – 3	♠ 4 – 3
	♥ 3 – 3	♥ 3 – 3	♥ 2 – 4	♥ 2 – 4	♥ 4 – 2	♥ 3 – 3
	♦ 3 – 4	♦ 4 – 3	♦ 4 – 3	♦ 3 – 4	♦ 3 – 4	♦ 2 – 5
	♣ 3 – 3	♣ 2 – 4	♣ 3 – 3	♣ 4 – 2	♣ 2 – 4	♣ 4 – 2
Weights	100	75	75	56	56	45

The relative weights are based on the number of card combinations that result from a random deal of the other three suits. From the weights we can get a rough estimation of the probability of each condition having been dealt. The assumption is the same as that behind the vacant place calculation, but here we display only the most common combinations of splits in hearts, diamonds, and clubs. This focuses the mind on what is most probable given the information available so far from the bidding and the opening lead. It is of interest to note that the single most likely distribution (Condition I) is the actual distribution encountered at the table, the flattest of flat 4-3-3-3 on both sides of the table. If declarer had to choose just one condition to play for, that would be it.

If we were to add more possibilities, we would be adding more uneven splits, but the bidding has not indicated that there are extreme distributions to worry about. Also, if North were to hold two four-card suits, we can imagine that the opening lead might have been, at least part of the time, a passive lead in the other four-card suit, rather than in a suit headed by the ♠AQ, which might give away a trick immediately. Even so, it appears that the ♥Q is more likely to have been dealt to South than to North, the odds being much the same as the vacant place odds.

However, at this level of the game, South may make the error of covering the ♥J as often as one time in ten, balancing the odds as to the location of that card. Playing the ♥J from dummy and overtaking with the ♥A when it is not covered appears to be a dangerous and unreliable way to collect information, but if North doesn't follow to the second heart, declarer wins the ♥K in dummy and leads a spade. The odds for the diamond finesse have improved, as North's most likely shape is now 4=1=4=4, and the ♥103 still stand guard if the diamond finesse loses. There is another consideration at matchpoints, which is this: how will the majority of declarers play the hand? I imagine that most will take the 'natural' heart finesse, playing to the ♥A in hand and finessing through North, in part to protect against an early diamond switch. It risks little to go this route even if the odds are against it. The strategy is to play for an average and hope the defenders make a mistake along the way.

My error was not so much in trying for a cover of the ♥J, but in not following through and playing the ♥A when South played low. When North won the ♥Q and cashed his spade winners, my luck changed and I was presented with the very mistake for which I should have been hoping all along. Now North played the expected diamond, and I fell from grace by playing the queen. I didn't follow through by rejecting the finesse and playing for the better odds of four tricks in clubs or a diamond-club squeeze. I should have been focused from the very beginning on the most probable conditions with regard to both minors taken simultaneously. (Just envisioning Condition I would have been a good start.) Taking the ♦A and playing off the hearts would determine the split in the heart suit to have been 3-3, so the remaining possibilities are Conditions I, II, and VI. Now South may evince some discomfort on the play of the last spade under Condition II when he holds the ♦K and four clubs to the jack. Indeed, in this case we have a show-up squeeze, as South is forced to hold on to the ♦K, and his ♣J will show up on the third round with no further need for guessing. As at worst only one trick remains to be lost, there is an average score to gain and nothing to lose by refusing the diamond finesse. With all hands revealed, failure to count becomes the most glaring of errors.

The bidding and the play are important factors in determining probabilities. Generally speaking the *a priori* odds represent a reasonably accurate guess of one's chances during the play as long as even splits are encountered. That may change significantly if some splits are known to be extreme. There was no evidence that such was the case on this deal as bid and played. If we calculate from Conditions I, II and VI the chance of obtaining four club tricks by playing ace, queen, king in that order, we actually find it to be 70%, much better than the *a priori* odds, because of the squeeze possibilities that have evolved.

Chapter 3

NINE-NEVER AND BASIC BRIDGE PROBABILITY

The risk of a wrong decision is preferable to the terror of indecision
— Maimonedes (1134-1204)

There are many bad ideas floating around the bridge world concerning probability theory. Generally there is an over-reliance on the application of the *a priori* odds that apply to the deal of the cards before any action is taken. The oft-quoted Nine-Never Rule* has this flaw. The *a priori* odds are appropriate when one is in a state of maximum uncertainty, or if you prefer, in a state of minimum information with regard to the placement of the cards. We do know something about the probable consequences of the dealing of the cards, but the actions at the table are going to tell us a lot more about the particular layout that is being played. Naturally, as more is revealed, the odds will change. I'm going to continue the discussion of how to replace the *a priori* odds with odds more in keeping with current circumstances. This time the examples are in reference to the Nine-Never Rule, but the methodology is applicable to any card play situation.

Treating the problem of changing odds in a theoretical manner often leads to misunderstandings, and in any case that approach requires some assumptions that get hidden in the mathematical jargon. The purpose here is to present a general framework for the calculation of odds. To achieve that purpose let's work through a simple, but realistic, example using only basic concepts that every player can understand. Along the way I shall make reference to familiar concepts, like vacant places, which may not be fully understood. Lack of basic understanding often leads to misapplication, and that in turn may lead to a distrust of some methods that are put forward under the guise of science. True science, first and foremost, reflects reality by complying with observation.

* With an eight-card fit missing the queen, always finesse; with nine, never.

A Flannery Hand†

Competitive bidding is a two-edged sword: if an opponent overcalls on a weak hand and his side doesn't end up declaring, the distributional information provided can be used by the eventual declarer to the overcaller's disadvantage. That's also true of opening bids if you end up defending. In our example I'm going to assume the dealer (West) opens a Flannery 2♦, advertising a hand with four spades, five hearts and 10-15 HCP. This bid, of course, being highly descriptive, reveals a large amount of information about the distribution of the cards that can be helpful in constructive bidding. However, if North-South end up declaring the hand, that information can be used for planning the play. Here we'll see an extreme example of that process at work. These are the North-South hands.

♠ 9
♥ A 7
♦ K 10 8 7 2
♣ A 10 9 8 5

♠ A 5 4 3
♥ 10 9 5
♦ A J 9 3
♣ K 3

West	North	East	South
2♦*	dbl	3♠	5♦
pass	6♦	all pass	

The lead is the ♥K. South wins the first trick with the ♥A and contemplates how best to draw trumps. The usual slogan is 'Nine Never!', meaning that declarer should play off the ♦AK hoping to drop the ♦Q doubleton. Not thinking beyond that, a novice may cash the ♦A and play towards the ♦K. A more knowledgeable player would play low towards the ♦K to guard against a 0-4 split when West is void in diamonds. Then, if both defenders followed suit, he would lead a diamond from dummy and play the ♦A when East followed with a second low diamond. Both approaches are wrong because they are based on a blind rule, Nine Never, that has its origins in the *a priori* odds. Let's examine first the effects of the bidding on the recommended play in diamonds.

† The original example deal published in my blog was slightly different. I am indebted to Richard Pavlicek for suggesting a couple of improvements that are incorporated in this version.

Vacant Places

First we register the distribution of the sides as revealed when the dummy appears. For North-South the suits are numbered 5=5=9=6, so the East-West side is numbered 8=8=4=6. Furthermore, the bidding tells us that the distribution in the majors is as follows:

	W – E
♠	4 – 4
♥	5 – 3
♦ & ♣	4 – 6

Given what we know about the majors, there are 4 vacant places in the West hand and 6 in the East hand. The relative probability that the ♦Q is on declarer's right (QR) rather than on the left (QL) is the ratio of the number of vacant places available. That is, the ♦Q is more likely to be on the right in the ratio of 3:2. This assumes that neither clubs nor diamonds have been played, and that the minor suits can be lumped together as indeterminate cards. This ratio is the odds of the deal with regard to those two suits. The missing diamonds and clubs can be reshuffled and dealt four to West and six to East without affecting the information currently available with regard to their content.

We chose to emphasize the location of the ♦Q, but the same relative probability would apply to any missing club or diamond regardless of rank — the ♣7, for example. However, if we assume the ♦Q is on the right, that assumption changes the number of vacant places to 4 and 5, so the odds for the ♣7 also being on the right has been reduced, although the vacant places still favor that, now in the ratio of 5:4.

So given the information concerning the probable location of the ♦Q, declarer should not play for the drop, but should plan to finesse East for that card, beginning play in the diamond suit with a low card to the ♦K in the dummy. This is not merely a safety play against a very rare void in the West hand, but a percentage play of greater relevance. What we want to calculate is the probability that declarer will succeed in his planned finesse.

The Magic of Vacant Places

From what does the magic of vacant places derive? If we are going to use it, we should understand it. Then we'll see there is really no magic involved. Probability is no more than a ratio of combinations. It is assumed that the ten missing minor-suit cards can be placed at random between the West and East hands. How many card combinations exist that feature the ♦Q on the left, and how many with the ♦Q on the right? The ratio of the card combinations gives us the odds, assuming the cards are randomly dealt. Here there are 4 vacant places on the left and 6 on the right. There are two possibilities for the distribution of the nine cards other than the ♦Q:

	Other cards		Other cards
♦ Q on the left	3L 6R	♦ Q on the right	4L 5R
Combinations	84		126

The ratio of the combinations is 6:4, which is also the ratio of the vacant places.

Thankfully we don't need to know the individual numbers of combinations in order to obtain the ratio. We just line up the splits as follows and choose the last number of the more even split and the first number of the less even split. Thus,

5-4 vs 6-3 gives a ratio of 6:4

This works for all *adjacent* splits. For 5-4 vs 7-2, which are not themselves adjacent, we need to look at three adjacent splits:

Split	5 – 4	6 – 3	7 – 2
Ratios	1	4:6	3:7

Ratio of combinations in 5-4 compared to 7-2 is

$$4/6 \times 3/7 = 2/7$$

Thus, the ratio of combinations for a 5-4 split relative to a 7-2 split is 7:2, not 7:4.

The Distribution of Sides

Once the dummy appears, declarer can count separately the number of diamonds and clubs held by the opponents: four diamonds and six clubs. It is now possible to state explicitly all five possible combinations.

	♠ 4 – 4	♠ 4 – 4	♠ 4 – 4	♠ 4 – 4	♠ 4 – 4
	♥ 5 – 3	♥ 5 – 3	♥ 5 – 3	♥ 5 – 3	♥ 5 – 3
	♦ 2 – 2	♦ 1 – 3	♦ 3 – 1	♦ 0 – 4	♦ 4 – 0
	♣ 2 – 4	♣ 3 – 3	♣ 1 – 5	♣ 4 – 2	♣ 0 – 6
Weights	90	80	24	15	1
	42.86 %	38.10 %	11.43 %	7.13 %	0.48 %
A priori	40.7 %	24.87 %	24.87 %	4.79 %	4.79 %

The *a priori* odds refer to the diamond splits. The even 2-2 split is more or less the same as the *a priori* value and the sum of probabilities for the 1-3 and 3-1 splits is 49.53%, very much the same as expected initially for those splits. The difference is that the 1-3 split is much more likely than the 3-1 split due to the imbalance in vacant places. This illustrates that the *a priori* odds have some value as approximations if used in the proper sense. (If that weren't the case, they would have gone out of use long ago.)

The Calculation of the Weights

The weights from which the probabilities in the above table are derived are merely the relative number of card combinations available on a random dealing of the cards. These weights apply before a card is played in either minor suit, so they are the probabilities of the deal only. We shall now illustrate how they come about from the numbers of card combinations available in diamonds and clubs considered separately. It is convenient to line up the diamond splits in consecutive order and place underneath the corresponding club splits that preserve the 4-6 split in vacant places.

Diamonds	4 – 0	3 – 1	2 – 2	1 – 3	0 – 4
Clubs	0 – 6	1 – 5	2 – 4	3 – 3	4 – 2

Combinations	♦	1	4	6	4	1
	♣	1	6	15	20	15
Product	♦ & ♣	1	24	90	80	15

Total = 210

The numbers of combinations are entries in the Pascal Triangle, familiar to many from their school days. (Was that too long ago?) The greater the number of combinations, the greater is the probability that particular configuration has been achieved. The individual percentages are merely the number of combinations divided by the total number of combinations possible (210). It is important to remember this arrangement because once cards are played in a suit, diamonds first, some combinations are ruled out of the realm of possibility. That reduces the number of combinations available in the diamond suit, while the number of possible club combinations remains unaffected. However, the changes in probability are not solely governed by the numbers of card combinations remaining. That is where some come to grief: the probability of the play is not the same as the probability of the deal as not all cards are treated equally under the rules of bridge — some are significant, some aren't, depending on the circumstances. That matters.

The Diamonds in Play

Here explicitly are the possible combinations in the diamond suit (excluding the 4-0 and 0-4 splits) with the number of club combinations associated with each possible diamond combination. The number of club combinations serves as the weighting factor for each individual diamond combination taken 1 by 1.

Split	**3 – 1**	**2 – 2**	**2 – 2**	**1 – 3**
	Q 6 5 – 4	Q 6 – 5 4	5 4 – Q 6	4 – Q 6 5
	Q 6 4 – 5	Q 5 – 6 4	6 4 – Q 5	5 – Q 6 4
	Q 5 4 – 6	Q 4 – 6 5	6 5 – Q 4	6 – Q 5 4
	6 5 4 – Q			Q – 6 5 4
Clubs	6	15	15	20

We shall assume declarer begins properly with a low diamond to the ♦K in dummy and that West follows with the ♦6 and East with the ♦4. Here are the possibilities that remain. The club weights are unchanged as no club has been played.

Split	**3 – 1**	**2 – 2**	**2 – 2**	**1 – 3**
	Q 6 5 – 4	Q 6 – 5 4	6 5 – Q 4	6 – Q 5 4
Clubs	6	15	15	20

The relative probability of the ♦Q on the left (QL) and on the right (QR) can be calculated from the weights from the associated club split.

QL = 6 + 15 = 21 QR = 15 + 20 = 35

The odds are now 5:3 that the ♦Q is on the right. What of the vacant places? They started at 4 and 6 and now they are 3 and 5, if we eliminate the formerly unknown diamonds that have now been exposed. That's great for simplifying the calculation of odds, but this is a special case as we shall see when we come to investigate more closely the plausible plays in the diamond suit. The special situation is this: the play of the ♦6 and ♦4 are equally probable for all remaining combinations.

Next we shall assume that declarer next leads the ♦2 from dummy and East follows with the ♦5. How does this change the probabilities? There are only two possibilities remaining:

Split	**2 – 2**	**1 – 3**
	Q 6 – 5 4	6 – Q 5 4
Club Combinations	15	20

The odds of the ♦Q being on the right are 4:3, in exact agreement once more with the number of vacant places remaining. Obviously declarer should take the finesse against East and not play for the drop.

Finally let's assume that declarer plays the suit the wrong way around by cashing the ♦A in hand and leading towards the dummy. Assume West follows to the second round with the ♦5. The remaining possibilities are as follows:

Split	**3 - 1**	**2 - 2**
	Q 6 5 — 4	6 5 — Q 4
Club Combinations	6	15

QL = 6   QR = 15 so QR:QL is 5:2

When West plays the second low diamond, the vacant places stand at 2 and 5, which reflects the current relative probabilities, and South should play for the drop. The Nine Never rule applies. So in one situation declarer should play for the drop and in the other he should finesse — both times, playing East for the ♦Q. Here in detail are the vacant place situations described above.

	♦K then low to hand			**♦A then low to dummy**	
Vacant Places	**West**	**East**		**West**	**East**
Before a ♦ play	4	6		4	6
West plays ♦6, East ♦4	3	5		3	5
East plays the ♦ 5	3	4	West plays ♦5	2	5

Preferential Plays and Vacant Place Calculations

The accuracy of the vacant place ratio in determining probabilities depends on the way in which low cards are played; in particular, the assumption is that the low cards are chosen equally at random. This reflects the conditions of the deal. From a doubleton ♦65, say, the choice of the ♦6 has a probability of 50%, as does the choice of the ♦5. From ♦654, each low card has a 1/3 probability of being chosen on the first round and any permutation over two rounds has a 1 in 6 chance of being chosen. This assumption is equivalent to a condition of maximum uncertainty (entropy) for which the amount of information transmitted by the sequence of plays is a minimum. Any rule that gives preference to one sequence over another transmits more information. Here is an illustrative example. Suppose that defenders always play their highest spot card, so from a holding of ♦65 or ♦64 they always play the ♦6;

Split	**3 – 1**	**2 – 2**
	Q 6 5 — 4	6 5 — Q 4

After one round the odds of the ♦Q on the right is 5:2, whereas the vacant place ratio is 5:3. Thus the direct correspondence between vacant place ratios and probabilities is broken. The additional information available has effected a reduction in the number of possible combinations remaining from 16 down to 2, which under the condition of maximum entropy would require, as we have seen, a third card to be revealed in order to achieve the same degree of reduction.

Permutations and Plausible Plays

The order in which a defender chooses to play his cards when following suit represents a selection of one sequence of plays chosen from all possible permutations on the cards he holds. If all permutations are equally likely, the probability of his having chosen the observed sequence is merely the reciprocal of the total number of equal choices. This is the normal assumption, but it is subject to revision under some circumstances. The mathematics can accommodate other assumptions when appropriate. I'll discuss the effects later.

Assume a defender holds ♦654 and is required to follow to two rounds of diamonds. There are six possible ways to follow: 6-5, 6-4, 5-4, 5-6, 4-6, and 4-5. We term these the plausible plays. If the defender chooses to play equivalent cards at random, the probability that a given sequence will emerge from the original holding is 1/6. On the other hand, if the defender always gives standard count, the only sequence possible under that restriction is 4 followed by 5 and there exists a condition of certainty. The probability of observing 4-5 is 1. Thus, in order to calculate probabilities we must specify initially the assumptions concerning a defender's method of choosing equivalent cards.

Suppose that a defender picks up each card as it is dealt. He ends up with ♦654. The order in which he gets to observe the arrival of the cards isn't important. The same is true if he is dealt ♦Q64. All six permutations on the deal are equally likely. When he gets to play the cards the order becomes important, because the play is subject to the rules of bridge. The defender is assumed to play his cards in such a way as to optimize his chances. It would be foolish in most situations to play the unsupported honor on the first round. The plausible plays over two rounds are 6-4 and 4-6, and the probability of observing either is 1/2. This is not a necessary assumption. There may be situations where the play of the ♦Q on the first round is called for. A defender may choose to cover an honor with an honor, or may attempt to create an entry to partner's hand. Declarer needs to be aware of these situations, as does anyone wishing to calculate the odds. The mathematical formulation can accommodate any assumption in this regard.

Why is it commonly assumed a defender will play equivalent cards at random? That is the optimum strategy when the objective is to keep declarer in the dark to the greatest extent possible. If the objective is to inform partner to the greatest extent possible, such as by giving count or suit preference, equivalent cards will not be played at random, but will conform to prior agreement. However, there is no guarantee the defenders will keep to such an agreement if they feel it is in their best interest to be uninformative.

Restricted Choice

Let's consider next the combination ♦QJ6. On the first round normally a defender will follow with the ♦6. On the second round he may play either the ♦Q or ♦J with equal effect. The number of plausible plays is two, 6-J and 6-Q, and the probability of each is 1/2. This is a common example used to illustrate the Principle of Restricted Choice. When a declarer sees an honor emerge, naturally he takes notice. Of course, the very same principle applies to the play of low cards, as we have shown above. The selection from two honors is merely the last element in a sequence of equal choices.

If a defender wishes to inform partner that he holds both honors, he will not play the ♦Q on the second round, he will play the ♦J. If that takes the trick, his partner can see that declarer hasn't got the ♦Q, otherwise he would have covered. If instead the defender takes the trick with the ♦Q, the possibility still exists that declarer holds the ♦J. Playing the ♦Q and ♦J at random creates a condition of maximum uncertainty in the minds of both declarer and the other defender, at least to the greatest extent that is possible given the cards they can see in their own hands.

It is understandable that the Principle of Restricted Choice is treated in the bridge literature solely as applying to the choice of honor cards. That is because its most dramatic application involves cards that have the potential of taking tricks. Also, low cards usually appear early in the play of a suit whereas honor cards emerge later at a critical stage. A defender will not usually part with an honor card early unless there is a distinct advantage to doing so, whereas it is easy to part with a low card whose only function appears to be to fill in the suit. Usually there are more low cards missing than honors. For these reasons, then, the usual illustrations of Restricted Choice involve honors. I'll show the application of Restricted Choice using a second Flannery hand.

A Second Flannery Hand

♠ 9
♥ A 7
♦ K 8 7 5 2
♣ A J 10 9 5

♠ A 5 4 3
♥ 10 9 5
♦ A 10 9 3
♣ K 3

West	North	East	South
2♦*	dbl	3♠	5♦
pass	6♦	all pass	

On this deal both the ♦Q and ♦J are missing. Suppose that this time the ♠K is led, won by declarer with the ace. Now a diamond is played to the ♦K. West follows with the ♦6 and East with the ♦4, just as in the previous deal.

After the first round these are the possibilities remaining:

Split	**3 – 1**	**2 – 2**	**2 – 2**	**1 – 3**
	Q J 6 – 4	Q 6 – J 4	J 6 – Q 4	6 – Q J 4
Clubs	6	15	15	20
Plays	1	1	1	1

Each combination is subject to just one plausible play, so they all have the same probability of being chosen. Under the condition of equality of permutations, we can apply the vacant places to the calculation of probabilities. Adding the weights as before, we find:

QL = 6 + 15 = 21 = JL; QR= 15 + 20 = 35 = JR

and the odds of either honor being on the right is in the ratio of the remaining vacant places, namely, 5:3.

However, on the next trick a low diamond is led from dummy, and East follows with the ♦J. What is the probability the ♦Q will fall doubleton from the West hand? The situation has changed dramatically.

Split	**2 – 2**	**1 – 3**
	Q 6 — J 4	6 — Q J 4
Clubs	15	20
Plausible Plays	1	2

With a 2-2 split the appearance of sequence 6, 4, J is a certainty, so the weight retains its full potential. However, the probability of the sequence 6, 4, J given a 1-3 split is reduced by half under the assumption that it is just as likely that East would have chosen the ♦Q on the second round, resulting in the sequence 6, 4, Q. These are two equally probable sequences. The odds of dropping the ♦Q from West has improved to 3:2. Before the second round of diamonds was played the odds favored the queen being with East by 3:2. Thus, the freedom of choice of one honor over the other greatly affects the odds.

Let's suppose that East is more likely to play the ♦J than the ♦Q, for whatever reason. At Trick 3 the situation is still unclear. If the probability of playing the jack rather than the queen were 75%, that would make the 2-2 split and the 1-3 split equally probable at this point in the play, so the odds of dropping the ♦Q in the West hand would be 50%. If East would always play the lower of two touching honors as a matter of general principle, alleviating the need for thought, the weight of the 1-3 split retains its full value as there is just one plausible sequence available. In that case we return to the odds based on the current vacant places, 3:2 against the drop being successful. (While, of course, if East plays the queen, the jack is known to be on the left.) This is true before a diamond is played and later whenever the number of potential card sequences is the same for all remaining combinations.

What about the ♣Q?

Once declarer determines the diamond split, the club split becomes a certainty and the play proceeds accordingly. Let's suppose that against the odds according to Restricted Choice East was dealt ♦QJ4, so now there is a diamond loser after declarer plays for the drop. The slam is still assured when a heart loser can be avoided by establishing two tricks in clubs on which two losing hearts can be discarded from declarer's hand. Declarer has been keeping count, so knows the clubs must be split 3-3. He makes his claim, showing his cards, after which the following exchange takes place.

South: I ruff the clubs good and pitch two hearts on the clubs. You can take your trump queen whenever you like, but I still make my twelve tricks.

East (showing his cards): Down one. I have four clubs to the queen.

North: Oh, partner, you could have made it by taking a finesse in clubs! If West holds the majors, surely East must hold the minors...
West: Sorry, I know I had six hearts, but it felt like a Flannery hand.
South: Grrrr...

Yes, it happens. The opening bid provided information, but it was inaccurate. Information is not always exact, but you should plan according to the information available. In some situations, before drawing trumps declarer may take some seemingly pointless ruffs in a side suit in order to confirm what he thinks he knows about the distribution. In a 3NT contract declarer may duck a trick or two to good effect. These are processes of gathering information, and it may happen that this gathered information contradicts what was assumed previously. At that point, a reevaluation of the assumptions is required.

If you are unable to gather more information safely, then it is correct to play according to what is most likely given the current information. On this deal West took an unusual action in the auction, but the probability that he had done so was very low. This is generally true. It doesn't pay to assume an unusual action and over-react in what is most likely a normal situation. More is lost by assuming that aggressive opponents are trying to swindle you than by assuming they are playing normally. However, it is wise to check if you can.

Bayes' Theorem

Without stating the governing equations explicitly, what has been demonstrated in these examples is simply the application of Bayes' Theorem to card play. The mathematics involved is not difficult; however, the notation is difficult to grasp on a casual reading. Putting the equations into words has caused confusion, beginning early in the 18th century with Reverend Thomas Bayes himself, who was hoping to add further confirmation of the existence of God by extending the results produced by Blaise Pascal, the 17th century co-founder of probability theory. Once philosophers got involved, matters became unnecessarily complicated. It wasn't until the 1950s that clarity was restored and probability was linked mathematically to information. We can hope that all that confusion is behind us now and common sense will prevail, at least as far as the analysis of bridge play is concerned.

Warning! The application of this analysis to cases where the defenders hold an odd number of trumps is more complicated than for the examples treated above, because the number of plausible plays reaches equality at a later stage of play than for an even number of missing trumps.

See what cannot be seen
— Miyamoto Musashi(1584-1645)

Mathematical models require assumptions, and it is the nature of the assumptions that limit their applicability to the real world. The Nine-Never Rule is based on a mathematically sound argument, but the assumptions behind it are not always appropriate to the situation at hand. The 2009 Prince Takamatsu Cup tournament was held in that beloved oasis of polite bridge in the heart of heartless Tokyo known as the Yotsuya Bridge Club. A slam deal arose that provided a rare example where theory was well-matched to reality, yet the seasoned BBO commentators did not clearly explain to the observers how the analysis should go. In fact, their analysis was somewhat confused; the simple conclusion being that here was another example where the Nine-Never Rule failed. Why it failed was not explained, and it was not clear whether the declarer had actually made a mistake when he played for the drop rather than finessing against the queen. This leads me to conclude that it is worthwhile to go through the play of the hand in detail in order to demonstrate the process that a declarer should follow when making a decision with regard to the play in a suit where nine cards are held with the queen missing.

The Basic Suit Configuration

The suit layout involved is very similar to the one shown in our first two examples. South is the declarer faced with the problem of finding the ♦Q in the following configuration:

♦ K 10 6

♦ A J 9 7 5 2

The ♦K is played from dummy; East follows with the ♦4 and West with the ♦3. Next the ♦10 is led, East follows with the ♦8, and the time of decision has arrived. Should declarer finesse or play for the drop? There are two possible holdings remaining, namely,

West	East	West	East
♦ 3	♦ Q 8 4	♦ Q 3	♦ 8 4

On the play so far in the suit there is no reason to choose one configuration over the other as they share the same number of plausible plays, so the question boils down to the question of which is more likely, a 1-3 split or a 2-2 split? Declarer should play for the drop when it can be assumed that an even split in diamonds is more likely than an uneven split. This assumption is supported by the *a priori* odds, but it may not hold after cards have been played in another suit.

The Effect of an Uneven Split in Another Suit

In the Prince Takamatsu Cup deal, hearts were trumps. The opening lead from West was a trump, and it was immediately discovered that the hearts were split 3-0, with East having a void in the suit. Does this information point to an uneven split in diamonds? Yes, it does.

	West East	**West East**
Hearts	3 – 0	3 – 0
Diamonds	1 – 3	2 – 2
Blacks	9 – 10	8 – 11
Weights	11	9

The weights are an expression of the number of combinations in the black suits that are available under the two possible diamond splits. The ratio of combinations yields odds of 11:9 in favor of the 1-3 split. Hence declarer should finesse under these circumstances, when nothing is known concerning the splits in the black suits. This is roughly the situation when trumps are drawn and diamonds are broached immediately thereafter.

The same odds are available from a vacant place analysis, following Kelsey's Rule, which allows the use of the current vacant places when only the location of the queen remains in doubt. This is no surprise, as the assumption of maximum uncertainty with regard to the black suits is common to both methods.

	Current Vacant Places	
	West	**East**
Initial vacant places	10	13
After first round of Diamonds	9	12
During second round, when East follows	9	11

The odds are 11:9 that the ♦Q is in the East hand. This is the basis of the criticism the BBO commentators directed towards declarer when he played for the drop — which turned out to be a losing line on this occasion. Given that the hearts had been seen to split 3-0, it seemed as if the odds favored an uneven split in diamonds. However, life is seldom that simple. At the table, diamonds were not broached immediately, and several rounds of both spades and clubs were played before the critical decision was made in diamonds, so more information was available than in the simple model indicated above. To gauge the effects, let's look fully at the play of the cards.

What Really Happened

	North	
	♠ J ♥ K J 10 8 7 3 ♦ K 10 6 ♣ 9 8 6	
♠ K 8 7 3 2 ♥ A 9 4 ♦ 3 ♣ A Q J 2	N W E S	♠ 10 9 6 5 ♥ — ♦ Q 8 4 ♣ K 10 7 5 4 3
	♠ A Q 4 ♥ Q 6 5 2 ♦ A J 9 7 5 2 ♣ —	

Shimizu	Imamura	Noda	Ino
	2♦*	pass	2NT*
pass	3♦[1]	pass	6♥
dbl	all pass		

1. Good hearts.

The bidding was very much to the point and South showed commendable faith in partner's non-vulnerable weak-two preempts. No doubt there were hopes of a helpful lead, hopes that were not realized when Shimizu, in the face of such uncertainty, chose a passive trump lead. The ♥4 rather than the ♥A indicated he had no fear of losing his ♣A in the early rounds. Let's look at some common distributions of sides with an established 3-0 heart split.

	I	II	III	IV	V
	♠ 4 – 5	♠ 4 – 5	♠ 5 – 4	♠ 5 – 4	♠ 3 – 6
	♥ 3 – 0	♥ 3 – 0	♥ 3 – 0	♥ 3 – 0	♥ 3 – 0
	♦ 2 – 2	♦ 1 – 3	♦ 1 – 3	♦ 2 – 2	♦ 2 – 2
	♣ 4 – 6	♣ 5 – 5	♣ 4 – 6	♣ 3 – 7	♣ 5 – 5
Weights	100	80	67	57	80

Even though the hearts are split 3-0, the even 2-2 diamond split is more likely than the uneven 1-3 split over this small selection. Why? Because the imbalance in the vacant places is more readily compensated for by uneven splits in the longer black suits. These five cases represent a snapshot rather than a panoramic view; however, as cards are played, it is to be expected that the play will 'zoom in' on the most likely possibilities shown above. Of course, the discovery of uneven splits in the black suits may alter our view.

The play in a slam contract is often a long journey of discovery. Here, declarer may postpone the critical play in the diamond suit until he has learned more about the splits in the black suits. Let's follow the perilous path as it occurred at the table.

Trick #	**West**	**North**	**East**	**South**
1	♥4	♥J	♣4	♥2
2	♥A	♥10	♣5	♥5
3	♣A	♣6	♣3	♥6
4	♠2	♠J	♠6	♠A
5	♠3	♥3	♠5	♠4
6	♣2	♣9	♣10	♥Q
7	♠7	♥7	♠9	♠Q
8	♥9	♥K	♣7	♦5
9	♠8	♥8	♠10	♦7
10	♦3	♦K	♦4	♦2
11	??	♦10	♦8	??

Both defenders have played four spades and only the ♠K is now missing. It could sit on either side. East has played five clubs, ♣107543, and the ♣KQJ are still missing. From the play in the club suit we may assume the clubs were not dealt 5-5, for that would mean West began with ♣AKQJ2, which is contrary to the evidence of the lack of bidding, the opening lead and the subsequent play. Thus it is reasonable to assume clubs were split 4-6, and it is only the split in spades that remains in question at Trick 11. At this point the split in diamonds depends solely on the split in spades. Here are the two remaining possibilities:

Under Condition I

	♠ — ♦ 10 6 ♣ 8	
♠ — ♦ Q ♣ Q J	N W E S	♠ K ♦ 8 ♣ K
	♠ — ♦ A J 9 ♣ —	

Under Condition III

	North	
	♠ — ♦ 10 6 ♣ 8	
West ♠ K ♦ — ♣ Q J		**East** ♠ — ♦ Q 8 ♣ K
	South ♠ — ♦ A J 9 ♣ —	

Of the five most likely distributions of sides listed above, only Conditions I and III remain as possibilities at the critical point in Trick 11. The problem has boiled down to this: does West or East hold the ♠K? The answer will determine who holds the ♦Q.

Restricted Choice

So far the numbers indicate that declarer should play for a 2-2 diamond split since on this deal Condition I is the more likely distribution of sides in the proportion of 3:2. However, as Shakespeare once wrote, 'the play's the thing'; there is the evidence of the play to be considered. The problem in probability is this: given the defender's sequence of plays, is it more likely that East holds the ♠K and, if so, by how much? This card play problem is related to the Principle of Restricted Choice.

At Tricks 4 and 5, East and West followed suit with low spades. At Trick 7, when declarer ruffed the ♠Q the defenders' ♠K109 were equivalent cards. If East had been dealt ♠K10965 he could have followed with any of the three equals, but if he had originally held only ♠10965 his choice would now be restricted to two cards. In retrospect, Conditions I and III had become equally probable when East played the ♠9. At Trick 9, both defenders had to find a discard on a trump lead. The ♠8, ♠10 play was one of eight possible plays under Condition III and one of nine under Condition I. Thus, at Trick 10 Condition III becomes the more likely in the proportion of 9:8. So the BBO commentators, along with everyone else who could see all four hands, were correct in maintaining declarer should have finessed at Trick 11.

At the other table, North opened with a weak 2♥ and also ended up in slam. However, West first competed in spades at the three-level on a poor five-card suit and in the end, egged on by East, doubled 6♥. With this wealth of information freely available to him, North was able to play the diamonds correctly and make his doubled contract for a gain of 16 IMPs. Lest this be construed as a condemnation of hyper-aggressive bidding practices, I hastily add that the aggressors' team won their semifinal match handily, only to lose by a narrow margin in the final.

Bayes' Theorem at Work

Truth will sooner come out of error than from confusion
— Sir Francis Bacon (1561 -1626)

Seldom can it be said that a model fits reality to perfection; models are perfect, but reality is messy. The concept of 'best chance' is an abstraction in itself that requires the incorporation of all the information that is available at the time of decision. Some simplification is required, how much partly depending on a declarer's powers of observation. Although you may not be able to reach the correct conclusion through common sense alone, in retrospect a mathematically-derived conclusion should be seen to reflect common sense. In simple language, Bayes' Theorem tells us the following:

(1) If a player hasn't played a particular card, it is more likely he hasn't got it than that he has it but chose to play an equivalent card instead.

(2) The more likely a player is to have played a particular card if he had it, the more likely it is that he doesn't have it if he hasn't played it.

The Assumption of Perfect Play

At the table it is a player's task to induce errors, and suppress the fear that an opponent can see through the cards, yet the mathematical model assumes the play is perfect. It is all too easy to assume a defender will not err, but then, how can you expect to win against a faultless opponent? In the above example it is in the defenders' best interest not to give away information with regard to the distribution of the cards. That means they should play equivalent cards at random rather than attempt to give partner the count, something which makes declarer's task much easier (if he believes them!).

It is not easy to remember at Trick 11 the exact sequence of every spot card played by the defenders, yet a complete analysis requires you to make use of this information. The task is made simpler if a declarer is alerted to the main features of interest. That is why when dummy comes down, declarer should focus on the most likely distributions of sides, which are the models for what is most likely to transpire. In the above example, it should be clear you must determine the diamond split, so declarer should make plans to achieve that objective.

With regard to Ino's journey of discovery, he might have tried to induce a revealing play in spades by leading the ♠J from dummy at Trick 2 to see whether East would cover. Of course, East does not cover, and declarer goes about his business. Suppose that subsequently East shows up with the ♠109. Now, looking back on Trick 2, declarer may make his decision based on the observation that if East held ♠K109 he might have covered the ♠J at Trick 2, although in an expert game that is highly unlikely. On this partial basis he might choose correctly to finesse for the ♦Q. This is a matter of anticipation and recall for which an understanding of the mathematical treatment prepares the mind.

Chapter 4

THE PROBABILITIES OF HCP DISTRIBUTIONS

The theory of probabilities is basically only common sense reduced to a calculus. It makes one estimate accurately what right minded people feel by a sort of instinct, often without being able to give a reason for it.
— Pierre-Simon Marquis de Laplace (1749-1827)

Imagine you are playing in a 2♠ contract, having got there without the opponents' interference. Dummy comes down and you count up 20 HCP for each side. Well bid! The defenders lead clubs and sooner or later you ruff and draw trumps. Now you are faced with playing on diamonds with KJxx in dummy and xxx in hand. So far RHO has shown up with 8 HCP and LHO with none. Who is the more likely to hold the ♦A? Thinking back on the hundreds of deals where you have played in 2♠ against silent defenders holding 20 HCP, your memory tells you that most of the time the HCP are pretty evenly divided between them. Does that mean that LHO is more likely to hold the ♦A, because so far he has shown up with fewer HCP? No it doesn't, even though the *a priori* odds tell you the HCP are likely to be split evenly.

This is similar to a classic problem that arose with coin tosses. It wasn't until the 17th Century that thinkers got it right. Consider a sequence of tosses with all heads, say, HHHH. How often does it arise? Not very often — the *a priori* odds are definitely against such an occurrence. So, if someone has thrown HHH would you think that on the next throw tails (T) is now more likely than heads? Of course not. The probability of throwing tails is 50%, just as it was at the beginning of the sequence and all the way through. At this point the relevant odds are the *a posteriori* odds, and HHHH and HHHT are equally likely. This is still very difficult for some to grasp. They think the more heads that appear in sequence, the greater the chances of tails on the next toss. They give odds against the string continuing. Wrong. You have to abandon the *a priori* odds, and concentrate on the current odds.

Consider four players sitting around the table as a dealer gives them cards one-by-one. The expectation is that by the end of the deal each will receive four honor cards (A,K,Q,J). As the cards are dealt one by one, you find that your first four cards are honors. Does that mean you are likely subsequently to receive fewer honor cards than a player who has received none as yet? Not at all. The chance of receiving an honor card in the next nine rounds is the same for all players regardless of their current holding.

Back to the 2♠ deal. Of the 20 HCP missing initially East has shown up with 8 HCP so far, so there are 12 HCP remaining. These are likely to be split in accordance with the number of vacant places. If there is a balance in vacant places, the odds favor 6 HCP on each side for an overall split of 6 HCP in the West hand and 14 in the East hand. However, suppose that East has passed in first seat. This constrains East to at most 12 HCP. He cannot have 14 HCP and have passed as dealer. This constraint acts to eliminate the possibility of some card combinations. To calculate the resulting probabilities we merely discard the now impossible combinations, and add up the numbers of combinations still within the bounds of the constraint. This is the way of conditional probability, and it is the proper guide to the play.

The best process with regard to estimating probabilities is to get (safely) as much information as possible concerning the distribution of cards before making a decision. Ideally, to find the ♦A, say, you would like to know how the diamond suit is split. If you can't accomplish that, then you may have to work with two suits, treating them as equals during the deal. This is the situation we shall examine to show how the calculation proceeds in several simple examples. We may be able to use the number of vacant places to calculate the current probabilities, but sometimes constraints apply and the vacant places won't yield a good approximation.

Let's look at declarer's problem on these cards, on various different auctions and sequences of play.

♠ Q J 8 4
♥ A 7 5 2
♦ K J 7 2
♣ 8

♠ A K 7 6 5
♥ K 10 6
♦ 8 6 5
♣ A 3

Case 1: Uncontested auction, East-West have 15 HCP

West	North	East	South
			1♠
pass	4♣*	pass	4♠
all pass			

In Case 1, West leads the ♣J (playing jack denies, so probably the top of a sequence). South takes his ace, ruffs a club and draws trumps in two rounds. With some luck in diamonds declarer just might get rid of a heart loser. There are 9 HCP outstanding in the red suits. West has shown up with 1 HCP (♣J) and East with 5 (♣KQ). Does that mean West is more likely to hold the ♦A than the ♦Q? No, it does not.

The key to the locations of the ♦A and ♦Q lies in the number of vacant places remaining before declarer broaches the diamond suit. Let's assume the following splits to this point based on the evidence of the opening lead and East's plays in the club suit.

	West	East
Spades	2	2
Clubs	5	5
Diamonds and Hearts	6	6

We cannot be certain of the club split as the defenders may be falsecarding, but when there exists a high degree of uncertainty, it is best to assume the most even split consistent with the play to this point, there being nothing to suggest otherwise. In the absence of opposition bidding, this is the most probable condition. Under that assumption, the probability of the ♦A being in the West hand is 50%. The same applies to the ♦Q, or any other red card.

The chance of finding the ♦A or the ♦Q in the West hand is 50%. This is because the vacant places are equal at 7-7. It doesn't matter that East has shown up with just 1 HCP to West's 5. The same would apply if West had led the ♣K from ♣KQJ. A random deal is blind with regard to the ranks of the cards. (Of course, the evidence of the bidding slightly affects the odds: with 14 HCP and ♣KQJxx, West might have overcalled.)

The equality of vacant places is a very significant feature. The *a priori* odds are also a consequence of symmetry with 13 vacant places to a side. The reduction from 13 a side to 7 a side changes the probabilities, but the conclusions are not unexpected. Before play in the suit starts, there is nothing to choose between playing for the ♦A on the right or the ♦A on the left.

Next we'll look at a case where the bidding tells us there is an imbalance in vacant places.

Case 2: East has long clubs and doubles the splinter, suggesting a save

♠ Q J 8 4
♥ A 7 5 2
♦ K J 7 2
♣ 8

♠ A K 7 6 5
♥ K 10 6
♦ 8 6 5
♣ A 3

West	North	East	South
			1♠
pass	4♣*	dbl	4♠
all pass			

The ♣J is led. Declarer wins, ruffs a club and plays on trumps, which prove to be split 2-2. The evidence of the bidding and play point to an imbalance of vacant places for the red suits. We don't know for sure how many clubs East has, but for the sake of argument let's assume he has a seven-card suit.

	West	East
Spades	2	2
Clubs	3	7
Diamonds and Hearts	8	4

Here are the probabilities for red-suit HCP held by West:

0 HCP	0%	5 HCP	17%
1 HCP	1.5%	6 HCP	11%
2 HCP	3%	7 HCP	23%
3 HCP	11%	8 HCP	11%
4 HCP	7%	9 HCP	14%

Obviously, symmetry is destroyed and West is likely to hold more HCP than East. The median value is 6.5 for West, and therefore 2.5 for East. The chance of finding the ♦A or the ♦Q in the West hand is 2:1, in accordance with the ratio of the vacant places (8 against 4). East has not limited his HCP by his double, so there is no restriction in that regard. Now let's look at the situation where East has limited his HCP range.

Case 3: East preempts and a constraint applies

♠ Q J 8 4
♥ A 7 5 2
♦ K J 7 2
♣ 8

♠ A K 7 6 5
♥ K 10 6
♦ 8 6 5
♣ A 3

West	North	East	South
		3♣	3♠
4♣	4♠	all pass	

Now the bidding tells us something about the distribution of the HCP. Let's bravely assume that East cannot hold the ♦A along with seven clubs to the king-queen. Here are the probabilities for red-suit HCP held by West.

0 HCP	0%	5 HCP	8%
1 HCP	0%	6 HCP	17%
2 HCP	0%	7 HCP	34%
3 HCP	0%	8 HCP	17%
4 HCP	2%	9 HCP	21%

Based on our working assumption, the probability of the ♦A being in the West hand is 100%. The probability of the ♦Q also being with West is 63.6% based on a compilation of the possible card combinations without an ace in the East hand. This result agrees exactly with a vacant place calculation. Here we have placed the ♦A in the West hand, so the vacant places are now 7-4. The probability of any red card other than the red ace being on declarer's left is 7/11.

A conflicting constraint

The above cases are simple illustrations of how probabilities are calculated using vacant places when there is no conflicting constraint. The mathematical procedure conforms to what can be classified as a common sense approach by a competent declarer. The distribution of the cards is the foremost consideration.

Let's go back to where we started this discussion — a 2♠ contract with the high cards split evenly between the two sides. With no opposition bidding, we may assume initially that their hands are balanced in shape and HCP content, but that may not be true. The play of the cards may reveal that an abnormal constraint applies. We'll now examine a case where the distribution of sides is the most common 8-7-6-5 configuration.

♠ Q 9 8
♥ 8 7 5
♦ K J 7 2
♣ 8 7 5

♠ A K 7 6 5
♥ K 10 6
♦ 8 6 5
♣ A 3

West	North	East	South
		pass	1♠
pass	2♠	all pass	

The ♣J is led. South ducks the opening lead and West, after some thought, helpfully continues with the ♣10, leaving declarer in peace to work things out for himself. We don't mind that. South wins the second club, goes to dummy with the ♠Q, ruffs a club, and draws trumps, West having been dealt ♠Jxx. East has dropped the ♣KQx so we determine that there are 5 vacant places in the West hand and 8 in the East hand. Since he passed initially, we assume East cannot hold two red aces to go along with his club honors. Under these restrictions the probabilities for East's share of the remaining 13 HCP are:

0 HCP	0%	5 HCP	14%
1 HCP	1%	6 HCP	14%
2 HCP	2%	7 HCP	27%
3 HCP	7%	8 HCP	14%
4 HCP	5%	9 HCP	17%

East has more red cards than West, but cannot hold more than 9 HCP (AQQJ) in the red suits, so the constraint on HCP in the East hand acts contrary to the tendency of the vacant place imbalance. The vacant place imbalance favors East to be the holder of the ♦A, but the HCP distribution makes that less likely. It is very important that declarer be aware of this conflict, which arises when a defender has revealed a long, topless suit. The HCP distribution appears truncated at the top end of the scale due, of course, to the constraint placed on the total number of HCP. East holds 7-9 red HCP in 58% of the cases, but did not open the bidding with 12+ HCP. This points to his having a flat shape. It was not very sporting of him to pass — reasonably conservative perhaps, albeit a rare and unexpected occurrence. Immediately, once a long topless suit is revealed, declarer should become aware that something unusual has occurred.

Errors in judgment by the opponents occur most commonly when a player holds a topless long suit. It is more probable that a long suit contains its fair share of HCP, so when a long suit is deficient in HCP, the distribution of HCP will not conform to expectations. Bidding systems are designed for normal conditions. When a player preempts in a long, topless suit with honors in his short suits, the other players will reasonably assume otherwise. Swing results are likely to occur. Similar considerations apply during the play of the hand. A declarer who finds evidence of a long, topless suit must adjust his prior expectations.

The probability of the ♦Q being in the East hand is 65%, of the ♦A being in the East hand, 40%. So how should declarer play the diamond suit? If he simply plays to the ♦K and that loses, the ♥A is sure to be offside. But if the ♦K holds, there is a very good chance the ♥A is also onside. So the optimist plays to the ♦K and then to the ♥K and claims eight tricks! This approach has a 48% chance of success.

Finally, we note that East made a discard on the third spade. Was it a low diamond? That points to a hand like this:

♠ x x ♥ A J x ♦ Q x x x x ♣ K Q x

Usually you would ignore such slim indications; however, an attempt should be made to speculate without being unduly swayed. Here a diamond discard supports the direct approach, so merely reinforces a decision rather than being an essential element in its construction.

Finally, a normal situation for a change

In the previous example West led from ♣J109xx and his partner held ♣KQx, an abnormal configuration. This led to constraints on the East hand that resulted in an abnormal distribution of HCP, far different from the *a priori* expectations. Now let's consider the more normal situation where West leads the ♣10 (coded nines and tens, 0 or 2 higher) from ♣KJ109x, and East has ♣Qxx. This is a somewhat venturesome attacking lead, but we'll assume that West didn't have a better one. The upshot of this is that the constraints on the total HCP held in the West or East hands are not onerous. In the previous case, we had to count up the excluded combinations, but in this case West could have opening points and not wish to overcall or balance in the club suit. Consequently, due to the lack of constraint, uncertainty is at a high level and declarer may use vacant places as the primary tool in estimating probabilities.

The probability that the ♦A sits in the West hand and the ♦Q sits in the East hand is

$$8/13 \times 5/12 = 26\%$$

so the optimistic approach of drawing trumps and playing low to the ♦K has lost its appeal.

On the other hand, the chance that West holds at least one of the diamond honors is

$$(8/13 \times 7/12) = 64\%$$

So rather that adopting a kamikaze attacking approach as before, it makes better sense for declarer to play a diamond to the jack initially and hope West holds a doubleton honor. This is a deal where declarer wishes he had better spot cards.

It pays to envision the distribution of the sides based on the evidence. What is the single most probable distribution?

	I	II	III
	♠ 3 – 2	♠ 3 – 2	♠ 3 – 2
	♣ 5 – 3	♣ 5 – 3	♣ 5 – 3
	♥ 4 – 3	♥ 3 – 4	♥ 2 – 5
	♦ 1 – 5	♦ 2 – 4	♦ 3 – 3
Weights	40	100	80

Condition I looks to be much less likely on the bidding and play than is indicated by the weights, which are based solely on the number of card combinations on the deal. Rule it out. It is better to play West for a doubleton diamond rather than a doubleton heart. If the diamonds are split 2-4, the chance of West's holding at least one top honor is 60%. So choosing to play for the single most likely distribution (Condition II) makes sense as it is in accordance with the vacant place analysis. Both methods are based on the assumption of an unconstrained random deal in the red suits.

According to Laplace, probabilities should be no more or less than a numerical expression of common sense. If an expert like Bob Hamman, say, plays the hands well, consistently, over a decade or more, we may assume that he has followed sound mathematical principles even though he may or may not express his procedures in mathematical terms. Artificial intelligence computer programs that adapt to the expert's procedures may grow to simulate his performance with greater and greater efficiency without providing less gifted players with any insight into how to improve their own game. It is the task of the mathematician to provide an explicit model that functions as a general guide for good decision making. Simplifying assumptions are necessary to make the model useful at the table, or, at the very least, to provide insight into an expert's approach.

The application of vacant place estimations as to the probability of the location of a card of interest is a consequence of such a model. Vacant places yield a good approximation under the right conditions, namely, when there is maximum uncertainty with regard to the distributions of suits which are yet to be played. The degree to which a mathematical model fits reality is the true test of its worth. Those who insist in applying the *a priori* odds inappropriately are doing themselves a disservice, because their simplistic model is inflexible with regard to changing circumstances, especially with regard to imbalances in the vacant places.

Chapter 5

A FOUR-DOOR MONTY HALL GAME

The gate is straight and deep and wide
Break on through to the other side
— The Doors (1967)

Bridge is a game of choices. A player ideally makes the choices that give him the best chance of success within the rules of the game. Decisions are based on evidence and prior knowledge, and usually there is a large degree of uncertainty involved. How to make decisions in the face of uncertainty is a topic shared by many games, some serious, some not, but the theoretically optimum approach can be expressed in terms of probabilities. It is probability theory that links many games. In order to understand the optimum decision-making process in a very complex game, one may investigate how probabilities apply in a simpler game, and then seek to generalize. For this purpose the game often chosen by bridge writers is the so-called Monty Hall scenario, which is an example of Restricted Choice, and is supposedly well understood.

The underlying link between Monty Hall and bridge is the Bayes' Equation, named after the British clergyman, Thomas Bayes (1702-1761), who first stated the ideas in the language of the time. As is common with entirely new ideas, the current language was inadequate for their expression. Intuitively, Bayes knew he was right, but he just couldn't find the phrases to convince the world at large. Ironically, British academics were among the last to concede that their countryman Bayes had been right. Is it only a problem with the vagaries of the English language?

The most famous demonstration of the validity of Bayes' Equation, the Monty Hall Problem, can be credited to the American journalist Marilyn von Savant, who in her column in the magazine *Parade*, applied Bayes' reasoning to the popular TV game show *Let's Make a Deal*, hosted by Monty Hall. The scenario for the game was a set of three doors behind one of which sat a large prize (a new car, for example). A contestant was asked to choose one door in the hope of winning the large prize. Initially, the chance of receiving the large prize was 1 out of 3. The MC then opened one of the two doors remaining to demonstrate that the prize did not sit behind it. The contestant was then given the option of sticking with his first choice or switching to the other unopened door. Von Savant took the fun out of the game by pointing out that the odds were 2:1 in favor of switching doors. This claim was met with much opposition from her readers. Even some professors argued that it could not be true, on the grounds that Monty Hall could always open a door with no prize, so with two doors remaining the chances should be 50-50 regarding which one hid the prize. Wrong! A 'free' choice has unexpected consequences.

The matter was resolved when von Savant had teachers play the game with schoolchildren and report the results. The statistical evidence overwhelmingly supported Bayes' 2:1 odds. To their credit, many dissenting experts publicly apologized and it appeared the matter was settled once and for all. Civilization moved a step forward.

Perhaps the most elegant explanation of the Monty Hall problem was given by Leonard Mlodynyk in *The Drunkard's Walk*, where he demonstrated the logic behind it in another context. His was denoted 'The Girl Named Florida Problem'. If you read the introduction to this book, you encountered a family with two children, one of whom was a girl named Florida. He asked the reader, what is the probability that the other child was also a girl? To understand the issue here, let's first look at a simpler case. Suppose that a couple have produced two naturally conceived children. What are the chances they are both girls, assuming that at the time of conception a boy is as likely to result as a girl? The event is mathematically equivalent to tossing a coin, and the correct answer is 1 in 4.

Next he asked: if one child is known to be a girl, what are the chances the other is also a girl? The 'obvious' but incorrect answer is that the fact that one is a girl doesn't affect the odds that the other is a boy, so the chances of the children being both girls should be 50%. Wrong. The correct argument is that the birth order of boy-boy has been removed from consideration, so there are now three possible sequences remaining (G-G, B-G, G-B), making the chances of two girls 1 in 3. Similarly, if we are assured that at least one finesse wins, the chance of the other also succeeding is 1 in 3.

Next we ask, what are the chances of the family consisting of two girls given that one of the children is a girl named Florida? There are those who would argue that whether the girl was named Florida or Jane or Sarah should make no difference to the odds that their other child is a boy. Although the name Florida is unusual, there is no causal effect at work. Consider the problem statistically and imagine going through U.S. census data looking for all parents with two children, one of whom is named Florida. Can we expect to find that the other child is a more likely also to be a girl? It doesn't make sense that we should.

Although there seems to be no causal link between the name and the probability of two girls, the argument doesn't solve the Girl-Florida problem as posed.

The correct solution is obtained by incorporating the information that a daughter is named Florida, condition FL, into the possible sequences of births. The possibilities are the following four:

Boy – FL   FL – Boy   FL – Girl   Girl – FL

in half of which the other child is a girl. (We'll ignore the tiny chance that these parents had two children and named them both Florida!) This corresponds to our intuitive feeling that it doesn't matter whether the girl was named Florida or Jane or Laura, the chances are 50-50 the other child is also a girl. *What does matter is that the naming of one child changes the chances for a second girl from 1 in 3 to 1 in 2.* The addition of this specific identification changes the condition from *a priori* to *a posteriori*.

The application to bridge probability is straightforward, keeping in mind that there are a limited number of cards to be considered. Suppose the opponents hold the ♣4, ♣3, and ♣2. If we lead the ♣A from hand and LHO follows with a low club, it may not matter on this trick whether the played card is the ♣2, ♣3, or ♣4, but what does matter is that the card is specified. There is a difference between seeing 'a low club' and the ♣2 being played, just as there is a difference between 'a girl' and 'a girl named Florida', the difference being in the amount of information being made available. Just as birth order must be taken into account in the two-child family problem, so the order of play must be taken into account when following in a suit. If our LHO is seen to have been dealt the ♣2, all the possible combinations for which RHO has that card have been eliminated.

What is the connection to the Monty Hall Problem? The formulation is the same. We begin with a set of conditions of known probabilities. Information is provided that reduces the number of possibilities. In the Monty Hall Problem, a door is opened to show that the prize does not sit behind that door. The probabilities are reevaluated on the basis of the remaining conditions. These conditional probabilities must add to 1, as only these possibilities remain. Mathematically the process is expressed by Bayes' Theorem. We must be careful when describing the process in normal language, which is not well suited to describing random activity. Hence the need for books to attempt to get around this fundamental difficulty, one that affects even the most intelligent readers.

In the game of bridge the well-known principle of Restricted Choice arises from the very same Bayes' Equation that governs the probabilities in the Monty Hall game. In order to explain its application to their readers, bridge writers discuss the solution to the Monty Hall problem in the hope that the reader will be convinced and can make use of evidence that comes their way during the play of the hand when a defender follows with one of two missing honors. Here we shall aim for a wider understanding and wider applications. So, let's consider a

Monty Hall game where four doors are available. The contestant chooses one door in the hopes of winning a large prize, and the MC opens one of the three other doors to show that no prize sits behind it. We'll show how to calculate the probabilities that the prize sits behind each of the three remaining doors, and we'll express the process in mathematical terms that have a general applicability. The process can be extended to as many choices as we wish to incorporate.

The Statistical Approach to Probability

The easiest way to understand how probabilities work is to imagine an experiment where choices are presented to a great number of participants. The choices are counted and put in a table of results forming patterns from which conclusions can be drawn. This can be a convincing approach if the results are clearly in favor of one circumstance over another, as von Savant demonstrated to her readers when the odds in the three-door game were indeed 2:1. We shall return to the idea of testing hypotheses from a collection of data, but first we shall assume that the contestants adhere strictly to our assumption concerning how their choices are made. This yields the expected or average numbers that may or may not be reflected in the actual results, since these are subject to variation due to sampling conditions. There are two assumptions to be studied for the purpose of exposition, illustrated below.

Initial Conditions				**Equal Choices**				**Biased Choices**			
					B	**C**	**D**		**B**	**C**	**D**
A	B	C	D	—	30	30	30	—	60	30	0
A	**B**	C	D	—	**0**	45	45	—	**0**	45	45
A	B	**C**	D	—	45	**0**	45	—	90	**0**	0
A	B	C	**D**	—	45	45	**0**	—	60	30	**0**

The box on the left indicates the four doors, behind one of which sits the prize. The bold-face letter denotes the door that hides the prize. We shall designate Door A as the one chosen by the contestant. Now let's consider the three doors from which the MC must choose one to open.

Each line is given 90 samples, 360 in all, so the initial probability that the prize is hidden behind a given door equals 1/4 for each door. Imagine that the options are presented to 360 schoolchildren who are asked to choose a door from B, C and D that does not hide the prize. Thus, from line 1 the 90 children have a choice of three doors, whereas for the other lines the other 270 children can choose from two doors only. First we consider the ideal experiment where the children choose exactly in accordance with a probability model. There are two models to consider.

Equal Choices

Under this model the children have an equal chance of choosing any door that is presented to them. So they will line up with 30 choices each when there are three choices available and 45 choices each when there are two choices. The results are shown in the middle box. The lines top to bottom represent the samples where the prize sits behind Doors A, B, C and D, respectively. The columns left to right represent, respectively, the number of times Doors B, C, and D were chosen. Now we may ask the question: what is the probability that the prize sits behind Door A (the contestant's pick) if Door B is chosen (by the MC)?

Door B has been chosen 120 times out of 360, that is, for 1/3 of the samples. For 30 samples, the prize was behind Door A. Thus, the probability that the prize was behind Door A after Door B is chosen is given by the ratio of 30/120 or 1/4. The same is true for opening Door C or Door D. This result is not general, but arises from our assumption of equally likely choices. This special condition is important theoretically and is referred to as the condition of maximum entropy or maximum uncertainty.

Given that Door B has been opened, the probabilities that the prize lies behind C and D must be equal and their sum must be 3/4. This is so because the prize cannot lie behind Door B. We get the same result by adding the frequencies (45 plus 45) and dividing by 120 (the total number of cases). Thus the probability that the prize lies behind Door C is 3/8, and similarly for Door D. Given that Door B has been opened, the chance that the prize lies behind Door C rather than Door A bears the favorable odds of 3:2 rather than 2:1 as in the three-door game.

We can also apply Mlodynyk's approach, as follows. There are two possible initial choices, one correct (RD) and three incorrect (WD). In condition RD, switching will always lose. In condition WD, we still have a choice to make when we switch, and we shall be right half the time — and half of the three WD situations is 3/2. So we will win by switching in a ratio of 1.5 to 1, or 3:2.

The non-mathematicians can skip this part, but for the remaining few readers, let's restate these evident truths in mathematical notation that provides the means to generalize. First we state that:

$$P(A \mid B) + P(C \mid B) + P(D \mid B) = 1$$

where $P(A \mid B)$ denotes the probability that the prize sits behind Door A given that Door B has been opened. The equation states the fact that because the prize is not behind Door B, it must be behind one of the other doors. Next,

$$P(A \mid B) \, \alpha \, P(B \mid A)$$

This line simply claims that the probability of a prize being behind Door A is proportional to the probability that Door B would be opened if the prize were behind Door A. It is common sense to say that the more likely one is to choose Door B when the prize is behind Door A, the more likely that this is indeed the situation when Door B is chosen. More generally,

$$P(A \mid B) \propto P(B \mid A)\, P(A)$$

$$P(C \mid B) \propto P(B \mid C)\, P(C)$$

$$P(D \mid B) \propto P(B \mid D)\, P(D)$$

where P (X) is the probability that the prize was initially behind Door X. It is not necessary to assume P (X) is the same for all doors, but that is an inherent condition of our experiment where each line represents 90 samples. The relevance to bridge is that any card initially has an equal chance of being dealt to any of the four players.

Biased Choices

Randomness does not require that all possibilities are equally likely. It is easy enough to introduce a bias in our experiment by assigning colors, Door B being red, the most popular color, Door C being chartreuse, and Door D being yellow, the least favored color. How exactly the addition of colors would bias the results is unknown, so we shall make some assumptions. These are reflected in the numbers given in the right-hand box of the previous diagram. Let's say that red is twice as popular as chartreuse, while yellow is never chosen when red is available, but is an equal choice with chartreuse otherwise. This assumption could be the subject of a test on results obtained in practice.

If we now go through the process indicated above for equal choices, we find the following:

$$P(A \mid B) = 2/7,\ P(C \mid B) = 3/7, \text{ and } P(D \mid B) = 2/7.$$

Thus, P (A | B) divided by P (C | B) is 2/3, as in the previous example, but P (A | B) is equal to P (D | B). Also, the probability that the prize sits behind Door A has changed from 1/4 to 2/7, even though every child had a 'free' choice of a door behind which there is no prize. What we really should say with regard to the first case is that the child made an unbiased choice. Some children may prefer chartreuse to red, and they were free to make that choice based entirely on personal preference.

If Door D is chosen:

$$P\ (A \mid D) = 0,\ P\ (B \mid D) = 1, \text{ and } P\ (C \mid D) = 0.$$

The effect of our assumption with regard to selection due to color is that Door D is chosen only when Door B is unavailable. In bridge this is equivalent to the conclusion that if a defender fourth to play takes a ten with a king, declarer may assume he was not dealt the jack. However, it cannot be said that he was not dealt the queen.

We now present the tables of conditional probabilities under each condition defined by choice of door in the two cases considered, equal choices and biased choices.

Conditional Probability Matrix				**Equal Choices**				**Biased Choices**			
—	P (A|B)	P (A|C)	P (A|D)	—	2/8	2/8	2/8	—	2/7	2/7	0
—	P (B|B)	P (A|C)	P (B|D)	—	0	3/8	3/8	—	0	3/7	1
—	P (C|B)	P (C|C)	P (C|D)	—	3/8	0	3/8	—	3/7	0	0
—	P (D|B)	P (D|C)	P (D|D)	—	3/8	3/8	0	—	2/7	2/7	0

The initial probability, P(A), that the prize sits behind Door A is given by Bayes' Equation:

$$P\ (A) = P\ (A \mid B) \cdot Q\ (B) + P\ (A \mid C) \cdot Q\ (C) + P\ (A \mid D) \cdot Q\ (D),$$

where Q (X) represents the probability that Door X be chosen. We use Q (X) because we must make a distinction between the possibility that Door X might be chosen blindly and the probability that it would be chosen. This distinction is not necessary if the choice of doors is unbiased, but such an assumption doesn't always apply. In bridge there is a difference between the probability of a card being dealt to a given defender and the probability that he would play it under the given circumstances if he had it.

Thus, based on the number given in the right-hand box,

$$P\ (A) = (2/7) \cdot (7/12) + (2/7) \cdot (7/24) + 0 = 1/4$$

Also,

$$\begin{aligned} P\ (B) &= P\ (B \mid C) \times Q\ (C) + P\ (B \mid D) \times Q\ (D) \\ &= (3/7) \times (7/24) + (1/8) = 1/4 \end{aligned}$$

Similarly, the initial probabilities for P (C) and P (D) are correctly given if we use Q (X), the probability of Door X being chosen, rather than P (X), the probability that the door might have been chosen if the choices were unbiased. In relation to cards, P (X) refers to the *a priori* odds of the deal where all cards are treated equally regardless of rank, and Q (X) refers to the probability that a card would be played by a defender from a given selection of cards. During the play you must refer the conditional probabilities as they apply to the cards in the suit to which a defender must follow. Probabilities depend on the *a priori* knowledge, which includes such information as an imbalance in the vacant places at the time of decision that imposes a bias on a defender's choices.

Testing a Hypothesis

Suppose that we examine 360 samples of choices from colored doors that were submitted by real schoolchildren. We compare the matrix of selections with the box on the right and ask whether the results can be said to conform to our model. Well, we can say immediately that there will be some (probably among those who choose to sit at the back of the class) who will prefer a hideous shade of sickening yellow to a bright and cheerful crimson. So we should never model human behavior under an assumption of 'never'. Furthermore, even if we assumed 5% would pick yellow over red, it would be hard to justify or reject that assumption on a small sample. It requires many experiments to assign a probability to a rare occurrence with a high degree of confidence. In fact, the matrix of the results may fit more closely the box for equal choices (perhaps the school colors of one set of happy students were green and yellow, whereas at another school the unpopular home room teacher was wearing a red dress). So we must always be suspicious of results from small samples where local conditions will affect the choices.

The scientific method is to gather a set of data, say, 360 samples, adjust one's hypotheses if necessary, then test them again on an entirely new set of data. As we play a bridge hand, we are gathering results to add to the collection of data stored in our memory. It is easy to be swayed by our emotions. Emotionally, successes outweigh the failures, and we may continue to pursue bad practices based on the good feelings we felt when successful. It is difficult to adjust solely on the basis of one's personal experiences. Rarely does one see a player start fresh and change his losing approach. With regard to conventions that rarely occur, you would have to play many years in order to establish how useful they really are. However, computer simulations can be useful for this if done carefully.

It is fair to say that the evidence from a small sample, potentially being highly variable, will often appear to arise from conditions of maximum uncertainty. This leads to conclusions such as 'system doesn't matter', or 'luck plays the biggest part in determining victory'. By now we should know this isn't so. However, while it is often the safest approach to assume a condition of maximum uncertainty when attempting to analyze the actions of mediocre players at the table, partners included, this may not be the optimum approach.

A Sequence of Choices

Now let's consider the situation in which the schoolchildren are asked to choose one door then another when there are three doors behind which there is no prize. It is possible, even likely, that the second choice between two doors may be biased in some way. If the doors are shown in a diagram, I would expect the middle door would be the most frequently chosen. However, let's assume the sequences are chosen equally. There are six permutations of choices from three doors as shown below.

Prize	B-C	C-B	B-D	D-B	C-D	D-C
A	15	15	15	15	15	15
B	0	0	0	0	45	45
C	0	0	45	45	0	0
D	45	45	0	0	0	0

The requirement to choose two doors results in a greater resolution of the location of the prize. Given the sequence Door B followed by Door C, there are only two possibilities remaining. Now the probability of the prize being behind the door not chosen is three times greater than that of its being behind Door A, the original choice. The same holds for any possible permutation.

Next we assume there is a bias introduced into the choice of doors, as defined by our previous assumption. The yellow door is never chosen if it can be avoided.

Prize	B-C	C-B	B-D	D-B	C-D	D-C
A	60	30	0	0	0	0
B	0	0	0	0	45	45
C	0	0	90	0	0	0
D	60	30	0	0	0	0

The prize sits behind Door A only when B and C are chosen, in which case there is an equal probability that the prize lies behind Door D, the yellow door.

Application to Card Play

We can translate the door matrix into a configuration in which a suit played by declarer has four missing cards: ♣Q, ♣8, ♣6, ♣2. It is assumed LHO holds three of the four cards. For this trial the participants play the role of LHO and are asked to choose any card they wish except the queen. We aim to estimate the probability that RHO holds the ♣Q. First, we assume the spot cards are chosen at random. We isolate the ballots that have chosen the ♣8.

LHO has these cards	RHO must have	Plausible plays	Number of ballots
♣8 ♣6 ♣2	♣Q	3	30
♣Q ♣6 ♣2	♣8	0	0
♣8 ♣Q ♣2	♣6	2	45
♣8 ♣6 ♣Q	♣2	2	45

Given that LHO cannot choose the ♣Q, we arrive at the result that the ♣Q sits on the right in 30 cases, and on the left in 90 cases, so the chances of the ♣Q being on the left are unchanged from the probability associated with the initial distribution (the deal). Of course, the chance of the ♣8 being on the right is now zero. If we use the same ballots to calculate the odds that the ♣2 is on the left, we find the probability has changed from 3:1 to 5:2. As bridge players, though, we are more interested in the location of the ♣Q than that of a spot card.

Suppose we next impose upon the participants the severe restriction of always playing the highest spot card (giving false standard count.) The numbers of ballots on which the ♣8 has been chosen are as follows:

LHO has these cards	RHO must have	Plausible plays	Number of ballots
♣8 ♣6 ♣2	♣Q	1	90
♣Q ♣6 ♣2	♣8	0	0
♣8 ♣Q ♣2	♣6	1	90
♣8 ♣6 ♣Q	♣2	1	90

The odds of the ♣Q (or the ♣2) being on the left are now 2:1, so the odds on the location of the ♣Q have been altered by rules that have greatly restricted the number of plausible plays. At the other extreme in the scale of permissibility, each card can be chosen without restriction as with the dealing of the cards, a rule that leads to the following configuration:

LHO has these cards	RHO must have	Plausible plays	Number of ballots
♣8 ♣6 ♣2	♣Q	3	30
♣Q ♣6 ♣2	♣8	0	0
♣8 ♣Q ♣2	♣6	3	30
♣8 ♣6 ♣Q	♣2	3	30

The number of plausible plays is the same for all remaining conditions. This is reflected in the number of vacant places. The process began with 3 vacant places on the left and 1 on the right. After a card is played by LHO, the vacant places are reduced to 2 on the left and 1 on the right. Thus, when the plausible plays have reached a status of equality between the remaining conditions, the vacant place ratio under the assumption of a given split equals the probability of the location of a particular missing card. Note that this same property is possessed as well by the results obtained under the previous rule, because, there too, the plausible plays attained equality.

Distinguishable Spot Cards

Spot cards cannot be said to be indistinguishable. Fans of Robert Darvas' *Right Through the Pack* have happily recognized this for a long time. Given a choice of cards from ♣862, many bridge players initially would play the ♣2 without giving the matter much thought. Some seeking to deceive would try the ♣8. Those with an innate love of obscurity are drawn inexorably to the ♣6. Taken together this unequal treatment of the spot cards alters the probabilities. Let's suppose the ♣8 appears with a frequency of 1 in 6, the ♣6, 1 in 3 and the ♣2, 1 in 2 times. Here is the matrix for the play of the ♣8.

LHO has these cards			RHO must have	Plausible plays	Number of ballots
♣8	♣6	♣2	♣Q	6	15
♣Q	♣6	♣2	♣8	0	0
♣8	♣Q	♣2	♣6	2	45
♣8	♣6	♣Q	♣2	2	45

Under the assumption that the spot cards are indistinguishable in the three combinations of Qxx, then when the ♣8 appears the odds become 6:1 that the ♣Q is on the left. When the ♣6 appears the odds are unchanged from the *a priori* odds of 3:1. When the ♣2 appears, the odds are 2:1. So, when the ♣2 appears we are less confident than initially that the ♣Q is on the left, whereas when the ♣8 appears we are more confident in that regard. When the ♣6 appears we are left to puzzle its significance. Here are full results in matrix form with the rows representing the cards held by LHO.

Initial Conditions				Trial Results				Probabilities P (X | Y)		
8	6	2		15	30	45		1/7	1/4	1/3
Q	6	2		**0**	45	45		**0**	3/8	1/3
8	**Q**	2		45	**0**	45		3/7	**0**	1/3
8	6	**Q**		45	45	**0**		3/7	3/8	**0**
			Totals	105	120	135	Chosen	♣8	♣6	♣2

Plausible Plays

In situations where the card choices are of equal probability, the number of plausible plays is equal to the number of choices. Therein lies the origin of the terminology. From the combination ♣Q82, it is assumed the ♣8 and the ♣2 would be chosen equally. There are two real and immediate alternatives, and the probability of either being chosen is the reciprocal of the actual number of apparent choices. However, the probability of the ♣8 appearing from ♣862 is 1 in 6. In this case there are not six immediate alternatives, there are only three, and the various choices have differing probabilities of being chosen. However, in a virtual experiment choices are made repeatedly as many times as we specify. The expected number of ballots expressing the choice of the ♣8 is one-sixth of the total number cast, in the above case 15 out of 90. The expected number of ballots expressing a particular choice is the total number cast divided by the number of 'plausible plays', where fractional plays are allowed. If the ♣2 was expected over a long sequence of choices to be chosen at a frequency of 2 out of 3 times, say, the effective number of plausible plays would be 1.5, which is the reciprocal of 2/3. The ♣2 is chosen on average once out of every 1.5 opportunities.

Random Choices

Let's go back to our matrix diagrams, but make a different assumption about how the spot cards are selected. Each line represents 90 samples, 360 in all, so the initial probability that the ♣Q is held by the RHO is 1/4. The samples are presented to 360 bridge players who are asked to imagine themselves defending a 3NT contract and to choose a card other than the ♣Q. Thus, from line 1 the 90 players have a choice of three cards, whereas for the other lines the other 270 players can choose from two cards only. In an ideal experiment the players would choose the spot cards equally at random, so as to minimize the information transmitted to a declarer. The expected numbers of ballots are in the middle box and the resultant conditional probabilities are shown in the box on the right. All very nice and regular.

Initial Conditions				**Trial Results**				**Probabilities P (X | Y)**		
8	6	2		30	30	30		1/4	1/4	1/4
Q	6	2		**0**	45	45		0	3/8	3/8
8	**Q**	2		45	**0**	45		3/8	0	3/8
8	6	**Q**		45	45	**0**		3/8	3/8	0
			Totals	120	120	120	Chosen	♣8	♣6	♣2

With regard to human behavior it is inappropriate to expect perfection. Inevitably one encounters natural variability. There is always an oddball in the crowd (maybe it's me!). To the extent that a statistical study can be thought of as being perfect, it is with regard to the conditions under which the study was conducted rather than to the results obtained. Even the conditions of an experiment may be questioned. 'Why 3NT?' we may ask. What does the rest of the deal look like? Let's not get sidetracked. The relevant question here is: why assume equally probable choices?

The Maximum Entropy Principle

In the 19th century, applications of statistics were condemned by those who prefer to think in terms of causes and effects. Pierre-Simon Laplace (1749-1827) caused a stir when he stated publicly to Napoleon that he had no need for divine intervention in his explanation of celestial mechanics. This amused Bonaparte but subsequently angered theologians and Newtonian scientists who maintained that some unseen hand was required to keep everything eternally rotating. With regard to statistical inference, Laplace maintained that as the sun had risen regularly for 500 years, he was willing to give odds of 1,826,214 to 1 that it would do so on the following morning as well. Some have taken this jest seriously, and have continued to argue about causality. There were no takers at the time despite the favourable odds.

More relevant to bridge (where the hidden hand is an integral part of the game) is the Laplacian concept that all possible unknown conditions are equally probable. Metaphysicians have argued that if nothing is known about conditions, they could just as easily be assumed to possess any probability distribution one might wish to assign. Modern information theory has given us this explanation: maximum ignorance concerning a set of conditions is a state of maximum entropy in which all probabilities are equal. If some knowledge is made available concerning these conditions, their probabilities must reflect this new knowledge, and so are equal no longer.

So we come, finally, to the play of the cards at bridge. By the time the opening lead is made, much information has been conveyed through the bidding that will affect the various probabilities. A declarer should adjust the probabilities in accordance with the information he has received on this particular deal as well as with his general knowledge of how the game is played. If we conduct a survey concerning the card selected from ♣862 for the opening lead, the results may indicate that the choices are equally probable, but this is true only in a statistical sense. Some players will lead the ♣2 (low from odd), others the ♣8 (top of nothing) or the ♣6 (MUD). Each lead is informative as there is a deterministic rule behind it, if declarer takes the time to look at the back of the convention card. The information is degraded to the extent to which players will deviate from the stated rules. The choices would be maximally uninformative if the opening lead were chosen at random every time. In summary, you do not play against everyone at once, but against one pair at a time.

The situation is different when a defender is required to follow to a lead by a declarer. The defender may choose to play low cards at random in order to reduce the information conveyed. The statistics of these plays are relevant in the analysis of cardplay in a way that a survey of opening leads is not. For opening leads, the statistics of opening leads tell us how many prefer to play MUD, the least informative of the three possibilities. As for following to declarer's lead, the statistics show us how random are the choices from insignificant cards. There is an essential difference.

The Testing of a Hypothesis

When you wish to devise a statistical test, you must first have in mind what you are attempting to discover. The conditions of a test should be tailored with a particular question in mind. Assume we have devised a scenario discussed above where LHO holds three of the four outstanding club cards and it doesn't matter in any practical sense which insignificant card he chooses. Does the probability that the ♣Q is held by LHO remain unchanged regardless of which low card appears on the first round from LHO? In other words, in this situation do players choose from their low cards equally at random (our null hypothesis) or is there a bias? We can collect the results from the 360 bridge players in the manner indicated previously and perform a test of the results to see to what degree the null hypothesis can be said to be confirmed. Here is a set of results we might obtain:

Initial Conditions				**Trial Results**				**Probabilities P (X ǀ Y)**		
8	6	2		20	34	36		0.19	0.27	0.28
Q	6	2		**0**	41	49		**0**	0.33	0.39
8	**Q**	2		48	**0**	42		0.45	**0**	0.33
8	6	**Q**		39	51	**0**		0.36	0.40	**0**
			Totals	107	126	127	Chosen	♣8	♣6	♣2

There are methods we can use to discover how likely it is that our results were generated by a uniform distribution of choices. In total we have 360 choices represented of which 107 were of the ♣8, 126 of the ♣6, and 127 of the ♣2. The expected number was 120 for each. The null hypothesis that each card is equally likely to be played can be accepted at the 25% level, meaning that such variation from the norm would be generated by a random sampling of 360 trials more than 25% of the time.

Of course, the numbers represent not just one experiment but four. The results in the first line give rise to suspicions that the ♣8 is less likely to be played from a combination of ♣862 than either the ♣6 or the ♣2. The variation evident in this mix would occur from a random sampling of equal distributions less than 10% of the time. We might conclude that more experiments are required for this combination in particular.

Alternatively, we might change the assumptions for this line. The hypothesis we are testing should not be formulated from the data itself, for in that case we would always get a good fit, but must be proposed before the experiment is performed. Let's assume the expected numbers are 15, 30, and 45, respectively: our guess expressed earlier. The goodness-of-fit for this hypothesis is very good as variations greater than this would occur in more than 50% of samples of the same size. The biased choice model is more acceptable than the unbiased choice model.

Our Bridge Experiences

The eye sees only what the mind is prepared to comprehend
— Robertson Davies (1941-2002)

Our encounters at the bridge table, successful or otherwise, represent but a very small sample of experiences from the great experiment which is Life. If it is difficult to draw conclusions from a controlled experiment, how much harder is it to do so from the chaotic conditions we encounter at the local bridge club? Our results, good or bad, are subject to a natural, random variability. Some impatiently attempt to 'time the market' by taking huge risks, thus increasing the variability, while others, akin to bond holders, ride out the storms with stable, but uninspired, adherence to standard textbook advice. Most tend to 'go with the field', which involves guessing the actions of the majority of the surrounding players. This acts to widen the statistical base, and has the advantage of minimizing variability at the cost of not attempting to maximize gain. Ideally we should prefer to employ methods based on the sound principles of probability theory, tempered by experience, without the egotistical expectation of always being right.

An Eight-Ever Example

Let's look at the effect of a false card on the probabilities. Look at this suit combination, where declarer is missing five to the queen.

♥ K J 9 4

♥ A 10 8 6

Declarer plays the ♥A followed by the ♥6 towards the ♥KJ9, the defenders following with small cards all the way. The missing cards we denote as ♥**Quwyz** and the observed sequence has been card **u** followed by card **z**, then card **w** from LHO on the second round, the sequence being denoted as **u-z; w**. What are the current relative probabilities of these two conditions?

Condition I Quw opposite yz Condition II uwy opposite Qz

If all low cards are chosen with equal probability, the probability of Condition I (QL) is 1/4 the probability of the 3-2 split, because there are four equally probable permutations. The probability of Condition II (QR) is 1/6 times the probability of the 3-2 split because there are six equally probable permutations. The relative probability in favor of the Q being on the left is 3:2. In mathematical terminology:

$$P\ (QL \mid u\text{–}z;\ w) = (3/2) \times P\ (QR \mid u\text{–}z;\ w)$$

This is part of the justification for the eight-ever rule of always finessing for the queen. However, what if the probability of RHO playing card **z** rather than card **y** on the first round was 1/3 rather than 1/2, due to a known tendency to falsecard? Then the current probability of QL and QR would be equal after the given sequence. Furthermore, if RHO would always falsecard from a combination of **yz**, or would always give the correct count playing card **z**, card **y** would be characterized by the same certainty of being withheld as the queen from the combination. In this case,

$$P\ (QL \mid u\text{–}z;\ w) = 3 \cdot P\ (QR \mid u\text{–}z;\ w)$$

This merely expresses the result that Condition I is characterized by two possible permutations and Condition II by six under the assumption that LHO needn't consistently favor one low card over the other in a three-card combination. Let's now consider an example of biased selection arising from a story by David Bird, who delights in situations where the probabilities are influenced by psychological factors that produce the effect that $P\ (X) \neq Q\ (X)$. The tale is funnier than the equation suggests.

A Muzzy Monk's Muddled Math

The man who can't be fooled can always be beaten
— Lou Brock (b. 1939)

The amusing series of monastic stories by David Bird continue to open our eyes to a dazzling display of squeezes and endplays (many devised by Tim Bourke, I understand) of which Abbot Hugo Yorke-Smith is the usual victim. It is clearly the author's design to show in subtle ways that the sophisticated Abbot gets it wrong, but when Bird's preferred alter ego, the clever Brother Lucius, speaks, one assumes the emphasis has shifted from spoof to proof.

Dealer South Both vul.	♠ A 10 5 ♥ K J 7 ♦ A 8 3 ♣ A 7 4 2	
♠ 9 7 3 2 ♥ 10 6 4 ♦ Q J 10 5 ♣ K 3	N W E S	♠ J 8 4 ♥ 8 5 3 2 ♦ 9 7 2 ♣ 10 8 6
	♠ K Q 6 ♥ A Q 9 ♦ K 6 4 ♣ Q J 9 5	

West	North	East	South
Brother Xavier	*Brother Paulo*	*The Abbot*	*Brother Lucius*
			1NT
pass	6NT	all pass	

In the story that appeared in the June 2009 ACBL Bulletin, Lucius has reached 6NT. The lead is the ♦Q, and the duplication in the North-South hands makes the contract a relatively poor one. Declarer needs to bring in the club suit for four tricks. After successfully running the ♣Q on the first round, to which Brother Xavier (West) follows with the ♣3 and the Abbot (East) with a highly suspicious ♣8, declarer has to decide whether on the second round to start with the ♣5, hoping West started with a doubleton ♣K, or to lead the ♣J hoping to pin a now bare ♣10 in the East hand.

The Abbot is notorious for his penchant for falsecards, so he would normally play the ♣8 from three possible holdings, ♣108, ♣1086, or ♣1083. As two of those combinations are three-card holdings, Brother Lucius erroneously concludes the odds are 2:1 that the Abbot was dealt a tripleton and Xavier a doubleton club. Perhaps he had spent considerable time in the cellar that day reclassifying the monastery's vast reserves of Home Counties claret because his thinking was rather muzzy. Why? Answer: because one of those tripletons was ruled out when Xavier followed with the ♣3. Does that mean the odds are now 50-50? No, not even the Abbot would think so, as he is an avid reader of the works of Terence Reese.

The situation as Brother Lucius ponders his second lead in the club suit has been reduced to two potentially favorable combinations:

	Xavier	**Abbot**
Condition I	♣K3	♣1086
Condition II	♣K63	♣108

If with ♣K63 Xavier would always play the ♣6 on the first lead (as Bird seems to imply in his narrative), then Condition II has been eliminated entirely as a possibility, so the ♣5 should be led on the second round, regardless of which club the Abbot has chosen. The ♣K will pop up to be taken by the ♣A in dummy. If Xavier only sometimes plays the ♣6 and the Abbot always plays the ♣8, Condition I is still the more probable, and the same low club play is indicated. Lucius had no problem choosing to play a low club on the second round based on the known proclivities of his opponents.

On first take, the theme of the episode appears to be a condemnation of defenders who always play 'false' cards, thus becoming their own worst enemies; however, that may be said also of players who always choose 'true' cards. If Brother Xavier always gives true count with the ♣3 from ♣K63 and the Abbot similarly with the ♣6 from ♣1086, the card position becomes a certainty. The defect lies in the defenders' predictability. Their best strategy is to follow with low cards at random, making plays of equal probability out of the appearances of the ♣3 on the left and the ♣8 on the right. This transmits the minimum possible amount of information to declarer. An insightful view of the defenders' unbiased random selection of equivalent cards is that they are merely simulating the act of dealing, thus keeping declarer in a state of maximum uncertainty to the extent possible under the circumstances.

The Inference from the Opening Lead

The *a priori* odds are that the 3-2 and 2-3 club splits are equally probable. Conditions I and II each have two plausible permutations when the defenders follow randomly with low cards to the first round. To make a decision that has better odds than a coin flip, declarer must decide whether a 3-2 split is more probable than a 2-3 split on this particular deal, and to do so he needs to consider the other suits. The only clue on this deal is that Xavier has led the ♦Q, placing him with touching honors in that suit and usually length. A 4-3 diamond split gives a higher probability to a 2-3 club split, which, perhaps fortuitously, leads to the correct decision.

Let's look at this lead of the ♦Q, with the defenders between them holding seven diamonds to the QJ10. We'll assume the lead against an uninformative auction to 6NT is from a sequence of QJ10. The fellow defender plays his most discouraging card. There are two cases to consider:

	Diamonds 4-3 QJ10x opposite xxx	Diamonds 3-4 QJ10 opposite xxxx
Combinations	4	1
Possible Plays	1	1

On the basis that falsecarding is not an option, it appears obvious that the diamonds are more likely to be split 4-3 than 3-4. Of course, there would be no funny story to tell if one thought that way… or maybe not. If the adventuresome Brother Cameron had been on lead and had chosen the ♦Q from queen doubleton, the outcome might have been a happier one for the Abbot. Fans of the monks know that such a lead would be uncharacteristic of Brother Xavier, who, despite his perpetual criticism of the Abbot's normal leads, wouldn't himself dare to deviate from orthodoxy with such a partner.

Against players who defend to best advantage, after the first round of clubs declarer should gather more information outside the club suit by cashing some top hearts and spades to see if the vacant places, like unreliable clarets, need to be reassessed. This may result in going down more than one trick in the slam, but it increases the probability of making the right decision in the club suit. By immediately playing a second round of clubs, Brother Lucius indicated that he was pretty sure his opponents were acting predictably.

Chapter 6

RESTRICTED CHOICE REVISITED

Probability is expectation founded upon partial knowledge. A perfect acquaintance with all the circumstances affecting the occurrence of an event would change expectation into certainty, and leave neither room nor demand for a theory of probabilities.
— George Boole (1815-1864)

Earlier, during the discussion of the Nine-Never adage, we ran into the Principle of Restricted Choice. It is time to examine the concept in a little more detail. There are two questions to be answered: What is the principle of Restricted Choice? and how should we apply it? The first question deals in generalities, the second with specifics. The principle arises from Bayes' Theorem, a mathematically precise statement relating to the calculation of probabilities after cards have been played. It can be difficult to restate this from the language of mathematics into plain English as something may get lost in the translation. In his masterwork, entitled in different versions either The Expert Game or Master Play (1960), Terence Reese put it this way:

It comes to this: that a defender should be assumed not to have had a choice rather than to have exercised a choice in a particular way.

From this we may gather that if the J10 in a suit are missing and one defender plays the jack, it affords the assumption he does not also hold the ten. It is not always correct to act on this assumption. Reese knew that, but he was trying to be helpful and may have inadvertently put the wrong idea in some readers' minds. When making decisions you should take into account all possible combinations remaining.

Here is my take on the application of Bayes' Theorem to card play:

After you observe a sequence of plays in a suit, the probability that the observed sequence arose from a particular card combination is in inverse proportion to the number of equally plausible plays available with that combination. The greater the number of plausible alternatives, the less likely it is that the observed sequence was chosen from that combination.

The *Official Encyclopedia of Bridge* gives the following cautionary example in the section on restricted choice:

North	A 2
South	K Q 9 8 4 3

Declarer plays the ace and then leads the two. East follows with two low cards, but West has followed with the ten on the first round. Should declarer finesse the nine on the assumption that West had no choice but to play the ten? No. That is not what Reese meant. Let's look at the combinations remaining after East follows to the second round. And as we discovered in the last chapter, when looking at probabilities we should specify which cards have appeared, and not state vaguely that East has followed twice with low cards. Let's say East has played the five followed by the seven. We'll assume that West would habitually play the 'obligatory' falsecard from J10x.

Here are the possible combinations remaining along with their probabilities:

West	**East**	**Plausible plays**	***A priori* %**	**% relative to plausible plays**	***A posteriori* %**
10	J 7 6 5	6	2.83	0.472	29
J 10	7 6 5	12	3.39	0.283	18
J 10 6	7 5	4	3.39	0.8475	53

The single most likely combination is the 3-2 split, at 67.8%, giving each of our 3-2 situations a probability of 3.39% *a priori*. Now if we reduce the likelihood of each combination by dividing by the number of plausible plays available from each, we get the *a posteriori* figures shown in the last column. You can see that on this sequence of plays, J10 or J106 in the West hand is more likely than the singleton 10 in the ratio of about 12:5, so it's not even close: declarer should play to drop the jack in three rounds.

The question arises: does the same logic apply if West follows first with the jack? That depends on how often a player would play the jack from J10. If he always plays the lower of two touching honors, then J10 is impossible. Always playing low from touching honors is more informative than playing either honor at random. In the above example partner need not be informed, so the random approach is best.

I believe in intuition and inspiration
— Albert Einstein (1879-1955)

Of course, this kind of suit analysis in isolation is unrealistic, as some bids have to be made and some cards have to be played before declarer gets around to leading his ace. If the vacant places are equal, as they are when the cards are unseen, the results based on the *a priori* odds will provide reasonably sound guidance. However, the vacant places may be unbalanced — for example, if there has been preemptive bidding by the opponents — and this significantly affects the odds. All suits may have to be included in the calculation, as in this example from the 2013 Spingold Final.

Both vul.

	North	
	♠ 10 7 6 5 3 ♥ K ♦ 6 4 ♣ K 10 8 4 2	
West ♠ J 2 ♥ 10 9 8 6 3 ♦ J 8 ♣ Q J 6 5	N W E S	**East** ♠ A K Q 8 4 ♥ A 5 4 ♦ K 3 ♣ A 7 3
	♠ 9 ♥ Q J 7 2 ♦ A Q 10 9 7 5 2 ♣ 9	

Nowasadski	**Grue**	**Kalita**	**Moss**
			3♦
pass	pass	dbl	pass
3♥	pass	3NT	all pass

Brad Moss led the ♣9, covered by the ♣Q, ducked by Joe Grue (♣2). Jacek Kalita led the ♥10 from dummy, covered by the ♥K and ♥A, Moss following with the ♥2. The ♠4 was played towards dummy, Moss playing the ♠9, a significant card. The ♠J won the trick, Grue playing the ♠3. On the lead of the ♠2 from dummy Grue followed with the ♠5, and Kalita was faced with the decision of whether or not to finesse the ♠8.

A declarer blindly following the guidance of Reese, as quoted as above, finesses, playing for the ♠10 and ♠9 to have been dealt to separate hands. *The Official Encyclopedia of Bridge* has warned us that such simplistic thinking can be wrong. You have to examine the remaining combinations before deciding.

Let's simplify and assume on the bidding that Moss holds seven diamonds and Grue two, and that Moss has led passively from a tripleton club. The vacant places are three in the South hand and eight in the North. The probabilities before a major suit has been played relate to the number of combinations of hearts and spades possible on the deal. The hearts and spades may be split in the following manner to fill the vacant places.

	North – South	**North – South**
Heart Splits	3 – 2	4 – 1
Spade Splits	5 – 1	4 – 2
Combinations	60	75

If there were no restrictions on the play of the cards, then one round of hearts and one round of spades would not affect the relative probabilities. Thus, it is still more likely that South was dealt a doubleton spade rather than a singleton in the ratio of 5:4. If there are restrictions on the play, probabilities can change drastically as an adjustment must be made for the number of plausible plays for each remaining combination. The table below shows the number of plausible plays at the time of decision when the ♠2 is led from dummy and Grue follows with the ♠5.

	North – South	**North – South**	**North – South**
Heart Division	KQJ 72	KQ7 J2	KQJ7 2
Spade Division	107653 9	107653 9	7653 109
Plausible Plays	72	24	72
Probability (%)	20	60	20

I have assumed North would cover randomly with an honor on the lead of the ♥10 from dummy. Many possible combinations have been reduced to a mere three. The results show that a singleton spade in the South hand is four times as likely as a doubleton, so the finesse is the correct play.

It is said that only a genius or a fool leads from a worthless doubleton and it doesn't pay to assume your opponent is a genius. Let's suppose the lead was from a singleton, as was the case. If the clubs were dealt 1-5, there are 6 vacant places in the North hand and 5 in the South. The hearts and spades may be split in the following manner to fill the vacant places.

	North – South	**North – South**	**North – South**
Heart Splits	1 – 4	2 – 3	3 – 2
Spade Splits	5 – 1	4 – 2	3 – 3
Combinations	30	150	200

On an *a priori* basis the chances of a singleton spade in the South hand are slim indeed, but let's again look at the situation at the time the ♠2 is led from dummy and Grue follows with the ♠5. We assume that North would split his heart honors randomly from ♥KQx on the lead of the ♥10 from dummy. Under those assumptions there are just these combinations left to consider:

	North	South	Plausible Plays	Probability (%)
I				
Hearts	K	QJ72		
Spades	107653	9	24	33
II				
Hearts	KQ	J72		
Spades	7653	109	96	7
III				
Hearts	KQJ	72		
Spades	76x	109x	108	15
IV				
Hearts	KQ7	J2		
Spades	75x	109x	36	44

The odds are 2:1 in favor of playing for the drop. In order to justify deciding to finesse against the ♠10, as Kalita did, declarer would have to assume that the ♥K would be played from ♥KQ(x) much less often than half the time; in particular, at Trick 2 Grue would play the ♥Q about 7 times out of 8. In fact, the ♥K would be the book play, as normally when splitting honors a defender plays the card that would have been led. If the lower honor would always be played in Cases II-IV, the only remaining possibility is Case I with the singleton ♥K.

We have observed on BBO that the experts usually strive to play the informative card, probably because they trust their partners to make good use of the information they transmit. So, it is quite possible that, early in the play especially, the ♥K would often be chosen over the ♥Q. This would reinforce the numbers and more strongly suggest playing to drop the ♠10 rather than taking a finesse.

In a comment on my original blog, Hall of Famer Bobby Wolff offered his view that Jacek Kalita simply made a great intuitive play when he played Brad for the singleton ♠9. He pointed out that if Kalita had known that Moss had only a singleton ♣9 (the opening lead) he might have had second thoughts about finessing, but sometimes the less knowledge the more effective the play. In any event, he said, the next time Kalita's intuition kicks in he may lose that spade trick to an original 109xx, with Grue having the ♥KQJ. Of course, with that layout declarer would have been increasing his downtricks from one to perhaps five, but in retrospect Wolff thought his gamble was justified since doing so, taking advantage of the actual layout of the deal, enabled the contract to be scored up.

This isn't the whole story, but this discussion has been long enough and as a two-finger typist I was happy that Nowosadzki didn't play a larger role. At least you can see how the calculations should proceed — even computers have been known to make mistakes. (That's a comforting thought in the nuclear age!) Bayes' Theorem is correct but we must assign realistic probabilities to the play options. As Voltaire noted, a logical conclusion is only as valid as the assumptions that went into it. Detractors of Restricted Choice will be glad to discover that in the end judgment is the determining factor, and mathematics acts merely as the Handmaiden to Success (or the Mop-Lady to Failure.)

CHAPTER 7

TWO CHANCES ARE BETTER THAN ONE

If you can't describe what you are doing as a process, you don't know what you are doing.
— W. Edwards Deming (1900-1993)

Eddie Kantar, a favorite author of many, won the 2009 American Bridge Teachers' Association Book of the Year award for a work titled, *Take All Your Chances in Bridge*. He has since produced a second volume on the same topic that is just as good. The recurring theme is that in declarer play two chances are better than one, if we can take the first chance for success without losing the advantage of a second chance should the first one fail. There is a process for doing just that. The solution is simple after the answer is known.

Of course you won't get a second chance if you don't look for it at the very beginning unless you are really lucky and it falls into your lap after a narrow escape. (That's how I met my wife, come to think of it.) Kantar takes the sensible approach of leading the student repetitiously to the correct approach without piling on abstract mathematical arguments. Of course, mathematics is what we are all about. Mathematically the odds change during the process, but once you're committed it doesn't matter that much.

As we have seen in the previous two chapters, one of the conceptual difficulties encountered in bridge literature is that a suit is often treated as being independent of the other suits. So we encounter lists of percentage plays in a suit taken in isolation. This can be helpful, but it gives the wrong impression from a theoretical perspective. Suits are intertwined. Here is an example in which expert Tim Bourke asked, 'Is there a clear-cut solution?'

♠ A 8 6 5 3 2
♥ A Q 9
♦ 7 2
♣ K 9

♠ K 9 7
♥ K J 6 4
♦ A 9
♣ A J 5 3

The bidding is unknown, but let's assume South opened 1NT, and North transferred to spades and drove to slam.

The lead against 6♠ was a not unexpected ♦K. Declarer won and played two rounds of spades, hoping for a 2-2 split. The opening leader showed out on the second round, so East had a sure trump trick. Unlucky. The problem now was how to arrange a diamond discard on a club or heart winner before East won his trump trick.

There are three obvious lines that achieve a happy result:

1) Play the ♣K and finesse the ♣J, discarding a diamond on the ♣A;
2) Play three rounds of hearts hoping to discard a diamond on the fourth heart;
3) Play the ♣K, ♣A and ruff a club, hoping the queen falls and that East can't ruff.

Viewed in this light you might be inclined to hope the hearts are favorably split, but that is the wrong approach. Perhaps the club finesse is a reasonable shot, as LHO has already shown up with the ♦KQ? Wrong again. Let's see why #3 is the correct approach, as noted by Tim Bourke.

The Less Often You Lose, the More Often You Win

Rather than search for the winning percentage, it is often easier to look at the losing percentage. The probable location of the ♣Q depends on the number of clubs held by each defender, and there is no indication that RHO has many more clubs than LHO. We conclude the finesse has roughly a 50% chance of losing.

Playing off the hearts will work as long as RHO has three or more hearts. Let's look at the combinations available in hearts taken in isolation:

Split	0 – 6	1 – 5	2 – 4	3 – 3	4 – 2	5 – 1	6 – 0
Combinations	1	6	15	20	15	6	1

Total = 64

The 3-3 split stands right in the middle and is included in the winning category, so East will hold three or more hearts better than 50% of the time. That observation alone makes the heart play superior to the club finesse. East will follow to three rounds of hearts in 42 out of 64 combinations, which translates to roughly a 33% chance of failure. Playing on hearts clearly has a lesser chance of failure than the club finesse.

The chance of failure when playing three rounds of clubs hoping to drop the ♣Q can be calculated in the same way. A full calculation is burdensome at the table, so let's consider only a few of the more even splits to get a rough estimate. Whereas the even-numbered heart distribution was shaped like Mt Fuji with one peak in the middle, the odd-numbered club distribution is shaped like Table Mountain with an extensive central plateau.

Split	2 – 5	3 – 4	4 – 3	5 – 2	
Relative Weights	60	100	100	60	Total = 320

The third round of clubs will be ruffed by East (West has no more trumps!) in the ratio of 60 to 260, roughly 19% of the time. This clearly represents a lesser risk than playing on hearts. Success may depend on what transpires during the next phase of play.

The Chance of Success

With regard to the first two strategies (taking the club finesse or playing on hearts), declarer either wins or loses on the completion of the play in the suit. However, with regard to playing three rounds of clubs, there are card combinations for which declarer neither wins nor loses: the queen doesn't drop, but East doesn't ruff. Failure has been avoided, so there is now an additional chance to win, namely hearts may be played in the hope that East holds at least three hearts. It has become a case of two chances are better than one. Or has it?

Let's assign some terminology:

A represents the number of combinations where the initial club play works.
B represents the number of combinations where the initial club play fails.
C represents the number of combinations without a resolution.
A + B + C represents the total of all combinations allowed.

We start with a division of sides for the defenders of 4=6=9=7. The lead is the ♦K. Early in the play West shows out of spades so we assume he began with the ♦KQ and a singleton spade, whereas East has three spades and at least one diamond. This leaves us with 10 vacant places in the West hand and 9 in the East hand. However, West had to discard something on the second round of spades. What information does that give us, and by how much does it affect the odds?

If West were to discard a heart, declarer would be happy enough to play on clubs as planned. If West discarded a low club, it might raise concerns that West had five clubs and, therefore, that East had only two clubs. If West discarded a diamond, declarer would proceed happily with his plan in a neutral state of mind. In theory the discard is assumed to be a random selection from the unseen cards in West's hand. Indeed, the appearance of a diamond, heart or club doesn't materially affect the odds, so let's stick with the vacant places as being 9 and 10 for the sake of convenience. On that basis we do our probability calculations.

Probability of initial failure	16.8%	(15631 combinations)
Probability of success	36.5%	(33682 combinations)
Probability of neither	46.6%	(43065 combinations)

The club plays have left us with combinations where the club splits of 3-4 and 4-3 represent over 80% of the remnant combinations. We are now working on the high plateau of card distributions in accordance with the general tendency of proportionally more even splits as the cards are played out without incident. It is matter of calculating how many of these combinations show the hearts evenly split as well. This is tedious to do by hand, but I did it. I can tell you that the probability of the heart play succeeding as a backup plan is 67.4%. So the probability of success overall is given by the following:

Probability of success = 0.365 + 0.466 x 0.674 = 0.679
Probability of failure = 0.321

A rough approximation is suggested:

Probability of success = 1/3 + (1/2) x (2/3) = 2/3
Probability of failure = 1/6 + 1/6 = 1/3

Curiously, this is essentially identical to the chances of success if we just play off the hearts in the first place.

And Now for a Fourth Idea...

Some readers may have been annoyed by the above analysis, since they will have realized that I have been ignoring a fourth possible line of play, which encompasses the possibility that the ♣Q will be doubleton or singleton. That line is to play only two rounds of clubs before falling back on hearts.

The ♣Q will be kind to us about 10% of the time, and East will ruff the second round (without the queen having appeared) in about 2.5% of cases. Of the remaining 87.5%, the hearts will be favorable 67.4% of the time. This gives us a combined chance of making our contract playing this way of about 68.3% — superior to any of the other three, but not by a lot. Other than the finesse, which at 51% is much the worst option, there is little to choose among the alternatives.

And In Real Life...

In real life, as so often, virtue had to be its own reward, as the (admittedly marginally) optimal line would not have succeeded. The full deal was as follows:

	♠ A 8 6 5 3 2 ♥ A Q 9 ♦ 7 2 ♣ K 6	
♠ 4 ♥ 8 5 3 2 ♦ K Q 10 9 3 ♣ Q 9 4	N W E S	♠ Q J 10 ♥ 10 7 ♦ J 8 5 4 ♣ 10 8 7 2
	♠ K 9 7 ♥ K J 6 4 ♦ A 6 ♣ A J 5 3	

Thanks go to Tim Bourke for bringing an interesting problem to my attention along with the correct solution. He is developing a computer program that should prove very useful for calculating compound probabilities for complex situations. In the end it may be possible to develop rules which will serve as good guidance at the table from such results.

A Kantar Problem

Being a contrarian, and a mathematician to boot, I love to pile it on to see if it is in fact true that two chances are always better than one. If not, why not? How come?

Tim Bourke, a man who has cast a critical eye on many a declarer play, was again responsible for bringing to my attention to the following deal, presented by Eddie Kantar in a 1981 book entitled *Test Your Declarer Play, Volume 2*. Tim asks whether Kantar's suggested line is always the correct one.

♠ Q 3
♥ Q 10 7 2
♦ 2
♣ A K J 10 4 3

♠ A 2
♥ J 6 5
♦ A K J 10 9 8
♣ 6 2

West	North	East	South
			1♦
pass	2♣	pass	2♦
pass	2♥	pass	2NT
pass	3NT	all pass	

The lead is the ♠5 — not surprising. Declarer puts up the ♠Q from dummy, but East produces the ♠K. Now declarer must find nine tricks without losing the lead. Taking the club finesse twice, without cashing the ace first, is obvious (it is four times as likely that West has ♣Qxxx than that East has a singleton queen, so it is correct not to cash the ♣A before taking the finesse). However, Kantar suggests that it is better to play off the ace and king in the minor with more cards between the two hands, in this case clubs, to see whether the ♣Q falls. If so, declarer has nine tricks to cash. If the ♣Q doesn't fall, declarer is in the dummy to take the finesse in diamonds, and may still come to nine tricks that way.

Mathematically, we can express this idea in the following equation:

$$P1 + (1 - P1) \times P2 > P3$$

where P1 is the probability the drop play will succeed, (1-P1) is the probability it will not succeed, P2 is the probability the subsequent diamond finesse will produce six tricks, and P3 is the probability that the clubs produce six tricks on their own via a finesse or two.

The *A Priori* Odds

The probabilities depend on what we assume about the distribution of the sides. Let's calculate the odds for the case of maximum uncertainty which is represented by the *a priori* odds. These are shown below for the cases where declarer has a chance of succeeding.

The *A Priori* Odds of the E-W Club Splits

Split	4 – 1	3 – 2	2 – 3	1 – 4	Total
Probability (%)	14.3	33.9	33.9	14.3	
P1 Component	2.8	13.6	13.6	2.8	32.8
P3 Component	11.3	20.3	13.6	2.8	48.0

The *A Priori* Odds of the E-W Diamond Splits

Split	3 – 3	4 – 2	5 – 1	Total
Probability (%)	35.5	24.4	7.3	
P2 Component	17.7	8.1	1.2	27.0

Under these conditions: P1 + (1-P1) x P2 = 52%, and P3 = 48%.

So it is better to take the two chances as described rather than stake everything on the finesse in clubs. We conclude that Kantar's line is better by 4%, but only if the *a priori* odds can be applied with sufficient accuracy.

An essential characteristic of the *a priori* odds is that the numbers of vacant places are equally divided. What happens if there are more vacant places in the West hand, which will tend to favor the club finesse and be detrimental to the diamond finesse?

When South Preempts

Let's go through the mathematics when East enters the auction with a 2♠ preempt. South still plays in 3NT for the lack of a better alternative. The play to the first trick is the same, but now declarer must take into account that the spades are split 3-6. The *a priori* odds no longer apply, and instead the club splits have the following probabilities.

The *A Posteriori* Odds of the E-W Club Splits‡

Split	4 – 1	3 – 2	2 – 3	1 – 4	Total
Probability (%)	23.8	40.7	25.5	5.7	
P1 Component	4.8	16.3	10.2	1.1	32.4
P3 Component	19.0	24.4	10.2	1.1	54.7

The probability of succeeding by finessing in clubs twice (P3) has improved to over 50%, since the ♣Q is more likely to be in the West hand in the ratio of 10 to 7. And indeed, the same is true of the ♦Q, so we expect a lesser probability for the success of the diamond finesse.

‡ From J. P. Roudinesco's *The Dictionary of Suit Combinations*

The *A Posteriori* Odds of the E-W Diamond Splits§

Split	3 – 3	4 – 2	5 – 1	Total
Probability (%)	33.9	12.7	1.7	
P2 Component	17.0	4.2	0.3	21.5

Based on these numbers, the chance of success via the club finesse is about 55%. The chance of success for the alternative procedure is 47%, so the odds now favor the club finesse by a substantial margin. The one chance in the club suit is significantly better than the combination of two chances, playing for the drop in clubs and subsequently finessing in diamonds.

The Effect of the Opening Lead

If the only information declarer has concerning the distribution of the defenders' sides comes from the opening lead, the situation is uncertain. As there are nine spades in the opponents' hands, the suit cannot split evenly. In Kantar's book the lead was the ♠5 from a five-card suit, so the imbalance in the spade suit made the finesse in clubs an even worse proposition than the *a priori* odds indicate. The normal lead is in the unbid suit, so it is possible that West has led from a four-card suit, in which case the probability of success for the club finesse is 50.8% and that of the two chances, 51.4%. Thus, whether the imbalance of one vacant place favors West or East, it is better to play for the drop in clubs.

Some would argue that playing to drop the ♦Q is a safety play of sorts that does not risk immediate defeat on this particular deal. If the difference in the probabilities of success is only a few percentage points, consistently choosing the lower percentage play may not entail a significant cost over the short term. However, in the case of a 3-6 spade split, the margin of superiority for the club finesse is substantial, so declarer should believe the bidding. He won't be far wrong even if the spades are split 4-5. Entries are an issue too: if you cash the diamonds first, you only have the entries for one club finesse, not two if you need them.

For some the lesson is 'Don't put all your eggs in one basket.' For others, 'Always pursue the line that offers the greatest chance of success, given what you know at the time of decision.'

§ From J. P. Roudinesco's *The Dictionary of Suit Combinations*.

CHAPTER 8

EXPERTS PREFER ENDPLAYS

Until you're ready to look foolish, you'll never have the possibility of being great
— Cher (b. 1946)

So far our application of probability to cardplay has led to several conclusions. We know now not to apply the *a priori* odds willy-nilly when choosing our line of play. We also know that two chances are not necessarily better than one, and that any information we have received from the auction and the opening lead will affect the odds, and must be considered when choosing our line of play. Let's look at a more general application of these ideas.

In the December 2009 issue of the *ACBL Bulletin*, Mark Horton, editor of *Bridge Magazine*, gives this advice to the readers: 'As declarer, eliminate a suit if you can, even if you don't know why you are doing it. Good things can happen.' There is no warning label; Horton leaves it to the reader not to get carried away and to use some common sense in its application. I like that. Another good piece of advice would be, 'Always make the play that has the greatest probability of success', but here the difficulty is that a declarer may not know which choice fits the description. That is where Jeff Rubens comes in. In his book *Expert Bridge Simplified*, Rubens presents the reader with methods to sift out the plays that stand the best theoretical chance of being successful, always assuming that the player is capable of the necessary rapid mental arithmetic.

The following deal from Rubens' book combines these two pieces of advice as it involves the possibility of a triple elimination with the estimation of the probability of success. Will Horton's advice prove its worth on careful examination?

West	East	Opponent's Cards	Even Split
♠ A Q J 10 9 8	♠ 7 5 4 3	3	2 – 1
♥ A K	♥ J 7	9	5 – 4
♦ 7 3	♦ A K	9	5 – 4
♣ A K 4	♣ 7 6 5 3 2	5	3 – 2

West becomes declarer in a spade slam after an uninterrupted auction, and receives the opening lead of the ♥10. Going to dummy in diamonds to take a winning spade finesse would assure the contract, but that play has a probability of success of just under 50%. The alternative is to play the ♠A, hoping but not expecting to drop the ♠K, then cash all the top side winners before endplaying a defender who started with doubletons in both black suits. The central question is, how likely is it that a player holding the ♠K doubleton was also dealt a doubleton club. Rubens recommends that the latter play should be chosen as the success rate is about 54%.

For most declarers, this may be too close to call. The best approach for many may be to save energy, take the finesse, and move quickly to the next board. I object to that. A margin of 4% is not too low to matter — it equates to taking a winning view about once a session, enough to move you up several places in the standings. Good players should attempt to make the distinction at least part of the time.

The simplest advice one can imagine with regard to going with the odds is Bob's Blind Rule: always play for the most likely splits possible given what you know at the time of decision. How much easier can it get?

As we can see from the diagram above, declarer controls a division of sides 10=4=4=8 (all even), so the defenders hold a sides of 3=9=9=5 (all odd). There has been no opposition bidding and the opening lead is a nondescript ♥10, so no alarm bells are ringing, and there is no reason to assume that the suits aren't splitting normally. Let's look at pairs of hands for North-South composed of the most even splits in each suit, which are also the most probable combinations on the evidence so far.

I	II	III	IV
♠ 2 – 1	♠ 1 – 2	♠ 2 – 1	♠ 2 – 1
♥ 5 – 4	♥ 5 – 4	♥ 6 – 3	♥ 6 – 3
♦ 4 – 5	♦ 4 – 5	♦ 3 – 6	♦ 4 – 5
♣ 2 – 3	♣ 3 – 2	♣ 2 – 3	♣ 1 – 4

Condition III is less likely than Condition I by a factor of 4/9, and Condition IV is less likely by a factor of 1/3. After the play of a small spade by LHO on the lead towards the long spades, Condition I becomes twice as likely as Condition II on the basis of Restricted Choice. Under Conditions I – III, the doubleton spade is in the same hand as the doubleton club. Note that it doesn't matter which defender has the two black-suit doubletons. Therefore, under Bob's Blind Rule, declarer should adopt Rubens' suggestion: play the ♠A and, if the ♠K doesn't drop, play off the side-suit aces and kings (as Mark Horton suggests) then exit with a second spade hoping the defender who wins this trick is out of clubs. Of course, we cannot say the odds are anything as good as 2:1 in favor, but it appears likely that this sequence gives better odds than an initial trump finesse.

Let's look at the suit combinations that lie at the bottom of this simple method. What are the probabilities? By ignoring the implications, if any, of the red cards played to the first two tricks, we may use the initial vacant places as a rough guide to the number of combinations. These are as follows:

Spades	3 – 0	2 – 1	1 – 2	0 – 3
Others	10 – 13	11 – 12	12 – 11	10 – 13
Weights	11	13	13	11

The weights are the relative number of card combinations that accompany the given splits in spades. Next we shall consider the play of low trump to which East follows with a low card, say the ♠2 (we are missing the ♠K, ♠6 and ♠2). These are the possibilities at the time of decision:

Spades	2 – 1	1 – 2	1 – 2	0 – 3
	K6 – 2	6 – K2	K – 62	0 – K62
Weight	26	26	26	22
Plausible Plays	1	1	2	2
Adjusted Weights	26	26	13	11
				Total = 76

To obtain the current odds the combinatorial weights must be adjusted through division by the number of plausible plays available for the play of the ♠2. If South held both the ♠6 and the ♠2, he might have played the ♠6 instead, so the probability that he would play the ♠2 is halved. Now we can calculate the probability of the finesse winning by comparing the number of combinations for which the finesse wins (FW) to the number for the finesse losing (FL) or the drop winning (DW).

FW = 26 + 11 = 37; FL = 26 + 13 = 39; and DW = 13

The finesse has a winning percentage of 49%, the drop, 17%. Thus, if it were just a matter of finesse versus drop, declarer would finesse; however, as the saying goes, two chances are better than one. The probability of success for the subsequent endplay is the product of the probability of the king now being singleton (52/76) multiplied by the probability that the doubleton club sits with the singleton ♠K. The following configuration shows the weights to be attached to the relevant conditions:

I	Spades	1-2	**II**	Spades	1-2
	Clubs	2-3		Clubs	3-2
	♥ & ♦	10-8		♥ & ♦	9-9
	Weight	9		Weight	10

The chance of the doubleton club having been dealt to the hand containing the ♠K doubleton is 10/19, slightly more than 1/2. So we find the probability, PE, that the endplay will be successful after a small spade appears:

$$PE = 13/76 + (52/76) \times (10/19) = 0.53 > 0.49$$

The conclusion is that the endplay is the preferred strategy by about 4%. This is in close agreement with Rubens. The calculation confirms the more general advice of Horton and Bob's Blind Rule. We might conclude that happiness in bridge as well as life depends largely on our ability to recognize possibilities and take advantage as they arise.

Here is an obvious variation on the Rubens hand that involves an elimination play when the defenders hold seven cards in a suit. It is more readily recognized as there will be a loser in clubs should the spade finesse fail. How does Bob's Blind Rule fare?

		Opponent's Cards	**Even Splits**
♠ A Q J 10 8	♠ 9 7 5 4 3	3	2 – 1
♥ K Q 2	♥ A 7 4	7	4 – 3
♦ A 3	♦ 8	10	5 – 5
♣ A 10 8	♣ K 9 5 3	6	3 – 3

One of the interesting features of the above hand is that the numbers of missing cards in a suit are a combination of evens and odds. This profoundly affects the card combination probabilities as shown in the following list of the five most likely distributions.

	I	**II**	**III**	**IV**	**V**
	♠ 2 – 1	♠ 2 – 1	♠ 2 – 1	♠ 2 – 1	♠ 2 – 1
	♥ 3 – 4	♥ 4 – 3	♥ 4 – 3	♥ 3 – 4	♥ 2 – 5
	♦ 5 – 5	♦ 4 – 6	♦ 5 – 5	♦ 4 – 6	♦ 6 – 4
	♣ 3 – 3	♣ 3 – 3	♣ 2 – 4	♣ 4 – 2	♣ 3 – 3
Weights	1	5/6	3/4	5/8	1/2

Given that the spades split 2-1, creating an imbalance in vacant places of 1, the other odd-numbered suit, hearts, can split 3-4, allowing the even-numbered suits to attain an even-split status (Condition I). If the longer card lies to the same side in both hearts and spades, it creates a vacant place imbalance of 2, so one of the even-numbered suits must split unevenly to fill the vacant places, thus reducing the number of possible card combinations (Conditions II thru IV). It is much more probable that the longer minor (diamonds) is split unevenly (conditions II and V). Taken all together, we see that on these five common conditions, the clubs are most likely to split 3-3.

Usually, playing in a matchpoint contest, simple is best if you play to match the field result. If the contract is 4♠, taking the spade finesse is the recommended line, as the field may make twelve tricks on this line of play. If the contract is 6♠, other factors require consideration. What proportion of the pairs will reach this slam? If fewer than half, you will score above average just by making the contract, whereas going down one will be a disaster. The situation is such that assuring the contract is the paramount consideration, just as it is at IMPs. Rather than maximizing the number of tricks won, you may strive to minimize the number of tricks lost.

If we return to the situation where a spade is led towards the trump tenace and South follows with a small card, the probability of the drop succeeding is 17%, so under those fortunate circumstances twelve tricks are assured. When the drop fails to produce the desired result, declarer must revert to the endplay hoping not to lose a trick in clubs. It would be extremely unethical at this point to mutter 'Oh, damn!' or inject here your accustomed equivalent. The red suits are eliminated and a spade ducked to the ♠K. A club is returned and declarer plays for the ♣QJ to be split between North and South. Are the odds good enough to give an overall probability of success greater than 49%?

Let's first suppose that declarer always plays for split club honors. The probability, PE, that the endplay will be successful on those grounds alone:

PE = (13/76) + (52/76) times the probability that the ♣QJ are split

We may estimate the probability of the ♣QJ being split on an *a priori* basis as follows:

Split	Probability	Probability with Split Honors	
3-3	35.5%	21%	
4-2	48.5%	26%	
5-1	14.5%	5%	
6-0	1.5%	0%	Total 52%

In the case of a 6-0 split, the endplay will be successful at least half of the time. So we arrive at an estimated overall rate of success, PE, of 53% before a club is played. This is about the same chance as we had in the original problem from Rubens' book. The next question is whether anything can be gained or lost from the enforced return in the club suit.

Suppose South wins the ♠K and returns the ♣Q. Should declarer play for split honors, or does that lead make it more likely South also holds the ♣J? If so, declarer can win in dummy and finesse against the jack presumed to be in the South hand. Here is the situation with a 3-3 split. The ratios are a measure of the relative conditional probabilities once the queen has been placed on the table.

South	North	Combinations	Plausible Plays	Ratio
Qxx	Jxx	6	3?	2
QJx	xxx	4	2	2

With QJx South must lead an honor, but he could choose either, so there are two plausible plays with four possible combinations. With Qxx South must find North with the jack, so he can lead with equal effect any of three cards with six possible combinations. On the assumption of perfect knowledge, the lead of the queen is equally probable from Qxx or QJx. This illustrates the general principle that maximum uncertainty is achieved by defenders who choose equally between equivalent cards.

Well, that's the way one might program a computer to defend, but reality is different. The computer strategy may represent the distillation of results for defenders of wide-ranging abilities, but in practice you play against one pair at a time. Weaker players, seeing only their own cards, would lead an honor only when obviously necessary, that is, when holding both queen and jack. It is (almost) certain that the lead would be low from Qxx, in which case declarer should let it ride, win the jack with the king, and finesse for the queen on the way back.

A stronger player can see that declarer must hold the ♣10 in order to spurn the spade finesse, so that leading low from Qxx is a losing proposition. Such a player will most often lead the queen from Qxx, hoping declarer will play him for both honors. Against such a player declarer should assume the lead is from Qxx, win the ace and finesse North for the jack. The reason is that the queen will be played from more combinations of Qxx than of QJx. There may be a nasty surprise in store, but you must expect occasionally to pay off to an unusual play. Advice to declarers: play for split honors unless a poor player leads an honor.

The above argument is based on a 3-3 split, the most likely situation even when South has shown out on the third round of hearts (Condition V above), the reason being that the longer diamonds will split unevenly more readily than the shorter clubs. Here is the rare situation where South leads the queen from Qx:

South	North	Combinations	Plausible Plays	Ratio
Qx	Jxxx	4	1	4
QJ	xxxx	1	2	1/2

The probability that a lead of the queen comes from Qx greatly outweighs the probability for QJ, the reason being that a QJ doubleton is a rare occurrence. On any lead, declarer should play for split honors.

Chapter 9

PROBABILITY, INFORMATION, AND THE OPENING LEAD

If ignorance is bliss, why aren't more people happy?
— Thomas Jefferson (1743-1826)

At the start of play, other than from the auction, the most useful clues for declarer are found in the opening lead. In the next few pages we'll show how the link between information and probability applies to the opening lead. The demonstration is given in terms of possible card combinations after a spot card lead against a notrump contract.

Here are the basic conclusions:

1) The less probable an opening lead, the more information it provides, *and*
2) The greater the amount of information in the opening lead, the fewer the number of card combinations that remain to be considered.

These conclusions are simple enough and amount to common sense when one thinks about it, provided always that one has thought about it. Nonetheless, a demonstration is desirable to bolster the statement without extensive computer simulation (the results of which may not convince the skeptical mind), so we must make some simplifying assumptions. The major assumption I'll make is that the lead is a low card from the defender's longest suit. Not always true, of course, but true most of the time I venture to say. For the great majority of cases where it is true, the results follow like clockwork.

Here are the bridge rules that arise from this analysis:

1) If the opening lead is in the shortest suit jointly held by declarer's side, the information content is relatively low, and many distributions of sides remain as possibilities;
2) If the opening lead is in the longest suit jointly held by declarer's side, the information content is relatively high, and relatively few possible distributions of sides remain.
3) The difference between the information conveyed in one suit relative to another increases with the difference in the number of cards held in the suits.

To Assume Nothing is to Assume Something

How can nothing be something? This question has plagued philosophers between breakfast and lunch for centuries. We recall that the concept of zero was slow in gaining general acceptance and that the symbol for zero didn't appear in the Western World until the year 967 (AD, that is). More recently, philosophers have argued whether or not zero probability can exist in an infinite universe: an important point, I gather. They have no such problems after supper when they go out to play bridge and enter the finite and interdependent world of cards. With just fifty-two cards to cope with they can see that more of this means less of that, and vice versa. A zero score on a deal is not only possible, but frequent. Nonetheless, the idea that nothing can be something may appear strange at first glance. Here is the context.

An opening lead is made, play continues, cards are revealed — yet many believe that the *a priori* odds still apply. By staying with the probabilities of the deal, the only action that is guaranteed to be random, they feel they have made no assumptions, yet by ignoring the evidence before them, they are assuming no information has been provided since that would necessarily affect the probabilities. Their assumption is one of total mistrust. To give such arguments some leeway, we might instead say that the assumption is that the information provided is so unreliable as not to be trusted until a defender shows out of a suit, at which time vacant places can be adjusted on the basis of an incontrovertible fact. If the opening lead is excluded from consideration, one prefers to make a decision based on the basis of maximum uncertainty rather than be swayed by meager evidence. Variability swamps the mean. This attitude is akin to the advice that in a storm at sea you should stay with a foundering ship rather than trust a leaky lifeboat. A more fruitful way of thinking of this is that there must be information in the opening lead because the reasonable choices are few. The fact that the lead is not random is what provides the information!

I often have wondered why some people have a predisposition to ignoring the evidence. Then recently I saw on television that some people who have lost limbs still feel pain in their missing members. They can experience itchiness and cramps where there is neither flesh nor bone. Their brains do not accept what is apparent even to their eyesight. Reasoning doesn't help, because logically they already know the facts. Recently, doctors have learned that mercifully the brain can be fooled by mirrors. By showing the patient their other hand or arm in a mirror when a corrective action is applied to the remaining limb, it is possible to fool the brain into thinking the action has been applied to the missing one, and the symptoms are relieved. To translate this into bridge terms, irrational distrust may not go away even if we know logically that we are not being continually deceived. The cure is to look at yourself in the mirror and ask how often you make opening leads with deceit uppermost in your mind. The next question is, when you do this, how often does it pay off? Just as the best spies are those least suspected, the best deceptions are rare and disguised as normal. If you continually treat a lead as though it has been made with intent to deceive, then on the basis of frequency of occurrence it can be said that you have found another way to lose by defying the odds.

When the dummy appears it presents declarer with an incontrovertible fact in the form of the division of sides. This by itself shifts the odds, a fact hardly mentioned in the bridge literature, but it doesn't change the principle characteristic, which is that the missing cards are most likely to be divided according to the most even distributions still possible. The opening lead shifts the vacant places temporarily, but a complete round to which both defenders follow with low cards in the suit maintains a balance of possibilities between the defenders' hands. Nonetheless, some possibilities have been eliminated and fewer remain. That is due to an acquisition of knowledge.

The Lead from Length

Now let's look at a common situation from what to most will be a new perspective. The bidding has gone 1NT-3NT, the opening lead is a low spade, and as declarer we think automatically of 'fourth highest from longest and strongest'. Before drawing inferences, let's look at the cards in dummy and form the distribution of sides to see if, indeed, the spade lead is what we should expect. Suppose from the dummy we can calculate that the defenders hold eight spades, seven hearts, six diamonds, and five clubs, thus a sides of 8=7=6=5. Spades is their most plentiful suit, so it is no surprise that a spade was led.

There will be times when shorter suits are led. You will have formed, consciously or unconsciously, a set of prior probabilities for leads in the various suits. At this point, you might like to estimate based on experience how often a lead would be made in each suit. Express this estimate in terms of a probability, P(suit). I'm going to assume on a tentative basis the following set for the sake of illustration:

$$P(\spadesuit) = 0.50$$
$$P(\heartsuit) = 0.35$$
$$P(\diamondsuit) = 0.10$$
$$P(\clubsuit) = 0.05$$

Maybe you have a better estimate. When a low card is led against the auction 1NT-3NT, I estimate it will be a spade half the time. A club lead would be unusual, occurring just once in twenty occasions. A heart is expected once in three occasions, and a diamond once in ten. The numbers could be the subject of a test using computer simulations, but let's assume they are close enough for now.

Next we consider the most likely distributions in each suit. The weights are the relative number of combinations.

	♠	♥	♦	♣	♣
	♠ 4 – 4	♠ 3 – 5	♠ 3 – 5	♠ 3 – 5	♠ 3 – 5
	♥ 3 – 4	♥ 4 – 3	♥ 3 – 4	♥ 3 – 4	♥ 3 – 4
	♦ 3 – 3	♦ 3 – 3	♦ 4 – 2	♦ 3 – 3	♦ 2 – 4
	♣ 3 – 2	♣ 3 – 2	♣ 3 – 2	♣ 4 – 1	♣ 5 – 0
Weights	100	80	60	40	6

As the number of cards available to the defenders decreases from spades to clubs, the probability of that suit being led decreases (the manner of this being open to question), and the number of combinations for the most likely distribution decreases. Overall there will be fewer combinations available that are attributable to the variations that lie behind these most frequent distributions. The possibilities are reduced as the number of available cards in the suit led is reduced. The fewer the possibilities, the more severe the restrictions on the properties of the hidden hand, and the greater the information conveyed by a lead in that suit.

The last two columns are the two most frequent cases where clubs is the leader's longest suit. Of low probability, the last column will be quickly eliminated when RHO follows suit. So we may conclude that a club lead on the first round establishes the full distribution of the defenders' cards. There was an impressive amount of information in that lead.

Mathematical Models for Opening Leads

Mathematical models are based on assumptions. Simple is best, but if a model doesn't predict accurately enough, we must question first the underlying assumptions. There is a relationship between the choice of an opening lead and the lengths of the suits held, but such a relationship observed from the outside must be of a statistical nature. In this section we study the consequences of three rules that relate suit lengths to the frequency of choices of the opening lead.

Random Rule

From a deck made up of 8 spades, 7 hearts, 6 diamonds and 5 clubs, draw a card, then lead a low card in that suit. The chances of a spade being led are 8 out of 26.

Rule A

a) Lead a spade if it is the longest suit or when it is equal in length to a minor, and half the time when it is of equal length with hearts;

b) lead a heart when it is the longest suit or when it is equal in length to a minor, and half the time when it is of equal length with spades;
c) lead a minor only when it is the longest suit or equal longest with the other minor.

Rule B

A popular idea with some logic behind it is, 'When in doubt, lead hearts'. We shall look at the consequences to information when the following rule is applied:
a) Lead a spade if it is the longest suit, or equal in length to a minor;
b) lead a heart when it is the longest suit or equal longest with another suit;
c) lead a minor only when it is the longest suit or equal longest with the other minor.

Given those rules it is an easy but tedious task to count up the number of combinations that will produce a lead in a given suit. I did so, excluding hands that contained a void, cases of low probability made lower under the circumstance of no opposition bidding. Here are the probabilities that result:

Suit Led	**Random**	**Rule A**	**Rule B**	**My Guess**
Spades	32%	49%	43%	50%
Hearts	27%	37%	43%	35%
Diamonds	23%	11%	11%	10%
Clubs	19%	3%	3%	5%
Entropy	0.595	0.463	0.466	0.475

Entropy is a measure of the degree of uncertainty given that the only information transmitted is the suit denomination. The mathematical expression for entropy is the sum over the four suits of

$$- P(\text{suit}) \times \log [P(\text{suit})]$$

The uncertainty is a maximum when all four suits have an equal probability of being led. A random choice from cards whose composition restricted to 8=7=6=5 is just slightly less than the theoretical maximum. If a major-suit lead is much more likely than a minor-suit lead, uncertainty is reduced, and there is not much difference in that regard between Rules A and B. Where Rule B gains an advantage is that the probabilities of a heart lead and a spade lead are equal and provide the same amount of information, as there is (near) equality in the number of combinations associated with each play. Thus, there is an information-theoretic reason for adopting that strategy as a defender on these most common choices (86%), the same reason that governs Restricted Choice and tells defenders to follow to declarer's suit plays with low cards chosen at random.

What Should Declarers Assume?

It is obvious that the selection of an opening lead is a more complex operation than the simple application of Rule B. It is also obvious that the opening lead is *not* chosen at random. Our experience amounts to a statistical survey of leads made against us and the evidence points to probabilities not far different from Rules A and B. The analysis above may serve to direct our attention to implications of which we were only vaguely aware. If so, our models may have served a purpose, and may lead to further refinements. At best, Rule B should be adopted as a working assumption until further information becomes available. The leading candidates for the distribution of sides are the most likely distributions listed above for each suit.

We can observe the effect without discerning the cause, so why not adopt an *ad hoc* model for the sake of convenience? If we assume nothing, but note the division of sides, the amount of information we can gather from an opening lead is near minimum. Yet our experience tells us that Rule B better reflects our observations overall. One card will not tell us everything, but it tells us something, and that is the best attitude with which to start. Hope to gather more information before a critical decision is required.

Another Division of Sides

The 8-7-6-5 division is the most common, so it is easier to gauge than the rarer types. However, the same principles apply throughout, so we can calculate the information available under the different divisions and there should be no surprises. A computer program is desirable for going through the process, but here is one more set of results got by hand calculation, this time for a sides of 8=6=6=6. Declarer's division is 5=7=7=7.

Suit Led	**Random**	**Rule A**	**Rule B**
Spades	31%	56%	51%
Hearts	23%	21%	25%
Diamonds	23%	12%	12%
Clubs	23%	12%	12%
Entropy	0.598	0.505	0.521

Although the same number of spades is held as in the previous example, there is less of an inclination to lead a major suit, and more inclination to lead a minor suit, so the overall effect is to increase uncertainty, as can be seen by a comparison of the entropies involved.

A Warning Sign Lead

Sometimes after 1NT-3NT you get an unusual lead that is not in your short major. This is the time to ponder the circumstances. Here are the most likely splits with 9=7=6=4.

	♠	♥	♦	♣
	♠ 5 – 4	♠ 3 – 6	♠ 3 – 6	♠ 3 – 6
	♥ 3 – 4	♥ 4 – 3	♥ 3 – 4	♥ 3 – 4
	♦ 3 – 3	♦ 4 – 2	♦ 4 – 2	♦ 3 – 3
	♣ 2 – 2	♣ 2 – 2	♣ 3 – 1	♣ 4 – 0
Weights	100	50	33	11

If you don't get a spade lead, it's good news, bad news, as you are going to get a very bad split in that suit. If a diamond is led, expect uneven splits in both minors. If a club is led, expect a violent signal from LHO. If you don't get one, think again.

The Message

Think of the opening lead as a message that relates to the operation through which it has been selected. If the lead is to be considered a random choice from a group of cards jointly held by the defenders, it has the status of the first low card dealt to LHO. Knowing the first card tells us nothing about the relationships between the cards that will be dealt subsequently, and very little that a declarer doesn't know already. On the other hand, there are many practical advantages to choosing a lead from the longest suit held, so it is reasonable to assume the message most often relates, in some mysterious way, to the relative lengths of the suits held by the opening leader. Start there.

Chapter 10

THE MOST LIKELY DISTRIBUTION

In making inferences on the basis of partial information we must use the probability distribution which has maximum uncertainty subject to whatever is known.
— after E. T. Jaynes (1957)

The central idea behind Jaynes' Principle is that you accept whatever information is available then interpret it in the widest sense consistent with the evidence at hand. The consequences to the play of a bridge hand are easily stated. Whether we think of maximum uncertainty or ratios of card combinations, it comes to the same thing: assume the most even splits possible under the current set of circumstances. Of course, calls and plays provide additional amounts of partial information, so the most likely distribution of the sides may change. You start with a candidate distribution consisting of even splits, but you must not be over-reliant on just one distribution. That having been said, you must start somewhere, and the most likely distribution of sides is the preferred choice. This focuses the mind in the correct manner, at least as far as probability is concerned.

Let's see how this works in the simple situation where the bidding has gone 1NT-3NT and a spade is led. This is the classical 'blind lead' situation where the fourth highest lead in a major suit is the common choice. We discussed in the previous chapter how an opening lead places restrictions on distributions: the more informative the lead, the more severe the restrictions and the fewer the distributions that remain to be considered. Now we'll consider how the most likely distribution of sides relates to declarer's inferences.

The 8=7=6=5 Division of Sides

The opening lead is chosen, the dummy is tabled, and declarer attempts to form a plan based on the information that has been made available to him. With regard to the division of sides, declarer knows immediately how many cards the opponents hold in each suit. In the absence of bidding, the opening lead may be the best indicator of how the cards in each suit are distributed between the defenders. The distribution of the cards that encompasses the greatest number of card combinations is the most probable single distribution. We call this distribution the *maximum likelihood estimate*. As an example, let's consider the sides of

8=7=6=5. The most even splits are: spades 4-4, hearts 4-3 or 3-4, diamonds 3-3 and clubs 3-2 or 2-3. These even splits have to be assembled into combinations that make up thirteen cards to a side, so some restrictions apply. Here are some of the more likely splits:

I	II	III	IV	V
♠ 4 – 4	♠ 4 – 4	♠ 5 – 3	♠ 4 – 4	♠ 5 – 3
♥ 3 – 4	♥ 4 – 3	♥ 3 – 4	♥ 3 – 4	♥ 4 – 3
♦ 3 – 3	♦ 3 – 3	♦ 3 – 3	♦ 4 – 2	♦ 2 – 4
♣ 3 – 2	♣ 2 – 3	♣ 2 – 3	♣ 2 – 3	♣ 2 – 3
100	100*	80	75	60

The number below each distribution is the relative number (on a scale of 100) of the card combinations encompassed by that distribution. Condition I is the most likely single distribution after a spade is led from length. Although Condition II (marked with an asterisk) encompasses the same number of card combinations, the opening leader's hearts are of equal length to his spades, so there is a good chance that a heart might have been led instead of a spade. In the absence of clues from the bidding, it is reasonable to assume that a heart would be chosen for half of the combinations encompassed, so the weight of II after a spade is led should be reduced by half to 50, making it less likely than Conditions III – V.

The splits in the minors are of interest as well. The five conditions shown encompass the most even split in diamonds, 3-3, but also the 2-4 and 4-2 splits. The fact that a spade was led from length alters the odds in favor of the even split. The club splits divide between 3-2 and 2-3, with the latter more favored on the limited selection shown.

The Spot Card Effect

For the purposes of our illustration, the assumption is that a spade will be led whenever spades is the longest suit or spades are equal in length to a minor, and half the time when spades are of equal length with hearts. When you read in an analysis of a deal that 'a low spade was led', you have been poorly informed. Which spade is selected makes a difference to the odds, because sometimes declarer can make a pretty good guess as to whether or not the lead was from a four-card or a five-card suit. Let's assume the lead is the ♠2, which looks very much like a lead from a four-card suit. Given this is a long-suit lead, the possible distributions are reduced in number to ten; four of them contain a four-card heart suit, and for these we reduce the number of combinations by half. We can calculate the number of combinations for the relevant splits in hearts, diamonds and clubs and express them as percentages of the total available for a given suit, as follows:

Hearts	4 – 3	3 – 4	2 – 5	1 – 6	Heart Left	43%
	28%	52%	18%	2%	Heart Right	57%
Diamonds	4 – 2	3 – 3	2 – 4	1 – 5	Diamond Left	52%
	36%	44%	19%	2%	Diamond Right	48%
Clubs	4 – 1	3 – 2	2 – 3	1 – 4	Club Left	56%
	20%	45%	30%	5%	Club Right	44%

The column on the right gives the overall probability that a given card in that suit will be on declarer's left or right.

It is quite according to expectations that the hearts are more likely to split 3-4, as some combinations of 4-3 are partially eliminated by our assumption of a heart lead 50% of the time when the majors are of equal length. However, it is also expected that the diamonds split evenly at 3-3, but there is an unexpected bias to the left in favor of the 4-2 split. Clubs also exhibit a left-hand bias with the 3-2 split half again as likely as a 2-3 split. The conclusion is that, although the spade lead appears at first glance to imply an extra vacant place on the right, it cannot be concluded that a missing honor in a minor is more likely to be on the right. In fact, the contrary tendency applies.

How do we make sense of this conclusion? Easily, if we consider the maximum likelihood estimate expressed under Condition I shown above. Putting together the most frequent splits (modes) in each suit as shown above, we obtain ♠ 4-4, ♥ 3-4, ♦ 3-3, and ♣ 3-2, which constitutes Condition I. It is good practice to consider the maximum likelihood distribution as a starting point as it makes you conscious of the modes of the distributions of the suits taken individually. Remember this: the suit combinations can vary independently only to the degree that the total number of cards in the suits must come to thirteen in the end, and the degree of variation depends on the number of cards held in the suit.

Of course, bridge players are keenly interested in the probability that a given card in a suit, say a queen, will be dealt to the opening leader on the left or to his partner on the right. In our example, with the ♠2 led, the ♥Q is more likely to be on the right roughly in the proportion of a 3-4 split. The ♦Q is actually just slightly more likely to be on the left. The ♣Q is a fair bit more likely to be with the opening leader, roughly in the ratio of a 3-2 split. Thus Condition I provides a first approximation of the suit-dependent odds of finding a queen on the left or right.

The Lead from a Sparse Suit

The restrictions are more severe after a diamond is led, under the assumption that it must be the longest suit or in a tie with clubs.

Spades	4 – 4	3 – 5	2 – 6		Spade Left	37%
	20%	63%	17%		Spade Right	62%
Hearts	4 – 3	3 – 4	2 – 5	1 – 6	Heart Left	40%
	12%	61%	22%	4%	Heart Right	60%
Clubs	4 – 1	3 – 2	2 – 3	1 – 4	Club Left	51%
	7%	49%	29%	15%	Club Right	49%

The maximum likelihood estimate of the distribution given the diamond is a long-suit lead is ♠ 3-5, ♥ 3-4, ♦ 4-2, ♣ 3-2 (weight =60), which is also the combination of the most likely splits taken individually in each suit. The information from the diamond lead indicates that a major-suit honor card is strongly favored to be on the right, that is, not in the hand of the opening leader. The most likely splits give a fairly good approximation of the probability of catching a major-suit queen on the right (3:2).

The uncertainty is again a maximum for the club suit, as it is nearly 50-50 that the ♣Q would be on the right. The average number of clubs on the right or left is 2.5, an impossible number of cards to be dealt, which reflects the uncertainty. It goes against intuition, perhaps, that a 3-2 club split is more likely than a 2-3 split, that is, the longer club is more frequently with the opening leader. This apparent inconsistency can be resolved by considering the maximum likelihood estimate, which is a result of taking into account all four suits and how they interact.

The Effects of Uncertain Length

The effect of uncertainty is to increase the number of possible combinations that must be taken into consideration. If the spot card lead is unreadable, we must allow more suit combinations to enter the mix. Suppose we can only infer that the lead is from four, five or six spades, rather than exactly four. Adding these possibilities we find the following frequency of splits:

Hearts	5 – 2	4 – 3	3 – 4	2 – 5	1 – 6	Heart Left	43%
	2%	28%	44%	2%	2%	Heart Right	57%
Diamonds	5 – 1	4 – 2	3 – 3	2 – 4	1 – 5	Diamond Left	48%
	4%	29%	44%	22%	1%	Diamond Right	52%
Clubs	4 – 1	3 – 2	2 – 3	1 – 4		Club Left	49%
	12%	35%	38%	14%		Club Right	51%

The modes for the splits are: ♥ 3-4, ♦ 3-3, and ♣ 2-3 as before. The 3-4 split in hearts is favored greatly, and overall any particular heart, the ♥Q say, is more likely to sit on the right with a probability approximating that in a 3-4 split. The probability of a particular diamond (say the ♦Q) or a particular club (♣Q) sitting on the right is close to 50%, but with a slight bias towards the right. The margin is roughly that provided by a vacant place split of 12 on the left and 13 on the right, in keeping with the *a priori* odds adjusted by the exposure of one card on the left.

Why has Condition I lost the status of the modal distribution? The reason is obvious: the inclusion of the possibilities of five-card and six-card spade suits, but not of three-card suits, results in the average length of West's spades being 4.6, even though the leader's spades will be only four cards in length nearly half of the time (48%). The pressure of 'virtual' vacant places on the right is overcome by a flip from a 3-2 club split to a 2-3 club split, as we find in Condition III. The heart and diamond splits are the same for both conditions, so the flip in the shortest odd-numbered suit, clubs, by itself accommodates the additional spade length. Common sense tells us that we shouldn't put all our money on Condition I, which requires a lead from a four-card suit, but keep in mind Condition III just in case the lead was from a five-card suit. (Frivolous overcalls help.)

Opening Leads and Honors

Many books have been written on how to choose an opening lead. The defender must take into account the opposition bidding and the location of his high cards. It is considered dangerous to choose a suit with gaps in the honors, because even a lead such as the queen from QJ98 can come to grief. Declarer knows the division of sides and which high cards are missing, but he doesn't know on which side of the table they sit unless the opening lead is from an honor sequence. Generally, in the absence of bidding, the honors are divided evenly between the defenders, and consideration of the most likely distribution of sides remains a valid approach. If the lead is an honor card, that occurrence constitutes additional information that isn't dictated by length alone, but it is more likely to have been made from a sequence in a longer suit rather than a shorter one. The most difficult situation to read is when a long-suit lead is avoided because of gaps in the honors held.

Symmetry and the *A Priori* Argument

Some bridge analysts are reluctant to use the evidence of the opening lead as justification for an adjustment of the odds on the location of a particular card of interest, say the ♥Q. A characteristic of the *a priori* conditions is symmetry, which is destroyed on the opening lead, but this feature remains central in the minds of some. To see how the opening lead has affected the odds, let's look at the most likely candidates involving the 4-3 and 3-4 splits in hearts, which initially are equally probable.

I	II	III	IV	V	VI
♠ 4 – 4	♠ 4 – 4	♠ 5 – 3	♠ 3 – 5	♠ 5 – 3	♠ 3 – 5
♥ 3 – 4	♥ 4 – 3	♥ 3 – 4	♥ 4 – 3	♥ 4 – 3	♥ 3 – 4
♦ 3 – 3	♦ 3 – 3	♦ 3 – 3	♦ 3 – 3	♦ 2 – 4	♦ 4 – 2
♣ 3 – 2	♣ 2 – 3	♣ 2 – 3	♣ 3 – 2	♣ 2 – 3	♣ 3 – 2
100	100	80	80	60	60

The distributions form symmetric pairs with equal probabilities before a lead is made, but the opening lead destroys the symmetry in probabilities. Under Condition II a heart is as likely to have been led as a spade, hence the number of combinations represented must be reduced by half. Under Conditions IV and VI a spade would not be led. So for these six conditions a spade lead adjusts the probabilities as follows:

4-3 split	50 + 60 = 110	38%	Heart Left	48%
3-4 split	100 + 80 = 180	62%	Heart Right	52%

Of course, if the spade lead were to come from the right instead of the left, the odds would be reversed with the 4-3 split the more probable. As the opening lead can be made from either side with equal probability, it is correct to say that the 4-3 and 3-4 splits are equally probable before a lead is made. But such is not the case after a spade is led from the left. Hypothetically, the lead could have been a spade from the right, not the left, but there is no evidence to support that assumption on this particular deal.

We note that on average the probability of the location of the ♥Q corresponds to 12 vacant places on the left and 13 on the right. This average involves all remaining possible heart splits, but because the number of hearts is an odd number, there is no single split that closely reflects those odds, as there is with a 3-3 split in diamonds. The hearts can be split 4-3 or 3-4, but not both at the same time. The odds on the location of the ♥Q with the hearts taken in isolation will be either 4:3 or 3:4. Rather than look at averages over several possible splits, we should consider the mode of the splits, and initially a 3-4 split clearly represents the greatest frequency of occurrence. As play progresses the general trend is to retain the more even splits. If a split in another suit gets established, that information will affect the odds of the heart splits. It may turn out eventually that a situation is reached where the 4-3 and 3-4 splits again become equally likely, but we can't assume that will happen. An example: with ♠ 5-3 and ♦ 2-4, the distribution ♠ 5-3 ♥ 4-3 ♦ 2-4 ♣ 2-3 has the same weight as the non-symmetric ♠ 5-3 ♥ 3-4 ♦ 2-4 ♣ 3-2. This situation will be reached rarely, as it is normal for declarer to play on clubs before diamonds.

Chapter 11

VIRTUAL REALITY AND OPENING LEADS

A property in a 100-year flood plain has a 96% chance of being flooded in the next 100 years. The fact that several years go by without a flood does not change that probability.
— Earl Blumenauer (b. 1948)

(This explains why flood insurance never goes down during a drought.)

We have seen that while a ‘normal’ lead conveys some intelligence to declarer (and the defenders, of course), an unusual lead is much more informative. In this chapter we look at the connection between information and probability in the case of an unusual opening lead for the not uncommon 7=7=6=6 division of sides. There has been confusion on the effect of the opening lead on probabilities, which I’m going to try to set straight through an example that is of interest for its own sake. The probabilities on opening lead are transitory, but it is important for declarer to start play in the right frame of mind and to proceed logically from that point. At the end we’ll look at ‘virtual vacant places’, an interesting concept that provides insight and may have a wider range of applicability.

The Improbable Can Happen

The improbable grabs our attention. ‘Man bites dog’ is newsworthy, but not vice versa. You shouldn’t fear the improbable, but you shouldn’t ignore it, either. It is a matter of degree. ‘Don’t undress in front of your dog’, is sound advice as such action poses an unnecessary risk with little to gain, but it is not a rule over which one becomes obsessive. I suppose it depends to some extent on the height of the dog. On the other hand, if your dog growls in the other room, you should pay attention: there must be a reason, even if the dog can’t reason. But I digress. At the bridge table, a predictable opening lead doesn’t greatly affect normal expectations, whereas an unusual opening lead does. It is newsworthy, and there is more than the ordinary amount of information to be gleaned from it. You shouldn’t be afraid of taking a risk if the circumstances merit it, but you should evaluate the unusual situation realistically. This is where probability comes in.

The 7-7-6-6 Division of Sides

It is said that Dame Fortune favors the bold, which, in my opinion, is fair and just. Why should Dame Fortune, or any other dame for that matter, favor a do-nothing who is afraid to get involved? The suggestion that the meek will eventually inherit the earth sounds to me like a shameless appeal to the latent greed that lies dormant within the indolent breast. A one-sixth portion is a more reasonable expectation, and we are not talking sea-view property. In the here and now the meek get rewarded far beyond anything commensurate with the risks they are willing to take. At the local club, and it is bridge we're mainly thinking about, you may score well enough by sitting quietly and taking profit from the opponents' errors, but it is sheer greed to expect to win without ever stepping outside your comfort zone. In addition there is a small domain where the rewards for a lack of initiative are immediately forthcoming due to a low expectation of total tricks.

The center of the domain to which I refer is defined by a 7-7-6-6 division of sides where the number of total trumps is a measly 14. This situation constitutes 10.5% of all deals, so is encountered about three times per session. This is approximately what the meek deserve, besides which, it is a reasonable proportion for keeping a lid on excesses. Within its bounds, those who don't open a perfectly good 12 HCP because they don't like the look of their cards get rewarded for their timidity, and those who don't balance get praise from their partners rather than the customary scorn. Here players who bid to normal contracts that run aground in the shoals of the distribution are forced to listen respectfully as the meek excitedly explain how unfounded suspicions led them to underbid profitably.

Bold ones mustn't resent yielding up this small portion of infrequent victories. The best we can do as declarers is modify our procedures and take a different tack, one designed to take into account that we are in a strange land where aggressive play may not be superior to a wait-and-see approach that risks less. When the dummy comes down and we note the 7-7-6-6 division of sides, it is time to reconsider our strategy with a fresh mind. What does the opening lead tell us? In what follows we'll consider the blind minor-suit lead against an uninformative 1NT -3NT auction when the defense holds 7=7=6=6.

Rule A and Rule B

In order to calculate probabilities after a blind lead is made, we need to relate the opening lead strategy to the lengths of the suits held by the opening leader. The rule adopted is generally statistical in nature, and may be modified by knowledge based on the proclivities of the player involved. This facet of Bayesian probability is beneficial because it is sensible. We'll consider two rules formulated as follows:

Rule A The longest suit is led. Suits of equal length have an equal chance of being led.

Rule B A major suit is led whenever it is equal in length to or longer than either minor suit. Suits of equal length have an equal chance of being led.

These rules represent extremes. Rule A amounts to 'Fourth highest from the longest and strongest' regardless of suit rank. Rule B gives full preference to the major suits. Some habitually play that way. There are intermediate variations.

The frequency of an opening lead in each of the four suits is given below (on the assumption neither hand features a void). 'Random' refers to a random choice from a deck of 7-7-6-6 cards.

	Random	**Rule A**	**Rule B**
Spades	27%	33%	39%
Hearts	27%	33%	39%
Diamonds	23%	17%	11%
Clubs	23%	17%	11%

The probabilities of Rule B may be closest to our everyday experiences, but this can be tested using statistics from actual bridge deals. The question here is, 'How unusual is a minor-suit lead?' It depends on which rule is the more likely to be employed by a particular player. Rule B players are strongly affected by the bidding and will not lead a minor suit unless there is no reasonable alternative. They are sometimes the victims of those who open 1NT with a five-card major. I do that in third position, and am surprised at how often the lead is in my long major. Rule A players operate on a hand-by-hand basis, putting less trust in the bidding and more on the quality of their suits. They tend to be more passive than Rule B players. A major-suit lead is still normal, so it's a matter of degree.

A Low Diamond is Led

The lead is likely to be from either a four-card or a five-card suit. It is best to start with a look at some of the more probable distributions of sides and their initial weights.

	I	**II**	**III**	**IV**	**V**	**VI**
♠	3 – 4	4 – 3	4 – 3	3 – 4	3 – 4	3 – 4
♥	3 – 4	3 – 4	2 – 5	2 – 5	3 – 4	2 – 5
♦	4 – 2	4 – 2	4 – 2	4 – 2	5 – 1	5 – 1
♣	3 – 3	2 – 4	3 – 3	4 – 2	2 – 4	3 – 3
	75	56	45	34	22	18

On the deal alone, the first four conditions for which the diamonds split 4-2 are more likely than the conditions for which the diamonds split 5-1. In addition there are companions to Conditions II-IV for which the hearts are longer than the spades. Under Rule B, Conditions II and III don't get counted as possibilities, since a major suit would have been led instead of a diamond. Under Rule A they are included but their weights are reduced by half. The weights for the 5-1 diamond split are not affected by the difference in the rules, so their relative contributions increase under Rule B more than under Rule A. Condition I represents the most likely distribution regardless of which rule is applied.

The most likely distribution on an *a priori* basis has a weight of 100. It is missing because it consists of the following splits: ♠ 4-3, ♥ 3-4, ♦ 3-3 and ♣ 3-3, and a spade would have been led. A companion shape has four hearts instead of four spades.

In order to calculate probabilities we merely amass the distributions that apply under the rules, adjust the weights accordingly, and add up the total weights under the various conditions of interest. Once we have defined the process, it is easy enough to have a computer program do the work (and do it better) for all possible division of sides and all possible reduction factors, but for the present case the calculations were done by hand and distributions with voids were excluded.

Under Rule A we find:

Diamonds	4 – 2	5 – 1				Average length 4.38
	62%	38%				

Spades	5 – 2	4 – 3	3 – 4	2 – 5	1 – 6	Spade Left 42%
(Hearts)	<1%	25%	48%	21%	4%	Spade Right 58%

Clubs	5 – 1	4 – 2	3 – 3	2 – 4	1 – 5	Club Left 45%
	0%	16%	46%	30%	8%	Club Right 55%

Under Rule A the modal distribution is ♠ 3-4 ♥ 3-4 ♥ 4-2 ♣ 3-3, which corresponds to the maximum likelihood estimate (Condition I). The distributions of the major suits are centered about a 3-4 split. It is perhaps the club suit that is of most interest to a declarer as usually he will aim to develop tricks in that suit. The probability of any particular club, the ♣Q say, being on the right is much greater than that of its being on the left, but the 3-3 split far outweighs the 2-4 split. There are no surprises here.

For Rule B, we find:

Diamonds	4 – 2	5 – 1	Average length 4.55
	45%	55%	

Spades	5 – 2	4 – 3	3 – 4	2 – 5	1 – 6	Spade Left 40%
(Hearts)	0%	12%	60%	24%	4%	Spade Right60%
Clubs	5 – 1	4 – 2	3 – 3	2 – 4	1 – 5	Club Left 48%
	0%	21%	51%	21%	7%	Club Right 52%

The 5-1 split in diamonds is the most likely and the average number of diamonds lies closer to five than to four. There is no correspondence between the modes of all four suits to any one distribution of sides. There is a strong tendency for the majors to be split 3-4 and the clubs to be split 3-3, which tends to place the diamonds at 4-2. There is an inconsistency due to the great reduction in the number of leads from a four-card diamond suit. If we look at the weights of the distributions, the most likely distribution stands out like a giant among the pygmies, but the short people outweigh it on accumulation.

Virtual Vacant Places

A fistful of numbers may not come out and hit you between the eyes. How can we discern some useful order in the above probabilities? It would be convenient if we could translate the numbers in terms of vacant places, but life isn't always so convenient. Let's give it a try nonetheless. The concept of vacant places relating to probabilities is based on the following argument. There are six diamonds held by the defenders. Suppose these are split 5-1, leaving 8 vacant places on the left and 12 on the right. The difference in vacant places is 4. If the remaining spades, hearts and clubs are shuffled and dealt, the chance of any card ending up on the right is 12 out of 20, or 60%, regardless of the rank of that card. If the diamonds are split 4-2, the probability of a card being dealt to the right becomes 55%. The difference in vacant places is 2.

In practice, the diamond lead is either from a four-card suit or a five-card suit and there are just two vacant place differentials possible, but on average the number of cards in the leader's diamond suit could lie anywhere between four and five. This idea gives rise to the concept of a 'virtual vacant place differential' that represents an average that can't exist in reality. Let's see how it works with six cards in a suit and twenty other cards divided between the two defenders.

Virtual Split	**Vacant Places**	**Probability**	**Vacant Place Differential**
3 – 3	10 – 10	50.0 – 50.0	0
3.5 – 2.5	9.5 – 10.5	47.5 – 52.5	1
4 – 2	9 – 11	45.0 – 55.0	2
4.5 – 1.5	8.5 – 11.5	42.5 – 57.5	3
5 – 1	8 – 12	40.0 – 60.0	4

The interpretation of the probabilities I calculated under Rules A and B in terms of virtual vacant places fits the results rather well. Differentials of 1, 2, 3 and 4 are represented. An interesting facet is that clubs have lesser differentials than the majors, so the indication is that the number of vacant places that need to be filled varies with the available length of the suits. In other words, the probability of a club being on the right is different from the probability of a heart or a spade being on the left, which is contrary to the classical approach to vacant places with regard to probabilities, where each card is treated as making an independent contribution.

The most remarkable result is under Rule B where the virtual split in diamonds with regard to clubs is 3.5 – 2.5, remarkable due to the fact that the diamond suit must be at least four cards in length. But it makes sense. If the diamonds are 4-2, there are 2 vacant places to be filled on the right. The 3-4 splits in the majors fill those vacant places quite nicely, leaving the clubs to maintain a balance at 3-3. This is the characteristic of the most likely distribution, Condition I given above. If the diamonds are split 5-1, the majority situation under Rule B, then there are 4 vacant places to be filled, most readily accomplished by 3-4 splits in the majors and a 2-4 split in clubs. That is the characteristic of Condition IV, which is the most likely distribution when diamonds are known to split 5-1. On average, then, the clubs tend to fill 1 vacant place on the right.

Under Rule A, a large majority of distributions feature a 4-2 diamond split. The major-suit distributions correspond to a virtual diamond split of 4.5 – 1.5, a difference of 3 virtual vacant places, the additional vacant place arising from the possible 5-1 split. Removing six diamonds from the defenders' hands with a 4-2 split is not the same as noting that the opening lead was from a four-card diamond suit. Why? Simply because the six cards removed by the opening lead are akin to Shylock's pound of flesh: there is spillover involved. The connecting tissues between suits means the combinations with longer spades, hearts, and clubs are no longer possible. Under Rule B, many weighty distributions featuring a four-card major are removed entirely from consideration because the preferred lead is a major rather than a diamond from an equal-length suit. This accounts for the differences in probabilities under the application of different rules for the opening leads.

Conclusion

It appears that in our pursuit of mathematical beauty we have wandered far from our starting point, which was how to take advantage of the unusual minor-suit lead when the division of sides is 7=7=6=6. Not so. As long as you keep to a reasonable path, you won't get lost in the undergrowth. Good mathematics will support reasonable decisions. The advice is always the same: be aware of the circumstances which have been thrust upon you. Focus on the most likely distribution of sides. This direct approach puts the emphasis on counting out the hand with all suits involved. Each suit should be treated not individually, but in conjunction with the other suits. When the lead is from a suit of indeterminate length more than one distribution of sides needs to be kept in mind, but the most even splits are the most likely and these are closely related. The first goal is to safely resolve uncertainty, so ducking a diamond lead will often combine safety with resolution of the diamond split. Losing a tempo is not likely to be costly.

Be valiant, but not too venturous
— John Lyle (1554-1606)

Virtual Vacant Places with 8=6=6=6

The concept of virtual vacant places should work well when the opening lead is very often from the most plentiful suit where the restrictions on the distributions are not severe. The most obvious candidate is the 8=6=6=6 division of sides, for which a low spade lead is not at all surprising. The total trumps equal 15, so we find ourselves in the borderlands of the Meek. Here are some of the more likely distributions of sides:

I	II	III	IV	V	VI
♠ 4 – 4	♠ 5 – 3	♠ 4 – 4	♠ 5 – 3	♠ 6 – 2	♠ 5 – 3
♥ 3 – 3	♥ 2 – 4	♥ 4 – 2	♥ 4 – 2	♥ 2 – 4	♥ 1 – 5
♦ 3 – 3	♦ 3 – 3	♦ 3 – 3	♦ 2 – 4	♦ 2 – 4	♦ 4 – 2
♣ 3 – 3	♣ 3 – 3	♣ 2 – 4	♣ 2 – 4	♣ 3 – 3	♣ 3 – 3
100	60	56	34	22	18

Condition I is unique, whereas Condition II has three variations, any suit being capable of splitting 2-4. This tells us that a spade lead will often be from a five-card suit. Condition III has six variations, two of which contain a four-card heart suit, and so are subject to a reduction by half on the grounds that a heart could have been led as well as a spade. A lead from a six-card suit will not be uncommon as Condition V has three variations.

Here are the overall results under Rule A, which doesn't distinguish between the ranks of the suits. They support the application of virtual vacant place approximations.

Spades	4 – 4		5 – 3	6 – 2		Average Length	4.83
	34%		49%	17%			

Others	5 – 1	4 – 2	3 – 3	2 – 4	1 – 5	Club Left	45%
	<1%	18%	43%	30%	9%	Club Right	55%

The most likely distribution of sides is Condition I, but because it is unique, it gets outweighed by the variations on Condition II. The modal distribution features 3-3 splits in three suits, but this is not matched by a 4-4 split in spades, which is impossible. In practice then, declarer anticipates a 3-3 split in any one of the sparse suits, but is aware that spades are probably not split 4-4. If spades are 5-3, look for 2-4 in one of the other suits.

With regard to virtual vacant places, it so happens that the probabilities of a non-spade card being on the left or right is given by the virtual vacant places resulting from the average split in spades, which is 4.83 – 3.17. The virtual vacant places are 8.17 and 9.83, so the probability of a card being on the right is approximated by 9.83 divided by 18 (54.6%).

If we apply Rule B, which distinguishes between hearts and the minors with regard to the choice of opening lead, differences arise, because the distributions with four of a minor are given full weight. A minor on the right has a probability of 53%, whereas a heart on the right has a probability of 55%. The average spade length is 4.72 (54%).

Chapter 12

TOTAL TRUMPS WITH A 4-4-4-1 SHAPE

Hope is the confusion of the desire for a thing with its probability
— Arthur Schopenhauer (1788-1866)

We turn now to a neglected area of study: the application of probability to bidding. In *To Bid or Not To Bid*, his book on the Law of Total Tricks, Larry Cohen often estimates the number of total trumps on the assumption that partner holds a singleton in the opponents' suit and a 4-4-4-1 shape. On the surface this is a rather a strange assumption, because a 5-4-3-1 shape is more likely on an *a priori* basis. You might surmise vaguely that 4-4-4-1 is reasonable as the average of 3, 4, and 5 is 4, but that is not a fruitful way of thinking. In this chapter, we'll look at the exact consequences of Cohen's assumption with regard to probability considerations to be taken into account during the auction.

Here are six consequences of assuming partner holds a 4-4-4-1 shape:

1) Your side's best fit is in the longest suit in your own hand;
2) the total number of trumps is 16 + the difference in length between your longest suit and the opponents' trump suit;
3) the estimate of total trumps thus obtained is optimistic on average;
4) given that partner has a singleton in the opponents' trump suit, the resultant distribution of sides is the single most likely one on a random deal;
5) the 4-4-4-1 shape represents a condition of maximum uncertainty with regard to the number of cards held by partner in the three potential trump suits.
6) The assumption conforms to Jaynes' Principle, which states that in a condition of partial knowledge one should assume the distribution that is most probable, that is, the one associated with the greatest number of card combinations on the basis of a random deal and whatever else is otherwise known.

Let's see how these consequences apply to common competitive bidding situations in which the Law of Total Tricks plays a central role.

The Total Trump Calculation

When the dummy appears, declarer can count the total trumps by subtracting the number of cards in the longest combined suit from the number of cards in the shortest and adding the result to 13. The total number of trumps is an unambiguous characteristic of the division of sides.

Total trumps = 13 + (longest combined suit – shortest combined suit)

Of course, the defenders' hands produce that same number of total trumps albeit generally with different independent hand distributions. Their division of sides may be different from that of the declarer. For example, if declarer has an 8-7-6-5 division, so will the defending side, but in the order 5-6-7-8. The total number of trumps is 16. If declarer has a 9-6-6-5 division, the defenders will have a 4-7-7-8 division, and the total number of trumps is 17, no matter which division of sides we use to calculate it. A difference in the division of sides is a characteristic of deals that produce an odd number of total trumps.

The usual problem faced by a player is to estimate the total number of trumps in the middle of an auction knowing only for sure his own hand shape. Naturally, he would attempt to use the estimate that is most probable given what is assumed at the time from the auction. Suppose the opponents have preempted and partner has doubled for takeout. The calculation of total trumps is easiest for a 4-4-4-1 shape opposite. That shape contributes a difference of 3 between the long suit and the short suit regardless of which of three suits provides the best fit. This gets us to a minimum of 16 total trumps. To obtain an estimate of the total trumps with both hands taken into account, the player needs to add to 16 the difference between his longest suit and the presumed length of the opponents' trump suit. Thus, with a 5-3-3-2 shape the number of total trumps will be either 18 or 19 depending on whether the player holds three or two cards in the opponents' trump suit.

The Most Probable Conditions

To illustrate how to estimate relative probabilities, let's consider the following simple example from Cohen's book where a player holds this hand:

♠A 4 3 ♥Q J 10 5 4 ♦9 6 3 ♣8 2

a 3=5=3=2 shape. LHO opens the bidding with a preemptive 5♣. Partner doubles and RHO passes. Cohen assumes LHO holds eight clubs for his bid and partner holds a singleton in that suit for his double. What is the number of total trumps we should use as a basis for the decision of whether or not to pass the double for penalty? Cohen assumes partner holds a 4=4=4=1 shape, so the division of sides should be 7=9=7=3. The total trumps add up to 19 (13 + [9-3]). The division of sides for the opponents is 6=4=6=10. Cohen concludes that one should pass the double with such a low number of total trumps available. But this decision depends on the assumption of that 4-4-4-1 hand opposite.

Let's look at the relative probabilities of a 4-4-4-1 or a 5-4-3-1 shape opposite. Here are six possible distributions.

	I	II	III	IV	V	VI
	♠ 3 – 4	♠ 3 – 4	♠ 3 – 5	♠ 3 – 5	♠ 3 – 3	♠ 3 – 4
	♥ 5 – 4	♥ 5 – 3	♥ 5 – 3	♥ 5 – 4	♥ 5 – 4	♥ 5 – 5
	♦ 3 – 4	♦ 3 – 5	♦ 3 – 4	♦ 3 – 3	♦ 3 – 5	♦ 3 – 3
	♣ 2 – 1	♣ 2 – 1	♣ 2 – 1	♣ 2 – 1	♣ 2 – 1	♣ 2 – 1
Sides	7=9=7=3	7=8=8=3	8=8=7=3	8=9=6=3	6=9=8=3	7=10=6=3
Trumps	19	18	18	19	19	20
Weights	100	96	96	69	69	46

The probability weights are a reflection of the number of card combinations available for each pairing. Once a player sees his hand, the distribution on the left is fixed (at 3=5=3=2 in Cohen's example) and it becomes a question of how many card combinations are available to his partner on the right-hand side. The more combinations available on a random deal basis, the greater the probability that the given condition exists. Condition I, encompassing a 4-4-4-1 shape, is the single most likely distribution; however, it is much more likely overall that partner holds a 5-4-3-1 shape as there are six such possibilities (only five are shown).

Calculation of Weights

Given partner has only one club, these are the cards available to fill out the hand: 10 spades, 8 hearts and 10 diamonds. Here are the totals of card combinations for Conditions I and II chosen at random from the pool of unknown cards:

Condition I (10! X 8! X 10!) divided by (4! X 6!) x (4! X 4!)x x(4! x 6!)
Condition II (10! X 8! X 10!) divided by (4! X 6!) x (3! X 5!) x (5! X 5!)
Ratio of II to I is 24 divided by 25, which is the same as 96 to 100.

Similarly for Conditions III through VI. These calculations don't take into account high-card content, but they provide reasonably accurate guidelines in this free-wheeling age where shape plays the dominant role in competitive bidding.

Note that the most even distribution in the three suits, 4-4-4, produces the maximum number of card combinations. Condition I is the condition of maximum likelihood: it is the single most likely configuration given the player has counted his own hand. The fit in the heart suit is the key factor. There is a nine-card heart fit for Conditions I, IV, and V, hence 19 total trumps, and a total weight of 238 (46% of all cases). There is an eight-card heart fit for Conditions II and III, hence 18 total trumps and a sum of weights of 192 (37%). It is most likely that the total number of trumps is 19, least likely is that the total trumps are 20, two conditions with a total weight of 92 (18%). There is a low probability that the longest suit in one hand matches the longest suit in the hand opposite. The average number of total trumps is 18.5, so 19 represents a slightly optimistic estimate on average.

Subjectivity

These probabilities are based on the dealing of the cards, a situation of maximum uncertainty with regard to the placement of the cards. The auction may provide clues that provide information giving greater weight to one condition over another. There is another factor to be taken into account, which is: with which shape is partner most likely to have doubled? The most advantageous situation for takeout is represented by Condition VI for which the doubler holds nine cards in the majors with longer hearts than spades. If partner takes out to 5♥, all is well, as from his point-of-view that should represent the best fit. Not surprisingly, this is also the condition that represents the greatest number of total tricks. Thus, it is also the condition under which it is most dangerous to pass the double for penalty.

The degree to which you might wish to adjust the probabilities depends on the known behavior of your partner. If his doubles are generally penalty-oriented and send the message, 'They can't push us around', then you might favor leaving the double in. If his message is more likely to be, 'We should play this hand', then Condition VI becomes more likely and you would be more inclined to take out to 5♥.

Theoretically, there is nothing wrong with adjusting probabilities according to what you expect from the known inclinations of your partner and/or your opponents. To be realistic, probabilities must reflect the current state of partial knowledge. That is 'un-mathematical' unless we can assign some numerical percentages to our subjective bias. So, in the above example, you should make an estimate of how likely it is that the doubler is operating under Condition VI rather than Condition I. Probabilities are adjusted accordingly. If you are maximally uncertain about partner's action, you accept the weights as shown that represent the probabilities of the deal, the *a priori* condition of maximum uncertainty. Partners being partners, that may be the best policy overall. However, Larry Cohen has given us many examples where experts bid one more than the rest of us, which is not always best.

OFF-SHAPE DOUBLES AND THE LAW

You don't concentrate on risks. You concentrate on results
— Chuck Yeager (b.1923)

There are other situations where it is useful to be able to estimate the number of total trumps in a deal, as an aid to making better competitive decisions. In this chapter we'll consider the implications with regard to card distributions of doubling an opponents' three-level preemptive raise in hearts, a raise that implies a nine-card fit. In particular, we want to determine the probabilities of different numbers of total trumps based on an assumption of a random deal of the unknown cards, a flawed assumption to be sure, but one that enables a reasonably accurate estimate as a first approximation.

This time, we're going to assume that the (potential) doubler has a 5-4-3-1 hand, with a singleton heart. To simplify, we'll only consider the most likely shapes opposite since extreme distributions are rare, and in such cases we'll assume that partner will take appropriate action independently.

	I	II	III	IV	V
	♠ 5 – 4	♠ 5 – 4	♠ 5 – 5	♠ 5 – 5	♠ 5 – 6
	♥ 1 – 3	♥ 1 – 3	♥ 1 – 3	♥ 1 – 3	♥ 1 – 3
	♦ 4 – 3	♦ 4 – 4	♦ 4 – 3	♦ 4 – 4	♦ 4 – 2
	♣ 3 – 3	♣ 3 – 2	♣ 3 – 2	♣ 3 – 1	♣ 3 – 2
Sides	9=4=7=6	9=4=8=5	10=4=7=5	10=4=8=4	11=4=6=5
Also	8=4=8=6	9=4=6=7	10=4=6=6	10=4=5=7	7=4=10=5
	8=4=7=7	7=4=8=7	8=4=9=5	9=4=9=4	7=4=7=8
			8=4=6=8	9=4=5=8	
			7=4=9=6	6=4=9=7	
			7=4=7=8	6=4=8=8	
Trumps	18	18	19	19	20
Total Weights	257	169	231	90	37
Percentages	33%	22%	29%	11%	5%

Here are the five most frequent conditions when the opponents have a nine-card heart fit and you hold a 5=1=4=3 hand.

It may be surprising to those who rely on *a priori* odds that a 5-3-3-2 shape opposite is more likely than a 4-4-3-2 shape. The above figures are estimates of the *a posteriori* odds under the given restrictions. There are six possible divisions of sides with a 5-3-3-2 shape and only three with a 4-4-3-2 shape. More conveniently, you should deal with the total number of trumps. The percentages are as follows: 18 trumps (55%), 19 trumps (40%) and 20 trumps (5%). In 11 out of 20 deals, a player holding 5-4-3-1 shape will be in a situation where the total trumps number 18.

The most important suit is spades, in which it is assumed the player holds five. We would like to estimate the percentages for combined holdings of seven, eight or nine cards in the suit when consideration is limited to these five most common conditions.

Number of Spades Held	6	7	8	9	10	11
Percentages	6%	27%	34%	24%	8%	1%
Number of Diamonds Held	5	6	7	8	9	10
Percentages	3%	20%	32%	30%	13%	1%
(4 Spades)			42%	40%	17%	1%
Number of Clubs Held	4	5	6	7	8	9
Percentages	2%	14%	29%	33%	20%	3%
(3 Spades)				40%	38%	21%

There is maximum uncertainty as to whether there are 6-7 spades, 8 spades, or 9-11 spades opposite: each grouping has a probability of 1/3. An eight-card fit is smack in the middle of the expectations, in which case the total number of trumps is 17. It would appear then that it is okay to bid 3♠ on a 5-4-3-1 hand in the expectation of partner's shape, or even to stretch to 4♠ with appropriate controls.

What if you hold just four spades? There is a 53% chance that partner will hold three or fewer diamonds. We may convert the diamonds to spades and apply the percentages given above for diamonds, but if we assume the opponents would not preempt to 3♥ when holding as many as seven spades between them, we can eliminate the lower two holdings when considering a four-card spade suit. It would be better to double to show four spades and let partner decide whether to support spades or not. The odds are roughly 60-40 that you will find a good fit there, so a double is not aggressive since it is significantly on the right side of the odds.

What if you hold just three spades? There is only a 23% chance the partnership holds eight or more clubs. We may convert the clubs to spades and apply the percentages. Normally it is not best to suggest playing in spades when holding

5-4 in the minors. If one assumes the opponents don't hold as many as seven spades for their preemptive action, the odds swing to roughly 60-40 that you will find a good fit in spades. It is acceptable to double with only three spades if the opponents are assumed to be short in spades, dangerous otherwise. After 2♥ – P – 3♥, you may double hopefully, but after 1♥ – P – 3♥, you have to be more cautious. Remember that partner did not take action over 1♥.

The Mode and the Most Likely Distribution of Sides

The mode is the most likely number of trumps for each suit. The modes for spades, diamonds and clubs are 8, 7 and 7, which is in agreement with the single most likely side, 8=4=7=7. Note that given the 3-1 heart split, there are twenty-two cards remaining to be divided between three suits. The most even division is the most probable, that being an 8-7-7 split. In order of probability the component sides are 8=4=7=7 (13%), 7=4=8=7 (9%) and 7=4=7=8 (8%). You may wait a long time for a specific distribution to turn up at the table, but in total the most likely division of sides (8-7-7-4) occurs roughly 30% of the time.

Competing over 3♠

After the opponents have bid preemptively to 3♥, the odds favor taking action when holding 5-4-3-1 shape with a singleton heart, even with only three spades. The same percentages can be used to gain a rough estimate of the number of hearts held when the opponents preempt to 3♠. The difference is that you need more to bid a level higher, so a nine-card fit is the norm for bidding at the four-level, only a 33% chance with a 1=5=4=3 shape. At IMPs scoring, vulnerable against not, bidding 4♥ is not unreasonable even holding only four hearts when the high-card content otherwise justifies the risk.

Larry Cohen Defies the Law

The tradition in North America is that a double of one major promises at least four cards in the other, but there are cases where an off-shape double is made on general strength. This can lead to problems later. The idea is that the doubler can get around to describing the nature of his call on the next round, but this method is fraught with danger as it is susceptible to preemptive action. The following example involves Larry Cohen and David Berkowitz playing in the finals of the 2004 Vanderbilt Cup against a famously active Italian pair. On Board 60, the opponents preempted, Cohen held a 4-4-4-1 hand, but it was Berkowitz who doubled first to show strength. It appeared they held at most eight hearts between them; nonetheless, Cohen bid to 4♥, vulnerable against not.

	North	
	♠ 10 8 7 6 3 ♥ 7 2 ♦ 5 4 ♣ J 10 6 2	
West ♠ A K ♥ K Q 4 ♦ Q 3 2 ♣ A Q 9 8 5	N / W E / S	**East** ♠ 2 ♥ A J 8 5 ♦ J 10 9 8 ♣ K 7 4 3
	♠ Q J 9 5 4 ♥ 10 9 6 3 ♦ A K 7 6 ♣ —	

Berkowitz	Versace	Cohen	Lauria
	pass	pass	1♠
dbl	3♠	4♥	4♠
dbl	all pass		

Michael Rosenberg commented dourly on this deal in the November 2004 issue of *The Bridge World*. It turns out that 4♥ is off one and 4♠ is off two, so the total (major-suit) tricks are 17 and the total trumps are 17, a perfect match. The best fit for East-West is in the never-mentioned club suit (how common is that?), making ten tricks, but the method of scoring dictates that East-West play in the inferior fit. There is one other consideration: East-West can make 3NT. The only alternative Cohen had in order to maintain the possibility of reaching the right contract was a space-saving double of 3♠. That would make sense if the proper interpretation could be placed on that bid. It can't be a penalty double, can it?

Cohen's decision to bid 4♥ is wrong in theory, but the Italian pair could not afford to gamble that East-West might make their game. Bidding 4♠ was 'good insurance'. How could Lauria tell that the defense can take the first four tricks off the top? There is enough uncertainty that South makes his decision on general principles. Cohen's bold move has paid off.

This illustrates how the game is played today. South opens light in third seat, West doubles on an off-shape hand too strong for an overcall, North preempts with a poor hand driving the opponents to an unmakeable contract, after which partner saves them by pulling to another unmakeable contract. Everyone feels they have routinely made the right decision. Only those who can see all four hands feel some improvement can be made.

Let's consider Cohen's decision to bid 4♥. As he holds a singleton spade, Cohen can place Berkowitz with at least two spades, possibly three. Berkowitz may well have a hand too strong for a notrump overcall. Of course, Cohen can't know for sure the best available alternative. It would be convenient if 4♥ proved the best spot, but there is no reason for thinking it will be, except on general grounds. This is a good situation for a responsive double that leaves the decision to partner, who, after all, stands to have the best hand at the table. Double cannot be unilaterally for penalty, but all alternatives remain in place.

Perhaps the best answer to frivolous preempts is to bid an uninformative 3NT directly and see if the opponents can find the defense to beat it. Stoppers? Who needs stoppers? After all, the opponents are reducing the information content of their bids, so why should the stronger side help them out by expressing doubt and giving away information unnecessarily? Bidding what you think you can make is the other side of the application of the doubtful double. An imaginative action perhaps, and it would certainly be embarrassing if the opponents simply cashed the first five spade tricks.

Total Trumps and 5-4-2-2

The 5-4-2-2 shape has gained a bad reputation over the years even though the *a priori* probability of finding an eight-card fit in one of the long suits is the same as for the 5-4-3-1 shape (74%). The total trumps are one fewer for the former shape, and the latter shape has the advantage of a built-in shortage.

We can analyze the distribution of sides for 5-4-2-2 in the same way as described above for the 5-4-3-1 shape when the opponents have advertised a nine-card heart fit. I'll spare the reader the details, and only show below the most likely distributions that illustrate the potential problems encountered with the 5=2=4=2 shape.

	Division	**Partner**	**Occurrence**	**Total Trumps**	**Estimated T.T.**
I	8=4=8=6	3=2=4=4	13%	17	18
II	8=4=7=7	3=2=3=5	12%	17	19
III	9=4=7=6	4=2=3=4	11%	18	18
IV	7=4=8=7	2=2=4=5	9%	17	19

The estimated total trumps are what partner would calculate if he assumed the standard 4=1=4=4 shape for the double. This would be a very bad estimate when he held five clubs, the suit in which the doubler holds only two. The mathematical reason why Cases II and IV have such a high probability of occurrence derives from the fact that the most even split between eleven unknown clubs is 6-5, in the ratio of 7:5 with regard to a 7-4 split. This argues against doubling for takeout with a 5-4-2-2 shape. The temptation is to double and try to find a fit in one of the long suits, while the danger is that partner's best suit is the one in which you hold a doubleton.

On the other hand, with 5-4-2-2 shape, if you simply overcall in your five-card suit, there is a 20% chance that the four-card suit provides the better fit. Of course, if the five-card suit is spades, there is no concern about that when competing for a partscore or a game. If the five-card suit is diamonds and the four-card suit is spades, it is a different situation. You can double and correct to diamonds if partner bids clubs. (Partner must remember you are not trying to be difficult, merely bidding your longest suit.) If the five-card suit is clubs, you needs must correct 4♣ to 4♠, and partner must interpret this as suit correction and not an indication of slam invitational values.

Finally we may ask whether, over a preempt to the three-level in spades, doubler's partner should bid a four-card heart suit at the four-level as Larry Cohen did in the above example. Even if it is wrong, the opponents may 'save'. Otherwise, provided that the opponents don't hold more than six hearts, there is roughly a two in three chance that you will find an eight-card or longer heart fit in doubler's hand.

As noted above, if you're going to make an off-shape double, it pays to have a good holding in the doubleton suit when that is probably partner's longest suit. Here is a case where doubleton honors do not constitute 'wasted' values, but, in fact, are an essential component in the hands taken as a complementary whole. (In terms of the losing trick count, one doesn't like to gamble with two losers in the short suit.)

Chapter 14

GOING TO A BETTER PLACE

Put a good person in a bad system and the bad system wins
— W. Edwards Deming (1900-1993)

Nothing personal, but sometimes it just doesn't feel right to let partner play in 1NT. Most responders with a flat hand and fewer than 8 HCP will pass, reasoning that it makes sense to close one's eyes and think of +120. But when partner opens 1NT and you have a bad hand without an entry, the standard approach is to try to get to a better place. Alas, the path to bottoms is paved with such good intentions. Maybe there isn't a better place.

The 1NT opening bid has a preemptive effect that works in its favor. A small minus score in 1NT may represent a good result versus a makeable partscore for the opposition. On the other hand experts have often pointed out that, given a 4-4 major fit, game in the major usually plays better than 3NT. Even more so in a partscore, where in addition the minor suits come into their own whenever they represent the best chance of getting a plus score. Alas, normal Stayman is not geared for that eventuality.

Here is an unconvincing example of an optimistic undertaking from *No Trump Bidding — the Scanian Way* (Mats Nilsland & Anders Wirgren), discussing responding to 1NT on this hand.

♠ 10 9 5 3 ♥ J 8 3 ♦ K J 9 4 3 ♣ 5

It is recommended that responder employ Stayman on this hand and pass opener's reply. Of course, 1NT might play better than 2♥ on a 4-3 fit, as on the following example:

♠ K J 4			♠ 10 9 5 3
♥ 10 6 4 2		N	♥ J 8 3
♦ A Q 2	W		E ♦ K J 9 4 3
♣ A Q 10 8		S	♣ 5

West	North	East	South
1NT	pass	2♣	pass
2♥	pass	?	

Pass is the recommended action. You would be more optimistic about its being correct if the diamonds were poorer (♦QJ943, say), as dummy might be dead in notrump with no entry available for diamond winners or for taking an essential finesse in a major. So, another case of less is more (when not vulnerable).

Here is an example of what is sometimes derisively referred to as Garbage Stayman, as reported by Bart Bramley in his Vanderbilt report in the December, 2015 issue of *The Bridge World*. The five-card boss suit seemed to provide some degree of safety.

West		East
♠ 10 6 4	N	♠ K 9 5 3 2
♥ A Q 3	W E	♥ J 10 8 6
♦ K J 10	S	♦ 6 5
♣ A Q 8 4		♣ 9 7

Zia	North	Duboin	South
1NT	pass	2♣	pass
2♦	pass	2♥	all pass

This time responder had to make two bids in his attempt to reach a better place to play. With such weakness it is necessary to make responder's second bid non-forcing. This made the weaker hand declarer, and after a diamond lead through the broken suit in dummy, Duboin went down one. Interestingly, by a different path at the other table East (Greco) also became the declarer in 2♥, making on a clever deduction that his RHO held ♥K doubleton. That gained 4 IMPs. Neither pair was able to reach the optimum contract of 2♠ played by the opening bidder (making three), preferring to mess about in hearts. Even 1NT makes eight tricks.

One Notrump and a Weak 4-4-4-1

The classic situation is where responder has a singleton club and can in good conscience pass the opener's reply to Stayman. If you hold a 4-4-4-1 shape, the *a priori* odds are that there is at least one eight-card fit with partner 80%

of the time. The Law of Total Tricks indicates that if one partner holds a balanced hand with 15-17 HCP and the other partner holds 3 to 7 HCP, ideally they should be able to compete gainfully for a partscore at the two-level. If you really believe that, the question you must ask yourself is: how good is my bidding system at getting me to the right spot? Of course, the right spot may not be in diamonds, and there's the rub.

Normally, Stayman is limited to three replies, 2♦, 2♥ and 2♠, thus removing clubs from consideration at the two-level. If responder uses Stayman, how often will he immediately hit a playable fit? To get an approximate answer using paper and pen, we'll limit the shape of the 1NT opening bid to 4-3-3-3, 4-4-3-2 or 5-3-3-2 (five-card minor only). Here are the approximate fractions we obtain for direct hits (4-4 or better).

Reply	4=4=4=1	4=4=1=4	4=1=4=4	1=4=4=4
2♠	2/9	2/9	1/5	—
2♥	3/10	3/10	—	3/10
2♦	2/9	—	1/3	1/3

If the reply is two of your major, the job is done, but if it is 2♦ more information is required. The ambiguous situation could be improved if a fourth reply (2NT) showed a maximum with five rebiddable clubs and no four-card major, allowing for the hand to be played there or in clubs. I'll call this Explicit Stayman. Notice that the 2NT response rules out playing in a 4-3 major fit at the two-level if responder has the four-card major.

Let's look at the likelihood of this method locating a reasonable place to play opposite various shapes for responder.

Responder is 4=4=4=1

Half the time responder will find a 4-4 major-suit fit. That is a gambler's position, but after a 2♦ reply it is probable that the opening bidder has good clubs. This is because opener is more likely to have longer clubs opposite a singleton than long diamonds opposite a four-card suit. When the reply is 2♦, opener will hold four or five clubs without four diamonds more than twice as often as four diamonds without four clubs. Therefore, if the Stayman reply is 2♦, responder most likely has taken a backward step and worsened the contract, so he should relay to 2♥ in a search of a 4-3 major-suit fit at the two-level. Thus, responder's escape from 2♦ to 2♥ must be non-forcing, but correctable to 2♠ if opener holds better spades. Here are the total tricks expected.

Division of Sides	Total Tricks	Percentage	A Priori
7-7-6-6	14	17%	10%
7-7-7-5	15	8%	5%
8-7-6-5	16	32%	24%
8-7-7-4	17	11%	7%
8-8-6-4	17	8%	5%
9-7-6-4	18	5%	7%
9-7-7-3	19	2%	3%

The number of total tricks is somewhat lower than the *a priori* odds indicate. There are 14-16 total tricks 57% of the time as opposed to the expected percentage of 39%. This argues for caution. The 8-7-6-5 division of sides is still the most frequent, with a total trick sum of 16, but that is achieved when the singleton club sits opposite a four-card club suit in the opener's hand. The deadly 7-7-6-6 division of sides occurs when opener holds five clubs.

Responder is 4=4=1=4

Often on BBO we see commentators grow impatient when an opening 1NT ends the auction. 'Next!' says Joey Silver, but life is what you make it. With a singleton diamond responder will find an eight-card fit in at least one of the majors about half the time. The probabilities of the division of sides with diamond shortage give much the same picture as for club shortage, with a 25% chance that there is no eight-card fit. A 2♦ response makes it clear that responder has made an unlucky move from 1NT and needs must scramble.

Here is a recent example from the GORDON versus BECKER match late in the 2015 Reisinger (BAM scoring), where opener took it upon himself to do the correcting.

	♠ J 6 5 4 ♥ J 5 3 2 ♦ 10 ♣ J 8 7 2	
♠ K 10 8 7 2 ♥ A 6 4 ♦ A 8 3 ♣ K 10		♠ A ♥ 10 8 7 ♦ J 9 6 4 2 ♣ Q 9 6 3
	♠ Q 9 3 ♥ K Q 9 ♦ K Q 7 5 ♣ A 5 4	

West	Kamil	East	Coren
			1NT
pass	2♣	pass	2♦
pass	2♥	pass	2♠
all pass			

Kamil's 2♥ was announced as being for play, but Coren removed to 2♠ anyway. Keeping the strong hand hidden had a detrimental effect on the defense as 2♠ went down only one. At the other table, after a similar start Sontag-Berkowitz stopped in 2♥, down two. (Interestingly, Sontag had the option to correct to 2♠, but chose not to, presumably hoping his partner had five hearts.)

If the normally wary David Berkowitz took it upon himself to enter the auction freely with three jacks, it must be okay. The division of sides was a moderate 7-7-7-5. Deep Finesse tells us that East-West can score nine tricks in a diamond contract, but I didn't find a pair who managed it. Six tricks in hearts and nine in diamonds add up to 15 total tricks — right on!

Responder is 4=1=4=4

It is trickier when responder holds four cards in only one major. Shortage in hearts is the worst situation. There is a one in three chance of having a 4-4 spade fit, but only a one in five chance of getting a 2♠ response. There is a two out of three chance of at least one eight-card fit, but you may have to go to the three-level in a minor to find it. There will be at least 17 total trumps five times out of nine, partly justifying such a move. Here are the percentages.

Division of Sides	Total Tricks	Percentage	A Priori
7-7-7-5	15	9%	5%
8-7-6-5	16	36%	24%
8-7-7-4	17	19%	7%
8-8-6-4	17	12%	5%
8-8-7-3	18	9%	2%
9-7-6-4	18	11%	7%
9-7-7-3	19	4%	3%

When opener bids 2♥ over 2♣ (just below half the time), he will be hiding a four-card spade suit a quarter of that time, and responder must take further action and bid 2♠ with the agreement that it is non-forcing. There are in fact several ways to play this sequence; perhaps the most popular is to use it to show an invitational hand with four spades, allowing the partnership to end the auction there if opener so wishes. However, opener may be hesitant to let 2♠ be played in a 4-3 fit from the wrong side. Many are reluctant to give up the captaincy when holding at least 40% of the HCP. There are hands on which opener may decline the option to play in spades and bid 2NT to play, and he may even be correct in that some of the time. However, if opener had replied to Explicit Stayman with 2NT showing a good hand with five clubs and no four-card major, responder would have been informed immediately of the good nine-card club fit.

Responder is 1=4=4=4

The expected division of sides is the same as for the previous case, but there is the advantage this time that if opener bids the short suit (spades), denying four hearts, responder knows there is a good chance of at least one eight-card fit in a minor, perhaps even a nine-card fit. There is a better than 50% chance that the total trick count is at least 17, backing a move to the three-level. That's good. It is best to think of Stayman here as a move to play in a minor-suit contract with the added bonus of hitting a heart fit one-third of the time.

If the reply is 2♦, responder knows he has a fit in a minor, but can't be sure that clubs aren't a better trump suit than diamonds; however, the opponents have at least a nine-card fit in spades. The number of total tricks is at least 18. That's even better. This is a time to take action before the opponents catch on — bid 2NT as a takeout to the opener's better minor.

After a 2♠ reply (which occurs one-third of the time) responder may take out to a minor by bidding 2NT over 2♠. After that move you will get to an eight-card fit at the three-level about three out of four times, only failing when opener has an unlucky 4=3=3=3. I call this 2NT a Transcendental Elevation. In this way, happy heretics may reach a far, far better place than perhaps they deserve or expect.

Stronger hands

Skeptical readers will have noticed that Explicit Stayman and the Transcendental Elevation eliminate the invitational use of 2NT. How serious is that? If you were dealt a balanced hand with 16 scattered HCP, and had just one bid to make (imagine partner has committed some infraction that bars him from the auction), what should it be? 3NT! The single most likely situation is that the remaining HCP are divided evenly around the table, 8 HCP for each player, so it makes sense for you to bid 3NT, your best chance for a good score. Some theorists feel the invitational 2NT is a wasted bid as little useful information has been given, so they think of other uses for that bid. How many pairs actually choose to stop in 2NT and profit thereby?

The Notrump Zone, by Danny Kleinman, gives a conversational overview of theory mingled with personal views that vary between extreme fussiness and extreme fuzziness. Kleinman suggests the notrump bidder holding:

♠7 5 ♥A Q 8 5 ♦Q J 10 ♣A K J 7

should bid 3NT over an invitational 2NT, unconcerned about the weak spade holding. Attempts at refinement may do more harm than good, he notes. Responder probably has a spade stopper, but even if a spade is the killing lead, the opening leader may choose a passive heart lead after an uninformative auction. Also, bridge is easier if, rather than watch partner sweat, you guess early yourself, going with the odds given the information you have at the time.

1=4=3=5 Hands

In the discussion above, we used the expected numbers of total trumps as a guide for competing with a weak 4-4-4-1 hand opposite a strong notrump opening bid. We noted that a second non-forcing bid by responder is required in order to stop at a low level if the initial reply to Stayman doesn't uncover a major 4-4 fit. Let's look at the case of responder's holding a 1=4=3=5 hand and less than game-forcing values.

Again we calculate the *a posteriori* probabilities under the assumption that the opener holds a hand that is 4-3-3-3, 4-4-3-2 or 5-3-3-2 with a five-card minor. From these we obtain the probabilities of the various divisions of sides, hence probabilities of the number of total trumps.

Total Tricks	1=4=4=4	1=4=3=5
15	9%	8%
16	36%	29%
17	31%	37%
18	20%	15%
19	4%	9%
20	0%	1%

The most pronounced difference between the two hand shapes is that for 1=4=3=5 the most frequent number of total trumps is 17, not 16, as the *a priori* probabilities predict. This is expected, as for 1=4=3=5 the difference between the longest suit and the shortest suit is 4, the maximum contribution to total trumps being 17 (13+4) instead of 16 (13+3). Indeed, there is a 2/3 chance of 16 or 17 total trumps and an 80% chance of having 16, 17 or 18 total trumps. This encourages activity even on weak hands that carry the auction to the three-level.

The Replies to Stayman

With the 1=4=3=5 distribution the three standard replies to Stayman occur equally 1/3 of the time. The 2♥ reply discloses the 4-4 major fit immediately and opener doesn't deny also holding four spades. The 2♦ reply will deliver three hearts 80% of the time, so a non-forcing 2♥ over 2♦ may land responder in a 4-3 fit. Of course, as we have seen, he may not be left to play there.

The number of forcing bids required depends on whether you are describing your own hand or asking partner to describe his. One forcing bid is all that is needed when one player has a good appreciation of what to expect in his partner's hand, as here. The five possibilities for the opener are ranked according to frequency as follows.

Case	Shape	Sides	Occurrence
1	4=3=3=3	5=7=6=8	28%
2	4=3=4=2	5=7=7=7	24%
3	4=3=2=4	5=7=5=9	21%
4	4=2=4=3	5=6=7=8	15%
5	4=2=3=4	5=6=6=9	13%

As responder holds five clubs, he can bid 3♣ to play, indicating invitational values. Opener may pass or take a shot at 3NT if he has a maximum with club support. That being the case, responder needs a forcing bid for his better hands and 2NT can be used in that capacity. Opener is asked to bid 3♣ or 3♦, the latter case only when his diamonds are longer than his clubs. This reply scheme is necessary in case responder has 1=4=4=4 shape and just wants to play in the better minor suit fit. Responder may take further action if he has a good hand, perhaps by bidding 3♥, forcing. The meanings of these sequences are different from the current standard practice, where, for example, 3♣ over 2♠ might be a slam try with long clubs and four hearts. (I don't recall ever having reached slam by that route, but, who knows, it could happen.)

Having reached this point, you cannot expect the opponents to enter the auction and rescue you from a bad contract, so it is important to have a suitable hand for declarer to play in three of a minor. What might that look like? As 3♣ would be a descriptive bid, inviting further action by opener, it is best if the hand conforms to expectations, that is, no top honor in spades but honors in clubs. Hearts needn't be strong, but should be better than diamonds, as hearts may sometimes be required to be trumps in a 4-3 fit.

If the opening bidder doesn't hold a four-card major his response will be 2♦, so responder will know immediately that the opponents hold at least a nine-card spade fit. It is a situation where several live possibilities exist. The final contract will be resolved in an atmosphere of uncertainty with the responder in the best position to resolve it. Uncertainty is manifest in the multiplicity of the possible divisions of sides that remain.

Case	Shape	Sides	Occurrence
1	3=3=4=3	4=7=7=8	23%
2	3=3=3=4	4=7=6=9	17%
3	3=3=5=2	4=7=8=7	14%
4	3=2=5=3	4=6=8=8	12%
5	2=3=4=4	3=7=7=9	9%
6	2=3=5=3	3=7=8=8	8%
7	3=2=3=5	4=6=6=10	6%
8	3=3=2=5	4=7=5=10	5%
9	2=3=3=5	3=7=6=10	4%
10	3=2=4=4	4=6=7=9	3%

It is an exciting moment for the responder as he, and only he, knows the opponents have a workable nine-card fit in spades. Passing 2♦ is not a good idea. There is a 1/3 chance of an eight-card fit, but such passivity may only set LHO in motion. However, bidding 3♣ is likely to find club support in dummy, and the better the support, the more likely it is that partner will push to game. Not so good. A non-forcing bid of 2♥ is a way to keep the ball rolling with all options alive. The overall chances of hitting a 4-3 heart fit are over 80%, but the downside is that the door is still left open for the opponents to make a move.

Chapter 15

COMMENTS ON ZAR POINTS

Not everything that can be counted, counts
— Albert Einstein (1879-1955)

The basis of all bidding systems is hand evaluation, something that has been the focus of mathematical analysis for the best part of a hundred years. How valuable are the high cards, alone or in combination? What kind of adjustments should we make for long and short suits, or for finding a fit? And how do we make this evaluation system not only as accurate as we can, but simple enough for players to be able to use at the table?

The 4-3-2-1 Milton Work HCP scale popularized by Charles Goren has become a standard descriptor in the definition of opening bids. It represents hard information on the basis of which the opponents can make deductions on which to base their actions. The HCP content by itself is not a good method of hand evaluation in the case of a hand with short suits, so distribution points have been added to the HCP scale that reflect that fact. These points do not represent hard information, as the opponents cannot know their source until after the hand is played. Thus, even a 2/1 player may open 'light' with 10 HCP upon occasion when the shape is particularly attractive. It then becomes a question of how far a player may carry this action of adding points for distribution.

In the case of so-called Zar points, the answer is: a long way. Even a hand with 8 HCP can be considered for constructive action. To determine HCP in the Zar method, control values (A=2, K=1) are added to the traditional 4-3-2-1 HCP. With an eight-card fit, extra points are added for extra trumps, and there are other adjustments available for honors in partner's suit, honors in short suits and so forth. For complete details of the development of the Zar point-count process, the reader is referred to the article by the originator, Zar Petkov of Ottawa (2003), available from the Bridge Guys website under 'zar points'.

The Statistical Basis

Before I discuss the theoretical basis for the Zar formula, I want to critique the arguments that Petkov gives to justify his claims for superiority over the traditional Goren methods. First I should say that criticizing the 4-3-2-1 count is akin to flogging a dead horse, as for many years experienced players have treated it only as a very rough initial guide to hand evaluation.

Time for another analogy. Suppose TV reporters interview Occupy Wall Street protesters, one hundred of whom have beards. They ask the question, 'Do you support a special tax on executives earning over $1 million?' After the first fifty are questioned, the interrogators believe they are on to something, as forty-nine have confessed that they do support such a tax. It was not surprising that ninety-eight out of the hundred bearded protesters felt the same.

Armed with this information, those who do opinion polls decide to stop bearded men in the vicinity of Wall Street and ask the same question. Can they expect 98% accuracy in predicting that bearded men support such a tax? No, they cannot. After a few 'no' answers, the analysts add a modification: they exclude bearded men in suits who carry briefcases. Accuracy improves after the police release some protesters and a number of them return to the area. So there is some validity to the conclusion that bearded New Yorkers tend to support a super-tax, but it would be wrong to think it applies in most cases — that would constitute prejudicial judgment based on tainted evidence.

This is a simple example of how deductions from a test group cannot always be taken as predictors over a wider sample. The Petkov statistical results come from a narrowly chosen set of deals that satisfy a certain criterion. Let's take as an example the set of 70,000 deals in which the correct contract is 3♥ or 3♠. By 'correct' I mean that on a double-dummy basis, nine tricks and only nine tricks can be made. The Goren points method overbids on 21,931 deals (30%) whereas the Zar points method overbids on 2,439 deals. In that sense the Zar method provides a much more accurate evaluation. However, there is an advantage to overbidding at IMP scoring where a vulnerable game should be bid if it has a 3 in 8 (or greater) chance of success. Even at matchpoints, you gain by bidding games that come home on a defense that falls short of double dummy. The more uncertainty in the bidding, the better the chance of a faulty defense, so the Goren methods may work advantageously in practice. So if we choose a sample of 70,000 results from deals played by those with a wide variety of skills, we can expect quite different results with a greater degree of fluctuation.

The above argument against the validity of the statistical justification for Zar points as predictors does not mean that they do not constitute a good method of hand evaluation. There are theoretical reasons why they should work better than the Goren points, and I'll go into those next. The first advantage and perhaps the greatest, is that the method allows for light opening bids — a clear practical advantage. Petkov points out that there are more hands that fall in the narrow range of 8-11 HCP than fall in the wider range 12-37 HCP. Traditionally the former fall in the category of an initial pass, while the latter are divided into five main categories of opening bids. On an information-theoretic basis, this is a bad arrangement. It would be a better situation if those 8-11 HCP hands were also divided into five categories, which would increase the average information conveyed by an opening call. This is not really practical (there just aren't enough bids available), but the more passed hands that can be moved to opening bid status, the more informative the system becomes. This is a justification for aggressive systems in general — being aggressive also means being more informative, and more accurate in prediction as well. So, some special arrangements for those 'good-bad' hands have been made at the two-level, while the HCP limits for one-level bids have been lowered.

Zar Points and the Law

The Law of Total Tricks is a principle used by many to guide their bidding. A hand does not exist in isolation, so the playing potential depends on the degree of fit with partner's hand that you expect. The most common division of sides is 8-7-6-5. The number of total tricks is the sum of 13 and the difference between the length of the longest combined suit (8) and the shortest combined suit (5), so the number of total tricks (TT) equals 16. Less than 16 and the hands do not fit well, greater than 16, and we are taught to bid 'em up. Here are three examples with their *a priori* probabilities:

8-7-6-5	TT=16	23.6%
8-8-5-5	TT=16	3.3%
8-8-6-4	TT=17	4.9%

The occurrence of the 8-7-6-5 division of sides greatly outweighs the other two, and it is reasonable to base one's action on the assumption of this division, provided that it remains the most probable condition once you see your own hand. Sticking with this *a priori* assumption means that you will sometimes miss the opportunity to act on a more favorable division with a greater number of total tricks.

Judging a hand in isolation, we may consider the difference between the longest suit held and the shortest as an indication of playability in that it represents the maximum available contribution to TT. This sets a limit to what is possible. Thus, a 4-3-3-3 shape can contribute at most 1 to TT, whereas 5-4-3-1 can contribute up to 4, so has more potential.

Players have also learned from statistical studies that hands with a double fit play better than the TT predict. So when you consider opening a hand, you should take into account the probability that a double fit exists. Let's consider the division of sides when we are dealt a hand with 5=5=2=1 shape.

	♠ 5 – 3 (8)	♠ 5 – 2 (7)
	♥ 5 – 3 (8)	♥ 5 – 3 (8)
	♦ 2 – 3 (5)	♦ 2 – 4 (6)
	♣ 1 – 4 (5)	♣ 1 – 4 (5)
Weights	16	9

The probability that the division of sides is 8=8=5=5 relative to a division of 7=8=6=5 is in the ratio of 16 to 9 (64%). Based on which is more likely, it makes sense to act as if there is a double fit and the TT equal 17, not 16. That results in a greater than normal motivation to take action.

Double fits enhance the playing strength of the combinations. In the case above, we see that the 5-5-2-1 shape readily produces a double eight-card fit. Overall the *a priori* chance of a double fit is 44%. For a 5-4-3-1 shape the chance is 34%, and for 4-4-3-2 it is 22%. Petkov has taken this into account by adding as points the sum of the two longest suits, 10, 9 and 8, respectively for these hand patterns. The greater the sum, the more likely it is that a double fit exits.

It is possible to calculate the probabilities of Total Tricks and double fits for any given shape of hand, but the problem remains as to how to rank the distributions and provide them with a number of points that will reflect their relative degrees of playability. Petkov has assigned points in a simple manner, so 5-4-4-0 is ranked 1 point below 5-5-3-0. Is that a valid assessment when the former has a great probability of encountering a double fit? Is 1 point the correct differential?

Zar Points Formulation

In the simplest version we have this definition:

Zar Points = Honor Strength + Distribution
= HCP + Controls + (Longest – Shortest) + (Two Longest)

Zar points are divided into two main categories: honor strength and distribution, subdivided into the following four factors: the HCP on the scale of 4-3-2-1, the number of controls (Ace=2, King=1), the difference between the lengths of the longest suit and the shortest suit, and the sum of the lengths of the two longest suits. These four are not independent. The sum of the first two results in a points scale of 6-4-2-1, which favors the aces and kings over the queens and the jacks. This is appropriate for hands that are distributional in nature and are suitable for play in suit contracts. The third term relates to the potential contribution of the hand to the Total Tricks, and the fourth term relates to the probability of a double fit. Thus, the basic elements of hand evaluation as I have described them above are included in the Zar evaluation.

There is another factor that so far has not been considered: the losing trick count. This takes into account the placement of the honors. A combination of KQxx in one suit and xx in another counts as three losers, whereas a combination of Kxxx in one suit and Qx in another counts as four losers. Clearly the coincidence of the KQ in one suit is the more favorable situation. It is more likely that a suit with four cards has been dealt two top honors than it is that a suit with two cards has been dealt one top honor, so on that basis alone if we look at successful combinations more of them will be of the former type than of the latter. Generally, hands for which game is likely will have a suitable losing trick count, hence a well placed honor structure, so that factor is filtered out in the Petkov selection process.

Integration into a System

Bidding involves releasing information. A major question is, how can partner react systemically to the revelations? Opening light in third seat, even on a four-card major, is a feature of many systems, including the currently popular 2/1. Such actions have been justified on the grounds that partner has passed and will not overreact to a noise, or that the opponents may be about to enter the auction profitably with the balance of power. What Zar evaluation implies is that you shouldn't wait for partner to pass — you should pre-balance on speculation, as it were.

When a partner discovers a fit, he may jump preemptively (Bergen style) or he may ask for further definition through a checkback bid, such as Drury. So you could in theory use Drury over a first-seat opener and the best hand at the table may end up doing the asking. A problem may arise when there is no apparent fit. The probability of a fit with one of the longer suits has not been realized, which in the case of a shapely hand goes against the *a priori* odds. In this exceptional case, the bidders must find out more about the distribution and relays may be an effective solution, but there is still a danger of getting too high. Once the distribution points are known, a lower limit is set on the total of high card points.

There are some, myself included, who are prepared to go against the strictures of the 2/1 system by occasionally opening light in first seat. One danger is that the opponents may overcall; if the auction becomes competitive, partner may feel obliged to double the opponents in a contract that may prove unbeatable. To guard against this happening I prefer to open light on suits that I want led, if it comes to that. The same applies to my overcalls. Another danger is that partner may take us to 3NT. Again, if I can provide a good suit that represents a potential source of tricks, I am more inclined to open light.

Zar points do not provide a means of distinguishing between good suits and bad suits, so in that respect they share a fault with Goren points. My qualification for a light opening bid is at most seven losers and at least three controls in the long suits. The more points I have outside my best suit, the less inclined I am to take action with less than the normal complement of HCP. Here is a hand given by Petkov that qualifies by my standards:

♠ K Q x x x ♥ K J x x x ♦ x x x ♣ —

This hand has 9 HCP, 2 controls, but only 6 losers. However, I would be inclined to wait and see with this seven-loser hand:

♠ Q x x x x ♥ K J x x x ♦ K x x ♣ —

One consideration: if we end up defending at a high level, I am less sure that a spade lead will get us off to the right start.

What is a Void Worth?

To examine the difference in evaluation between a void and a singleton, let's compare the 5-4-3-1 shape to the 5-4-4-0 shape by looking at light opening bids with 10 HCP (26 Zar points constitute an opening bid).

♠ K Q x x x	♠ A Q x x x	♠ K Q x x x
♥ K Q x x	♥ K J x x	♥ K Q x x
♦ x x x	♦ x x x	♦ x x x x
♣ x	♣ x	♣ —
25 Zar points	26 Zar points	26 Zar points
7 losers	7 losers	6 losers

Because of the void, the hand on the far right has one fewer loser. That should be worth about 5 Zar points because a game bid in hearts or spades (10 tricks) requires 52 Zar points. Well, the void represents a contribution of 5 points in that it applies to Zar points through the term (Longest — Shortest). That is almost as good as an ace on the 6-4-2-1 point scale. The singleton contributes 4 points, only 1 point less, as good as a king. What is the significance? The hand on the left is not an opening bid by Zar standards, but the middle hand is. The difference in these borderline hands lies in the number of controls held: 2 on the left and 3 in the middle. The void delivers the equivalent of a difference between an ace and a king. Next we examine some frequent divisions of sides.

Hand	Opposite (Division)			
♠ 5	3 (8)	2 (7)	2 (7)	3 (8)
♥ 4	3 (7)	3 (7)	4 (8)	4 (8)
♦ 3	3 (6)	4 (7)	3 (6)	3 (6)
♣ 1	4 (5)	4 (5)	4 (5)	3 (4)
TT	16	15	16	17

Hand	Opposite (Division)			
♠ 5	3 (8)	2 (7)	2 (7)	3 (8)
♥ 4	3 (7)	3 (7)	4 (8)	4 (8)
♦ 4	3 (7)	4 (8)	3 (7)	3 (7)
♣ 0	4 (4)	4 (4)	4 (4)	3 (3)
TT	17	17	17	18

The *a priori* chance of at least an eight-card fit with a 5-4-3-1 shape is 74%, which is why it is generally considered a shape with which you strive to bid. We see that the mundane 8-7-6-5 division of sides is common, and there is a danger of a misfit division, 7=7=7=5.

A common division with a 5-4-4-0 shape is 8-7-7-4. Overall the *a priori* chance of at least an eight-card fit is 84%, so the prospects are clearly better than for 5-4-3-1 by an average of one card. That one card extra in a fit is equivalent to one fewer loser. It is not clear that Zar points give sufficient weight to the difference at the game-level. However, we must keep in mind that three losers (xxxx) is not typical of a four-card suit, and that Q10xx is much better.

Further adjustments

Petkov lists a whole range of further adjustments you can make, taking into account factors such as honors in short suits (a minus), honors in partner's suit (a plus) and the position of honors in suits the opponents have bid (plus or minus, depending). You can, I suppose, refine any system *post hoc* by adding or subtracting points to make the numbers come out the way you want.

In practical terms, you can also divide the Zar count by two, and then use that number to select a bid. Thus 26 Zar points (which divided by 2 comes to 13) is the minimum for an opening bid, and so forth. Alternatively you can go back in time to the point count for high cards proposed by the Four Aces in the 1930s: A=3, K=2, Q=1, J=1/2; you then add the length of your longest suit, then half the difference in length of the second and fourth longest suits — again, 13 is an opening bid. For example:

♠ A K x x x ♥ J x x ♦ K x x ♣ x x

The Zar total is 11+4+8+3=26, so constitutes an opening bid. The Four Aces counted it as 7.5+5+0.5 =13, so again an opening bid. This hand also has 2.5 quick tricks (an opening bid by that method), and 8 losers (not an opener in LTC). I suppose if this is an opening bid for you, you should be using one of the methods that agrees with your choice.

Chapter 16

THE PAJAMA GAME — A WINNING STRATEGY?

I have found that luck is quite predictable. If you want more luck, take more chances.
— Brian Tracy (b. 1944)

It is surprising how much attention bridge writers have paid to numbers as they relate to making decisions in the play of a hand. It is equally surprising how little attention is given to something more important — the correct strategy at the various forms of the game, and how that strategy is affected by the type of scoring.

Larry Cohen has confessed that he has quit playing high-level tournament bridge because he couldn't stand to continue playing a game in which he had only a 1 in 10 chance of winning. If everyone took that attitude there would be no more ACBL. Some experts prefer playing rubber bridge with poor players, where taking a conservative approach with good technique guarantees a more or less steady accumulation of rewards. But a tournament is a short-term affair and, no matter how good you are, you are bound to encounter some bad boards against inferior opposition. It is painful to hear players complain of bad luck after they score a bottom they feel they don't deserve. Over the length of a tournament one will inevitably encounter bad boards against bad players, which can be thought of as random events subject to a statistical analysis. In theory they should also be complaining about the tops when the opposition misplay the hand. I did encounter one expert, Jim McAvoy, who said to me, 'Take that card back, Bob, you don't want to play it'. That was in the spirit of the game, but illegal.

At the local club there are players who take lots of chances. They tend to play against the field and against the mathematical odds, and some get good results through the application of superior judgment. They also accept the bottoms they generate, reasoning that two tops for every bottom is a 67% game. That's looking on the bright side. Let's suppose that there is a special game coming up at my club and I would like to win it. I have two very good non-expert partners whom I could ask to play. One is very cautious (Player A) and the other is very aggressive (Player B). Which partnership is more likely to produce the score of three boards above average that is needed to stand a chance of coming away with the trophy?

I decided to do a simple mathematical analysis of our chances based solely on the expected distribution of tops and bottoms. Player A is a very good card player who never takes big risks. He seldom generates a bottom and similarly does not often generate a top through overt action. On most of the boards we are at the mercy of the opposition, who provide us with a scattering of average pluses and average minuses over the session. My role with him is to be a reliable partner who occasionally adds a dash of initiative for which normally I am chastised.

Player B is like a stock speculator who gambles on emerging markets with unsubstantiated optimism. He gets in early and often, forcing the action with aggressive bidding based largely on distribution rather than HCP. Many of the opposing pairs who would have made a mistake if given the chance are reduced to a silent but effective defense against a hopeless contract. Player B has a tendency to aim always for the best possible result — too narrow a target — but sometimes it works. Partners must get out of the way and accept some bottoms along with the more frequent tops — a result often referred to as a 'pajama game'.

You might think that a swinging player is more likely to produce a big score than a cautious player, on the grounds that swingers are masters of their own fate and may pile up many more tops when the conditions prove favorable. Let's examine that idea. We'll assume the upcoming session is 26 boards, scored on a top of 12. On similar sessions at the club, Player A and I produce on average an estimated three tops and one bottom through our own overt actions (that bottom probably being due to my over-reaction to his quiet approach). Let's suppose that Player B produces five tops and three bottoms, my role being reduced largely to that of scorekeeper. Both methods produce a final score of two boards above average, a decent 58%. The question is this: with which partner am I over 26 boards more likely to score more than three boards over average (61.5% or better)?

To obtain a rough estimate of our future chances let's assume the tops are randomly distributed in time so as to conform to a Poisson distribution with a given average. The average is also the variance of the distribution, so if our average game includes five tops we can produce a large number of tops on occasion. That's encouraging to those who habitually play for tops, but we can't overlook the bottoms. Psychologically, we may think of a top as a result of good practice and a bottom as a result of bad luck, but the two belong to the same family.

We'll assume the number of bottoms also conforms to a Poisson distribution, one independent of the distribution of tops. Thus the joint probability density function of tops and bottoms taken together is merely the product of their individual probabilities. This is not true in general for, as we all know, our early results affect our later actions. However, for simplicity we'll assume that each board is played on its own merits in a consistent manner.

Poisson Probability Distributions

	Player B		Player A	
Events	**Tops (5)**	**Bottoms(3)**	**Tops(3)**	**Bottoms(1)**
0	0.0067	0.0498	0.0498	0.3679
1	0.0337	0.1494	0.1494	0.3679
2	0.0842	0.2240	0.2240	0.1839
3	0.1404	0.2240	0.2240	0.0613
4	0.1755	0.1670	0.1670	0.0153
5	0.1755	0.1008	0.1008	0.0031
6	0.1462	0.0504	0.0504	0.0005
7	0.1004	0.0216	0.0216	—
8	0.0653	0.0081	0.0081	—
9	0.0363	—		
10	0.0181	—		

For those who love numbers as I do, the columns show how the number of tops and bottoms (events) are likely to be distributed over many sessions. The numbers in each column sum to 1, so all the possible products also sum to 1, as all cases are covered (0-0, 0-1, 0-2, 0-3, etc.). We wish to extract those cases for which the tops exceed the bottoms by 3 or more. The combinations are 3-0, 4-0, 4-1, 5-0, 5-1, 5-2, and so on. The sum of the products in these cases gives the proportion of sessions in which the condition for a good score holds.

I estimate that for Player B the proportion of good scores is just above 30%, whereas for Player A it is about 40%. Thus, swinging for tops lessens the chance of a good score when the associated number of bottoms is also high. Surprisingly, just playing a large number of hands for boring averages is a good policy if you can combine that with the ability to score well when the few opportunities arise. Failure to take advantage of erring opponents, for example by avoiding making penalty doubles, is carrying caution to an unacceptable extreme. You need to generate some tops in order to expect to win, but careful Player A still appears to be the better choice.

And yet… each week the winners seem to score many tops, so it can't be an entirely bad strategy to force the action during the auction. The secret is to avoid the bottoms while striving for tops. If we assume that the average number of tops created by overt action is five, what is the average number of bottoms we can live with to match the performance of Player A? It turns out that our average has to be reduced to 2.2 bottoms per session. Two bottoms are an acceptable number of disasters provided that you generate five tops through hyperactivity. So Player B is a little too error prone and doesn't always come up to his full potential. He has to pick his spots better.

Memo to Myself
When playing with cautious Player A, remind him that he shouldn't always play down to the field. Encourage him to take advantage of clearly advantageous positions, such as doubling part scores. Remind him that missed opportunities are mistakes characteristic of the chicken-flew-the-coop syndrome, although his reply might be, 'Oh, I could never do that.'

When playing with active Player B, don't get upset by the occasional foolish result and needlessly add to the number of bottoms by trying to recover the loss. Discouraging words are wasted on a character who seeks the thrill of a bad gamble paying off.

Confession

I must now confess that the results given above are restricted by practical considerations based on my experiences with these players. I have not included cases where we might score six or more boards above average (73%), as it essentially never happens. Quite probably that is as much, if not more, my fault than theirs, but there it is. You shouldn't be a slave to mathematical theory when practical experience overrides the idealistic assumptions. Thus, if the game were a world-wide pairs contest, I should choose Player B, as a big score is needed to place amongst all those unknowns who get over 70% playing in small clubs in remote locations.

Long-Term and Short-Term Strategy

Of course, these are the results for a one-session matchpoint event. One difficulty for swingers is that there may not be enough boards on which to get tops to make up for the inevitable self-generated bottoms. Eight tops and five bottoms constitute half the boards in play. There is a logical reason for playing a tight game in a short contest and the mathematics should reflect that.

In a two-session (or longer) event there is more scope for recovery from early bottoms. The Poisson distributions for larger averages more closely resemble a normal (Gaussian) distribution, symmetrical about the mean. The cautious player loses some of the advantage that results from the skew of the Poisson distribution concentrated between zero and two bottoms. (Refer to the right-hand extreme of the table above for evidence of this effect.) As a result of the symmetry the key characteristic becomes the difference between the average number of tops and bottoms, independent of the approach.

Swingers must combine patience with their aggression. A convenient way of achieving this is to play a system that differs from that of the majority and then to keep faithfully to that system. This removes the emotional element. You wait for the opportunities to generate different results that will inevitably arise because you are operating under different bidding constraints. At my club the 2/1 system is the almost exclusive choice. I prefer to play Precision, but suspect that Polish Club might work best. The measure of success should be how many more tops than bottoms are produced systemically. Moving to Poland, I might switch to Acol, but I very much doubt that would be a move in the right direction.

If your main concern is avoiding bottoms, you should adopt the communal bidding system, perhaps, for the sake of ego, adding a few trendy conventions. The worse the communal system, the better it is for experienced players who can make intelligent adjustments (sometimes incorrectly referred to as 'lies'), and who can take comfort in the belief that, come what may, they will seldom do worse than the dumbest pair in the me-too crowd.

Chapter 17

FIGHTING THE FIELD AT MATCHPOINTS

The biggest risk is not taking any risk.
— Mark Zuckerberg (b. 1984)

When playing in a competitive game, attitude is important. Matchpoint bridge is strangely abstract in that the players you meet at the table are not your enemies, they are your friends. Wish them well on the next round. You are competing against players you may never face at the table, so your attitude towards the field is important. At matchpoints as in free elections, the mediocre often takes on the guise of the safe choice. In a recent Regional my partner and I had a healthy lead after the first of two sessions in a Seniors Pairs game. The field was not distinguished. The second session began well, and about a third of the way through, when my partner scored well in a doubled partscore against our main rivals, I felt our position was unassailable.

At this point, thinking to consolidate our position, I decided to play down the middle and concentrate on reaching the par result on each board. In this I was mostly successful as a later study of the hand records confirmed. Yes, there was one deal where I kept the wrong card in the endgame and allowed 3NT to make, but that was only slightly below par as the declarer had given us that opportunity by misplaying the contract earlier. Imagine my surprise when after eleven rounds we found we had fallen behind in the standings. A bad last round where so-so opponents fell into a high scoring minor-suit partscore sealed our fate. It was as if we had been riding high, wide and handsome in a hot air balloon before I turned off the gas burner and set us drifting gently to the ground, helped along by a nasty downdraft at the end. Our skills had not diminished suddenly, but my attitude had, and this reduced my alertness towards opportunities that presented themselves. Too late, I recalled a quote from the late Paul Soloway: 'I never play for averages.'

It is common for experienced players to advise otherwise. In his classic book *Matchpoints* (1982) Kit Woolsey begins by stating, 'What we are trying to do when we play bridge (or any other game of imperfect knowledge) is to minimize the expected or average cost of being wrong.' This appears to suggest the need to minimize potential losses by going with the field in close bidding decisions. What many, like me, forget is his following definition: 'The cost of being wrong is the difference between the result of the action in question and the optimal possible result which would be achieved if the winning action were taken.' The cost is not calculated against the field action but against the odds present in the lie of the cards. You cannot afford to miss opportunities to get a good score even if that means going against the field.

More Theory

The total number of pairs doesn't affect the theoretical results concerning the expected scores, which depend solely of the probabilities of two events: PB, the probability that a given contract will be bid by the field, and PM, the probability that the given contract will make. You score 1 for every pair whose score is below yours, score 1/2 for each pair who have the same score as yours, and score 0 for each pair whose score exceeds yours. For ease of explanation let's illustrate the problem in terms of a slam being bid, but keep in mind that the same method applies to the other situations mentioned above.

Let's simplify our problem by assuming that the play of the hand is routine, that the opening lead does not affect it and nor does the side from which it is declared. Thus we eliminate cases where bidding game and making twelve tricks will already garner you an excellent score. We are left with four possible situations:

Situation 1	you bid slam and it makes;
Situation 2	you bid slam and it fails;
Situation 3	you stay in game and slam makes;
Situation 4	you stay in game and slam fails.

The probability of slam making is PM, so the probability of its failing is 1 – PM. The probability of the slam being bid is PB, so the probability that it won't be bid is 1 – PB.

In the Situation 1, you will get 1/2 for each pair that bids it and 1 for each pair that stops in game, reduced by the probability of the slam's making, i.e.

$$1/2 \times (PB \times PM) + 1 \times (1 - PB) \times PM$$

or

$$PM - 1/2 \times (PM \times PB)$$

Without going through the derivation process for each situation, I'll just present the results, which are as follows:

Situation 1	$PM - 1/2\ (PM \times PB)$
Situation 2	$1/2\ PB - 1/2\ (PB \times PM)$
Situation 3	$1/2\ PM - 1/2\ (PM \times PB)$
Situation 4	$1/2 + 1/2\ PB - 1/2\ PM - 1/2\ (PM \times PB)$

We'll denote these sums as S1, S2, S3 and S4, respectively. The sum S1 + S2 equals the expected score when you choose to bid the slam; S3 + S4 equals the expected score when you choose to stay in game.

$$S1 + S2 = PM + 1/2PB - PM \times PB$$
$$S3 + S4 = 1/2 + 1/2PB - PM \times PB$$

So that

$$[\text{Score for bidding the slam}] - [\text{Score for staying in game}] = PM - 1/2$$

Clearly, you should choose to bid the slam if there is a greater than 50% chance of making it, regardless of what the field is doing. This is a nice result for idealists, as you should in theory bid according to an evaluation based on the cards alone. An accurate evaluation requires accurate information, and inevitably there is uncertainty due to the inadequacies of the bidding system. The worse your bidding, the more you are inclined to go with the field. It helps in this regard if everyone bids according to the same rules.

The field has an expected score of average (1/2), so no matter how crazy the crowd, if you follow the crowd, you can expect a near-average score. That is part of the survival kit of the mediocre player. You might imagine that the probability of bidding a contract should reflect the probability of its making, that is, the field will tend to bid contracts where $PM>1/2$ and avoid those where $PM<1/2$. The critical decisions will occur where PM is somewhere in the vicinity of 1/2, that is, where there is maximum uncertainty as to whether the contract is more likely to make than not. However, the field has its preferences and tends to overbid to games. It also tends to overestimate the likelihood of the contract's making. A typical '50%' major-suit game usually does not depend on a simple finesse, the most common pitfall being the possibility of a 4-1 trump split, an eventuality most players ignore both in the bidding and in the play. On the other hand the field tends not to bid minor-suit slams, as most pairs do not have good enough methods to explore for slam and stop in 4NT.

Why, then, are so many good players affected by what the field is bidding? The reason lies in the way we learn the game, that is, by following simple rules and watching the veterans play the game. Most beginners respect experienced players and tend to imitate their actions, but in the real world the majority is often wrong, and scores are usually spread over a wide range with many contracts having been reached. Uncertainty is high on many hands, so it is natural for novices to be cautious. There is safety in the middle of the crowd.

Baseball and Bridge

Over the long run of a baseball season, it is normal for a team to experience ups and downs. So it is at matchpoint bridge: you don't score above average on every board, and sometimes a disaster occurs at random. The baseball strategy employed for making the playoffs is to beat up on the poor teams while breaking even with your rivals. Similarly, it is hard to win a matchpoint event if one gets a string of averages against pairs who are handing out tops to others. On the other hand, you should be content to achieve an average against the best pairs, as that means you haven't fallen behind them in the race for a top position. To achieve that average, you bid as the field bids.

Matchpoint games differ from team games in the same way that the baseball season differs from the playoffs. To get to the playoffs a team needs home run hitters, players who are considered good if they hit a home run once in twenty at bats. They hit the pitcher's mistakes. They are like the players who take advantage of poor pairs. Once a team gets to the playoffs, the game changes, as a team is facing another good team. Now accuracy and consistency are the most important attributes, and an ability to bunt may become critical. Often the heroes are steady players who never make the highlight reels during the season. So it is at bridge. Tactics vary with the opponents at the table and depend on the quality of the field.

Strategic Bidding and Maximum Uncertainty

At matchpoints, sometimes you may wish to minimize the potential loss and sometimes to maximize the potential gain. The two strategies can be analyzed mathematically as follows.

The difference (S1 + S2) – (S3 + S4) can be broken down into the following two components:

S1 – S4	representing the difference in gains for being in the right contract, and
S2 – S3	representing the difference in losses for being in the wrong contract.

You may attempt to maximize the gain for bidding correctly, or attempt to minimize the loss for bidding incorrectly. A very important condition is a probability of 1/2, which represents a condition of maximum uncertainty as to which contract the field will prefer (PB = 1/2), or which contract will make (PM = 1/2). In such cases, maximizing the gain and minimizing the loss are contrary strategies: as S4 goes up, S3 must go down. If S4 is greater than S1, then S3 must be less than S2. Here are some numerical illustrations, after a reminder of our three situations:

Situation 1	you bid the higher contract and it makes;
Situation 2	you bid the higher contract and it fails;
Situation 3	you stay in game and the higher contract makes;
Situation 4	you stay in game and the higher contract fails.

Conditions	**I**	**II**	**III**	**IV**	**V**
	PM = 1/2	PM = 3/8	PM = 3/4	PM = 1/2	PM = 1/2
	PB = 1/2	PB = 1/2	PB = 1/2	PB = 1/3	PB = 2/3

	I	**II**	**III**	**IV**	**V**
S1	3/8	0.28	0.56	0.42	0.33
S2	1/8	0.16	0.06	0.08	0.17
S3	1/8	0.09	0.19	0.17	0.08
S4	3/8	0.47	0.19	0.33	0.42

Condition I is the condition of total (legitimate) confusion. There is symmetry with regard to bidding slam or staying in game. It matters not one iota, on average, whether you bid on or not. The gains are the same, the losses are the same.

Condition II represents the situation where half the field bids to a contract with a below-average (37.5%) chance of making. The worst possible result is got by not bidding the popular game when it makes (Situation 3). So to minimize the loss you bid the game the field favors even though the chances of making it are poor. To maximize the gain, you sensibly avoid a game that has a poor chance of success.

Condition III represents a situation where half of the field misses a very good slam, possibly stopping in 3NT. To maximize the gain, you should bid the slam, even though you get the worst score if it happens to fail. To minimize the loss, you avoid the slam, but are likely to score poorly.

Condition IV represents the field's obvious blind spot, avoiding a so-so minor-suit game. The best score is got by bidding and making it, the worst by going down. It doesn't cost much to stay in a partscore (and there may be a bonus when some go down in 3NT).

Condition V is the reverse situation in which the field is eager to bid a 50-50 game. In this case the highest expectation is for bidding against the field, staying in a partscore, and making it, while the lowest expectation is for not bidding the popular contract that happens to make.

Minimizing the Effect of Being Wrong

Given a choice of mistakes, under what conditions is it better to bid a contract that fails (Situation 2) than not to bid a contract that succeeds (Situation 3)? The expected advantage for overbidding versus underbidding is 1/2 (PB – PM). If you are making a mistake, it is better if you have lots of company. The worst outcome possible is obtained by bidding an unlucky but unpopular slam, as represented by Condition III.

When in doubt you may decide to bid on the basis of what the field may be doing. That will minimize the potential loss, but it may not get you to the best contract, as illustrated by Conditions IV and V. Under these conditions the best and the worst scores are got by bidding against the tendency of the field. Under Condition IV, PM + PB < 1, whereas under Condition V, PM + PB >1. In the next chapter we shall show that this is an important distinction in a team game.

Maximizing the Effect of Being Right

There are those of us who prefer to back our own judgment rather than follow a field that is too often flawed in its approach. The question for us is this: which correct decision is likely produce the highest score on average, bidding a makeable slam (Situation 1) or staying out of a slam that doesn't make (Situation 4)? In what way does the decision depend on the probability of slam being bid by the field? The difference in the expected scores for staying safely in game or bidding successfully to slam (S1 – S4) is given by the following expression:

$$\begin{aligned}\text{Expected Gain} &= 1/2\,(3PM - 1 - PB)\\ &= PM - 1/2 \text{ when PM equals PB}\end{aligned}$$

The expected gain for bidding on is a positive quantity when 3PM is greater than 1 + PB (Condition IV). This tells us that you may justify bidding a slam that is less than 50% successful if the field tends to avoid it. For example, if two-thirds of the field will avoid a particular slam, you need only a 45% chance of making it to gain more by bidding slam than by stopping in game. This is swinging bridge, as it risks a large loss albeit in a good cause. On the other hand, if the field is more likely than not to bid the higher scoring contract, you need a better than 50% chance of success in order to justify not staying low as well. Thus, if two-thirds of the field is expected to bid a slam, you maximize the expected gain by avoiding the slam if it has less than a five in nine chance of making (PM=56%). Here is an example of this phenomenon.

Beating the Field

On this deal, the probability of making slam is far less than the probability of the field bidding it, so the hands represent a situation where one might profit hugely by going against the field.

West		East
♠ K Q 8 5	N	♠ A 9 3
♥ A Q 3	W E	♥ K J 5
♦ K 5 4 3	S	♦ A 10 7
♣ A K		♣ J 4 3 2

There are ten tricks off the top, and some luck is needed to move the total up to twelve. Favorable breaks in the pointed suits and/or some kind of squeeze or a doubleton ♣Q will be required, but we are well under 50% territory here.

Given enough information to make a choice, either partner might decide not to bid the slam and the pair will probably score very well indeed by merely taking their ten tricks off the top in 3NT. However, most players would not bid in this way — either they consider it dangerous, or they like to take charge, or they are not capable. And if West is dealer, a Standard auction of 2NT-6NT is highly likely. If the slam just happens to make, a pair that stays out of it will score poorly. Maybe unlucky, but one mustn't complain.

Bidding Maps

It is helpful to visualize the decision-making process by way of a bidding map with PM and PB as the coordinates. The simplest version is a flat map that displays the decision to bid on or not without indication of the elevations involved. Here are the flat maps for matchpoint decision as to whether to bid on (Yes) or not (No) or Flip a Coin (—).

Minimize Loss

PM/PB	.45	.50	.56	.60
.45	—	Yes	Yes	Yes
.50	No	No	—	Yes
.56	No	No	—	Yes
.60	No	No	No	—

Maximize Gain

PM/PB	.45	.50	.56	.60
.45	No	No	No	No
.50	Yes	—	No	No
.56	Yes	Yes	Yes	Yes
.60	Yes	Yes	Yes	Yes

The 'minimize loss' map merely reflects the symmetrical rule, 'Bid on when PB>PM, don't when PM>PB'. This yields a bad decision when poor games are being bid throughout the room, but many players go with the field in this situation, compounding the error. This is a situation where a brave judgment to pass based on poor trump quality can result in a good score.

The 'maximize gain' map reflects the rule, 'Bid on if 3 x PM is greater than 1+PB.' The maximize map yields the better approach as it conforms more closely to the optimal rule of 'Bid on if PM>1/2.' When PM equals 0.5 (maximum uncertainty as to whether the higher contract makes) and PB is greater than 0.5 (a suspicion most will bid it), you maximize the gain by stopping short and minimize the loss by bidding on.

When I hear a player reflect, 'I thought we might make slam (or game), but there won't be many in it', I think, 'There goes a player so good he can afford to pass up golden opportunities.' At the end of the game I find he usually scores above average, yes, but he is not near the top. Such a player does better at teams. In the next chapter we shall investigate why this is so.

Hugh Kelsey's Advice

In his book *Match-Point Bridge* (1970) Hugh Kelsey presented the conservative view with regard to bidding close games. He wrote, 'The game should normally be bid, for most players are healthily aggressive in their bidding habits and a fifty-fifty game will be bid more often than not. If there appears to be any reasonable chance of success you should wish to be in game.' His advice is equivalent to minimizing the loss when one makes the wrong decision. He notes, 'The good player does not like to gamble on close bidding decisions... He therefore chooses to play down the middle on such boards, relying on superior judgment on the competitive hands to pull his score above average.'

This advice seems to me to be poorly argued. There is a limit to the accuracy one can obtain from a bidding sequence, true, but that accuracy may be lessened by interference, so there is more scope for error, not less. We see that all the time. Bidding space is reduced. Superior judgment depends partly on the information made available by unreliable opponents, so you can be misled, whereas bidding in an uncontested auction depends on the information provided by your usually reliable partner who has your best interests at heart.

After the opening lead is made, declarer has a firmer grasp on the probabilities involved. He may feel more in command of the opportunities presented. That is an essential psychological factor. No one argues that declarer should do what the poorer players are doing, like finessing at every opportunity; rather, you concentrate your efforts on besting the average players. You must be prepared to take advantage of a particular lie of the cards that provides an overtrick, even if there is some risk involved due to the fact that most players will not make the same play. For example, a partial elimination and endplay is a common way of achieving this, but you may have to rely on a fairly even split in the side suits, otherwise an untimely defensive ruff could result in a poor score. The difficulty with adopting the same positive attitude during the auction is that the uncertainty when bidding is greater than the uncertainty when you can see twenty-six cards. Nonetheless there may be more clear mistakes made in the play than in the bidding.

The argument that you should bid with the field does not even assert that the risk outweighs the gain, for as we have seen, there may be great profit obtained from staying out of popular unmakeable contracts. No, the advice reflects what Pliny the Elder observed 2000 years ago, that the best plan is to rely on the mistakes of others. If you are a good player in a bad field, that cynical approach can be successful, but remember what happened to Rome — it fell to the barbarians in 476 AD.

Matchpoints as Democracy

Matchpoint scoring is a democratic process. On every board each pair has a vote on the best contract. Rarely is there complete agreement, but usually a consensus is achieved. The plebiscite may be worded as, 'Does such-and-such a contract have a better than 50% chance of success?' A player votes 'Yes' by bidding it. Insufficient punishments are handed out to those who made a mistake when they joined the majority, but the system comes down hard on dissenters who got themselves in trouble even though their motivation was sound and they would be correct more often than not. Their rewards are posthumous. In theory if a self-interested majority votes for a particular contract, it is most likely to be a sound one, but in practice a large number of voters may not have the foggiest idea, so they just go along with what they guess others think. The average player feels safest in the middle. He reacts conventionally the way he has been taught. Often he is left to ask himself, 'What went wrong?' That happens a lot, and not just in politics, where those who choose the middle of the road often end up as road-kill.

Chapter 18

EXPECTATIONS AT IMPS AND MATCHPOINTS

Luck, bad if not good, will always be with us. But it has a way of favoring the intelligent and showing its back to the stupid
— John Dewey (1859-1952)

In the last two chapters, we've been discussing optimal strategies at matchpoints. However, most players prefer to play in team matches: bad players because it's easier to win, good players because it's harder. An important difference from the matchpoint game is that the opponents you have to beat are not the diverse crowd situated throughout the room, but the two pairs your team is facing directly. You must play well to win against a team of four good players. A large field makes for an equitable game at matchpoints as the distribution of the scores on any given hand tends to reflect normal conditions. In a small field, unusual results have more of an effect and there is more need to shoot for top scores against the poorer pairs in order to win.

On many deals at teams the gains for bidding a higher scoring contract are equally balanced by the losses you encounter if the contract fails. On such deals the gain-loss ratio reflects the conditions at matchpoints, the difference being that at teams there is more at stake when bidding games. We shall study this important category of decisions in detail, the results being applicable to either matchpoint or team play.

At teams you are often facing opponents whose tendencies are well known if not entirely predictable. It makes sense to adapt your strategy to some degree to the known quality of the team you are facing. Overall, adopting the tendencies of the opponents is equivalent to playing for a tie. The swing deals are those where there is maximum uncertainty whether or not a contract will make and/or whether or not the opponents will bid it. Traditionally a large number of flat boards (or 'pushes') was considered an indication of a well-played match, but recently we find players frequently attempting to increase uncertainty, thereby creating swings one way or the other, much to the annoyance of idealists who would prefer matches to be won through accurate bidding and double-dummy card play.

In the previous chapter we considered the effect of the field on matchpoint strategy. Here we continue to explore the theory for team play, where the potential gains and losses are not the same for every hand. Just as in matchpoints, there are four possible situations:

1) You bid a higher-scoring contract and it makes;
2) you bid a higher-scoring contract and it fails;
3) you stay in a lower-scoring contract and a higher-scoring contract makes;
4) you stay in the lower-scoring contract and the higher-scoring contract fails.

As before, we'll label the probability of making the higher scoring contract as PM; the probability of the opponent bidding to that contract is PB.

Again, we'll consider deals with just two outcomes. If the opponent is in the same contract, we assume there is a tie, so the score on that board is 0. If the opponent is in the alternate contract, either you gain an amount G or you lose an amount L, where G and L are in general different numbers of IMPs. In the four situations listed above, the expected scores S1 through S4 are as follows:

$$S1 = G \times PM \times (1 - PB)$$
$$S2 = -L \times (1 - PM) \times (1 - PB)$$
$$S3 = -G \times PM \times PB$$
$$S4 = L \times PB \times (1 - PM)$$

The advantage to bidding the higher scoring contract that provides gain G is:

$$S1 + S2 - (S3 + S4) = PM \times (G + L) - L$$

This is the gain factor. The optimum strategy is to bid the higher scoring contract if this quantity is positive and not to bid it if the quantity is negative — i.e.

$$\text{Bid it if } PM > L/(G + L)$$

If we represent the potential loss, L, as kG, where k is the ratio of L to G, we bid on if PM > k/(k + 1)

Under normal circumstances k lies within the limits 0.5 and 2. If the gain and the loss are equal, k=1 and the optimal condition becomes PM > 1/2, as with matchpoint scoring. If the potential loss is twice the potential gain, optimally we bid on only if PM > 2/3, as in the case of bidding a grand slam at rubber bridge. At teams, the value of k associated with a vulnerable game is 0.6, so the game should be bid if PM > 3/8 (37.5%, an oft-quoted number).

At IMP scoring, the gain factor changes from board to board according to the number of IMPs available for making the correct decision. If you bid the higher contract and the opponents don't, the expected gain is (1 – PB) times the gain factor. If you bid the lower contract and they bid the higher one, the expected gain is PB times the gain factor. Of course, the gain factor turns into a loss factor if you make the wrong choice and the opponents make the correct one.

Minimizing the Loss, Maximizing the Gain

You may aim to minimize the loss when making the wrong bidding decision on a board, in which case you should avoid bidding the higher scoring contract under the following condition:

$$| S2 | — | S3 | > 0, \text{ such that}$$
$$(1 – PM) L > PB \times [L – (G – L) \times PM], \text{ which can be rewritten as}$$
$$1 > (PB + PM) – r \times PM \times PB, \text{ where r equals } (k–1)/k.$$

Note the symmetry with regard to PM and PB, which act interchangeably. If L equals G, r is zero, in which case you should bid the higher contract if the probability of making it plus the probability of the opponents bidding it is greater than 1. This is normal for uncontested auctions.

The gain for bidding the higher scoring contract and making it versus the gain got by bidding the lower scoring contract is given by the following expression:

$$S1 – S4 = G \times PM – L \times PB + (L – G) \times PM \times PB$$

To maximize the gain, bid on if PM > k x PB – (k – 1) x PB x PM.

Now if L equals G, bid on if PM > PB that is, if the probability of making the higher contract is greater than the probability that the opponents will bid it, even if PM is less than 1/2, which is contrary to the optimal strategy. To minimize the loss, don't bid on if the probability of the opponents bidding the higher contract is greater than the probability it will fail, that is if PB > (1 – PM). To maximize gain and to minimize loss are not incompatible aims at IMP scoring, and a 'comfort zone' achieving both ends is possible.

Comfort Zone

On boards where the potential loss and the potential gain are equal, bid the higher scoring contract if:

$$(PM + PB) > 1$$
$$\text{and}$$
$$PM > PB$$

The Max-Min Diagrams

In a previous chapter we introduced text maps as shown below. 'Yes' indicates you should bid higher to achieve the aim, 'No', that you should not, and the dashes signify a toss up.

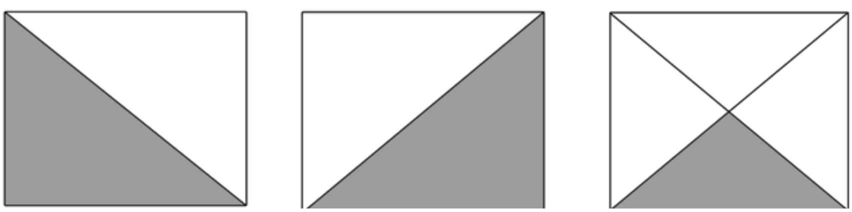
Maximize the Gain

PM/PB	**.45**	**.50**	**.56**	**.60**
.45	—	No	No	No
.50	Yes	—	No	No
.56	Yes	Yes	—	No
.60	Yes	Yes	Yes	—

Minimizing the Loss

PM/PB	**.45**	**.50**	**.56**	**.60**
.45	No	No	—	Yes
.50	No	—	Yes	Yes
.56	—	Yes	Yes	Yes
.60	Yes	Yes	Yes	Yes

The boxes that contain a 'Yes' in both diagrams are representative of the comfort zone. The 'maybe' boxes clearly lie along diagonals that separate the Yeses from the Nos.



The conditions for maximizing and minimizing can be represented graphically by lines in a more detailed PM/PB diagram, as sketched above. When G equals L the maximize line runs diagonally from the upper left corner to the lower right corner. The shaded area to the left of this line represents conditions in which the gain is maximized by bidding higher. The minimize line is a diagonal from the upper right corner to the lower left, the area to the right representing conditions in which the loss is minimized by bidding the higher contract. The diagonals cross at PM=PB=1/2, the point of maximum uncertainty. When the gain is not equal to the loss the point of intersection is elsewhere, where there is less uncertainty, as will be discussed in a later chapter.

A decision to bid the higher contract can be associated with a point in the diagram that reflects the *de facto* probabilities. If the point lies within the comfort zone, you have acted both to maximize the gain and to minimize the loss. If the point lies within the 'no-no' zone, you have chosen poorly on both counts. If the point lies in one of the other two zones, you have acted either to maximize the gain (on the left) or to minimize the loss (on the right). Another viewpoint is that any mistakes that are made are due to a miscalculation of PM, owing to a lack of information on how the cards lie or a poor prediction of the probable action of the opponents, PB. In a double dummy analysis PM is entirely dependent on the lie of the cards, but in practice the defense may benefit from information received during the auction.

The Full Picture

The three sketched diagrams above show the decision zones over a range of probabilities when the potential gain equals the potential loss, the situation that we encounter most often in constructive bidding, but which also applies to decisions made during play, for example, whether or not to take a finesse for an overtrick. As most think directly of matchpoint scores, let's present a map of the expected scores as functions of the probability that a slam will make (PM) and the probability that a slam will be bid by the opponents (PB). In a matchpoint game we may think of PB as the proportion of players in the slam, but the results apply to what is to be expected in similar situations occurring over several sessions. The numbers are multiplied by 1000 for convenience of display.

For each coordinate (PM, PB) we provide a cell consisting of six elements,

S1	S4
<u>S2</u>	<u>S3</u>
Sum	Sum

S1 is the score associated with bidding a slam and making it.

S2 is the score for bidding a slam and not making it.

Their sum approximates the total of the scores available from bidding slam.

S4 is the score associated with staying in game when slam goes down.

S3 is the score associated with staying in game when slam makes.

Their sum approximates the total of scores available from stopping in game.

There are certain characteristics that can be verified by observing these numbers closely.

$$(S1 + S4) - (S2 + S3) = 1/2$$

This says that the score obtained by making the correct decision in practice exceeds the score obtained by making the wrong decision by half of the matchpoints available. So if one scores 25% by matching other players who have also made the wrong decision, the players who made all the right decisions in practice will score 75% (50% more). It doesn't depend on PB or PM.

$$(S1 + S2) - (S3 + S4) = PM - 1/2$$

This says that there are more matchpoints available from bidding slam than from staying in game provided that PM>1/2. This means one should bid the slam if there is a better than 50-50 chance of it making, regardless of how many players are staying out.

$$S1 - S4 = 1/2 \times (3PM - PB - 1)$$

This tells us that if one aims to maximizing the score for being right, one should bid the slam when the quantity in brackets is positive. If one is maximally uncertain whether slam will make (PM=1/2) one should stay in game when more than half the players are expected to be in game.

$$S2 - S3 = 1/2 \times (PB - PM)$$

This tells us how to maximize the score when you have made the wrong decision: bid slam when there is a greater proportion not making slam (1–PM) than there are not bidding it (1–PB). This is a detail that haunts the pessimists who aim for the least bad score. Imagine if you can a field where players will open 1NT and have no way to explore a minor-suit slam. The great majority will bid 3NT straight up even when 6♣ is cold. They take their average and move on to the next hand unperturbed. This is the lazy way to play bridge. What does the theory tell us? PM is high but PB is low, so one stays in game in order to minimize the loss if one turns out to have made the wrong decision.

Matchpoint Map

PM	**PB = 2/5**		**PB = 4/9**		**PB = 1/2**		**PB = 5/9**		**PB = 3/5**	
2/5	320	420	311	433	300	450	288	467	280	480
	120	120	133	111	150	100	167	89	180	80
	440	540	444	544	450	550	455	556	460	560
4/9	356	389	345	401	333	416	321	432	311	444
	111	133	123	123	139	111	154	99	167	89
	467	522	468	524	472	527	475	531	478	533
1/2	400	350	388	361	375	375	361	389	350	400
	100	150	111	139	125	125	139	111	150	100
	500	500	500	500	500	500	500	500	500	500
5/9	444	311	432	321	416	333	401	346	388	356
	89	167	99	154	111	139	123	123	133	111
	533	479	531	475	527	472	524	469	521	467
3/5	480	280	467	289	450	300	433	311	420	320
	80	180	89	167	100	150	111	133	120	120
	560	460	556	456	550	450	544	444	540	440

If you are playing in an expert field, PB will be strongly correlated to PM, that is, the experts will be close to bidding good slams and staying out of the bad ones. They will be operating along the diagonal line upper left to lower right representing the condition of PM=PB. In a mixed field there will be more uncertainty in the evaluation of slam probabilities, and the field will often be operating along the horizontal line of maximum uncertainty, PM= 1/2 .

When one is going to make a decision it is important to know where you are on the map. You can only estimate your location. At the club regular partnerships have a good idea of what the field will be doing on the deal, that is, they have a good estimate of PB. In many slam decisions they will be working along the horizontal line representing maximum uncertainty of the chances of making it, whereas experts may have a better estimate based on superior bidding methods that reveal more information or are based on a better means of evaluation. Here is an example.

Bidding 6NT on 33 HCP

Standard bidders may get to slam on the auction 1NT – 4NT – 6NT. The decision may be based on the knowledge that both hands are flat and the partnership combined holds at least 33 HCP. That is a good indication that PM> 1/2. However, if a partnership has agreed to using Gerber, the auction may proceed 1NT – 4♣; 4♠ – 5♣; 5♦ – 5♠; 5NT – pass. The 5♠ bid asks opener to bid 5NT so that the partnership can play in 5NT when missing an ace and a king. The success of the slam depends largely on whether the finesse for the king is working, so a 50% slam in most cases.

In this case the use of Gerber may result in maximum uncertainty and now we must estimate how many in the field will reach 6NT because they have at least 33 HCP. Let's say there is a very high percentage of pairs who will be there. What should the expert do? To aim at maximizing his gain against the slam-bidding field, he should pass 5NT, going against the field. However, if he aims to minimize his loss when he is wrong, and slam makes, he should go with the field and bid the slam (PB>PM).

A most conservative piece of advice about slam bidding appeared recently in an article about IMP strategy in the *Daily Bulletin* of the 2016 Orlando Nationals in which Steve Gaynor emphatically advised players not to bid a slam that is less than 75% sure of making. What was he thinking? In theory a non-vulnerable slam should be bid if it has better than a 50% chance of making, in line with matchpoint strategy where gains and losses are balanced. Where on the map was he pointing?

It would make sense if he was focused on the situations where a slam is bid and makes (S1) and where staying in game makes exactly (S4). Let's suppose the expert opponents are sure to be in slam (PB= 1). This is off our map, nonetheless, we know

$$S1 - S4 = 3PM - 2$$

If PM = 2/3, the scores are the same for each condition so it doesn't matter if you bid it or not. One scores more by being in slam if PM>2/3. If we choose PM=3/4 as a safety margin due to the inaccuracy in estimation, then S1 is safely greater than S4. In a matchpoint game it is rare for everyone to be playing in the same contract, whereas in a team match it is reasonable to assume expert opponents will play in a slam if it is a favorite to make. The conclusion is that if the opponents are sure to bid 6NT on point count, you needn't necessarily join them, unless it is pretty sure that twelve tricks are assured with only an ace missing — one makes no allowance for bad breaks.

Chapter 19

WATCHING WITH WOOLSEY

The excitement that a gambler feels when making a bet is equal to the amount he might win times the probability of winning it
— Blaise Pascal (1623-1662)

Now let's turn our analytical eye to a less common form of teams scoring — Board-a-Match (BAM), or Point-a-Board is it is known in Europe. In BAM scoring, both teams get half a point on the board if their scores are identical. If not, the team with the higher score gets 1 point, the opponents 0. Effectively you are playing matchpoints in a two-table game.

At least in North America, we seldom encounter BAM scoring any more, which is a pity since every trick on every board counts. The IMPs game pales by comparison as an exciting and instructive contest of skill and a demonstration of card play, as there is none of this diversionary small talk from the online commentators after the opening lead, or yawns being expressed in print when the bidding stops at 1NT, a contract that should be one of the most exciting in bridge. My version of *The Official Encyclopedia of Bridge* states that BAM went out of favor largely because it is more difficult for good players to score an upset over better players. This tells me that, contrary to some opinions, the luck factor is reduced to a minimum. The annual Reisinger Cup at the Fall NABC is one of the few major events still scored using BAM. Watching the 2010 Reisinger final on BBO was a real treat as it featured one of my favorite commentators, Kit Woolsey, guiding viewers through the action.

A British BBO commentator felt it was easier to sacrifice at BAM than at IMPs as the cost is limited, unlike at IMPs where a large penalty risks a large loss. If the opponents can only make a vulnerable game, conceding a penalty of 1100 translates to a loss of 10 IMPs, and going for 800 will lose you 5 IMPs, while holding it to 500 represents a gain of 3 IMPs. Although at BAM scoring the gain and loss are equal, as Woolsey pointed out you shouldn't be inclined to sacrifice at BAM if you can pass and tie the board without risk since that adds a score of 1/2 to your running total. Sometimes sacrificing is 2:1 against the odds — if you are wrong and their contract does not make, you score a zero, whereas letting them play in their doubtful game or slam can have a positive outcome even if it is not theoretically optimal. In the last two chapters I applied some mathematics to bidding decisions of this sort, so for clarification I decided to apply the same methodology to the situation to which Kit was referring.

The probability of making slam (or game) we'll denote as PM and the probability that the opponents at the other table will take the sacrifice is denoted as PB. We'll assume that the result depends solely on the bidding — that if we do what the opponents do, the board will be tied, so the resultant score is 1/2 (on a scale of 1, 1/2, 0). There are eight possibilities to be considered if we assume the sacrifice will cost less than the slam.

Condition	**Action**	**Result**	**Expected Score**
Slam makes	we sac, they sac	1/2	1/2 PM x PB
(PM)	we sac, they don't sac	1	PM x (1–PB)
	we don't sac, they sac	0	0
	we don't sac, they don't sac	1/2	1/2 PM x (1 – PB)
Slam doesn't make	we sac, they sac	1/2	1/2 (1– PM) x PB
(1 – PM)	we sac, they don't	0	0
	we don't, they do	1	(1 – PM) x PB
	we don't, they don't	1/2	1/2 (1 – PM) x (1 – PB)

Expected score for sacrificing 1/2 PB + PM – (PM x PB)
Expected score for not 1/2 PB + 1/2 – (PM x PB)

The advantage to not sacrificing is the difference: 1/2 – PM.

Clearly, if the slam has a better than 50% chance of making, on average it pays to sacrifice, regardless of what the opponents are doing. This implies that the decision rests solely with your expectations as to the lie of the cards. However, Woolsey points out that you may be making a poor bet to sacrifice when the opposition is not likely to do so. Let's look at the conditions when we are certain they won't sacrifice (PB=0).

Expected score for sacrificing = PM
Expected score for not sacrificing = 1/2

Thus, you are guaranteed an average score by not sacrificing. In order to make an even bet, where risk equals gain, slam must be a certainty. That was Woolsey's point.

What's Wrong with this Picture?

Here is a little story. Fred has just inherited $10,000, so he goes to his accountant to ask advice as to how to invest it.

‘Fred, this is your lucky day,’ the accountant says. ‘Just this morning I read in the newspaper about this new sport franchise, whose stock is predicted to rise 20% in the next year. It’s a terrific investment just waiting for someone like you.’

‘Tom,’ Fred says, ‘you mean I should put in $10,000 in order to gain $2000? Seems to me I would need 5:1 odds in order to justify such a gamble. Thanks, but I think I’ll just stuff the cash in my mattress and take out some extra fire insurance.’

Playing bridge is not analogous to betting on a horse or putting all your money in one stock. It is more like investing in a mutual fund with many variable components, the sum of which serves on average to reduce the overall variability. Every deal is an investment for which there is a cost, a probable gain, and a probable loss. You will inevitably incur losses on some boards, the hope being that the cumulative gains will outweigh the losses by a sufficiently large margin so as to meet the aims set at the beginning. Your strategy should depend on what you wish to achieve: scoring above average, or competing for a top finish.

Back to the Reisinger

In the qualifying stages of an event like the Reisinger, there is evidence that indicates you should cautiously accept an average score. In the last two sessions of the 2010 event, the Jacobs team qualified for the final with scores of 52% and 53%. Non-qualifiers included Mahaffey at 35% and 65%, and Hampton at 42% and 61%. I am sure players on those teams wished they had scored a modest 45% on their first rounds, but it is difficult to control violent swings. Of course, eliminating the lows doesn’t mean you’ll get to keep the highs, as there is a different strategy involved. As 50% is not a qualifying score, you cannot be content with an average on every board. You should approach every board with cautious optimism, not reckless abandon.

With regard to sacrificing against a slam, the even bet limit depends on the probability that the opponents will sacrifice. PB may not be independent of PM, as the more likely the slam is to make, the more likely it is that the opponents will find the sacrifice. Nonetheless, the relationship between the two depends on the bidding. If one side opens 2NT at one table, say, the opponents may never get into the auction, and it may not become obvious that a sacrifice is a good risk. On the other hand, if the opening bid were a big club, a preemptive overcall could point in that direction. In many cases it will be difficult to guess the auction at the other table. The situation in which you are maximally uncertain as to whether the opponents will sacrifice or not (PB = 1/2) constitutes a reasonable approach in unexceptional circumstances. Under this assumption, the expected scores are as follows:

Expected score for sacrificing = 1/2 PM + 1/4
Expected score for not sacrificing = 3/4 – 1/2 PM

For PM=5/6 you have an even bet (the potential gain of 1/6 above average equals the potential loss below) but the expected score for not sacrificing is a lowly 1/3. You score 2/3 on average by sacrificing, so it seems obvious to do so. There is something very wrong with the argument that you should act only when the gain is greater than the risk as only in one case out of six will it be wrong to sacrifice, and if you don't sacrifice you are willing to accept a frequent poor score. So Fred may have doubts about the stock purchase, but it would not be reasonable to expect the shares to become worthless overnight. If he trusts the available information, it makes sense to invest for probable gains rather than take losses time after time, but the degree of risk depends on what Fred is trying to achieve.

Just reaching the Reisinger final may be good enough for some, but if you wish to be near the top you need to step it up. Here are some 2010 results for the final two sessions without the carryovers.

Team	Session 1	Session 2	Final position
Cayne	61%	52%	1
Smirnov	63%	50%	2
Rosenthal	50%	57%	3
Gordon	44%	61%	4
Jacobus	56%	52%	6

The top two teams were over 60% in first session and held on with slightly above average scores in the second. Rosenthal improved significantly by coming back with a 57% in the second session. Gordon's recovery was more spectacular but their earlier session was too damaging. The Jacobus results are interesting; they scored well in the finals, but were handicapped by being 2.5 carryover points behind the leaders after playing against the full field. So, scores in the low 50s in the qualifying rounds will prove a handicap; however, just getting to the finals is worth 60 or more platinum masterpoints. For most players, being happy with average boards early in the going makes good sense.

If a top team had been capable of two final sessions of 57%, they would have won, but, of course, it doesn't work that way — there will inevitably be random variability. If your target is 57% per session, you have developed the winning approach. Put yourself in the West seat in the first final session playing a board on which the two top finishers went face-to-face. Would you, as the front-runner, sacrifice in a non-vulnerable 5♦ or pass?

♠ 8 6 4
♥ A 7 5 2
♦ A 9 6
♣ A 10 7

♠ J 9 5 3
♥ J
♦ 10 7 5 4
♣ 9 8 4 3

N
W E
S

♠ A 2
♥ K 4
♦ K Q J 8 3 2
♣ K Q J

♠ K Q 10 7
♥ Q 10 9 8 6 3
♦ —
♣ 6 5 2

West	North	East	South
pass	1♣	dbl	1♥
pass	2♥	3♦	4♥
???			

At favorable vulnerability, would you as West pass or bid 5♦ ?

West for the eventual winning team, Michael Seamon, bid 5♦ in the hope that his partner, Jimmy Cayne, could hold the losses to four tricks, which he did. This was in keeping with the mathematical analysis given above that indicates, regardless of the action at the other table, you should be inclined to sacrifice if the probability of the opponents' contract making is close to 1. Seamon thought so, undoubtedly influenced by the presence of the singleton ♥J in his hand and the absence of anything else of much value.

This action created a pickup against the second place team, for which the West player took the conservative view of passing, scoring zero for doing so when 4♥ by North-South made easily. This was a swing of a full board and put CAYNE in the lead. They barely held on in the second session, so this earlier board proved critical to the end result. The result indicates that you should be prepared to take your chances as they arise rather than hope to get them near the end when the situation has become desperate and you cannot be as selective in choosing when to swing.

Chapter 20

THE MATTER OF FREQUENCY

Even very low probability events can and do occur
— Gavin Extence (1982 -)

In a previous chapter we considered the matter of frequency when choosing a partner to play in a matchpoint game that you particularly want to win. We saw that in a single session event it is better to choose a partner who avoids tops and bottoms whereas in a longer event it is better to choose a more aggressive partner whose tops exceed his bottoms. The reasoning behind that rested on the observation that with few opportunities for an appropriate swinging action, a bottom may be unrecoverable, whereas over many boards there are sufficient opportunities for a bottom (or two) near the beginning to be overcome. In the long run the governing factor is not the number of bottoms expected but the probability of getting a healthy difference between the number of tops and the number of bottoms. Mathematically, over a large number of boards the governing probability distribution function (Gauss) tends to be bell-shaped, symmetric about the median difference, whereas for a small number of boards the probability distribution function (Poisson) is skewed towards the number of bottoms.

It is time to talk about one of the most popular forms of scoring, at least in North America — Swiss Teams on a Victory Point scale. There are two differences to consider: the gains to be had from bidding vulnerable games, and the length of the matches. A few months ago I played in a Saturday night Swiss consisting of four seven-board matches. In the second match, even before comparing scores I knew our team had lost by a considerable margin because my scorecard showed three boards where we had -120. On two of these boards I expected our normally aggressive partners to be in a vulnerable game going down. Those adverse swings pretty well ruled out our team's chances of winning the event, but we finished respectably by winning the next two matches by large margins. My experience tells me that in order to win these events you have to bid every close vulnerable game in an attempt to maximize your gain, even when the odds are against making it. Suppose, however, you are playing for first place in the event with just seven boards left to play; should you rein it in, try to avoid minus scores, and bid only those games that have a better than 50% chance of success? Jeff Rubens thinks so.

In the December 2010 issue of *The Bridge World*, Rubens introduced the concept that the strategy should depend on the length of the match being played. He argued that with only a few boards remaining to be played, the best strategy is to try to win more boards than you lose; that is to say, the frequency of success outweighs the potential gain, just as it does throughout a Board-A-Match contest. He maintained that the standard odds in favor of bidding a vulnerable game are an exaggeration in a short match, where it is more important to be right rather than to 'have a good bet for an average result'. Presumably, he would aim to bid only to vulnerable games with at least a 50% chance of success. The argument is the same as the one against using a 'tops and bottoms' strategy in a short matchpoint game, namely, the odds are against recovering a loss, even when you gain 10 IMPs if a game is successful and lose only 6 IMPs if it is not. The context of Rubens' example was the final seven-board round of a National Swiss scored on Victory Points where a 20-IMP victory would see his team place third, and a 5-IMP victory would see them in twelfth place.

We'll assume that a partnership decides before a match which strategy they will pursue. That may depend on what they assume the opponents will do in the same circumstances. Or not: Rubens is silent on that aspect. Let's suppose that the partners decide to play according to the long-term odds and that they will bid all vulnerable games with at least a 37.5% chance of success, 3 chances out of 8. That means they are prepared to fail 5 times out of 8, because the opponents, following Rubens' advice, have decided not to bid such games. If the game succeeds they gain 10 IMPs (G = 10), and if it fails, they lose 6 IMPs (L = – 6 IMPs). Thus succeeding on just three deals balances the losses on five deals, and if the odds of making game are better than 37.5% then we expect to gain in the long run. However, in a short match you will not encounter eight such deals. Let's look at the situation where only a few deals provide the opportunity to bid an inferior game with a 3 out of 8 chance of success, the lower limit usually recommended for bidding vulnerable games.

One Decision G frequency 3/8, or L frequency = 5/8

So you will lose IMPs 5 times for every 3 times you gain.

Two Decisions	Results	IMPs	Frequency (x64)
	G-G	20	9
	G-L	4	30
	L-L	–12	25

Here you gain IMPs 61% of the time and lose 39% of the time, which is highly encouraging for bidders. In terms of the frequency of gaining the advantage, there is a large difference between making just one decision and having the opportunity to decide twice under similar circumstances. This is not the case when G equals L, as in Board-A-Match, where the aggressive bidders will suffer 25 losses for every 9 victories.

Three Decisions	Results	IMPs	Frequency (x512)
	G-G-G	30	27
	G-G-L	14	135
	G-L-L	–2	225
	L-L-L	–18	125

The most frequent result is a loss of 2 IMPs. This may not matter much in the final victory point tabulation. The significant swings are more frequently to the plus side, 162 versus 125, 56%, for an average gain of 1.5 IMPs on those selected boards. Between gaining 14 IMPs (52% of the time) and losing 18 IMPs, there is an expected net loss of 1.4 IMPs over the long run, but overall there is a 5% chance of scoring 30 IMPs. Thus, with three opportunities the bidders will occasionally gain a very large margin of victory at a relatively low cost, as well as gaining significantly more times than losing significantly.

Four Decisions	Results	IMPs	Frequency (x4096)
	G-G-G-G	40	81
	G-G-G-L	24	540
	G-G-L-L	8	1350
	G-L-L-L	–8	1500
	L-L-L-L	–24	625

We consider 8 IMPs to be a significant amount, so you lose on 52% of the significant deals when four decisions are made. In that sense, you approach a 50-50 split on a frequency basis between gain boards and loss boards. The conclusion is that with close decisions you should bid games, as frequency of failure will not be a major concern.

Next we'll consider bidding games that have a 50% chance of success. First, assume we face ultra-conservative opponents who will avoid such games.

One Decision	Results	IMPs	Frequency (x2)
	G	10	1
	L	–6	1
			Net Gain = 2 IMPs

Two Decisions	Results	IMPs	Frequency (x4)
	G-G	20	1
	G-L	4	2
	L-L	–12	1
			Net Gain = 4 IMPs

Three Decisions	Results	IMPs	Frequency (x8)
	G-G-G	30	1
	G-G-L	14	3
	G-L-L	–2	3
	L-L-L	–18	1
			Net Gain = 6 IMPs

Four Decisions	Results	IMPs	Frequency (x16)
	G-G-G-G	40	1
	G-G-G-L	24	4
	G-G-L-L	8	6
	G-L-L-L	–8	4
	L-L-L-L	–24	1
			Net Gain = 8 IMPs

Clearly it does not pay to be ultra-conservative. With multiple decisions there are more plus boards than minus boards got by bidding the game.

Let's assume that the opponents will bid 50% games 50% of the time. Whether the game makes or not is maximally uncertain, as is whether they will bid it or not. Maximum uncertainty implies that success or failure depends on how the cards were dealt, an unpredictably random process, not on the accuracy of the bidding or on the skill of the declarer. Half the time, when they bid game, they will tie those who always bid game. Otherwise, they lose 10 IMPs a quarter of the time and gain 6 IMPs a quarter of the time, for a net loss of 1 IMP. Does it make sense to play to lose 1 IMP rather than to tie? No.

Let's suppose both teams are neutrally selective, bidding half of their 50% games. Assume the choices are random and independent and the results are random. Half the time both teams make the same decision, so there is no resultant advantage. On one-quarter of the deals, one team gains 4 IMPs and on another quarter, the other team gains 4 IMPs. Perfect balance has been achieved. The greater the likelihood that one team will bid its 50% games, the greater its advantage over a neutrally selective team. Finally, if ideally you always bid games that have a probability of success greater than 50%, under those conditions you can only match those simple souls who blast away and bid some bad (37.5%) games along with all the good (50%) games. Note that bidding systems are based on probable outcomes and seldom provide enough information for a player to estimate the probability of success with great accuracy. So bidding a 50% game or not is more a matter of inclination rather than science, which gets you to the point of decision but doesn't tell you with certainty which way to go on any particular deal. That is the beauty of bridge as a game.

The Expected Number of Decisions

Let's look at the example of an eight-board match within which your side is vulnerable on four boards. On half of those boards your side holds the advantage. We'll assume (without detailed numerical evidence) that the chance of having to make a close decision as to whether to bid game is 1 in 4, an exciting proposition. How many such decisions are expected over one match, two matches, or three matches? That can be calculated using the binominal probability distribution function. Let P(n) represent the probability of n boards requiring a close decision. P(0) indicates the probability of not having to decide.

Probability	**Over 8 Boards**	**Over 16 Boards**	**Over 24 Boards**
P (0)	0.32	0.10	0.03
P (1)	**0.42**	0.27	0.13
P (2)	0.21	**0.31**	0.23
P (3)	0.05	0.21	**0.26**
P (4)	0.00	0.09	0.19
P (5)	—	0.02	0.10

Over an eight-board match the greatest expectation is for one decision (42%). The probability of having to make two decisions is half the maximum (21%). The probability of encountering no such decisions is 32%, resulting in a skew towards the lower number, characteristic of a small number of possibilities. Over two matches the greatest expectation is for two decisions, and over three matches, for three decisions. Over twenty-four boards the probability of having to make two, three or four decisions is 78%. At the beginning of the session it makes sense to assume an attitude of bidding all close vulnerable games. The same applies over sixteen boards.

The situation similar to that considered by Rubens is that where eight boards remain to be played, and your team is in contention for a decent finish, but out of contention for the top spot. The odds are 2:1 that your partnership will have to make one close decision rather than two. If early in the last match you bid game and lose 6 IMPs, that loss may drop your team from 16th place to oblivion as there is little hope of a second chance. If you bid the game and gain 10 IMPs, you reach the dizzying heights of fifth place, and if you get another chance, you may actually gain another 10 IMPs and attain third place, although the odds are well against being so lucky. So what kind of player are you — one who strives for excellence, or one who is afraid of dropping out of the money?

I like people who take risks
— Billie Jean King (b. 1943)

Like the ill-fated Icarus of Greek mythology my preference is to rise as high as possible without fear of precipitously losing elevation. So I adopt the strategy of bidding close games, regardless of the point in the match at which they occur. Using Victory Point scoring each board, regardless of the order in which it is played, contributes to the end result. Many matches are pitifully lost when near the end the leading team grows cautious and attempts to sit on their lead. It is a common occurrence in competitive sports that the eventual winners come from behind at the end by vigorously striving to maximize their gains rather than passively waiting for the leader to make a mistake.

Consider the four-match Victory Point game at our local club. The winning score is usually slightly greater than 60 out of 80. You can attain 60 by the route 15-15-15-15, but that would be unusual since the last match will be against a team that has been playing with luck on their side. If both teams play with caution, your scores may turn out to be 15-15-15-10, which adds to a respectable score, but one that is short of a winning total. So to come first in the event you must try to win the last match by at least 10 IMPs, attaining a sequence of 15-15-15-15. A vulnerable game in the last match is a god-given opportunity to pick up the margin of victory on one board.

Two swings of 6 IMPs, each got by not bidding close vulnerable games, will also achieve a victory point margin of 10, but the odds against two such swings are low. First, you may not get two chances, and second, the chances of gaining 6 IMPs on both is low, being the product of probabilities. Consider the case of a lower limit chance of success.

Chance of a 10 IMP swing on 1 board	0.42 x 0.375 = 0.16
Chance of a 12 IMP swing over 2 boards	0.21 x 0.625 x 0.625 = 0.08

So it is reasonable when going into a final match to attempt to swing for 10 IMPs if the opportunity arises, rather than hope for two chances to gain 6 IMPs by keeping out of close vulnerable games that aggressive opposition may bid. You hope for at least one chance to bid a close vulnerable game and get it right. The first wish has a probability of 68%; the second wish is in the lap of the gods. I look at it this way: if there are no close decisions encountered in a match, my chances of winning the event are reduced.

Chapter 21

GAMES AND SLAMS — TO BID OR NOT TO BID?

Take calculated risks. That is quite different from being rash.
— General George S. Patton (1885-1945)

Two situations in which the expected action of the opponents plays a major part in constructive bidding have to do with vulnerable games and grand slams, the first being common and the second relatively rare. The game situation engenders reckless abandon, the grand slam situation, extreme caution. You would think that holding hands that have the combined assets to make thirteen tricks would be a pleasurable and momentous occasion for any pair, but many nervously anticipate the event more with apprehension than joy. Some come ill-prepared. Alan Truscott in his book, *Grand Slams*, suggested that grand slams that at worst depend on a finesse should be bid, yet it is common enough to hear veteran players advise novices not to bid a grand slam unless they can count fourteen tricks. Larry Cohen has commented on BBO to the effect that grand slams are overvalued in IMPs, as a single board can produce a disproportionate swing that may determine the match winners. In effect, there is widespread feeling that bridge would be better if grand slams didn't exist, and many bid as if they don't. That shortcoming influences the strategy.

Standard bidding methods are poorly suited to exploring grand slam possibilities. If the bidding starts at the two-level, 2NT being the worst case, the auction has eaten up valuable bidding space, so it becomes difficult to obtain sufficient information on which to base a critical decision, especially with regard to the minor suits. This is where more artificial systems have an advantage, always assuming, of course, that the opponents aren't encouraged to interfere, and that the grand slam will not depend on the opening lead or defense.

Grand slam decisions are therefore based partly on the perceived capabilities of the opposition. This is expressed mathematically by PB, the probability the grand slam will be bid. If I am playing a complex relay system, I am aware that against standard bidders I have a great advantage. As the bidding progresses I will be encouraged to move towards a grand slam as my superior methods can land me in a good contract others are unlikely to reach. On the other hand, against opponents with similar methods, I would bid the grand slam in a close decision expecting to tie the board. The theoretical point of neutrality is where it is 50-50 that the opposition will bid the grand slam (PB=1/2), and the probability of making it, PM, equals 17/30 (a 57% chance.)

Vulnerable Game Strategy

Grand slams are the nectar of the bridge gods, whereas vulnerable games are the ham sandwiches of the masses. Sometimes they survive on baloney alone. The neutral point is at PM=3/8, PB=1/2, a point far removed from the point of maximum uncertainty, PB=1/2 and PM=1/2. The dice are heavily loaded in favor of bidding the game. You don't need much hope when bidding on, especially when most opponents will be bidding the game as well, poor as it is. The gain if successful is 10 IMPs, the loss if not is 6 IMPs. The optimal rule is to bid the game if PM>3/8 (37.5%). Above that threshold, on aggregate the expected score is greater for bidding the game. Plainly put, you should bid a vulnerable game on any excuse, the quicker the better. That is not the whole story, as there is a downside. The risk is minimized under the prevailing rule of not doubling mutually agreed contracts at IMPs, so you can hope to enjoy the benefit of making the contract without fearing unduly the consequences of arriving in a hopeless one. This leads to loose action that provides thrills, suspense, anguish and rapture — exactly what wise mothers warn against.

The Full Picture

The full map of the critical decision zone over a range of PB and PM is shown below. The numbers given are the expected scores in IMPs for the game on the left and the partscore on the right. The aggregate for each contract is given below the line. This is the pattern:

Game	**Partscore (ps)**	
S1	S4	(expected gain)
S2	S3	(expected loss)
S1 + S2	S3 + S4	(aggregate)

This matrix, for various values of PM and PB, is shown in the table following:

PM	PB = 3/8		PB = 4/9		PB = 1/2		PB = 5/9		PB = 3/5	
	Game	ps	Game	ps	Game	ps	Game	ps	Game	ps
2/7	1.79	1.61	1.59	1.90	1.43	2.14	1.27	2.38	1.44	2.57
	-2.68	-1.07	-2.38	-1.27	-2.14	-1.43	-1.90	-1.59	-1.71	-1.71
	-0.89	0.54	-0.79	0.63	-0.71	0.71	-0.63	0.79	-0.57	0.86
1/3	2.08	1.50	1.85	1.78	1.67	2.00	1.48	2.22	1.33	2.40
	-2.50	-1.25	-2.22	-1.48	-2.00	-1.67	-1.78	-1.85	-1.60	-2.00
	-0.42	0.25	-0.37	0.30	-0.33	0.33	-0.30	0.37	-0.27	0.40
3/8	2.34	1.41	2.08	1.67	1.88	1.88	1.67	2.08	1.50	2.25
	-2.34	-1.41	-2.08	-1.67	-1.88	-1.88	-1.67	-2.08	-1.50	-2.25
	0	0	0	0	0	0	0	0	0	0
4/9	2.78	1.25	2.47	1.48	2.22	1.67	1.98	1.85	1.78	2.00
	-2.08	-1.67	-1.85	-1.98	-1.67	-2.22	-1.48	-2.49	-1.33	-2.67
	0.69	-0.42	0.62	-0.49	0.55	-0.55	0.49	-0.62	0.44	-0.67
1/2	3.12	1.13	2.78	1.33	2.50	1.50	2.22	1.67	2.00	1.80
	-1.88	-1.88	-1.67	-2.22	-1.50	-2.50	-1.33	-2.78	-1.20	-3.00
	1.24	-0.73	1.11	-0.89	1.00	-1.00	0.88	-1.11	0.80	-1.20

The comfort zone, in which the criteria to maximize the gain and minimize the loss both require that the higher contract be bid, consists of the boxes below the critical boundary line at PM=3/8. Two exceptions are highlighted. On the left we minimize the loss by staying in the partscore, but the gain is 0.41 IMPs against a potential loss of 1.53 IMPs, so even if the opposition is not bidding this game, the odds favor doing so at IMP scoring. On the right we maximize the gain by staying in a partscore when most will be bidding the game. We gain 0.22 IMPs, gambling a potential loss of 1.33 IMPs. Clearly we should go with the opposition on this one in order to minimize the potential loss. This is entirely consistent with the optimal rule.

What if in their enthusiasm the opposition has bid a filthy game, represented by the line PM=1/3? If the probability of their doing so lies between PB =4/9 and PB=5/9, there is less than one-tenth of an IMP to be lost on average by bidding on. This reduces the penalty for being foolish. The effect is to degrade the game of bridge in such a way that good judgment based on hand evaluation is not well rewarded — boldness is.

Minor-Suit Slams at Matchpoints

Recently at my club, a local expert examined the scores and shook his head sadly after my partner and I bid a cold 6♣ against him. 'People just don't bid minor-suit slams anymore', he moaned. Yes, it was an unfair result, but that is matchpoints — a game where many players are happy to dumb down and go with the field. Why is that, after we have been taught that we should bid a slam when the chance of its making is better than 50-50? Let's have a mathematical look at this situation.

A bidding system, like cheap insurance, doesn't cover all contingencies. There is a built-in bias of spades before hearts, hearts before diamonds, and diamonds before clubs, which applies to partscores and games, but is less relevant to slams. If you choose to bid a slam, normally you opt for the safest strain. Trying to get to a minor-suit slam involves swimming against the current of popular practice.

In an earlier chapter we noted that in a team game you score a zero if you end up in the same contract as the opponents, so on the face of it there doesn't appear to be anything to lose by bidding a higher-scoring contract when the probabilities favor it. At matchpoints, there is a tangible loss compared to the near-average score that you get by playing in a common contract. Under such conditions some players think as follows:

1) The final score is an accumulation of matchpoints won over several rounds.
2) Winning bridge entails never risking a bottom score.
3) If I end up playing in 3NT, I will have lots of company and may score an average.
4) If I end up in slam, I will have little company, and may score a bottom, in which case I will fall badly behind the field.
5) Rather than trying to win matchpoints on this deal, I will wait for a situation where I can profit without risk from my superior playing skills.

Doing the Math

We'll assume that there are just two alternatives: to bid a small slam or to stop in game. Let's have these symbols take on the specified meanings:

PM	Probability of the slam making
PG	Probability the opponents will stop in game
YB	You Bid the slam
YD	You Don't bid the slam
TB	The opponents Bid the slam
TD	The opponents Don't bid the slam

Here are the expected scores under the various conditions:

When the slam makes
YBTB 1/2 x PM x – (1 – PG)
YBTD PM x PG
YDTB 0
YDTD 1/2 x PM x PG

When the slam fails
YBTB 1/2 x (1 – PM) x (1 – PG)
YBTD 0
YDTB (1 – PM) x (1 – PG)
YDTD 1/2 x (1 – PM) x PG

The difference between expected scores when you bid the slam and when you don't gives the result: YB – YD = PM – 1/2.

This expected score difference > 0, when PM> 1/2.

The condition for achieving on average a better result by bidding the slam is independent of whether or not the opponents bid it. On that basis you should bid slam when the probability of making is more than 50%, just like the books tell us.

PM is a concept of convenience. Ideally PM can be calculated with a computer program once we feed in declarer's hand and the dummy. The calculation involves the probability of the various card combinations for the opposition. (This assumes perfect defense and perfect play. Of course, that is not realistic as the opening lead is often critical, but it is dangerous to bid a slam on the assumption you will benefit from imperfect defense.)

At the table we have only an estimate of PM available. As the auction progresses our estimate changes as more information is received. This also affects PG, as the higher the estimate of PM the less likely one is to prefer game to slam. The estimates depend on the nature and quality of the information being processed. That in turn depends on the structure of the bidding system. It is an over-simplification to assume that PM depends solely on the HCP total, but some believe that a partnership should bid slam only when holding at least 33 combined HCP, because that is what they have been taught. Usually, in a cooperative slam auction, the estimated PM is related to the number of bids being made, on the basis that one player or the other will end the process if he thinks game is the limit.

In a matchpoint game, the one deal is played at several tables by different pairs, so the average score is in reference to the distribution of scores across the field on that particular board. If the field is playing the same simple bidding system, it may be easier to estimate PG than PM. In a good field, the two are closely related.

The Effect of Superior Technique

PM is the theoretical probability that twelve tricks can be taken, but taking them may require a certain delicacy of technique, such as a squeeze. Perhaps not all players in the field will be capable of reading the situation, so you can gain matchpoints against the inept pairs when you wrongly stay out of slam, but manage that elusive twelfth trick. Let's assume that a player knows he is a better technician than one-quarter of the field. In the YDTD category when slam makes on a squeeze, say, he will make twelve tricks in a game, but one-quarter of the declarers will hold themselves to eleven. Against those opponents he scores a full matchpoint, so the overtrick is put on the same footing as the slam bonus. This favors individual effort. Now, the break-even point is achieved when PM equals 1/2 + (1/8) x PG. If PG is 3/4, so that the slam is unlikely to be bid across the field in the ratio of 3:1, the justification for bidding the slam requires PM to exceed 60%, even though a good player is quite capable of taking the necessary twelve tricks when conditions allow it.

Some players consider this effect to be a contamination. They consider that deliberately going against one's better judgment because the field contains a few inept players, and, moreover, profiting from it, degrades the game. So say some — but is that the case? Luckily bridge is not politics, so you needn't dumb it down to be successful. You may choose to ignore the effect of the field and imagine you are playing against the best at all times. That noble approach is not optimal. It makes good sense to vary your play according to what the field is likely to do on any given board. That adds an intriguing extra dimension to the matchpoint game.

Major-Suit Slams

The same mathematics applies to major-suit games and slams, but there is less need for adjusting to the field. The fact that so few players reach a good minor-suit slam indicates that either: (1) everyone is convinced they are better players than most, so don't need to bid slams, or (2) most systems are deficient in this aspect. I strongly suspect the latter, because the problem does not occur to the same extent with major-suit slams. Most systems (and especially the currently popular 2/1) are geared towards notrump and major-suit contracts and most responders know when and how to look for slam possibilities right from the beginning. That was the whole idea behind 2/1, wasn't it? But now the main effect is to simplify the bidding for mediocre players by narrowing the number of choices they are expected to make — which brings us back to the beginning of this book.

AFTERWORD — BACK TO BASICS

What you don't know won't hurt you.
— Old adage, largely discredited today.

'Bridge is a partnership game.' So observed Al Roth, who was notorious for his violent arguments at the table with his regular partner, Tobias Stone. One of the problems that must be (but may not be) resolved during an auction is: who decides? One player describes and the other decides, but which and when? When asked, my usual advice is: 'let the strong hand decide', to which many vehemently object.

Recently I heard someone approach our club expert with a sad story. 'My partner opened 2♣ and I jumped to 6NT on a balanced 12 HCP and it went down.' His partner turned out to have a huge two-suiter — not what this man had envisaged at all.

When we say that you should choose to maximize the gain when it is right to do so, that means you need at least a 50% chance of being right, the percentages being based on what you know at the time of decision. Of course, you never know everything, only what the system has allowed you to know with partner's cooperation. His impulse to make the decision was largely psychological, and one wonders if he will ever learn that the stronger hand is usually the one best placed to make the final decision. Probably not.

Different systems give different information, as do different partners. Let's look at what might be referred to as a bad slam recently bid in a matchpoint game at the local club. The question is: did the opening bidder make the right decision given what he knew at the time?

♠ A ♥ A K 10 5 3 ♦ K Q J 4 2 ♣ 8 6	N W E S	♠ Q J 9 ♥ 9 8 7 ♦ A 9 3 ♣ A Q 5 3
4 losers		8 losers

1♥	2♣
2♦	2♥
2♠	3♦
4NT	5♥
6♥	pass

The hands should be well suited to 2/1 methods, but problems quickly arise, at the first response. Responder can treat his hand as a three-card limit raise and start with 1NT, forcing, planning to rebid 3♥, leaving the decision up to partner whether or not to carry on to game or higher, or he can bid 2♣ as a game force and have a greater input into the final decision. He has 8 losers, 3=3=3=4 shape, but 13 HCP. In accordance with 2/1 dogma the ♠J tips the scales in favor of 2♣, even though an 8-loser hand is hardly adequate to force to game opposite a possibly light opening bid with 7 (or even 8) losers.

The choice of 2♣ moves opener with 4 losers in the direction of a slam in a red suit. The ♦J is an important card. Slam is almost inevitable after responder shows heart support with the ♦A on the side. Opener takes charge with RKCB and must make the final decision. What does he know at this point? Does slam appear to be better than 50%?

Opener knows responder has three-card support, two aces and no ♥Q. He can expect at least one loser in the trump suit. With eleven tricks in the bag the twelfth may come from the ♠K or the ♣K or a club finesse. The bidding is too high to find out without bidding the slam regardless, so optimistically opener bids 6♥, a contract that on the surface may depend on a club finesse (50%) and not losing two heart tricks (90%). The pressure of the 2♣ response has pushed him in that direction.

In theory he shouldn't bid the slam as it appears to have less than a 50% chance of making. In practice the club finesse worked. There were fourteen pairs whose results were: six in 6♥ making, three in 6♥ failing, two in 4♥ making six, two in 6NT failing; and one in 7♥, failing. Bidding and making 6♥ was worth 11 matchpoints, whereas stopping safely in 5♥ and making six was worth 7 matchpoints, an average score.

The really bad scores were obtained by three above-average pairs. My guess is that responders 'corrected' 6♥ to 6NT partly to protect their club tenace, not knowing that the opener had a singleton spade. In other words it was a failure in the establishment of the captaincy, at least in the minds of those involved. One can imagine the comment: 'Sorry, partner, I thought you'd have more in order to bid slam.' This is a trait of the point-counter who can't stand being taken out of the loop. We don't believe he's sorry, rather we believe that he will do the very same the next time. At the time of decision responder could have had a better hand than he had shown. It was his decision to respond 2♣ on an eight-loser hand that induced opener to push on. The system structure did the rest.

Yin: the Shady Side of the Mountain

It is not correct to conclude that the slam depended on a 50% finesse and a 3-2 split in trumps. This is the argument one often hears at the table or from BBO experts who warn against bold action. What one should keep foremost in mind is the number of tricks needed and where they might come from. In slam declarer requires six tricks in hearts and clubs by way of four hearts and two clubs or five hearts and one club. One must focus attention on the hidden factors, very often in the guise of spot cards that are not counted within the bidding system rules, yet are very important in determining the eventual outcome of the process. A hand is made up of thirteen cards. Only a few are control cards (Yang); most are hidden (Yin.) The Yin-factors make their presence most strongly felt in the probabilities. Like water, their natural movement is away from disparity and towards uniformity.

To simplify, suppose a club is led and declarer puts in the ♣Q from dummy. If this wins the trick, declarer needs only four heart tricks, a 90% chance. If the ♣Q were to lose to the king he would need five tricks from hearts, a 22% chance. Rather than a 3-2 split, declarer hopes for a 4-1 split as that gives the better chance of finding the ♥QJ together in one hand. Put those chances together and we find there is a 56% chance of success overall. The hidden factor is the ♥987 in the dummy. If the dummy's hearts were ♥642, there is an 83% chance of at least four tricks and only a 14% chance of five tricks, 48% overall. So responder was correct in his initial assumption that his hand was worthy of further consideration.

One other point: if declarer knows he needs only four tricks in hearts, his best play when dummy's trumps are poor is to cash a top heart before finessing the ♥10. With ♥987 his best play for four or five tricks is to finesse early and often, which is convenient as the transportation is problematic.

Information from the Auction

There are several factors apart from HCP that affect the number of tricks declarer can expect to make. Everyone knows that. The question is: how does one incorporate these factors into a bidding system that has boundaries defined largely by HCP? This is the delicate matter of hand evaluation. If both players adjust in the same direction, the bidding structure comes under strain.

The most important factor is trump quality. Having a good trump suit (nine-card fits are so much better than eight-card fits), allows for a variety of maneuvers in the side suits. For one thing the drawing of trumps may not be critical, so the timing is on declarer's side. The trump suit itself may provide the necessary transportation. In the hand above, the lack of trump quality was a problem that wasn't discovered early in the bidding.

Another important property is the number of controls. Declarer may need the timing to perform maneuvers which require uninterrupted transportation. In the above hand a ♠K would be so much better to find in dummy than the queen-jack. The ♠QJ could be useful, most obviously in a ruffing finesse, but the transportation isn't ideal. The ♦9 was an entry as the cards were dealt, but not a chance declarer would happily depend upon.

A third factor that is essential in the matter of hand evaluation is shortness, a Yin characteristic as it does not in itself generate tricks but may prevent losses. Often a RKCB bid is a means of seizing the captaincy while withholding information from the opponents. Nondisclosure withholds critical information from partner as well, so he should not take it upon himself to make the final decision. In this case the ♠QJ9 lost value opposite the singleton ♠A, but responder may have expected ♠Ax opposite, which would have complemented his holding in a NT contract. He didn't know.

The 2/1 auction was not efficient in disclosing the essential characteristics. The trump quality was not known until the bid of 5♥ revealed the absence of the ♥Q. The number of controls was unknown as there was uncertainty about the black kings. All this came too late to allow for any judicious adjustment. One could argue that the leap to 4NT was premature, but it would still be hard for opener to get all the information he requires before committing to slam.

The Precision Auction

Here is a Precision auction on the same cards as an example of how the essential factors are addressed. The opening bid is 1♣, showing 16+HCP.

West		East
♠ A	N / W E / S	♠ Q J 9
♥ A K 10 5 3		♥ 9 8 7
♦ K Q J 4 2		♦ A 9 3
♣ 8 6		♣ A Q 5 3
4 losers		8 losers

1♣*	2NT
3♥	4♣
4♦	4♥
6♥	pass

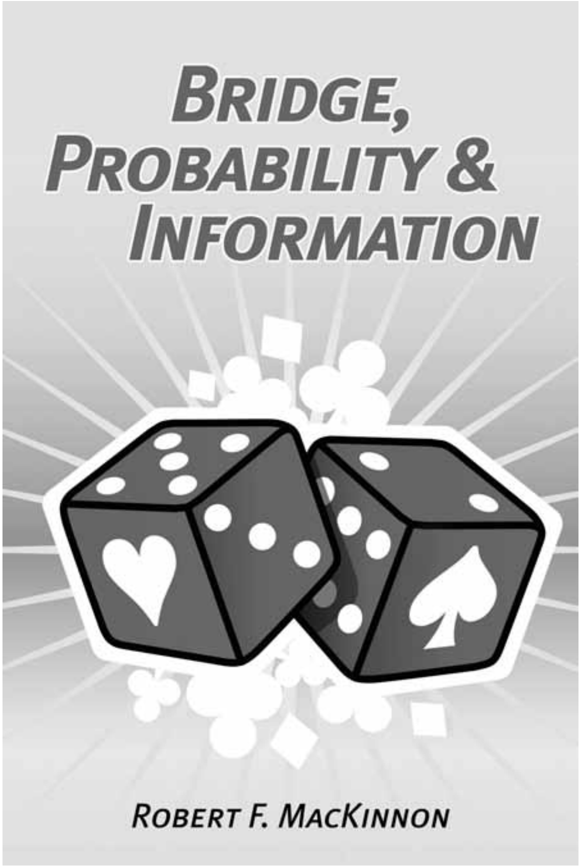
Responder immediately shows a flat hand within the range of 11-13 HCP. Normally the hand has 8 losers with 4 controls, which was the exact case here. The 4♣ bid shows a fit in hearts with values in clubs. This is an optimistic bid, but within the prescribed limits. Now 4♦ is waiting and 4♥ denies extras (because of the lack of a heart honor.) At this point opener was clearly in charge. Rather than seek further information, opener jumped to 6♥ with 4 losers opposite an expected 8 losers. The hands have at most 29 HCP in total, eight total trumps, and dummy will not have any shortage. Responder had no problem passing 6♥ as he had already shown his values.

The opener could argue that slam should be bid with 4 losers opposite 8 losers, but that applies best when there is a nine-card trump fit. There was no way to find out if responder held ♥987 or ♥642. Opener was encouraged by the 4♣ cuebid, which indicated some slam interest, promising good controls and decent trumps. However, perhaps the trumps were not as good as he might have expected.

In both the 2/1 auction and the Precision auction, although the responder did not make the final decision, it was the earlier choice by the weaker hand to show values in clubs that tipped the balance in favor of bidding the slam. That choice should to some extent have depended on the quality of the heart support. In 6 cases out of 14, ambition was matched by capacity, but just barely.

All is well that endes well
— English proverb, John Heywood (1497-1580)

More from Robert F. MacKinnon



While firmly rooted in sound mathematics, this book aims to be accessible to any bridge player. Concepts such as Vacant Spaces, Restricted Choice, and how splits in one suit affect the probabilities in other suits, are discussed in depth. Readers will emerge with a better understanding of how to apply these ideas at the table, and with some very practical rules and advice that will make them more successful players.

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ADVANCED

In Bob MacKinnon's bestseller, *Bridge, Probability and Information*, the author introduced readers to the mysteries of information theory and Bayes Theorem, and their surprisingly practical applications for bridge players. In this sequel, he takes these same ideas further, exploring the application of the concepts to opening leads, declarer play, bidding theory, hand evaluation and the correct strategy at different forms of scoring.

Along the way you'll meet a girl named Florida, whose sister is remarkably probable. You'll discover the right way to play *Let's Make a Deal* if Monty Hall's game had involved four doors and not three. And you'll find out which partner to choose if you absolutely must win that special club matchpoint game: Steady Eddy, who usually finishes a board or two above average, but occasionally much better, or Swinging Sam, who is as likely to score a bottom as a top on any deal.

Praise for *Bridge, Probability and Information*

"A combination of charm and savvy. This is not a dry collection of tables and formulas. MacKinnon liberally sprinkles the text with amusing quotes and interesting anecdotes about math and bridge."

— ACBL Bulletin

"A superb treatment of bridge probability and statistics. We do the best we can with the information we have, and this book teaches you how to improve your 'guesstimating' at the bridge table."

— The Denver Post

ROBERT F. MacKINNON (Victoria, Canada) is the author of *Samurai Bridge, Richelieu Plays Bridge*, and *Bridge, Probability and Information.*

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