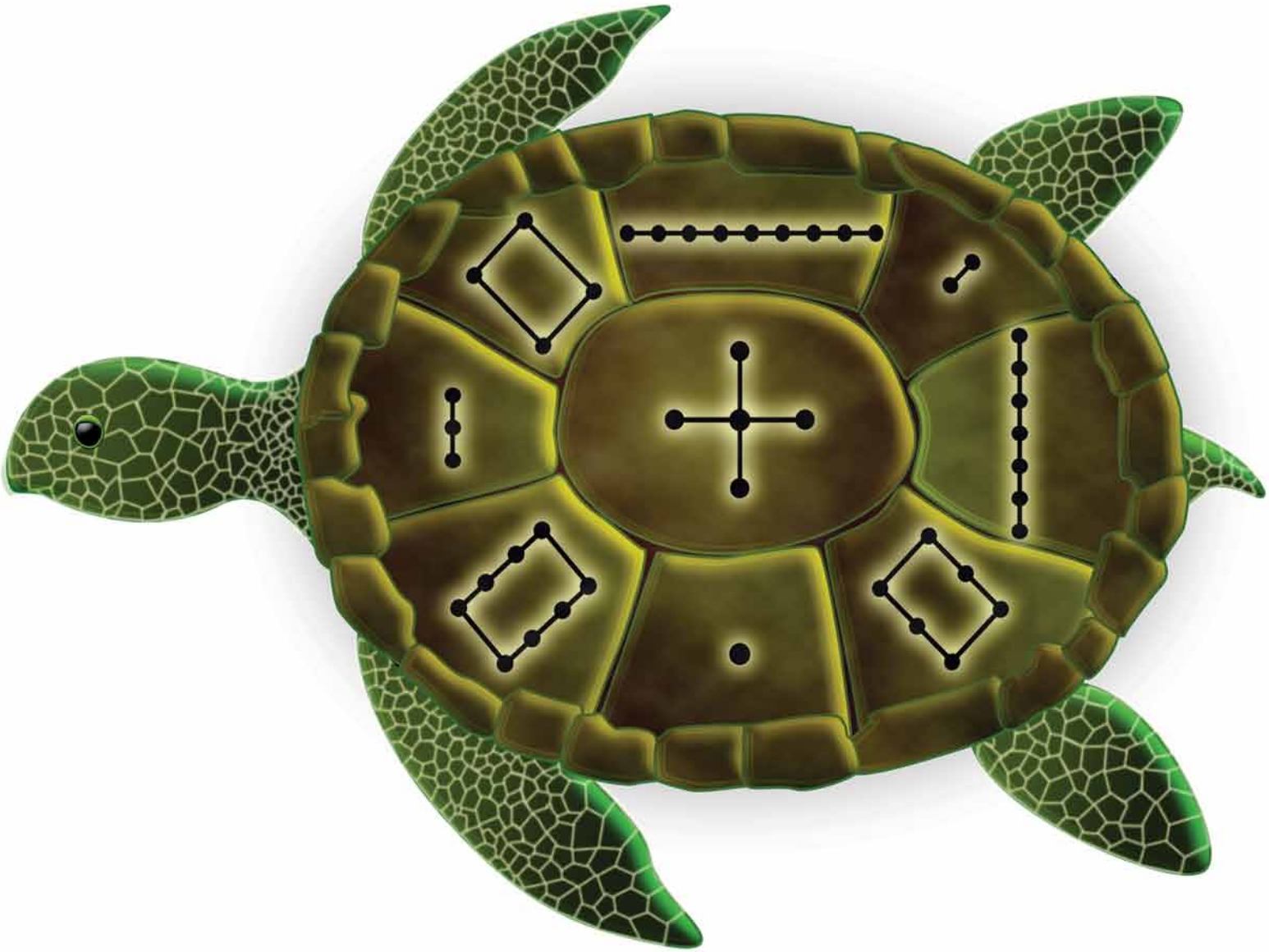
Ian McKinnon

Duplicate Bridge Schedules, History and Mathematics



AN HONORS eBOOK FROM MASTER POINT PRESS

By the same author

Bridge Directing Complete, 1979

Duplicate Bridge Schedules, History and Mathematics

by
Ian McKinnon

Edited by
Ron Klinger

Foreword by
Dr. Ross Moore

Text © 2012 Ian McKinnon

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior permission of the copyright owner.

Honors eBooks is an imprint of Master Point Press. All contents, editing and design (excluding cover design) are the sole responsibility of the author.

Master Point Press
331 Douglas Ave.
Toronto, Ontario, Canada
M5M 1H2
(416) 781-0351

Email: info@masterpointpress.com

Websites:
www.masterpointpress.com
www.bridgeblogging.com
www.teachbridge.com
www.ebooksbridge.com

ISBN: 978-1-55494-525-2

Layout and Editing: Ian McKinnon
Cover Image: Stuart Blythe
Cover Design: Lisa-Marie Wilson

This book is dedicated to my mother.
Born in 1917 and she still plays a great game of cards.

Graphic design work and the enhancements of the old photographs was done by Stuart Blythe.
The turtle on the front cover was designed and produced by Stuart Blythe (2012).

Summary contents

PART 1 SQUARES 5

PART 2 DUPLICATE 17

PART 3 WHIST SCHEDULES 35

PART 4 MODERN SQUARES 55

PART 5 SCHEDULE CATEGORIES 63

PART 6 BALANCE DEBATE 81

PART 7 PAIRS' SCHEDULES 95

PART 8 TEAMS' SCHEDULES 213

PART 9 INDIVIDUALS' SCHEDULES 253

PART 10 SCISSOR SCHEDULES 285

PART 11 TEAMS OF MANY PAIRS 341

PART 12 LARGE PAIRS' MOVEMENTS 349

PART 13 ADDITIONAL INFORMATION 365

Full contents

FOREWORD XVII
PREFACE XIX
SPECIAL THANK YOU XXIII
INTRODUCTION 1
PROLOGUE 3

PART 1 SQUARES 5

1.1 LO SHU SQUARE 6
1.2 MELANCHOLIA I 7
1.3 PLEASING PROBLEMS 8
1.4 SPREADING THE WORD 9
1.5 TODAY'S RECREATIONS WITH SQUARES 10
1.6 THE HEADMASTER OF MATHEMATICS 11
1.7 USING LATIN SQUARES 15

PART 2 DUPLICATE 17

2.1 DUPLICATE BRIDGE 18
2.2 THE FIRST DUPLICATE 18
2.3 CAVENDISH, 1857 19
2.4 THE FIRST DUPLICATE IN THE U.S.A. 22
2.5 THE ALLISON IMPROVEMENT 23
2.6 THE FIRST DUPLICATE TOURNAMENT 24
2.7 THE JOHN T. MITCHELL STORY 26
2.8 THE FIRST BOOK ON DUPLICATE 27
2.9 THE NEED FOR BOARDS 29
2.10 DEVELOPMENT OF TRAYS AND BOARDS 31

PART 3 WHIST SCHEDULES 35

3.1 THE FIRST SCHEDULES 36
3.2 ELAIKIM HASTINGS MOORE 38
3.3 THE SAFFORD SYSTEM 40
3.4 HOWELL'S LAW 41
3.5 THE MITCHELL SYSTEM 44
3.6 EARLY MITCHELL MODIFICATIONS 46
Hadlock modification 46
Mitchell modification 47
Baker modification 47
Howell modification 48
3.7 THE CLAY UNIVERSAL SYSTEM 49
3.8 CLAY MODIFICATIONS 51
Howell modification 51
Barney modification 51
3.9 SAFFORD VERSUS HOWELL 53

PART 4 MODERN SQUARES 55

4.1 THE ROOM SQUARE 56
4.2 THE ROUND-ROBIN TOURNAMENT 57
4.3 THE HOUSE SQUARE 58
4.4 THE HOWELL DESIGN 59
4.5 THE BRIDGE DESIGN 60

PART 5 SCHEDULE CATEGORIES 63

5.1 THE SAFFORD COMPARATIVE SYSTEM 64
5.2 THE HOWELL SYSTEM 69
5.3 WHITFELD SCHEDULES 73
5.4 THE COOK PLAN 75
5.5 THE SCHEVENINGEN SYSTEM 76
5.6 THE SNOW SYSTEM 76
5.7 MOVEMENT CLASSES 77
Safford class 78

Mitchell class *78*
Howell class *78*
Barney class *79*
McKinnon class *79*

PART 6 BALANCE DEBATE 81

6.1 COMPARISON OF RESULTS 82
6.2 THE FIRST PROPOSITION 82
6.3 BALANCED INDEPENDENT BLOCK DESIGN 83
6.4 BALANCED MOVEMENTS 85
6.5 THE MATHEMATICS OF BALANCE 88
Measure of competition *88*
Defining balance *88*
Movement calibre *89*
Calibre of two winner movements *90*
Imbalance in Individual movements *90*
Seeding players *91*

PART 7 PAIRS' SCHEDULES 95

7.1 FURTHER MITCHELL MODIFICATIONS 96
More on Baker modification *96*
Share-and-relay modification *97*
Simple half-table modification *99*
Displacement or bump modification *99*
Rover modification *99*
1½ table appendix modification *101*
2 table appendix modification *101*
Stationary board 2-table appendix modification *102*
Ewing modification *103*
Web modification *103*
Web hesitation modification *104*
Pivot modification *105*
Modifications in small movements with half table *106*
Beynon appendix modification *107*
Rover-table modification *108*
Bowman-Ewing modification *108*
Relay-rover table modification *109*
Web rover-table modification *109*
3-table appendix modification *111*
5-table appendix modification *112*
Coffin modification *113*
Parker modification *114*
Extra-board modifications *115*
Hesitation modification *116*
Twin modification *117*
Stagger modification *118*
Trisect modification *118*
Snow modification *120*
Appending-movements modification *123*
Incomplete Mitchell movements *125*
7.2 STATIONARY-BOARD MITCHELLS 143
Stagger Mitchell *144*
Double-weave Mitchell *144*
Oddly-even Mitchells *145*
Self-orthogonal Mitchells *151*
7.3 SWITCHED MITCHELLS 154
Low numbers of tables *154*
Odd numbers of tables *155*
Even numbers of tables *157*
7.4 MODERN HOWELL MOVEMENTS 160
Comparison of pairs *163*

Deriving Howell movements *165*
Producing Howell movements *166*
Howells and the Snow method *174*
Double (or interwoven) Howells *178*
7.5 INCOMPLETE HOWELLS 182
Half table issue *182*
Appendix table *182*
Appendix pair *182*
¾-Howells *183*
Incomplete interwoven Howells *185*
Incomplete two session Howells *187*
7.6 FLOWER MODIFICATION 193
7.7 EXTRA-COMPLETE HOWELLS 194
Expanded Howell *194*
Rover-board Howell *195*
Extra-board Howell *196*
7.8 TWO SESSIONS IN HOWELL MOVEMENTS 199
Worger two Double Howells *199*
7.9 THREE SESSIONS IN HOWELL MOVEMENTS 201
10 pairs, 3 sessions *201*
16 or more pairs over 3 equal sessions *202*
Three sessions and three groups (Snow method) *203*
7.10 FIVE SESSIONS IN HOWELL MOVEMENTS 204
7.11 HYBRID PAIRS MOVEMENTS 204
Two-session movements *204*
Two-sessions, one round short *206*
Three-sessions movements *208*
7.12 THE AMERICAN WHIST LEAGUE MOVEMENT FOR PAIRS 210
Odd number of tables *210*
Even number of tables *211*
Half tables *211*
Rover pairs *211*
Inverted movement *212*

PART 8 TEAMS' SCHEDULES 213

8.1 ROUND-ROBIN TEAMS 214
Bicyclic round-robin schedules *217*
Oddly-even House round-robin schedules *218*
45 teams round-robin schedule *220*
53 and more odd teams round-robin schedules *222*
8.2 THE AMERICAN WHIST LEAGUE SYSTEM 224
Two stanza modification *224*
Same-board whist modification *225*
Split modification *226*
Double-skip modification *227*
Stationary-board modification *227*
8.3 NEW ENGLAND RELAY SYSTEM 229
8.4 HASLER SYSTEM 230
8.5 CLAY SYSTEM 231
8.6 TEAM MITCHELL SYSTEM 233
Worger modification *233*
Stationary-board modification *234*
Two-stanza modification *235*
Same board two stanza modification *235*
8.7 MCKINNON MODIFICATION 237
Hanner-Snow modification *239*
8.8 THE STAGGER SYSTEM 240
Part A stagger movement *241*
Part B stagger movement *241*
Modification *241*
8.9 PATTON SYSTEM 241

8.10 DOUBLE-HOWELL SYSTEM 242
8.11 THURNER SYSTEM 244
8.12 MCKINNON VARIATIONS 245
Oddly-even teams *245*
Sixteen teams *246*
8.13 SWISS SYSTEM 247
Odd number of teams *251*
Organization suggestions *251*
8.14 TEAMS OF THREE 252

PART 9 INDIVIDUALS' SCHEDULES 253

9.1 MITCHELL TYPE 254
Prime tables Mitchells *254*
Appendix Mitchell type *255*
Non-prime tables Mitchells *256*
6, 10 and 14 Tables *258*
9.2 HOWELL TYPE 261
1 group *262*
2 Groups interwoven *266*
2 Groups mingled *266*
3 Groups *267*
4 Groups *269*
5 Groups *269*
9.3 IRREGULAR TYPE 269
12 players and 5 rounds *269*
12 players and 6 rounds *270*
12 players and 9 rounds *270*
12 players and 11 rounds *270*
13 players and 6 rounds *271*
14 players and 6 rounds *271*
16 players and 5 rounds *271*
16 players and 6 rounds *271*
16 players and 15 rounds *272*
The Shomate movement *272*
9.4 COMBINATION TYPE 276
McKenney-Baldwin *276*
52-players Mitchell-Howell *277*
4 Sessions with interwoven Howells *277*
9.5 SNOW METHOD 279
Whitfeld individual schedules *279*
Hanner schedules *279*
Two groups in parallel *281*
9 players and 9 rounds *283*

PART 10 SCISSOR SCHEDULES 285

10.1 COMPLETE COUPLING ROUND-ROBIN SCHEDULES 288
10.2 EVEN NUMBERS OF TEAMS 290
Two teams *290*
Four teams *290*
6 teams *291*
8 teams *292*
10 teams *292*
12 teams *293*
14 teams *293*
18 teams *293*
20 teams *294*
24 teams *294*
10.3 ODD NUMBERS OF TEAMS 295
Three teams *295*
5 teams *296*
7 teams *297*

9 teams *298*
11 teams *299*
13 teams *300*
15 teams *300*
17 teams *300*
19 teams *300*
21 teams *300*
23 teams *301*
25 teams *301*
10.4 TWO SESSIONS, HALF EVEN 301
12 teams, two sessions of 10 rounds *302*
16 teams, two sessions of 14 rounds *303*
20 teams, two sessions of 18 rounds *303*
24 teams, two sessions of 22 rounds *304*
28 teams, two sessions of 26 rounds *304*
36 teams, two sessions of 34 rounds *305*
40 teams, two sessions of 38 rounds *305*
10.5 TWO SESSIONS, HALF ODD 306
10 teams, two sessions *306*
14 teams, two sessions *307*
18 teams, two sessions *308*
22 teams, two sessions *309*
10.6 TWO SESSIONS, ODD NUMBERS OF TEAMS 309
Two sessions, technique one *309*
Two sessions, technique two *313*
Two sessions, technique three *319*
10.7 THREE SESSIONS 320
13 teams, three sessions of 8 rounds *320*
15 teams, three sessions of 10 rounds *321*
16 teams, three sessions of 10 rounds *322*
18 teams, three sessions of 12 rounds *324*
19 teams, three sessions of 12 rounds *325*
21 teams, three sessions of 14 rounds *326*
10.8 FOUR SESSIONS 328
16 teams, four sessions *328*
20 teams, four sessions *329*
24 teams, four sessions *330*
28 teams, four sessions *331*
32 teams, four sessions *332*
36 teams, four sessions *334*
40 teams, four sessions *335*
10.9 PAIRS' BICYCLIC SCHEDULES 337
6 tables, 10 rounds *338*
8 tables, 14 rounds *338*
10 tables, 18 rounds *338*
12 tables, 22 rounds *338*
14 tables, 26 rounds *338*
18 tables, 34 rounds *339*
20 tables, 38 rounds *339*
24 tables, 46 rounds *339*

PART 11 TEAMS OF MANY PAIRS 341

11.1 TWO TEAMS OF PAIRS 342
Two teams of an even number of pairs *342*
The stagger movement *343*
Two teams of an odd number of pairs *343*
11.2 MULTIPLE TEAMS OF PAIRS 344
Three teams of four pairs *344*
Three teams of six pairs *345*
Three teams of eight pairs *345*
Four teams of two pairs *345*

Four teams of three pairs ... *345*
Four teams of four pairs ... *345*
Four teams of five pairs ... *346*
Four teams of seven pairs ... *346*
Five teams of two pairs ... *346*
Five teams of four pairs ... *346*
Six teams of two pairs ... *347*
Six teams of three pairs ... *347*
Six teams of five pairs ... *347*
11.3 TEAMS OF EIGHT PLAYERS ... 347
Two teams of eight ... *347*
Multiple teams of eight ... *348*

PART 12 LARGE PAIRS' MOVEMENTS ... 349

12.1 MULTI-SECTION CONGRESS EVENTS ... 350
Congress movements ... *350*
12.2 BLOCK MITCHELL ... 351
Five to eight tables ... *351*
Three tables ... *352*
Four tables ... *352*
Five tables ... *352*
Six tables ... *352*
Seven tables ... *353*
Eight tables ... *353*
12.3 BOWMAN FOR LARGE PAIRS MOVEMENTS ... 353
12.4 HESITATION SUPER-BOWMAN ... 355
12.5 COMBINED HOWELL-MITCHELL MOVEMENT ... 356
Perfect Howell-Mitchell ... *356*
Stationary-Boards Howell-Mitchell ... *357*
Imperfect Howell-Mitchell ... *358*
12.6 DOUBLE HOWELL PLUS MITCHELL MOVEMENT ... 359
Even-tables Mitchell ... *359*
Odd-tables Mitchell ... *360*
Snow modification ... *360*
12.7 DOUBLE MITCHELL-HOWELL MOVEMENT ... 362
12.8 DOUBLE COMBINED HOWELL-MITCHELL MOVEMENT ... 363

PART 13 ADDITIONAL INFORMATION ... 365

EPILOGUE ... 366
APPENDIX A ... 367
Pairs' movements ready reckoner ... *367*
APPENDIX B ... 372
John Manning's 1979 paper ... *372*
APPENDIX C ... 381
Constructing 17 teams CCRRS over 2 sessions ... *381*
APPENDIX D ... 383
Constructing self orthogonal latin squares (SOLS) ... *383*
APPENDIX E ... 389
Balanced tournament design ... *389*
APPENDIX F ... 391
4 Table appendix modification ... *391*
APPENDIX G ... 392
The Clay universal movement ... *392*
APPENDIX H ... 394
Olof Hanner semi-barometer ... *394*
APPENDIX I ... 395
Solutions ... *395*
APPENDIX J ... 396
Golf problems ... *396*
20-player 5 × 5 affine plane ... *400*
8-player Fano plane ... *401*

APPENDIX K 402
Kirkman's 'Schoolgirl Problem' *402*
APPENDIX L 403
Ross Moore's 1992 paper *403*
APPENDIX M 412
Why the best man loses *412*
APPENDIX N 419
CCRRS Howell designs *419*
GLOSSARY 421
BIBLIOGRAPHY 422
References *422*
INDEX 424

Reproduced from the original picture by Maxfield Parrish,
The whist reference book, William Mill Butler 1898

'YE ROYALL RECEPCIOUN'

Ye King and Quene with plesaunce looke
Uppon ye grete Whiste Ref 'rence Booke.
"Now, wyffe," quoth he,"let all ye playeres
We meet in bataile say their prayeres!"
Whereat ye solemn Knaves bowe low;
And quoth ye Quene, "Aye, truly so!"

(Chaucer Redivivus.)

List of illustrations

'YE ROYALL RECEPCIOUN' printed in colours, from original. Page xiv

First duplicate tray, Kalamazoo, 1891. Colour photograph. Page 5

Lo Shu turtle. Computer drawn colour image. Page 6

Set of 12 Kalamazoo duplicate whist trays, circa 1896. Colour photograph. Page 17

Cavendish's 'card-table talk' published in 1897. Henry Jones, B&W photograph. Page 19

Secretary of the American Whist League, 1893, Theodore Schwarz, B&W photograph. Page 21

The first duplicate in the U.S.A. Milton C. Work, B&W photograph. Page 22

The first duplicate tournament. Nicholas Browse Trist, B&W photograph. Page 25

The John T. Mitchell story. John T. Mitchell, B&W photograph. Page 26

The invention of trays. Cassius M. Paine, B&W photograph. Page 30

St. Louis 'method'. Whist magazine B and W advertisement. Page 31

Kalamazoo 'method'. Advertisement. Page 32

Cincinnati 'method'. Advertisement. Page 32

One page ad from June 1896 *Whist* journal. Page 32

Culbertson deluxe board, advertisement 1933. Page 33

Set of 20 Paine duplicate whist trays, circa 1896. Colour photograph. Page 35

The first schedules. A. G. Safford, B&W photograph Page 37

'Foster's duplicate whist'. R. F. Foster, 1894. B&W photograph. Page 41

Clay's universal system. Charles M. Clay, B&W photograph. Page 49

Fourth president of the American Whist League, Walter H. Barney. B&W photograph. Page 51

Octagon duplicate whist trays, circa 1896. Colour photograph. Page 55

Duplicate whist and (auction) bridge boards, circa 1910. Colour photograph. Page 63

Safford comparative system. Example card. Page 64

Pivot movement, flyer, circa 1920. Colour print. Page 65

Howell method, B&W advertisement. Page 70

Howell guide cards. Original, B&W. Page 70

The Howell system. Edwin C. Howell, B&W photograph. Page 71

Whitfeld schedules. William H. Whitfeld, B&W photograph. Page 73

Maridey duplicate (auction) bridge boards, circa 1920. Colour photograph. Page 81

Bicycle bridge boards, circa 1925. Colour photograph. Page 95

At deco duplicate contract bridge boards, circa 1930. Colour photograph. Page 213

Little Slam contract bridge boards, circa 1933. Colour photograph. Page 213

Bridge boards by Ferd Piatnik of Vienna, Austria, circa 1932. Colour photographs. Page 253

1932 par point world championship boards from Olympic. Colour photograph. Page 285

Art deco boards from 'Bridgette'. Colour photograph. Page 285

Boards manufactured by Index sales corporation, circa 1950. Colour photograph. Page 341

Deluxe boards in box, from Taylor, circa 1950. Colour photograph. Page 341

'J-R' aluminium boards circa 1960. Colour photograph. Page 349

Congress plastic boards, circa 1970. Colour photograph. Page 349

Recent plastic boards used in dealing machines. Colour photograph. Page 365

Foreword

Ian McKinnon is well known on the Australian bridge scene, not only as a player, but also as the author of scoring software, which necessarily entails having a deep understanding of bridge movements. This book is his masterwork — the pièce de résistance, or Meisterwerk.

For tournament organizers and bridge directors, it serves as a compendium of all the 'best' movements to employ, for whatever number of entrants is desired in any particular kind of tournament. It also contains advice on how to modify movements into others, when something unforeseen occurs; e.g., a pair or team fails to arrive, or must drop out part-way through. Thus this book is very valuable as a director's guide to running bridge tournaments. However it is much more than this. The early chapters include some history and the personalities behind the development and use of different whist movements, explaining why some became popular while others fell out of favour. Many quotations and reproductions of old photographs allow the reader to appreciate the mind-set of people like Mitchell, Butler and Howell — names which are familiar, though the people themselves are not — and many others, now all but forgotten. Also there are reproductions of early marketing materials, including original card holders, board-sets, and movement instructions; the original attempts at these, from long before the mass-produced sets that are in use these days.

The third aspect of this book is the way in which the mathematics of bridge/whist movements and tournament schedules more generally, is presented. Starting from the recreational curiosity on magic squares, and leading into Latin squares and Graeco-Latin squares (which have considerable significance in Algebra and Combinatorics), the reader is introduced to the concept of Room square, which governs the structure of some Howell movements and which is named after the distinguished Australian mathematician Thomas Gerald Room. These are an example of the more general combinatorial structure known as an Incomplete Block Design, which has an important application in the design of experiments. In some fashion, every bridge tournament can be regarded as an experiment whose outcome is to determine the relative abilities of the pairs or teams taking part. So it is no surprise that such combinatorial structures should play a role and that there is much theoretical knowledge already available to be used when working out movements for bridge tournaments. Part of this knowledge is the concept of 'balance' for an Incomplete Block Design. In the context of a bridge tournament, using a Howell movement, say, this equates with a notion of the movement's 'fairness', having regard to how frequently two different pairs play as effective opponents or as effective team-mates. Measuring this appropriately for all couples of pairs, the fairest movements are those which minimize the discrepancy in these values, so that the presence of particularly good, or poor-performing, other players in the movement tends to have a roughly equal effect on all the participants. This aspect is discussed in some detail, and clearly applies to all kinds of movements, not just the Howell-based ones. Every movement presented in this book comes with a number called its 'Calibre', indicating how close it comes to the ideal 'best possible' of its type. Those with calibre of 100 (which means 100%) are ideal for that number of pairs or teams competing. However, in many cases the ideal is unachievable for practical reasons; nevertheless the movements in this book are of very high calibre indeed. Finding these was no easy feat, requiring much computing power to search among the myriad possibilities. This is work that has occupied Ian McKinnon over many years, with the results being recorded in his software, as well as in this book.

Extending the idea of Room square is the so-called House square. This is used primarily with teams' tournaments, having an odd-number of teams. In each round, three teams play a 'triangle' consisting of two half-matches against each opponent. These half-matches are completed in the next round, so that two full matches are completed over two rounds, and nobody needs not to play at all for any length of time. The House square was introduced by mathematicians D. R. Stinson and Walter Denis Wallis in 1984. Ian had faced Wal Wallis across the bridge table in Sydney in the early 1970s, and later came to learn about some of his work relating to bridge movements. More recently Ian established contact again, leading to Wal contributing much in the way of explanations and theory to the contents of this book. As recently as 2006,

building on the work of Wallis, mathematicians Gennian Ge, Esther Lamken and Alan Ling described a new class of movements called ‘complete coupling round-robin schedules’ (CCRRS), where the acronym is pronounced as ‘scissors’ for convenience. Thus was born the ‘Scissor schedules’, which allow a teams’ tournament to be also scored as a pairs’ movement; thereby providing an interesting new kind of bridge event.

In the Appendices of this book, Ian pays tribute to the mathematicians whose work has provided many new schedules, not previously known, or who have supplied perspectives on the combinatorial structure of movements allowing new and better examples to be found. These include reprints of old papers — including one of my own, which was never published academically, though distributed in some bridge circles. Such a collection, and this book as a whole, should inspire further research. Who knows what new kinds of bridge movement may result? The book, while not likely to be read by every bridge player, certainly belongs on the shelves of every club and every Tournament Director, and those players (and maybe some non-players) also interested in the history and/or mathematics of tournament schedules, particularly of bridge movements.

Ross Moore
February 2012

Preface

Behold the turtle. He makes progress only when he sticks his neck out.
James Bryant Conant

Since my first book 'Bridge Directing Complete' (BDC) things have changed extensively in the world of electronics, publishing and particularly computers and their use in bridge. BDC was designed to be used by tournament directors so they might conduct their events well. Before and since that time (1979) there have been several books published for the tournament directors to assist them in doing their job well whether they are at a local club or running a national event. This book, as the title implies, is not an attempt to replace any of these books. I hope the title 'Duplicate bridge schedules, history and mathematics' describes the nature of the contents. I also hope it can be used by bridge tournament directors to complement the director's book they use regularly.

You will find many historical details throughout the text including details on many of the people involved. Therefore I think it only appropriate that I expose my background and credentials, for the want of a better description. I was born on a Thursday in February 1945 at a hospital in Parramatta at the edge of the western suburbs of Sydney. The location is now the centre of the greater city of Sydney, Australia. My early education at the Wentworthville infants and primary school was far from ideal and can only be described as a blackboard jungle. I did manage to dodge the flying blackboard dusters, though many of my fellow students in the class of 40 were not so fortunate, ending up with split foreheads.

At the age of 11 the family moved to Tamworth in northern New South Wales where things took a turn for the better. Once I had finished my high school education at Tamworth, where I had passed mathematics with honours, I went to the University of New England at Armidale, an hour's drive, north of Tamworth. There were a number of significant events that occurred while there, setting the course of my life and career. Two of these things happened in 1964. The first was the mathematics faculty took delivery of their first computer which I learnt to program. The second was I moved to the Lawson house of the Sir Earle Page College where the resident tutor needed a second, third and fourth for bridge. Alan Walsh and I were two of the starters and we commenced our pursuit of the game with vigour.

Over the next two years my academic efforts were quite poor and my interests in bridge affected this significantly. Alan who clearly knew what was important in life better than I did, gave up University at the end of 1965 and so did I at the end of 1966. Ironically, if I had completed my degree in mathematics I would have automatically entered the NSW Education Department teaching mathematics because they were paying my university fees. Instead I moved to Sydney where I immediately got a job and career in computers – "you know what a computer is?" was the only qualification I needed.

Alan and I now worked together, played bridge together and by 1968 were representing NSW at the national championships with the greatest of team mates, Denis Howard and Tim Seres. In doing this we were playing regularly at the NSW Bridge Association where the tournament director was Bill Schaufelberger (1902 – 1972). In the last years of Bill's life he and I became very good friends. He wanted to give up the tournament directing and encouraged me to take over the reins at the club. He was a wonderful mentor and taught me many things about directing. I became very interested in movements, their structure and mathematics. From this time on I became less interested in playing bridge and more involved in directing.

Bill and I used to talk about lots of things while we were waiting for the players to finish a round or session and he told me many stories. One that sticks in my mind was about Tim Seres (1925 – 2007), who is considered the greatest bridge player ever to have played for Australia. Tim arrived in Australia in 1947 from Hungary though he was born in Austria. Tim could not speak English, but when he walked into the club run by Bill and he started playing, it was obvious to Bill how good Tim was. As Bill said "Tim could

not bid to save his life but he could really play the cards". Bill was Hungarian, arriving in Australia in the 1930s, so communication between the two was easy and Bill took Tim under his wing. Tim said later "my first taste of serious bridge was with the N.S.W. team of 1948, captained by Bill. I owe a great deal to the lessons of those early days." I can also say that I owe a lot to Bill for getting me interested in directing and those lessons of the early days.

All through the 1970s I was working as a computer specialist and directing tournaments at the state and national level. In 1972 we had the first Australia-wide pairs with almost 1,300 entries which I match-pointed using the results of the winning pairs at each of the clubs. In 1973 this event was scored by me over the entire field of 416 tables using a Honeywell computer. This annual event continued for many years averaging around 1000 pairs. These were reported each year in the 'Australian Bridge' magazine, edited by my friend, Ron Klinger, whom I had met in the late 1960s and played bridge with on occasions.
During the mid-1970s I did all my early research on movements using a Honeywell computer, owned by my most generous employer. The computer I used was a powerful mainframe (for the time) worth millions of dollars, but even so the power was nowhere near that available to me in recent years using an off-the-shelf PC, costing a couple of thousand dollars. These Howell movements were first reported in BDC in 1979. In 1976 I traveled to Europe for the World Bridge Olympiad and caught up with Frank Farrington, who was in the process of putting together his final edition of 'Duplicate Bridge Movements', also published in 1979.

The one thing that Frank and I did not understand well at that time was the balance of movements. John Manning of the U.K. published his paper 'Mathematics of duplicate bridge tournaments' in September, 1979, just missing BDC and Farrington by a few months. I received a copy of that paper from John, but the horse had bolted. In the 1980s I was too busy making money and raising a family to consider bridge movements, but did correspond frequently with Olof Hanner, who was continuing research into many different types of movements and their balance.

John Rowland Manning was born in Surry in 1925. He obtained his MA degree in mathematics from Cambridge University. By 1963 he was the head statistician at the British Boot, Shoe and Allied Trades Research Association, Kettering, Northhamptonshire, the town where he lived. One of his papers of that time was 'the changing shape of high-heeled shoes' where John made the point that the optimum heel height for a size 5 shoe is 2.5 inches. This has particular meaning to me as my father (died 1976 aged 61) was a cobbler and I do remember him having problems dealing with the much higher and fashionable 'stiletto' heels being produced for women's shoes at that time.

I believe the subject of balance is tackled thoroughly and correctly in this book. The meaning of 'balanced' has changed significantly since 1979, with the pure mathematics underlying this issue being fully defined by Mr. John R. Manning. More recently a measure of imbalance has been defined precisely by Dr. Ross Moore of Macquarie University, Sydney, in 1992 when he published 'Too many switches spoil the balance'. It was this paper that brought my attention to the inadequacies of many movements in my first book. These finding are included in this edition, as well as adding some more ideas. In recent years the contributions on balance by Peter Smulders of Holland are of particular significance.

The advances in pure mathematics are also most significant. In particular the advances made in the branch of mathematics known as combinatorics are of particular importance to the production of bridge schedules. Combinatorics concerns the study of finite or countable discrete structures and covers many areas such as graph theory, algebra, probability, topology and geometry and is used in many fields of application such as optimization, computer science and statistical physics. These many applications have meant there has been significant funding available to the pure mathematicians working in this area.
The bridge community has also been lucky that one of the leading mathematicians in combinatorics was keen to advance those areas that apply to bridge schedules. His name is Professor Emeritus Walter Wallis and a fellow Australian who attended University of Sydney at the same time as Ron Klinger. I also met Wal in the early 1970s, but only across the bridge table. Walter Denis Wallis was born on 26 June 1941 and currently is Professor Emeritus, Department of Mathematics, Southern Illinois University. He obtained his PhD at the University of Sydney in 1968.

During the 1970s and 1980s I met Ross Moore a number of times, particularly across the bridge table as a fierce opponent, but our mutual interest in mathematics and bridge movements made the relationship a bit special. Ross was born on 21 May 1955 and completed his BSc(Hons) at the University of Melbourne in 1977. He then went on the complete his DPhil at Oxford University in 1981, returning to Australia and working at Australian National University until 1986 and then to the Macquarie University where he is today. Up until October 2009 Ross Moore was gladly answering my questions and guiding my research in the right directions. I found the Internet a great source of papers, particularly those published by Wal Wallis. It was when I was struggling with the mathematics published by Wal and his colleagues that I established contact with Wal Wallis again, and from that time on he became a major contributor to the content of this book.

In this book I hope to expose the link between mathematics and bridge schedules in a way that is understandable to most people. I hope this book also supplies the groundwork for any future research into bridge schedules.

The use of computers has allowed me to dispense with the appendices found in the first book. Percentage tables are no longer needed, as computers do all the scoring of bridge events today. The expansion of bridge movement details are no longer needed because this edition should be used in conjunction with a computer software program that I have developed that allows you to view or print out those details at a click of a button. With this you can produce table guide-cards, player guide-cards and director's instructions for any movement. It also means that any pairs' movement can be balanced.

This computer program, called 'Jeanie', is clearly a play on words for Genie, the wizard that will supply your needs as you wish. It also recognizes my mother, Jean McKinnon, as a major contributor to the first book when she sat for many months typing in the text for the photocomposition program that I was responsible for at Honeywell and used for that publication. She used a telephone line and modem many years before the Internet was a public facility.

Let me make one thing very clear about copyright. Bridge movements are used for the pleasure of bridge players throughout the world, making their games as comfortable and fair as possible. Many movements in this book have been published before and the original source of some of them is likely to be open to debate. In fact one of the objectives of this book is to resolve most of the debate. Where I am certain of the origin I will make that very clear but for some it is not clear. Therefore while the book has a copyright held by me, and reproduction is forbidden without my permission, the movements can be used by any organizers to conduct their bridge tournaments.

There are many new movements included, too. Some are the result of my own research and development and some are from research carried out by other mathematicians. Some are the result of manipulation of existing movements from various sources, including my first book. It is possible to take just about any movement and change it to appear almost nothing like the original. This is one of the things illustrated very clearly in this book. Some of the movements come from mathematical papers that are completely unusable by bridge organizers until they are manipulated and presented in an understandable form. Some are a result of manipulation of an existing movement so that the movement is easier to use. This could be considered plagiarism in some cases, but there is definitely no intention on my part to do that. Any changes are for presentation, simplicity or consistency or all three.

Because the number of movements included in this book is extremely high it is not possible to show all in an expanded form. Therefore it is important for the reader to become familiar with the shorthand methods of representation, so that they can use any movement. In addition many are difficult to edit and proofread and would be prone to error, except that all movements are computer tested and generated. Any manual data entry is done with the aid of computer software and fully checked and validated by the computer before being exported to the textual form shown in this book. For this reason the reader can be very confident in the integrity of the content.

Duplicate bridge schedules for tournament directors and organizers are the objectives of the book but there is an attempt to make the text entertaining for other readers, particularly those who solve Sudoku puzzles.

There are three subjects: schedules, their history and their mathematical origins. I do realize that many readers will not be able to understand some of the mathematics or may not be interested in the mathematics, in which case simply skip over those chapters.

This book is the result of many years of computer software development, research and writing. The first draft was produced early in 2003 and the first version of Jeanie appeared on the Internet in October 2008. During the compilation of the information held in this book many sources and persons were used and consulted. The Internet has a lot of details about many aspects discussed in this book. Many ideas and concepts have been developed over time by discussions between bridge organizers and where possible, I have tried to summarize those ideas into a reasonably concise document. Where there is debate I have tried to present both sides and leave it up to you, the reader, to make your own decisions.

Special thank you

I am only wise insofar as what I do not know, I do not think I know
Socrates

Without the help of two special people this book would not have been produced. They both gave their time and extensive knowledge on their respective subjects without consideration of any monetary rewards or acknowledgment in authorship. Even so without their contributions this book would not have appeared. They are Wal Wallis and Ron Klinger.

Wal Wallis. My first email to Wal on this matter was sent during October 2009. Since then he has worked tirelessly, answering all my questions and supplying me with any mathematics missing from my repertoire. Many of his papers are available on the Internet, but knowing how to interpret much of the material was another matter.

When I would ask a question he often answered with the complete notes, supplying me with lecture notes given to his University students. In addition Wal was able to come up with many papers that I had never seen or have been unable to locate. Sometimes he would send a paper "have you seen this one?" Of course I had not and it would open up a whole new path of research.

Ultimately when I hit a brick wall on a particular movement I would ask "can you find this for me?" and he would spend many hours chasing it down without expecting any rewards apart from the joy of solving the mathematical puzzle.

All of this comes from a man who is the leading mathematician in the field of combinatorics and so has many demands on his time from commerce and research all around the world.

Bridge organizers, tournament directors and bridge players of the world will always be in debt to Wal.

Ron Klinger. Ron is a well known author of bridge books, videos and magazine articles as well as being an excellent bridge teacher and an expert player with countless successes in all sorts of events around the world.

Clearly Ron is a very busy man. Even so when I asked him to sort out all my scruffy writing in this book he did not hesitate to offer full assistance. With my mathematical background, my grasp of written English leaves a lot to be desired. Ron on the other hand has a background in Law and is a very experienced writer and able to identify faulty sentence structures in a blink of an eye.

He spent many hours reading through my efforts correcting grammar, punctuation and structure. Without his editorial contributions the book would not be anywhere near as tidy as it is.

As a long time friend and mentor I must thank Ron for his encouragement and support, without which this book would not have become reality.

Introduction

A question that sometimes drives me hazy: am I or are the others crazy?
Albert Einstein

This book is primarily aimed at bridge tournament directors. Even so I am hoping there is enough to amuse and entertain those who play duplicate bridge or those interested in mathematical squares or those interested in the history of duplicate bridge. It also applies to those wanting to know more about sporting tournament organization schedules particularly where it applies to duplicate bridge.

Techniques used in organizing tournaments change over time as well as differing in most countries so there is no intention to address these issues. Also computers are used today for scoring bridge events and so no attempt is made to cover scoring apart from its relationship to choosing movements. Most tournament directors when running club events want to supply the best movement to their players and handle any difficulties quickly and efficiently. When conducting larger events the issues of organization become more demanding and the planning is the prime concern. This is where the director will find this book invaluable in the provision of the best movements for all situations.

The approach taken is to supply all the available movements for teams, pairs, teams of pairs and individuals together with all modifications of these that can prove useful. As comprehensiveness is one of the aims, movements are included which are useful only in very rare cases.

All tournament directors should be encouraged to understand the principles and mathematics behind movements, so that a movement can be changed at the last minute with the minimum of trouble. Equally important is the confidence instilled in the players through their awareness of the director's ability to organise their events smoothly and quickly.

The director's primary considerations at all times should be player comfort and accuracy of scores. The major factors in achieving these are simplicity of movements (within the bounds of the tournament), playing of all boards by all contestants in the event and adequate player comparison on all boards.

Wherever possible, simple movements should be a sine qua non for the director, as the players are there to play bridge, not to solve bridge movement problems. In other words the director should choose the movement that gives the desired results (one winner or two, etc.) with minimum fuss and maximum accuracy.

Player comparison is an issue that has received much debate over recent years and that is dealt with thoroughly. The use of balanced or fair movements should be selected over ones that are unfairly imbalanced. All players want to feel that their efforts at the table are not unfairly influenced by imbalanced movements that favor other players. With the aid of this book the director can now choose to use a balanced movement in preference to others.

This book is about duplicate bridge schedules, their history and the mathematics behind them. Before we get into bridge schedules though we must digress somewhat so that we can get a real feel and understanding of its origins. We start with the history of the mathematics, then the history of duplicate bridge and the recognition of the need for player schedules.

Let me make one thing clear: each chapter can be read without reference to the previous ones. If you find the subject of any chapter, particularly the mathematics, is beyond your comprehension, simply skip forward to the next chapter.

For the tournament director you will find there are many new movements. Most of these are not needed very often. They are as a result of the research of the mathematicians over the past 30 to 40 years and in

particular the findings of Professor Emeritus Walter Wallis. The most recent research has delivered a completely new event type and its new schedules and movements.

By way of introduction we should consider the differences in the branches of mathematics. The modern school of pure mathematics was established by the famous English Mathematician Arthur Cayley in the mid 1800s. Pure mathematics is motivated by pursuit of reasoning and logic without any interest in the application. All the mathematicians we refer to in this book are from the school of pure mathematics. They are pursuing their subject with ever-increasing detail building upon previous theorems and structures such that there is never duplication of effort. One of the consequences of this inherent approach is the need to scan an apparent never-ending pyramid of papers that reference previous papers.

Applied mathematics is a branch of mathematics that concerns itself with the mathematical techniques used in the application of mathematical knowledge to other domains, in our case, bridge schedules. Having done the research my job with this book is to present the structure of the mathematics as applied to duplicate bridge. In doing so I am hoping to give the reader some understanding of the history of bridge schedules both mathematically and as required by the organizers over the last 120 years.

The modern computer is an excellent tool for supplying us with new variations on the themes we already know as well as finding better and new movement schedules for tournament directors around the world to conduct their events with precision and perfection.

Prologue

God always takes the simplest way.
Albert Einstein

The content of this book contains many different subjects, interspersed with each other, but amongst these there are many items that are essentially mathematical. These are included to complete the study of bridge schedules and make it possible for future students of the subject to pursue their goals. Making these mathematical claims, defining new theorems and designs, can be included in this book with confidence only after peer review.

As any scientist knows, their work is not complete until there has been a thorough peer review. Mathematics is a science, or rather, Carl Friedrich Gauss (1777–1855) referred to mathematics as "the Queen of the Sciences".

During the many years of writing this manuscript, many mathematicians who were involved in their particular subjects were called upon to review my work. These various subjects are from the main fields of mathematics: Pure, Applied and Statistical. I am sure this was done with as much rigour as possible, but it is not until the manuscript is ready for the publisher that a final check can be done. This is where Ross Moore stepped in and was extremely valuable. He gave up many hours of his valuable time to read through the complete manuscript, picking over the details, questioning claims I have made and correcting any errors.

In addition to this essential task Ross offered a lot more. His professional career is Senior Lecturer at Macquarie University, near Sydney. As well as pursuing research in algebraic geometry, Ross has made significant contributions to mathematical typesetting and typography, by devising ways to encode the features of complicated mathematical diagrams and figures easily for reproduction on the printed page and electronically for websites. With this knowledge and skills Ross was extremely valuable in the refinement of the final product. The quality of the final draft of the book has been raised to excellent standards thanks to his involvement.

When I first set out to produce this book I had no idea what discoveries I would make. Initially much of what I was doing was for my own enlightenment and entertainment. As time went on it became clear to me that most bridge players should also enjoy the findings.

When I first started the research it was fortunate that most of the old and well established newspapers around the world were supplying free access to their archives on the Internet. Within a few years this access was restricted and certainly not free. During this time of easy access I made many discoveries, both historical and mathematical, and enjoyed the sleuthing very much. I do hope you, the reader, enjoy these findings as much as I did.

Part 1 Squares

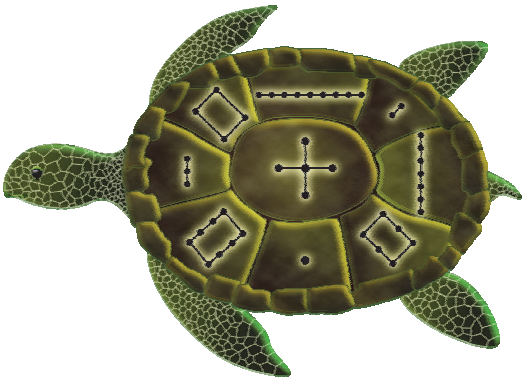
An example of the first duplicate whist board (called a tray at the time), circa 1891.
Lead on the right.
Trumps is recorded on a slip of paper, placed on top of the dealer's cards and travels with the board.
Vulnerability had not been invented.

1.1 Lo Shu Square

The question was heatedly debated of how much Western culture should be brought into China.
Chen Ning Yang

A Chinese legend has the Gods very angry in about 650BC when they ordered a huge flood. Out of the waters of this flood emerged a turtle with a curious pattern on its shell. This pattern was called *Lo Shu* or 'scroll of the river'.

Using this pattern the locals were able to control the flow of the river because it had magical powers. This is the first documented magic square. On the back of the turtle were the numbers 1 through 9 represented in a series of dots and circles joined by lines. These gave a unitary depiction as figurative numbers in geometric patterns. These numbers were arranged in a three by three array so that they formed a square.



In Arabic numerals this magic square can be shown as:

4	9	2
3	5	7
8	1	6

Let us consider the properties of the magic square.

- Each number between 1 and 9 appears only once.
- Each row, when all three numbers are added together, comes to a total of 15.
- Each column, when all three numbers are added together, comes to a total of 15.
- Each diagonal, when all three numbers are added together, comes to a total of 15.
- The odd numbers form a cross in the centre.
- The even numbers are at the four corners.
- The totals are 15 which is the number of days in each of the 24 cycles of the Chinese solar year.
- The opposite numbers (ignoring the centre 5) add to 10 a most significant number in I Ching. It is the number of Ho Tu or the Yellow river map.
- The first number 1, is at the bottom centre and the last number 9, is at the centre top. The number 5 in the centre is the perfectly balanced number (101 in binary) and the centre of the Ho Tu.

This square is called a magic square of order 3. Mathematicians, artists and authors have been fascinated with squares ever since. Their applications seem to be endless.

1.2 Melancholia I

All religions, arts and sciences are branches of the same tree.
Albert Einstein

One of the more famous magic squares is that contained in the engraving produced by German Renaissance master Albrecht Dürer in 1514. It is called Melancolia I or in the modern spelling Melancholia I. It is one of the most famous old master prints and it featured in the recent book by Dan Brown called *The Lost Symbol*.

The magic square contained in this engraving is order 4:

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

The date of the engraving is 1514 and is found in the bottom centre two squares. The outer two numbers 1 and 4 correspond to the 1st and 4th letters of the alphabet, A and D, the initials of Albrecht Dürer. The monogram is 1514AD which is the date and AD again Albrecht Dürer initials. All the rows, columns and diagonals sum to 34 as expected with any magic square.

This one goes a lot further with many combinations of 4 numbers adding to 34:

- the 4 quadrants;
- the four centre squares (10+11+6+7);
- the four corner squares (of the 4x4 square as well as all the 3x3 squares);
- the four outer number clockwise (3+8+14+9);
- the four counter clockwise (2+5+15+12);
- the four diagonally symmetrical number (2+8+9+15 and 3+5+12+14);
- the middle entries in the two outer columns and rows (3+2+15+14 and 5+9+8+12);
- the four kites or crosses (3+5+11+15, 2+10+8+14, 3+9+7+15, 2+6+12+14).

1.3 Pleasing Problems

As far as the laws of mathematics refer to reality, they are not certain,
and as far as they are certain, they do not refer to reality.
Albert Einstein

Claude Gaspard Bachet de Méziriac was a French mathematician born October 9 1581. One of the many things in mathematics that intrigued him was magic squares. He devised one of the first methods found in Europe for deriving magic squares. His method is to place the first number in the square (box) above the centre. The numbers in sequence are placed in the box to the right and up one square. Consider the magic square of order 5. Place 1 in the box located in column 3 row 2. Then place the numbers 2, 3, 4, 5 in the boxes above and to the right. Where this move leaves the confines of the magic square, move to the last row or first column.

			2	
		1		
	5			
4				
				3

It is not possible to place 6 in the next box as that is occupied by the first number, 1. When this occurs Bachet said move up two boxes instead and continue as before.

23	6	19	2	15
10	18	**1**	14	22
17	5	13	21	9
4	12	25	8	16
11	24	7	20	3

The rows, columns and diagonals of this magic square all add to 65.

In 1612 Bachet wrote a book called *Problèmes plaisants et delectables qui se font par les nombres* (pleasing and delectable numerical problems). In this were included many mathematical problems or questions including numbers in other than base 10, card tricks, watch dials, think of a number, river crossings using ferries, juggling weights, pouring liquids, magic squares and so forth, many of which appears in books of this type today.

Included in these problems was this one: “Take the ace, king, queen, jack of each of the four suits and arrange them in four rows of four cards each so that no row, column, or diagonal contains two cards of the same suit or of the same value.” One solution is:

A♠	**K♥**	**Q♦**	**J♣**
Q♣	**J♦**	**A♥**	**K♠**
J♥	**Q♠**	**K♣**	**A♦**
K♦	**A♣**	**J♠**	**Q♥**

This problem has moved away from the magic square problems in one major way. **There are two values in each box of the square**. In addition there are symbols used instead of numbers though it is possible to use numbers.

1.4 Spreading the word

There are only two ways to live your life. One is as though nothing is a miracle.
The other is as though everything is a miracle.
Albert Einstein

Jacques Ozanam was a French mathematician born in 1640, just after the death of Bachet. Ozanam was not a great mathematician, but he was a prolific writer on the subject. He published many books which were in turn translated into several languages.

He was happily married, had a large family and earned a good income from teaching mathematics but spent most of it on gambling. All his books sold well and there were many editions of each. Of all his books he is best remembered for *Récréations mathématiques et physiques* (Recreational mathematics and physics), published in 1694.

Récréations, the precursor to many mathematical books published over the next 200 years, ran to four volumes and went through at least 10 editions. It was based on the earlier works of many mathematicians including Bachet. It was revised, updated and enlarged by others over time and was published in English by the famous English mathematician, Charles Hutton, in the early 1800s.

In *Récréations* he dealt with many of the problems of Bachet including magic squares and other squares. Of these writings one question posed by Ozanam was to do with the Bachet's 16 face card problem.

How many ways can the 16 cards be arranged to produce a new and different square?

This *riddle* persisted for a long time. Walter Rouse Ball, born August 14 1850, claimed the number of ways was 72. This persisted for many years until it was shown to be 144 by Dame Kathleen Timpson Ollerenshaw, DBE (born 1 October, 1912, in Withington, Manchester), aged 99 at the time of writing. Each of these 144 solutions has 8 reflections and rotations, giving 1152 solutions in total. The 144×8 solutions can be categorized into the following two groups or classes:

A♠	**K♥**	**Q♦**	**J♣**
Q♣	**J♦**	**A♥**	**K♠**
J♥	**Q♠**	**K♣**	**A♦**
K♦	**A♣**	**J♠**	**Q♥**

A♠	**K♥**	**Q♦**	**J♣**
J♦	**Q♣**	**K♠**	**A♥**
K♣	**A♦**	**J♥**	**Q♠**
Q♥	**J♠**	**A♣**	**K♦**

Each of these two solutions produces a total of 576 (24×24) solutions. They can be derived by permuting the four suits and the four face values independently. No permutation will convert one solution to the other.

Mr. E. Riddle in the preface to *Recreations in Science and Natural Philosophy* (1844), the English translation of *Récréations* by Hutton, quotes Ozanam as saying: "It is the business of the Sorbonne to discuss, of the Pope to decide, and of a mathematician to go straight to heaven in a perpendicular line."

1.5 Today's recreations with squares

All God does is watch us and kill us when we get boring. We must never, ever be boring.
Chuck Palahniuk

One of the more popular pastimes with squares today is called Sudoku. This is a modern name of a puzzle that dates back to (at least) an example that appeared in the newspaper *La France* on July 6 1895. In the *Daily entertainment* section a square appeared under the title *Carré Magique Diabolique* (diabolical magic square) by M. B. Meyniel.

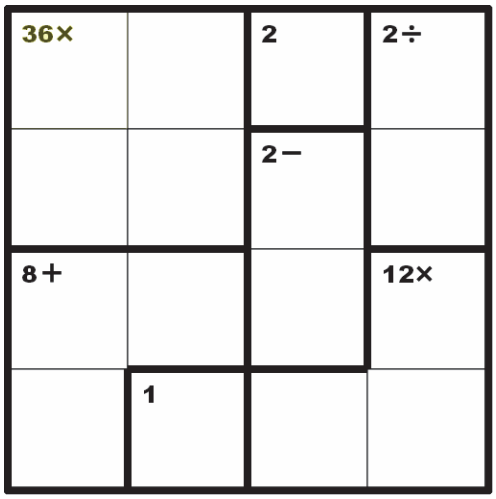
7	8	9	1	2	3	4	5	6
3				4				8
5				9				1
8				3				4
1	2	3	4	5	6	7	8	9
6				7				2
9				1				5
2				6				7
4	5	6	7	8	9	1	2	3

Even though this square is called *magic* it does differ from the earlier definition. Only the numbers 1 to 9 appear in the square. If it were truly magic the numbers 1 to 81 would appear. Even so the restriction exists that the numbers 1 through 9 appear only once in each row and column. Also the numbers 1 through 9 only appear once in each 3 × 3 square as shown by the thick lines above. These are called sub-squares. It should be noted in the diagram of the original square that the bold lines around each 3 × 3 sub-square were not shown.

Still this can be called a magic square because each row, column *and diagonal* add to the same total. In a modern Sudoku square that is not a requirement. If you do solve this Sudoku you will find the two diagonals will contain all the digits 1 through 9, also making this a magic square. The total of each row, column and diagonal is 45.

The most modern of these puzzles (2004) is the KenKen (also called KenDoku, MathDoku, Calcudoku and others) and is a style of arithmetic or logic puzzle invented by the Japanese maths teacher, Tetsuya Miyamoto, a leader in teaching that he calls "the art of teaching without teaching". The name KenKen is derived from the Japanese word for cleverness.

Like Sudoku the objective is to fill the square with digits so that there are no digits repeated in any row or column.



The heavily outlined set of squares, called cages, must combine in any order to produce the target number in the top corner using the algebraic operation shown next to the number. The cage with one number shown contains only that number. The only example of that is shown here with 1. A number can be repeated in a cage so long as it does not appear in the same row or column of the cage.

1.6 The headmaster of mathematics

There was something beautiful in the truth revealed by maths – something perfect and just.
Julian Assange

Leonhard Paul Euler was a Swiss mathematician born in 1707, 10 years before the death of Ozanam. Euler (pronounced Oyler and sometimes written that way) is considered the master of mathematics, the teacher from whom we must learn all the basics of mathematics.

Any attempt to cover the work of Euler in this book would be pointless. He was the most prolific writer in mathematics to this day. Even listing the things *named* after Euler would run to many pages. We owe to Euler the notation $f(x)$ for a function (1734), e for the base of natural logarithms (1727), i for the square root of −1 (1777), π for pi as well as many others.

As a small digression let us look at *Euler's identity*, one of the many things named after him:

$$e^{i\pi} + 1 = 0$$

This formula was described by Richard Feynman (a physicist involved in the development of the atomic bomb) as "the most remarkable formula in mathematics" and voted "the most beautiful mathematical formula ever" by the readers of the *Mathematical Intelligencer.* It uses addition, multiplication, exponentiation, and equality, and includes the single use of the important constants 0, 1, e, i and π. Of these constants zero is a concept and is not negative or positive, one is unity or oneness, e and π are irrational and transcendental numbers and i imaginary.

The mathematicians before Euler had described various squares: magic squares, squares with single numbers in each box and squares with 2 numbers in each box, though extremely rare. The 16 picture cards problem from Bachet was the first.

Euler came up with the name *Latin square* and the way to construct them. First let us look at a square of order 3 with *two symbols* in each cell.

A α	B β	C γ
C β	A γ	B α
B γ	C α	A β

This is the way Euler depicted his squares and he called them Graeco-Latin squares. He used two sets of symbols, the first were Latin letters and the second Greek letters, hence the name Graeco-Latin squares. The name *Latin square* was a derivative of this name and was for squares with one symbol in each square.

How many squares are there for each order? It is easy to change a square from one to another, for example swapping rows or columns is easy enough. Swapping rows 1 and 2 of the above example gives us a new square.

C β	A γ	B α
A α	B β	C γ
B γ	C α	A β

There are other ways of converting this to more examples, such as change A to B and vice versa. Euler was very interested in creating these and found ways to create magic squares. Continuing with this simple case if we change the symbols to 0, 1, 2 for both the Greek and the Latin letters we get:

21	02	10
00	11	22
12	20	01

Working in base 3 (the order of the square) multiply the first digit by 3 and add it to the second digit and add 1 to get a familiar magic square:

8	3	4
1	5	9
6	7	2

Now consider the Latin square. It is a square where there are $n \times n$ (rows × columns), constructed so there are n different values or symbols and each symbol appears exactly once in each row and each column. With no other limitations we said above there are many squares of each order, the higher the order the more there are. In fact Ian Wanless of Monash University has found for order 11 Latin squares there are:

776,966,836,171,770,144,107,444,346,734,230,682,311,065,600,000

It is a bit hard to dispute that.

A normalized, or reduced, Latin square is a Latin square with the first row and column given by $\{1,2,\ldots n\}$. The number of normalized order 11 Latin squares is:

5,363,937,773,277,371,298,119,673,540,771,840

Multiplying this number by 11! (11 × 10 × 9 × … × 1) gives the first number.

We have also said that many (most) examples are essentially the same mathematically. Those that are the same are called *isomorphic*. What we need to make our next step is to identify all the Latin squares that are *essentially* different (non-isomorphic) or to come up with the idea that they are as different as is possible. One way to do this is take a square and superimpose it on top of the other with which we are comparing it. If none of the digits in corresponding cell positions are the same then they are different. For order 4 Latin squares there are only 3 examples that satisfy this:

3	4	1	2
2	1	4	3
4	3	2	1
1	2	3	4

A

4	1	2	3
3	2	1	4
1	4	3	2
2	3	4	1

B

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

C

Euler spent a good part of his life in St. Petersburg, Russia. The story goes that around 1779, the ruler (that would have been Catherine the Great) called on him to come up with a unique marching order. The challenge facing Euler was: given six ranks of officers, and six regiments, arrange 36 officers in six rows and six columns so that each row and column has only one of each rank and one of each regiment.

This has a familiar ring about it. It is similar to the problem originally posed by Bachet with his 16 cards, 4 suits and 4 ranks. Using the two squares A and C shown above one answer to Bachet's problem is:

31	42	13	24
22	11	44	33
43	34	21	12
14	23	32	41

For this solution it is also essential that no pair of numbers is repeated anywhere in the Latin square. If we did this with A and B there would be pairs of numbers repeated on different rows. Using B and C we get another valid and unique answer to Bachet's puzzle. These two answers are the only unique answers to the problem and the orthogonal squares (A,C) and (B,C) are said to be *two-orthogonal*, meaning that each pair of two numbers are unique in each of the squares. These are called mutually two-orthogonal Latin squares, some mouthful, or 2MOLS for short.

Now consider these three Latin squares order 4.

2	1	4	3
3	4	1	2
4	3	2	1
1	2	3	4

A

3	4	1	2
4	3	2	1
2	1	4	3
1	2	3	4

B

4	3	2	1
2	1	4	3
3	4	1	2
1	2	3	4

C

These squares are not isomorphic (row 4 has the same values in all four columns) but ‘A’ can be changed to ‘B’ by taking row 1 and placing it at row 3. Similarly ‘B’ can be changed to ‘C’. These are called *isotopic* Latin squares. Where the ‘Latiness’ of the square is maintained when the rows, columns or symbols are permuted the squares are called *isotopic*.

Also any pair of these squares can be used together to produce a Graeco-Latin square and an answer to Bachet’s puzzle. ‘A’ with ‘B’, ‘A’ with ‘C’ and ‘B’ with ‘C’ are all 2MOLS. Together the three are called a set of mutually orthogonal Latin squares.

Consider this order 4 Latin square:

1	2	3	4
3	4	1	2
4	3	2	1
2	1	4	3

This square has one other interesting property: the diagonal does not have a symbol repeated. If it is transposed (use a mirror through the diagonal) you get another Latin square that is orthogonal to the original. This Latin square is called ***self orthogonal*** and we will see more of these later. The transpose is

1	3	4	2
2	4	3	1
3	1	2	4
4	2	1	3

These two orthogonal Latin squares produce the Graeco-Latin square

11	23	34	42
32	44	13	21
43	31	22	14
24	12	41	33

Euler knew all this. So when he considered the 36 officer’s problem he found it could not be done. He could not find a Graeco-Latin square of order 6. Not only that, he believed it could not be done for squares of the orders 10, 14, 18, and so on, every four. These numbers are also called oddly-even numbers (half the number is odd) or more precisely where the order n is defined by any $n = 4m + 2$ for $m > 0$. The name oddly-even also came from Euler.

That was ***Euler's conjecture***, but he didn't prove it, and for years it remained one of mathematics' famously unsolved problems.

1.7 Using Latin squares

Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.
Albert Einstein

Latin square *Definition*: An $n \times n$ Latin square consists of n sets of the numbers 1 to n arranged in such a way that no orthogonal (row or column) contains the same number twice.

Graeco-Latin square *Definition*: A square array made by combining n objects of two types such that the first and second elements form Latin squares.

Pure mathematicians have been developing theories for thousands of years that do not seem to have any practical application apart from entertaining the curiosities of the mind. Orthogonal Latin squares were almost certainly considered by Euler for his entertainment and the 36 officers problem was very likely a figment of his imagination rather than real. Even so, one day an application for the theories does come along. So there is the question: what are the uses of Latin squares?

Today Latin squares are used in many ways. It applies to many fields of mathematics including Balanced Incomplete Block designs. The theory of experimental designs, finite fields and finite geometries all use Latin squares.

In the practical world they are great for scheduling of round-robin tournaments. As a matching procedure they relate to problems in graph theory, job assignment and processor scheduling in large computers.

Clearly our main interest is in the tournament application. A tournament can be in any sport but tennis is a good example where round-robin events are required. Another good one is golf where the requirements can be more complicated.

One problem posed to Professor Emeritus Walter Wallis was the following. He was asked for a golf schedule for 24 players, playing in six groups of four, for six rounds, in such a way that no two players are in the same group of four more than once. See the solution in the appendix J (page 396).

In bridge the scheduling problems for players may be quite complex in some cases and very simple in others. The need for scheduling bridge tournaments only arose once the players were playing duplicate bridge.

Part 2 Duplicate

Set of 12 Kalamazoo duplicate whist trays, circa 1896.

2.1 Duplicate bridge

Make everything as simple as possible, but not simpler.
Albert Einstein

The term *Duplicate Bridge* may apply to any bridge competition where the deals are played more than once. The play of the deal is repeated or duplicated. Events or tournaments are conducted so the players may compare their performances. These events can be for the comparison of pairs, teams or individuals.

Duplicate bridge is distinct from rubber bridge where the hand record is destroyed during the first playing. For a deal to be played more than once the cards in each hand need to be recorded or retained in some way and reproduced for playing a second time.

In modern duplicate bridge the cards are played by placing the card being played immediately in front of the player and face up on the table. After the trick is completed the card is turned over and placed in front of the player, near the edge of the table, 'pointing' in the direction of the pair winning the trick. Subsequent tricks overlap the previous ones so as to give an accurate record of the play.

After the completion of the deal the cards are placed in a 'Board' with each player's hand inserted in the slot reserved for the player sitting at that compass position at the table.

This was not always the case and it took many years for the organizers of bridge to come up with the best solutions to these problems.

2.2 The first duplicate

Necessity ... the mother of invention.
Plato

Just as the game of bridge developed out of whist so did the need for duplicate play. There has been much debate over where the first duplicate game was played or even where the idea first appeared. In John T. Mitchell's first book 'Duplicate Whist' published in 1891 he makes the claim:

"Duplicate whist is no new thing. The writer has it on good authority that it was played in Berlin and Paris as far back as 1840, and later in Philadelphia and New York; but it seems to have been dropped for the want of a simple method of play…".

In response to this Foster in 'Foster's duplicate whist' 1894 says:

> "Some loose writers on the subject claim that duplicate whist was played in Paris as far back as 1840; but so far as I have been able to ascertain, they have nothing on which to base this assertion, and so far from being at all inclined to credit it, I am of the opinion that the French story is a pure fiction."

Foster asked 'Cavendish' what he thought of these claims. 'Cavendish' (Henry Jones) replied:

> "I never heard of the Frenchmen's duplicate in 1840, until I read of it in Mitchell's book; and unless I can obtain some more positive information on the subject, I shall regard its occurrence as doubtful. Frenchmen do not play whist. When they play what they call whist, it is always dummy. Four form a table and one sits out. In my opinion they do not care enough for four-handed whist to try duplicate experiments."

When asked of his opinion Thierry Depaulis, the French authority on the history of bridge, replied:

"I share Foster's and Cavendish judgment. Mitchell's statement seems to be pure rigmarole. It is true the form of Whist that became popular in France was 'le Mort', i.e. Dummy Whist. I cannot imagine duplicate being used for this kind of game play. I fear Berlin is just a name: the Germans were no great Whist players. Vienna was probably a better place for Whist, but here too straight Whist was little played. German-speaking players rather played 'Cayenne Whist'.".

2.3 Cavendish, 1857

The direction in which education starts a man will determine his future in life.
Plato

'Cavendish' is the pseudonym under which Henry Jones, Member of the Royal College of Surgeons (M.R.C.S.), of London, England, is known wherever the language of whist is spoken. The name was taken by him from a club to which he belonged at the time he first took up his pen on behalf of the modern game. 'Cavendish', universally recognized as the leading whist authority of the day, was born in London, Nov. 2, 1831 and died in 1899.

This following extract is taken from Cavendish's 'card-table talk' published in 1897. It reports his memories of 1857.

'Cavendish' (Henry Jones).

Page 248:
"Between 1850 and 1860 a knot of young men at Cambridge, of considerable ability, who had at first taken up Whist for amusement, found it offer such a field for intellectual study, that they continued its practice more systematically, with a view to its complete scientific investigation. Since the general adoption of Short-Whist, the constant practice of adepts had led to the introduction of many improvements in detail, but nothing had been done to reduce the modern play into a systematic form, or to lay it clearly before the public. Its secrets, so far as they differed from the precepts of Hoyle and Matthews, were confined to small coteries of clubplayers. The little Whist-school held together afterwards in London, and added to its numbers; and, in 1862, one of its members brought out the work published under the name of

'Cavendish'. This passage has been the subject of comment during the past week in several newspapers of high standing; the points raised being, Did the 'knot of young men' originate, or elaborate, or compass anything? or did they draw their inspiration from other sources, and merely arrange what was well known and procurable before?

"I think the knot of young men did originate something, and I believe that the result of their discussions when put into book form was more than a re-arrangement of previously existing matter.

…

"When we used to meet in London, notes were occasionally made of points which interested us, and some experiments were tried, such as matching two bad players against two good ones, an account of which has already appeared in *The Field*.

…

"The experiment just referred to, *viz*., the matching two good players against two bad ones, was undertaken in order to ascertain approximately the advantage of skill. It arose and was conducted as follows:

"In the latter part of the winter of 1857, during an after-dinner conversation, it was remarked by some of the party that Whist is a mere matter of chance, since no amount of ingenuity can make a king win an ace, and so on. This produced an argument as to the merits of the game; and as two of the disputants obstinately maintained the original position, it was proposed to test their powers by matching them against two excellent players in the room. To this match, strange to say, the bad players agreed, and a date was fixed. Before the day arrived, it was proposed to play the match in double, another rubber of two good against two bad players being formed in an adjoining room, and the hands being played over again, the good players having the cards previously held by the bad ones, and vice versa, the order of the play being, of course, in every other respect preserved. The difficulty now was to find two players sufficiently bad for the purpose; but two men were found, on condition of having odds laid them at starting, which was accordingly done.

"On the appointed day, a table was formed in room A, and as soon as the first hand was played, the cards were re-sorted and conveyed into room B. There the hand was played over again, the good players in room B having the cards that the bad players had in room A. At the end of the hand, the result was noted for comparison, independently of the score, which was conducted in the usual way. Thirtythree hands were played in each room. In room A, the good players held very good cards, and won four rubbers out of six; in points, a balance of eighteen. In room B, the good players had, of course, the bad cards. They played seven rubbers with the same number of hands that in the other room had played six, and they won three out of the seven, losing seven points on the balance. The difference, therefore, was eleven points, or nearly one point a rubber in favour of skill.

"A comparison of tricks only showed some curious results. In seven of the hands the score by cards in each room was the same. In eighteen hands the balance of the score by cards was in favour of the superior players; in eight hands in favour of the inferior. In one of these hands the bad players won two by cards at one table, and three by cards at the other.

"The most important result is, that at both tables the superior players gained a majority of tricks. In room A, they won on the balance nineteen by tricks; in room B, they won two by tricks.

"It will be observed that this experiment does not altogether eliminate luck, as bad play sometimes succeeds. But by far the greater part of luck, *viz*., that due to the superiority of winning cards, is, by the plan described, quite got rid of. "

This account was also reported in the *Whist* of May 1893 and led to a question from the current Secretary of the American Whist League, Mr. Theodore Schwarz. Then in September *Whist* 1893 it was reported:

> "At the request of Mr. Theo. Schwarz, of Chicago, 'Cavendish' has furnished the names of the players who participated in the game of duplicate Whist described in the cutting sent us by 'Cavendish', which was published in May *WHIST*.

Theodore Schwarz

> "AUGUST 17, 1893.
> My DEAR SCHWARZ : You wanted to know the names of the first duplicate Whist players. The date has already been given in WHIST. I have heard from my brother, and he states as follows:

	Daniel Jones	
Chinery	Table 1	Lewis
	Edward Wilson	

	Bullock	
Matthias Boyce	Table 2	Henry Jones
	Izard	

> "Daniel Jones is my brother. Chinery was afterwards editor of the *Times*. John D. Lewis was afterwards M.P. for Devonport. Boyce is known in Whist circles as 'Mogul'.
> *HENRY JONES*."

The editor of *Whist* then felt compelled to say:

> "Notwithstanding this early record, we think John T. Mitchell, of Chicago, is entitled to all credit for introducing the game of duplicate Whist to the public. It is to the discredit rather than to the credit of players who made earlier trials and failed to realize the merit of the game and reduce it to practice."

One thing is clear from this 1857 event, the duplication of play was purely to compare a weak pair's performance with that of a strong pair. It was not a form of competition, tournament or otherwise an event to produce a winner.

In this first duplicate the procedure of play was not modified in any way. The hands were played and tricks gathered as in every other game of whist. The hands were then re-sorted before being played at the other table.

2.4 The first duplicate in the U.S.A.

There has to be this pioneer, the individual who has the courage, the ambition to overcome the obstacles that always develop when one tries to do something worthwhile, especially when it is new and different.
Alfred P. Sloan

Milton C. Work (1864–1934), a noted authority on whist, bridge and auction bridge, had seen an account of the Cavendish experiment in an 1880 London newspaper. At that time Work was a freshman at college at the University of Pennsylvania.

Milton C. Work

Work wrote of his experience in Bridge World, November 1932:

> "Armed with the Duplicate idea in embryo, I, an exceedingly fresh freshman, issued a challenge to The Saturday Night Club to play a Duplicate match against the team of the University of Pennsylvania Whist Club. The Saturday Nighters were quite willing to give the presumptuous youths a lesson, and in due course the match was played at the residence of Captain John P. Green, then vice-president of the Pennsylvania Railroad. The date was February, 1881, and I am satisfied that this was the first Duplicate card game ever played between two clubs… . A most interesting experiment for the players and also quite a triumph for the winners, as in twenty-four hands the boys defeated their more experienced rivals by the liberal margin of 17 tricks.
>
> "Merely as a record, I will note that the players on the two teams that participated in the first Duplicate game between clubs were Messrs. John P. Green, E. Price Townsend, J. C. Pinkerton and Dr. Jones for The Saturday Night Club, and Messrs. Ellis Ames Ballard, James P. Cowperthwaite, Gustavus Remak, Jr., and Milton C. Work for the University of Pennsylvania. I am glad to add that Messrs. Townsend, Remak and Ballard are now among the most active Contract Bridge players of the Quaker City."

Copyright 1932, by The Bridge world Inc.

2.5 The Allison improvement

You can discover more about a person in an hour of play than in a year of conversation.
Plato

The first person to see the folly in re-sorting the hands was James Allison. He was born in Glasgow, Scotland, February 21, 1848. He was in business as a merchant in Glasgow and first became interested in whist in 1880, and in 1886 assisted as one of the 'sorters' in a duplicate match between eight players of his club, the Carleton.

> "The old method employed by 'Cavendish' and his friends in 1857 was used, but it was very slow and cumbersome work, and nettled Mr. Allison when he found that the 'sorters' could not keep up with the players. Soon thereafter he devised his method by which the hands were kept separate as they were played, and the necessity of registering and re-sorting them was done away with."

The following paragraph appeared in the *The Field, the country gentleman's newspaper* No. 1,843 – April 21, 1888 page 570. It is reproduced in its entirety:

> **"MATCH AT WHIST. – TRIAL OF NEW DUPLICATE SYSTEM** – A match at whist was played in the Albany Hotel, Sauchiehall-street, Glasgow, between teams of the Carlton and Wanderers Clubs, on Monday evening last, 16th inst., when a new system of duplicate play, the invention of Mr. James Allison, was tested. The cards, at the commencement of each hand, were dealt in the usual manner; but in the course of play they are not formed into tricks. Each player keeps his thirteen cards before him till the finish of the hand, and after playing to each trick he places his card either longwise or shortwise, to show by which side the trick has been won. This arrangement prevails at two tables, the hands being simultaneously played. The hands as soon as finished are gathered by each player and placed backs up on the tables, the dealer leaving the trump card face up on the top of his packet. The players then change tables, and re-play the hand, the players being reversed. The players were: Wanderers, Messrs Gourlay, Allison, Finlay, and Thompson; Carlton, Messrs Rogerson, Irwin, Miller, and McCall. On changing tables, Miller and McCall took hands previously held by Allison and Gourlay; Irwin and Rogerson those held by Thompson and Finlay; and vice versa. A. deal was only necessary every two hands, and, but little more time, after some practice, was taken than in playing one hand in the usual way. The secretary of the Blythswood Club and the presidents of the St. George's and Carlton were present, and

expressed a high opinion of the 'Allison' system as a great saving of time, and also as tending to eliminate luck, and had no doubt it would be adopted as the system on which the proposed inter-Club tournament in Glasgow would be carried out next season. As in all matches on the duplicate system, the score was close, the final result being: Carlton, 39; Wanderers, 34. A vote of thanks to Mr. Allison terminated the proceedings."

The *London Figaro* reported this new duplicate whist in the following way:

"AS TO DUPLICATE WHIST
The new way of playing whist – duplicate whist as they call it – is already causing the most intense excitement in whist playing circles, and wherever I go, in my whist playing capacity I mean, I find the pros and cons of the new method of playing the game being vigorously – nay, in some cases, almost fiercely – discussed. The alterations started by Mr. R. A. Proctor as to 'signaling' and 'echoing' and other modern devices of that kind was mildness itself compared to that which is raging as to the 'duplicate whist' which a whist playing doctor in the north of London has invented. His 'duplicate whist', I may explain, eliminates at a stroke the element of chance, or luck, or whatever you choose to call it, which has always been a feature of whist playing. For this now you play 'duplicate whist'. Having dealt the cards as usual, you play them according to the existing rules; but then, when the game is over, instead of dealing the cards afresh, the same hands which have been played are again taken by the four players; A and C, however, now having the cards which B and D held, while B and D take the hand just played by A and C. Thus the same hands are played out a second time, and a score is kept so that it may be seen which pair of partners has made the most of the cards they have successfully held. And this process being repeated with every game, the rubber is finally said to be won by the two players who, under the above conditions, have shown the greater skill.
…

"So far as I can judge, however, duplicate whist is not likely to be generally adopted instead of the ordinary game. The element of chance which enters into whist as now played gives it its chief charm in the eyes of many players, who often trust to their luck to enable them to hold their own with far stronger and more scientific players. Luck, indeed, serves not infrequently to make a fair handicap out of what would otherwise be a very one sided race, and though Cavendish and other whist players of the first rank are said to be ardent admirers of duplicate whist, I think it will only oust ordinary whist to a comparatively small extent."

Whoever the 'London Doctor' was he clearly had been in Glasgow in the past month or had read *The Field*. This was also published in *The New York Times* of June 6, 1888, acknowledging the *London Figaro*. Duplicate whist with no re-sorting had moved across the Atlantic very quickly.

2.6 The first duplicate tournament

The way of the pioneer is always rough.
Harvey S. Firestone

Duplicate whist tournaments were being played in the U.S.A. before the Allison improvement despite the need to re-sort the hands. The first reported duplicate event is found in *Whist Sketches, April 1891* by C. S. B. (Charles Boutcher):

"Trist's club appropriates every year a certain sum for prizes for whist tournaments. He has taken part in eight of these tournaments and his partners and himself have been prize winners in seven.

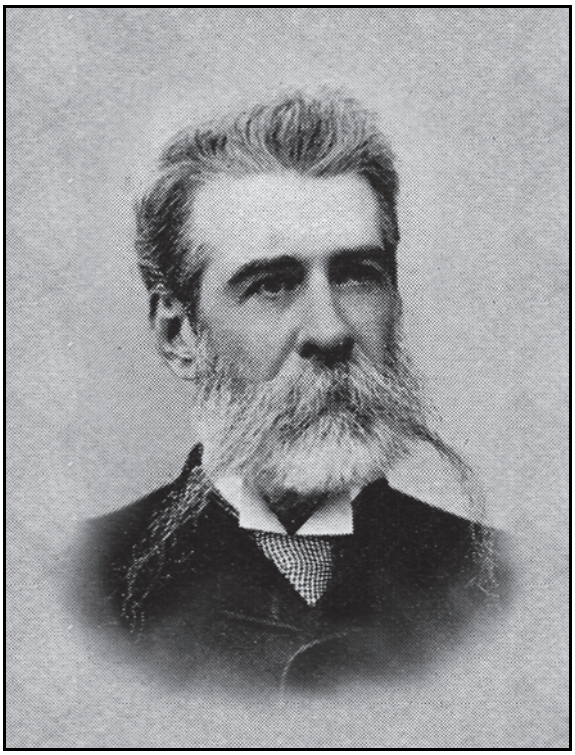
"The result of the tournament of 1882, in which Kennedy and Hare captured the first prize and Bringier and Trist the second, gave rise, probably, to the first duplicate contest ever played in this country. It came to pass in this way. The defeated contestants kept hinting that 'big luck' had been

the main cause of the winners' success. The four named winners, however, were confident that luck was not the controlling reason of their victories, and that their play would beat the play of any team which could be found in the club.

"Trist had read in Cavendish's Card Essays about an experiment of the overplay of hands and he suggested to his three friends to challenge any four members of the club to play, for a prize, twenty-four hands, which were to be overplayed. The challenge offered the odds of eight points; each side to be credited with all tricks made by it. The challenge was posted, but accepted there from two strong players on account of the odds.

"It was accepted and the match was played in July of that year with the following result:
Trist's team scored …….321 tricks.
The opponents scored ……303 tricks.

"The result showed pretty clearly to those who had charged their previous shortage to a bad run of cards, that it was due to the character of play they encountered, and not to big luck or bad luck on one side or the other."



Nicholas Browse Trist

This is the first appearance of a duplicate whist tournament in America and was held in New Orleans in 1882. Unfortunately there are no other details of that contest. Nicholas Browse Trist was an important force in whist and is known as the inventor (with Cavendish) of the system of American leads. His club in New Orleans often conducted duplicate events throughout the 1880s, re-sorting hands and well before the James Allison innovation.

2.7 The John T. Mitchell story

In a word I was a pioneer, and therefore had to blaze my own trail.
Major Taylor

The influence that John T. Mitchell had on whist and particularly duplicate whist should not be underestimated. He was the author of the first book on duplicate whist and the leader of the duplicate whist movement in America. Mr. Mitchell was born in Glasgow, Scotland, April 3, 1854, and went to America in 1875. For five years he was at Milford, Conn., and after a year in Detroit, Mich., he went to Chicago in 1882, where he worked with the Union National Bank. He commenced playing whist in 1888 after reading the 'clipping' from *The Field*. This led to his taking up the study of the duplicate game and in the same year he organized the Chicago Duplicate Whist Club.

John T. Mitchell

Mitchell writes in *Whist* in 1891:

> "It is just a little more than three years ago that the now famous 'clipping' from the *London Field* was going the rounds of the newspapers of this country, and attracted the attention of some of the unlucky Whist players, myself among the number. In a weak moment I had been induced to commence the study and practice of the game of Whist and while I became fascinated with its manifest beauty and variety, there was always a strong undercurrent of dissatisfaction as to the result of my endeavors, for do what I might it seemed impossible to win. But as soon as I had read the 'clipping' I perceived that a new sun had arisen on the Whist horizon, and that hereafter no man would have a right to complain of his luck.
>
> "Of course, the game of Duplicate Whist as now played is very different from that original experiment described by the *Field,* having been modified from time to time as weaknesses appeared or improvements suggested themselves, and there is still room for further improvement and a great opportunity awaiting the player who shall invent some simple mode of attaining the desired result.
>
> "Inter-club Duplicate Whist is briefly this: Contestants are divided into fours, instead of pairs, one

pair being the copartners of the other pair, and sitting in the opposite positions at another table. After every original deal the hands are passed from one table to another and replayed, each pair's copartners playing their opponents hands, thus giving both sides exactly the same hands.

"The only objection to this game is the fact that the resulting loss or gain cannot be imputed to either of the pairs, and must be shared by both alike. There are two ways now in vogue for overcoming this defect. One is the progressive or multi-table game, in which every original deal is played as many times as there are tables engaged, and the loss or gain to individual teams is computed by comparing the actual score with the average of all the scores made by the players who played the same hands. The other is the retrospective or direct duplicate game, in which an agreed number of original hands are played continuously, and afterwards replayed by the same players, the positions of the hands being, of course, reversed in the duplicate. I do not know who is entitled to the credit of first proposing the progressive game but if any credit is to be accorded for the direct duplicate one I claim it for myself, as until the articles which I wrote on the subject appeared in the *Chicago Globe* of December 2, 1888, and *Tribune* of January 20, 1889, no game of this nature had ever been proposed to the Whist fraternity that I was aware of… We have the tools now, and there is nothing to prevent us from playing the finest of games in the fairest manner to all, as Duplicate Whist most certainly puts every pair on the same footing, affords an exact and scientific test of skill, and raises Whist into the category which leaves luck behind and includes Draughts and Chess in its list."

Butler reported:

"The true rise and popularity of duplicate in America also dates from the year of the Glasgow match, for among the whist enthusiasts in the city of Chicago who read the account in the London Field and were charmed with it, was John T. Mitchell, who soon set about and organized a match to be played in the same manner. It took place at the residence of Dr. Camp, in the spring of 1888, and is the first match of the kind on record in the West. The eight players participating were: John T. Mitchell, Harry and Ezra Booth, G. K. Mitchell, J. W. Mitchell, Frank Huntress, George Owen, and G. C. Aldrich. The play was so satisfactory that it resulted in the organization of the Chicago Duplicate Whist Club, which created a great furor by successively defeating half a dozen other organizations at duplicate. The matches attracted wide attention."

2.8 The first book on duplicate

On every side, and at every hour of the day, we came up against the relentless limitations of pioneer life.
Anna H. Shaw

John T. Mitchell published the first book on organizing duplicate events: *Duplicate Whist* (1891). It is quite clear from this book the Allison improvement was not being used for some events. In chapter 1 he describes the game he calls 'retrospective':

"Two independent games are played simultaneously, requiring eight players. The original hands – fourteen or sixteen usually constituting a game – are played from first to last without interruption; then the hands are reproduced and distributed in the reverse position, giving to each team the cards their opponents held originally…

"The rearrangement of the hands for the duplicate play is accomplished in the following manner. Each player is provided with a hand register, on which he records the cards that are dealt to him, the record, of course, being taken before the first lead is made. The hand is then played, and the score (by tricks) recorded. The cards are then shuffled and dealt, and again recorded on the hand register, and so on, until the stipulated number of hands have been played. All this time the players have not left their seats, or departed from the practices which obtain in 'straight' whist, except that they have taken a record of their hands. Now, however, in order to effect the arrangement of the hands for the duplicate play, it is necessary to have the players at one table take the hand registers of those at the other table, and re-sort the hands for them, beginning with the first, and continuing in order to the

last. The position of the hands is reversed during the duplicate half of the game, and care must be taken to place each hand on the proper side of the dealer. After each hand is rearranged, the players change tables, and proceed to play the hands as they find them, each player taking the corresponding seat at the other table."

This is a rather quaint game. You and your partner play each of the 16 hands as North-South and then later as East-West. You are required to be honorable in this game and try not to remember cards. Clearly with no dummy as in bridge the task of remembering is more difficult. The final word from Mitchell on this type of game is:

> "No talking in regard to cards held or leads made must be indulged in while the original hands are being played. Every word might emphasize a point which would tell for or against the player in the duplicate, and the player who breaks this rule should be fined a point. ***Duplicate whist must be a game of 'whist' indeed***."

There were other variations on this 'retrospective' game for two tables. Individual and pairs events called 'Progressive duplicate whist' were also explained in this first book. Given how complicated they were to conduct I am sure they were not very popular with the players. There were hand records (registers), the score cards and complicated moves that were confusing at best.

The inter-club and progressive inter-club events were also conducted along similar lines for many players from each club.

In chapter 4 'Inter-club duplicate whist' Mitchell makes the observation:

> "Some objection has been made to the Retrospective game for the reason that mistakes in recording the hands are so apt to occur that it is difficult to rearrange the hands for the duplicate play, and also because players with phenomenal memories can remember the cards and leads of the original hand when they are playing the duplicate. These objections are entirely overcome in the Inter-Club game, in which the duplicate hands are arranged as soon as the original hands are played, and by the same players who held them, and are played off by other players at another table who are co-partners of the original players. Any multiple of eight may participate in this game, each eight being separate and distinct from the others."

Next is the 'Progressive inter-club game':

> "This game is played to establish the relative strength of the different teams. Every original hand is duplicated as many times as there are tables, thus giving every player a turn at every hand."

With the conclusion:

> "Sometimes it is found that a slow recorder will delay a whole club, especially in the progressive game; and when this is the case he should be excused from marking altogether, and the other three players at his table can assort their hands, he taking the remainder. Any whist player can remember whether the cards so given him are the correct ones. This exemption from recording the hand should be insisted upon in such an emergency without hesitation, as it is manifestly unfair to keep a number waiting for the sake of one, when the difficulty can be gotten over in so simple a way."

The first suggestion of using a player movement was for these inter-club games where there were 3, 4 or 5 tables involved. There were 3, 4 or 5 pairs from each club playing 3, 4 or 5 hands in each round.

> "The two table Inter-Club game does not indicate the relative merit of the different teams; and as this is a most desirable thing to establish, the number of players should be increased to twelve whenever possible, the third table affording the factor necessary to strike an average for the weak and the strong hands, thus enabling the gain and loss to be computed by the difference between the actual and the average score. In playing this game it is not necessary to leave the hand register at the

original tables, as in the many table progressive game. Instead of changing in regular rotation, however, the players change in the following manner.

"After playing and rearranging the original hand, the players at first table move to second, those of second table to third, and the third table players to first. After playing the hands they find at these tables, they rearrange the hands they *originally* held, and move back to their original tables, where they find the remaining hand of the three. The accompanying table will illustrate this, perhaps, more clearly. Let A represent the players at first table; B, the players at second table; and C, the players at third table.

Table	**1**	**2**	**3**
Original	**A**	**B**	**C**
First change	**C**	**A**	**B**
Second change	**A**	**B**	**C**

"Each player carries his register and scorecard with him, and marks the score in triplicate. After every second change, the players find themselves back at their original table, and, after playing the triplicate hand, start a new deal and move as before, – comparing scores after each triplicate hand."

So each table deals, records, plays a hand and re-sets the deal. They all move to a new designated table, play the hand there and re-set their *first* hand and move again to play the last deal. These rounds of play can be repeated any number of times. A similar arrangement can be followed for 4 or 5 tables with similar 'movements'.

2.9 The need for boards

Before anything else, preparation is the key to success.
Alexander Graham Bell

Mitchell says, in the concluding paragraph of chapter 4:

"If, for any reason, the players object to rising from one table and walking to another, the cards can be passed by means of trays, on which the positions are marked; and each player must see that he takes the cards off his 'corresponding' number."

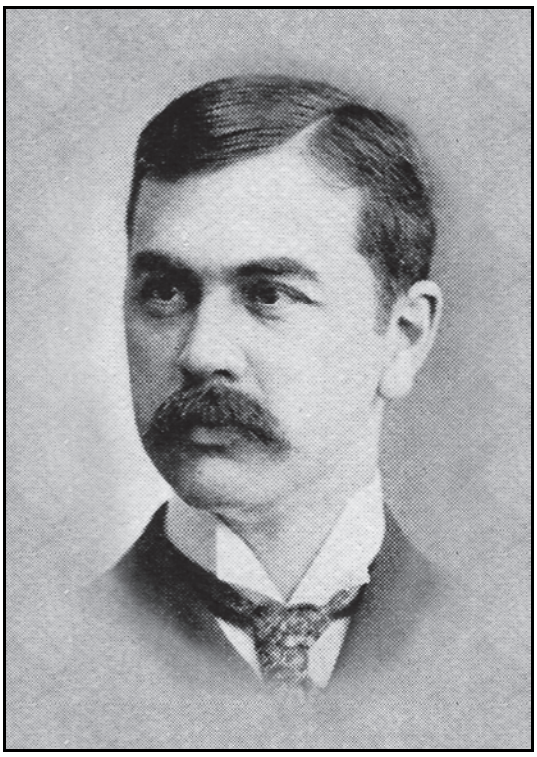
A refined version of the Allison system was clearly needed to eliminate a lot of the problems. Butler says:

"The Allison system was generally adopted in this country…" and… "The invention of trays formed still another important step in the progress of duplicate whist. The credit for this improvement largely belongs to Cassius M. Paine, to whom, jointly with James L. Sebring, a patent was granted on November 3, 1891, for the device which to-day is in universal use, and which assured the success and permanent popularity of the new form of whist."

In chapter 7 Mitchell presents the reader with 'Duplicate whist without the hand register':

"The manner in which this game is now played is as follows. The players are reversed prior to the commencement of the game, the players of one four sitting, say, north and south at the first table and east and west at the second table, while the players of the other four sit, say, east and west at first table and north and south at second table; then, when they change tables, each takes the corresponding seat at the other table. Instead of designating the winner of a trick by placing the card longwise or shortwise, thirteen chips or counters are placed in the centre of the table, and as each trick is won a chip is taken, and the number of chips in the possession of either side at the end of the hand shows the number of tricks taken.

"The single table game can also be played in this manner, and twenty hands can be played instead of sixteen, a pack of cards being of course necessary for every deal, the hands being put away in envelopes…



Cassius M. Paine

"Boxes may be used instead of envelopes for the single table game, one box for each player. The boxes should be made square, with a block in one corner the size of the difference between the length and the breadth of the cards, and with lids both at top and bottom. The hands are placed in the boxes, backs down, first lengthwise, then crosswise, alternately; and when all the original hands have been played, the boxes are turned upside down and passed to the right. Boxes should be made deep enough to hold five packs of cards.

"Another method of playing duplicate whist without the hand register has lately come into vogue, which dispenses with the changing of tables. Players retain their seats, and pass the cards from one table to another on trays or boards, on which are marked the positions of the hands and the number of the table, one tray being required for every table.

"It may be remarked that in this style of duplicate game the second half of the game is played exactly like ordinary whist, the cards being thrown into the centre of the table and gathered into tricks."

Mitchell does deal with the problems of organizing man-against-man events (called individuals today) for retrospective games and did supply schedules for this type of play. The 8 player schedule was shown as:

Original position	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Deals 1 and 2, or 1 to 4	**A**	**C**	**B**	**D**	**E**	**G**	**F**	**H**
Deals 3 and 4, or 5 to 8	**A**	**E**	**C**	**G**	**B**	**F**	**D**	**H**
Deals 5 and 6, or 9 to 12	**A**	**E**	**D**	**H**	**B**	**F**	**C**	**G**
Deals 7 and 8, or 13 to 16	**A**	**B**	**E**	**F**	**C**	**D**	**G**	**H**
Deals 9 and 10, or 17 to 20	**A**	**D**	**F**	**G**	**B**	**C**	**E**	**H**
Deals 11 and 12, or 21 to 24	**A**	**B**	**G**	**H**	**C**	**D**	**E**	**F**
Deals 13 and 14, or 25 to 28	**A**	**C**	**H**	**F**	**B**	**D**	**G**	**E**
Duplicate position	**2**	**3**	**4**	**1**	**6**	**7**	**8**	**5**

This schedule is not a square. Positions 1, 2, 3 and 4 correspond to the seats N, E, S and W at the table and there is no cyclic movement of the players. Mitchell also produced a similar schedule for 16 players in this first book. Clearly there is a long way to go with 'movements'.

2.10 Development of trays and boards

America is a country of inventors, and the greatest of inventors are the newspaper men.
Alexander Graham Bell

The first attempt at retaining original the hands for over-play or re-play was a box. The boxes were generally for two table games, so playing of hands at each table was simple. See Mitchell's description of their use in the previous chapter. These were called the St. Louis 'method':

DUPLICATE WHIST.

THE ST. LOUIS METHOD IS THE SIMPLEST, CHEAPEST, MOST DURABLE AND BEST.

PATENTED 1892.

Half the Shuffling and Dealing, Ten Times the Interest.

RULES OF WHIST UNCHANGED.

The above cut represents one of our D. W. Boxes with one cover removed and not shown in cut. The Box is reversible, double-ended, both ends exactly alike. having two covers, one for each end.

If your Dealer hasn't it we will send you Full Outfit, consisting of 4 DUPLICATE WHIST BOXES, 12 DECKS BICYCLE CARDS, 50 PRINTED SCORES, 13 CELLULOID COUNTERS, and Full Directions, all in handsome case, prepaid, for.....$3.00
Without Cards, prepaid, for..... 1.50
4 D. W. Boxes large size, capacity 48 hands, Duplicate Play (without Cards), in case, prepaid.....$2.00

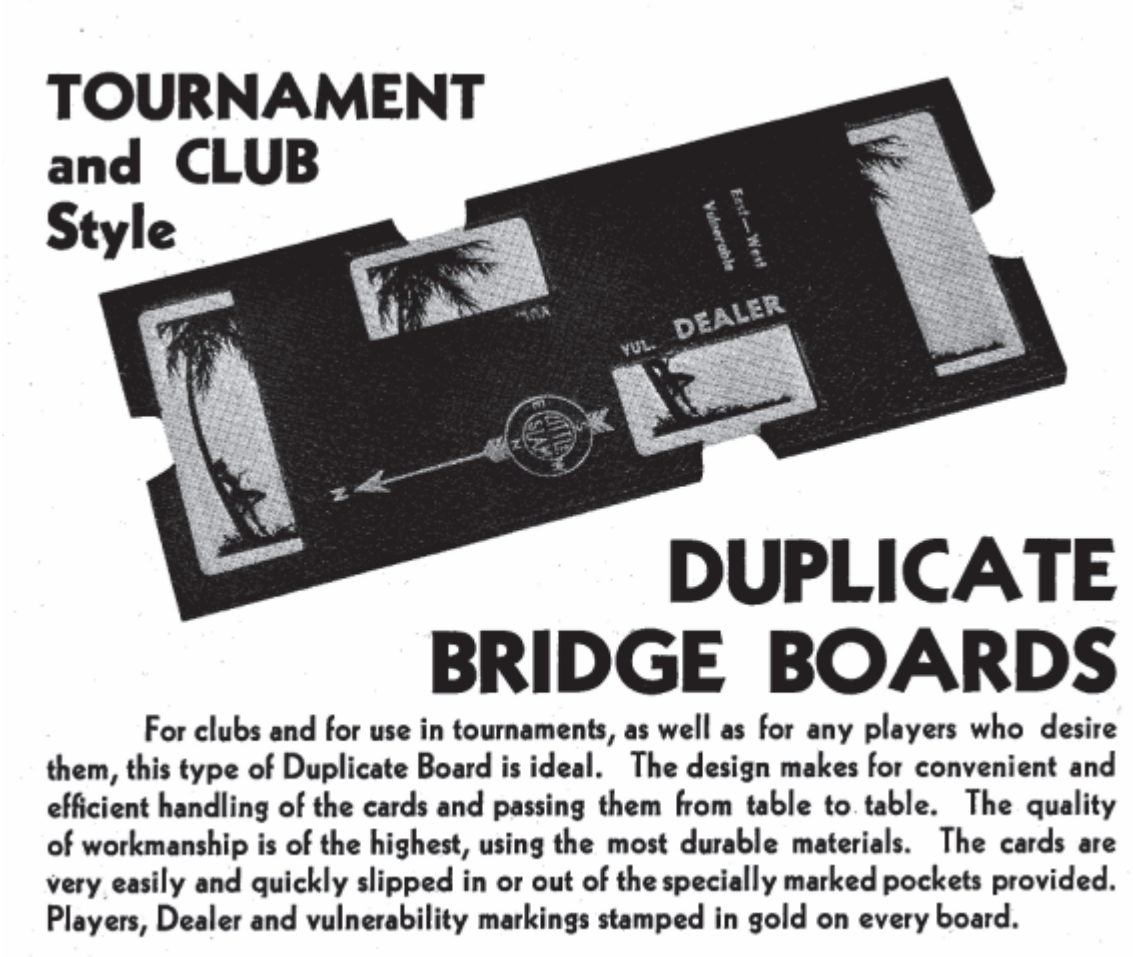
We guarantee Complete Satisfaction in every respect. Bicycle Cards are the BEST Cards sold anywhere for 25 cents.

ST. LOUIS DUPLICATE WHIST CO.,
2322 Albion Place, St. Louis, Mo.

By 1891 Ihling Bros. & Everard had patented their Kalamazoo Duplicate whist tray. This was a simple device in principle with an elastic band at each compass point to hold the hand. The advertisements were revealing the strong support for this 'method', though there was also the Cincinnati 'method'.

The competition increased. This shows one page of advertisements from June 1896 *Whist* journal.

Once the very popular whist game had shifted to bridge the development of boards had taken another turn and became standardized on something like the following. This is an advertisement from 1933 (copyright Bridge World) showing the Culbertson deluxe board. Once contract bridge was the game of choice the boards included vulnerability details.



Today a duplicate board may be made of various materials, the most common being aluminium, plastic, wood and cardboard. A board may be a solid piece of equipment as with aluminium, wood and hard plastic, or a soft wallet type of equipment with soft plastic or cardboard. Aluminium is certainly the most durable material but is also the most expensive at the initial outlay. The disadvantage of aluminium is that it is very wearing on cheaper cards. Thus it is almost imperative to use the more expensive plastic cards with these boards, but for long-term use this combination is certainly the most economical, lasting for many years even with regular daily use. For low volume use the plastic wallet coupled with good quality cardboard cards is an ideal compromise.

Since the advent of the dealing machines the boards need to be able to fit with the dealing machines and are generally made from plastic which is cheap and disposable.

The markings on the boards indicate the dealer and the vulnerability for that deal. The standard ordering of the dealer and vulnerability as used by the World Bridge Federation as well as most countries is as follows.

Dealer: North on Board 1, East on 2, South on 3 and West on 4. This pattern continues for all subsequent sets of four boards.

Vulnerability: starting from board 1, in order:

Nil	N-S	E-W	All
N-S	E-W	All	Nil
E-W	All	Nil	N-S
All	Nil	N-S	E-W

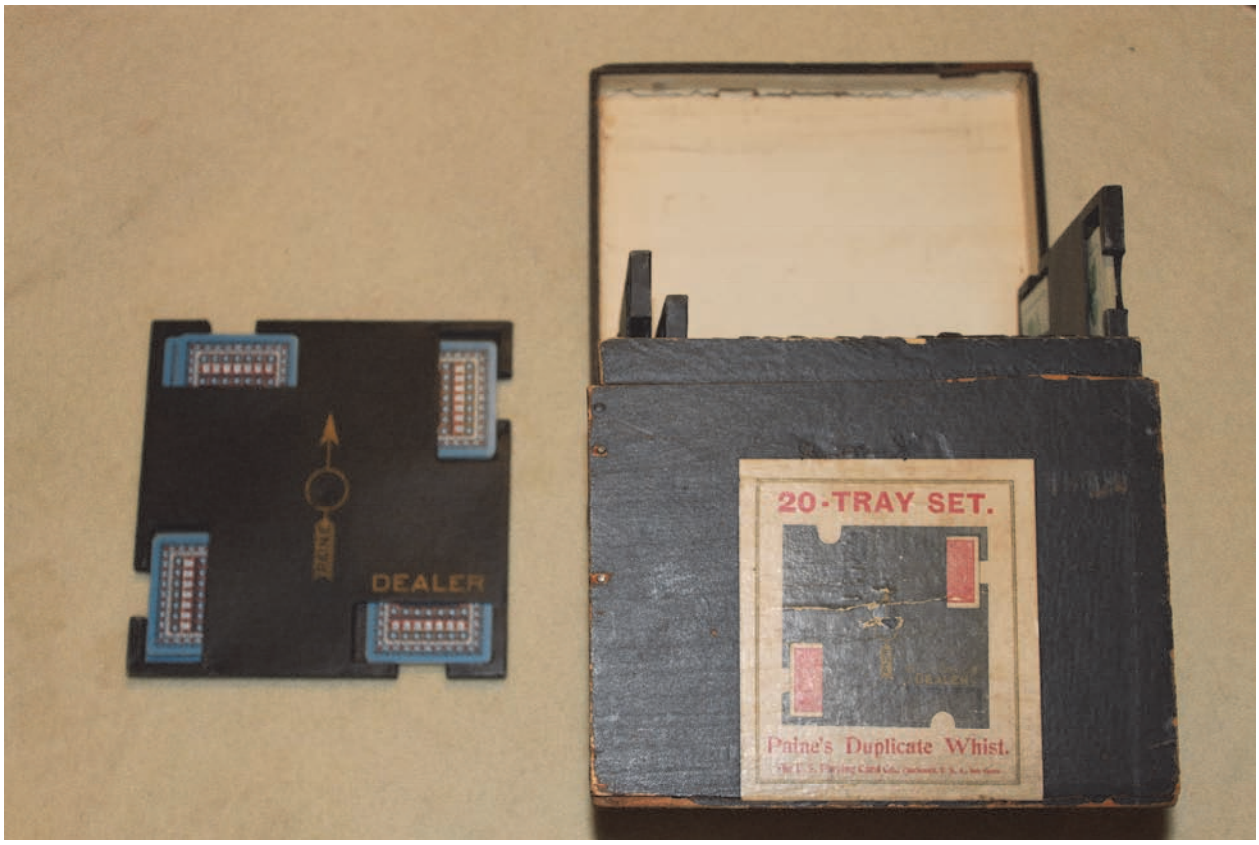
As an aid to memory, use *ONEB:*

O	N	E	B
N	E	B	O
E	B	O	N
B	O	N	E

Where
O = zero or Nil;
B = both or All;
N = N-S and
E = E-W.

This pattern continues for each set of 16 boards. Note this is a Latin square.

Part 3 Whist schedules



Set of 20 Paine duplicate whist trays, circa 1896.
These were the most common boards of the time and one of the most popular designs for boards later used in bridge.

3.1 The first schedules

There are basically two types of people. People who accomplish things, and people who claim to have accomplished things. The first group is less crowded.
Mark Twain

There was a lot of investigation and specification of movement schedules to cater for these man-against-man events during and after 1891. The key players in this quest were Mitchell, Safford, Howell, Moore, Whitfeld and Foster.

Also at this time terminology was in the making. The first term used for 'movement' was 'schedule' and applied to individual events. The term 'system' tended to apply to the results of the efforts of individuals and 'method' was used to summarize systems into groups, including 'modifications' to the systems. Eventually the term 'movement' became the favorite.

A very interesting and telling letter to the editor of *Whist* was:

> "CHICAGO, November 12, 1891.
> *Editor Whist:*
> In fixing up schedules for the progressive duplicate game, it has been found possible to arrange eight players so that at the end of seven hands each player will have played once with every other player, and twice against him. Sixteen players can also be made to go through a like experience in fifteen hands, but the difficulty comes in when it is attempted to arrange for twelve players to accomplish this feat in eleven hands. It is easy enough to arrange them so that they will play once with every other player in that number of hands, but although the writer has made, probably, a hundred attempts, he has not yet been able to arrange the players so that they will be opposed to every other player twice during the game. During his experiments, he has found that twenty-one out of the thirty-three combinations is as far as he can get, and he would like some of the readers of Whist, who are also mathematicians, to either help him out of his difficulty or put an end to his suspense by informing him of the law in figures which makes the solution of the problem an impossibility. Will anyone oblige? F. T. M."

We do not know who F.T.M. is but we can assume the schedules for 8 and 16 players he mentions are those published in Mitchell's first book. A month later *Whist* reports:

> **"Formula for Table Arrangement.**
>
> "WASHINGTON, D. C., December 10, 1891.
> *Editor Whist:*
> In your edition for December 'F.T.M.' inquires whether, in fixing up schedules for the progressive duplicate game, it is possible to arrange twelve players so that each player shall have played once with every other player and twice against him.
>
> "I give below the arrangement of players for 8, 12, and 16. At the end of each set between partners, the players take the seat next lower in number, except No.1, who always sits still, and No.2, who goes to the highest numbered seat."

Note the stationary player 'number 1' and the cyclic procedure for the other players that any bridge player of today will recognize as that applicable to a 'Howell' movement. He then goes on to supply the seating positions for the starting position.

For the record these arrangements published in *Whist* December 1891 read as follows, with the 'v' meaning versus:

8 players: 1–2 v 5–3, 8–4 v 6–7
12 players: 1–2 v 6–3, 12–11 v 7–9, 10–5 v 8–4
16 players: 1–2 v 12–5, 16–15 v 9–13, 14–6 v 5–3, 8–11 v 10–4

The arrangements given here are E-W v N-S with the tables separated by commas.

It should be quickly noted there is a misprint in the 16 player schedule. Player 5 is repeated and no player 7 is shown. Table 1 should read 1–2 v 12–7. Today authors of movements have the luxury of computers where the movements are checked for validity and the published schedule taken directly from the original.

Remember these are 'movements' to be used in games where all players play the same hands. There are issues with duplicating boards etc., with which this letter also deals. In other words these are not movements using trays or boards. For example, on round 1 in the 8-player game, table 1 plays board 1 and table 2 plays board two and they reset them and exchange hands to be played at the other table. It is signed:
"Yours truly, A.G.S.".

Note: 'A. G. S ' refers to Mr. A. G. Safford. Alfred Greeley Safford was born at St. Albans, Vt., August 17, 1844. He was educated at the University of Vermont, class of 1863, and received the degree of A. M. (Masters) from that institution. He left college in 1862, and entered the military telegraph corps of the army, serving as chief operator of the Department of the South, and at the headquarters of General Grant, at City Point, Va., during the last year of the war. After the war Mr. Safford took up the study of law. He was admitted to the bar in September, 1867, in his native State, and practiced there until 1886, when he removed to Washington, D. C.

Alfred Greeley Safford.

Immediately after this letter and in the same edition of *Whist* is another letter to the editor:

"DECEMBER 12, 1891.
Dear Whist:
In reply to F. T. M. in December *Whist*, allow me to say that the rule he seeks for the arrangement of

> any number of players, divisible by four, forms an important part of my forthcoming work on 'Duplicate Whist.' The law is not in any work on Mathematics, but was discovered by E. C. Howell, a Harvard man, who was paid by me to study it out, if he could. The formula is very simple, and shows that there are thousands of ways in which the players may be arranged…
> Yours truly, R. F. Foster."

John T. Mitchell in his second edition of his book, *Mitchell's duplicate whist* (1897) said:

> "Formulas for eight, twelve, and sixteen players were published in Whist of January, 1892, by Mr. A. G. Safford, of Washington, D. C., who was the first in the field with solutions of problems of this nature; but until Mr. W. H. Whitfeld came to the rescue, in 1895, I was unable to obtain a formula for any number higher than sixteen, with the exception of thirty-two."

Mr. W. H. Whitfeld is a very significant contributor to movements. We will see more of him later.

3.2 Elaikim Hastings Moore

Pure mathematics is, in its way, the poetry of logical ideas.
Albert Einstein

Elaikim Hastings Moore (born: 26 Jan 1862 in Marietta, Ohio, USA, Died: 30 Dec 1932 in Chicago, Illinois, USA). Moore was appointed professor and acting head of the mathematics department at Chicago when the university first opened in 1892.

Butler later reported in relation to cyclic schedules:

> "Professor Moore, of Chicago University, published some elaborate and highly original calculations on the subject in the American Journal of Mathematics (volume 18, pp. 264–303, 1896, Baltimore), and his labors are also utilized by Mr. Mitchell. Professor Moore originated what he calls a '*triple*-whist tournament arrangement,' by means of which, in combination with known arrangements, he is able to construct other arrangements. "Thus," says he, "from the known arrangement of four and sixteen players I constructed for the Denver Whist Club an arrangement for sixty-four players."

Clearly in the early 1890s the individual events were very significant in the whist calendar. The other very significant contribution was Professor Moore's paper 'Tactical memoranda I-III' mentioned above. In his bibliography Moore acknowledges all the above writers and articles. It should be noted the absence of Howell in those references because at the time Howell had no publications. Moore delivered a mathematical method of defining cyclic rotations in matrix theory, extending his areas of interest in the original work of the famous English Mathematician Arthur Cayley (1821–1895), the founder of the school of pure mathematics.

Some of the conclusions were defined in his new matrix specification called Wh[n] for a cyclic whist tournament arrangements. Note n is the number of players in the tournament and of the letters in the cycle * is stationary and the other values i are replaced by $i + 1$ in each round of the cycle.

Wh[4]	| * 0 1 2|				(only class)
Wh[8]	| * 0 4 5|	| 1 3 2 6|			(Safford)
Wh[12]	| * 0 4 5|	| 1 10 2 8|	| 3 7 6 9|		(Safford)
Wh[12]	| * 0 2 3|	| 1 8 5 7|	| 4 10 6 9|		(New class)
Wh[16]	| * 0 10 5|	| 1 2 4 8|	| 3 11 14 12|	| 6 9 13 7|	(Safford)

His paper takes this subject, mathematically, a lot further, but from our point of view the significant point is the clear acknowledgement of Safford's ground work.

To clarify, the 8 player schedule supplied by Safford above, was shown as: 1–2 v 5–3, 8–4 v 6–7.

For Moore's Wh[8] matrix subtract 2 from each number and make the stationary 1 = * so that we get

| * 0 3 1| | 6 2 4 5|

These are identical mathematically to Moore's representation of the class. The pairs of numbers can be switched and repositioned without changing the cyclic nature of the schedule.

This *Safford* matrix can be expanded to be shown as:

1	2	5	3
1	3	6	4
1	4	7	5
1	5	8	6
1	6	2	7
1	7	3	8
1	8	4	2

8	4	6	7
2	5	7	8
3	6	8	2
4	7	2	3
5	8	3	4
6	2	4	5
7	3	5	6

These cyclic individual player movements were not only the first published, but with some manipulation, were the first Latin squares used in bridge. There is no doubt that Safford had no idea what he was dealing with mathematically. Euler had laid all the ground work for these more than 100 years earlier but no one had made the connection.

The eight player movement given above can be depicted as a Latin square. The first column and first row show the player number and the order 8 Latin square formed inside the thick border shows the round numbers where each of the players play with each other. Clearly players do not play with themselves and so a zero is entered in that cell.

Player	1	2	3	4	5	6	7	8
1	0	1	2	3	4	5	6	7
2	1	0	4	7	2	6	5	3
3	2	4	0	5	1	3	7	6
4	3	7	5	0	6	2	4	1
5	4	2	1	6	0	7	3	5
6	5	6	3	2	7	0	1	4
7	6	5	7	4	3	1	0	2
8	7	3	6	1	5	4	2	0

This depiction of the movement is not very useful. It simply says who you are playing with on each round. There is no indication of which table to sit at or who your opponents are. Therefore it is no wonder the Latin square connection did not exist.

This same cyclic matrix can be shown in the following way. Take away the first column (where player 1 is stationary), then subtract 1 from all other numbers and we get a Latin square. This is very similar to a Sudoku square, each row and each column has each number (1 to 7) once and only once. Again, this is not very useful for bridge organizers. See appendix J for more details (page 401).

1	4	2	7	3	5	6
2	5	3	1	4	6	7
3	6	4	2	5	7	1
4	7	5	3	6	1	2
5	1	6	4	7	2	3
6	2	7	5	1	3	4
7	3	1	6	2	4	5

3.3 The Safford system

All systems are oligarchy. There is no other.
Tom Metzger

Organising teams' events was one of the earliest requirements of duplicate whist. Clearly the round-robin type of event was the most obvious answer to this type of event.

Mitchell expressed the solution to deriving the draw for this event as:

> "Safford's method of placing the players in numerical order down one position and up the other applies to every even number of teams, without changing the numbers of the players to correspond with the previous occupants of the seats; and as there are only two positions in the case of teams, it is a very simple matter writing out the formula for a team schedule. All you have to do is to write 1, 2, 3, then the last number, then 4 and the second last number, then 5 and the third last number, and so on until all your numbers are exhausted, and your formula is made. The succeeding numbers follow each other in the same order as in the individual schedules. For instance, take six teams; the formula would be:

Game	Table 1	Table 2	Table 3
	N-S …E-W	N-S …E-W	N-S …E-W
1	1…2	3…6	4…5
2	1…3	4…2	5…6
3	1…4	5…3	6…2
4	1…5	6…4	2…3
5	1…6	2…5	3…4

> "By following up the numbers it will be found that the teams that start east and west keep moving towards the last table until they get to the foot, they then change over to north and south and move up towards the first table, and after playing north and south at the second table they go to the east and west seats at the first table, and then down the east and west seats to where they started. Similarly, the teams who start north and south keep moving- up towards the first table to No. 2, then down the east and west seats from the first to the last table, and back along the north and south seats to where they started. In their travels they meet the other teams in a regular order. If the first team played against is odd, the next team will be the next higher odd number, then the next higher, and so on until the highest odd number is met, after which the first even number (No. 2) will be the team played against, then the even numbers will be met in succession, until all of them have been exhausted, when No. 3 will be met and the balance of the odd numbers. At any point in the procession where a team's own number would come in, No. 1 will come in to fill up the blank.
>
> "It will thus be seen that if players could be depended on to follow directions a schedule would be unnecessary. All that has to be done is to make them change tables in accordance with the movement

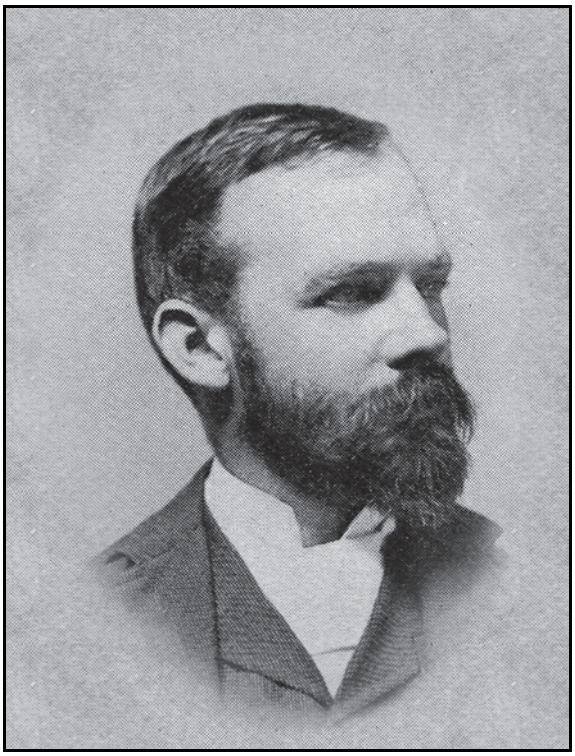
described in the schedule, and they could seat themselves any way at all for the first game and go ahead. Unfortunately, however, players cannot be depended upon in such a manner, and even if they could a schedule would be necessary to settle the disputes which would arise."

Today's tournament directors will appreciate these comments.

3.4 Howell's Law

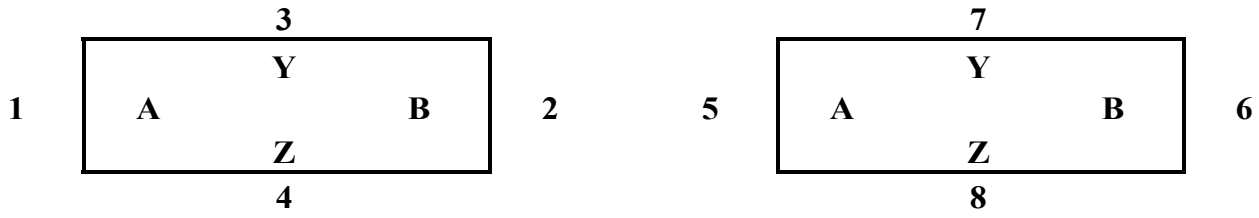
The best way to get a bad law repealed is to enforce it strictly.
Abraham Lincoln

In 1894 Foster published *Foster's duplicate whist* and included in the individual movements the following explanation of *Howell's law*. It is immediately apparent that the explanation is far from simple, but it is given here in its entirety to complete the record on this subject.



R. F. Foster

"Let us suppose this to be the first arrangement, although any other will do as well:



"The partners are 1-2, 3-4, 5-6 and 7-8, while the oppositions are 1-5, 2-6, 3-7 and 4-8. Seven sets will exhaust the possible partnerships, and also the oppositions. The law governing the changes necessary in order to bring this about was first discovered by Mr. E. C. Howell, a graduate of Harvard, who was one of many to whom I explained the difficulty, and the only one who ever offered a solution of it. As his law is not to be found in any work on mathematics, I give it here in full, so that those who wish to arrange other schedules than the one here given may do so:

"Instead of arranging the figures as above, we shall arrive at the same result if we place them in a straight line as follows:

A-B	Y-Z	A-B	Y-Z
1-2	3-4;	5-6	7-8

"The pairs are joined by hyphens, and the two tables are divided by a semicolon. The same positions at each table are opposed, such as 1 and 5, 2 and 6, etc.

"To arrive at the second arrangement of the seven required, we shall follow the rule that each pair in the first arrangement shall be opposed in the second; that is, that 1 and 2 shall now be opposed, and so on. This is not enough, however. The second pair, 3-4, in being opposed must also be *reversed.* The same is true of the third pair, 5-6; and finally the fourth pair, 7-8, must be opposed before the third. Thus we have the full rule:

"Oppose the first pair, reverse and oppose the second, oppose the fourth, reverse and oppose the third. Applying this to our first arrangement, we take 1 and 2, and oppose them by putting 2 where 5 was, and allowing 1 to sit still; then we reverse and oppose according to the rule, which gives us this for our second arrangement:

	A-B	Y-Z	A-B	Y-Z
2nd	1-4	7-6;	2-3	8-5

"This would arrange the tables thus, No. 1 always sitting still as a pivot for the others:

	7				7	
1		4		2		3
	6				5	

"Continuing the process we should get the five following:

	A-B	Y-Z	A-B	Y-Z
3rd	1-6	8-3;	4-7	5-2
4th	1-3	5-7;	6-8	2-4
5th	1-7	2-8;	3-5	4-6
6th	1-8	4-5;	7-2	6-3
7th	1-5	6-2;	8-4	3-7

"If we repeat the process once more, we shall arrive at the original arrangement, showing that the combinations are exhausted.

"It is obvious that we can produce by the aid of this rule, several hundred schedules for eight players; for instead of putting the numbers in regular order originally, they might be in any order, such as 5-3, 7-1; 4-6, 8-2, No. 5, always sitting still. Again, they need not be arranged at the tables as we have given them, but the straight lines of figures can be written *round* the table in either direction, one of which would give this for the initial arrangement:

	2				6	
1		3		5		7
	4				8	

"This being the second:

	4				**3**		
1		**7**		**2**		**8**	
	6				**5**		

"On the next page will be found the complete schedule for eight individual players, arranged at two tables, according to the arrangement first suggested.

"In order to facilitate the changing of so many players at a time, it will be best to follow the plan given on page 144(*), placing on the tables, opposite the position occupied by each player, a number indicating to him the number of the seat he is next to occupy. As the changes are made in regular order, each follows the same course; and if each player has a number on his lapel showing where he sat in the first set, any confusion or mistake can be set right by referring to the schedule, which will show where he should be in any set. The numbers on the tables are as follows for the arrangement given:

	6				**5**		
0		**2**		**3**		**7**	
	1				**4**		

"The zero always sits still, while each of the others goes to the number next higher than the one he occupies, 7 going to 1. It will be found that the players get accustomed to these changes in a few rounds, and find their new positions without difficulty.

"SCHEDULE FOR EIGHT INDIVIDUAL PLAYERS.
"CHANGING EVERY FOURTH HAND.

	3				**7**		
1		**2**	**First set**	**5**		**6**	
	4				**8**		

"No. 1 sitting still, the others change as directed by the numbers on the tables given on page 155 (*).

	7				**8**		
1		**4**	**Second set**	**2**		**3**	
	6				**5**		

"No. 1 sitting still, the others change as before.

	8				**5**		
1		**6**	**Third set**	**4**		**7**	
	3				**2**		

	5				**2**		
1		**3**	**Fourth set**	**6**		**8**	
	7				**4**		

	2				**4**		
1		**7**	**Fifth set**	**3**		**5**	
	8				**6**		

	4				**6**		
1		**8**	**Sixth set**	**7**		**2**	
	5				**3**		

	6				**3**	
1		**5**	**Seventh set**	**8**		**4**
	2				**7**	

"This exhausts the combinations for eight individuals."

This completes Howell's law published by Foster. It should be noted that this is 1894 and his law really only covers the subject of cyclic rotation and no Howell movement is mentioned for pairs.

Note the numbers shown above with * are the original references in Foster's book. The incredible thing about these references is they are not correct in the original book and in fact the pages cannot be found anywhere in the book. It appears this subject was both rushed and incomplete. The subject did not appear in his next book in 1901, most likely because of the unnecessary complexity.

3.5 The Mitchell system

Data is a precious thing and will last longer than the systems themselves.
Tim Berners-Lee

During this same time Mitchell had made great inroads into conducting pair's events in whist. His contribution to 'Progressive duplicate whist' is documented by Mitchell in his 1897 book, *Mitchell's duplicate whist*:

> "The form of the progressive game introduced by the writer into the Chicago Whist Club in 1892 was adopted by the American Whist League for the preliminary play for the Hamilton Trophy at the Congress of 1893 and has been a feature of all succeeding Congresses. The number of hands to be played is divided equally among the number of tables, and the quota at each table is played before any progression takes place. When a set has been played the east and west players move to the next higher numbered table, while the hands are passed to the next lower numbered table and another set is played. Then the east and west players move again in the same direction as before, and the hands are passed in the opposite direction as before, and so on until a complete round of the tables has been made. The hands are passed from the first to the last table, while the east-west players move from the last table to the first. At the conclusion of the game every east and west pair has played against every north and south pair and every pair has played all the hands.
>
> "**The above directions apply only to games in which the number of tables is odd**."

Let us clarify what we are dealing with in a duplicate pairs' tournament movement. Pairs' movements have three basic interacting groups: the boards, the tables and the pairs. The tables are the basic or trivial group and are stationary. In a Mitchell movement, the pairs consist of two sub-groups: the N-S pairs and the E-W pairs.

Therefore a Mitchell movement has 4 groups in all: N-S pairs, E-W pairs, boards and tables. The number of tables equals the number of N-S pairs which in turn equals the number of E-W pairs. The number of boards is a multiple of the number of tables. It is also possible for N-S pairs to equal E-W pairs minus 1 or E-W pairs plus 1, commonly called the half table movement.

The objective of the Mitchell movement is for all the N-S pairs to meet all the E-W pairs while all pairs play all the boards. This will be so only for an odd number of tables. This is achieved by one group remaining stationary (together with the tables), while another group moves up one table and the final group moves down one table.

The normal procedure, as defined by Mitchell, is for the N-S pairs to remain stationary, the E-W pairs to

move up one table, and the boards to move down one table. The advantage of moving the boards, instead of their remaining stationary, is less movement of people around the room at change time.

With the boards stationary and both groups of pairs moving, there is the advantages the boards will never be misplaced, and the pair numbers (both N-S and E-W) are in sequence and errors are obvious in entering pair numbers on traveling score-sheets.

Too much movement during the change tends to spoil the event. On the other hand, if a movement is using more than one room, then keeping the boards stationary is highly recommended (also see *switched Mitchells,* page 154).

As it is quite clear that no pair meets another pair sitting in the same direction, there will be two winners. Mathematically, this does produce the ideal situation: each pair being compared with the same set of pairs on each board and all pairs playing all boards. *This movement is then said to have perfect comparisons*.

The following schedules will help clarify these points. This is the very simple case of a three table Mitchell. The N-S and E-W pairs are shown, for example, as 2-3 respectively. The board-set 1 indicates a set of boards numbered 1 through 8 or 1 through 9 etc., depending on the number of boards in the movement (24 and 27 in these two cases).

Table	1	2	3
Round 1 Board–set	1–1 1	2–2 2	3–3 3
Round 2 Board–set	1–3 2	2–1 3	3–2 1
Round 3 Board–set	1–2 3	2–3 1	3–1 2

Consider the variation where the boards are stationary and the N-S and E-W pairs move in opposite directions, one up and the other down one table.

Table 1 – set 1	Table 2 – set 2	Table 3 – set 3
1 – 1	2 – 2	3 – 3
3 – 2	1 – 3	2 – 1
2 – 3	3 – 1	1 – 2

Consider the darker bound square and use the symbols Euler would have used. Change N-S 1–3 to A, B, C and E-W 1–3 to α, β, γ and we get:

A α	B β	C γ
C β	A γ	B α
B γ	C α	A β

This is a Graeco-Latin square as defined by Euler in the 1780s and transforms into the original magic square (see *The headmaster of mathematics*, page 11).

All *odd-table* Mitchell movements can be represented as Graeco-Latin squares. Where there are even numbers of tables it is also possible, but with a few qualifications.

3.6 Early Mitchell modifications

I tried a dozen different modifications that were rejected. But they all served as a path to the final design.
Mikhail Kalashnikov

In the 1897 book Mitchell goes on to say:

> "If the number of tables is even, after the east and west players have made half the circuit they strike the same hands that they started with."

The solution to this problem was designed so the players did not have to move outside their comfort zone. They could easily handle the 'E-W move up one and pass boards down one' in the odd table movement. In the modern 'bridge' world the players are familiar with other methods of movement or tricks including guide-cards and sharing boards between tables. For 'whist' though things like that had not even been considered for a Mitchell movement. All the solutions of the time were modifications to the standard Mitchell movement procedure.

Hadlock modification

Mitchell continues:

> "This difficulty has been very cleverly overcome by the invention of Professor A. Hadlock, of Kalamazoo, Mich., whose method is as follows:
>
> "Divide the total number of hands to be played into two equal parts, and finish one half before starting the other. When the east and west players get half way around in the progression they remain for another hand or set of hands at the table at which they find themselves, and then pass on to the next table. This makes them finish the first half with one table yet to play. When the second half of the game begins, the east and west players move back to the table next following the one at which they played the double set and proceed as before. When they get half way around the second time they find themselves at the table at which they did not play at all the first time, and as that is where they now play a double set of hands before passing to the next table, when they get around the second time they have played an equal number of hands at each table. Prof. Hadlock's game will apply to any number of even tables, and in the case of four tables, it may be remarked, there is no change of positions between the first and second halves of the game, as at the third change the east and west players finish the first half and are seated at the proper tables for commencing the second."

See the depiction of this movement with 4 tables to the right. Each column represents a table and N-S pair (1 to 4) and each row the rounds 1 to 8. The two numbers in each cell represents the E-W pair (1 to 4) and the board played (1 to 8) on the round. Clearly this is not a square. There are more rows than columns. The reason the inventors of these movements needed to take this approach was the inability to have a square where the movement of the pairs was regular and simple.

1 1	2 2	3 3	4 4
4 2	1 3	2 4	3 1
4 3	1 4	2 1	3 2
3 4	4 1	1 2	2 3
3 5	4 6	1 7	2 8
2 6	3 7	4 8	1 5
2 7	3 8	4 5	1 6
1 8	2 5	3 6	4 7

There are a number of interesting points about this modification for an even number of tables:

- the E-W pairs move around all the tables twice, playing all the N-S pairs twice;
- the boards in play are divided into two equal lots, playing and completing the first half before playing the second half;
- boards are never shared between tables;
- all boards of each half are in play during their half;
- the E-W move is mostly 'up one table';
- the N-S pairs are stationary;
- the boards move down one table.

Hadlock's solution is not that desirable in today's terms though generally acceptable. The main disadvantages are:

- the E-W pairs traverse the movement twice;
- there is a special move when the second half-of-the-boards are introduced;
- the movement cannot be shortened easily.

Otherwise the solution is neat: every N-S pair meets every E-W pair and all pairs play all boards.

Mitchell modification

Mitchell continues:

> "If the number of tables is an even number not divisible by three (such as 4, 8, 10, 14, etc.,) there is another way, the invention of the writer, to get around the difficulty, which is as follows:
>
> "Start the game exactly as described in the directions for odd tables, but divide the quota to be played at each table into two equal portions, and let the east and west players move after every half set of hands has been played, and go the round of the tables twice. The hands, however, only make the round once and are played in exact succession by the north and south players, who play the unplayed half of the first set against the next comers; then the first half of the next set, and so on."

The depiction for 4 tables on the right is similar to that in Hadlock's modification, but it is still not a square.

1 1	2 3	3 5	4 7
4 2	1 4	2 6	3 8
3 3	4 5	1 7	2 1
2 4	3 6	4 8	1 2
1 5	2 7	3 1	4 3
4 6	1 8	2 2	3 4
3 7	4 1	1 3	2 5
2 8	3 2	4 4	1 6

This modification has some of the disadvantages of Hadlock's but the particular disadvantage is the restriction on the number of tables. The main points are:

- the E-W pairs move around all the tables twice, playing all the N-S pairs twice;
- boards are never shared between tables;
- all boards are in play except only half are played on each round;
- the E-W move is 'up one table';
- the N-S pairs are stationary;
- the boards move down one table on every second round.

The novel or even cute way the boards are handled is a forerunner to the use of relay or bye-stand tables. Today we would normally place a bye-stand between each pair of tables so there is no confusion over which boards are to be played and when to move the boards.

Baker modification

Mitchell continues:

> "If the number of tables is so great that only one deal can be started at each table, neither of the above methods of getting around the even table difficulty can be adopted, in which case the skipping-one-table game invented by Mr. E. T. Baker (born July 12 1853), of Brooklyn, comes into play. This is as follows:
>
> "After the east and west players have gone half way around they skip one table and continue the progression. This brings them to the table at which they started with one hand to play, and they play that hand against their original opponents."

The depiction of this movement for 4 tables is shown to the right. This is a square but not a Graeco-Latin square. It satisfies some of the requirements of a Graeco-Latin square but not all.

1 1	2 2	3 3	4 4
4 2	1 3	2 4	3 1
2 3	3 4	4 1	1 2
1 4	2 1	3 2	4 3

- All pairs of numbers are different; all pairs play all boards.
- The first number is not repeated on any row; each pair plays on each round.
- The second number is not repeated on any row; boards are not shared.
- The second number is not repeated in any column; N-S plays all boards.

It fails in the last requirement: in each column the first number has one number repeated. This is seen by the players as playing against one pair for a second time. They also miss one pair.

Most bridge players today are familiar with the *skip Mitchell*. It is surprising Mitchell only recommends this movement when there are one-board rounds. Clearly the disadvantage of not playing one pair and playing one pair twice was considered more serious than moving through the movement twice. See page 96 for further discussion of this movement.

Howell modification

In these early days conducting teams' events was very common and the Mitchell system was the only real movement they had available for this purpose. This was reported in *Mitchell's duplicate whist* in 1897:

> "The other progressive 'four' game is the invention of Mr. E. C. Howell, of Boston, and was called in the programme of the fifth American whist congress, held at Minneapolis in 1895, 'Howell's Modification of the John T. Mitchell Progressive System.' It provides for the playing of every four against every other four and also for complete matches between them. In the writer's pair game no attempt is made to have the east and west co-partners of any north-south team play the same hands in duplicate against the north and south co-partners of the east-west team that started them. This is accomplished in the Howell method by the following process.
>
> "The players move as in Mitchell's system, but the trays are carried, not to the next table, but from the middle to the head, with the others in natural order. With only three tables in play the two systems are identical. With five tables Howell moves the trays up two tables, with seven tables, three; with nine tables, four, and so on.
>
> "When the number of players engaged make an even number of tables an extra table is put in, and during the play there are always two tables which are idle, but the hands are passed back and forth just the same as though the tables were fully occupied.
>
> "The effect of this scheme is to bring about the overplay of the same deals by the same teams, and the score is usually counted by matches won instead of tricks gained. In case of a tie the tricks decide.
>
> "Schedules showing movement of trays under Howell system:"

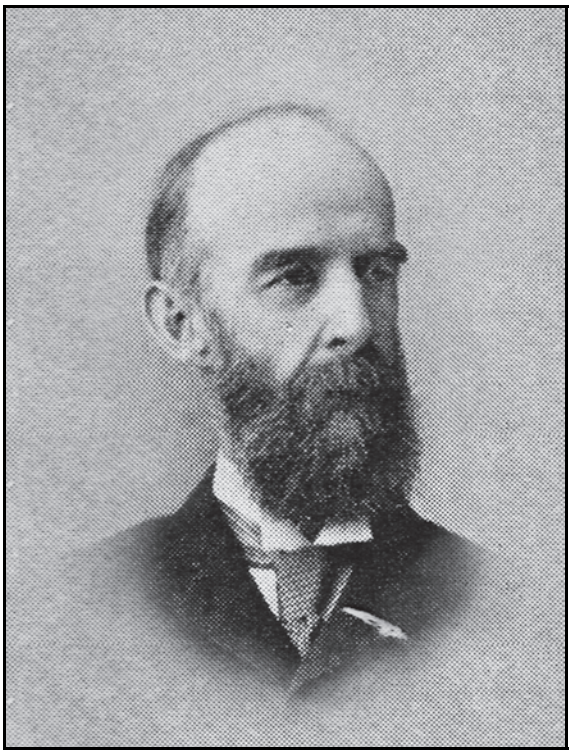
Note in the original version there were arrays up to 29 tables, but we only show up to 13 tables here.

4-5 Tables	6-7 Tables	8-9 Tables	10-11 Tables	12-13 Tables
3 to 1	4 to 1	5 to 1	6 to 1	7 to 1
4 to 2	5 to 2	6 to 2	7 to 2	8 to 2
5 to 3	6 to 3	7 to 3	8 to 3	9 to 3
1 to 4	7 to 4	8 to 4	9 to 4	10 to 4
2 to 5	1 to 5	9 to 5	10 to 5	11 to 5
	2 to 6	1 to 6	11 to 6	12 to 6
	3 to 7	2 to 7	1 to 7	13 to 7
		3 to 8	2 to 8	1 to 8
		4 to 9	3 to 9	2 to 9
			4 to 10	3 to 10
			5 to 11	4 to 11
				5 to 12
				6 to 13

3.7 The Clay universal system

I'm not familiar with the metric system.
Patton Oswalt

In Mitchell's first book it is important to note the absence of tables sharing boards and no use of bye-stand tables. It is surprising that Mitchell does not use a movement known to everyone at the time, invented by Charles M. Clay (born October 7, 1847). In fact this movement seems to have disappeared from the repertoire of all directors since that time. I like to think of it as the lost movement or the missing link. There is still a descendant of this movement widely used in bridge.



Charles M. Clay

Most modern tournament directors will be completely unaware of this movement and its simplicity. There is no board sharing and it works for both pairs and teams. The most surprising thing is it works for ***any number of tables.***

Set up the movement as you would with any Mitchell movement but with a relay table placed between each pair of tables. The number of boards in each set is halved and the number of sets doubled. For example with 4 tables it is set up as 1 R 2 R 3 R 4 R, with one board-set on each table and one on each relay. As can be readily seen this does not increase the number of boards; for example instead of 4 × 6 board-sets you would have in a Mitchell, there are 8 × 3 boards. Each E-W pair traverses the movement twice, playing against each N-S pair twice without a skip and without a special round. The movement is fairly standard with the N-S pairs stationary, the E-W pairs moving ***up*** 1 table, and the boards ***up*** one *station* (i.e. table or relay) at each change.

The only real disadvantage is the extra moves by the pairs to complete the movement. There is one issue not addressed so far: what happens when you want to play an odd number of boards against each pair? Simple, divide the sets into two lots, one lot an even number and the other an odd number. For example to play 5 board-sets in the 4 table example above, divide each set into 2 and 3 boards respectively. The first set on table one consists of boards 1 and 2, and the next relay contains boards 3, 4 and 5. Then table 2 has boards 6 and 7 and the next relay 8 to 10 and so forth. The first round is played over two boards, the second round 3 boards, the third round 2 boards and so forth.

The movement can be curtailed after any even number of rounds as all the boards will have been played the same number of times.

In the July 1897 *Whist* journal it is reported there were new methods of duplicate whist being tested. It goes on to say:

> "The other scheme used for the first time is a universal system for compass matches between pairs, for multiple teams of four, or for two teams of any number, and with any number of tables whatever, though its practical limit is probably sixteen tables, on account of the time required.
>
> "When the match is between fours, each four seat themselves together at a table, then the east-west pairs move up one table before beginning play.
>
> "After each round the east-west pairs move up one table.
>
> "When the match is between two teams of more than four on a side, the visiting team seat themselves at tables 1, 3, 5, etc., and the home team at tables 2, 4, 6, etc., then the east-west pairs move up one table before beginning play. After each round the home pairs move, the visitors sitting still throughout the play. When the match is between pairs, no preliminary arrangement is necessary.
>
> "The system of play consists in placing upon each table more than one board, and playing, each round, only one-half the boards, or, in case of five boards, say, playing only two the first round, and the remaining three the next round. This system requires that the moving players shall make the circuit of the tables twice, so that there are always twice as many rounds as tables."

The article then goes into the details on how to organize to play an odd number of boards in matches involving an even number of teams. This is a serious problem because without the proper organization of boards a different number of boards are played in some matches. See Appendix G for details (page 392).

> "After each round the players move up one table, always taking with them the deals just played, and placing them beneath the others which they find there. Of course, they are to play those which they have not played before. In matches between fours the east-west players will meet, at the end of the first circuit, with their north-south partners. They should seat themselves at the table with them and

note what boards should be played according to the schedule. These boards they may play against their north-south partners, if they wish to, but, evidently, they will not affect the result. Therefore it is better to consider these boards as played that is, to omit playing them and to pass on to the next table, playing there the next boards in order. When the second circuit is completed, it will be found that each team plays and overplays against each other team the same boards; also, that the same set-of-boards has been played by more than two teams."

In essence this movement is the same as the Mitchell modification for an even number of tables (see page 46) but the boards are moving in the same direction as the E-W pairs.

3.8 Clay modifications

"Obvious" is the most dangerous word in mathematics.
E. T. Bell

Even though the Clay system is universal we have seen there is one shortcoming, the E-W pairs repeating their journey around the section. In particular the inability to handle an odd number of boards split into two lots for teams events with an even number of tables.

Howell modification

Howell offered the simplest of solutions for this problem. Add one table to make the total number of tables odd. Of course one N-S pair and one E-W pair are sitting out on each round.

Barney modification

Walter H. Barney was the fourth president of the American Whist League and was born September 20, 1855, at Providence, Rhode Island.

Walter H. Barney

Walter Barney noticed that when playing teams events using the Clay system it was not necessary to change the order of the boards. In fact as he pointed out the board-sets are played in sequence by both the E-W and N-S pairs, N-S in descending order and E-W in ascending order. With both pairs entering their scores on a score-sheet the integrity of the sequence is maintained and checked by the players. Consider a simple example of 4 teams expanded below with no play taking place on rounds 1 and 5.

Round	Table 1	relay	Table 1	relay	Table 1	relay	Table 1	relay
1	1–1 1	2	2–2 3	4	3–3 5	6	4–4 7	8
2	1–4 8	1	2–1 2	3	3–2 4	5	4–3 6	7
3	1–3 7	8	2–4 1	2	3–1 3	4	4–2 5	6
4	1–2 6	7	2–3 8	1	3–4 2	3	4–1 4	5
5	1–1 5	6	2–2 7	8	3–3 1	2	4–4 3	4
6	1–4 4	5	2–1 6	7	3–2 8	1	4–3 2	3
7	1–3 3	4	2–4 5	6	3–1 7	8	4–2 1	2
8	1–2 2	3	2–3 4	5	3–4 6	7	4–1 8	1

Barney also came up with a similar solution for the ‘even tables’ – ‘odd board-sets’ problem. By placing the boards sets in the same way as before, 1 on table 1, 2 on relay, 3 on table 2 etc and then all the remaining sets on the last relay the event would proceed in the exact same manner but with 3 lots of boards. The only nuisance was the need for the E-W pairs to traverse the movement three times. For example with 4 teams and 12 board-sets (3 per match) the arrangement is:

Round	Table 1	relay	Table 2	relay	Table 3	relay	Table 4	relay
1	1–1 1	2	2–2 3	4	3–3 5	6	4–4 7	8 9 10 11 12
2	1–4 12	1	2–1 2	3	3–2 4	5	4–3 6	7 8 9 10 11
3	1–3 11	12	2–4 1	2	3–1 3	4	4–2 5	6 7 8 9 10
4	1–2 10	11	2–3 12	1	3–4 2	3	4–1 4	5 6 7 8 9
5	1–1 9	10	2–2 11	12	3–3 1	2	4–4 3	4 5 6 7 8
6	1–4 8	9	2–1 10	11	3–2 12	1	4–3 2	3 4 5 6 7
7	1–3 7	8	2–4 9	10	3–1 11	12	4–2 1	2 3 4 5 6
8	1–2 6	7	2–3 8	9	3–4 10	11	4–1 12	1 2 3 4 5
9	1–1 5	6	2–2 7	8	3–3 9	10	4–4 11	12 1 2 3 4
10	1–4 4	5	2–1 6	7	3–2 8	9	4–3 10	11 12 1 2 3
11	1–3 3	4	2–4 5	6	3–1 7	8	4–2 9	10 11 12 1 2
12	1–2 2	3	2–3 4	5	3–4 6	7	4–1 8	9 10 11 12 1

Of course there is no play on rounds 1, 5 and 9 but the E-W and board-set movements on those rounds are still required.

Barney also came up with another modification along the same lines. For an odd number of tables he noticed that the need for the relay tables disappeared. The E-W pairs simply move up two tables, the boards up one table and the pairs only need to traverse the section once. As he was the president of the American Whist League at the time it was easy for him to make it the standard movement for odd numbers of teams and it then became known as the *American Whist League movement.* This movement is suitable for both pairs and teams events, the same as the original Clay, and is still used extensively today.

3.9 Safford versus Howell

Before I fight, I always pray that no one gets hurt.
Sugar Ray Leonard

Clearly there were two groups of developers of individual movements in the early 1890s. There was not the same urgency for developing pairs' movements because those types of events were not that popular. Besides the most popular and simple movement to use were the Mitchell movements published in Mitchell's 1891 and 1897 books under the title of 'Progressive duplicate whist'. Even so the lines had been drawn. The Mitchell/Safford/Whitfeld/Moore alliance was proceeding along one path and the Foster/Howell along another.

It cannot be imagined that the work done by Safford and Howell were completely independent of each other apart from Foster's claim made in the *Whist* journal. There was nothing published by or about Howell before the Safford and Foster letters to *Whist*. In fact there was nothing ever published by Howell other than his table guide-cards which were first printed in 1897. The so called 'Howell's law' was not published until 1894 in *Foster's Duplicate Whist*. Every indication is that Safford came up with the first of the cyclic movements. Also there is no indication that Howell came up with the first 'Howell' movement.

Butler reported:

> "Mr. Safford's attention having been drawn to the duplicate game, he quickly saw the necessity for correct and equitable methods of arranging and moving the players, individuals as well as pairs, and teams of four. He made the subject a study during his spare moments, and the first result consisted of formulas for moving eight, twelve, and sixteen players, which were published in *Whist* January, 1892, for the benefit of all lovers of the game. For five years more he continued his labors, and in 1897 appeared his series of schedules for pairs which he named the 'comparative system.' Minute directions are given for the movements of the pairs and trays, and the final comparative scores are quickly ascertained. By the 'comparative system' the players are arranged in pairs, and moved about at the different tables in such a manner that each pair plays once with every other pair as adversaries, and each pair plays one side or the other of every deal, and is compared for results with the pair holding the same cards. To accomplish this without duplicating cards is the special feature of the system, which may be illustrated by the following diagram for the movement of eight pairs of players, four tables, kindly sent us for this purpose by Mr. Safford:"

		1				3				2				5		
First .	8	A–I	8	6		B–V	6	7		C–III	7	4		D–II	4
		1				3				2				5	
		2				4				3				6	
Second.	8	A–II	8	7		B–VI	7	1		C–IV	1	5		D–III	5
		2				4				3				6	
		3				5				4				7	
Third.	8	A–III	8	1		B–VII	1	2		C–V	2	6		D–IV	6
		3				5				4				7	
		4				6				5				1	
Fourth.	8	A–IV	8	2		B–I	2	3		C–VI	3	7		D–V	7
		4				6				5				1	
		5				7				6				2	
Fifth.	8	A–V	8	3		B–II	3	4		C–VII	4	1		D–VI	1
		5				7				6				2	
		6				1				7				3	
Sixth.	8	A–VI	8	4		B–III	4	5		C–I	5	2		D–VII	2
		6				1				7				3	
		7				2				1				4	
Seventh.	8	A–VII	8	5		B–IV	5	6		C–II	6	3		D–I	3
		7				2				1				4	

The above is a copy of the original diagram. Rounds 1 to 7 are shown as tables, with the pair 8 stationary E-W at table 'A'. The tables are A, B, C and D. The board-sets are shown as roman numerals I, II through VII. Why he chose to place the tables in that order is not clear. The order A, D, C, B does seem to be more logical (the order of the board-set numbers). Once that is done the movement of the boards from one table to the next via bye-stands is obvious but bye-stands were never mentioned at that time.

Once that change is made we get what is commonly called a Howell movement today. It can be abbreviated using the Howell rules of definition (see *Deriving Howell movements*, page 165, for more details) to be:

1–<u>8</u> 5–4 2–7 R 3–6 2R

All the essential information is depicted in this abbreviated form. The board-sets are numbered from 1 to 7 from the left. The tables are numbered from 1 to 4 from the left. The sets of boards on the relays are shown by R and 2R shows two relays. Pair 8 is stationary, shown by the underline. At each table the N-S pair is shown first and E-W pair second separated by a hyphen.

This can depicted as a square:

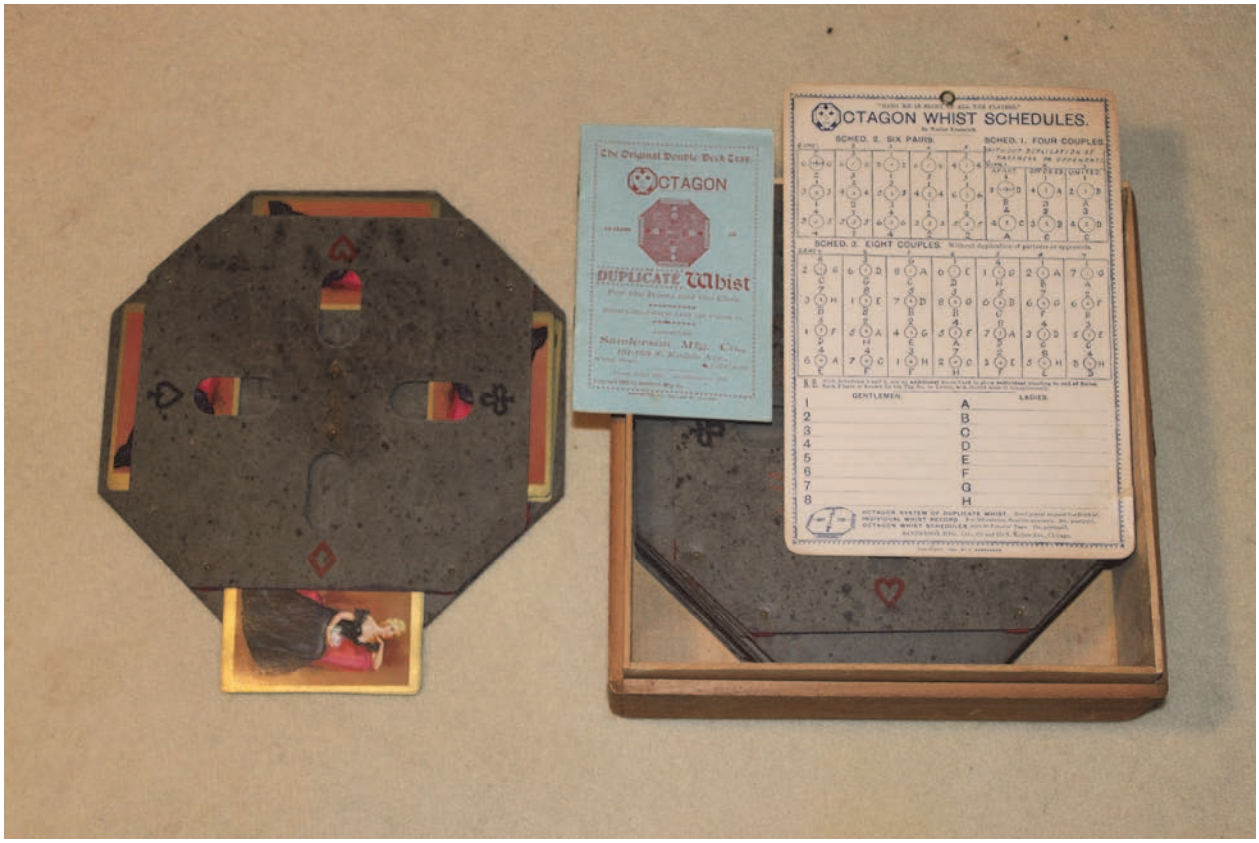
Sets: Rounds	1	2	3	4	5	6	7
1	1–8	5–4	2–7		3–6		
2		2–8	6–5	3–1		4–7	
3			3–8	7–6	4–2		5–1
4	6–2			4–8	1–7	5–3	
5		7–3			5–8	2–1	6–4
6	7–5		1–4			6–8	3–2
7	4–3	1–6		2–5			7–8

The dark line outlined area is a square, but not a Graeco-Latin square. It fails immediately because it is not made up of two Latin squares of order 7. We need a new square for this type of movement and it is supplied in the next part: *Modern squares*.

It is clear at this point that Butler is reporting this as the first example of this type of movement he has seen. There is no example of a 'Howell' movement supplied by Howell in this definitive document of its time. Butler goes on to say: "If the plan is examined, it will be found that each pair meets every other pair as adversaries, and each pair has played one side or the other of every deal; and only seven boards or trays are required. Further examination of the schedule will show that each pair plays one side or the other of every one of the seven deals, and holds the same cards with every other pair the same number of times… Scorecards are prepared… Mr. Safford was the first to apply numbers to the pairs, and to provide for the movement from formation to formation, by increasing the number of the player or pair playing at a particular position by one. These formulas were published from time to time, and Mr. Safford's method of designation is now generally employed in the making of schedules, whether individual, pairs, or teams."

There is every indication the 'Howell movement' should be called the 'Safford movement'. Howell's name clearly became associated with the Howell movement because he published the guide-cards. In addition, it is very likely Howell did independently derive most of, if not all, the movements he published.

Part 4 Modern squares



Set of 8 Octagon duplicate whist trays , circa 1896.
The arrow in the centre points to the opening leader and indicates the trump suit.
The ‘red’ play the ‘black’.

‘The original double-deck tray, Octagon system of duplicate whist’

4.1 The Room square

I've never let my school interfere with my education.
Mark Twain

All right, so every one now calls these cyclic movements 'Howells'. What about the square we use to depict them? It is a different square from those Euler defined and they are called a *Room square.*

Thomas Gerald Room, Professor of Mathematics in the University of Sydney from 1935 until his retirement in 1968, died at his home near Sydney on 2 April 1986, at the age of 83. Born and educated in England, he came to Sydney in his early thirties and made it his adopted home. He was a classical geometer with unusual gifts of intuition and combinatorial skill. He exerted enormous influence on the course of mathematics in New South Wales at both school and university levels.

He is best known for the Room square which of course is named after him. Even so he wrote only one paper (read 'one page') on the subject where he formally defined the square.

Definition: ***Room square*** is an $n \times n$ array filled with $n + 1$ different symbols in such a way that:

- each cell of the array is either empty or contains an unordered pair from the set of symbols;
- each symbol occurs exactly once in each row and column of the array;
- every unordered pair of symbols occurs in exactly one cell of the array.

An example of a Room square of order seven follows, where the set of symbols are integers from 0 to 7:

7,0			1,5		4,6	2,3
3,4	7,1			2,6		5,0
6,1	4,5	7,2			3,0	
	0,2	5,6	7,3			4,1
5,2		1,3	6,0	7,4		
	6,3		2,4	0,1	7,5	
		0,4		3,5	1,2	7,6

It is known that a Room square exists if and only if n is odd but not 3 or 5. Proof of this was produced in 1975 by Mullin and Wallis.

Even though Professor Room is given credit for this square he was not the first to use it, if he ever did. Thomas Penyngton Kirkman was born 31 March 1806 in Bolton, England. He died 4 February 1895. He was a significant mathematician of his time, publishing over 60 substantial mathematical papers. He lived at the same time as English mathematician Arthur Cayley (1821–1895), see *Eliakim Moore,* page 38, and Kirkman was recognized for his work by Cayley who had recommended he be made a fellow of the Royal Society in 1857.

Extensive as his work was he will be best remembered for his Fifteen Schoolgirls Problem. He published this question in the *Lady's and Gentleman's Diary* of 1850. *Fifteen young ladies of a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk abreast more than once.*

The solution to the fifteen schoolgirls problem was not hard for a mathematician like Kirkman, but Cayley published a solution first. Then Kirkman published his own solution, which of course he knew before asking the question. James Sylvester (1814–1897) also studied aspects of this problem and later argued with Kirkman about who had thought of it first.

Kirkman's original solution to his 'Schoolgirl Problem' used a Room square, although he didn't define those objects. See the appendix K, page 402, for the solution.

Also the first infinite classes of Room squares were constructed by Robert Richard Anstice (born 9 April 1813 and died 17 December 1853) in 1852. Anstice, however, appears to have failed to realise the importance of his own work as he ends one paper with the comment: *But too much space has been already devoted to such a trifle.*

Even though the mathematics were available to Safford and Howell at the time they were investigating movements it is very unlikely they even knew about it, let alone used it.

Towards the end of Room's career one of his fellow researchers Wal Wallis was to take Graeco-Latin squares and Room squares to an entirely new plane. Wal has been the leader in the quest for Euler's oddly-even Graeco-Latin squares throughout his career which started at the time he was with Room. Further investigation into Room squares was also carried out by Wal Wallis and R. C. Mullin when they produced *The existence of Room squares* published in 1975.

4.2 The round-robin tournament

Cricket needs brightening up a bit. My solution is to let the players drink at the beginning of the game, not after. It always works in our picnic matches.
Paul Hogan

A round-robin tournament (also called all-play-all tournament) is a type of tournament in which each contestant meets all other contestants in turn. In a single round-robin schedule, each participant plays every other participant once. If each participant plays all others twice, this is sometimes called a *double round-robin*. The term *round-robin* is derived from the French term *ruban*, meaning ribbon, tape or wire. The first to the wire is the winner.

In bridge there are two basic situations where the round-robin is used. One is for barometer pairs (the Snow method) and the other is for round-robin teams. The schedules for both these events are identical and the standard Safford cyclic method is used.

The first round is found by numbering the teams from left to right until half the number of teams and then back to the first. On subsequent rounds one must be added to each team number except the highest. This is also called the starter-adder method.

For example 8 teams:

8 v 1	2 v 7	3 v 6	4 v 5
8 v 2	3 v 1	4 v 7	5 v 6
8 v 3	4 v 2	5 v 1	6 v 7
8 v 4	5 v 3	6 v 2	7 v 1
8 v 5	6 v 4	7 v 3	1 v 2
8 v 6	7 v 5	1 v 4	2 v 3
8 v 7	1 v 6	2 v 5	3 v 4

Team 8 would be the bye round if there were only 7 teams. If the bye is not desired with an odd number of teams and a round-robin tournament is required by the organisers the following method can be adopted. It does not apply to pairs events. This is generally only applicable to bridge tournaments because other sports do not have the same comparison of results in scoring.

This was first published in Bridge Directing Complete (BDC), 1979, by this author. Since then it has become know as a triangle match. A draw is used where three teams play two half matches on each round rather than one full match which the rest of the teams are playing. After two rounds these three teams will have completed two full matches.

The three tables playing the two half matches are arranged in a row so that a simple Mitchell-like movement can be conducted. The first change is E-W move up one table and play for the first half match commences; the next move at the end of the first session is E-W up one table and the boards down one table to play the second half match.

The number of boards on each table is the same as used in one full match and the 3 way triangle matches will need three sets of boards with the same number as those used in the other matches. The example for 7 teams that follows was also first shown in BDC, with the first three named teams being the ones involved in the triangle matches.

7 teams round-robin

1 v 2 v 3	7 v 4	6 v 5
1 v 2 v 3	4 v 6	5 v 7
1 v 4 v 5	2 v 6	3 v 7
1 v 4 v 5	7 v 2	6 v 3
1 v 6 v 7	2 v 4	3 v 5
1 v 6 v 7	5 v 2	4 v 3

It can be seen that the starter-adder method does not apply to this draw, though such a draw does exist. For other such draws and further discussion on odd numbers of teams, see *Round-robin teams,* page 214.

4.3 The house square

If God existed He would have made 6 a prime number
Wal Wallis

The mathematics of odd numbers of teams round-robin draw was covered in a research paper published by D. R. Stinson and W. D. Wallis in 1984, called *An even side analogue of Room squares*. See *the Room square,* page 56, for an introduction to this subject and its underlying mathematics used in Howell movements. In Wallis's publication of 1975 he states:

Theorem 1.1
There exists a Room square of side r if and only if r is an odd positive integer other than 3 or 5.

In the research paper of 1984 we are introduced to a new concept, a Room square with a side of an even number. One of the consequences of this new requirement is the need for some numbers to appear in the square twice. The third property of the Room square 'every unordered pair of symbols occurs in exactly one cell of the array' is therefore violated and so such a square cannot be called a Room square. We are now introduced to the **house square**.

Definition: house square. Let n be a positive integer, and let S be a symbol set of size $2n$. Let F be a partition of S into unordered pairs. A house of order n is a $2n$ by $2n$ array H which satisfies the following:

1. every cell of H either is empty or contains and unordered pair of symbols S;
2. every symbol occurs in precisely one cell of each row and each column of H;
3. the pairs in F each occur in precisely two cells of H, whereas every other pair of symbols occurs in exactly one cell of H;
4. the pairs in the first and second rows of H are precisely those in F;
5. every column of H contains one pair from F.

Thus a house of order n is an approximation to a Room square of side $2n$. We will say that F is the set of repeated pairs in H.

Let us consider the consequences of this formal definition. Consider an order 3 house square.

12		34		56	
	12		34		56
35		16			24
46			15	23	
	45		26		13
	36	25		14	

The numbers 1 to 6 appear in each row only once and each column only once. The pairs (12), (34) and (56) appear twice in the array, once in row 1 and once in row 2. Using this house square we can now produce a draw for 7 teams by using the first 2 rows for the 3-way matches over two rounds and introducing team 7. The columns then become the rows or sessions while eliminating the empty cells.

Session	3 way teams	other	teams
1	7-1-2	3 5	4 6
2	7-1-2	4 5	3 6
3	7-3-4	1 6	2 5
4	7-3-4	1 5	2 6
5	7-5-6	2 3	1 4
6	7-5-6	2 4	1 3

House squares are generally only used in teams events, though they are also used in scissor schedules, see page 285.

4.4 The Howell design

Design is everything. Everything!
Paul Rand

Mathematically a *design* is defined as:

"A formal description of the constraints on the possible configurations of an experiment which is subject to given conditions. A design is sometimes called an experimental design."

Definition: *Howell design*.

Let V be a set of $2n$ symbols, then a Howell design $H(s, 2n)$ on symbol set V is an $(s \times s)$ array H such that:

1. every cell of H is either empty or contains an unordered pair of symbols from V;
2. every symbol of V occurs once in each row and column of H;
3. every unordered pair of symbols occurs in at most one cell of H.

Let us be quite clear here. The Howell design defines many of the squares we use in bridge schedules. We have seen Graeco-Latin squares and Room squares. All of these can be called a Howell design. Clearly mathematicians have honoured Howell by naming this design after him.

A Howell design $H(n, 2n)$ is a Graeco-Latin square or Mitchell movement.

A Howell design $H(2n-1, 2n)$ is a Room square or Howell movement.

A reduced Howell movement is commonly called a ¾-Howell and is a Howell design $H(r, 2n)$ where r is less than $2n - 1$ and greater than n. Consider the following ¾-Howell with 5 tables and 6 rounds, being a Howell movement reduced from the maximum possible 9 rounds.

10–1 R 9–6 8–5 4–3 7–2

This is a $H(6, 10)$:

10–1		9–6	8–5	4–3	7–2
7–3	10–2		9–1	8–6	5–4
6–5	7–4	10–3		9–2	8–7
8–1	1–6	7–5	10–4		9–3
9–4	8–2	2–1	7–6	10–5	
	9–5	8–3	3–2	7–1	10–6

The Howell design and bridge design (see next) generally only apply to pairs' schedules.

4.5 The Bridge design

I didn't want to be a slave to any passion anymore. I gave up card playing altogether, even bridge and gambling–more or less. It took me a few years to get out of it.
Omar Sharif

We have seen that Howell designs cover all the Graeco-Latin and Room squares but there are movements that do not fall under that umbrella. The first are the Share-and-relay Mitchell movements, where two tables share boards on the one round. Similarly for Howell movements where two tables share boards. The other is the curtailed Mitchell, where play stops before all the rounds have been completed.

Definition: Bridge pairs design.

Let V be a set of $2n$ symbols, then a Bridge pairs design $B(s, 2n)$ on symbol set V is an $(s \times s)$ array B such that:

1. every cell of B is contains either zero, one or two unordered pair of symbols from V;
2. every symbol of V occurs at most once in each row and column of B;
3. every unordered pair of symbols occurs in at most one cell of B.

This design does not allow for more than 2 tables sharing boards, a restriction that in practical terms is not a problem. Where all tables share boards, using the Snow method, the movements can be depicted as a Graeco-Latin square and is a $B(n, 2n)$.

A Mitchell movement with fewer rounds than tables is called a curtailed Mitchell. This is a Bridge pairs design $B(r, 2n)$ where r (the rounds) is less than n, the number of tables.

For example a 5 Table Mitchell ($n = 5$) playing only 4 rounds, reduced from 5, is $B(5, 10)$. This can be depicted as an array with the boards stationary, each column representing a table and E-W pairs numbered from 6 to 10:

1–6	2–7	3–8	4–9	5–10
2–10	3–6	4–7	5–8	1–9
3–9	4–10	5–6	1–7	2–8
4–8	5–9	1–10	2–6	3–7

It is obvious that the last round is empty. If the movement were to be completed the last row would be populated and the square would still be a *B*(5, 10) as well as *H*(5,10) and a Graeco-Latin square.

The 4 table relay Mitchell is *B*(4, 8):

1 5 4 8	2 6		3 7
3 6	1 8 4 7	2 5	
	3 5	1 7 4 6	2 8
2 7		3 4	1 6 4 5

A reduced Howell, 4 tables, 6 rounds is *B*(6, 8):

8 1		7 5 4 6	2 3		
	8 2		7 6 5 1	3 4	
		8 3		7 1 6 2	4 5
5 6			8 4		7 2 1 3
7 3 2 4	6 1			8 5	
	7 4 3 5	1 2			8 6

Part 5 Schedule categories

A set of 12 duplicate whist and (auction) bridge boards, circa 1910.

Vulnerability does not exist and the board number is on the back.
The word ‘tray’ has now disappeared.

5.1 The Safford comparative system

A person who never made a mistake never tried anything new.
Albert Einstein

Even though Safford had the movement for all pairs worked out he concentrated on his *comparative system* which is essentially a scoring method for whist. Once the whist players moved to bridge in the early 1900s clearly the scoring methods changed and all Safford's work was lost to the simplicity of the movement cards supplied by Howell. It is not even clear that Safford ever played bridge seriously.

The US Playing Card Co. by Walter H. Barney describes the Safford score-card:

> "It requires, for its successful conduct, specially prepared score-cards, which have been copyrighted, and are for sale in sets at a very reasonable price. We give below, as a sample, one of the four-table set, from which the method of using it and the necessity of the special score-cards can readily be seen:

PAIR No. 5. **8 Pairs—Duplicate Whist.**

M.....					OTHER PAIRS No.						
M.....					1	2	3	4	6	7	8
East D.	2	9	16	23							
North D.	3	10	17	24							
North B.	7	14	21	28							
North C.	6	13	20	27							
North A.	5	12	19	26							
East C	1	8	15	22							
East B.	4	11	18	25							
Add together score of the hands and extend amount into each blank space.	TOTAL, - - -										
	SCORE OF OTHERS,										
	PLUS, - - -										
	MINUS, - - -										

> "The tables are marked with letters, in alphabetical order, instead of being numbered. The words 'East D,' 'North D,' etc., indicate the positions which the pair take at each change. The numbers opposite these letters indicate the deals to be played in each position. The higher numbers can be disregarded, if it is desirable to play fourteen or twenty-one deals, instead of twenty-eight, for the match.
>
> "The scores made are recorded under the number of the deal, and the total made on the entire set of four boards (or less, if the match is curtailed, as above suggested,) is carried out and entered into each of the blank squares in the tables in the same horizontal row. These entries will come under the numbers of the pairs which play these particular deals the same way of the table as the pair whose score is recorded on this score-sheet."

The movement used in this card is the same as that shown in *Safford versus Howell* apart from table D where 5 and 4 are switched. Why he made that change is not obvious.

Sets: Rounds	1	2	3	4	5	6	7
1	1–8	4–5	2–7		3–6		
2		2–8	5–6	3–1		4–7	
3			3–8	6–7	4–2		5–1
4	6–2			4–8	7–1	5–3	
5		7–3			5–8	1–2	6–4
6	7–5		1–4			6–8	2–3
7	3–4	1–6		2–5			7–8

Consider line 1 of the score-card, sitting East-West at table D and playing board-set 2, on round 1. Pair 5 plays again N-S 4 (not shown on the card) and they play the same cards as pairs 8 (on round 2), 3 (on round 5) and 6 (on round 7). This is whist, so the score is the number of tricks they won. Pair 5 enters their score under these three pairs (the blanks) to the right on line 1.

At the end of the session all the columns for the other pairs are totaled and entered under the *Total* row. Each other pair is then consulted and their score entered in the score of others, taken from their respective pair 5 columns. Net plus or minus is calculated for each pair and a tally made to the right.

While this is not a match-point score, it is a score based on comparison with other pairs on each board. The players do the scoring and know the results within minutes of finishing play. Very neat for its time, but with the introduction of bridge and a different scoring method this method disappeared. Safford was ahead of his time.

There is no doubt that Safford came up with the cyclic rotation of players before anybody else. Also his ideas were still around well into the 1920s. A document published specifically mentioning his name was a flyer about his pivot movement.

first couple will be given tallies *a*-1, the second *b*-2, the third *c*-3, and so on, the final couple getting tallies *h*-8—*h* going to the woman and 8 to the man.

Place indicators on the four tables and have the players find their seats according to to the letter or figure on their tallies. The players are placed as follows:

6 A* [Table 1] 2 G	3 D [Table 2] 8 E
F 1* [Table 3] B 7	C 4 [Table 4] H 5

Players *a* and 1 are pivots and sit still throughout the play. After one deal round (four hands) at each table, the other players progress, the women moving to the next higher letter, and the men to the next higher figure. Player *b* goes to place marked *c*, player 2 goes to 3. Since players *a* and 1 sit still, player at place *h* moves to *b*, and the one at place 8 moves to 2. At each succeeding move, the player goes to the next higher place from the one he is then occupying.

After seven moves—28 hands—each woman will have had every man but her husband once for a partner and once for an opponent; every man will have played with every woman but his wife. Husbands and wives will never play at the same table.

For Card Parties, Gifts and Prizes

The New TWO-PACK CASES of PLAYING CARDS

Two-Pack CONGRESS Cards

Art Picture Backs, Gold Edges

Ideal for parties and all social play. Perfect for prizes and gifts. Costs no more than two separate packs. Each handsome gold-stamped telescope case holds two contrasting packs of Congress Cards, narrow (bridge) size. The cards have rich full color picture, decorative, or initial backs, with gold edges.

Two-Pack BICYCLE Cards

One red and one blue pack of the same design, narrow (bridge) size, in attractive case. Perfect finish and big indexes make Bicycles the favorites for general play.

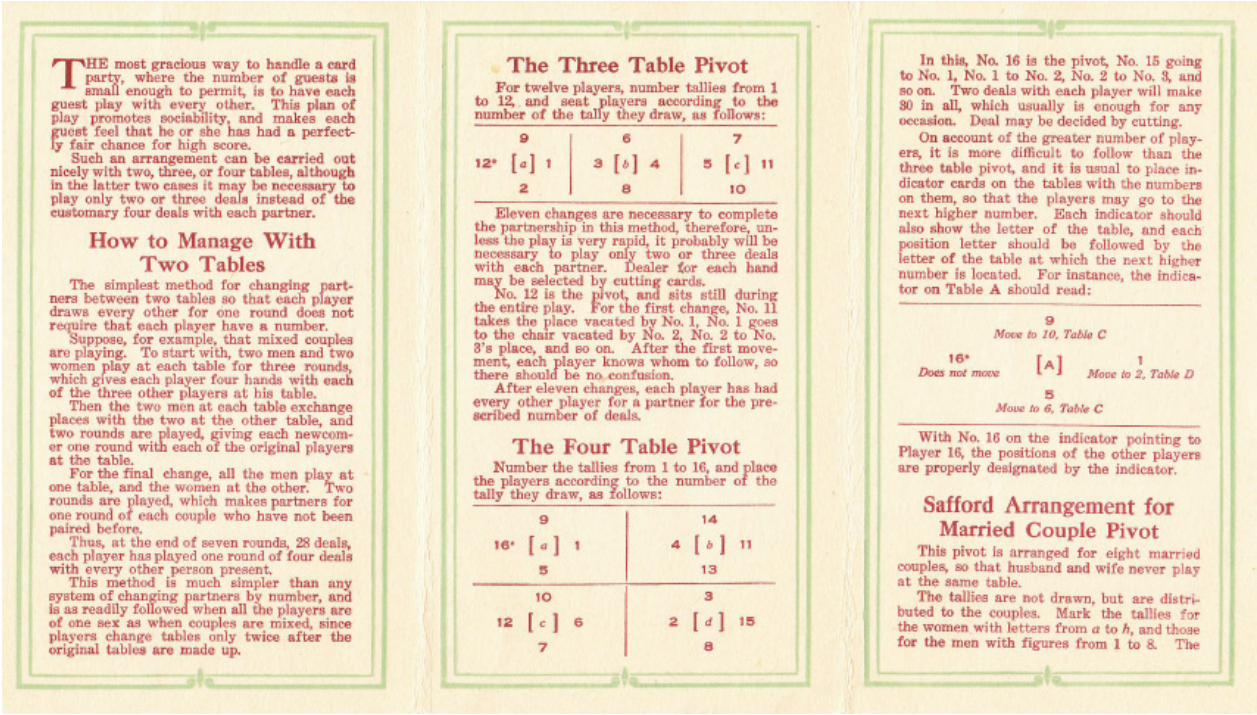
THE U. S. PLAYING CARD CO.
CINCINNATI, OHIO, U. S. A.

Operating Broadcasting Station WSAI

Regular programs are broadcast on Monday 10:00 to 12:00 P. M.; Tuesday 7:00 to 10:00 P. M.; Thursday 10:00 to 12:00 P. M.; Saturday 8:00 to 10:00 P. M. and at 12 midnight. Central Standard time. WSAI operates on a wave length of 309 meters, 970 kilocycles. Suggestions are invited. Let us know if you hear our programs

How to Pivot at Card Parties

– so that each player draws every other as a partner ♣♥♠♦



THE most gracious way to handle a card party, where the number of guests is small enough to permit, is to have each guest play with every other. This plan of play promotes sociability, and makes each guest feel that he or she has had a perfectly fair chance for high score.

Such an arrangement can be carried out nicely with two, three, or four tables, although in the latter two cases it may be necessary to play only two or three deals instead of the customary four deals with each partner.

How to Manage With Two Tables

The simplest method for changing partners between two tables so that each player draws every other for one round does not require that each player have a number.

Suppose, for example, that mixed couples are playing. To start with, two men and two women play at each table for three rounds, which gives each player four hands with each of the three other players at his table.

Then the two men at each table exchange places with the two at the other table, and two rounds are played, giving each newcomer one round with each of the original players at the table.

For the final change, all the men play at one table, and the women at the other. Two rounds are played, which makes partners for one round of each couple who have not been paired before.

Thus, at the end of seven rounds, 28 deals, each player has played one round of four deals with every other person present.

This method is much simpler than any system of changing partners by number, and is as readily followed when all the players are of one sex as when couples are mixed, since players change tables only twice after the original tables are made up.

The Three Table Pivot

For twelve players, number tallies from 1 to 12, and seat players according to the number of the tally they draw, as follows:

9 12* [a] 1 2	6 3 [b] 4 8	7 5 [c] 11 10

Eleven changes are necessary to complete the partnership in this method, therefore, unless the play is very rapid, it probably will be necessary to play only two or three deals with each partner. Dealer for each hand may be selected by cutting cards.

No. 12 is the pivot, and sits still during the entire play. For the first change, No. 11 takes the place vacated by No. 1, No. 1 goes to the chair vacated by No. 2, No. 2 to No. 3's place, and so on. After the first movement, each player knows whom to follow, so there should be no confusion.

After eleven changes, each player has had every other player for a partner for the prescribed number of deals.

The Four Table Pivot

Number the tallies from 1 to 16, and place the players according to the number of the tally they draw, as follows:

9 16* [a] 1 5	14 4 [b] 11 13
10 12 [c] 6 7	3 2 [d] 15 8

In this, No. 16 is the pivot, No. 15 going to No. 1, No. 1 to No. 2, No. 2 to No. 3, and so on. Two deals with each player will make 30 in all, which usually is enough for any occasion. Deal may be decided by cutting.

On account of the greater number of players, it is more difficult to follow than the three table pivot, and it is usual to place indicator cards on the tables with the numbers on them, so that the players may go to the next higher number. Each indicator should also show the letter of the table, and each position letter should be followed by the letter of the table at which the next higher number is located. For instance, the indicator on Table A should read:

	9 *Move to 10, Table C*	
16* *Does not move*	[A]	1 *Move to 2, Table D*
	5 *Move to 6, Table C*	

With No. 16 on the indicator pointing to Player 16, the positions of the other players are properly designated by the indicator.

Safford Arrangement for Married Couple Pivot

This pivot is arranged for eight married couples, so that husband and wife never play at the same table.

The tallies are not drawn, but are distributed to the couples. Mark the tallies for the women with letters from *a* to *h*, and those for the men with figures from 1 to 8. The

This was probably from the 1920's and was published by The U.S. Playing Card Co., Cincinnati, Ohio, U.S.A. The text in its original and unedited form reads:

"THE most gracious way to handle a card party, where the number of guests is small enough to permit, is to have each guest play with every other. This plan of play promotes sociability, and makes each guest feel that he or she has had a perfectly fair chance for high score.

"Such an arrangement can be carried out nicely with two, three, or four tables, although in the latter two cases it may be necessary to play only two or three deals instead of the customary four deals with each partner.

"How to Manage With Two Tables

"The simplest method for changing partners between two tables so that each player draws every other for one round does not require that each player have a number.

"Suppose, for example, that mixed couples are playing. To start with, two men and two women play at each table for three rounds, which gives each player four hands with each of the three other players at his table.

"Then the two men at each table exchange places with the two at the other table, and two rounds are played, giving each newcomer one round with each of the original players at the table.

"For the final change, all the men play at one table, and the women at the other. Two rounds are played, which makes partners for one round of each couple who have not been paired before.

"Thus, at the end of seven rounds, 28 deals, each player has played one round of four deals with every other person present.

"This method is much simpler than any system of changing partners by number, and is as readily followed when all the players are of one sex as when couples are mixed, since players change tables only twice after the original tables are made up.

"The Three Table Pivot

"For twelve players, number tallies from 1 to 12, and seat players according to the number of the tally they draw, as follows:

	9			6			7	
12*	[*a*]	1	3	[*b*]	4	5	[*c*]	11
	2			8			10	

"Eleven changes are necessary to complete the partnership in this method, therefore, unless the play is very rapid, it probably will be necessary to play only two or three deals with each partner. Dealer for each hand may be selected by cutting cards.

"No. 12 is the pivot, and sits still during the entire play. For the first change, No. 11 takes the place vacated by No. 1, No. 1 goes to the chair vacated by No. 2, No. 2 to No. 3's place, and so on. After the first movement, each player knows whom to follow, so there should be no confusion.

"After eleven changes, each player has had every other player for a partner for the prescribed number of deals.

"The Four Table Pivot

"Number the tallies from 1 to 16, and place the players according to the number of the tally they draw. as follows:

	9			14	
16*	[*a*]	1	4	[*b*]	11
	5			13	
	10			3	
12	[*c*]	6	2	[*d*]	15
	7			8	

"In this, No. 16 is the pivot, No. 15 going to No. 1, No. 1 to No. 2, No. 2 to No. 3, and so on. Two deals with each player will make 30 in all, which usually is enough for any occasion. Deal may be decided by cutting.

"On account of the greater number of players, it is more difficult to follow than the three table pivot, and it is usual to place indicator cards on the tables with the numbers on them, so that the players may go to the next higher number. Each indicator should also show the letter of the table, and each position letter should be followed by the letter of the table at which the next higher number is located. For instance, the indicator on Table A should read:

	9 *Move to 10, Table C*	
16* *Does not move*	[A]	1 *Move to 2, Table D*
	5 *Move to 6, Table C*	

"With No. 16 on the indicator pointing to Player 16, the positions of the other players are properly designated by the indicator.

"Safford Arrangement for Married Couple Pivot

"This pivot is arranged for eight married couples, so that husband and wife never play at the same table.

"The tallies are not drawn, but are distributed to the couples. Mark the tallies for the women with

letters from *a* to *h,* and those for the men with figures from 1 to 8. The first couple will be given tallies *a*-1, the second *b*-2*,* the third *c*-3*,* and so on, the final couple getting tallies *h-8 – h* going to the woman and 8 to the man.

"Place indicators on the four tables and have the players find their seats according to the letter or figure on their tallies. The players are placed as follows:

	6			3	
A*	[*Table 1*]	2	D	[*Table 2*]	8
	G			E	
	F			C	
1*	[*Table 3*]	B	4	[*Table 4*]	H
	7			5	

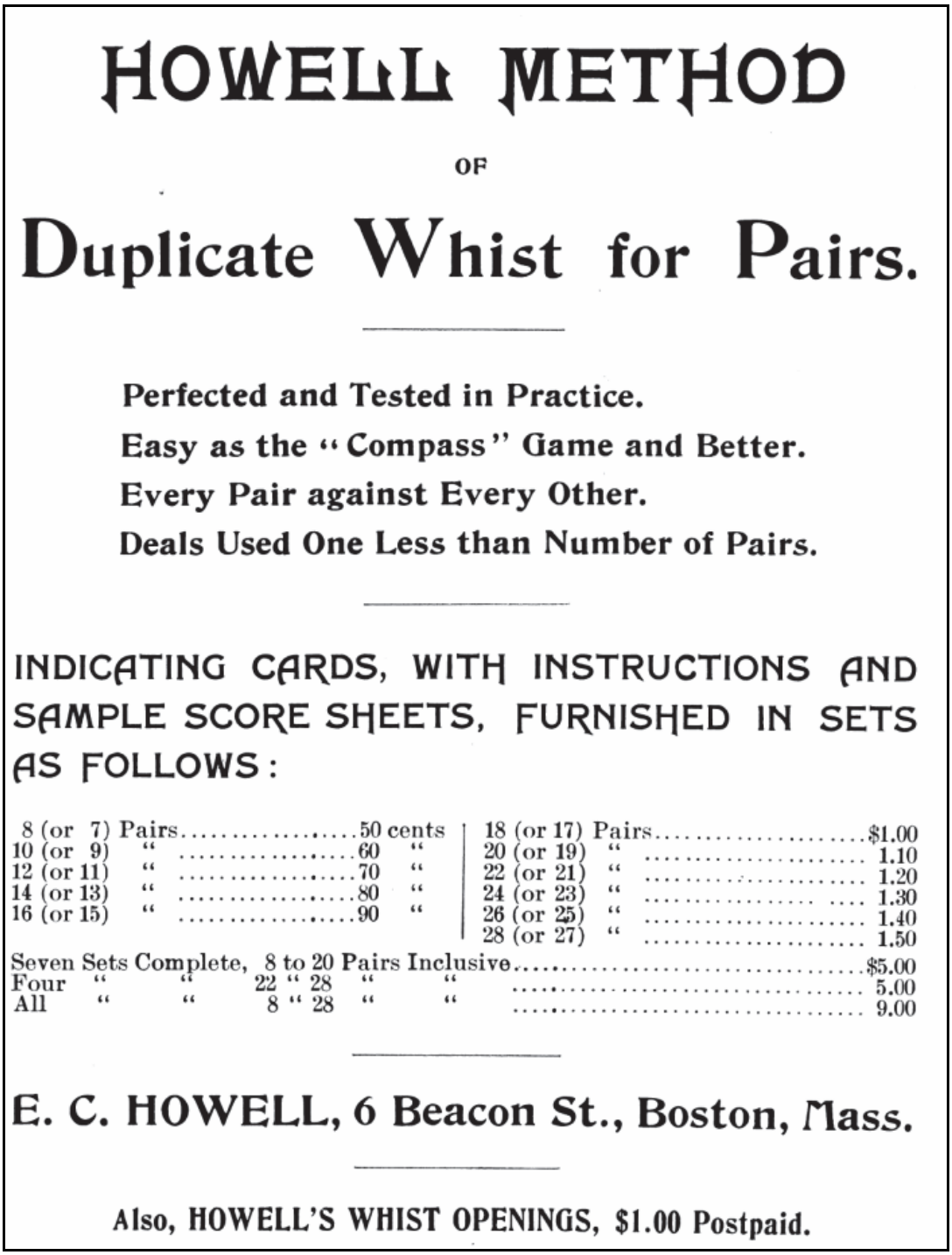
"Players *a* and 1 are pivots and sit still throughout the play. After one deal round (four hands) at each table, the other players progress, the women moving to the next higher letter, and the men to the next higher figure. Player *b* goes to place marked c, player 2 goes to 3. Since players *a* and 1 sit still, player at place *h* moves to *b,* and the one at place 8 moves to 2. At each succeeding move, the player goes to the next higher place from the one he is then occupying.

"After seven moves-28 hands-each woman will have had every man but her husband once for a partner and once for an opponent; every man will have played with every woman but his wife. Husbands and wives will never play at the same table."

5.2 The Howell system

One of the penalties for refusing to participate in politics
is that you end up being governed by your inferiors.
Plato

Just as Safford started publishing his method in1897, Howell also published his table guide-cards during that year. An advertisement for these was first shown in August 1897 edition of *Whist*:



HOWELL METHOD

OF

Duplicate Whist for Pairs.

Perfected and Tested in Practice.
Easy as the "Compass" Game and Better.
Every Pair against Every Other.
Deals Used One Less than Number of Pairs.

INDICATING CARDS, WITH INSTRUCTIONS AND SAMPLE SCORE SHEETS, FURNISHED IN SETS AS FOLLOWS:

8 (or 7) Pairs	50 cents	18 (or 17) Pairs	$1.00
10 (or 9) "	60 "	20 (or 19) "	1.10
12 (or 11) "	70 "	22 (or 21) "	1.20
14 (or 13) "	80 "	24 (or 23) "	1.30
16 (or 15) "	90 "	26 (or 25) "	1.40
		28 (or 27) "	1.50

Seven Sets Complete, 8 to 20 Pairs Inclusive	$5.00
Four " " 22 " 28 " "	5.00
All " " 8 " 28 " "	9.00

E. C. HOWELL, 6 Beacon St., Boston, Mass.

Also, HOWELL'S WHIST OPENINGS, $1.00 Postpaid.

These table guide-cards were the same size as the Paine boards which were quite large. An example of the Howell guide card, reduced, is shown below.

SOUTH—Go next to Table 4, West.

EIGHT PAIRS.

TABLE 2.

Original North and South Pair, ... No. 6.

Original East and West Pair, ... No. 3.

Original Deals, ... Nos. 4, 5.

SCHEDULE.

N. & S. E. & W.	N. & S. E. & W.
1st Round, Pair 6 *vs.* Pair 3, Deal 4.	5th Round, Pair 3 *vs.* Pair 7, Deal 1.
2d Round, Pair 7 *vs.* Pair 4, Deal 5.	6th Round, Pair 4 *vs.* Pair 1, Deal 2.
3d Round, Pair 1 *vs.* Pair 5, Deal 6.	7th Round, Pair 5 *vs.* Pair 2, Deal 3.
4th Round, Pair 2 *vs.* Pair 6, Deal 7.	

EAST—Go next to Table 3, East.

WEST—Go next to Table 3, West.

NORTH—Go next to Table 4, East.
(Take deal just played to Table 1.)

The complete schedule of the 4-table Howell was shown as:

ORIGINAL ARRANGEMENT FOR EIGHT PAIRS.

TABLE.	N. & S. PAIR.	E. & W. PAIR.	DEAL IN PLAY.	DEALS, NOT IN PLAY.
1	8	1	1	2, 3
2	6	3	4	5
3	7	2	6	....
4	4	5	7	....

This is not the same as Safford's 4-table movement, but very close in that the pairings for the first round are the same, but some are switched. The starting boards are different. See *producing Howell movements,* page

166, for all the possible movements for 4 tables. It can be seen these two designers could easily have come up with completely different answers. One should doubt the independence of their efforts in the early days, but it is very likely Howell extended his efforts a lot further than Safford. The Howell method was also applicable to bridge, which would have stamped his name on the Howell movement forever more.

This 8-pair schedule can be summarized as a Howell for 4 tables:

8–1 2R 6–3 R 7–2 4–5 (100)

The resultant Room square is:

Sets: Rounds	1	2	3	4	5	6	7
1	8–1			6–3		7–2	4–5
2	5–6	8–2			7–4		1–3
3	2–4	6–7	8–3			1–5	
4		3–5	7–1	8–4			2–6
5	3–7		4–6	1–2	8–5		
6		4–1		5–7	2–3	8–6	
7			5–2		6–1	3–4	8–7

Edwin C. Howell was born April 21, 1860, at Nantucket, Massachusetts, USA.

Edwin C. Howell

Starting with 10 pairs and all the way up to 36, the original Howell movements for 5 to 18 tables were:

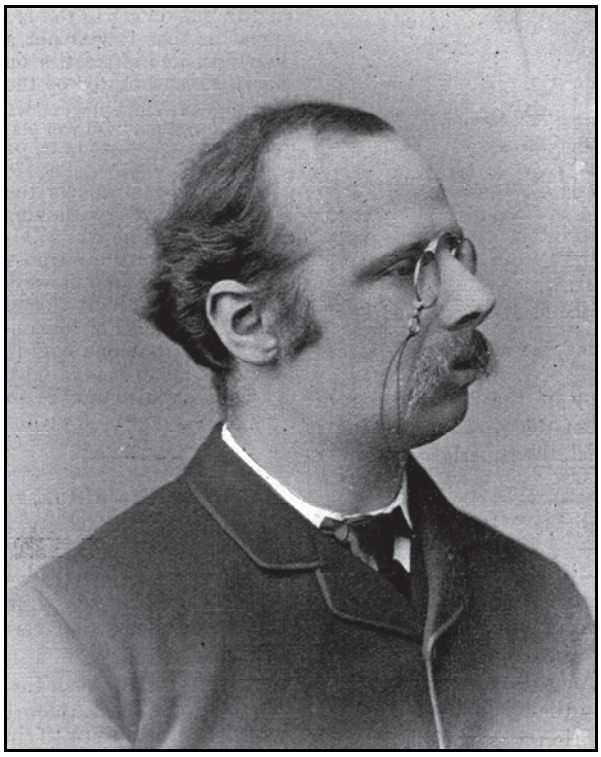
5 Tables
10–1 R 5–2 R 9–8 7–3 R 4–6 R (83)
6 Tables
12–1 2R 6–7 R 11–4 R 3–9 2–5 R 10–8 (100)
7 Tables
14–1 3R 11–4 2R 6–9 3–12 R 13–2 7–8 5–10 (91)
8 Tables
16–1 R 5–11 2R 13–15 2R 8–12 7–6 R 10–3 14–2 4–9 R (100)
9 Tables
18–1 3R 15–4 R 10–9 R 16–3 R 7–12 R 2–17 R 13–6 11–8 14–5 (95)
10 Tables
20–1 4R 9–12 17–4 19–2 10–11 R 15–6 R 18–3 R 8–13 R 5–16 14–7 R (96)
11 Tables
22–1 5R 15–8 18–5 2R 17–6 4–19 16–7 13–10 3–20 14–9 2R 2–21 R 12–11 (94)
12 Tables
24–1 20–16 13–15 R 10–7 22–17 2R 12–3 23–11 2R 9–19 2R 14–8 2R 5–4 R 18–2 21–6 R (96)
13 Tables
26–1 16–11 19–8 15–12 R 18–9 2–25 R 24–3 R 17–10 2R 21–6 2R 14–13 R 20–7 22–5 4R 4–23 (93)
14 Tables
28–1 23–6 R 22–7 17–12 26–3 4R 20–9 18–11 R 19–10 R 27–2 25–4 2R 8–21 16–13 24–5 2R 14–15 2R (86)
15 Tables
30–1 19–12 29–2 24–7 R 11–20 2R 18–13 4R 27–4 3R 26–5 17–14 22–9 28–3 23–8 16–15 3R 21–10 R 25–6 (96)
16 Tables
32–1 2R 8–25 R 12–21 18–15 5–28 R 6–27 10–23 R 4–29 R 24–9 2R 20–13 11–22 30–3 19–14 3R 7–26 2–31 2R 17–16 2R (100)
17 Tables
34–1 17–18 R 27–8 32–3 R 26–9 R 21–14 2R 22–13 2R 33–2 R 12–23 3R 30–5 25–10 31–4 20–15 2R 19–16 R 28–7 R 24–11 29–6 R (92)
18 Tables
36–1 31–6 22–15 R 23–14 34–3 4R 25–12 R 26–11 3R 32–5 3R 17–20 28–9 2R 27–10 29–8 35–2 4–33 19–18 16–21 R 24–13 2R 30–7 (93)

The calibre (see movement calibre, page 89) of all these movements is excellent, with the exception of the 14 table example, which is still very good. The calibre is shown in brackets after each movement. This effort by Howell should not be underestimated. It was before 1900 and he had no computers or other tools to help him. Today we would choose Howells where all the relays are on one table (if possible), but it was not until 1974 that such movements (up to 40 pairs) were found by the author using a computer.

5.3 Whitfeld schedules

The second player, holding two cards (not the ace) only of the suit led, should, if possible, cover the nine, or any higher card led
William H. Whitfeld

William H. Whitfeld was a foremost inventor of double-dummy problems, and a whist mathematician and analyst of great ability. Mr. Whitfeld was born at Whist Villa, Ashford, Kent, England, October 15, 1856. He did point out the name of the house had reference to its retired character, and not to the game.



William H. Whitfeld

We know that Safford developed the first individual schedules, but for numbers greater than 32 players Mr. Whitfeld was responsible. Butler reports: "Mr. Whitfeld, although without practical experience in duplicate whist, took the matter of arranging progressive duplicate schedules up as an interesting mathematical puzzle. For individuals, he produced schedules for twelve, twenty, twenty-four, twenty-eight, thirty-six, forty, forty-eight, fifty-two, fifty-six, and sixty-four players. For teams, he produced schedules for eight, twelve, fourteen, sixteen, eighteen, and twenty, and the same are published in Mitchell's 'Duplicate Whist.' Other team schedules for four, six, and ten will be found therein, as well as additional schedules covering the same ground as those of Mr. Whitfeld.

> "Again, using Mr. Whitfeld's ordinary arrangement for one hundred players, and my triple arrangement for sixteen and one hundred and sixty players, we can construct two ordinary arrangements for sixteen by one hundred, and one hundred and sixty by one hundred players, and one triple arrangement for sixteen by one hundred and sixty players."

Mitchell's book of 1897 published schedules as devised by Mr. W.H. Whitfeld. They were on one page (67) titled *PROGRESSIVE DUPLICATE SCHEDULES*: *each pair against every other pair*. Only schedules for 4, 6 and 7 tables are given. The 4-tables schedule is shown as:

No of tables	N. & S. pair at start	E. & W. pair atstart	No. of 1st deal at table	Nos. of trays placed at table	No. of table tray passed to.	No. of table tray received from
1	x	1	1	1-2-3	4	3
2	7	2	6	6	3	4
3	6	3	4	4-5	1	2
4	5	4	7	7	2	1

On closer inspection of this clumsy description it is clear this is a 'Howell' movement, as it is known today. It can be abbreviated to:

8–1 2R 6–3 R 7–2 5–4 (53)

This is the exact same movement used and published by Howell the same year, but with the pairs at table 4 switched. The other two Whitfield cyclic movements are not the same as those used by Howell:

The 6 table Whitfeld schedule is: 12–1 R 9–4 3R 10–3 7–6 11–2 R 8–5 (82).

The 7 table Whitfeld schedule is: 14–1 R 9–6 R 13–2 8–7 12–3 3R 10–5 R 11–4 (62).

These three schedules are the first Howell movements published in a duplicate bridge book and ironically it is Mitchell's. This same book publishes individuals' schedules designed by Safford and Whitfeld. The teams' round-robin schedules were specified by Safford and are based on what is now known as the endless Howell system. See the *Whitfeld individual schedules*, page 279, for his schedules using the Snow method.

At about the same time Butler reports:

> "During the summer of 1897, Mr. Howell published the *Howell Method of Duplicate Whist for Pairs*, consisting of indicating cards, with instructions and sample score-sheets, which adapt to general use the system of playing every pair against every other. The schedules on which the method is based are essentially the same as Safford's, but were discovered quite independently."

This sounds like a very diplomatic and politically-correct statement but it must be taken on face value. All the key players involved with this subject were alive and well at this time, members of the same club and promoting the game of duplicate whist. There are no movements prior to this time that have been published in the name of Howell. There is no copyright on *movements* of any type. Therefore, as publisher of table guide-cards, Howell was making available to the bridge-playing community the facility to organise their events quickly and easily.

5.4 The Cook plan

An idea isn't responsible for the people who believe it.
Don Marquis

In 1928 Wilburn C. Whitehead and Frank A. Cook published *Whitehead's duplicate auction bridge*. Using Howell's law Cook devised his own set of Howell movements. As these were the first set of Howell movements devised after Howell and Safford, they are included here.

Originally they were defined with the stationary pair as 1, but they are reproduced here with the highest pair stationary. At the time it was fashionable to have pair 1 stationary. Just like the original Howell movements the number of relay tables is unnecessarily high but the calibre is generally excellent.

4 Tables
8–1 2R 6–3 R 7–2 4–5 (100)
5 Tables
10–1 R 5–2 R 9–8 7–3 R 4–6 R (83)
6 Tables
12–1 2R 6–7 R 11–4 R 3–9 2–5 R 10–8 (100)
7 Tables
14–1 3R 11–4 2R 6–9 3–12 R 13–2 7–8 5–10 (91)
8 Tables
16–1 R 5–11 2R 13–15 2R 8–12 7–6 R 10–3 14–2 4–9 R (100)
9 Tables
18–1 3R 15–4 R 10–9 R 16–3 R 7–12 R 2–17 R 13–6 11–8 14–5 (95)
10 Tables
20–1 R 8–13 2R 3–18 2R 11–10 4–17 2R 5–16 12–9 2–19 15–6 R 14–7 R (100)
11 Tables
22–1 5R 15–8 18–5 2R 17–6 4–19 16–7 10–13 3–20 14–9 2R 2–21 R 12–11 (96)
12 Tables
24–1 20–16 13–15 R 10–7 22–17 2R 12–3 23–11 2R 9–19 2R 14–8 2R 5–4 R 18–2 21–6 R (96)
13 Tables
26–1 4R 25–2 R 14–13 2R 8–19 24–3 15–12 R 5–22 R 21–6 16–11 7–20 23–4 2R 18–9 R 17–10 (94)
14 Tables
28–1 3R 4–7 2R 24–19 2R 17–16 3R 5–11 23–8 25–12 3–21 2–10 R 14–18 9–26 R 22–6 R 27–20 13–15 (96)
15 Tables
30–1 20–13 4–2 27–10 R 16–25 2R 21–26 4R 11–17 3R 14–22 6–3 28–12 19–23 29–15 9–8 3R 18–7 R 24–5 (98)

5.5 The Scheveningen system

If at first, the idea is not absurd, then there is no hope for it.
Albert Einstein

The Scheveningen system is a method of organizing a chess match between two teams. Each player on one team plays each player on the other team. The team with the highest number of games won is the winner.

The system was first used in a chess tournament in Scheveningen in 1923. The event was organized to celebrate the 50th anniversary of the Royal Chess Federation. Ten foreign masters were to play against 10 players from The Netherlands.

A brand new system invented by Dr. A. Rueb, then chairman of Dutch Chess Federation, was introduced in this tournament, which since then is known as the *Scheveningen system*, named after the district and beachside resort of The Hague where the tournament took place. Every player from the first team played against a player of the second team in a ten round all from one team play all from another event.

As for using this name in bridge the only definite link we have is the first European Team Championships held in Scheveningen in 1932. The unique aspect of this type of event is the boards remain stationary, as of course they also do in chess. Just the players move about the room after each round and meet new opponents on each round. In 1932 money was very short and rumour has it the organizers chose to use this system to save on the cost of board caddies. By the 1950s the name Scheveningen was almost synonymous with 'duplicate bridge' in Holland.

Essentially the movement when used in pairs' events is a stationary-board Mitchell, because the Scheveningen system has two groups meeting each other but not playing within the group. In teams it could be considered the Gruenther relay movement, devised in 1930 by Major General A. M. Gruenther with the help of Major L. R. Gracy of the U.S.A. Army. It is hard to imagine this did not have any influence on the first European Teams' Championships held in Scheveningen

More recently Olof Hanner, born 1922 in Stockholm and is currently living in Gothenburg, has called these types of movement ring-dance Mitchell because of the way the two groups N-S and E-W move around the room in opposite directions like two rings.

It is unlikely the name Scheveningen will ever become popular outside the Dutch-speaking communities because of the pronunciation difficulty. The name is still commonly used in the Netherlands.

5.6 The Snow system

The way to get good ideas is to get lots of ideas, and throw the bad ones away.
Linus Pauling

Charles F. Snow was a player and administrator of whist but was not remembered for his system for duplicate bridge. His system was extremely simply and can be applied as an alternate method to most other systems available. Today we call such movements 'barometers' because of the scoring method: posting the scores after each round of play. The US Playing Card Co. in 1900 describes the Snow system:

> "Play one set of deals without moving the players; then move the East and West players one table and play another set of deals, and so on until the East and West players have been opposed to all of the North and South. If desired, more deals can be played in each round than there are tables. This system is not applicable except with a small number of tables or a large number of deals. It has an advantage in that it can be played in several sessions, as an intermission can be taken at the end of the play of any set of deals. When applicable, it is preferable to any other system.

With 2 tables, it is possible to play 4, 6, 8, 10, 12, 16, etc., deals.

3	tables	9, 12, 15, 18, 21, 24	deals
4	tables	16, 20, 24, 28, 32, 36	deals
5	tables	25, 30, 35, 40	deals
6	tables	36, 42, 48	deals
7	tables	49, 56, 63	deals".

From now on all the systems have the Snow method applied as an alternative and the subject is usually treated separately when there are any related issues.

5.7 Movement classes

Language is the means of getting an idea from my brain into yours without surgery.
Mark Amidon

So far we have seen a variety of schedules that have developed out of need. Within these schedules I have shown many types of movements that are generally classified by the 'system' developed by different people over time. The term 'system' was first chosen by tournament organizers in the Whist days (when bidding systems had not been thought of) and so we have the following list of systems:

- Safford
- Mitchell
- Howell
- Clay
- Scheveningen
- Snow

These systems have given us a variety of techniques to construct schedules leading to many different types of movements that tournament directors use in their day-to-day work. Note I do not use the terms 'schedule' and 'movement' interchangeably, even though they may be the same in many cases. A movement may be constructed from one, two or even more schedules. A simple example is the double-Howell movement made up from two Howell schedules.

Now I want to introduce the term 'class'. This is a term used to define a construct that is used as a blueprint or template of an object. In our context we start with the parent class *BridgeMovement*. The name is chosen and shown in this way so there is no confusion as to what is being referred to when we use the term. The class *BridgeMovement* has a number of properties with particular attributes that will have different values depending on the **instance** of the bridge movement that has been **derived** from the class *BridgeMovement*. Examples of the properties follow, and are shown as, **Number_Of_Players**, **Number_Of_Tables**, **Number_Of_Board_Sets**, **Number_Of_Boards_Per_Set** and so forth. Note the use of the '_' in these properties is used to distinguish the names from other meanings.

Now I want to define the next level of *classes* that have been derived from the *BridgeMovement* class. These classes are:

- *Safford*
- *Mitchell*
- *Howell*
- *Barney*
- *McKinnon*

Safford class

The *Safford* class is an individual **Movement_Type**. Within this class many instances of movements have been developed, some cyclic and others not. Depending on how the field of players is divided there may be one, two, three or four **Number_Of_Winners**. The **Scoring_Unit** is the individual players. The **Scoring_Method** varies depending on the needs of the organizers but typically it is match pointed.

***Safford* class**

Property	value
Movement_type	individuals
Scoring_unit	player
Scoring_method	match_point +
Number_of_winners	1 to 4
Fields	1 to 4
Schedule	cyclic or non-cyclic*

Mitchell class

The *Mitchell* class is a pairs **Movement_Type**. Within this class many instances of movements have been developed and have been covered above with the many varieties of modifications mentioned earlier. The most important property is the **Number_Of_Winners** which is always two. The **Number_Of_Fields** is always two. The **Scoring_Unit** is the pairs of players. The **Scoring_Method** varies depending on the needs of the organizers but typically it is match pointed.

***Mitchell* class**

Property	value
Movement_type	pairs
Scoring_unit	pair
Scoring_method	match_point +
Number_of_winners	2
Fields	2
Schedule	cyclic *

Howell class

The *Howell* class is a pairs **Movement_Type**. Within this class many instances of movements have been developed and have been covered above with the many varieties of modifications mentioned earlier. The most important property is the **Number_Of_winners**, which is always one. The **Number_Of_Fields** is always one. The **Scoring_Unit** is the pairs of players. The **Scoring_Method** varies depending on the needs of the organizers but typically it is match pointed.

***Howell* class**

Property	value
Movement_type	pairs
Scoring_unit	pair
Scoring_method	match_point +
Number_of_winners	1
Fields	1
Schedule	cyclic *

Barney class

The *Barney* class is a teams' **Movement_Type**. Within this class many instances of movements have been developed and have been covered above with the many varieties of modifications mentioned earlier. The most important property is the **Number_Of_Winners** which is always one. The **Number_Of_Fields** is always one. The **Scoring_Unit** is the teams of players. The **Scoring_Method** varies depending on the needs of the organizers but typically it is IMP. The usual movements are American Whist League but others are used such as twin Mitchell.

***Barney* class**

Property	value
Movement_type	teams
Scoring_unit	team
Scoring_method	IMP +
Number_of_winners	1
Fields	1
Schedule	cyclic *

McKinnon class

The *McKinnon* class is new to bridge. It is a descendent of the *Howell* and *Barney* classes. Movements that satisfy this class are covered later (see Scissor schedules, page 285).

***McKinnon* class**

Property	value
Movement_type	teams and pairs
Scoring_unit	teams and pairs
Scoring_method	IMP and match_point
Number_of_winners	1 (twice)
Fields	1 (twice)
Schedule	bicyclic *

+ {match point, IMP, total point, VP}
* {cyclic, non-cyclic, bicyclic}

Part 6 Balance debate

Set of 16 Maridey duplicate (auction) bridge boards, circa 1920.
The metal clips are difficult to use but they are very strong spring steel.

6.1 Comparison of results

There's always merit to having a debate.
Ward Churchill

A major consideration of a tournament must be the comparison of the players with each other. Obviously the comparison of two teams playing against each other is perfect with each board being played in the N-S and E-W direction by each team. In an event where there is more than one team the comparison of teams will remain equal so long as the same number of boards is played in each match.

In an individual movement the comparison of players is rarely equal because of other more important considerations such as playing with all other players. In fact, because of the nature of the event, an individual result is affected by many factors other than the movement but it is still the responsibility of the director to supply a movement that is as fair as possible.

In a pairs event the comparison of players is a very important consideration of the director. In the Mitchell event the comparison of pairs is perfect within each group (N-S and E-W fields being treated as separate groups) as long as all the N-S pairs play all the E-W pairs over an equal number of boards and play all the boards in the event. The consideration of comparison of pairs applies only to the pairs with which a pair is being compared (i.e., the pairs sitting in the same direction as the pair being considered in a Mitchell movement) and does not apply to the opponents. For all things to be equal and fair each pair should play an equal number of boards against the same set of opponents. A movement that has fair comparisons is said to be *balanced*.

In the Howell-type tournament the comparison of pairs becomes a lot more complicated and is dealt with thoroughly later. In particular movements that are incomplete (e.g. switched Mitchell or ¾-Howell) the balance of the movement is a subject on its own.

Since the publication of my first book, the debate on balanced movements has taken on a new meaning. For this reason the next few chapters attempt to put this matter in perspective and hopefully can put the matter to rest.

6.2 The first proposition

Before I refuse to take your questions, I have an opening statement.
Ronald Reagan

In the early 1930s duplicate contract bridge was a very new game. At this time the Mitchell movement was the most popular and generally used in most large events in the USA. For finals of most events the Howell movement was used to give fairer outcomes, where all pairs met all other pairs, and played all boards. Even so, with this in place, there was much rumblings amongst the better players, claiming the movements were unfair, and their failure to win was due to unfair movements.

The investigations into fair movements started at this time with Charles T. Kennedy and Jacques L. Ach. It was in response to many players in America asking questions about movements in relation to their performance when Howell movements were used. See '*Why the best man loses*' in Appendix M, page 412, for the first published (*The Bridge World*, December 1934) account of the issue. Reading between the lines, and considering the Howell movements that were published around that time, it is clear there was much debate and infighting to claim the title to the best movements.

One of the ironies of that article was the so-called Ach-Kennedy Howell movement for 12 pairs, used in their examples, is the same as the original Howell movement (see page 69), published before 1900. That movement is:

12–1 2R 6–7 R 11–4 R 3–9 2–5 R 10–8 (100)

A month later, in January 1935, *The BridgeWorld* published an article, '*Testing Pair Schedules*', by Jacques Ach, that explained how to determine the quality of a movement. This explanation was quite good, but only to a limited extent.

The McKenney-Baldwin movement that was the standard of the day was quite poor, as Kennedy proved. That movement was: 12–1 R 11–6 3R 5–9 2–3 10–8 R 4–7 (48). The surprise is that the 1943 edition of Beynon's 'Duplicate bridge direction' contains a well balanced version of the same movement, namely: 12–1 R 11–6 3R 9–5 3–2 8–10 R 7–4 (100), where the last 4 tables of the original are switched. It appears McKenney-Baldwin were listening to the debate, understood the explanation supplied by Ach, and had the political influence to have their movements accepted as the standard. Even so the published movement gave no acknowledgement to McKenney-Baldwin and by 1955 Beynon included movements developed by Sam Gold of Montreal, these being preferred because all idle sets of boards were on one relay table and playing 13 rounds in all cases. The 7 and 8 table examples are shown here, and clearly the issue of balance had been lost with these far from perfect. See 13 rounds, ¾-Howells, for the rest of Gold's movements (page 184). Balance of incomplete movements had not been solved.

[7] 5–12 2–4 9–10 14–1 8–13 7–11 6–3 6R (91) — Gold
[8] 16–1 4–15 11–13 14–7 10–9 5–12 3–8 2–6 5R (80) — Gold

Also in 1944, Beynon introduces the subject of balance of comparisons by showing three movements of 4–table Howells. They are claimed to be all perfect:

8–1 2R 6–2 R 5–7 3–4 (53) — Ach-Kennedy
8–1 2R 6–3 R 7–2 4–5 (100) — Gruenther
8–1 5–6 2–4 R 3–7 2R (100) — McKenney-Baldwin

The irony is that the only imperfect example is the Ach-Kennedy, and is particularly bad. Switching table 2 makes it perfect.

6.3 Balanced Independent Block Design

Probability is expectation founded upon partial knowledge. A perfect acquaintance with all the circumstances affecting the occurrence of an event would change expectation into certainty, and leave neither room nor demand for a theory of probabilities.
George Boole

The balance of a movement is essentially a statistical measure of the quality of the movement. Another way to view this is as an experiment, where we have a group of pairs and bridge hands to play and we would like to know which pairs are the best players. The experiment must be fair for all the pairs.

First of all let us consider a block design. Block design was a result of two mathematicians R. A. Fisher and F. Yates in the 1930s. Suppose you are doing agricultural experiments — trying 7 new varieties of wheat, A, B, …, G. The ideal method would be to plant all 7 and see which did best. Do it several times over, once where you have good soil, once in poor soil, etc. Problem: you do not have enough soil. There is a certain minimum amount of wheat — say 10 plants — if you plant less than that, the random variations make your readings unreliable (you think variety A is better than B, but really there was just enough random variation, rocks in the soil just where the roots went, bad seedlings, etc, that variety B did worse than it should have done); and the areas you have where the soil is uniform (called 'blocks') are only big enough for 30 plants. Solution: plant 3 varieties in each block. We talk about 3 plots. For example, Block 1 might contain A, B, E. If you had enough uniform soil to test all 7, you would call the experiment 'complete'; the case we are talking about is 'incomplete'.

In bridge movement terms we have a Howell movement where the event is complete (all pairs play against

all other pairs) and then we have a Mitchell movement, with some switching to produce one winner, and we have an incomplete movement: all the pairs do not play against all other pairs.

Consider two statistical points. For the most reliable testing the blocks should all be the same size (in the example, 3 plots) every pair of varieties should occur together in the same number of blocks (this is called 'balance').

A Balanced Incomplete Block Design with parameters v, b, r, k, λ is a set of b blocks (sets of size k chosen from a set of v varieties) such that every variety is in r of the blocks and any two belong to λ common blocks.

The mathematicians usually say BIBD or BIB and normally write something like **(v, k, λ) BIBD** because this is enough to derive b and r:

$$b\,k = v\,r \qquad \text{and} \quad \lambda\,(v-1) = r\,(k-1)$$

Wal Wallis writes in his paper *Balanced Tournament Design*: In bridge tournaments, the most common form of competition is where the competitors are fixed partnerships or pairs. The competition is a simple round-robin consisting of spanning rounds. Each deal of the cards is known as a board (because the deals were originally preserved using wooden containers) and is played in several rounds. The two pairs who play against each other in a match are allocated a (numbered) table, and the two pairs are designated by the directions north-south and east-west.

Example: A bridge tournament for eight pairs on four tables {1, 2, 3, 4} playing boards {A, B, C, D, E, F, G} in seven rounds. For example, in Round 1, pair 8 (NS) plays board 'A' against pair 7 (EW) at Table 1. (In practice A, B, … would each represent a set of three or four boards).

Round	1	2	3	4	5	6	7
Table	1234	1234	1234	1234	1234	1234	1234
NS	8241	8352	8463	8574	8615	8726	8137
EW	7653	1764	2175	3216	4327	5431	6542
Boards	ABCE	BCDF	CDEG	DEFA	EFGB	FGAC	GABD

Remarks: there are two combinatorial aspects to these competitions. The movement is the arrangement which assigns boards to each table, assigns directions to each pair, and directs pairs to tables in each round. (For practical purposes, the number of boards in each set must be equal.) The above example is a complete movement for eight pairs.

The second aspect is the scoring. The scores of all east-west pairs on the same set-of-boards are compared, as are the scores of all north-souths. In the above example the boards in 'A' lead to comparisons between the sets {8, 4, 2, 1} and {7, 6, 3, 5}. The whole tournament gives rise to 14 sets:

{8421}, {2853}, {4386}, {5487}, {1658}, {2768}, {3178},
{7635}, {6174}, {5721}, {6132}, {3724}, {4135}, {5246}.

These sets constitute the *underlying block design* of the tournament. The tournament is *balanced* if the underlying design is (pairwise) balanced. For example, the above tournament is balanced because its underlying design is an (8,4,3) BIBD.

The values in this BIBD are based on the properties of the (Howell) movement which has n tables, $2n$ pairs and $n-1$ boards. For this movement to be balanced the **(v, k, λ) BIBD** has values of:

$v = 2n$ (the number of pairs)
$k = n$ (the number of tables)
$\lambda = n - 1$ (the number of board-sets)

6.4 Balanced movements

The only thing more dangerous than ignorance is arrogance.
Albert Einstein
It ain't what you don't know that gets you into trouble. It's what you know for sure that just ain't so.
Mark Twain

We have seen that the balance of a movement is based on the statistical analysis of the experiment we are conducting each time we run an event. The Balanced Independent Block Design applies to any ***complete*** movement.

The controversial issue of balance in bridge movements has been around for nearly as long as duplicate bridge. Since the 1930s people have been searching for the best balanced movements, with claims for the best coming from Mr. Charles Kennedy in the 1930s, Dr. Lawrence Rosler in the 60s, Mr. J. R. Manning in the 70s and more recently Professor Olof Hanner.

The pure mathematics of balance and the measure of competition have been known since John Manning published a paper on the subject in 1979 (see appendix B, page 372) and more recently a measure of imbalance has been defined precisely by Dr. Ross Moore in 1992 (see appendix L, page 403). Even so, and despite knowing the existence of this applied mathematics, Hans-Olof Hallen, Olof Hanner and Per Jannersten (HHJ) published their most excellent book 'Movements – a fair approach' in 1994 and claimed their movements were the best available. Closer examination shows that their ***incomplete*** movements disagree with this mathematics and their reasons are very questionable.

So what is balance? Balance refers to the fairness of the movement: 'Do all pairs have the same advantages during the event?' A completely balanced movement is the fairest of all. For any given situation and requirements, the movement with the least imbalance is the most desirable and should be used in preference to any other.

Another term used for this is 'balanced comparisons'. Manning also calls this 'the measure of competition' between all pairs and Ross Moore 'match-point comparisons'.

Let us consider what is involved in 'comparisons' or comparing scores between pairs on a board. The match-point scoring method underlies the understanding of comparison and by definition is the comparison of results of all pairs who play a board.

Other than how well you play a hand, there are three other potential influences on the result, all outside your control:

1. The quality of the pairs sitting in the same direction as yourself (say, group 1).
2. The quality of the pairs sitting opposite to those in group 1 (say, group 2).
3. The quality of the pair sitting opposite to you (your head-to-head opponent).

Call these factors 1, 2 and 3. Each member of group 1 can *play well* and take one match-point from your score. Call these pairs your ***foes***. Each member of group 2 can *play well* against group 1 opponents and add one match-point to your score. Call these pairs your ***allies***. Your head-to-head opponent can *play well* and take as many match-points from your score as there are other tables in the movement (by scoring a 'top'). Call this pair your ***opponent***. A good pair will *play well* more often and a weak pair less often. Ideally each pair should have a fair share of these influences for the movement to be balanced.

Balance is not an issue where all pairs have the same opposition and the same comparisons. The *completed* Mitchell movement is the best example. There are two winners and two fields, the stationary pairs playing against the moving pairs and being compared with the other stationary pairs and vice versa. Only external factor 1 has any influence as 2 and 3 are equal for all pairs.

The *complete* one winner event is more complicated but balance is achievable. The Howell movement is the typical example. Carefully selected Howell movements can be very well balanced, particularly with an even numbers of tables. In these *balanced* cases factors 1, 2 and 3 are the same for each pair giving a movement that is as equitable as possible.

Note in these Mitchell and Howell movements all rounds are played and the movements are defined as *complete*. The real problem for balance arises with *incomplete* movements. This is where there is one winner but all pairs do not meet. Typical examples are the ¾-Howell and the switched Mitchell. In these cases there are two groups of pairs, those moving and those stationary. The stationary pairs do not meet each other and the moving pairs meet a mixture of the stationary and moving pairs. This is where applied mathematics and 'Movements – a fair approach' disagree.

John Manning's paper 'Mathematics of Duplicate Bridge Tournaments' (1979) says that the measure of competition (M) between two pairs is found by considering the three factors.

A = the number of times the pairs play in the same direction
B = the number of times they play in opposite directions
C = the number of times they play as opponents

$$M = A - B + (n - 1)\,C$$

Where n = the number times a board is played.

For a fair movement, it is the values of M which ideally should be the same for all comparisons. In other words a movement is perfectly balanced when all these values are the same for each two pairs being compared. This perfection is only achievable in *complete* movements like a Howell.

Dr. Ross Moore of Macquarie University, in his paper 'Too many arrow switches spoil the balance' (1992) (see appendix L, page 403), called this value (M) the matchpoint comparison. Using these values for all pairs of pairs in a movement he produces an 'average imbalance' for a movement. This can be called a measure of imbalance of a movement, the lower this number the less imbalance in the movement.

A surprising and counter intuitive conclusion from this study is there should be less switching than previously thought in switched Mitchells. For example, switching only one round in a 9 table Mitchell produces the lowest imbalance and best balance by a clear margin. This example is put forward by HHJ as a test case for this imbalance measurement being incorrect. Let us consider their position in more detail.

They maintain that the measure of comparison between pairs should not include their head-to-head encounter. Per Jannersten says: *"Let us assume that you participate in a 9-table Mitchell with one arrow switch. Then you will never be compared to the pair you meet at the table in the switching round. That is to say, your result will be dependent on how the pair you meet at the table in the switch round performs in the other rounds. But you have no influence over this pair's performance, so why should you allow it to influence your score?"*

"***Never be compared***"? I think most bridge players would disagree; if you want to beat a pair, the best way to do that is to play against them.

In fact, the head-to-head encounter is not only a part of the comparison, but also has (n – 1) times the effect of being 'compared' by playing in the same direction on a board. HHJ, in all their movements, choose to leave out the influence of factor 3. Fortunately this only affects the incomplete movements, but unfortunately this is where it counts the most.

Another point raised by HHJ is the balance of the field (how are the pairs seeded). Clearly if the field is biased then having a movement which is balanced will not solve all your problems. In practical terms, if the

pairs sit arbitrarily or randomly at the start of the event then the field is balanced well enough.

Olof Hanner says: *"My favorite example is as follows: 10 strong pairs and 8 weak pairs are going to play a 9-table Mitchell. All the weak pairs are starting E-W. You and your partner constitute a strong pair. You get the option to start N-S or to be the only strong E-W pair. What will you choose? If there is no arrow switching this may seem to be an even choice. In one case you get weak opposition but strong comparison, in the other case it is just the opposite. But at closer study you find that it is completely irrelevant which pairs you meet. All the pairs you are compared to will meet the same opponents. So of course you take the EW-seat. This situation is not changed by switching in only one round."*

Sorry, I must disagree. The situation does change; the matchpoint comparisons are balanced when you switch one round. Of course it cannot be denied the ***field*** is biased when you do this, but the ***movement*** is not biased.

Once you have a balanced movement then any aberrations in the field balance are not significant. Most organizers of events have their arbitrary method of selecting the seating of pairs. Any reasonable method is acceptable. You certainly do not have only the strong or only weak players sitting in the stationary or moving seats. This is a sure way to get a biased field.

All the pairs' movements in this book are published with an indication as to 'quality of balance'. This 'measure of quality of balance' is called ***calibre***.

The imbalance or 'measure of competition' is not as useful because the size of the imbalance does not necessarily lead to a definitive conclusion on how good or bad the balance of the movement is. The imbalance is a raw figure that is generally higher for a higher numbers of tables and out of context says nothing about 'measuring the quality of balance'.

The ***calibre*** of each movement published in this book is supplied and is an integer number between 1 and 100. A movement with a calibre of 100 is perfectly balanced. A calibre less than 50 is particularly poor. The calibre is calculated as a percentage of the theoretical perfect example of the same movement, with the same number of tables played over the same number of rounds.

Peter Smulders of Holland was the first person to put this quality of a movement into a mathematical calculation based on the Manning groundwork. Since that time Peter and I have worked towards delivering the best results for both pairs and individual movements. John Manning never did consider individual movements.

Ideally the field should be seeded and that seeding needs to be based on a sound and reliable measurement. The best way to balance the field is to use a player-rating scheme to determine the correct seeding for any movement.

All this prompts the obvious question, *'how do you know how well a particular movement is balanced'*? Over the past few years I have developed a computer program that will do just that and a lot more. To determine the imbalance and calibre of your movements, use ***Jeanie – the movement Wizard***. Jeanie contains most of the movements in this book. There are references to the publication, page and author of each movement as well as the calibre. You can also use these movements automatically in your computer scoring program, print guide-cards, define new movements, modify existing movements, as well as many other features too numerous to mention.

There are a few other computer programs that have produced the imbalance and 'quality of balance' features. Examples are those developed separately by Paul Vermaseren, Gerrit van der Velde and Peter Smulders. None of these consider all the available movements and are not available as Windows programs.

6.5 The mathematics of balance

There are lies, damned lies and statistics.
Mark Twain
If a man is talking in the forest, and there is no woman there to hear him, is he still wrong?
Jenny Weber

Measure of competition

John Manning gave us the first thorough analysis of the balance of movements in 1979. See the previous chapter for an overview of the subject and also see John Manning's complete paper (appendix B, page 372) for full details. Also it should be noted that Peter Smulders of Holland contributed significantly into further research on this subject. The first step in considering balance is the measure of competition.

Let us define the notation to be used.

- P is the number of pairs in a competition
- V is the number of boards in play
- n is the number of times each board is played
- T is the number of tables in play
- k is the number of boards in each set
- r is the number of rounds

The derived values are:

- $P = 2T$ *or* $P = 2T + 1$, according to whether the number of pairs P is even or odd;
- the total number of hands played is $T\,r\,k = V\,n$;
- a movement is *complete* if all the pairs play all the boards, when $P = 2T,\ n = T,\ V = k\,r$.

Consider the competition between two pairs, numbered i and j, with $i < j$.
Let:

- a_{ij} be the number of boards where i and j encounter one another;
- b_{ij} be the number of boards which i and j play in the same direction;
- c_{ij} be the number of boards which i and j play in opposite directions, but not as a direct encounter;
- d_{ij} be the number of boards not played by i and j (d_{ij} is always zero in a complete movement);
- s_{ij} be the amount of competition between i and j.

The number of boards in play for each i and j we have $V = a_{ij} + b_{ij} + c_{ij} + d_{ij}$
and s_{ij} is given by $s_{ij} = (n - 1)\,a_{ij} + b_{ij} - c_{ij}$

Defining balance

There are P pairs, and thus there are $\frac{1}{2}P\,(P - 1)$ values of s_{ij}. Call these values 'scores'. We require these values of s_{ij} all to be equal, or failing that, their dispersion as measured by their standard deviation ought to be as small as possible. If the s_{ij} are all equal and the movement is also complete, the movement is said to be *perfect*.

Let $N = \frac{1}{2}P\,(P - 1)$ the number of scores.

By summing over all pairs of pairs, we arrive at the following relations, valid for incomplete as well as complete movements:

$$\sum_{i<j} a_{ij} = n\,V,$$

$$\sum_{i<j} b_{ij} = \sum_{i<j} c_{ij} = n\,(n-1)\,V,$$

$$S = \sum_{i<j} s_{ij} = (n-1)\sum_{i<j} a_{ij} + \sum_{i<j} b_{ij} - \sum_{i<j} c_{ij} = n\,(n-1)\,V,$$

$$SS = \sum_{i<j} (s_{ij})^2,$$

$A = n\,(n-1)\,V\,/\,\tfrac{1}{2}P\,(P-1)$ where A is the average amount of competition between two pairs.

For a *complete* movement in which $P = 2T$ and $n = T$, the expression for A simplifies to

$$A = V\,(T-1)\,/\,(2T-1).$$

Note that k is typically considered to be 1 as that does not affect the balance of the movement.

For a *perfect* movement, all the values of sij are equal, so that A must be integral, and therefore V must be a multiple of $(2T-1)$.

The quality S is the sum of all scores s_{ij} and is called the 'total competition'.

The quality SS is called the 'sum of squares' of all the scores.

By considering all these scores we have a finite population with equal probabilities on all points and so the standard deviation (sd) is equal to the square root of (the average of the squares less the square of the average). As Manning says, we need the standard deviation to be as small as possible for good balance. When zero, the balance is perfect. If less than 1 it is good.

$$sd = \sqrt{(SS - S^2/N)\,/\,N}$$

Movement calibre

To determine a measure of quality of any movement we need to normalize the standard deviation so that all movements may be compared. Let us call the normalized standard deviation of the movement *imbalance B*. This B is derived by dividing the sd by the mean of all the scores. Call this mean m.

$m = S\,/\,N$ and $B = sd\,/\,m$

Note that the standard deviation and imbalance are both smaller for better balance but the imbalance takes into consideration the size of the movement. The larger the mean, the smaller the imbalance. The larger the number of scores, the smaller the mean and the larger the imbalance. Both the standard deviation and the imbalance are open-ended values and do not give a comparable measure of quality.

To convert the imbalance to a percentage let us define the *quality Q* as:

$$Q = 100\,/\,(1 + B^2)$$

Essentially what we are doing here is comparing the SS (which we need to be a minimum) with those of a perfect example of the same movement. In the perfect movement each point in the scores would have the value S/N or m. The SS for the perfect movement is

$$SS = N \times (S / N)^2 = S^2 / N$$

Then the simple form of Q given above can be also expressed as:

$$Q = 100 \times S^2 / (N \times SS)$$

To get the ultimate measure of any movement we need to take one more step. *S/N* is very rarely an integer value but in reality the scores are always integers. Therefore we need to find the Q value for the best movement where all the scores are integers. Let us call this *Qb*.

Note *Qb* is based on the assumption that the 'best' score-matrix consists of elements with at most 2 different values that differ by 1. The lowest one is *S/N* truncated to an integer value, the higher one is 1 more.

Then the ***calibre*** *C* of the movement is found by comparing its estimated quality with the quality of the best possible physical movement. *C* is defined as an integer value.

$$C = (100 \times Q / Qb)_{\text{rounded}}$$

Calibre of two winner movements

The point was made earlier that when you have a two winner movement, such as a standard Mitchell movement, the comparison of pairs is perfect within each group (N-S and E-W fields being treated as separate groups) as long as all the N-S pairs play all the E-W pairs over an equal number of boards and play all the boards in the event.

Therefore a completed Mitchell movement, by definition, has a calibre of 100. One problem is not all Mitchell movements are completed, and some by the very nature of the schedule used cannot be completed, such as the appendix Mitchell.

In the 10 table skip Mitchell the calibre is 100 if you play the revenge round. Playing against the same person is not an issue, or at the very least hard to quantify.

Without the revenge round it is 90, because you need to multiply the calibre by rounds and divide by tables.

$$C' = C * R / T$$

The appendix Mitchell is another example. Say you have 17 tables playing 13 sets over 13 rounds.

$$C = 100 * 13 / 17 = 76$$

The share and relay Mitchell is perfectly balanced when completed. C = 100. Consider 8 Tables, where the movement is not completed. Say the last round is not played. Some boards are played only 6 times, some 7 and some 8.

$Q = 98.29$ based on comparisons but the final value is

$$C = 98.29 * 7 / 8 = 86$$

Imbalance in Individual movements

When we consider an individuals' movement there is one more issue to deal with. Each player plays with all or at least some of the other players as partner. In an individual movement the players can play against

others more than once. The value a_{ij} may be more than 1. In addition the value of a_{ij} should be reduced each time a player partners another.

In incomplete movements the partnering is an important issue. Consider the case where a player partners the best player in the club but does not play against them. The advantage would be very significant..
In individuals there are twice the participants, four players at each table instead of two pairs.

$$P = 4T,\ 4T + 1,\ 4T + 2 \text{ or } 4T + 3$$

In other words N is higher than in pairs and S is lower because of partnering. Therefore to measure the calibre of the movement we must scale the imbalance to meet these differences. Call this scaling g. Then we note that essentially $SS = 16\ (P - 4)$ for Individuals and $SS = 8\ (P - 2)$ for Pairs.

If we assume that these two cases should lead to the same value of B, then using algebra one obtains:

$$g = \sqrt{(8\ (P - 2) / (P - 4))}$$

This number is remarkably close to 3 for a large range of values of P and therefore $B = sd / m / 3$. For individuals' movements, use this value of B in the same formulae for Q and C shown above for pairs.

Seeding players

Now we have a calibre measurement of the quality of any movement a new question arises.

When two similar movements have the same calibre is one better than the other?

First, let us consider two 4-table Howell movements both with the same calibre of 100. These are both perfect, but is there any difference?

8–1 2R 3–6 R 2–7 5–4
8–1 6–5 4–2 R 7–3 2R

Joop van Wijk and Peter Smulders discuss this very issue in this way:

To demonstrate the effect of the balance some authors use the example of one pair of very good players, who always score a top, in a field of normal players, who always get average results. Sometimes the strong pair is replaced by a pair of very bad players, but that does not change the idea. In a movement with perfect balance only 2 scores will matter, the good scores and the bad scores. With a not so perfectly balanced movement the good and bad scores will vary around the average.

Consider a model where *two* strong pairs play in a field of average pairs. Average pairs always get average results when they meet, strong pairs always get the top. Since a top may now be a shared top and a bottom may be a shared bottom, there is more variation in the score cards. We will see how this works out for these perfectly balanced Howells.

We have a 4-table Howell, 8 pairs, 7 rounds of 1 board (using more boards does not change the discussion). The balance is perfect: each pair of pairs play 3 boards in the same direction, 3 boards in opposite directions, and 1 board against each other. Every board is played 4 times, so the top is 6. If this board was played by the 2 strong pairs in different directions, they will each have a 6, their opponents a 0, and all other pairs a 3. If the strong pairs were playing in the same direction, they have a shared top of 5, and their opponents a 1.

It is more interesting to look at the scores of the average pairs that did not meet the strong pairs directly. The 2 pairs that played in the same direction as the strong pairs are unlucky. They are left with 1 point each, since the strong pairs already took 10 out of the 12 available MPs. Consequently their opponents score 5,

just as much as the strong pairs. This illustrates how pairs that play a board in opposite direction from you are your allies. Finally, there is one board where the 2 strong pairs meet, and on this board everyone gets 3.

So the score of the strong pairs will be $3 \times 6 + 3 \times 5 + 1 \times 3 = 36$ out of the possible $7 \times 6 = 42$, and therefore score 85.71%. The rest of the pairs' scores are divided up in this way:

6 scores of 0
12 scores of 1
18 scores of 3
6 scores of 5

In the ideal case each of them would get $1 \times 0 + 2 \times 1 + 3 \times 3 + 1 \times 5 = 16$.

It now turns out that it is possible to devise Howell movements that have this property, but also Howell movements that do not, but still are perfectly balanced movements in the customary sense of the word.

Using the first movement above we have (board-sets 1 to 7 are shown as A to G respectively).

8–1	A	3–6	D	2–7	F	5–4	G
8–2	B	4–7	E	3–1	G	6–5	A
8–3	C	5–1	F	4–2	A	7–6	B
8–4	D	6–2	G	5–3	B	1–7	C
8–5	E	7–3	A	6–4	C	2–1	D
8–6	F	1–4	B	7–5	D	3–2	E
8–7	G	2–5	C	1–6	E	4–3	F

No matter which way you choose the 2 strong pairs from the numbers 1 to 8, the average pairs will have a score of 16, i.e.,

- 1×0: direct meeting with strong pair
- 2×1: shared bottom, either by playing against a strong pair or by having both strong pairs in your direction.
- 3×3: strong pairs played against each other or they played this board in different directions
- 1×5: both strong pairs played this board in a direction opposite from yours.

Now the second movement.

8–1	A	6–5	B	4–2	C	7–3	E
8–2	B	7–6	C	5–3	D	1–4	F
8–3	C	1–7	D	6–4	E	2–5	G
8–4	D	2–1	E	7–5	F	3–6	A
8–5	E	3–2	F	1–6	G	4–7	B
8–6	F	4–3	G	2–7	A	5–1	C
8–7	G	5–4	A	3–1	B	6–2	D

Here some pairs score a point more or less then the average. Let us assume, for instance, the strong pairs are 1 and 2, and look at the score for pair 5. It is $3 \times 1 + 4 \times 3 = 15$. Even though they never had a full bottom, they had no shared top either, ending up 1 below the score of 16 they were entitled to.

These authors describe the first as 'super-perfect'. As to whether these super-perfect movements exist for higher numbers of tables is debatable. We have not yet been able to spot one for 6 tables or for that matter any other number of tables, but this outcome could simply be a result of the modeling techniques we are using.

This is a nice discussion, but it does not consider the reality of a bridge tournament where the players are all

different and it is possible to consider all the pairs as having a ranking from 1 to 8. Another approach is to use the Bussemaker model. See '*Why the best man loses*' in Appendix M, page 412, for the origins of this concept.

The Bussemaker model is described in an article by Pieter Bussemaker in WEKOwijzer (#58), a journal published by the Netherlands Bridge Organization. It is assumed that the pairs are ordered according to strength, and the difference in strength between any two pairs is equal. The score on a board is simply taken as the difference in pair numbers times 100.

For example, if pair 4 plays a board against pair 8, pair 4 obtains a score of +400, and pair 8 obtains -400. This is found by 8 - 4 = 4 and multiplied by 100 for a score of 400. If pair 5 plays the same board against 1, the score would also be 400. These scores are converted to matchpoint scores in the usual way.

In the ideal case pair 1 would end up first, followed by pair 2, then 3, and so forth. The difference between consecutive pairs should be equal. The results show differently. These tables show boards 1 to 7 with NS and EW pairs shown like 8-1 and matchpoint scores like 0_6. The first is the super-perfect example.

The total matchpoint scores for each pair are

1	2	3	4	5	6	7	8
36	32	26	23	19	14	11	7

Board 1		Board 2		Board 3		Board 4		Board 5		Board 6		Board 7	
8-1	0_6	8-2	0_6	8-3	0_6	8-4	0_6	8-5	0_6	5-1	0_6	6-2	0_6
7-3	2_4	5-3	2_4	6-4	2_4	7-5	2_4	3-2	2_4	8-6	2_4	3-1	2_4
4-2	4_2	7-6	4_2	2-5	4_2	2-1	4_2	4-7	4_2	4-3	4_2	8-7	5_1
6-5	6_0	1-4	6_0	1-7	6_0	3-6	6_0	1-6	6_0	2-7	6_0	5-4	5_1

The other movement gives the total matchpoint scores for each pair are

1	2	3	4	5	6	7	8
32	31	29	22	21	16	12	5

Board 1		Board 2		Board 3		Board 4		Board 5		Board 6		Board 7	
8-1	0_6	8-2	0_6	8-3	0_6	8-4	1_5	7-3	0_6	8-6	1_5	b	1_5
5-4	2_4	3-1	2_4	5-1	2_4	6-2	1_5	8-5	2_4	7-5	1_5	8-7	1_5
3-6	4_2	6-5	4_2	4-2	4_2	5-3	4_2	6-4	4_2	3-2	4_2	2-5	4_2
2-7	6_0	4-7	6_0	7-6	6_0	1-7	6_0	2-1	6_0	1-4	6_0	1-6	6_0

Clearly these two perfect movements are not the same *when using this model*. The most important difference is that the pairs play different pair numbers on each board. Essentially we are using a *seeding* of the pairs in the order 1 to 8. Changing the seeding we will get a different result. For example if we make pair 5 the second seed and pair 2 the 5th seed we get:

2	3	4	5	6	7	1	8
34	28	24	16	15	12	32	7

Now pair 5 is the winner at the expense of pairs 2, 3, 4 and 6. The two movements are the same in terms of quality and this quality is the same based on statistical analysis.

To further analyze the various possibilities of seeding of the pairs we will consider these results using the Bussemaker model for the scoring. The following two probability tables are generated using 1,000,000

random seeding orders. The probability table of the results for the super-perfect Howell is

Pair/Pos	1	2	3	4	5	6	7	8
1	92	8						
2	8	82	10					
3		10	73	8	8			
4			17	72	12			
5				12	72	17		
6				8	8	73	10	
7						10	82	8
8							8	92

This means the best pair, pair 1, has a 92% chance of ending up in the first place, and 8% for second place.

For the non-super-perfect Howell the probability table is:

Pair/pos	1	2	3	4	5	6	7	8
1	87	13	·					
2	12	77	10	·				
3	1	10	81	8	·			
4		·	8	77	15	·		
5			·	15	77	8	·	
6				·	8	81	10	1
7					·	10	77	12
8						·	13	87

Here the best pair has 13% chance of finishing second, and even a finite chance of ending third. All numbers are rounded, a dot means larger then 0 but less then 0.5.

This shows that if the pairs sit at random the best pair has a high probability of winning in both cases but in the first the chances are just a little better. Generally the probability table for all perfect calibre movements (100) will show a very high probability of the best pair as the winner and typically greater than 80%. Bridge is such a good game because of the remaining chances that any pair is able to win an event.

All we can do as organizers of tournaments is supply the best balanced movements with the fairest seeding of the players. We have shown here that movements with a calibre of 100 make it possible to ignore the seeding, but the default seeding of 1 to 8 shown above means these pairs have a high probability of finishing in positions 1 to 8 in both movements. The organizers of such events could not be found at fault when they choose this seeding.

Where the seeding is shown in Jeanie the default seeding of 1 to *N* is used to score the event based on the Bussemaker model. The recommended seeding is based on these results and generally the same as the default value (1 to *N*) for Howell movements of a high standard.

With switched Mitchell movements this is not the case. For example, for the 7-table switched Mitchell with the last round switched the seeding order is 1 2 3 4 9 8 7 5 10 6 11 12 13 14. Note N-S pairs are 1 to 7 and E-W 8 to 14. Switching another round instead of the last does not change the calibre (92), but the seeding does vary.

Part 7 Pairs' schedules

Set of 4 duplicate (auction) bridge boards, circa 1925.
This set is brand new and made by the U.S. playing card company.
This was the time that contract bridge was invented, and probably the reason that this set remained unused.

7.1 Further Mitchell modifications

Cards are war, in disguise of a sport.
Charles Lamb

The objective of the Mitchell movement is for all the N-S pairs to meet all the E-W pairs while all pairs play all the boards.

The normal procedure is for the N-S pairs to remain stationary, the E-W pairs to move up one table, and the boards to move down one table. This is the original Mitchell system and the basis for all the modifications that have occurred over the years since 1892 when the first needs arose in duplicate progressive Whist. See *The Mitchell system*, page 44 for the original description.

There are two basic ways of organising the movement for an even number of tables: the SKIP Mitchell and the RELAY Mitchell. See other solutions below as well as the whist movement and multiple teams movements for even tables.

More on Baker modification

E.T. Baker devised the skip Mitchell circa 1892. See the *Baker modification* above in the chapter on *Early Mitchell modifications* (page 46). It proceeds the same way as the standard Mitchell with the exception that after the half way round, the E-W pairs skip one table by proceeding up two tables rather than one. The boards move the same as the standard Mitchell throughout.

Clearly, the E-W pairs will reach the table before the one at which they started, one round before the end. If the last round is played, the E-W pairs will play the pairs against whom they started. This is called the *revenge* round. Note that the E-W pairs do not play the N-S pair that they skip at the halfway mark. This means that the comparison of players is not equal, but on the other hand the movement is very hard to foul up, whereas the relay Mitchell has many areas in which things may go wrong.

When there is an even number of tables and an incomplete Mitchell (curtailed Mitchell) is needed, then the skip Mitchell should be used. This can be terminated at any point during the movement and all the boards will have been played an equal number of times.

It was pointed out that the skip should occur just after the half way point in the tournament. This means, for example, that in a 14 table Mitchell, the skip would occur after the 7th round has been played.

It is also possible to skip earlier than this point. This is handy if you have two or more sections of even tables, but of different sizes, and all sections are going to play the same number of rounds. This means that the skip can be announced for all sections at the same point. The formula for finding the earliest round at which the skip can be announced is:

$$E = R - T / 2$$

Where E = the earliest round; R = the number of rounds to be played; T = the number of tables in a section.

If, for example, you have two sections, one of 14 tables and one of 16 tables and you want to play 13 rounds, making a 26 board tournament, then clearly, the skip for the 14 table section will be made after the 7th round. The play will finish when the E-W pairs have played the table just before the one at which they started. In the 16-table section the earliest round at which the skip may occur is:

$$E = 13 - 16 / 2 = 5$$

Therefore the skip may also be called after round seven in the 16-table section. Further curtailing of the movement is possible if desired, such as terminating after 12 rounds in the above example.

If the early skip has been used it is also possible to continue the movement right to the end if it is decided not to terminate the movement early. All that is changed is the pair against whom the revenge round is played. If the skip is made one round earlier than the normal skip point, then the E-W pairs remain at the table of the second last round and only the boards move for the change into the last round.

If the skip was made 2 rounds before the normal change point then the E-W pairs play against the same N-S pair for the 3rd last and the 2nd last rounds and move normally into the last round. With the skip three rounds before normal, the repeat occurs on the 3rd and 4th last rounds and so forth.

In the example above, the 16-table movement could continue normally to the 15th round; then for the last round, only the boards would move.

An alternative procedure for the skip Mitchell could be called the REPEAT Mitchell. The movement is basically the same as the normal Mitchell movement, but instead of the skip at halfway, the E-W pairs repeat play against the same pair. For example, in a 14 table Mitchell the move after the 7th round is:

- N-S remain stationary;
- E-W remain stationary;
- boards move down (up) 1 table.

This procedure eliminates the skip and moves the repeat play from the first pair played to the middle pair. It is mentioned above that it is possible to have stationary boards in a Mitchell movement. As this is occasionally needed or desired (see switched Mitchells, page 154) then the method for having stationary boards with an even number of tables, using the Baker modification, is as follows.

The pairs move normally (for example N-S up one table, E-W down one table) up to the halfway point at which the N-S pairs continue to move as usual and the E-W pairs skip one table. For the last round the E-W pairs move to the table that they skipped with the N-S pairs still moving normally. Unfortunately the revenge round still occurs in this movement.

Share-and-relay modification

The origins of this modification are difficult to locate. It was first mentioned in *Auction bridge of 1924* by Milton C. Work. Note *mentioned* and only that. When discussing the skip Mitchell in the **Laws of duplicate Auction** and the inherent weakness of missing a pair, it is added: "This may be avoided by what is termed the 'Relay system'". By 1926 the **Laws of duplicate Auction** in Work's *Auction Bridge Complete* include a description of the procedure. In *Duplicate contract complete* by Alfred M. Gruenther (1933) and *Duplicate bridge auction or contract* by Alexander T. Hasler (1933), the first two serious tournament director books for bridge from each side of the Atlantic, they cover the subject fully.

The relay Mitchell adopts the same movement procedure as the standard Mitchell, with the N-S pair stationary and the E-W pairs moving up one table and the boards moving down one table. The difference lies in the arrangement of the tables and the distribution of the boards. The boards are distributed as usual, with set one on table one, set two on table two, and so on until exactly half the sets of boards have been distributed.

The next set is placed on a relay table, at which there are no players and which is usually a small side table placed there for this specific reason. The rest of the sets of boards are distributed normally with the next available set going on the next table in play and so forth.

With the director starting out with the same number of sets of boards as there are tables, it is obvious that there will not be a set for the last table as one set is on the relay table. This last table (without a board-set) will share boards with the first table for the duration of the movement.

The boards still move down one table at each change except now the boards that were shared at table one and the last table, move down to the second last table. With N equal to the number of tables in the movement, the boards on table $\frac{1}{2}N + 1$ will move on to the relay table and the boards on the relay table will move on to the table $\frac{1}{2}N$.

With the share-and-relay it is not possible to have stationary boards.

Relay Mitchell problems

There are several areas in this movement that need close attention. The first of these is the moving of the boards on and off the relay table. The second is the placement of the relay table in the correct position; occasionally even the most experienced director slips up. If this does happen, then it is possible to correct the error by the following procedure. If the byestand table is misplaced, hopefully it will only be by one table higher or lower than the correct position.

If the byestand table has been placed one position too low then the players will notice the boards are incorrect on the $(\frac{1}{2}N + 1)^{th}$ round at which point the correction will have to take place.

With the change into the $(\frac{1}{2}N + 1)^{th}$ round having been made:

- move the byestand up one position to its correct position;
- exchange the sets of boards on the byestand and the $\frac{1}{2}N^{th}$ table (Board-set one now will be on the byestand);
- have the N-S pairs at tables N and $\frac{1}{2}N$ swap places for this round only, returning to their original tables for the rest of the movement;
- for this round $(\frac{1}{2}N + 1)$ and subsequent rounds the highest numbered table (N) should share boards with the second highest table ($N - 1$) rather than table 1.

If the byestand has been originally placed one position too high it will also be noticed on the $(\frac{1}{2}N + 1)^{th}$ round at which point the correction will have to take place. With the change into the $(\frac{1}{2}N + 1)^{th}$ round having been made:

- move the byestand down one position to its correct position;
- exchange the sets of boards on the byestand and the $(\frac{1}{2}N + 1)^{th}$ table (Board-set one will now be on the byestand);
- for this round and subsequent rounds table 1 will share boards with table 2 rather than the highest numbered table (N);
- on the last round have the N-S pairs at tables 1 and $\frac{1}{2}N + 1$ swap places.

When arranging the tables for any Mitchell movements the horseshoe pattern is usually adopted, for this takes any burden off the players as to their next table when they reach the highest-numbered table in the movement. This arrangement is also very useful in the share-and-relay movement as the positions of the share-and-relay tables are clearly defined.

Examples:

7 table Mitchell:	1	2	3	4
		7	6	5

8 table Mitchell:	1	2	3	4
	Share			Relay
	8	7	6	5

Simple half-table modification

The standard Mitchell movement caters for half tables quite well. The table at which the odd pair is sitting (usually the last table) is treated as a full table for the purposes of distributing boards and determining the type of movement that is to be used. For example an 8½-table movement is treated as a 9-table movement, with three boards placed on each table making a total of 27 boards. Making the N-S pair at table 9 the phantom pair, the E-W pair at that table sits out on each round so that the movement proceeds quite normally to the end. All the N-S pairs will have played 27 boards while all the E-W pairs will have played 24 boards. Note that if you are using stationary boards, the phantom pair should be an E-W pair to facilitate the scoring.

With an even number of tables (including the half table), the standard SKIP Mitchell should not be used as the pair that sits out on the first round will also have to sit out on the last round. On the other hand, if the movement is to be curtailed before the last round, then it is possible to use this movement. Remember that this method introduces score-factoring problems. If the movement is to be curtailed, the rover Mitchell should be used.

Another alternative with the even number of tables is the standard RELAY Mitchell. Here the odd pair is placed E-W at the highest numbered table (one of the share tables) and all the E-W pairs sit out there. The advantage of this movement is that the share is effectively removed from the game as the boards usually shared are not being played at one of the share tables.

For more on half tables see the *pivot modification*, page 105.

Displacement or bump modification

This was an early attempt at the rover Mitchell and does work quite effectively. For this movement to work a basic Mitchell plus a pair is needed. The SKIP Mitchell is used with an even number of tables.

The basic Mitchell proceeds normally (including the skip at the halfway point in the SKIP Mitchell) and only the odd pair needs any special consideration. After the first round the extra pair, (which is assigned the N-S number one higher than the highest numbered table in the movement) replaces the N-S pair at table 1 while that pair sits out for the second round. The highest numbered pair remains at that table as N-S for the rest of the session. On the third round the N-S pair 1 replaces the N-S pair at table 2 and stays there for the rest of the event. This pattern is followed for the rest of the event with each sit-out pair returning to the next higher numbered table. It is important that the N-S pairs keep the same pair number throughout the event. The disadvantage with this movement is the last round of the event cannot be played, as the majority of the pairs will have played the boards on their table on the last round. When scoring, care must be taken that the scores of the pairs who sat out are adjusted to those of the pairs who did not.

This movement can be altered so that the last round can be played: the sit out pair replaces the next lower numbered N-S pair, rather than the next higher numbered pair. The disadvantage is all but two N-S pairs will repeat play against one pair in an odd-table movement and all but three pairs will repeat against two pairs in an even-table movement.

Rover modification

The replacement Mitchell (now called the rover or NS rover Mitchell) was originally devised by D. A. Sweet of Chicago and was applicable only to movements with a prime number of tables, plus the odd pair. i.e., the movement is useful for 5½, 7½, 11½, 13½, l7½, 19½, 23½, 29½ and 31½ tables.

The base movement is set up as though the ½ table does not exist. All boards are always in play with one board-set on each table.

The odd pair is assigned a pair number one higher than the highest numbered table They sit out on the first round and enter the movement on the second round by replacing one of the N-S pairs in the movement (usually N-S table 2), but only for that round. After this round the original N-S pair return to their table and the 'roving' pair move on by going up two tables (or down two tables, but preferably in the same direction that the E-W pairs are moving). This procedure, of moving up by two tables, is continued to the completion of the movement, or the curtailment of the movement, whichever is first.

The real advantage of using rover Mitchells is that all boards are played the same number of times, and as such, the *Top* is always the same on all boards.

The obvious need for the extension of this movement to other numbers of tables was devised by Sam Gold in 1961. The simple case of this extension is for an even number of tables, but not those divisible by 3, plus the half table i.e. 4½ (but see below), 8½, 10½, 14½, 16½, 20½, 22½, 26½, 28½ and32½**.** With the SKIP Mitchell being used for the basic movement and the odd pair sitting out on the first round, they enter the movement on the second round by displacing the N-S pair at table 2, but only for that round. The rover pair then moves up 2 tables at each change until the round when SKIP is called. When the skip is called the rover pair moves to table 3 and proceeds normally for the rest of the movement by moving up two tables at each change. The SKIP Mitchell can proceed to the last round where the normal revenge round is played and the rover pair will repeat against their first opponents (E-W pair 1).

The next type of rover is for movements where the base is divisible by 3 i.e. 6½, 9½, 12½, 15½, 18½, 21½, 24½, 27½ and 30½. Unfortunately there is no set pattern for these rover Mitchells and consequently it is necessary to supply the roving pair with a guide card to use the movement.

The following rover guide-list assumes that in even-table movements the N-S pairs are stationary, while the E-W pairs move up 1 table and skip after the half way round. The boards move down one table. Occasionally the roving pair will have to repeat against one E-W pair. This is shown by an asterisk (*) next to the number of the table where the pairs meet the second time. It should be noted that the movement cannot always be completed. To do so the rover would have to repeat displacement of a N-S pair.

The roving pair is assumed to have sat out for the first round and the following guides start at the table at which the rover displaces the N-S pair on round 2.

6 Tables:
2 4 1 3
9 Tables:
2 4 6 9 3 5 7 8*
12 Tables:
2 10 8 6 4 1 11 9 5 3
15 Tables:
2 13 11 9 7 5 1 14 12 10 8 4 (curtailed at 13 rounds)
15 Tables:
2 13 11 9 7 5 1 14 15* 10 6 4 8* (curtailed at 14 rounds)
18 Tables:
2 16 14 12 10 8 6 3 1 17 15 13 11 9 7* 4*
18 Tables:
2 16 14 12 10 8 6 3 1 17 15 13 11 9 5 (curtailed at 16 rounds).

The following list is for even tables with the share-and-relay movement, again with the guide starting on round 2.

4 Tables:
1 4 2
6 Tables:
5 3 6 4 1
8 Tables:
1 6 2 7 3 8 4
10 Tables:
1 7 2 8 3 9 4 10 5
12 Tables:
3 9 4 10 5 11 6 12 7 1 8
14 Tables:
1 9 2 10 3 11 4 12 5 13 6 14 7
16 Tables:
1 10 2 11 3 12 4 13 5 14 6 15 7 16 8
18 Tables:
13 5 14 6 15 7 16 8 17 9 18 10 1 11 2 12

1½ table appendix modification

This movement is a very useful movement when the previous movements for half-table Mitchells produce events of cumbersome or unsuitable length. For example, the 10½ table event can be run as straight Mitchell with 22 or 33 boards in play, or as a rover Mitchell on base 10 with 20 or 30 boards in play. When using the 1½ table appendix Mitchell the number of boards in play is 27.

To construct this movement, first of all subtract 1½ tables from the number you have, and set up a Mitchell for that number. The SKIP version is used for even numbers of tables. The 1½ tables are then appended on to this base by placing the whole table next to table 1, so that they are in a position to share boards. In the 10½ table example above, the base is 9 tables long, with 3 boards on each table, and table 10 shares boards with table 1 while the ½ table pair takes number 11 as their pair number and sit out for the first round. The move as far as the pairs are concerned is the same as if they were in a straight 11 table Mitchell, moving up one table at each change and going to table 1 after the bye table. The boards on the other hand, move as though they were in a standard 9 table Mitchell, moving down one table at each change, *going to table 9 after table 1.* When the base is an even number of tables, the SKIP occurs after the round number that is equal to half the number of tables in the *base.* Also the number of rounds in the event is equal to the number of tables in the base and there is no revenge round.

2 table appendix modification

The two table appendix movement is essentially a 1½ table appendix with the half table filled out with another pair. Even so, when considering this movement the *Ewing*, *Bowman-Ewing*, *web* and *appending movement* modifications all should be reviewed so that you have a complete understanding of the principles involved. The *Bowman-Ewing* modification is essentially the same movement with different board-sets in play on the last two tables.

The movement is simple and it works for all numbers of tables. This is an important movement where you want to conduct a Mitchell movement with two rounds less than the total number of tables. See *Incomplete Mitchell movements*, page 125.

The base movement is set up with two tables less than the total. The second-highest table starts with board 1 and plays the boards in sequence, the same as table 1. The highest table starts with the highest board-set and plays them in descending order. The E-W pairs move as usual for a Mitchell up one table at each

change. The only exception is with an even number of tables in the base when a skip occurs after half the number of tables in the base.

If you do not have duplicated boards then the second highest tables shares with table 1 throughout. The highest table shares boards with the third highest on the first round and subsequently two tables lower on each round. The only difficulty is with an odd number of tables when a three-way share occurs.

Stationary board 2-table appendix modification

Stationary boards in the base

The 2-table appendix movement can have stationary boards in the base movement, but is limited to odd numbers of tables. It is desirable to use pre-duplicated boards to eliminate sharing, preferably having three sets.

The base movement is set up in the same way as the original, with moving boards but the N-S pairs move down one table and always remain in the base movement. The boards remain stationary in the base, one set per table. The N-S pairs at the two appendix tables remain stationary as above. The E-W pairs always move up one table, moving from the last to the first.

The boards are played in descending sequence at the appendix tables, starting with the last set on round 1. The appendix tables can share boards where there are enough boards in each set otherwise it would be better for these tables to use one set each.

Stationary boards at all tables

The 2-table appendix movement can have stationary boards at every table, but only to a limited degree. Some tables will play one of two sets during the event and others only one set. All the pairs are moving and will require guide-cards. Examples follow.

5 tables, 3 rounds

Round/table:	1	2	3	4	5
1	1–1 A	4–4 A	2–2 B	3–3 C	5–5 C
2	3–2 A	1–5 B	4–3 B	5–4 B	2–1 C
3	2–5 A	5–3 A	3–1 B	1–4 C	4–2 C

This is shown as N-S and E-W pairs separated by hyphen and board-sets 1 to 3 as A, B, C. Note only two board-sets are needed if no sharing is required. If sharing is allowed, then the tables next to each other play the same sets. Careful examination will show that there are distinct patterns to the player movement, but for practical purposes guide-cards are essential.

Note with an odd number of tables one board-set is played 3 times on the same round, set B in this case.

6 tables with 4 rounds

Round/table:	1	2	3	4	5	6
1	1–1 A	5–5 A	2–2 B	3–3 C	4–4 D	6–6 D
2	4–3 A	1–6 B	5–4 B	2–1 C	6–5 C	3–2 D
3	3–6 A	4–1 B	6–3 B	1–4 C	5–2 C	2–5 D
4	2–4 A	6–2 A	3–5 B	4–6 C	1–3 D	5–1 D

7 Tables and 5 rounds

Round/table:	1	2	3	4	5	6	7
1	1–1 A	6–6 A	2–2 B	3–3 C	4–4 D	5–5 E	7–7 E
2	5–4 A	1–7 B	6–5 B	2–1 C	3–2 D	7–6 D	4–3 E
3	4–2 A	5–3 B	1–6 C	6–4 C	7–5 C	2–7 D	3–1 E
4	3–7 A	4–1 B	7–4 B	5–2 C	1–5 D	6–3 D	2–6 E
5	2–5 A	7–3 A	3–6 B	4–7 C	5–1 D	1–4 E	6–2 E

8 Tables and 6 rounds

Round/table:	1	2	3	4	5	6	7	8
1	1–1 A	7–7 A	2–2 B	3–3 C	4–4 D	5–5 E	6–6 F	8–8 F
2	6–5 A	1–8 B	7–6 B	2–1 C	3–2 D	4–3 E	8–7 E	5–4 F
3	5–3 A	6–4 B	1–7 C	7–5 C	2–8 D	8–6 D	3–1 E	4–2 F
4	4–8 A	5–1 B	6–2 C	8–4 C	1–5 D	7–3 D	2–6 E	3–7 F
5	3–6 A	4–7 B	8–3 B	5–8 C	6–1 D	1–4 E	7–2 E	2–5 F
6	2–4 A	8–2 A	3–5 B	4–6 C	5–7 D	6–8 E	1–3 F	7–1 F

Ewing modification

This modification is named after Mr. W. Ewing of the U.K and was first reported by E.E. Blandon in *Bridge Magazine*, February 1973 where he presented the Bowman-Ewing modification (see page 108). It was formally named as an independent modification in *Duplicate bridge movements* by Frank Farrington in 1979. John Manning also called this *Hesitation Bowman* in the EBU manual.

This is a modification suitable only for a one-winner event. The N-S and E-W pairs are not kept separate. Given this limitation, and using switching to balance the comparisons, this does not prove to be a problem. A normal switch-Mitchell is set up for the number of tables (N) in play. The N-S pair numbers are the same as table numbers except for the last two tables. The E-W pairs add $N - 1$ to their table number to get their pair number. The highest-numbered N-S pair at table N has number $N - 1$. The N-S pair at table $N - 1$ is a moving pair (normally E-W) with number $2N$.

The number of board-sets in play equals $N - 1$, one set on each table but the last. The last table shares boards with table $N - 2$ on the first round and then two tables lower on each subsequent round.

The movement proceeds as a normal Mitchell, N-S stationary, E-W move up one table and the boards down one table. The complication only occurs at the highest tables, where E-W at table N move to N-S at table $N - 1$ and N-S at table $N - 1$ moves to E-W at table 1.

So the E-W pairs move normally to the last table playing one round there, then to the second highest table as N-S for one round, then proceed as a normal E-W pair going to table 1 as E-W. The maximum number of rounds that can be played is the same as the number of board-sets.

With an odd number of tables ($N - 1$ is even) the normal skip Mitchell is used, skipping after half the rounds. At the highest tables the E-W at table N and the N-S at $N - 1$ move two steps: the E-W pair at the second highest table moves to N-S at the same table, N-S at the second highest table to E-W at table 2, the highest table E-W pair to N-S at table 1. No revenge round occurs. This should not be confused with the hesitation modification from Blandon, see page 116.

Web modification

This movement was devised by John ‘Spider’ Harris and hence the name ‘web’. This is useful for a medium to large one-session event where a Mitchell type of tournament is required. An even number of

tables is required in the whole field, though there are ways to cater for odd numbers. See web rover-table (page 109) and web hesitation (page 104) modifications for odd numbers of tables.

As far as the pairs are concerned the movement appears the same as any Mitchell movement. The tables are divided into two groups for the purpose of board moving.

The number of sets of boards dictates the length of the event and must be equal to or more than half the number of tables. You might have 7 or 8 sets of 3 or 4 boards, 9 or 10 sets of 3 boards, 11 or 12 sets of 2 boards, 13 or 14 sets of 2 boards. The tables are numbered from 1 to 2*N* and there must be 2 copies of each board in play. The duplication of the boards can be done by the players or they can be pre-duplicated.

Note: Where the number of board-sets is greater than the number of tables, pairs will meet more than once.

Tables 1 to ½ *N* play the first set-of-boards and tables ½ *N* + 1 to 2*N* play the second set-of-boards. In the first group of tables the first board-set goes on table 1, the second on 2, etc., with the excess sets on a relay table at the end. In the second set of tables the highest numbered set goes on table 2*N*, set 1 goes on table 2*N* – l, set 2 goes on table 2*N* − 2, etc., with the excess again on a relay table. For example, with 7 sets of boards and 10 tables the following arrangement is made:

Tables:	1	2	3	4	5		
Boards:	1	2	3	4	5	6	7
Tables:	6	7	8	9	10		
Boards:	4	3	2	1	7	6	5

In the first group, the boards move down one table at each change so that tables 1 through 5 play the boards in ascending order.

In the second group the boards move down one table also, but tables 6 through 10 play the boards in descending order.

The E-W pairs always move up 1 table ignoring any sectional boundaries and irrespective of the number of tables. *When using an even number of board-sets the E-W pairs must skip after playing half the boards.*

All players play all boards in this event, but all N-S pairs do not meet all E-W pairs when there are less board-sets than tables. As such, if this is used in qualifying events, the seeding of pairs is necessary for the starting positions so that all seeded pairs N-S meet all seeded pairs E-W.

Where there is an even number of board-sets and the number of tables equals the number of sets a revenge round, like in the skip Mitchell, is required to complete the movement.

One curious feature of this movement is the ability to handle any number of sets of boards. For example a 4-table web can be conducted with 13 board-sets, but the pairs do meet each other many times, in this case either 3 or 4 times. This ability does allow the use of the web as an appendix movement with any other Mitchell. See *appending movements modification,* page 123.

Web hesitation modification

For an additional information on the origins of this modification, see hesitation super-bowman, page 355.

Where you have a large number of tables, and only a very limited time for one session of play, and the need to produce one winner, then this modification is ideal. With an even number of tables, use a switched version of the web Mitchell. Note the balance is always poor with so much reduced competition.

With an odd number of tables, setup a standard web Mitchell for the number of board-sets you wish to play and one less table than the total. Then add an extra table using the hesitation Mitchell rules defined in the Ewing modification, see page 103. The only difference is the board-sets are played in order on the last table, but it is possible to share boards between table 1 and the last table.

For example with 15 tables and the need for 12 rounds, and one winner in one session, setup a 14-table web Mitchell, playing 12 rounds. Add the last table using the same procedures that are used for the normal hesitation Mitchell. The resulting expanded movement is shown here. To eliminate board sharing a separate set-of-boards are needed for the last table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1–15 (1)	2–16 (2)	3–17 (3)	4–18 (4)	5–19 (5)	6–20 (6)	7–21 (7)	8–22 (6)	9–23 (5)	10–24 (4)	11–25 (3)	12–26 (2)	13–27 (1)	30–28 (12)	14–29 (1)
2	1–30 (2)	2–15 (3)	3–16 (4)	4–17 (5)	5–18 (6)	6–19 (7)	7–20 (8)	8–21 (5)	9–22 (4)	10–23 (3)	11–24 (2)	12–25 (1)	13–26 (12)	29–27 (11)	14–28 (2)
3	1–29 (3)	2–30 (4)	3–15 (5)	4–16 (6)	5–17 (7)	6–18 (8)	7–19 (9)	8–20 (4)	9–21 (3)	10–22 (2)	11–23 (1)	12–24 (12)	13–25 (11)	28–26 (10)	14–27 (3)
4	1–28 (4)	2–29 (5)	3–30 (6)	4–15 (7)	5–16 (8)	6–17 (9)	7–18 (10)	8–19 (3)	9–20 (2)	10–21 (1)	11–22 (12)	12–23 (11)	13–24 (10)	27–25 (9)	14–26 (4)
5	1–27 (5)	2–28 (6)	3–29 (7)	4–30 (8)	5–15 (9)	6–16 (10)	7–17 (11)	8–18 (2)	9–19 (1)	10–20 (12)	11–21 (11)	12–22 (10)	13–23 (9)	26–24 (8)	14–25 (5)
6	1–26 (6)	2–27 (7)	3–28 (8)	4–29 (9)	5–30 (10)	6–15 (11)	7–16 (12)	8–17 (1)	9–18 (12)	10–19 (11)	11–20 (10)	12–21 (9)	13–22 (8)	25–23 (7)	14–24 (6)
7	1–24 (7)	2–25 (8)	3–26 (9)	4–27 (10)	5–28 (11)	6–29 (12)	7–30 (1)	8–15 (12)	9–16 (11)	10–17 (10)	11–18 (9)	12–19 (8)	13–20 (7)	23–21 (6)	14–22 (7)
8	1–23 (8)	2–24 (9)	3–25 (10)	4–26 (11)	5–27 (12)	6–28 (1)	7–29 (2)	8–30 (11)	9–15 (10)	10–16 (9)	11–17 (8)	12–18 (7)	13–19 (6)	22–20 (5)	14–21 (8)
9	1–22 (9)	2–23 (10)	3–24 (11)	4–25 (12)	5–26 (1)	6–27 (2)	7–28 (3)	8–29 (10)	9–30 (9)	10–15 (8)	11–16 (7)	12–17 (6)	13–18 (5)	21–19 (4)	14–20 (9)
10	1–21 (10)	2–22 (11)	3–23 (12)	4–24 (1)	5–25 (2)	6–26 (3)	7–27 (4)	8–28 (9)	9–29 (8)	10–30 (7)	11–15 (6)	12–16 (5)	13–17 (4)	20–18 (3)	14–19 (10)
11	1–20 (11)	2–21 (12)	3–22 (1)	4–23 (2)	5–24 (3)	6–25 (4)	7–26 (5)	8–27 (8)	9–28 (7)	10–29 (6)	11–30 (5)	12–15 (4)	13–16 (3)	19–17 (2)	14–18 (11)
12	1–19 (12)	2–20 (1)	3–21 (2)	4–22 (3)	5–23 (4)	6–24 (5)	7–25 (6)	8–26 (7)	9–27 (6)	10–28 (5)	11–29 (4)	12–30 (3)	13–15 (2)	18–16 (1)	14–17 (12)

This modification also supplies us with a method for handling an awkward number of tables with half a table involved. Consider the 15-table 12-round movement shown above, but where you are catering for 14½ tables. The N-S pair at table 15 is deemed the sit-out. Scoring is somewhat complicated but at least the pairs only sit out for 2 boards.

Pivot modification

The Displacement, Rover and 1½ table appendix Mitchells all have their uses in handling half tables. The N-S and E-W fields are always separate and the boards are played an equal number of times and so the tops on all boards are equal. The number of rounds can be reduced if needed.

Unfortunately the number of rounds that can be played is less than the number of tables in most cases and a need to have some pairs repeat against others in some other cases.

The standard odd table Mitchell, share-and-relay Mitchell and the skip Mitchell all have the disadvantage of

a board-set out of play on each round and so cannot be shortened. Choosing the right movement for the number of tables and the length of the event is generally critical at the start of the event. The pivot Mitchell does address these issues, but is suitable only for a one-winner event. The N-S and E-W pairs are not kept separate.

Given this limitation and using switching to balance the comparisons, this does not prove to be a problem. This movement is based on the Ewing modification and is called the pivot Mitchell by Olof Hanner.

Essentially the highest N-S pair is removed from the last table in the standard Ewing modification and the E-W pair that normally plays at that table sits out for one round.

A normal switch-Mitchell is set up for the number of whole tables in play. The N-S pair numbers are the same as table numbers and the E-W pairs add the number of tables to their table number to find their pair number. The highest numbered pair ($2N + 1$) sits out of play for the first round. The number of board-sets equals the number of tables, one set on each table.

The movement proceeds as a normal Mitchell, N-S stationary, E-W move up one table and the boards down one table. The complication only occurs at the highest table.

- E-W move to the sit out for one round.
- The sit-out pair moves to N-S at the highest table.
- N-S move to E-W at table 1.

So the E-W pairs move normally to the last table playing one round there, then sit out for one round and then return to the highest table as N-S for one round, then proceed as a normal E-W pair going to table 1 as E-W. The maximum number of rounds that can be played is the same as the number of board-sets and tables.

With an even number of tables (excluding the odd pair) the normal skip Mitchell is used, skipping after half the rounds. At the highest tables the E-W sit out and the N-S pairs move two steps: the E-W pair at the second highest table moves to the sit out, E-W to N-S at the highest table and the sit out goes directly to E-W at table 1. There is no revenge round.

Modifications in small movements with half table

When the number of tables is low and you still require a Mitchell movement even though you have a half table, there is one problem; the sit-out pairs have to sit out for a large number of boards. With 3½ tables a Howell is the only movement that should be used. With 4½, 5½ and 6½ tables, the appendix Howell is still the best, with the possible exception of 5½ tables where there are either 22 or 33 boards in play in the Howell. With these last three cases, playing standard Mitchells, the minimum number of boards that a pair sits out for is 4 (about half an hour). The same thing happens with rover Mitchells. However, if you do not have Howell guide-cards, this problem can be solved in the following way.

Set up a Mitchell for the base number of tables (i.e., 4, 5, or 6 tables), using the share-and-relay version for even tables. Use only an even number of boards per table (i.e., place 4 or 6 boards on each table). This basic movement will be run as a standard Switch-Mitchell (see page 154). The extra pair will play as a roving pair, but they start play after the first pair they are going to replace has played *half* the boards in play on the first round. From then on the roving pair replaces the N-S pair for half the boards on each round and the E-W pair for the other half of the boards. This way the sit-out pairs only sit out for 2 or 3 boards. Care must be taken with the scoring, as one pair does not sit out at all so that factoring of scores is required.

The following schedules show the tables that the roving pairs move to on each round (with the E-W pair they replace after half the boards shown in brackets). The E-W pairs are assigned a pair number equal to their original table number plus the number of tables in the movement.

In the movement the N-S pairs are assumed to remain stationary, the E-W pairs move up one table and the boards down one table.

4 Tables:
2, 3(6), 1(7), 4(5)*
5 Tables:
1, 3(7), 5(8), 2(9), 4(10)*
6 Tables:
1, 3(8), 5(9), 2(11), 4(12)*, 6(7)*
*These are switch rounds and the pairs are replaced in reverse order.

The roving pair in these movements will be assigned a pair number equal to $2N + 1$ where N = the number of tables in the base.

For 3½ tables, see extra-board movements, page 115.

Beynon appendix modification

This modification came about when George W. Beynon (1864–1965) and his fellow directors identified the need. This was first published in his book of 1955:

> "The Appendix Mitchell is a fine example of 'necessity is the mother of invention'. With large sections of 14 to 18 tables and 26 boards established as the standard length of games, pairs missed playing certain sets of boards. Inasmuch as all the pairs should have an equal chance to play all boards a solution became imperative and we came up with the Appendix Mitchell".

This movement has two advantages:

- a predetermined number of boards can be played irrespective of the number of tables (within a given number);
- a large number of tables can be accommodated with a relatively low number of boards.

The disadvantages are that many pairs are moving, and the large amount of board-sharing tends to slow the movement down.

An appendix table is a table added on to a basic movement without changing the movement of the boards. This table shares boards with one of the tables in the movement (the appendage table).

The Beynon Appendix Mitchell will only work where the basic Mitchell movement has a prime number of tables (N). From one table (use rover table in preference, see page 108) to N tables (use twin Mitchell in preference, see page 117) can be appended on to the base.

First of all, a standard Mitchell of 5, 7, 11, 13, 17 or 19 tables is set up. More than this is impractical because of the large number of boards, even if they are twinned to alleviate sharing. The first appendix table is positioned next to table 1 and so becomes appended to it. The second is appended to table 2 and so on. These appendix tables are numbered $N + 1$, $N + 2$ etc. It is not critical for the appendix tables to be appended in order from table 1, but for ease of instructions to the pairs on their movement, that setting is recommended. Both the appended and the appendage tables contain one stationary pair, but one must be a N-S pair and the other an E-W pair.

For the sake of describing the rest of the movement we will assume that the N-S pairs in the base movement, and the E-W pairs at the corresponding appended tables, are stationary.

Apart from these stationary pairs, the rest of the pairs and the boards are moving. The boards as usual move down one table, but remember they stay within the base movement, never actually 'moving to' the

appendix tables, only going there when 'being shared'. The N-S (or E-W) pairs move up one table, and the E-W (or N-S) pairs move either up two tables or down two tables. The latter choice is better as that group can be given the responsibility of moving the boards ("E-W move down two tables, dropping the boards off one table down"). The N-S pairs move up one table at each change, proceed through the appendix tables then switch over to the next available N-S place in the base movement. The E-W pairs on the other hand move down two tables, staying within the base movement throughout the event.

With a half table, the odd pair is placed at the last of the appendix tables as a moving pair, but remember to keep the N-S pair stationary at the appendage table. A scrambled appendix-Mitchell is possible, switching no more than the last 1/8th of the rounds. The players can change directions at the switch round, but continue to move in the same direction as before the switch.

With a low number of tables (for example a five-table base) the half table again becomes a problem, with that pair required to sit out for at least five boards. Even the 7-table-base appendix movement with a half table is not attractive, as they would normally have to sit out for 4 boards. The switching rover can again be used by having the rover pair replace the N-S pair for half the boards and the E-W pair for the other half of the boards. The rover pair moves only within the base movement, sits out for the first half round and replaces the N-S pair at table 1 for the second half of the first round. The roving pair then moves up two tables (assuming the other pairs are moving down two tables; if not it will be vice-versa), first replacing N-S, then E-W. For example, in a movement with a 7-table base, the roving pair moves 1, 3, 5, 7, 2, 4, 6. This is usually only used in conjunction with the switch-Mitchell.

Rover-table modification

This is a modification and extension of the principles involved in the rover (replacement) Mitchell. This movement works only when there is a prime number of tables (N) in the base. 1 to $N - 1$ tables can be appended to this base, but for practical considerations it is best not to have more than ⅓ N appendix tables. The standard base-movement is set up and conducted as normal: N-S stationary, E-W move up one table at the change, and the boards move down one table. At the appendix table the N-S pair remains stationary. The E-W pair from the appendix table proceeds as a roving pair, replacing an E-W pair at one of the tables in the base. The replaced pair then moves to the appendix table and plays there as the E-W pair. The appendix table always shares boards with the table at which the roving pair is sitting. The roving pair then moves up 2 tables (or down 2 tables) at each change. The replaced pairs continue moving normally and move from the appendix table as though they had played at the table from where they were replaced.

Several appendix tables can be added by his method with each roving pair starting at a different table. A half-table rover can also be accommodated, with it starting at yet another table in the base, and proceeding as a normal rover.

This movement can further be improved with the base movement moving with an American whist movement. (N-S stationary and E-W move up two tables, dropping the boards off one table up). This way the roving pairs will move down one table at each change. Another alternative (slightly better, as there is less congestion) is the inverted whist: E-W up one table and the boards up two tables and the rovers down one table.

Bowman-Ewing modification

This modification is named after Mr. Bowman and Mr. W. Ewing of the U.K and was first reported by E.E. Blandon in *Bridge Magazine*, February 1973. It is an extension of the 1½ table appendix Mitchell to a Mitchell with two appendix tables rather than 1. It has the advantage over previous appendix modification of not needing a prime number of tables in the base.

After setting up the base movement, with two tables less than the total number, the first appendix table (with a table number two more than the highest table in the base), is placed so as to share boards with the

highest-numbered table in the base. The other appendix table (with a number one more than the highest table in the base) shares boards with the second highest table in the base for the first round and then sharing two tables further down at each change. For example on a 5-table base, table 7 shares with 5 throughout and table 6 shares first with 4 then 2, 5, 3, 1. On a 6-table base, table 7 shares with 5, 3, 1, 5, 3, 1 and table 8 with 6 throughout. Note the repeated share tables with an even base, and the three-way share in one round with the odd base.

As with the 1½-table Appendix Mitchell the pair movement for E-W is a pure Mitchell, moving up one table at each change; 5, 6, 7, 1, 2 etc. for a 5-table-base movement. The only problem with this movement is the positioning of the second-highest table to minimise the problem of the roving share. The boards move down one table at a time, but always remain in the base.

Relay-rover table modification

This movement follows the general principles of the rover-table movement, by having the basic movement with one or more appendix tables sharing with different tables on different rounds. There are unfortunately several restrictions on this movement.

The number of tables in the base must be divisible by 4. The base movement is set up as a share-and-relay Switch-Mitchell. With N tables the number in the base, up to ½ N rover tables can be accommodated. It is normal for the first rover table to share with table $N - 1$ and then move down two tables at a time until sharing with table ½ $N + 1$. After this they share with table ½ N and they again move down two tables at a time so that they traverse the same circuit again.

For example:

4 tables:	3, 2, 3, 2
8 tables:	7, 5, 4, 2, 7, 5, 4, 2
12 tables:	11, 9, 7, 6, 4, 2, 11, 9, 7, 6, 4, 2

Second and subsequent rovers share on the first round with any one of the other tables in the circuit and then proceed twice around the circuit in the same order as given above. On the second encounter of the tables the rover table pairs exchange roles, thus in the same instance performing a switch.

Web rover-table modification

The rover-table principles can be applied to the web Mitchell. This caters for difficult numbers of tables where you want a reduced number of rounds, such as a 7-table Mitchell where you want to play 3 or 4 rounds. Continuing with this example a 6-table web Mitchell is set up to play the required number of rounds and board-sets. The E-W pair from table 7 visits a different table on each round replacing the E-W pair at that table for the round.

The replaced E-W pair plays against N-S 7 and table 7 plays the same boards as the roving E-W pair. For 3 rounds the rover E-W visits 1, 6 and 4. For 4 rounds the rover E-W visits 1, 3, 6, 4.

There are some numbers of tables where this method does not work but the list on the next page covers the possibilities and is in round order. Where the number of rounds is an even number the usual skip by all E-W players occurs after the half way round but the roving E-W pair ignores that issue.

Tables	Web	Rounds	Highest-table roving E-W pair visits
7	6	3	1, 6, 4
7	6	4	1, 3, 6, 4
9	8	4	1, 3, 8, 4
9	8	5	1, 3, 6, 8, 4
11	10	5	1, 3, 5, 2, 4
11	10	6	1, 3, 8, 2, 5, 9
13	12	6	1, 3, 6, 10, 2, 4
11	10	7	1, 3, 5, 8, 10, 4, 6
13	12	7	1, 3, 5, 2, 9, 12, 4
15	14	7	1, 3, 5, 2, 8, 10, 4
11	10	8	1, 6, 10, 5, 9, 2, 4, 7
13	12	8	1, 3, 5, 7, 10, 12, 2, 6
15	14	8	1, 3, 5, 7, 4, 11, 13, 6
17	16	8	1, 3, 5, 7, 2, 11, 13, 4
11	10	9	4, 8, 1, 3, 5, 7, 2, 10, 6
13	12	9	1, 3, 5, 7, 4, 10, 12, 2, 6
15	14	9	1, 3, 5, 8, 4, 9, 12, 14, 7
17	16	9	1, 3, 5, 2, 4, 11, 16, 12, 15
19	18	9	1, 3, 5, 2, 4, 10, 12, 14, 17
13	12	10	1, 4, 8, 5, 12, 10, 7, 6, 2, 9
15	14	10	1, 3, 5, 9, 2, 11, 4, 8, 13, 6
17	16	10	1, 3, 5, 2, 12, 13, 11, 6, 4, 7
19	18	10	1, 3, 5, 2, 4, 14, 12, 6, 18, 7
21	20	10	1, 3, 5, 2, 4, 12, 15, 13, 16, 19
13	12	11	1, 3, 8, 10, 12, 4, 9, 11, 6, 2, 7
15	14	11	1, 3, 5, 8, 13, 11, 4, 6, 12, 2, 7
17	16	11	1, 3, 5, 7, 4, 11, 16, 2, 6, 8, 15
19	18	11	1, 3, 5, 2, 4, 15, 13, 11, 6, 18, 7
21	20	11	1, 3, 5, 2, 4, 15, 12, 14, 6, 13, 7
15	14	12	1, 3, 5, 13, 11, 9, 2, 14, 6, 8, 10, 12
17	16	12	1, 3, 5, 2, 8, 13, 14, 4, 6, 16, 9, 7
19	18	12	1, 3, 5, 2, 4, 13, 14, 12, 6, 15, 17, 10
21	20	12	1, 3, 5, 2, 4, 15, 16, 14, 6, 17, 19, 10
15	14	13	1, 3, 6, 9, 11, 2, 5, 10, 13, 4, 8, 7, 12
17	16	13	1, 3, 5, 2, 13, 11, 4, 14, 12, 9, 6, 16, 7
19	18	13	1, 3, 5, 2, 8, 13, 16, 14, 17, 7, 10, 4, 9
21	20	13	1, 3, 5, 2, 4, 15, 17, 14, 16, 13, 6, 20, 7
17	16	14	1, 3, 5, 7, 12, 14, 11, 2, 16, 9, 6, 8, 10, 4
19	18	14	1, 3, 5, 2, 8, 16, 12, 17, 14, 10, 6, 9, 11, 4
21	20	14	1, 3, 5, 2, 4, 17, 15, 13, 16, 14, 6, 9, 19, 10
17	16	15	1, 4, 2, 5, 13, 15, 12, 14, 16, 6, 9, 7, 10, 8, 3
19	18	15	1, 3, 5, 2, 16, 9, 6, 15, 17, 14, 7, 4, 18, 8, 10
21	20	15	1, 3, 5, 2, 4, 9, 16, 13, 17, 14, 7, 18, 6, 11, 10
19	18	16	1, 3, 5, 7, 12, 14, 16, 18, 8, 4, 9, 17, 2, 10, 15, 11
21	20	16	1, 3, 5, 2, 4, 16, 18, 13, 6, 17, 15, 8, 11, 9, 20, 10
19	18	17	1, 3, 6, 15, 17, 10, 16, 13, 8, 2, 18, 14, 11, 9, 12, 4, 7
21	20	17	1, 3, 5, 2, 8, 15, 17, 19, 16, 18, 6, 11, 7, 10, 20, 13, 9
21	20	18	1, 3, 5, 2, 16, 9, 15, 18, 8, 6, 19, 17, 20, 7, 13, 11, 14, 4
21	20	19	1, 3, 5, 2, 9, 19, 14, 16, 6, 20, 17, 11, 18, 8, 4, 7, 12, 10, 15

3-table appendix modification

This is based on the 2-table appendix modification with an additional table appended. Essentially an additional table is appended using similar principles to those used in the rover table modification. The last table is added with the N-S pair stationary and the E-W pair displaces a pair at the other tables on each round, the displaced pair playing against the N-S pair at the highest table. They play the same boards as those at the displaced table.

For example, 7 tables with 4 rounds based on the 4-table skip Mitchell with 2-table appendix modification. The E-W pair from table 7 displaces pairs at tables 1, 3, 6 and 4 on rounds 1 to 4.

round/table:	1	2	3	4	5	6	7
1	1–7 A	2–2 B	3–3 C	4–4 D	5–5 A	6–6 D	7–1 A
2	1–6 B	2–1 C	3–7 D	4–3 A	5–4 B	6–5 C	7–2 D
3	1–4 C	2–5 D	3–6 A	4–1 B	5–2 C	6–7 B	7–3 B
4	1–3 D	2–4 A	3–5 B	4–7 C	5–1 D	6–2 A	7–6 C

In the following example there are 8 tables and 5 rounds. The E-W pair from table 8 displaces E-W from tables 1, 3, 5, 2, 4 in the same way as pairs are displaced in the rover table modification where the base has a prime number of tables.

round/table:	1	2	3	4	5	6	7	8
1	1–8 A	2–2 B	3–3 C	4–4 D	5–5 E	6–6 A	7–7 E	8–1 A
2	1–7 B	2–1 C	3–8 D	4–3 E	5–4 A	6–5 B	7–6 D	8–2 D
3	1–6 C	2–7 D	3–1 E	4–2 A	5–8 B	6–4 C	7–5 C	8–3 B
4	1–5 D	2–8 E	3–7 A	4–1 B	5–2 C	6–3 D	7–4 B	8–6 E
5	1–4 E	2–5 A	3–6 B	4–8 C	5–1 D	6–2 E	7–3 A	8–7 C

This movement can also be run with stationary boards, similar to the 2-table appendix. There are three-way shares (or sets) required on some rounds:

round/table:	1	2	3	4	5	6	7	8
1	1–8 A	6–6 A	8–1 A	2–2 B	3–3 C	4–4 D	5–5 E	7–7 E
2	5–4 A	1–7 B	6–5 B	2–1 C	3–8 D	7–6 D	8–2 D	4–3 E
3	4–2 A	5–8 B	8–3 B	1–6 C	6–4 C	7–5 C	2–7 D	3–1 E
4	3–7 A	4–1 B	7–4 B	5–2 C	1–5 D	6–3 D	2–8 E	8–6 E
5	2–5 A	7–3 A	3–6 B	4–8 C	8–7 C	5–1 D	1–4 E	6–2 E

The advantage of this movement over the rover-table modification is the larger number of tables to ‘rove’ and so there are not the same limitations found in the rover-relay table where the E-W pairs revisit some tables. See the 7-table example above where the rover pair visits table 6 on round 3, where 6 is outside the base 4 tables.

The usefulness of this movement is really limited to an odd number of tables. With an even number of tables the web Mitchell can be used with the required number of rounds. For example, with 8 tables conduct a web Mitchell with 5 rounds.

Starting with 2-table appendix movements, add one table and the roving pair (EW at the highest table) replacement summary is shown here.

Tables	Rounds	Rover replaces EW at
6	3	1, 5, 4
7	4	1, 3, 6, 4
8	5	1, 3, 5, 2, 4
9	6	1, 8, 4, 7, 5, 3
10	7	1, 3, 5, 7, 2, 4, 6
11	8	1, 4, 10, 9, 7, 5, 2, 8
12	9	1, 4, 6, 11, 9, 7, 5, 2, 8
13	10	1, 3, 5, 10, 8, 11, 4, 2, 7, 9
14	11	1, 3, 5, 7, 9, 11, 2, 4, 6, 8, 10
15	12	1, 3, 5, 12, 10, 13, 6, 4, 2, 8, 11, 9
16	13	1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12
17	14	1, 3, 6, 9, 15, 2, 11, 7, 16, 4, 14, 5, 13, 10
18	15	1, 3, 5, 9, 13, 10, 14, 6, 15, 2, 4, 7, 12, 8, 11
19	16	1, 3, 5, 8, 18, 14, 10, 17, 7, 16, 11, 2, 6, 9, 12, 15
20	17	1, 3, 5, 7, 9, 11, 13, 15, 17, 2, 4, 6, 8, 10, 12, 14, 16
21	18	1, 3, 5, 9, 12, 20, 17, 14, 18, 8, 10, 4, 6, 19, 15, 13, 11, 7

5-table appendix modification

The principles of the 5-table appendix modification are the same as those for the 3-table appendix modification. Similar to the 3-table appendix, in practical terms, this is only useful for an odd number of tables.

Where you need 5 rounds short of the total in a Mitchell movement and you have an odd number of tables, start with a standard skip-Mitchell, append a 4-table web Mitchell and then apply a rover table. Consider 11 tables where you need to play only 6 rounds. Start with a 6-table skip-Mitchell. Append a 4-table web Mitchell that plays 6 rounds. Both these movements skip after round 3. Then applying the methods of the rover-table modification, append another table to the movement for a total of 11 tables. The roving E-W pair from the last table visits the tables in the order 1, 3, 7, 8, 2, 9 with the displaced E-W pair playing against N-S 11 on those rounds. Table 11 plays the same board-sets as their E-W roving pair. All N-S pairs remain stationary and the E-W pairs (other than 11) always move up one table apart from the skip after round 3. The resulting movement is shown here with the board-sets in brackets (note 4 copies of set 1 are required). Tables 1 and 7 can share boards as can 2 and 8.

	1	2	3	4	5	6	7	8	9	10	11
1	1–11(1)	2–2(2)	3–3(3)	4–4(4)	5–5(5)	6–6(6)	7–7(1)	8–8(2)	9–9(1)	10–10(6)	11–1(1)
2	1–10(2)	2–1(3)	3–11(4)	4–3(5)	5–4(6)	6–5(1)	7–6(2)	8–7(3)	9–8(6)	10–9(5)	11–2(4)
3	1–9(3)	2–10(4)	3–1(5)	4–2(6)	5–3(1)	6–4(2)	7–11(3)	8–6(4)	9–7(5)	10–8(4)	11–5(3)
4	1–7(4)	2–8(5)	3–9(6)	4–10(1)	5–1(2)	6–2(3)	7–3(4)	8–11(5)	9–5(4)	10–6(3)	11–4(5)
5	1–6(5)	2–11(6)	3–8(1)	4–9(2)	5–10(3)	6–1(4)	7–2(5)	8–3(6)	9–4(3)	10–5(2)	11–7(6)
6	1–5(6)	2–6(1)	3–7(2)	4–8(3)	5–9(4)	6–10(5)	7–1(6)	8–2(1)	9–11(2)	10–4(1)	11–3(2)

Where there are a high number of boards per set and using stationary (or near stationary) boards, sharing between neighboring tables is possible:

1–11 A	7–7 A	9–9 A	11–1 A	2–2 B	8–8 B	3–3 C	4–4 D	5–5 E	6–6 F	10–10F
6–5 A	1–10 B	7–6 B	2–1 C	8–7 C	3–11 D	11–2 D	4–3 E	10–9 E	5–4 F	9–8 F
5–3 A	6–4 B	1–9 C	7–11 C	11–5 C	2–10 D	8–6 D	10–8 D	3–1 E	9–7 E	4–2 F
4–10 A	5–1 B	6–2 C	10–6 C	1–7 D	7–3 D	9–5 D	2–8 E	8–11 E	11–4 E	3–9 F
3–8 A	4–9 B	10–5 B	5–10 C	9–4 C	6–1 D	1–6 E	7–2 E	2–11 F	8–3 F	11–7 F
2–6 A	8–2 A	10–4 A	3–7 B	9–11 B	11–3 B	4–8 C	5–9 D	6–10 E	1–5 F	7–1 F

The rover schedules for 11 through 23 tables are:

Tables	Rounds	Rover replaces EW at
11 (6+4+1)	6	1, 3, 7, 8, 2, 9
13 (8+4+1)	8	1, 3, 5, 7, 10, 12, 2, 6
15 (10+4+1)	10	1, 3, 5, 10, 8, 11, 4, 2, 7, 9
17 (12+4+1)	12	1, 3, 5, 7, 13, 15, 12, 4, 6, 16, 2, 10
19 (14+4+1)	14	1, 3, 5, 7, 9, 14, 6, 2, 8, 17, 4, 18, 10, 12
21 (16+4+1)	16	1, 3, 5, 2, 8, 10, 13, 19, 6, 18, 20, 7, 4, 12, 15, 9
23 (18+4+1)	18	1, 3, 5, 2, 8, 10, 12, 19, 22, 7, 4, 20, 9, 21, 6, 14, 11, 18

Coffin modification

This movement was first published by G.S. Coffin (U.S.A.) in 1971 when he called it the criss-cross Mitchell, with the 4-table version being modestly named *Coffin Mitchell*. This was in a spiral bound booklet titled *New Mitchells for everyone in 1971* and the first draft (print run of 25) of a supplement to George W. Beynon's *Bridge Director's Manual*. A short time later he was calling it a double-weave Mitchell. Both names imply a pattern of crossing lines in both directions. It is applicable to all Mitchells with an even number of tables, but is best for movements where the number of tables is divisible by 4.

When not divisible by 4 the movement has no advantages over the skip Mitchell and many disadvantages and as such will not be considered here (see *stationary-board Mitchells*, page 143).

The E-W pairs move in two directions and the boards move in two directions at each change. The movement is set up with one board-set on each table as in a standard Mitchell. The pairs are numbered the same as in any Mitchell. The move instructions are as follows:

- the N-S pairs remain stationary ;
- the odd-numbered E-W pairs move up one table the whole movement;
- the even-numbered E-W pairs move down one table the whole movement;
- the odd-board-sets (those starting at odd-numbered tables) move down one table and at the half-way stage (after ½ N rounds), they move up ½ N tables after which they move up one table for the rest of the event;
- the even-board-sets (those starting at even-numbered tables) move up one table and at the half-way stage (after ½ N rounds), they move up ½ N tables after which they move down one table for the rest of the event.

So long as the boards are moved by the E-W pairs the procedure is simple. The E-W pairs move the boards in the opposite direction to which they are moving resulting in the neighbouring tables exchanging boards (1 with 2, 3 with 4 etc). The halfway board-move is also simple enough, with the tables 1 and ½ N swapping boards, 2 and ½ N + 1 swapping boards, etc.

Parker modification

This was first published in the *The Bridge World,* August 1941, page 26, in an article called 'The awkward Mitchells' by Leslie E. Parker of Melbourne, Australia. The main objective of the article was to supply a movement for 15 to 18 tables with more than 15 sets of boards, at the time typically playing 21 boards in one sitting. These were later published with the name *Extra-board Mitchells*.

Any Mitchell can have extra sets of boards inserted in the movement by placing them on byestand tables at the correct positions.

The advantages of these are they lengthen the game without changing the basic movement and with an even number of tables they eliminate the share or the skip, and yet all E-W meet all N-S.

The disadvantage is that the E-W pairs meet some N-S pairs twice and in some cases three times. There are two different cases: odd-table and even-table movements.

Odd tables

The standard Mitchell is set up and an even number of sets of boards are placed on a relay at the end of the movement (or for that matter anywhere in the movement). For example, in a 5-table Mitchell (with 2 extra sets to make a 28-board movement) set 1 (boards 1-4) is placed on table 1, set 2 (5-8) on table 2, … set 5 (17-20) on table 5, and set 6 (21-24) and 7 (25-28) on the relay. The move is exactly the same as a standard Mitchell, N-S stationary, E-W up 1 table and the boards down 1 table, moving through the relay after table 1. In this form the movement is also called a Blackpool.

Unfortunately there is always one special move round. With 2 sets on the relay, the E-W pairs on the ultimate round find the table to move to by subtracting their pair number from $N + 1$(where N = number of tables in the movement). The boards still move normally.

With 4 sets on the relay, they subtract their pair number from N to find their seats on the penultimate round and then move normally (up 1) into the last round. With 6 sets, subtract their pair number from $N - 1$ for the table on the ante-penultimate round, and then return to normal, moving up 1.

When the E-W pairs subtract their pair number from N, for one pair (highest-numbered pair) zero is the answer; they must subtract their number from $2N$. In the $N - 1$ case, for two pairs (N and $N - 1$) the answer is less than 1. They then subtract from $2N - 1$. These pairs should be given their instructions separately before or after general announcement.

In the example given above for 5 tables, the E-W pairs will be instructed to subtract their pair number from 6 to find the table at which to sit on the last round.

This extra-board movement supplies us with a Mitchell for 3½ tables. The movement is set up with three standard tables plus four extra board-sets at the end so that there are 7 sets of boards in play. As there are seven pairs, a rover movement can be used and each pair can sit out for one set. Original N-S pairs are numbered 1, 2 and 3; the E-W pairs 4, 5 and 6; the rover pair number is 7. A switch is needed at the 6th round, but as this is the round of subtracting their pair number from 3 *(pair number minus 3* for E-W i.e., 6 minus pair number) to get their table numbers, the pairs can also be asked to change directions at the tables. The N-S pairs are stationary throughout (except for the switch round) and the E-W pairs move up 1. The rover pair sits out on the first round. They then go to table 1 replacing N-S pair 1 on round 2, and E-W pair 5 on round 3. Then they go to table 2 to replace N-S pair 2 on round 4 and E-W pair 4 on round 5; table 3 (after the switch has been called) to replace the N-S pair 6 on round 6 and E-W pair 3 on the last round.

Therefore the rover guide card is:

Round	1	2	3	4	5	6	7
Table	sit out	1	1	2	2	3	3
Displace		NS1	EW5	NS2	EW4	NS6	EW3

A similar movement is available for a 5½-table Mitchell with 11 rounds (i.e., 2-board sit-out).

Six relay sets are placed after the last table. E-W pairs move up 1 table and boards move down 1 table but moving through the relay. Pair 11 is a rover.

Rover guide card:

Round	1	2	3	4	5	6	7	8	9	10	11
Table	1	3	5	2	4	1	3	Sit	5	2	4
Displace	EW1	EW2	EW3	EW4	EW5	NS1	NS3	out	NS5	NS2	NS4

The unusual change occurs after round 8.

Even tables

Any movement for an even number of tables (including those divisible by 4) can be extended by placing an odd number of sets at the end relay and the same number of sets at a halfway point relay (or for that matter anywhere in the movement in opposite positions). For example, in a 6-table movement, one set is placed between tables 1 and 6 and one set between 3 and 4. The movement is normal (E-W up 1, boards down 1, moving through the relays) with *no skip.*

The special-move rounds are:

- with 2 one-set relays, the E-W pairs subtract their pair number from $N + 1$ to find their seats for the ultimate round;
- with 2 three-set relays, the E-W pairs subtract their numbers from $N - 1$ to find their seats for the ante-penultimate round, and then they move normally again, up 1 table.

It is worth pointing out that with 4 and 6 tables, other procedures with 2 three-set relays are possible. With 4 tables it is possible to omit the move on the ante-penultimate round and the E-W pairs subtract their numbers from $N - 1$ on the ultimate round. With 6 tables simply omit all special rounds; they are not necessary.

Remember the number of rounds is equal to the number of sets of boards; not the number of tables.

Extra-board modifications

Fattened movement

This idea originated from Al Oszy (U.S.A.) and is a very simple method of lengthening a game without disrupting the current movement.

The director takes the next available board not in play and then on any round, the board is played by all tables. The board is moved as soon as it is finished by each table, and is played immediately a table receives it. This of course can be done for more than one board.

The director can score such boards immediately the round has finished, as they have been played at all tables. This may require the board to be duplicated by the director.

Rover boards

This idea originated from H.O. Worger (Great Britain) and is used to add one extra board-set to any movement with a prime number of tables. The standard movement is set up and conducted in the normal way. An extra board-set then roves through the movement (like the rover pair) replacing the board-set currently in play at the table at which the board-set arrives. The board-set starts at table 1 and then goes up two tables at each change. For the last round the players play the board-set that they did not play on the round in which they played the rover board-set. They play them against the same opponents against whom they played the rover board-set.

For example with a 7-table Mitchell with 3 boards per table (1-3 on table 1, 4-6 on table 2, etc.) the rover board-set is boards 22-24 which are played on table 1 on round 1 instead of boards 1-3 and then move up two tables at each change (1, 3, 5, 7, 2, 4, 6). The replaced board-set return to the movement and move as though they have been played at the table at which they were replaced.

With E-W moving up one table and the boards down 1 table, the last (8th) round sees E-W pairs move

1 to table 1 playing boards 1–3
2 to table 3 playing boards 10–12
3 to table 5 playing boards 19–21
4 to table 7 playing boards 7–9
5 to table 2 playing boards 16–18
6 to table 4 playing boards 4–6
7 to table 6 playing boards 13–15

The players can be asked, of course, to note the boards they miss and the opponents against whom they are to play the missed boards.

Snow method

With a small number of tables to be run as a Mitchell (say 3, 4 or 5 tables) it is possible to have one board-set played by all tables on a round simply by sharing all the boards of the board-set among all tables. For example 5-board-sets can be shared among 4 tables quite easily. At the end of the round, the E-W pairs move up one table and all tables play the next board-set by sharing them while the director scores the previous set. The scores for each preceding round can be announced during the play.

The number of board-sets to be played is limited only by time and play can stop after any round. In the above example 15, 20, 25, 30 or 35 boards can be played in 3, 4, 5, 6 or 7 rounds. The use of the endless Howell movement can improve this concept.

Hesitation modification

E.E. Blandon was the first to publish this movement under this name in *Bridge Magazine*, December 1971.

This is not a true Mitchell in that the E-W pairs sit as N-S on one round. Using switches at other tables, however, it is unnoticed, for the movement is then effectively a switched Mitchell.

This movement is based on the share-and-relay Mitchell with one table removed (one of the sharing tables). The number of tables is always odd and the number of rounds (and the number of sets of boards) is equal to one more than the number of tables.

The movement is set up as a normal Mitchell, with one board-set on each table. The only exception is the insertion of a relay table (with one board-set on it) placed at ½ N (N = number of tables in the movement), i.e. with 3 tables at 3/2 (1.5) or between tables 1 and 2; with 5 tables between 2 and 3, etc. The movement is

along standard Mitchell principles, E-W up one table and boards down one table, not forgetting the relay table. The only complication is in the movement of the E-W pairs, for when they have played as E-W at the highest numbered table, they switch to N-S at the same table, to play there for one round before moving to E-W at table one for the next round.

This movement has very few real applications, but one use is in combination with the Block Mitchell principle to produce a movement useful for 12 to 24 tables. See *large pairs' movements,* page 349.

This should compared with the Ewing modification (see page 103) and must not be confused with it. This version has one more board-set than the number of tables whereas the Ewing has one less set.

Rover-table hesitation

A rover table may be added to the Hesitation Mitchell in a similar fashion to the standard Mitchell. This is not of much practical use as the standard even-table movements are at least as easy to organize. A hesitation Mitchell is set up as above and an extra table added. This extra table operates as a rover table. The E-W pair from the highest table replacing E-W at table 1 on round 1. The E-W pair replaced at table 1 plays at the highest table as E-W and that table and tables 1 share boards. The Rover E-W moves up 2 tables on each change, the replaced pair moving to the highest table as E-W. The replaced pairs, after playing at the highest table, move as if they had played in their original position.

The complication arises after the rover pair has played as E-W at the hesitation table. The rover pair then moves to ***N-S*** at the hesitation table, just as the E-W pairs do normally at that table. The same pair is replaced as in the previous round. They stay as E-W at the rover table. The N-S pair at the rover table move to E-W opposite the roving pair who are now in the N-S seats. The E-W pair they replace play as ***N-S*** at the Rover table.

All pairs move after this round as though this strange move had not occurred. This is clearly complicated and using guide-cards would be best.

Twin modification

This was first published by George W. Beynon in *The Bridge World*, January, 1938, page 24, in an article called *Twin Mitchells*. Even though this movement belongs in large pairs' movements it has some uses in smaller Mitchells. For small Mitchells, with an even number of tables where half the number is odd (6, 10 and 14 tables) this can be very useful. The movement is divided into two sections of standard Mitchells, with one section having tables 1 to ½ N and the other ½ N + 1 to N. Tables 1 and ½ N + 1 share boards, 2 and ½ N + 2 share boards, etc.

The two sections move independently, so that when the E-W pairs and the boards reach one end of the row, they go to the other end of the same row. Only half the boards are in play, for when the E-W pairs have played all the N-S pairs in that section (thereby also having played all the boards), they swap places with the E-W pairs in the other section and a new set-of-boards is introduced into play and the same procedure as before is followed. There are two advantages with this movement:

- no relay tables (to misplace) and no skip;
- half the boards can be scored after half time.

The disadvantage is there are shares throughout the event, but this can be overcome by duplicated boards. Extra boards can be added by using standard extra board methods.

Where half the number of tables is even, the same procedure can be followed by placing the share and relays of the two sections opposite each other. Better still, use the modified multiple-teams movement that follows.

Stagger modification

This modification is based on the Stagger movement used in teams' events. The roots of this movement are found in the Clay universal movement.

Two rows of tables are set up beside each other so that board sharing is possible. For example, with 8 and 10 tables the arrangement is:

1	2	3	4
5	6	7	8

1	2	3	4	5
6	7	8	9	10

The pairs move as in a whist movement, boards up 1 table, E-W up 2 tables (or down 1 and 2 respectively) and because there is sharing, the boards do not need to stay on the one side of the movement. The E-W pairs must be careful when they reach the end of a row as they must swap rows *and go to the other end*.

When the boards have been played by all tables (i.e., after ½ *N* rounds), they are removed and a new set introduced; at the same time the E-W pairs move up 1 table. The movement then continues as before while the director may score the first half. The advantages of this movement are exactly the same as those in the standard twin Mitchell, with the added advantage that it works for all even numbers of tables.

Trisect modification

This movement was first published by G.S. Coffin (U.S.A.) in 1971 when he called it the Trisect Mitchell. See *Coffin modification*, page 113, for more details. Subsequent to that, E.E. Blandon published it under the name of Block Mitchell in *Bridge Magazine*, April 1973, page 244, the article titled *Large pairs movements(2)*.

The advantages apply when a large number of tables are spread over several rooms, as they can be run as one movement with a minimum of fuss; if the number of tables is odd and *not* prime, a rover pair or rover tables can be accommodated quite easily.

The principle in the trisect is to divide the total number of tables into three groups. The Block Mitchell is dealt with in more detail in the section on large pairs movements where it discusses the possibility of diving into 4 or more groups.

Therefore 9 tables are considered as 3 × 3, 12 tables 3 × 4, and 15 tables 3 × 5.

As conducting a Mitchell (including rovers) for 3, 4 or 5 tables has been covered, the organisation of the groups of movements is as follows.

The first sub-Mitchell has N-S pairs N1 playing against E-W pairs E1 with boards B1 over rounds Rl. Similarly the second group has N2, E2, B2 over R2, and so on. So for the movement with three blocks of tables the schedule is as follows.

N-S remain stationary.

Rounds	N1	N2	N3
R1	E1/B1	E2/B2	E3/B3
R2	E3/B2	E1/B3	E2/B1
R3	E2/B3	E3/B1	E1/B2

Rover tables are possible, so in a 9-table movement use the following guide to decide which tables share boards.

rover 1 – 1, 4, 7, 5, 8, 2, 9, 3, 6
rover 2 – 4, 7, 1, 8, 2, 5, 3, 6, 9
rover 3 – 7, 1, 4, 2, 5, 8, 6, 9, 3

A rover pair can use the schedule for one of these, sitting out on the first round, and entering as E-W at the second table in the list.

Using the same principle it is possible to divide movements into different sizes and numbers of groups. For example 12 tables can be made into 4 groups of 3 tables.

The expanded trisect for 9 tables is:

	1	2	3	4	5	6	7	8	9
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (8)	9–9 (9)
2	1–3 (2)	2–1 (3)	3–2 (1)	4–6 (5)	5–4 (6)	6–5 (4)	7–9 (8)	8–7 (9)	9–8 (7)
3	1–2 (3)	2–3 (1)	3–1 (2)	4–5 (6)	5–6 (4)	6–4 (5)	7–8 (9)	8–9 (7)	9–7 (8)
4	1–7 (4)	2–8 (5)	3–9 (6)	4–1 (7)	5–2 (8)	6–3 (9)	7–4 (1)	8–5 (2)	9–6 (3)
5	1–9 (5)	2–7 (6)	3–8 (4)	4–3 (8)	5–1 (9)	6–2 (7)	7–6 (2)	8–4 (3)	9–5 (1)
6	1–8 (6)	2–9 (4)	3–7 (5)	4–2 (9)	5–3 (7)	6–1 (8)	7–5 (3)	8–6 (1)	9–4 (2)
7	1–4 (7)	2–5 (8)	3–6 (9)	4–7 (1)	5–8 (2)	6–9 (3)	7–1 (4)	8–2 (5)	9–3 (6)
8	1–6 (8)	2–4 (9)	3–5 (7)	4–9 (2)	5–7 (3)	6–8 (1)	7–3 (5)	8–1 (6)	9–2 (4)
9	1–5 (9)	2–6 (7)	3–4 (8)	4–8 (3)	5–9 (1)	6–7 (2)	7–2 (6)	8–3 (4)	9–1 (5)

The trisect for 12 tables (4 lots of 3) is shown here, but see *one round short*, page 125, for the rover.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (8)	9–9 (9)	10–10 (10)	11–11 (11)	12–12 (12)
2	1–3 (2)	2–1 (3)	3–2 (1)	4–6 (5)	5–4 (6)	6–5 (4)	7–9 (8)	8–7 (9)	9–8 (7)	10–12 (11)	11–10 (12)	12–11 (10)
3	1–2 (3)	2–3 (1)	3–1 (2)	4–5 (6)	5–6 (4)	6–4 (5)	7–8 (9)	8–9 (7)	9–7 (8)	10–11 (12)	11–12 (10)	12–10 (11)
4	1–4 (7)	2–5 (8)	3–6 (9)	4–1 (10)	5–2 (11)	6–3 (12)	7–10 (1)	8–11 (2)	9–12 (3)	10–7 (4)	11–8 (5)	12–9 (6)
5	1–6 (8)	2–4 (9)	3–5 (7)	4–3 (11)	5–1 (12)	6–2 (10)	7–12 (2)	8–10 (3)	9–11 (1)	10–9 (5)	11–7 (6)	12–8 (4)
6	1–5 (9)	2–6 (7)	3–4 (8)	4–2 (12)	5–3 (10)	6–1 (11)	7–11 (3)	8–12 (1)	9–10 (2)	10–8 (6)	11–9 (4)	12–7 (5)
7	1–7 (10)	2–8 (11)	3–9 (12)	4–10 (7)	5–11 (8)	6–12 (9)	7–1 (4)	8–2 (5)	9–3 (6)	10–4 (1)	11–5 (2)	12–6 (3)
8	1–9 (11)	2–7 (12)	3–8 (10)	4–12 (8)	5–10 (9)	6–11 (7)	7–3 (5)	8–1 (6)	9–2 (4)	10–6 (2)	11–4 (3)	12–5 (1)
9	1–8 (12)	2–9 (10)	3–7 (11)	4–11 (9)	5–12 (7)	6–10 (8)	7–2 (6)	8–3 (4)	9–1 (5)	10–5 (3)	11–6 (1)	12–4 (2)
10	1–10 (4)	2–11 (5)	3–12 (6)	4–7 (1)	5–8 (2)	6–9 (3)	7–4 (10)	8–5 (11)	9–6 (12)	10–1 (7)	11–2 (8)	12–3 (9)
11	1–12 (5)	2–10 (6)	3–11 (4)	4–9 (2)	5–7 (3)	6–8 (1)	7–6 (11)	8–4 (12)	9–5 (10)	10–3 (8)	11–1 (9)	12–2 (7)
12	1–11 (6)	2–12 (4)	3–10 (5)	4–8 (3)	5–9 (1)	6–7 (2)	7–5 (12)	8–6 (10)	9–4 (11)	10–2 (9)	11–3 (7)	12–1 (8)

The trisect for 15 tables (5 lots of 3) is shown here, but see *one round short*, page 125, for the rover.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (8)	9–9 (9)	10–10 (10)	11–11 (11)	12–12 (12)	13–13 (13)	14–14 (14)	15–15 (15)
2	1–3 (2)	2–1 (3)	3–2 (1)	4–6 (5)	5–4 (6)	6–5 (4)	7–9 (8)	8–7 (9)	9–8 (7)	10–12 (11)	11–10 (12)	12–11 (10)	13–15 (14)	14–13 (15)	15–14 (13)
3	1–2 (3)	2–3 (1)	3–1 (2)	4–5 (6)	5–6 (4)	6–4 (5)	7–8 (9)	8–9 (7)	9–7 (8)	10–11 (12)	11–12 (10)	12–10 (11)	13–14 (15)	14–15 (13)	15–13 (14)
4	1–13 (4)	2–14 (5)	3–15 (6)	4–1 (7)	5–2 (8)	6–3 (9)	7–4 (10)	8–5 (11)	9–6 (12)	10–7 (13)	11–8 (14)	12–9 (15)	13–10 (1)	14–11 (2)	15–12 (3)
5	1–15 (5)	2–13 (6)	3–14 (4)	4–3 (8)	5–1 (9)	6–2 (7)	7–6 (11)	8–4 (12)	9–5 (10)	10–9 (14)	11–7 (15)	12–8 (13)	13–12 (2)	14–10 (3)	15–11 (1)
6	1–14 (6)	2–15 (4)	3–13 (5)	4–2 (9)	5–3 (7)	6–1 (8)	7–5 (12)	8–6 (10)	9–4 (11)	10–8 (15)	11–9 (13)	12–7 (14)	13–11 (3)	14–12 (1)	15–10 (2)
7	1–10 (7)	2–11 (8)	3–12 (9)	4–13 (10)	5–14 (11)	6–15 (12)	7–1 (13)	8–2 (14)	9–3 (15)	10–4 (1)	11–5 (2)	12–6 (3)	13–7 (4)	14–8 (5)	15–9 (6)
8	1–12 (8)	2–10 (9)	3–11 (7)	4–15 (11)	5–13 (12)	6–14 (10)	7–3 (14)	8–1 (15)	9–2 (13)	10–6 (2)	11–4 (3)	12–5 (1)	13–9 (5)	14–7 (6)	15–8 (4)
9	1–11 (9)	2–12 (7)	3–10 (8)	4–14 (12)	5–15 (10)	6–13 (11)	7–2 (15)	8–3 (13)	9–1 (14)	10–5 (3)	11–6 (1)	12–4 (2)	13–8 (6)	14–9 (4)	15–7 (5)
10	1–7 (10)	2–8 (11)	3–9 (12)	4–10 (13)	5–11 (14)	6–12 (15)	7–13 (1)	8–14 (2)	9–15 (3)	10–1 (4)	11–2 (5)	12–3 (6)	13–4 (7)	14–5 (8)	15–6 (9)
11	1–9 (11)	2–7 (12)	3–8 (10)	4–12 (14)	5–10 (15)	6–11 (13)	7–15 (2)	8–13 (3)	9–14 (1)	10–3 (5)	11–1 (6)	12–2 (4)	13–6 (8)	14–4 (9)	15–5 (7)
12	1–8 (12)	2–9 (10)	3–7 (11)	4–11 (15)	5–12 (13)	6–10 (14)	7–14 (3)	8–15 (1)	9–13 (2)	10–2 (6)	11–3 (4)	12–1 (5)	13–5 (9)	14–6 (7)	15–4 (8)
13	1–4 (13)	2–5 (14)	3–6 (15)	4–7 (1)	5–8 (2)	6–9 (3)	7–10 (4)	8–11 (5)	9–12 (6)	10–13 (7)	11–14 (8)	12–15 (9)	13–1 (10)	14–2 (11)	15–3 (12)
14	1–6 (14)	2–4 (15)	3–5 (13)	4–9 (2)	5–7 (3)	6–8 (1)	7–12 (5)	8–10 (6)	9–11 (4)	10–15 (8)	11–13 (9)	12–14 (7)	13–3 (11)	14–1 (12)	15–2 (10)
15	1–5 (15)	2–6 (13)	3–4 (14)	4–8 (3)	5–9 (1)	6–7 (2)	7–11 (6)	8–12 (4)	9–10 (5)	10–14 (9)	11–15 (7)	12–13 (8)	13–2 (12)	14–3 (10)	15–1 (11)

Snow modification

The *Snow system* can be applied to the Mitchell movement as easily as any other type of movement. This is very similar to the Howell barometer events where the Safford cyclic movement of players is used. Olof Hanner calls this a *ring-dance Mitchell* because the N-S and E-W pairs cycle about each other in a ring when the tables are organized in a circle.

This is used where you need to divide the field into two or more groups while using barometer scoring after each round. Compare this with the *combined Howell-Mitchell* and similar large pairs' movements where three or more groups of players are considered.

The *stationary-board Mitchells* and the associated player-movement schedules can be used in this type of event, but a simpler method is available. The pairs are divided into two groups, N-S and E-W. All pairs move after each round, N-S up one table and E-W down one table or vice versa, while a new set-of-boards is introduced to all tables. The number of rounds is equal to the number of tables.

This works for all numbers of tables, with a small modification with an even number of tables. After

playing the half-way round there is a pause for one of the groups. This is similar to the skip Mitchell where you are avoiding playing pairs twice. The pause move involves one group remaining seated (say, the N-S pairs) and the E-W pairs move as usual.

Where the event is to be scored as one field then a switched version of the movement is needed. See *combined Howell-Mitchell*, page 356, as an example of where this is needed. The switching is on a table basis and is the same as applied to the *Boufos Mitchell* found under *switched Mitchells* (see page 154). For example, in a normal 13-table Mitchell movement where rounds 1 and 2 are switched, in this movement tables 1 and 2 are switched throughout the event.

Reduced rounds

To play less rounds than the number of tables make two pairs stationary, one in the N-S field and one in the E-W field. These pairs need to be positioned at particular tables in the movement and all other pairs move as usual except they skip (move past) the stationary pair moving on to the next table at that point. The number of rounds is reduced by one and the pause move is not needed with an even number of tables. Where you have an odd number of pairs one of the stationary pairs can be deemed a sit out making the management of the movement so much easier with a missing pair.

For an odd number of tables one pair is stationary at table one and the other (in the opposite direction) in the middle of the row. By 'middle' I mean the same number of tables before and after them. For example, with 11 tables and E-W stationary at table 1, then N-S is stationary at table 6. For an even number of tables one pair is stationary at each end of the row. For example with 12 tables and E-W stationary at table 1 then N-S is stationary at table 12.

Where switching is required, the same schedules for switch-Mitchells round numbers can be used to switch the tables, though it should be realized that with an incomplete movement the balance will not be the most desirable.

Expanded groups

Olof Hanner demonstrated it is possible to use this movement with different size groups. This could be used where one club is visiting another for competition but they have different numbers of pairs in their teams. For example, two groups of pairs, one with 10 pairs and one with 12, can play in the one movement over 10 rounds, the lesser number of pairs. The setup is shown with the same number of pairs from each team sitting N-S.

Example for 10+12 pairs, without stationary pairs, play over 10 rounds and pause after 5:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
NS	1	21	3	4	18	17	16	8	9	13	12
EW	22	2	20	19	5	6	7	15	14	10	11

Pair 1 follows 10, 11 follows 22. The remaining moving pairs follow the pair with next lower number. Pairs 1-10 remain seated (pause) after round 5.

Example for 11+13 pairs without stationary pairs, 11 (or 12) rounds:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
NS	1	23	3	4	20	18	17	16	8	9	13	12
EW	24	2	22	21	5	19	6	7	15	14	10	11

Pair 1 follows 11, 12 follows 24. All other pairs follow the pair with the next lower number. This movement may be played with a 12th round with no repeated meetings. The pairs in group 1-11 do return to their starting positions, but get new opponents.

Example for 11+15 pairs, with stationary pairs, 10 rounds:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
NS	1	25	3	23	5	11	18	7	16	15	14	21	12
EW	26	2	24	4	20	19	6	17	8	9	10	13	22

Pairs 11 and 26 are stationary. Pair 1 follows 10, 12 follows 25. The remaining moving pairs follow the pair with next lower number and there is no pause after round 5. Note if you change EW at table 5 to 22, the pairs remaining for the two last tables will be 12-13-20-21. To start with meetings 12-21 and 13-20 would have the effect that pairs 13 and 20 would meet again in round 8.

If you wish to expand with only one home pair it is better to imagine an extra guest pair, choose the variation with stationary pairs (*reduced rounds*) and make the non-existing guest pair vacant. If you extend with 2 pairs as described above and make one of these pairs vacant, your guests will get the sit-out rounds. A similar approach can be used where you want to expand the home group by 3 pairs.

Example 11+14 pairs, with stationary pair, 11 rounds:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
NS	1	24	3	22	5	6	19	17	8	15	14	13
EW	25	2	23	4	21	20	18	7	16	9	10	11

Pair 25 is stationary. Pair 12 starts as the sit-out. Pair 1 follows 11 and 12 follows 24. The remaining moving pairs follow the pair with next lower number.

Four groups - three sessions

This illustrates the way to handle multiple groups over several sessions or stanzas. We are considering the four groups as separate entities that do not need to meet the pairs within their group. A typical example is where you have 4 clubs meeting in a competition where they only wish to meet the other clubs and not their own club members.

The four groups are divided into two *ring-dance Mitchell* movements, one group sit as N-S and another as E-W in the two movements. These examples show a switch at some tables that attempts to improve the overall balance. Not switching has the advantage that all pairs in each group play the same cards and so results are comparable.

The examples shown here are for an odd number of pairs in each group. With an even number in each group the pause move is required. See *Howells and the Snow method*, page 174, for a similar solution for *four groups - 3 sessions* which works very simply for odd and even numbers in each group.

4x5 pairs, 10 tables, 3 stanzas. Stanza 1, round 1:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
NS	20	19	3	17	16	10	9	13	7	6
EW	1	2	18	4	5	11	12	8	14	15

Pair 1 follows pair 5, 6 follows 10, 11 follows 15 and 16 follows 20. The remaining pairs follow the pair with next lower number. To get the starting positions for a new stanza keep numbers 16-20 in the original positions and increase numbers 1-10 by 5 and reduce 11-15 by 10.

4x7 pairs, 14 tables, 3 stanzas. Stanza 1, round 1:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
NS	28	2	26	25	24	6	7	15	13	12	11	10	20	8
EW	1	27	3	4	5	23	22	14	16	17	18	19	9	21

Pair 1 follows pair 7, 8 follows 14, 15 follows 21 and 22 follows 28. The remaining pairs follow the pair with next lower number. To get the starting positions for a new stanza keep numbers 22-28 in the original positions, increase numbers 1-14 by 7 and reduce 15-21 by 14.

4x9 pairs, 18 tables, 3 stanzas. Stanza 1, round 1:

Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
NS	36	2	3	33	32	6	30	29	28	19	17	16	22	14	13	12	26	10
EW	1	35	34	4	5	31	7	8	9	18	20	21	15	23	24	25	11	27

Pair 1 follows pair 9, 10 follows 18, 19 follows 27 and 28 follows 36. The remaining pairs follow the pair with next lower number. To get the starting positions for a new stanza keep numbers 28-36 in the original positions, increase numbers 1-18 by 9 and reduce 19-27 by 18.

Appending-movements modification

Any Mitchell movement can be appended to another Mitchell movement so long as a few simple rules are followed. Actually more than one can be appended, but that is unlikely to have many practical applications.

The 1st or base movement can have any number of tables, but an odd number of rounds is the easiest to manage. When the appended movement requires a prime number of tables then the base also must have a prime number. The second movement must play the same number of board-sets and rounds as the first. See twin Mitchell, page 117, as an example of the principle.

Note it is possible to append a movement onto an even-table Double-Weave Mitchell, but the resulting movement of the E-W pairs is so difficult there is no practical use. For example, if you needed to have a four-round event with 10 tables, a 6-Table, 4-round web could be appended to a 4-table double-weave. For organization it would be better to run these as two separate movements using duplicated board-sets.

Another example is the Rover-table modification appended to standard Mitchell of the same size. Start with a 5-table Mitchell, then append a 6-table rover-table Mitchell. Number the appended tables 6 to 11 with E-W 11 replacing the E-W pairs at tables 6 to 10 in the normal predefined order 6, 8, 10, 7, 9.

Sharing boards between the two movements is an option, but also with two board-sets they can be distributed with the first set on tables 1 to 5 and the second set on tables 6 to 10. E-W pairs move up one table and the boards down 1 as in a normal Mitchell. After 5 rounds all the boards have been played. At this time the board-sets can be removed and new sets introduced and play continues. This supplies a movement for 11 tables playing either 5 or 10 rounds. All pairs play all boards, which is not the case if you play an 11-table Mitchell shortened to 10 rounds. You can tailor the number of boards in the session to suit your needs. For example playing 3-board-sets in the first half and then 2-board-sets in the second half gives a session of 25 boards, which is reasonably fair.

Using similar principles you can append a Bowman-Ewing movement to a standard Mitchell. The Bowman and web modifications have a lot of similarities. There are normally more tables than rounds and two sets of boards move in opposite directions. The web has two parts to the movement and the Bowman has a base movement with tables appended, a very similar concept. See later in the super-Bowman, page 353, where the parts of the movement are extended to 3 and where two lots of boards move in one direction and a third in the other direction.

The main objectives with these appendix movements are very similar to the original Beynon appendix Mitchell. The organizers of such events wanted to be able to handle a broad range of table numbers with a predefined number of board-sets. The Bowman and the web both have all the North-South pairs stationary.

Often today we have pre-duplication of boards so that the number of copies of each board is not a limitation and sharing board-sets is a thing of the past. This is where the web and super-Bowman show their strengths.

If we revisit the Bowman and change the tables that share boards slightly we find the Bowman is looking more like a web.

Using the appended movement principles you can take any Mitchell (with an even number of tables a skip Mitchell is required) and append a web Mitchell. The number of board-sets is dictated by the size of the Mitchell. Where the total number of tables is even, a normal web movement can be used instead. The appended web can be any size (but limited to even numbers of tables) so long as half the number is less than or equal to the number of tables in the Mitchell. More than one web may be appended to the Mitchell. When two tables are appended as a 2-table web (essentially a 2-table appendix) it is no different than using a Bowman, though the sequence of the boards is different at the top two tables.

This arrangement is a vast improvement over the Beynon appendix-Mitchell because of the large number of pairs that are typically required to move in the Beynon. Consider a 9-table event where you need to play 21 boards. In the Beynon you have 7 tables, 2 appended and most pairs moving. Using the appended web (essentially the 2-table appendix modification) you have all NS stationary and EW always move up one table. The director needs to take charge of the board changes, but from the players' point of view they move the same as in a standard Mitchell movement.

An even more extreme example is a standard 7-table Mitchell with a 10-table web appended. This allows for 4-board rounds of play, 28 boards in the session of play. Only 2 sets of boards are required as rows 1 and 2 can share easily. All the Mitchell pair movement attributes are retained, N-S and E-W pair numbers are the same as their table number, the N-S pairs are stationary and E-W pairs move up one table while the boards move down one (but stay in their row). If two sessions of play are required, the E-W pairs remember where they played last and start at the next table in the second session, and a second set of hands are introduced for a further 7 rounds of play.

Tables:	1	2	3	4	5	6	7
Boards:	1	2	3	4	5	6	7
Tables:	8	9	10	11	12		
Boards:	1	2	3	4	5	6	7
Tables:	13	14	15	16	17		
Boards:	4	3	2	1	7	6	5

For the standard length games, 13 rounds of two boards being one of the most popular, the organizers can cater for any number of tables with a few sets of pre-dealt boards.

While this type of movement may be useful for the organizers it is not completely fair. The problem is the E-W pairs do not meet all the N-S pairs and so the seeding of pairs is paramount.

Another possibility is to use a Parker modification in the appended Mitchell. The Parker modification

supplies a second Mitchell with the right number of rounds to match the base Mitchell. For example, start with a 7-table base Mitchell with 7 rounds. Append a 5-table Mitchell with an additional 2 board-sets (using the Parker modification) so that the two movements have 7 rounds each. Unfortunately this movement does not solve the last-round special move found in the Parker. In this example the top 6 E-W pairs (7 to 12) need to subtract their number from 13 to find their last table and the top 3 repeat play against N-S 1, 2 and 3. The full movement looks like this

	1	2	3	4	5	6	7	8	9	10	11	12
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (1)	9–9 (2)	10–10 (3)	11–11 (4)	12–12 (5)
2	1–12 (2)	2–1 (3)	3–2 (4)	4–3 (5)	5–4 (6)	6–5 (7)	7–6 (1)	8–7 (2)	9–8 (3)	10–9 (4)	11–10 (5)	12–11 (6)
3	1–11 (3)	2–12 (4)	3–1 (5)	4–2 (6)	5–3 (7)	6–4 (1)	7–5 (2)	8–6 (3)	9–7 (4)	10–8 (5)	11–9 (6)	12–10 (7)
4	1–10 (4)	2–11 (5)	3–12 (6)	4–1 (7)	5–2 (1)	6–3 (2)	7–4 (3)	8–5 (4)	9–6 (5)	10–7 (6)	11–8 (7)	12–9 (1)
5	1–9 (5)	2–10 (6)	3–11 (7)	4–12 (1)	5–1 (2)	6–2 (3)	7–3 (4)	8–4 (5)	9–5 (6)	10–6 (7)	11–7 (1)	12–8 (2)
6	1–8 (6)	2–9 (7)	3–10 (1)	4–11 (2)	5–12 (3)	6–1 (4)	7–2 (5)	8–3 (6)	9–4 (7)	10–5 (1)	11–6 (2)	12–7 (3)
7	1–12 (7)	2–11 (1)	3–10 (2)	4–9 (3)	5–8 (4)	6–7 (5)	7–1 (6)	8–2 (7)	9–3 (1)	10–4 (2)	11–5 (3)	12–6 (4)

Incomplete Mitchell movements

The main problem with stopping play before all rounds are complete in a Mitchell movement is that all pairs have not played all boards. For example, if you have 5 tables with 5 sets of boards and stop after 3 rounds then each pair has played a different set of three boards. Each board-set is played three times but by different pairs. This is generally unacceptable and downright unfair.

Rule: *the number of board-sets in play should equal the number of rounds.*

One consequence of this rule is the boards must be played more than once on some rounds when the number of sets is less than the number of tables. Today this is typically not a problem because we have computers to deal boards, allowing us to simply have many copies of each board in play.

It is important to realize these movements are not balanced and, because they are not complete, no amount of switching will help. Therefore only use these when you have two winners, one from the N-S field and one from the E-W field.

Another approach is to use the Snow modification. Where you are sharing boards at all tables on each round the movement can be stopped after any round. See the *Snow modification* to Mitchell movements (page 120) with *reduced rounds* for more ideas.

One round short

Where you need to play one round less than the number of tables and you want one winner the best solution is to use the Ewing modification, but for the normal Mitchell movement with two winners you have a variety of ways of handling this, depending on the number of tables. The movements presented here are titled with the ‘number of tables’ plus 1. The ‘number of tables’ represents the number of rounds and the total number of tables in play is one more. Therefore 3 tables plus 1 means there are 4 tables in the movement but only three rounds are played.

Odd tables plus 1.

For a prime number of tables in the base use the rover-table modification. This is tidy, reliable and easy to organize and requires an appendix table attached to the base with the prime number of tables and rounds.

Where you can use stationary boards on the prime base the rover-table movement can be modified so that the rover moves up one table. For example, where the 5-table base plus 1 has the boards stationary, the rover starts at table 1 and moves up one table at each change and table 6 plays the boards sets in ascending order. N-S pairs move up two tables within the base and the E-W pairs down 1 within the base. The starting positions of the pairs do not match the table numbers, but even so they are regular. N-S pair 1 is placed at table 1, the highest N-S pair at table 2 then descending from there in sequence. For the E-W pairs, seat pair 1 at table 1 on round 1 and the others are placed in sequences going up two tables at a time. Additional rovers are possible, each one starting at a different table and moving up one table at each change.

	1	2	3	4	5	6
1	1 6	5 4	4 2	3 5	2 3	6 1
2	3 4	2 6	1 5	5 3	4 1	6 2
3	5 2	4 5	3 6	2 1	1 4	6 3
4	2 5	1 3	5 1	4 6	3 2	6 4
5	4 3	3 1	2 4	1 2	5 6	6 5

Similar rover-table solutions exist for other than prime-table base movements but the *relay-rover table* modification is not suitable for this purpose as it delivers one winner.

For 9, 12, 15 and 21 tables (those divisible by 3) see *trisect modification* (page 118) for one solution. Also see the self-orthogonal Latin square solutions (page 151).

For even numbers of tables (plus one appendix) each case needs to be considered. Note these movements can also be used for an extra half-table roving pair.

3 tables plus 1.

This was called the ¾ relay by George Coffin in 1971. It is key to the trisect movements where the rover pair (E-W 4) is required to play at one of three tables with the displaced pair playing at table 4. This is also used in 12 and 15 tables plus 1 below. With only three tables to rove it is not possible to use the technique in other odd- table Mitchells plus 1. At least two tables on each round are sharing boards.

	1	2	3	4
1	1–4 (1)	2–2 (2)	3–3 (3)	4–1 (3)
2	1–2 (3)	2–4 (3)	3–1 (1)	4–3 (2)
3	1–1 (2)	2–3 (1)	3–4 (2)	4–2 (1)

4 tables plus 1.

This example has stationary boards at all tables apart from table 5 that shares with the E-W rover 5 who plays at tables 1, 2, 3, 4 on rounds 1 to 4, playing the same board-set numbers on those rounds. This 4-table base movement is derived from the self-orthogonal Latin Square first introduced in discussing Euler *The headmaster of mathematics.* The intriguing quality of the rover E-W playing at tables in ascending sequence and also playing the board-sets in ascending sequence is found in all movements based on self-

orthogonal Graeco-Latin squares. The appendix table also plays the boards in ascending sequence.

	1	2	3	4	5
1	1 5	3 4	4 2	2 3	5 1
2	4 3	2 5	1 4	3 1	5 2
3	2 4	4 1	3 5	1 2	5 3
4	3 2	1 3	2 1	4 5	5 4

Another solution is available, but the first appendix does not play the pairs in order but the boards are the same.

	1	2	3	4	5
1	1 5	2 3	3 4	4 2	5 1
2	3 2	4 5	1 3	2 1	5 4
3	4 3	3 1	2 5	1 4	5 2
4	2 4	1 2	4 1	3 5	5 3

The advantage is the additional rover tables that can be added if needed. The rovers start at tables 2 or 4 moving down one table at each change or table 3 moving up one table. All four appendix tables are shown here but 1, 2, 3, or 4 may be used.

1–5 A	2–6 B	3–7 C	4–8 D	5–1 A	6–3 B	7–4 C	8–2 D
3–6 A	4–5 B	1–8 C	2–7 D	5–4 B	6–2 A	7–1 D	8–3 C
4–7 A	3–8 B	2–5 C	1–6 D	5–2 C	6–4 D	7–3 A	8–1 B
2–8 A	1–7 B	4–6 C	3–5 D	5–3 D	6–1 C	7–2 B	8–4 A

By reordering the appendix tables on each round we have all boards stationary, all pairs moving in a regular pattern and with tables sharing boards (1 with 5 etc) a very neat movement that rivals the 8-table web Mitchell of 4 rounds:

1–5 A	2–6 B	3–7 C	4–8 D	5–1 A	6–3 B	7–4 C	8–2 D
3–6 A	4–5 B	1–8 C	2–7 D	6–2 A	5–4 B	8–3 C	7–1 D
4–7 A	3–8 B	2–5 C	1–6 D	7–3 A	8–1 B	5–2 C	6–4 D
2–8 A	1–7 B	4–6 C	3–5 D	8–4 A	7–2 B	6–1 C	5–3 D

Another solution is to use the double-weave Mitchell where the E-W rover visits tables 1, 3, 2, 4.

6 tables plus 1.

You should not be surprised to learn that this is an irregular movement. Euler's conjecture raises its head again. The boards are stationary at tables 1 to 6 with sets 1 to 6 respectively. Table 7 plays board-sets 1 to 6 on rounds 1 to 6. Switch 1 round for calibre (65).

	1	2	3	4	5	6	7
1	1 1	2 2	3 3	4 4	5 5	6 6	7 7
2	4 2	6 4	2 5	5 7	1 3	3 1	7 6
3	2 6	5 1	1 2	6 3	3 7	4 5	7 4
4	5 3	3 5	6 7	2 1	4 6	1 4	7 2
5	3 4	4 3	5 6	1 5	6 2	2 7	7 1
6	6 5	1 7	4 1	3 6	2 4	5 2	7 3

8 tables plus 1.

The standard Coffin modification criss-cross movement does not allow for rover tables. Even so Coffin did supply us with a special version of the criss-cross for 8 tables that will allow for the rovers. It allows not only one rover, but up to eight. He described this as an 8-channel criss-cross. The neat thing is the first roving pair (9) moves to tables 1, 2, 3, 4, 5, 6, 7, 8 in sequence and all N-S pairs are stationary. Switch 1 round for (71).

	1	2	3	4	5	6	7	8	9
1	1–9 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (8)	9–1 (1)
2	1–7 (5)	2–9 (6)	3–5 (7)	4–6 (8)	5–3 (1)	6–4 (2)	7–1 (3)	8–2 (4)	9–8 (6)
3	1–8 (7)	2–7 (8)	3–9 (5)	4–5 (6)	5–4 (3)	6–3 (4)	7–2 (1)	8–1 (2)	9–6 (5)
4	1–2 (3)	2–1 (4)	3–4 (1)	4–9 (2)	5–6 (7)	6–5 (8)	7–8 (5)	8–7 (6)	9–3 (2)
5	1–3 (8)	2–4 (7)	3–1 (6)	4–2 (5)	5–9 (4)	6–8 (3)	7–5 (2)	8–6 (1)	9–7 (4)
6	1–5 (4)	2–6 (3)	3–7 (2)	4–8 (1)	5–1 (8)	6–9 (7)	7–3 (6)	8–4 (5)	9–2 (7)
7	1–6 (2)	2–5 (1)	3–8 (4)	4–7 (3)	5–2 (6)	6–1 (5)	7–9 (8)	8–3 (7)	9–4 (8)
8	1–4 (6)	2–3 (5)	3–2 (8)	4–1 (7)	5–8 (2)	6–7 (1)	7–6 (4)	8–9 (3)	9–5 (3)

The other 7 possible rovers are:

8,7,6,5,4,3,2,1
4,3,2,1,8,7,6,5
5,6,7,8,1,2,3,4
2,1,4,3,6,5,8,7
3,4,1,2,7,8,5,6
6,5,8,7,2,1,4,3
7,8,5,6,3,4,1,2

Alternatively, use the self-orthogonal square with stationary boards. The rover table 9 is appended and the roving E-W pair visits the tables in ascending sequence 1 through 8. Table 9 can share boards with E-W 9 or play a second copy of all boards, playing sets 1 to 8 in rounds 1 to 8. Switch 1 round for (71).

	1	2	3	4	5	6	7	8	9
1	1 9	3 5	4 6	5 7	6 8	7 2	8 3	2 4	9 1
2	5 3	2 9	7 4	1 8	8 7	4 5	6 1	3 6	9 2
3	6 4	4 7	3 9	8 5	1 2	2 8	5 6	7 1	9 3
4	7 5	8 1	5 8	4 9	2 6	1 3	3 2	6 7	9 4
5	8 6	7 8	2 1	6 2	5 9	3 7	1 4	4 3	9 5
6	2 7	5 4	8 2	3 1	7 3	6 9	4 8	1 5	9 6
7	3 8	1 6	6 5	2 3	4 1	8 4	7 9	5 2	9 7
8	4 2	6 3	1 7	7 6	3 4	5 1	2 5	8 9	9 8

9 tables plus one.

This is based on the 9 table trisect movement. The roving E-W 10 moves to tables 1, 4, 7, 3, 6, 9, 2, 5, 8. Switch 1 round for (74).

	1	2	3	4	5	6	7	8	9	10
1	1–10 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (8)	9–9 (9)	10–1 (1)
2	1–3 (2)	2–1 (3)	3–2 (1)	4–10 (5)	5–4 (6)	6–5 (4)	7–9 (8)	8–7 (9)	9–8 (7)	10–6 (5)
3	1–2 (3)	2–3 (1)	3–1 (2)	4–5 (6)	5–6 (4)	6–4 (5)	7–10 (9)	8–9 (7)	9–7 (8)	10–8 (9)
4	1–7 (4)	2–8 (5)	3–10 (6)	4–1 (7)	5–2 (8)	6–3 (9)	7–4 (1)	8–5 (2)	9–6 (3)	10–9 (6)
5	1–9 (5)	2–7 (6)	3–8 (4)	4–3 (8)	5–1 (9)	6–10 (7)	7–6 (2)	8–4 (3)	9–5 (1)	10–2 (7)
6	1–8 (6)	2–9 (4)	3–7 (5)	4–2 (9)	5–3 (7)	6–1 (8)	7–5 (3)	8–6 (1)	9–10 (2)	10–4 (2)
7	1–4 (7)	2–10 (8)	3–6 (9)	4–7 (1)	5–8 (2)	6–9 (3)	7–1 (4)	8–2 (5)	9–3 (6)	10–5 (8)
8	1–6 (8)	2–4 (9)	3–5 (7)	4–9 (2)	5–10 (3)	6–8 (1)	7–3 (5)	8–1 (6)	9–2 (4)	10–7 (3)
9	1–5 (9)	2–6 (7)	3–4 (8)	4–8 (3)	5–9 (1)	6–7 (2)	7–2 (6)	8–10 (4)	9–1 (5)	10–3 (4)

Alternatively use the self-orthogonal square with stationary boards. The rover table 10 is appended and the roving E-W pair visits the tables in ascending sequence 1 through 9. Table 10 can share boards with E-W 10 or play a second set playing sets 1 to 9 in rounds 1 to 9. Switch 1 round for (74).

	1	2	3	4	5	6	7	8	9	10
1	1 10	3 6	4 7	5 8	2 9	7 3	8 4	9 5	6 2	10 1
2	6 3	2 10	5 6	7 9	9 4	4 5	1 8	3 1	8 7	10 2
3	7 4	6 5	3 10	2 7	8 6	9 8	5 2	1 9	4 1	10 3
4	8 5	9 7	7 2	4 10	3 8	5 1	6 9	2 3	1 6	10 4
5	9 2	4 9	6 8	8 3	5 10	1 7	2 1	7 6	3 4	10 5
6	3 7	5 4	8 9	1 5	7 1	6 10	9 3	4 2	2 8	10 6
7	4 8	8 1	2 5	9 6	1 2	3 9	7 10	6 4	5 3	10 7
8	5 9	1 3	9 1	3 2	6 7	2 4	4 6	8 10	7 5	10 8
9	2 6	7 8	1 4	6 1	4 3	8 2	3 5	5 7	9 10	10 9

10 tables plus 1.

The stationary boards *oddly-even Mitchell* Graeco-Latin square for 10 tables works very well. The rover table 11 is appended and the roving E-W pair visits the tables in ascending sequence 1 through 10. Table 11 can share boards with E-W 11 or play a second set and play sets 1 to 10 in rounds 1 to 10. Switch 1 round for (74).

	1	2	3	4	5	6	7	8	9	10	11
1	1 11	4 2	10 3	8 4	9 5	2 6	3 7	6 10	7 9	5 8	11 1
2	2 4	7 11	1 6	10 5	3 1	4 9	6 8	5 2	8 10	9 3	11 7
3	3 10	6 1	2 11	7 8	10 9	5 7	1 4	8 3	9 6	4 5	11 2
4	4 8	5 10	8 7	6 11	2 3	10 4	9 2	7 1	3 5	1 9	11 6
5	5 9	1 3	9 10	3 2	8 11	6 5	10 1	4 6	2 7	7 4	11 8
6	6 2	9 4	7 5	4 10	5 6	3 11	8 9	10 7	1 8	2 1	11 3
7	7 3	8 6	4 1	2 9	1 10	9 8	5 11	3 4	10 2	6 7	11 5
8	10 6	2 5	3 8	1 7	6 4	7 10	4 3	9 11	5 1	8 2	11 9
9	9 7	10 8	6 9	5 3	7 2	8 1	2 10	1 5	4 11	3 6	11 4
10	8 5	3 9	5 4	9 1	4 7	1 2	7 6	2 8	6 3	10 11	11 10

12 table plus 1

Similar to 9 tables this is also based on the trisect movement, but is more complicated. The roving E-W 13 starts at tables 13 then 3, 1, 10, 11, 12, 4, 5, 6, 7, 8 and 9. There is board sharing at some tables on each round, but not always with table 13. George Coffin called this a ¾ relay. Switch 1 round for (75).

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)	7–7 (7)	8–8 (8)	9–9 (9)	10–10 (10)	11–11 (11)	12–12 (12)	13–13 (3)
2	1–2 (3)	2–1 (3)	3–13 (1)	4–6 (5)	5–4 (6)	6–5 (4)	7–9 (8)	8–7 (9)	9–8 (7)	10–12 (11)	11–10 (12)	12–11 (10)	13–3 (2)
3	1–13 (2)	2–3 (1)	3–1 (2)	4–5 (6)	5–6 (4)	6–4 (5)	7–8 (9)	8–9 (7)	9–7 (8)	10–11 (12)	11–12 (10)	12–10 (11)	13–2 (1)
4	1–4 (7)	2–5 (8)	3–6 (9)	4–1 (10)	5–2 (11)	6–3 (12)	7–10 (1)	8–11 (2)	9–12 (3)	10–13 (4)	11–8 (5)	12–9 (6)	13–7 (6)
5	1–6 (8)	2–4 (9)	3–5 (7)	4–3 (11)	5–1 (12)	6–2 (10)	7–12 (2)	8–10 (3)	9–11 (1)	10–8 (6)	11–13 (6)	12–7 (4)	13–9 (5)
6	1–5 (9)	2–6 (7)	3–4 (8)	4–2 (12)	5–3 (10)	6–1 (11)	7–11 (3)	8–12 (1)	9–10 (2)	10–7 (5)	11–9 (4)	12–13 (5)	13–8 (4)
7	1–7 (10)	2–8 (11)	3–9 (12)	4–13 (7)	5–11 (8)	6–12 (9)	7–1 (4)	8–2 (5)	9–3 (6)	10–4 (1)	11–5 (2)	12–6 (3)	13–10 (9)
8	1–9 (11)	2–7 (12)	3–8 (10)	4–11 (9)	5–13 (9)	6–10 (7)	7–3 (5)	8–1 (6)	9–2 (4)	10–6 (2)	11–4 (3)	12–5 (1)	13–12 (8)
9	1–8 (12)	2–9 (10)	3–7 (11)	4–10 (8)	5–12 (7)	6–13 (8)	7–2 (6)	8–3 (4)	9–1 (5)	10–5 (3)	11–6 (1)	12–4 (2)	13–11 (7)
10	1–10 (4)	2–11 (5)	3–12 (6)	4–7 (1)	5–8 (2)	6–9 (3)	7–13 (10)	8–5 (11)	9–6 (12)	10–1 (7)	11–2 (8)	12–3 (9)	13–4 (12)
11	1–12 (5)	2–10 (6)	3–11 (4)	4–9 (2)	5–7 (3)	6–8 (1)	7–5 (12)	8–13 (12)	9–4 (10)	10–3 (8)	11–1 (9)	12–2 (7)	13–6 (11)
12	1–11 (6)	2–12 (4)	3–10 (5)	4–8 (3)	5–9 (1)	6–7 (2)	7–4 (11)	8–6 (10)	9–13 (11)	10–2 (9)	11–3 (7)	12–1 (8)	13–5 (10)

It is possible to conduct the above trisect without guide-cards, but in reality no director would try it. This also gives a situation where the boards are also very complicated to distribute. Therefore the following 12 table plus 1 appendix built on a self-orthogonal Latin square is very attractive alternative. Guide-cards are needed for all pairs apart from the appendix table pairs and the boards are stationary apart from the appendix table which plays the boards in ascending order. N-S 13 is stationary playing E-W 1 to 12 and E-W 13 move up one table at each change.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1 13	9 11	4 5	7 10	3 2	10 8	12 6	2 12	11 4	6 9	8 3	5 7	13 1
2	11 9	2 13	10 12	5 6	8 11	4 3	6 8	7 1	3 7	12 5	1 10	9 4	13 2
3	5 4	12 10	3 13	11 7	6 1	9 12	10 5	1 9	8 2	4 8	7 6	2 11	13 3
4	10 7	6 5	7 11	4 13	12 8	1 2	3 12	11 6	2 10	9 3	5 9	8 1	13 4
5	2 3	11 8	1 6	8 12	5 13	7 9	9 2	4 7	12 1	3 11	10 4	6 10	13 5
6	8 10	3 4	12 9	2 1	9 7	6 13	1 11	10 3	5 8	7 2	4 12	11 5	13 6
7	6 12	8 6	5 10	12 3	2 9	11 1	7 13	3 5	10 11	1 4	9 8	4 2	13 7
8	12 2	1 7	9 1	6 11	7 4	3 10	5 3	8 13	4 6	11 12	2 5	10 9	13 8
9	4 11	7 3	2 8	10 2	1 12	8 5	11 10	6 4	9 13	5 1	12 7	3 6	13 9
10	9 6	5 12	8 4	3 9	11 3	2 7	4 1	12 11	1 5	10 13	6 2	7 8	13 10
11	3 8	10 1	6 7	9 5	4 10	12 4	8 9	5 2	7 12	2 6	11 13	1 3	13 11
12	7 5	4 9	11 2	1 8	10 6	5 11	2 4	9 10	6 3	8 7	3 1	12 13	13 12

14 and other oddly-even tables plus 1.

Use the *oddly-even Mitchell* Graeco-Latin for 14 tables and append table 15 in the same way as 10 tables above. The E-W rover plays at tables 1 to 14 in rounds 1 to 14 and table 15 plays board-sets 1 to 14 in rounds 1 to 14 against E-W pairs 1 to 14. The same method applies to 18, 22, 26 and 30 tables.

15 table plus 1

Similar to 12 tables this is also based on the trisect movement and ¾ relay. The roving E-W 16 starts at tables 16 then 3, 1, 7, 8, 9, 13, 14, 15, 4, 5, 6, 10, 11 and 12. There is board sharing at some tables on each round, but not always with table 16.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1– 1 (1)	2– 2 (2)	3– 3 (3)	4– 4 (4)	5– 5 (5)	6– 6 (6)	7– 7 (7)	8– 8 (8)	9– 9 (9)	10–10 (10)	11–11 (11)	12–12 (12)	13–13 (13)	14–14 (14)	15–15 (15)	16–16 (3)
2	1– 2 (3)	2– 1 (3)	3–16 (1)	4– 6 (5)	5– 4 (6)	6– 5 (4)	7– 9 (8)	8– 7 (9)	9– 8 (7)	10–12 (11)	11–10 (12)	12–11 (10)	13–15 (14)	14–13 (15)	15–14 (13)	16– 3 (2)
3	1–16 (2)	2– 3 (1)	3– 1 (2)	4– 5 (6)	5– 6 (4)	6– 4 (5)	7– 8 (9)	8– 9 (7)	9– 7 (8)	10–11 (12)	11–12 (10)	12–10 (11)	13–14 (15)	14–15 (13)	15–13 (14)	16– 2 (1)
4	1–13 (4)	2–14 (5)	3–15 (6)	4– 1 (7)	5– 2 (8)	6– 3 (9)	7–16 (10)	8– 5 (11)	9– 6 (12)	10– 7 (13)	11– 8 (14)	12– 9 (15)	13–10 (1)	14–11 (2)	15–12 (3)	16– 4 (12)
5	1–15 (5)	2–13 (6)	3–14 (4)	4– 3 (8)	5– 1 (9)	6– 2 (7)	7– 5 (12)	8–16 (12)	9– 4 (10)	10– 9 (14)	11– 7 (15)	12– 8 (13)	13–12 (2)	14–10 (3)	15–11 (1)	16– 6 (11)
6	1–14 (6)	2–15 (4)	3–13 (5)	4– 2 (9)	5– 3 (7)	6– 1 (8)	7– 4 (11)	8– 6 (10)	9–16 (11)	10– 8 (15)	11– 9 (13)	12– 7 (14)	13–11 (3)	14–12 (1)	15–10 (2)	16– 5 (10)
7	1–10 (7)	2–11 (8)	3–12 (9)	4–13 (10)	5–14 (11)	6–15 (12)	7– 1 (13)	8– 2 (14)	9– 3 (15)	10– 4 (1)	11– 5 (2)	12– 6 (3)	13–16 (4)	14– 8 (5)	15– 9 (6)	16– 7 (6)
8	1–12 (8)	2–10 (9)	3–11 (7)	4–15 (11)	5–13 (12)	6–14 (10)	7– 3 (14)	8– 1 (15)	9– 2 (13)	10– 6 (2)	11– 4 (3)	12– 5 (1)	13– 8 (6)	14–16 (6)	15– 7 (4)	16– 9 (5)
9	1–11 (9)	2–12 (7)	3–10 (8)	4–14 (12)	5–15 (10)	6–13 (11)	7– 2 (15)	8– 3 (13)	9– 1 (14)	10– 5 (3)	11– 6 (1)	12– 4 (2)	13– 7 (5)	14– 9 (4)	15–16 (5)	16– 8 (4)
10	1– 7 (10)	2– 8 (11)	3– 9 (12)	4–16 (13)	5–11 (14)	6–12 (15)	7–13 (1)	8–14 (2)	9–15 (3)	10– 1 (4)	11– 2 (5)	12– 3 (6)	13– 4 (7)	14– 5 (8)	15– 6 (9)	16–10 (15)
11	1– 9 (11)	2– 7 (12)	3– 8 (10)	4–11 (15)	5–16 (15)	6–10 (13)	7–15 (2)	8–13 (3)	9–14 (1)	10– 3 (5)	11– 1 (6)	12– 2 (4)	13– 6 (8)	14– 4 (9)	15– 5 (7)	16–12 (14)
12	1– 8 (12)	2– 9 (10)	3– 7 (11)	4–10 (14)	5–12 (13)	6–16 (14)	7–14 (3)	8–15 (1)	9–13 (2)	10– 2 (6)	11– 3 (4)	12– 1 (5)	13– 5 (9)	14– 6 (7)	15– 4 (8)	16–11 (13)
13	1– 4 (13)	2– 5 (14)	3– 6 (15)	4– 7 (1)	5– 8 (2)	6– 9 (3)	7–10 (4)	8–11 (5)	9–12 (6)	10–16 (7)	11–14 (8)	12–15 (9)	13– 1 (10)	14– 2 (11)	15– 3 (12)	16–13 (9)
14	1– 6 (14)	2– 4 (15)	3– 5 (13)	4– 9 (2)	5– 7 (3)	6– 8 (1)	7–12 (5)	8–10 (6)	9–11 (4)	10–14 (9)	11–16 (9)	12–13 (7)	13– 3 (11)	14– 1 (12)	15– 2 (10)	16–15 (8)
15	1– 5 (15)	2– 6 (13)	3– 4 (14)	4– 8 (3)	5– 9 (1)	6– 7 (2)	7–11 (6)	8–12 (4)	9–10 (5)	10–13 (8)	11–15 (7)	12–16 (8)	13– 2 (12)	14– 3 (10)	15– 1 (11)	16–14 (7)

This has the same problems as 12 plus 1 based on the trisect. Therefore the 15 plus 1 based on the self-orthogonal Mitchell is an excellent solution. This uses the same principles as those for 9 and 12 tables above. See *self-orthogonal Mitchells* (page 151) for the complete schedule.

16 tables plus 1

Use the double-weave with stationary boards where the E-W rover visits tables 1, 3, 9, 6, 15, 11, 2, 7, 12, 14, 5, 10, 4, 16, 8 and 13.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1 17	2 2	3 3	4 4	5 5	6 6	7 7	8 8	9 9	10 10	11 11	12 12	13 13	14 14	15 15	16 16	17 1
2	16 2	3 1	2 17	5 3	4 6	7 5	6 8	9 7	8 10	11 9	10 12	13 11	12 14	15 13	14 16	1 15	17 4
3	3 15	16 4	5 1	2 6	7 3	4 8	9 5	6 10	11 17	8 12	13 9	10 14	15 11	12 16	1 13	14 2	17 7
4	14 4	5 15	16 6	7 1	2 8	9 17	4 10	11 5	6 12	13 7	8 14	15 9	10 16	1 11	12 2	3 13	17 3
5	5 13	14 6	7 15	16 8	9 1	2 10	11 3	4 12	13 5	6 14	15 7	8 16	1 9	10 2	3 17	12 4	17 11
6	12 6	7 13	14 8	9 15	16 10	11 1	2 12	13 3	4 14	15 5	6 17	1 7	8 2	3 9	10 4	5 11	17 16
7	7 11	12 17	9 13	14 10	11 15	16 12	13 1	2 14	15 3	4 16	1 5	6 2	3 7	8 4	5 9	10 6	17 8
8	10 8	9 11	12 10	11 13	14 12	13 15	16 17	15 1	2 16	1 3	4 2	3 5	6 4	5 7	8 6	7 9	17 14
9	9 16	10 3	11 2	12 5	13 4	14 7	15 6	16 9	1 8	2 11	3 10	4 17	5 12	6 15	7 14	8 1	17 13
10	8 3	11 16	10 5	13 2	12 7	15 4	14 9	1 6	16 11	3 8	2 13	5 10	4 15	7 17	6 1	9 14	17 12
11	11 14	8 5	13 16	10 7	15 17	12 9	1 4	14 11	3 6	16 13	5 8	2 15	7 10	4 1	9 12	6 3	17 2
12	6 5	13 14	8 7	15 16	10 9	1 2	12 11	3 4	14 13	5 17	16 15	7 8	2 1	9 10	4 3	11 12	17 6
13	13 12	6 7	15 14	8 17	1 16	10 11	3 2	12 13	5 4	14 15	7 6	16 1	9 8	2 3	11 10	4 5	17 9
14	4 7	15 12	6 9	1 14	8 11	3 16	10 13	5 2	12 15	7 4	14 1	9 6	16 3	11 8	2 5	13 17	17 10
15	15 10	4 9	1 12	6 11	3 14	8 13	5 16	10 17	7 2	12 1	9 4	14 3	11 6	16 5	13 8	2 7	17 15
16	2 9	1 10	4 11	3 12	6 13	5 14	8 15	7 16	10 1	9 2	12 3	11 4	14 17	13 6	16 7	15 8	17 5

Another attractive solution is based on the 16 table self-orthogonal plus 1. See 12 and 15 plus 1 above and *self-orthogonal Mitchells* (page 151).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1 17	4 3	2 4	3 2	13 9	16 11	14 12	15 10	5 13	8 15	6 16	7 14	9 5	12 7	10 8	11 6	17 1
2	3 4	2 17	4 1	1 3	15 12	14 10	16 9	13 11	7 16	6 14	8 13	5 15	11 8	10 6	12 5	9 7	17 2
3	4 2	1 4	3 17	2 1	16 10	13 12	15 11	14 9	8 14	5 16	7 15	6 13	12 6	9 8	11 7	10 5	17 3
4	2 3	3 1	1 2	4 17	14 11	15 9	13 10	16 12	6 15	7 13	5 14	8 16	10 7	11 5	9 6	12 8	17 4
5	9 13	12 15	10 16	11 14	5 17	8 7	6 8	7 6	13 1	16 3	14 4	15 2	1 9	4 11	2 12	3 10	17 5
6	11 16	10 14	12 13	9 15	7 8	6 17	8 5	5 7	15 4	14 2	16 1	13 3	3 12	2 10	4 9	1 11	17 6
7	12 14	9 16	11 15	10 13	8 6	5 8	7 17	6 5	16 2	13 4	15 3	14 1	4 10	1 12	3 11	2 9	17 7
8	10 15	11 13	9 14	12 16	6 7	7 5	5 6	8 17	14 3	15 1	13 2	16 4	2 11	3 9	1 10	4 12	17 8
9	13 5	16 7	14 8	15 6	1 13	4 15	2 16	3 14	9 17	12 11	10 12	11 10	5 1	8 3	6 4	7 2	17 9
10	15 8	14 6	16 5	13 7	3 16	2 14	4 13	1 15	11 12	10 17	12 9	9 11	7 4	6 2	8 1	5 3	17 10
11	16 6	13 8	15 7	14 5	4 14	1 16	3 15	2 13	12 10	9 12	11 17	10 9	8 2	5 4	7 3	6 1	17 11
12	14 7	15 5	13 6	16 8	2 15	3 13	1 14	4 16	10 11	11 9	9 10	12 17	6 3	7 1	5 2	8 4	17 12
13	5 9	8 11	6 12	7 10	9 1	12 3	10 4	11 2	1 5	4 7	2 8	3 6	13 17	16 15	14 16	15 14	17 13
14	7 12	6 10	8 9	5 11	11 4	10 2	12 1	9 3	3 8	2 6	4 5	1 7	15 16	14 17	16 13	13 15	17 14
15	8 10	5 12	7 11	6 9	12 2	9 4	11 3	10 1	4 6	1 8	3 7	2 5	16 14	13 16	15 17	14 13	17 15
16	6 11	7 9	5 10	8 12	10 3	11 1	9 2	12 4	2 7	3 5	1 6	4 8	14 15	15 13	13 14	16 17	17 16

20 tables plus 1

Use the double-weave with stationary boards where the E-W rover visits tables 1, 3, 5, 2, 4, 11, 13, 15, 12, 14, 8, 20, 17, 7, 10, 18, 6, 9, 19 and 16.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1 21	2 2	3 3	4 4	5 5	6 6	7 7	8 8	9 9	10 10	11 11	12 12	13 13	14 14	15 15	16 16	17 17	18 18	19 19	20 20	21 1
2	20 2	3 1	2 21	5 3	4 6	7 5	6 8	9 7	8 10	11 9	10 12	13 11	12 14	15 13	14 16	17 15	16 18	19 17	18 20	1 19	21 4
3	3 19	20 4	5 1	2 6	7 21	4 8	9 5	6 10	11 7	8 12	13 9	10 14	15 11	12 16	17 13	14 18	19 15	16 20	1 17	18 2	21 3
4	18 4	5 21	20 6	7 1	2 8	9 3	4 10	11 5	6 12	13 7	8 14	15 9	10 16	17 11	12 18	19 13	14 20	1 15	16 2	3 17	21 19
5	5 17	18 6	7 19	20 21	9 1	2 10	11 3	4 12	13 5	6 14	15 7	8 16	17 9	10 18	19 11	12 20	1 13	14 2	3 15	16 4	21 8
6	16 6	7 17	18 8	9 19	20 10	11 1	2 12	13 3	4 14	15 5	6 21	17 7	8 18	19 9	10 20	1 11	12 2	3 13	14 4	5 15	21 16
7	7 15	16 8	9 17	18 10	11 19	20 12	13 1	2 14	15 3	4 16	17 5	6 18	19 21	8 20	1 9	10 2	3 11	12 4	5 13	14 6	21 7
8	14 8	9 15	16 10	11 17	18 12	13 19	20 14	15 1	2 16	17 3	4 18	19 5	6 20	1 7	8 21	3 9	10 4	5 11	12 6	7 13	21 2
9	9 13	14 10	11 15	16 12	13 17	18 14	15 19	20 16	17 1	2 18	19 3	4 21	1 5	6 2	3 7	8 4	5 9	10 6	7 11	12 8	21 20
10	12 10	11 13	14 12	13 15	16 14	15 17	18 16	17 19	20 18	19 1	2 20	1 3	4 2	3 21	6 4	5 7	8 6	7 9	10 8	9 11	21 5
11	11 20	12 3	13 2	14 5	15 4	16 7	17 6	18 21	19 8	20 11	1 10	2 13	3 12	4 15	5 14	6 17	7 16	8 19	9 18	10 1	21 9
12	10 3	13 20	12 5	15 2	14 7	17 4	16 9	19 6	18 11	1 8	20 13	3 10	2 15	5 12	4 17	7 14	6 19	9 16	8 1	11 21	21 18
13	13 18	10 5	15 20	12 7	17 2	14 9	19 4	16 11	1 6	18 13	3 8	20 15	5 10	2 17	7 12	4 19	9 21	6 1	11 16	8 3	21 14
14	8 5	15 18	10 7	17 20	12 9	19 2	14 21	1 4	16 13	3 6	18 15	5 8	20 17	7 10	2 19	9 12	4 1	11 14	6 3	13 16	21 11
15	15 16	8 7	17 18	10 9	19 20	12 11	1 2	14 13	3 4	16 21	5 6	18 17	7 8	20 19	9 10	2 1	11 12	4 3	13 14	6 5	21 15
16	6 7	17 16	8 9	19 18	10 11	1 20	12 13	3 2	14 15	5 4	16 17	7 6	18 19	9 8	20 1	11 10	2 3	13 21	4 5	15 14	21 12
17	17 14	6 9	19 16	8 11	1 18	10 21	3 20	12 15	5 2	14 17	7 4	16 19	9 6	18 1	11 8	20 3	13 10	2 5	15 12	4 7	21 13
18	4 9	19 14	6 11	1 16	8 13	3 18	10 15	5 20	12 21	7 2	14 19	9 4	16 1	11 6	18 3	13 8	20 5	15 10	2 7	17 12	21 17
19	19 12	4 11	1 14	6 13	3 16	8 15	5 18	10 17	7 20	12 19	9 2	14 1	11 4	16 3	13 6	18 5	15 8	20 7	17 21	2 9	21 10
20	2 11	1 12	4 13	3 14	6 15	5 16	8 17	7 18	10 19	9 20	12 1	11 2	14 3	13 4	16 5	15 21	18 7	17 8	20 9	19 10	21 6

Another attractive solution is based on the 20-table self-orthogonal plus 1. See 12 and 15 plus 1 above and *self-orthogonal Mitchells* (page 151).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1 21	4 3	2 4	3 2	17 9	20 11	18 12	19 10	13 17	16 19	14 20	15 18	9 5	12 7	10 8	11 6	5 13	8 15	6 16	7 14	21 1
2	3 4	2 21	4 1	1 3	19 12	18 10	20 9	17 11	15 20	14 18	16 17	13 19	11 8	10 6	12 5	9 7	7 16	6 14	8 13	5 15	21 2
3	4 2	1 4	3 21	2 1	20 10	17 12	19 11	18 9	16 18	13 20	15 19	14 17	12 6	9 8	11 7	10 5	8 14	5 16	7 15	6 13	21 3
4	2 3	3 1	1 2	4 21	18 11	19 9	17 10	20 12	14 19	15 17	13 18	16 20	10 7	11 5	9 6	12 8	6 15	7 13	5 14	8 16	21 4
5	9 17	12 19	10 20	11 18	5 21	8 7	6 8	7 6	1 13	4 15	2 16	3 14	17 1	20 3	18 4	19 2	13 9	16 11	14 12	15 10	21 5
6	11 20	10 18	12 17	9 19	7 8	6 21	8 5	5 7	3 16	2 14	4 13	1 15	19 4	18 2	20 1	17 3	15 12	14 10	16 9	13 11	21 6
7	12 18	9 20	11 19	10 17	8 6	5 8	7 21	6 5	4 14	1 16	3 15	2 13	20 2	17 4	19 3	18 1	16 10	13 12	15 11	14 9	21 7
8	10 19	11 17	9 18	12 20	6 7	7 5	5 6	8 21	2 15	3 13	1 14	4 16	18 3	19 1	17 2	20 4	14 11	15 9	13 10	16 12	21 8
9	17 13	20 15	18 16	19 14	13 1	16 3	14 4	15 2	9 21	12 11	10 12	11 10	5 17	8 19	6 20	7 18	1 5	4 7	2 8	3 6	21 9
10	19 16	18 14	20 13	17 15	15 4	14 2	16 1	13 3	11 12	10 21	12 9	9 11	7 20	6 18	8 17	5 19	3 8	2 6	4 5	1 7	21 10
11	20 14	17 16	19 15	18 13	16 2	13 4	15 3	14 1	12 10	9 12	11 21	10 9	8 18	5 20	7 19	6 17	4 6	1 8	3 7	2 5	21 11
12	18 15	19 13	17 14	20 16	14 3	15 1	13 2	16 4	10 11	11 9	9 10	12 21	6 19	7 17	5 18	8 20	2 7	3 5	1 6	4 8	21 12
13	5 9	8 11	6 12	7 10	1 17	4 19	2 20	3 18	17 5	20 7	18 8	19 6	13 21	16 15	14 16	15 14	9 1	12 3	10 4	11 2	21 13
14	7 12	6 10	8 9	5 11	3 20	2 18	4 17	1 19	19 8	18 6	20 5	17 7	15 16	14 21	16 13	13 15	11 4	10 2	12 1	9 3	21 14
15	8 10	5 12	7 11	6 9	4 18	1 20	3 19	2 17	20 6	17 8	19 7	18 5	16 14	13 16	15 21	14 13	12 2	9 4	11 3	10 1	21 15
16	6 11	7 9	5 10	8 12	2 19	3 17	1 18	4 20	18 7	19 5	17 6	20 8	14 15	15 13	13 14	16 21	10 3	11 1	9 2	12 4	21 16
17	13 5	16 7	14 8	15 6	9 13	12 15	10 16	11 14	5 1	8 3	6 4	7 2	1 9	4 11	2 12	3 10	17 21	20 19	18 20	19 18	21 17
18	15 8	14 6	16 5	13 7	11 16	10 14	12 13	9 15	7 4	6 2	8 1	5 3	3 12	2 10	4 9	1 11	19 20	18 21	20 17	17 19	21 18
19	16 6	13 8	15 7	14 5	12 14	9 16	11 15	10 13	8 2	5 4	7 3	6 1	4 10	1 12	3 11	2 9	20 18	17 20	19 21	18 17	21 19
20	14 7	15 5	13 6	16 8	10 15	11 13	9 14	12 16	6 3	7 1	5 2	8 4	2 11	3 9	1 10	4 12	18 19	19 17	17 18	20 21	21 20

21 tables plus 1

This has the same problems as 12 plus 1 based on the trisect. Therefore the following 21 plus 1 based on the self-orthogonal is an excellent solution.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1 22	4 3	5 4	2 5	3 2	16 11	13 19	15 17	12 20	14 18	21 16	20 9	19 7	18 10	17 8	6 21	8 15	10 14	7 13	9 12	11 6	22 1
2	3 4	2 22	1 5	5 3	4 1	15 19	12 17	20 14	11 18	13 16	18 9	17 7	16 10	21 20	19 6	10 13	7 12	9 11	6 15	8 21	14 8	22 2
3	4 5	5 1	3 22	1 2	2 4	14 17	11 20	13 18	15 16	19 12	20 7	21 19	18 8	17 6	16 9	9 15	6 14	8 13	10 21	7 11	12 10	22 3
4	5 2	3 5	2 1	4 22	1 3	13 20	18 15	12 16	14 19	11 17	17 10	16 8	20 6	19 9	21 18	8 12	10 11	7 21	9 14	6 13	15 7	22 4
5	2 3	1 4	4 2	3 1	5 22	12 18	14 16	11 19	17 13	15 20	19 8	18 6	21 17	16 7	20 10	7 14	9 21	6 12	8 11	10 15	13 9	22 5
6	11 16	19 15	17 14	20 13	18 12	6 22	9 8	10 9	7 10	8 7	1 21	5 18	4 20	3 17	2 19	21 11	14 3	12 5	15 2	13 4	16 1	22 6
7	19 13	17 12	20 11	15 18	16 14	8 9	7 22	6 10	10 8	9 6	3 20	2 17	1 19	5 16	4 21	14 5	12 2	21 15	13 1	11 3	18 4	22 7
8	17 15	14 20	18 13	16 12	19 11	9 10	10 6	8 22	6 7	7 9	5 19	4 16	3 18	2 21	1 17	12 4	15 1	13 3	11 5	21 14	20 2	22 8
9	20 12	18 11	16 15	19 14	13 17	10 7	8 10	7 6	9 22	6 8	2 18	1 20	5 21	4 19	3 16	15 3	21 13	11 2	14 4	12 1	17 5	22 9
10	18 14	16 13	12 19	17 11	20 15	7 8	6 9	9 7	8 6	10 22	4 17	3 21	2 16	1 18	5 20	13 2	11 4	14 1	21 12	15 5	19 3	22 10
11	16 21	9 18	7 20	10 17	8 19	21 1	20 3	19 5	18 2	17 4	11 22	14 13	15 14	12 15	13 12	1 6	4 10	2 9	5 8	3 7	6 16	22 11
12	9 20	7 17	19 21	8 16	6 18	18 5	17 2	16 4	20 1	21 3	13 14	12 22	11 15	15 13	14 11	4 8	2 7	5 6	3 10	1 9	10 19	22 12
13	7 19	10 16	8 18	6 20	17 21	20 4	19 1	18 3	21 5	16 2	14 15	15 11	13 22	11 12	12 14	2 10	5 9	3 8	1 7	4 6	9 17	22 13
14	10 18	20 21	6 17	9 19	7 16	17 3	16 5	21 2	19 4	18 1	15 12	13 15	12 11	14 22	11 13	5 7	3 6	1 10	4 9	2 8	8 20	22 14
15	8 17	6 19	9 16	18 21	10 20	19 2	21 4	17 1	16 3	20 5	12 13	11 14	14 12	13 11	15 22	3 9	1 8	4 7	2 6	5 10	7 18	22 15
16	21 6	13 10	15 9	12 8	14 7	11 21	5 14	4 12	3 15	2 13	6 1	8 4	10 2	7 5	9 3	16 22	19 18	20 19	17 20	18 17	1 11	22 16
17	15 8	12 7	14 6	11 10	21 9	3 14	2 12	1 15	13 21	4 11	10 4	7 2	9 5	6 3	8 1	18 19	17 22	16 20	20 18	19 16	5 13	22 17
18	14 10	11 9	13 8	21 7	12 6	5 12	15 21	3 13	2 11	1 14	9 2	6 5	8 3	10 1	7 4	19 20	20 16	18 22	16 17	17 19	4 15	22 18
19	13 7	15 6	21 10	14 9	11 8	2 15	1 13	5 11	4 14	12 21	8 5	10 3	7 1	9 4	6 2	20 17	18 20	17 16	19 22	16 18	3 12	22 19
20	12 9	21 8	11 7	13 6	15 10	4 13	3 11	14 21	1 12	5 15	7 3	9 1	6 4	8 2	10 5	17 18	16 19	19 17	18 16	20 22	2 14	22 20
21	6 11	8 14	10 12	7 15	9 13	1 16	4 18	2 20	5 17	3 19	16 6	19 10	17 9	20 8	18 7	11 1	13 5	15 4	12 3	14 2	21 22	22 21

24 tables plus 1 The following is a 24 plus 1 based on the self-orthogonal rules.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1 25	4 3	5 4	2 5	3 2	16 11	13 19	17 15	20 12	18 14	21 16	23 20	19 7	22 18	24 17	6 21	8 24	10 22	7 13	9 23	11 6	14 10	12 9	15 8	25 1
2	3 4	2 25	1 5	5 3	4 1	19 15	17 12	20 14	11 18	16 13	18 9	22 17	24 16	21 20	23 19	10 24	7 22	9 11	6 23	8 21	14 8	12 7	15 6	13 10	25 2
3	4 5	5 1	3 25	1 2	2 4	14 17	20 11	18 13	16 15	19 12	24 20	21 19	23 18	17 6	22 16	9 22	6 14	8 23	10 21	7 24	12 10	15 9	13 8	11 7	25 3
4	5 2	3 5	2 1	4 25	1 3	20 13	18 15	12 16	19 14	17 11	23 17	16 8	22 20	24 19	21 18	8 12	10 23	7 21	9 24	6 22	15 7	13 6	11 10	14 9	25 4
5	2 3	1 4	4 2	3 1	5 25	18 12	16 14	19 11	17 13	15 20	22 19	24 18	21 17	23 16	20 10	7 23	9 21	6 24	8 22	10 15	13 9	11 8	14 7	12 6	25 5
6	11 16	15 19	17 14	13 20	12 18	6 25	9 8	10 9	7 10	8 7	1 21	5 24	4 22	3 17	2 23	21 11	14 3	24 12	23 15	22 13	16 1	20 4	19 2	18 5	25 6
7	19 13	12 17	11 20	15 18	14 16	8 9	7 25	6 10	10 8	9 6	3 24	2 22	1 19	5 23	4 21	23 14	22 12	21 15	13 1	24 11	18 4	17 2	16 5	20 3	25 7
8	15 17	14 20	13 18	16 12	11 19	9 10	10 6	8 25	6 7	7 9	5 22	4 16	3 23	2 21	1 24	12 4	24 15	23 13	22 11	21 14	20 2	19 5	18 3	17 1	25 8
9	12 20	18 11	15 16	14 19	13 17	10 7	8 10	7 6	9 25	6 8	2 18	1 23	5 21	4 24	3 22	22 15	21 13	11 2	24 14	23 12	17 5	16 3	20 1	19 4	25 9
10	14 18	13 16	12 19	11 17	20 15	7 8	6 9	9 7	8 6	10 25	4 23	3 21	2 24	1 22	5 20	24 13	23 11	22 14	21 12	15 5	19 3	18 1	17 4	16 2	25 10
11	16 21	9 18	20 24	17 23	19 22	21 1	24 3	22 5	18 2	23 4	11 25	14 13	15 14	12 15	13 12	1 6	4 10	2 9	5 8	3 7	6 16	8 19	10 17	7 20	25 11
12	20 23	17 22	19 21	8 16	18 24	24 5	22 2	16 4	23 1	21 3	13 14	12 25	11 15	15 13	14 11	4 8	2 7	5 6	3 10	1 9	10 19	7 17	9 20	6 18	25 12
13	7 19	16 24	18 23	20 22	17 21	22 4	19 1	23 3	21 5	24 2	14 15	15 11	13 25	11 12	12 14	2 10	5 9	3 8	1 7	4 6	9 17	6 20	8 18	10 16	25 13
14	18 22	20 21	6 17	19 24	16 23	17 3	23 5	21 2	24 4	22 1	15 12	13 15	12 11	14 25	11 13	5 7	3 6	1 10	4 9	2 8	8 20	10 18	7 16	9 19	25 14
15	17 24	19 23	16 22	18 21	10 20	23 2	21 4	24 1	22 3	20 5	12 13	11 14	14 12	13 11	15 25	3 9	1 8	4 7	2 6	5 10	7 18	9 16	6 19	8 17	25 15
16	21 6	24 10	22 9	12 8	23 7	11 21	14 23	4 12	15 22	13 24	6 1	8 4	10 2	7 5	9 3	16 25	19 18	20 19	17 20	18 17	1 11	3 15	5 14	2 13	25 16
17	24 8	22 7	14 6	23 10	21 9	3 14	12 22	15 24	13 21	11 23	10 4	7 2	9 5	6 3	8 1	18 19	17 25	16 20	20 18	19 16	5 13	2 12	4 11	1 15	25 17
18	22 10	11 9	23 8	21 7	24 6	12 24	15 21	13 23	2 11	14 22	9 2	6 5	8 3	10 1	7 4	19 20	20 16	18 25	16 17	17 19	4 15	1 14	3 13	5 12	25 18
19	13 7	23 6	21 10	24 9	22 8	15 23	1 13	11 22	14 24	12 21	8 5	10 3	7 1	9 4	6 2	20 17	18 20	17 16	19 25	16 18	3 12	5 11	2 15	4 14	25 19
20	23 9	21 8	24 7	22 6	15 10	13 22	11 24	14 21	12 23	5 15	7 3	9 1	6 4	8 2	10 5	17 18	16 19	19 17	18 16	20 25	2 14	4 13	1 12	3 11	25 20
21	6 11	8 14	10 12	7 15	9 13	1 16	4 18	2 20	5 17	3 19	16 6	19 10	17 9	20 8	18 7	11 1	13 5	15 4	12 3	14 2	21 25	23 24	24 22	22 23	25 21
22	10 14	7 12	9 15	6 13	8 11	4 20	2 17	5 19	3 16	1 18	19 8	17 7	20 6	18 10	16 9	15 3	12 2	14 1	11 5	13 4	24 23	22 25	21 24	23 21	25 22
23	9 12	6 15	8 13	10 11	7 14	2 19	5 16	3 18	1 20	4 17	17 10	20 9	18 8	16 7	19 6	14 5	11 4	13 3	15 2	12 1	22 24	24 21	23 25	21 22	25 23
24	8 15	10 13	7 11	9 14	6 12	5 18	3 20	1 17	4 19	2 16	20 7	18 6	16 10	19 9	17 8	13 2	15 1	12 5	14 4	11 3	23 22	21 23	22 21	24 25	25 24

25 tables plus 1 The following is a 25 plus 1 based on the self-orthogonal rules.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	1 26	4 3	5 4	2 5	3 2	16 11	19 13	20 14	17 15	18 12	21 16	24 18	25 19	22 20	23 17	6 21	9 23	10 24	7 25	8 22	11 6	14 8	15 9	12 10	13 7	26 1
2	3 4	2 26	1 5	5 3	4 1	18 14	17 12	16 15	20 13	19 11	23 19	22 17	21 20	25 18	24 16	8 24	7 22	6 25	10 23	9 21	13 9	12 7	11 10	15 8	14 6	26 2
3	4 5	5 1	3 26	1 2	2 4	19 15	20 11	18 13	16 12	17 14	24 20	25 16	23 18	21 17	22 19	9 25	10 21	8 23	6 22	7 24	14 10	15 6	13 8	11 7	12 9	26 3
4	5 2	3 5	2 1	4 26	1 3	20 12	18 15	17 11	19 14	16 13	25 17	23 20	22 16	24 19	21 18	10 22	8 25	7 21	9 24	6 23	15 7	13 10	12 6	14 9	11 8	26 4
5	2 3	1 4	4 2	3 1	5 26	17 13	16 14	19 12	18 11	20 15	22 18	21 19	24 17	23 16	25 20	7 23	6 24	9 22	8 21	10 25	12 8	11 9	14 7	13 6	15 10	26 5
6	11 16	14 18	15 19	12 20	13 17	6 26	9 8	10 9	7 10	8 7	1 21	4 23	5 24	2 25	3 22	21 11	24 13	25 14	22 15	23 12	16 1	19 3	20 4	17 5	18 2	26 6
7	13 19	12 17	11 20	15 18	14 16	8 9	7 26	6 10	10 8	9 6	3 24	2 22	1 25	5 23	4 21	23 14	22 12	21 15	25 13	24 11	18 4	17 2	16 5	20 3	19 1	26 7
8	14 20	15 16	13 18	11 17	12 19	9 10	10 6	8 26	6 7	7 9	4 25	5 21	3 23	1 22	2 24	24 15	25 11	23 13	21 12	22 14	19 5	20 1	18 3	16 2	17 4	26 8
9	15 17	13 20	12 16	14 19	11 18	10 7	8 10	7 6	9 26	6 8	5 22	3 25	2 21	4 24	1 23	25 12	23 15	22 11	24 14	21 13	20 2	18 5	17 1	19 4	16 3	26 9
10	12 18	11 19	14 17	13 16	15 20	7 8	6 9	9 7	8 6	10 26	2 23	1 24	4 22	3 21	5 25	22 13	21 14	24 12	23 11	25 15	17 3	16 4	19 2	18 1	20 5	26 10
11	16 21	19 23	20 24	17 25	18 22	21 1	24 3	25 4	22 5	23 2	11 26	14 13	15 14	12 15	13 12	1 6	4 8	5 9	2 10	3 7	6 16	9 18	10 19	7 20	8 17	26 11
12	18 24	17 22	16 25	20 23	19 21	23 4	22 2	21 5	25 3	24 1	13 14	12 26	11 15	15 13	14 11	3 9	2 7	1 10	5 8	4 6	8 19	7 17	6 20	10 18	9 16	26 12
13	19 25	20 21	18 23	16 22	17 24	24 5	25 1	23 3	21 2	22 4	14 15	15 11	13 26	11 12	12 14	4 10	5 6	3 8	1 7	2 9	9 20	10 16	8 18	6 17	7 19	26 13
14	20 22	18 25	17 21	19 24	16 23	25 2	23 5	22 1	24 4	21 3	15 12	13 15	12 11	14 26	11 13	5 7	3 10	2 6	4 9	1 8	10 17	8 20	7 16	9 19	6 18	26 14
15	17 23	16 24	19 22	18 21	20 25	22 3	21 4	24 2	23 1	25 5	12 13	11 14	14 12	13 11	15 26	2 8	1 9	4 7	3 6	5 10	7 18	6 19	9 17	8 16	10 20	26 15
16	21 6	24 8	25 9	22 10	23 7	11 21	14 23	15 24	12 25	13 22	6 1	9 3	10 4	7 5	8 2	16 26	19 18	20 19	17 20	18 17	1 11	4 13	5 14	2 15	3 12	26 16
17	23 9	22 7	21 10	25 8	24 6	13 24	12 22	11 25	15 23	14 21	8 4	7 2	6 5	10 3	9 1	18 19	17 26	16 20	20 18	19 16	3 14	2 12	1 15	5 13	4 11	26 17
18	24 10	25 6	23 8	21 7	22 9	14 25	15 21	13 23	11 22	12 24	9 5	10 1	8 3	6 2	7 4	19 20	20 16	18 26	16 17	17 19	4 15	5 11	3 13	1 12	2 14	26 18
19	25 7	23 10	22 6	24 9	21 8	15 22	13 25	12 21	14 24	11 23	10 2	8 5	7 1	9 4	6 3	20 17	18 20	17 16	19 26	16 18	5 12	3 15	2 11	4 14	1 13	26 19
20	22 8	21 9	24 7	23 6	25 10	12 23	11 24	14 22	13 21	15 25	7 3	6 4	9 2	8 1	10 5	17 18	16 19	19 17	18 16	20 26	2 13	1 14	4 12	3 11	5 15	26 20
21	6 11	9 13	10 14	7 15	8 12	1 16	4 18	5 19	2 20	3 17	16 6	19 8	20 9	17 10	18 7	11 1	14 3	15 4	12 5	13 2	21 26	24 23	25 24	22 25	23 22	26 21
22	8 14	7 12	6 15	10 13	9 11	3 19	2 17	1 20	5 18	4 16	18 9	17 7	16 10	20 8	19 6	13 4	12 2	11 5	15 3	14 1	23 24	22 26	21 25	25 23	24 21	26 22
23	9 15	10 11	8 13	6 12	7 14	4 20	5 16	3 18	1 17	2 19	19 10	20 6	18 8	16 7	17 9	14 5	15 1	13 3	11 2	12 4	24 25	25 21	23 26	21 22	22 24	26 23
24	10 12	8 15	7 11	9 14	6 13	5 17	3 20	2 16	4 19	1 18	20 7	18 10	17 6	19 9	16 8	15 2	13 5	12 1	14 4	11 3	25 22	23 25	22 21	24 26	21 23	26 24
25	7 13	6 14	9 12	8 11	10 15	2 18	1 19	4 17	3 16	5 20	17 8	16 9	19 7	18 6	20 10	12 3	11 4	14 2	13 1	15 5	22 23	21 24	24 22	23 21	25 26	26 25

27 tables plus 1 The following is a 27 plus 1 based on the self-orthogonal rules.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	1 28	14 25	27 13	18 26	19 14	5 2	23 15	9 3	10 27	21 20	4 17	17 5	8 18	12 6	22 21	13 4	26 19	3 16	11 12	24 9	7 24	25 7	2 22	15 10	6 23	16 11	20 8	28 1
2	25 14	2 28	15 26	6 3	16 27	20 15	11 25	24 13	7 1	18 6	19 21	5 18	23 19	9 16	10 4	1 17	14 5	27 20	8 22	12 10	22 7	13 11	26 8	3 23	21 9	4 24	17 12	28 2
3	13 27	26 15	3 28	21 13	4 1	17 25	8 2	12 26	22 14	6 16	16 4	20 19	11 5	24 20	7 17	25 21	2 18	15 6	23 8	9 23	10 11	1 24	14 12	27 9	18 10	19 7	5 22	28 3
4	26 18	3 6	13 21	4 28	17 19	21 16	12 20	22 17	8 5	16 7	20 22	6 10	24 23	7 11	11 8	2 12	15 9	25 24	9 26	10 14	23 2	14 15	27 3	1 27	19 1	5 25	18 13	28 4
5	14 19	27 16	1 4	19 17	5 28	18 20	9 6	10 21	23 18	4 11	17 8	21 23	12 9	22 24	8 12	26 22	3 10	13 7	24 3	7 27	11 15	2 25	15 13	25 1	16 14	20 2	6 26	28 5
6	2 5	15 20	25 17	16 21	20 18	6 28	24 16	7 4	11 19	19 24	5 12	18 9	9 10	10 7	23 22	14 8	27 23	1 11	12 13	22 1	8 25	26 2	3 26	13 14	4 27	17 15	21 3	28 6
7	15 23	25 11	2 8	20 12	6 9	16 24	7 28	11 22	24 10	5 15	18 3	19 27	10 1	23 25	9 13	27 26	1 14	14 2	22 4	8 19	12 16	3 20	13 17	26 5	17 18	21 6	4 21	28 7
8	3 9	13 24	26 12	17 22	21 10	4 7	22 11	8 28	12 23	20 25	6 13	16 1	7 14	11 2	24 26	15 3	25 27	2 15	10 17	23 5	9 20	27 6	1 21	14 18	5 19	18 16	19 4	28 8
9	27 10	1 7	14 22	5 8	18 23	19 11	10 24	23 12	9 28	17 2	21 26	4 14	22 27	8 15	12 3	3 13	13 1	26 25	7 21	11 18	24 6	15 16	25 4	2 19	20 5	6 20	16 17	28 9
10	20 21	6 18	16 6	7 16	11 4	24 19	15 5	25 20	2 17	10 28	23 7	9 22	27 8	1 23	14 11	5 24	18 12	19 9	3 2	13 26	26 14	17 27	21 15	4 3	22 13	8 1	12 25	28 10
11	17 4	21 19	4 16	22 20	8 17	12 5	3 18	13 6	26 21	7 23	11 28	24 8	15 12	25 9	2 24	20 7	6 22	16 10	27 15	1 3	14 27	5 1	18 25	19 13	10 26	23 14	9 2	28 11
12	5 17	18 5	19 20	10 6	23 21	9 18	27 19	1 16	14 4	22 9	8 24	12 28	3 22	13 10	26 7	17 11	21 8	4 23	15 25	25 13	2 1	20 14	6 2	16 26	7 3	11 27	24 15	28 12
13	18 8	19 23	5 11	23 24	9 12	10 9	1 10	14 7	27 22	8 27	12 15	22 3	13 28	26 1	3 25	21 2	4 26	17 14	25 16	2 4	15 19	6 5	16 20	20 17	11 21	24 18	7 6	28 13
14	6 12	16 9	20 24	11 7	24 22	7 10	25 23	2 11	15 8	23 1	9 25	10 13	1 26	14 28	27 2	18 15	19 3	5 27	13 20	26 17	3 5	21 18	4 6	17 21	8 4	12 19	22 16	28 14
15	21 22	4 10	17 7	8 11	12 8	22 23	13 9	26 24	3 12	11 14	24 2	7 26	25 3	2 27	15 28	6 25	16 13	20 1	1 6	14 21	27 18	18 19	19 16	5 4	23 17	9 5	10 20	28 15
16	4 13	17 1	21 25	12 2	22 26	8 14	26 27	3 15	13 3	24 5	7 20	11 17	2 21	15 18	25 6	16 28	20 4	6 19	14 24	27 12	1 9	19 10	5 7	18 22	9 8	10 23	23 11	28 16
17	19 26	5 14	18 2	9 15	10 3	23 27	14 1	27 25	1 13	12 18	22 6	8 21	26 4	3 19	13 16	4 20	17 28	21 5	2 7	15 22	25 10	16 23	20 11	6 8	24 12	7 9	11 24	28 17
18	16 3	20 27	6 15	24 25	7 13	11 1	2 14	15 2	25 26	9 19	10 16	23 4	14 17	27 5	1 20	19 6	5 21	18 28	26 11	3 8	13 23	4 9	17 24	21 12	12 22	22 10	8 7	28 18
19	12 11	22 8	8 23	26 9	3 24	13 12	4 22	17 10	21 7	2 3	15 27	25 15	16 25	20 13	6 1	24 14	7 2	11 26	19 28	5 16	18 4	9 17	10 5	23 20	14 6	27 21	1 18	28 19
20	9 24	10 12	23 9	14 10	27 7	1 22	19 8	5 23	18 11	26 13	3 1	13 25	4 2	17 26	21 14	12 27	22 15	8 3	16 5	20 28	6 17	24 21	7 18	11 6	2 16	15 4	25 19	28 20
21	24 7	7 22	11 10	2 23	15 11	25 8	16 12	20 9	6 24	14 26	27 14	1 2	19 15	5 3	18 27	9 1	10 25	23 13	4 18	17 6	21 28	12 4	22 19	8 16	26 20	3 17	13 5	28 21
22	7 25	11 13	24 1	15 14	25 2	2 26	20 3	6 27	16 15	27 17	1 5	14 20	5 6	18 21	19 18	10 19	23 16	9 4	17 9	21 24	4 12	22 28	8 10	12 7	3 11	13 8	26 23	28 22
23	22 2	8 26	12 14	3 27	13 15	26 3	17 13	21 1	4 25	15 21	25 18	2 6	20 16	6 4	16 19	7 5	11 20	24 17	5 10	18 7	19 22	10 8	23 28	9 11	27 24	1 12	14 9	28 23
24	10 15	23 3	9 27	27 1	1 25	14 13	5 26	18 14	19 2	3 4	13 19	26 16	17 20	21 17	4 5	22 18	8 6	12 21	20 23	6 11	16 8	7 12	11 9	24 28	15 7	25 22	2 10	28 24
25	23 6	9 21	10 18	1 19	14 16	27 4	18 17	19 5	5 20	13 22	26 10	3 7	21 11	4 8	17 23	8 9	12 24	22 12	6 14	16 2	20 26	11 3	24 27	7 15	25 28	2 13	15 1	28 25
26	11 16	24 4	7 19	25 5	2 20	15 17	6 21	16 18	20 6	1 8	14 23	27 11	18 24	19 12	5 9	23 10	9 7	10 22	21 27	4 15	17 3	8 13	12 1	22 25	13 2	26 28	3 14	28 26
27	8 20	12 17	22 5	13 18	26 6	3 21	21 4	4 19	17 16	25 12	2 9	15 24	6 7	16 22	20 10	11 23	24 11	7 8	18 1	19 25	5 13	23 26	9 14	10 2	1 15	14 3	27 28	28 27

28 tables plus 1 The following is a 28 plus 1 based on the self-orthogonal rules.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	1 29	7 3	6 5	5 7	4 2	3 4	2 6	15 22	21 24	20 26	19 28	18 23	17 25	16 27	22 8	28 10	27 12	26 14	25 9	24 11	23 13	8 15	14 17	13 19	12 21	11 16	10 18	9 20	29 1
2	3 7	2 29	1 4	7 6	6 1	5 3	4 5	17 28	16 23	15 25	21 27	20 22	19 24	18 26	24 14	23 9	22 11	28 13	27 8	26 10	25 12	10 21	9 16	8 18	14 20	13 15	12 17	11 19	29 2
3	5 6	4 1	3 29	2 5	1 7	7 2	6 4	19 27	18 22	17 24	16 26	15 28	21 23	20 25	26 13	25 8	24 10	23 12	22 14	28 9	27 11	12 20	11 15	10 17	9 19	8 21	14 16	13 18	29 3
4	7 5	6 7	5 2	4 29	3 6	2 1	1 3	21 26	20 28	19 23	18 25	17 27	16 22	15 24	28 12	27 14	26 9	25 11	24 13	23 8	22 10	14 19	13 21	12 16	11 18	10 20	9 15	8 17	29 4
5	2 4	1 6	7 1	6 3	5 29	4 7	3 2	16 25	15 27	21 22	20 24	19 26	18 28	17 23	23 11	22 13	28 8	27 10	26 12	25 14	24 9	9 18	8 20	14 15	13 17	12 19	11 21	10 16	29 5
6	4 3	3 5	2 7	1 2	7 4	6 29	5 1	18 24	17 26	16 28	15 23	21 25	20 27	19 22	25 10	24 12	23 14	22 9	28 11	27 13	26 8	11 17	10 19	9 21	8 16	14 18	13 20	12 15	29 6
7	6 2	5 4	4 6	3 1	2 3	1 5	7 29	20 23	19 25	18 27	17 22	16 24	15 26	21 28	27 9	26 11	25 13	24 8	23 10	22 12	28 14	13 16	12 18	11 20	10 15	9 17	8 19	14 21	29 7
8	22 15	28 17	27 19	26 21	25 16	24 18	23 20	8 29	14 10	13 12	12 14	11 9	10 11	9 13	1 22	7 24	6 26	5 28	4 23	3 25	2 27	15 1	21 3	20 5	19 7	18 2	17 4	16 6	29 8
9	24 21	23 16	22 18	28 20	27 15	26 17	25 19	10 14	9 29	8 11	14 13	13 8	12 10	11 12	3 28	2 23	1 25	7 27	6 22	5 24	4 26	17 7	16 2	15 4	21 6	20 1	19 3	18 5	29 9
10	26 20	25 15	24 17	23 19	22 21	28 16	27 18	12 13	11 8	10 29	9 12	8 14	14 9	13 11	5 27	4 22	3 24	2 26	1 28	7 23	6 25	19 6	18 1	17 3	16 5	15 7	21 2	20 4	29 10
11	28 19	27 21	26 16	25 18	24 20	23 15	22 17	14 12	13 14	12 9	11 29	10 13	9 8	8 10	7 26	6 28	5 23	4 25	3 27	2 22	1 24	21 5	20 7	19 2	18 4	17 6	16 1	15 3	29 11
12	23 18	22 20	28 15	27 17	26 19	25 21	24 16	9 11	8 13	14 8	13 10	12 29	11 14	10 9	2 25	1 27	7 22	6 24	5 26	4 28	3 23	16 4	15 6	21 1	20 3	19 5	18 7	17 2	29 12
13	25 17	24 19	23 21	22 16	28 18	27 20	26 15	11 10	10 12	9 14	8 9	14 11	13 29	12 8	4 24	3 26	2 28	1 23	7 25	6 27	5 22	18 3	17 5	16 7	15 2	21 4	20 6	19 1	29 13
14	27 16	26 18	25 20	24 15	23 17	22 19	28 21	13 9	12 11	11 13	10 8	9 10	8 12	14 29	6 23	5 25	4 27	3 22	2 24	1 26	7 28	20 2	19 4	18 6	17 1	16 3	15 5	21 7	29 14
15	8 22	14 24	13 26	12 28	11 23	10 25	9 27	22 1	28 3	27 5	26 7	25 2	24 4	23 6	15 29	21 17	20 19	19 21	18 16	17 18	16 20	1 8	7 10	6 12	5 14	4 9	3 11	2 13	29 15
16	10 28	9 23	8 25	14 27	13 22	12 24	11 26	24 7	23 2	22 4	28 6	27 1	26 3	25 5	17 21	16 29	15 18	21 20	20 15	19 17	18 19	3 14	2 9	1 11	7 13	6 8	5 10	4 12	29 16
17	12 27	11 22	10 24	9 26	8 28	14 23	13 25	26 6	25 1	24 3	23 5	22 7	28 2	27 4	19 20	18 15	17 29	16 19	15 21	21 16	20 18	5 13	4 8	3 10	2 12	1 14	7 9	6 11	29 17
18	14 26	13 28	12 23	11 25	10 27	9 22	8 24	28 5	27 7	26 2	25 4	24 6	23 1	22 3	21 19	20 21	19 16	18 29	17 20	16 15	15 17	7 12	6 14	5 9	4 11	3 13	2 8	1 10	29 18
19	9 25	8 27	14 22	13 24	12 26	11 28	10 23	23 4	22 6	28 1	27 3	26 5	25 7	24 2	16 18	15 20	21 15	20 17	19 29	18 21	17 16	2 11	1 13	7 8	6 10	5 12	4 14	3 9	29 19
20	11 24	10 26	9 28	8 23	14 25	13 27	12 22	25 3	24 5	23 7	22 2	28 4	27 6	26 1	18 17	17 19	16 21	15 16	21 18	20 29	19 15	4 10	3 12	2 14	1 9	7 11	6 13	5 8	29 20
21	13 23	12 25	11 27	10 22	9 24	8 26	14 28	27 2	26 4	25 6	24 1	23 3	22 5	28 7	20 16	19 18	18 20	17 15	16 17	15 19	21 29	6 9	5 11	4 13	3 8	2 10	1 12	7 14	29 21
22	15 8	21 10	20 12	19 14	18 9	17 11	16 13	1 15	7 17	6 19	5 21	4 16	3 18	2 20	8 1	14 3	13 5	12 7	11 2	10 4	9 6	22 29	28 24	27 26	26 28	25 23	24 25	23 27	29 22
23	17 14	16 9	15 11	21 13	20 8	19 10	18 12	3 21	2 16	1 18	7 20	6 15	5 17	4 19	10 7	9 2	8 4	14 6	13 1	12 3	11 5	24 28	23 29	22 25	28 27	27 22	26 24	25 26	29 23
24	19 13	18 8	17 10	16 12	15 14	21 9	20 11	5 20	4 15	3 17	2 19	1 21	7 16	6 18	12 6	11 1	10 3	9 5	8 7	14 2	13 4	26 27	25 22	24 29	23 26	22 28	28 23	27 25	29 24
25	21 12	20 14	19 9	18 11	17 13	16 8	15 10	7 19	6 21	5 16	4 18	3 20	2 15	1 17	14 5	13 7	12 2	11 4	10 6	9 1	8 3	28 26	27 28	26 23	25 29	24 27	23 22	22 24	29 25
26	16 11	15 13	21 8	20 10	19 12	18 14	17 9	2 18	1 20	7 15	6 17	5 19	4 21	3 16	9 4	8 6	14 1	13 3	12 5	11 7	10 2	23 25	22 27	28 22	27 24	26 29	25 28	24 23	29 26
27	18 10	17 12	16 14	15 9	21 11	20 13	19 8	4 17	3 19	2 21	1 16	7 18	6 20	5 15	11 3	10 5	9 7	8 2	14 4	13 6	12 1	25 24	24 26	23 28	22 23	28 25	27 29	26 22	29 27
28	20 9	19 11	18 13	17 8	16 10	15 12	21 14	6 16	5 18	4 20	3 15	2 17	1 19	7 21	13 2	12 4	11 6	10 1	9 3	8 5	14 7	27 23	26 25	25 27	24 22	23 24	22 26	28 29	29 28

32 tables plus 1 The following is a 32 plus 1 based on the self-orthogonal rules.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
1	1 33	3 5	4 6	5 7	6 8	7 2	8 3	2 4	17 25	19 29	20 30	21 31	22 32	23 26	24 27	18 28	25 9	27 13	28 14	29 15	30 16	31 10	32 11	26 12	9 17	11 21	12 22	13 23	14 24	15 18	16 19	10 20	33 1
2	5 3	2 33	7 4	1 8	8 7	4 5	6 1	3 6	21 27	18 26	23 28	17 32	24 31	20 29	22 25	19 30	29 11	26 10	31 12	25 16	32 15	28 13	30 9	27 14	13 19	10 18	15 20	9 24	16 23	12 21	14 17	11 22	33 2
3	6 4	4 7	3 33	8 5	1 2	2 8	5 6	7 1	22 28	20 31	19 27	24 29	17 26	18 32	21 30	23 25	30 12	28 15	27 11	32 13	25 10	26 16	29 14	31 9	14 20	12 23	11 19	16 21	9 18	10 24	13 22	15 17	33 3
4	7 5	8 1	5 8	4 33	2 6	1 3	3 2	6 7	23 29	24 25	21 32	20 28	18 30	17 27	19 26	22 31	31 13	32 9	29 16	28 12	26 14	25 11	27 10	30 15	15 21	16 17	13 24	12 20	10 22	9 19	11 18	14 23	33 4
5	8 6	7 8	2 1	6 2	5 33	3 7	1 4	4 3	24 30	23 32	18 25	22 26	21 29	19 31	17 28	20 27	32 14	31 16	26 9	30 10	29 13	27 15	25 12	28 11	16 22	15 24	10 17	14 18	13 21	11 23	9 20	12 19	33 5
6	2 7	5 4	8 2	3 1	7 3	6 33	4 8	1 5	18 31	21 28	24 26	19 25	23 27	22 30	20 32	17 29	26 15	29 12	32 10	27 9	31 11	30 14	28 16	25 13	10 23	13 20	16 18	11 17	15 19	14 22	12 24	9 21	33 6
7	3 8	1 6	6 5	2 3	4 1	8 4	7 33	5 2	19 32	17 30	22 29	18 27	20 25	24 28	23 31	21 26	27 16	25 14	30 13	26 11	28 9	32 12	31 15	29 10	11 24	9 22	14 21	10 19	12 17	16 20	15 23	13 18	33 7
8	4 2	6 3	1 7	7 6	3 4	5 1	2 5	8 33	20 26	22 27	17 31	23 30	19 28	21 25	18 29	24 32	28 10	30 11	25 15	31 14	27 12	29 9	26 13	32 16	12 18	14 19	9 23	15 22	11 20	13 17	10 21	16 24	33 8
9	25 17	27 21	28 22	29 23	30 24	31 18	32 19	26 20	9 33	11 13	12 14	13 15	14 16	15 10	16 11	10 12	1 25	3 29	4 30	5 31	6 32	7 26	8 27	2 28	17 1	19 5	20 6	21 7	22 8	23 2	24 3	18 4	33 9
10	29 19	26 18	31 20	25 24	32 23	28 21	30 17	27 22	13 11	10 33	15 12	9 16	16 15	12 13	14 9	11 14	5 27	2 26	7 28	1 32	8 31	4 29	6 25	3 30	21 3	18 2	23 4	17 8	24 7	20 5	22 1	19 6	33 10
11	30 20	28 23	27 19	32 21	25 18	26 24	29 22	31 17	14 12	12 15	11 33	16 13	9 10	10 16	13 14	15 9	6 28	4 31	3 27	8 29	1 26	2 32	5 30	7 25	22 4	20 7	19 3	24 5	17 2	18 8	21 6	23 1	33 11
12	31 21	32 17	29 24	28 20	26 22	25 19	27 18	30 23	15 13	16 9	13 16	12 33	10 14	9 11	11 10	14 15	7 29	8 25	5 32	4 28	2 30	1 27	3 26	6 31	23 5	24 1	21 8	20 4	18 6	17 3	19 2	22 7	33 12
13	32 22	31 24	26 17	30 18	29 21	27 23	25 20	28 19	16 14	15 16	10 9	14 10	13 33	11 15	9 12	12 11	8 30	7 32	2 25	6 26	5 29	3 31	1 28	4 27	24 6	23 8	18 1	22 2	21 5	19 7	17 4	20 3	33 13
14	26 23	29 20	32 18	27 17	31 19	30 22	28 24	25 21	10 15	13 12	16 10	11 9	15 11	14 33	12 16	9 13	2 31	5 28	8 26	3 25	7 27	6 30	4 32	1 29	18 7	21 4	24 2	19 1	23 3	22 6	20 8	17 5	33 14
15	27 24	25 22	30 21	26 19	28 17	32 20	31 23	29 18	11 16	9 14	14 13	10 11	12 9	16 12	15 33	13 10	3 32	1 30	6 29	2 27	4 25	8 28	7 31	5 26	19 8	17 6	22 5	18 3	20 1	24 4	23 7	21 2	33 15
16	28 18	30 19	25 23	31 22	27 20	29 17	26 21	32 24	12 10	14 11	9 15	15 14	11 12	13 9	10 13	16 33	4 26	6 27	1 31	7 30	3 28	5 25	2 29	8 32	20 2	22 3	17 7	23 6	19 4	21 1	18 5	24 8	33 16

Continued on the next page.

17	9 25	11 29	12 30	13 31	14 32	15 26	16 27	10 28	25 1	27 5	28 6	29 7	30 8	31 2	32 3	26 4	17 33	19 21	20 22	21 23	22 24	23 18	24 19	18 20	1 9	3 13	4 14	5 15	6 16	7 10	8 11	2 12	33 17
18	13 27	10 26	15 28	9 32	16 31	12 29	14 25	11 30	29 3	26 2	31 4	25 8	32 7	28 5	30 1	27 6	21 19	18 33	23 20	17 24	24 23	20 21	22 17	19 22	5 11	2 10	7 12	1 16	8 15	4 13	6 9	3 14	33 18
19	14 28	12 31	11 27	16 29	9 26	10 32	13 30	15 25	30 4	28 7	27 3	32 5	25 2	26 8	29 6	31 1	22 20	20 23	19 33	24 21	17 18	18 24	21 22	23 17	6 12	4 15	3 11	8 13	1 10	2 16	5 14	7 9	33 19
20	15 29	16 25	13 32	12 28	10 30	9 27	11 26	14 31	31 5	32 1	29 8	28 4	26 6	25 3	27 2	30 7	23 21	24 17	21 24	20 33	18 22	17 19	19 18	22 23	7 13	8 9	5 16	4 12	2 14	1 11	3 10	6 15	33 20
21	16 30	15 32	10 25	14 26	13 29	11 31	9 28	12 27	32 6	31 8	26 1	30 2	29 5	27 7	25 4	28 3	24 22	23 24	18 17	22 18	21 33	19 23	17 20	20 19	8 14	7 16	2 9	6 10	5 13	3 15	1 12	4 11	33 21
22	10 31	13 28	16 26	11 25	15 27	14 30	12 32	9 29	26 7	29 4	32 2	27 1	31 3	30 6	28 8	25 5	18 23	21 20	24 18	19 17	23 19	22 33	20 24	17 21	2 15	5 12	8 10	3 9	7 11	6 14	4 16	1 13	33 22
23	11 32	9 30	14 29	10 27	12 25	16 28	15 31	13 26	27 8	25 6	30 5	26 3	28 1	32 4	31 7	29 2	19 24	17 22	22 21	18 19	20 17	24 20	23 33	21 18	3 16	1 14	6 13	2 11	4 9	8 12	7 15	5 10	33 23
24	12 26	14 27	9 31	15 30	11 28	13 25	10 29	16 32	28 2	30 3	25 7	31 6	27 4	29 1	26 5	32 8	20 18	22 19	17 23	23 22	19 20	21 17	18 21	24 33	4 10	6 11	1 15	7 14	3 12	5 9	2 13	8 16	33 24
25	17 9	19 13	20 14	21 15	22 16	23 10	24 11	18 12	1 17	3 21	4 22	5 23	6 24	7 18	8 19	2 20	9 1	11 5	12 6	13 7	14 8	15 2	16 3	10 4	25 33	27 29	28 30	29 31	30 32	31 26	32 27	26 28	33 25
26	21 11	18 10	23 12	17 16	24 15	20 13	22 9	19 14	5 19	2 18	7 20	1 24	8 23	4 21	6 17	3 22	13 3	10 2	15 4	9 8	16 7	12 5	14 1	11 6	29 27	26 33	31 28	25 32	32 31	28 29	30 25	27 30	33 26
27	22 12	20 15	19 11	24 13	17 10	18 16	21 14	23 9	6 20	4 23	3 19	8 21	1 18	2 24	5 22	7 17	14 4	12 7	11 3	16 5	9 2	10 8	13 6	15 1	30 28	28 31	27 33	32 29	25 26	26 32	29 30	31 25	33 27
28	23 13	24 9	21 16	20 12	18 14	17 11	19 10	22 15	7 21	8 17	5 24	4 20	2 22	1 19	3 18	6 23	15 5	16 1	13 8	12 4	10 6	9 3	11 2	14 7	31 29	32 25	29 32	28 33	26 30	25 27	27 26	30 31	33 28
29	24 14	23 16	18 9	22 10	21 13	19 15	17 12	20 11	8 22	7 24	2 17	6 18	5 21	3 23	1 20	4 19	16 6	15 8	10 1	14 2	13 5	11 7	9 4	12 3	32 30	31 32	26 25	30 26	29 33	27 31	25 28	28 27	33 29
30	18 15	21 12	24 10	19 9	23 11	22 14	20 16	17 13	2 23	5 20	8 18	3 17	7 19	6 22	4 24	1 21	10 7	13 4	16 2	11 1	15 3	14 6	12 8	9 5	26 31	29 28	32 26	27 25	31 27	30 33	28 32	25 29	33 30
31	19 16	17 14	22 13	18 11	20 9	24 12	23 15	21 10	3 24	1 22	6 21	2 19	4 17	8 20	7 23	5 18	11 8	9 6	14 5	10 3	12 1	16 4	15 7	13 2	27 32	25 30	30 29	26 27	28 25	32 28	31 33	29 26	33 31
32	20 10	22 11	17 15	23 14	19 12	21 9	18 13	24 16	4 18	6 19	1 23	7 22	3 20	5 17	2 21	8 24	12 2	14 3	9 7	15 6	11 4	13 1	10 5	16 8	28 26	30 27	25 31	31 30	27 28	29 25	26 29	32 33	33 32

Two rounds short

The best solutions for two rounds short are the Bowman-Ewing, web and two-table appendix modifications. The two-table appendix movement will always work and is recommended. The *Bowman-Ewing* and *two-table appendix* modification applies to all numbers of tables. The only difference between these two movements is the sequence of play of the boards in the top two tables. The web modification generally only applies when the total number of tables is even.

Three rounds short

Use the three table appendix modification. This will always work and is based on the 2-table appendix with one additional appendix table. See the *3 Table appendix modification* (page 111) for more details. Where the total number of tables is even (6 or more tables), a web Mitchell can also be used, playing the required number of rounds.

Where the total number of tables is odd, the web rover table modification (see page 109) may apply.

Four rounds short

The first practical number of tables is 7, playing 3 rounds. This is best made up from a 3-table Mitchell with a 4-table web appended.

1–1 A	2–2 B	3–3 C	4–4 A	5–5 B	6–6 A	7–7 C
1–7 B	2–1 C	3–2 A	4–3 B	5–4 C	6–5 C	7–6 B
1–6 C	2–7 A	3–1 B	4–2 C	5–3 A	6–4 B	7–5 A

With 8 tables use an 8-table web Mitchell playing 4 rounds. For all other even numbers of tables use a web Mitchell playing 4 rounds short of the maximum.

For all odd numbers of tables append a 4-table web Mitchell to an odd-tables Mitchell. The web Mitchell must have the same number of rounds as the base Mitchell.

The *super Bowman-Ewing modification* is also suitable for this situation. See *large pairs' movements* (page 349) for more details. Subtract 4 from the total number of tables and implement a super Bowman-Ewing on that base, appending 4 tables.

Where the total number of tables is odd, the web rover table modification (see page 109) may apply.

Five or more rounds short

For 5 rounds short use either the *5-table appendix modification* or the *web modification*. The 5-table appendix Mitchell is suitable only for an odd number of tables and the web Mitchell for an even numbers of tables (10 or more).

For six rounds short where there is an odd number of tables append a 6-table web Mitchell to an odd-table base. For example, 13 tables playing 7 rounds use a 7-Table Mitchell and append a 6-table web Mitchell playing 7 rounds. This method can be applied to larger numbers of even numbers of rounds short.

For six rounds short and even numbers of tables equal to or greater than 12 tables, use a web Mitchell. Similar principles apply for larger numbers.

Where the total number of tables is odd the *web-rover table modification* may apply.

7.2 Stationary-board Mitchells

A good decision is based on knowledge and not on numbers.
Plato

The first known example of this is the *Scheveningen system*. Keeping the boards stationary in a Mitchell movement solves the problem of misplaced boards so long as the players do not mind moving. There is also an advantage when the movement is in more than one room. The player movement is slightly different from the standard Mitchell. The N-S pairs move down 1 table and the E-W pairs up 1 table or vice versa.

Any standard Mitchell with an odd number of tables can be conducted using stationary boards without any problems for the director. When needing a switched Mitchell the same switching schedule is used as shown for the normal switched Mitchell and generally the calibre will be the same or very close to the same.

The Snow system can also be used for board distribution, all tables playing the same boards on each round. See *Snow modification* (page 120) for more details.

Where the number of tables is even, the movement is not as simple and you must use a form of the SKIP Mitchell. The N-S pairs move up 1 table throughout and the E-W down one table, skipping one table after half-way so that they avoid playing the same pairs again. To play the revenge (last) round the E-W pairs move to the table they skipped at half-way. The other choices available for an even number of tables are the stagger, double-weave and oddly-even Mitchells. For switching in these movements see *switched Mitchells - even numbers of tables* (page 157).

The double-weave and oddly-even Mitchells have no board sharing and therefore can be shortened to any number of rounds and all the boards will have been played the same number of times. This is also true for the odd-table Mitchells.

Stagger Mitchell

Where you require all N-S pairs to play all the E-W pairs using stationary boards, the simplest way is to use the Stagger Mitchell. The player movement is N-S down 1 table and E-W up 1 table after each round. With *N* tables, *N* even, half the board-sets are distributed on the first half of the tables and they are shared by the second row as shown below in the examples for 8 and 10 tables.

1	2	3	4
5	6	7	8

1	2	3	4	5
6	7	8	9	10

With a second set-of-boards it is not necessary for sharing and all the tables can be in one row. After half the rounds have been played (½ *N*) the director introduces the second half of the board-sets in the same way. E-W pairs also skip one table.

For example, with 10 tables, the first 5 sets are placed on tables 1 to 5 and similarly tables 6 to 10 at the start of the game. After 5 rounds board-sets 1 to 5 are removed. Then sets 6 to 10 are placed on tables 1 to 5 and similarly tables 6 to 10.

Curtailing the movement is not as neat as odd numbers of tables because some board-sets will be played a different number of times. This can be overcome by introducing the second set-of-boards 1 round earlier (after round ½ *N* – 1) and cease play 2 rounds early (playing *N* − 2 rounds). The pair movements remain the same, E-W skipping after ½ *N* rounds.

Double-weave Mitchell

When using stationary boards the double-weave movement is easy to manage. Firstly we will consider the case where the number of tables is divisible by 4 (zero modulus 4). The movement is set up with one board-set on each table as in a standard Mitchell. The move instructions are as follows:

- the boards remain stationary;
- the odd-numbered N-S pairs move down one table for the whole movement;
- the even-numbered N-S pair move up one table for the whole movement;
- the odd-numbered E-W pairs move up one table for the first ½ *N* rounds (i.e., ½ *N* – l changes) and then move up ½ *N* tables after which they move down one table for the rest of the event;
- the even-numbered E-W pairs move down one table for the first ½ *N* rounds and then move down ½ *N* tables after which they move up one table for the rest of the event. ***Note:*** Moving down ½ *N* is the same as moving up ½ *N*.

Of course the director does not bother the players with the details of the later moves at the start, but announces the unusual change at half-time as in a skip Mitchell. For example, in an 8-table movement the director announces, "All E-W pairs move up 4 tables" after the fourth round. After the 5th round the instructions for the rest of the movement are given. Alternatively, guide cards can be provided.

This has the advantage over the stagger Mitchell in needing only one set-of-boards. The disadvantage is the more complicated player-movement, though in practice this is not a significant problem, particularly if you use guide-cards.

For completeness we will briefly consider the case where the number of tables is not divisible by 4, but is even (the number is 2 modulus 4). The changes are the same until the halfway point when the change for the E-W pairs is to move up (odd) or down (even) $\frac{1}{2}N - 1$ tables rather than $\frac{1}{2}N$. The moves for the second half for the E-W pairs are the same as the zero mod 4 case except that it is not possible to play the last round with the standard move. This ultimate move is the same as the half-way move of up (odd) or down (even) $\frac{1}{2}N - 1$ tables. Even when using this move into the last round, the E-W pairs must repeat against one pair as in the skip Mitchell. In preference use the skip Mitchell.

Oddly-even Mitchells

Let us return to Euler's conjecture which was introduced in the early chapter about the headmaster of mathematics, Leonhard Paul Euler. There we found he could not solve the *36 officers* problem. He said that this was unsolvable and that a Graeco-Latin square of order 6 was not possible. He could not find a Graeco-Latin square of order 6. Not only that, but he believed, it could not be done for squares of the orders 10, 14, 18, and so on, every four. These numbers he also called oddly-even numbers (half the number is odd).

The order 6 problem was first proved to be true in 1900 by Gaston Tarry. He was born in Villefranche-de-Rouergue, which is located in south central-France, on 27 September, 1843, and he died 21 June, 1913. His method was straight-forward He listed out all of the 812,851,200 Latin squares of order 6 and examined each pair for orthogonality and found none. Actually, by working with reduced squares he simplified the problem to checking only 9,408 pairs, but of course this was all done without the aid of computers.

In 1960, three mathematicians, R.C. Bose, S.S. Shrikhande and E.T. Parker produced the first proof of the falsity of Euler's conjecture for orders greater than 6. These men were called the 'Euler spoilers'. What does this mean for bridge schedules? The first important thing is that Euler has said we cannot have a 6-table Mitchell movement with stationary boards. Even so there are solutions that are acceptable but board sharing is required. One such solution is the *stagger modification* to the Mitchell movement, see page 118.

The following movement is what I call a pseudo Graeco-Latin square. Here the board sharing is reduced to its absolute minimum. It is set up like any Mitchell and the boards are stationary and all the pairs move. The only aberration in the movement is that the last two tables (5 and 6) where they share boards on the last two rounds (5 and 6). On round 5 they share board-set 5 and on round 6 they share board-set 6. Shown across the top of this square are the table numbers and down the side the round numbers. N-S pairs are shown first and the E-W next separated by a hyphen. The board-sets they play are shown in brackets. Note: To shorten this movement to 5 rounds it is important to play rounds 5 and 6 as rounds 1 and 2 so that board-sets 5 and 6 will be played the same number of times as the rest of the boards.

	1	2	3	4	5	6
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)
2	2–3 (1)	1–4 (2)	5–1 (3)	3–6 (4)	6–2 (5)	4–5 (6)
3	3–2 (1)	4–1 (2)	6–4 (3)	2–5 (4)	1–6 (5)	5–3 (6)
4	5–4 (1)	3–5 (2)	2–6 (3)	6–1 (4)	4–3 (5)	1–2 (6)
5	4–6 (1)	6–3 (2)	1–5 (3)	5–2 (4)	2–4 (5)	3–1 (5)
6	6–5 (1)	5–6 (2)	4–2 (3)	1–3 (4)	2–1 (6)	3–4 (6)

There are hundreds of possible such movements for 6 tables, but whatever one you use the players require guide-cards to navigate the session. It is possible to have this same movement with the N-S pairs stationary, but then the players need a schedule of boards to be played at each table as well as guide-cards for the E-W pairs. As the N-S pairs are stationary they can be given the responsibility to return and select the correct boards from a central location. Board sharing is still necessary between two tables (2 and 3) on rounds 5 and 6.

	1	2	3	4	5	6
1	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)	6–6 (6)
2	1–4 (2)	2–3 (1)	3–6 (4)	4–5 (6)	5–1 (3)	6–2 (5)
3	1–6 (5)	2–5 (4)	3–2 (1)	4–1 (2)	5–3 (6)	6–4 (3)
4	1–2 (6)	2–6 (3)	3–5 (2)	4–3 (5)	5–4 (1)	6–1 (4)
5	1–5 (3)	2–4 (5)	3–1 (5)	4–6 (1)	5–2 (4)	6–3 (2)
6	1–3 (4)	2–1 (6)	3–4 (6)	4–2 (3)	5–6 (2)	6–5 (1)

So now we have stationary-board Mitchells for all odd numbers of tables, even numbers of tables divisible by 4 using the criss-cross modification and Euler's *36 officers* impossible problem for an order-6 Graeco-Latin square using a pseudo version. Apart from the last one there is no board sharing required. What about all the other oddly-even numbers of tables?

We do know they exist, but to find them is a gigantic task. To start up a modern computer on this task to find just the first of these (10 tables) it would not find the answer in a lifetime. Clearly this is not the way to go.

Wal Wallis (see *Room squares*, page 56) produced a complete proof of the existence of all 2 MOLS for order 18 and above. He has supplied the following movements for the oddly-even Mitchells for order 18 and above. They are included here to complete the record on these movements, particularly as they have not been published before. The order 10 and 14 are generally available on the internet if you search hard enough, but are also included here for completeness. The squares show the table and board-set numbers across the top and the rounds are shown down the side. The N-S pairs are shown as the top or first number and E-W the lower or second number. The E-W pair numbers have had the number of tables added to their number for clarity.

In all these oddly-even Mitchells there is never any board sharing, but as a consequence we no longer have an orderly player movement. This is clearly a great disadvantage because the haphazard movement after each round requires player guide-cards. Fortunately Jeanie, the movement wizard, will produce them on demand.

10 Tables

	1	2	3	4	5	6	7	8	9	10
1	1 11	4 12	10 13	8 14	9 15	2 16	3 17	6 20	7 19	5 18
2	2 14	7 17	1 16	10 15	3 11	4 19	6 18	5 12	8 20	9 13
3	3 20	6 11	2 12	7 18	10 19	5 17	1 14	8 13	9 16	4 15
4	4 18	5 20	8 17	6 16	2 13	10 14	9 12	7 11	3 15	1 19
5	5 19	1 13	9 20	3 12	8 18	6 15	10 11	4 16	2 17	7 14
6	6 12	9 14	7 15	4 20	5 16	3 13	8 19	10 17	1 18	2 11
7	7 13	8 16	4 11	2 19	1 20	9 18	5 15	3 14	10 12	6 17
8	10 16	2 15	3 18	1 17	6 14	7 20	4 13	9 19	5 11	8 12
9	9 17	10 18	6 19	5 13	7 12	8 11	2 20	1 15	4 14	3 16
10	8 15	3 19	5 14	9 11	4 17	1 12	7 16	2 18	6 13	10 20

14 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1 15	9 28	4 21	13 19	10 17	3 26	6 18	11 27	7 25	12 20	2 24	5 16	14 22	8 23
2	14 23	2 16	10 28	5 22	1 20	11 18	4 27	7 19	12 15	8 26	13 21	3 25	6 17	9 24
3	7 18	14 24	3 17	11 28	6 23	2 21	12 19	5 15	8 20	13 16	9 27	1 22	4 26	10 25
4	5 27	8 19	14 25	4 18	12 28	7 24	3 22	13 20	6 16	9 21	1 17	10 15	2 23	11 26
5	3 24	6 15	9 20	14 26	5 19	13 28	8 25	4 23	1 21	7 17	10 22	2 18	11 16	12 27
6	12 17	4 25	7 16	10 21	14 27	6 20	1 28	9 26	5 24	2 22	8 18	11 23	3 19	13 15
7	4 20	13 18	5 26	8 17	11 22	14 15	7 21	2 28	10 27	6 25	3 23	9 19	12 24	1 16
8	13 25	5 21	1 19	6 27	9 18	12 23	14 16	8 22	3 28	11 15	7 26	4 24	10 20	2 17
9	11 21	1 26	6 22	2 20	7 15	10 19	13 24	14 17	9 23	4 28	12 16	8 27	5 25	3 18
10	6 26	12 22	2 27	7 23	3 21	8 16	11 20	1 25	14 18	10 24	5 28	13 17	9 15	4 19
11	10 16	7 27	13 23	3 15	8 24	4 22	9 17	12 21	2 26	14 19	11 25	6 28	1 18	5 20
12	2 19	11 17	8 15	1 24	4 16	9 25	5 23	10 18	13 22	3 27	14 20	12 26	7 28	6 21
13	8 28	3 20	12 18	9 16	2 25	5 17	10 26	6 24	11 19	1 23	4 15	14 21	13 27	7 22
14	9 22	10 23	11 24	12 25	13 26	1 27	2 15	3 16	4 17	5 18	6 19	7 20	8 21	14 28

18 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 19	3 22	4 20	2 21	18 31	5 32	6 33	7 34	8 35	9 36	10 23	11 24	12 25	13 26	14 27	15 28	16 29	17 30
2	4 21	2 20	1 22	3 19	8 28	9 29	10 30	11 31	12 32	13 33	14 34	15 35	16 36	17 23	18 24	5 25	6 26	7 27
3	2 22	4 19	3 21	1 20	17 27	18 28	5 29	6 30	7 31	8 32	9 33	10 34	11 35	12 36	13 23	14 24	15 25	16 26
4	3 20	1 21	2 19	4 22	7 29	8 30	9 31	10 32	11 33	12 34	13 35	14 36	15 23	16 24	17 25	18 26	5 27	6 28
5	13 36	10 26	9 35	11 25	5 23	12 27	18 19	17 20	16 21	15 22	7 24	6 31	14 33	4 28	3 30	2 32	1 34	8 29
6	14 23	11 27	10 36	12 26	9 30	6 24	13 28	5 19	18 20	17 21	16 22	8 25	7 32	15 34	4 29	3 31	2 33	1 35
7	15 24	12 28	11 23	13 27	1 36	10 31	7 25	14 29	6 19	5 20	18 21	17 22	9 26	8 33	16 35	4 30	3 32	2 34
8	16 25	13 29	12 24	14 28	2 35	1 23	11 32	8 26	15 30	7 19	6 20	5 21	18 22	10 27	9 34	17 36	4 31	3 33
9	17 26	14 30	13 25	15 29	3 34	2 36	1 24	12 33	9 27	16 31	8 19	7 20	6 21	5 22	11 28	10 35	18 23	4 32
10	18 27	15 31	14 26	16 30	4 33	3 35	2 23	1 25	13 34	10 28	17 32	9 19	8 20	7 21	6 22	12 29	11 36	5 24
11	5 28	16 32	15 27	17 31	6 25	4 34	3 36	2 24	1 26	14 35	11 29	18 33	10 19	9 20	8 21	7 22	13 30	12 23
12	6 29	17 33	16 28	18 32	13 24	7 26	4 35	3 23	2 25	1 27	15 36	12 30	5 34	11 19	10 20	9 21	8 22	14 31
13	7 30	18 34	17 29	5 33	15 32	14 25	8 27	4 36	3 24	2 26	1 28	16 23	13 31	6 35	12 19	11 20	10 21	9 22
14	8 31	5 35	18 30	6 34	10 22	16 33	15 26	9 28	4 23	3 25	2 27	1 29	17 24	14 32	7 36	13 19	12 20	11 21
15	9 32	6 36	5 31	7 35	12 21	11 22	17 34	16 27	10 29	4 24	3 26	2 28	1 30	18 25	15 33	8 23	14 19	13 20
16	10 33	7 23	6 32	8 36	14 20	13 21	12 22	18 35	17 28	11 30	4 25	3 27	2 29	1 31	5 26	16 34	9 24	15 19
17	11 34	8 24	7 33	9 23	16 19	15 20	14 21	13 22	5 36	18 29	12 31	4 26	3 28	2 30	1 32	6 27	17 35	10 25
18	12 35	9 25	8 34	10 24	11 26	17 19	16 20	15 21	14 22	6 23	5 30	13 32	4 27	3 29	2 31	1 33	7 28	18 36

22 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1 23	3 26	4 24	2 25	12 29	13 30	14 31	15 32	16 33	17 34	18 35	19 36	20 37	21 38	22 39	5 40	6 41	7 42	8 43	9 44	10 27	11 28
2	4 25	2 24	1 26	3 23	10 30	11 31	12 32	13 33	14 34	15 35	16 36	17 37	18 38	19 39	20 40	21 41	22 42	5 43	6 44	7 27	8 28	9 29
3	2 26	4 23	3 25	1 24	9 37	10 38	11 39	12 40	13 41	14 42	15 43	16 44	17 27	18 28	19 29	20 30	21 31	22 32	5 33	6 34	7 35	8 36
4	3 24	1 25	2 23	4 26	21 39	22 40	5 41	6 42	7 43	8 44	9 27	10 28	11 29	12 30	13 31	14 32	15 33	16 34	17 35	18 36	19 37	20 38
5	7 34	8 32	15 31	17 43	5 27	14 33	22 23	21 24	20 25	19 26	13 28	11 41	16 39	6 37	9 30	12 44	18 29	4 36	3 38	2 40	1 42	10 35
6	8 35	9 33	16 32	18 44	11 36	6 28	15 34	5 23	22 24	21 25	20 26	14 29	12 42	17 40	7 38	10 31	13 27	19 30	4 37	3 39	2 41	1 43
7	9 36	10 34	17 33	19 27	1 44	12 37	7 29	16 35	6 23	5 24	22 25	21 26	15 30	13 43	18 41	8 39	11 32	14 28	20 31	4 38	3 40	2 42
8	10 37	11 35	18 34	20 28	2 43	1 27	13 38	8 30	17 36	7 23	6 24	5 25	22 26	16 31	14 44	19 42	9 40	12 33	15 29	21 32	4 39	3 41
9	11 38	12 36	19 35	21 29	3 42	2 44	1 28	14 39	9 31	18 37	8 23	7 24	6 25	5 26	17 32	15 27	20 43	10 41	13 34	16 30	22 33	4 40
10	12 39	13 37	20 36	22 30	4 41	3 43	2 27	1 29	15 40	10 32	19 38	9 23	8 24	7 25	6 26	18 33	16 28	21 44	11 42	14 35	17 31	5 34
11	13 40	14 38	21 37	5 31	6 35	4 42	3 44	2 28	1 30	16 41	11 33	20 39	10 23	9 24	8 25	7 26	19 34	17 29	22 27	12 43	15 36	18 32
12	14 41	15 39	22 38	6 32	19 33	7 36	4 43	3 27	2 29	1 31	17 42	12 34	21 40	11 23	10 24	9 25	8 26	20 35	18 30	5 28	13 44	16 37
13	15 42	16 40	5 39	7 33	17 38	20 34	8 37	4 44	3 28	2 30	1 32	18 43	13 35	22 41	12 23	11 24	10 25	9 26	21 36	19 31	6 29	14 27
14	16 43	17 41	6 40	8 34	15 28	18 39	21 35	9 38	4 27	3 29	2 31	1 33	19 44	14 36	5 42	13 23	12 24	11 25	10 26	22 37	20 32	7 30
15	17 44	18 42	7 41	9 35	8 31	16 29	19 40	22 36	10 39	4 28	3 30	2 32	1 34	20 27	15 37	6 43	14 23	13 24	12 25	11 26	5 38	21 33
16	18 27	19 43	8 42	10 36	22 34	9 32	17 30	20 41	5 37	11 40	4 29	3 31	2 33	1 35	21 28	16 38	7 44	15 23	14 24	13 25	12 26	6 39
17	19 28	20 44	9 43	11 37	7 40	5 35	10 33	18 31	21 42	6 38	12 41	4 30	3 32	2 34	1 36	22 29	17 39	8 27	16 23	15 24	14 25	13 26
18	20 29	21 27	10 44	12 38	14 26	8 41	6 36	11 34	19 32	22 43	7 39	13 42	4 31	3 33	2 35	1 37	5 30	18 40	9 28	17 23	16 24	15 25
19	21 30	22 28	11 27	13 39	16 25	15 26	9 42	7 37	12 35	20 33	5 44	8 40	14 43	4 32	3 34	2 36	1 38	6 31	19 41	10 29	18 23	17 24
20	22 31	5 29	12 28	14 40	18 24	17 25	16 26	10 43	8 38	13 36	21 34	6 27	9 41	15 44	4 33	3 35	2 37	1 39	7 32	20 42	11 30	19 23
21	5 32	6 30	13 29	15 41	20 23	19 24	18 25	17 26	11 44	9 39	14 37	22 35	7 28	10 42	16 27	4 34	3 36	2 38	1 40	8 33	21 43	12 31
22	6 33	7 31	14 30	16 42	13 32	21 23	20 24	19 25	18 26	12 27	10 40	15 38	5 36	8 29	11 43	17 28	4 35	3 37	2 39	1 41	9 34	22 44

26 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	1 27	3 30	4 28	2 29	15 34	16 35	17 36	18 37	19 38	20 39	21 40	22 41	23 42	24 43	25 44	26 45	5 46	6 47	7 48	8 49	9 50	10 51	11 52	12 31	13 32	14 33
2	4 29	2 28	1 30	3 27	25 37	26 38	5 39	6 40	7 41	8 42	9 43	10 44	11 45	12 46	13 47	14 48	15 49	16 50	17 51	18 52	19 31	20 32	21 33	22 34	23 35	24 36
3	2 30	4 27	3 29	1 28	12 35	13 36	14 37	15 38	16 39	17 40	18 41	19 42	20 43	21 44	22 45	23 46	24 47	25 48	26 49	5 50	6 51	7 52	8 31	9 32	10 33	11 34
4	3 28	1 29	2 27	4 30	8 33	9 34	10 35	11 36	12 37	13 38	14 39	15 40	16 41	17 42	18 43	19 44	20 45	21 46	22 47	23 48	24 49	25 50	26 51	5 52	6 31	7 32
5	8 41	11 51	9 38	7 34	5 31	14 49	26 27	25 28	24 29	23 30	20 42	17 45	15 47	18 32	21 52	6 43	16 37	19 35	13 33	12 36	10 40	4 44	3 46	2 48	1 50	22 39
6	9 42	12 52	10 39	8 35	23 40	6 32	15 50	5 27	26 28	25 29	24 30	21 43	18 46	16 48	19 33	22 31	7 44	17 38	20 36	14 34	13 37	11 41	4 45	3 47	2 49	1 51
7	10 43	13 31	11 40	9 36	1 52	24 41	7 33	16 51	6 27	5 28	26 29	25 30	22 44	19 47	17 49	20 34	23 32	8 45	18 39	21 37	15 35	14 38	12 42	4 46	3 48	2 50
8	11 44	14 32	12 41	10 37	2 51	1 31	25 42	8 34	17 52	7 27	6 28	5 29	26 30	23 45	20 48	18 50	21 35	24 33	9 46	19 40	22 38	16 36	15 39	13 43	4 47	3 49
9	12 45	15 33	13 42	11 38	3 50	2 52	1 32	26 43	9 35	18 31	8 27	7 28	6 29	5 30	24 46	21 49	19 51	22 36	25 34	10 47	20 41	23 39	17 37	16 40	14 44	4 48
10	13 46	16 34	14 43	12 39	4 49	3 51	2 31	1 33	5 44	10 36	19 32	9 27	8 28	7 29	6 30	25 47	22 50	20 52	23 37	26 35	11 48	21 42	24 40	18 38	17 41	15 45
11	14 47	17 35	15 44	13 40	16 46	4 50	3 52	2 32	1 34	6 45	11 37	20 33	10 27	9 28	8 29	7 30	26 48	23 51	21 31	24 38	5 36	12 49	22 43	25 41	19 39	18 42
12	15 48	18 36	16 45	14 41	19 43	17 47	4 51	3 31	2 33	1 35	7 46	12 38	21 34	11 27	10 28	9 29	8 30	5 49	24 52	22 32	25 39	6 37	13 50	23 44	26 42	20 40
13	16 49	19 37	17 46	15 42	21 41	20 44	18 48	4 52	3 32	2 34	1 36	8 47	13 39	22 35	12 27	11 28	10 29	9 30	6 50	25 31	23 33	26 40	7 38	14 51	24 45	5 43
14	17 50	20 38	18 47	16 43	6 44	22 42	21 45	19 49	4 31	3 33	2 35	1 37	9 48	14 40	23 36	13 27	12 28	11 29	10 30	7 51	26 32	24 34	5 41	8 39	15 52	25 46
15	18 51	21 39	19 48	17 44	26 47	7 45	23 43	22 46	20 50	4 32	3 34	2 36	1 38	10 49	15 41	24 37	14 27	13 28	12 29	11 30	8 52	5 33	25 35	6 42	9 40	16 31
16	19 52	22 40	20 49	18 45	17 32	5 48	8 46	24 44	23 47	21 51	4 33	3 35	2 37	1 39	11 50	16 42	25 38	15 27	14 28	13 29	12 30	9 31	6 34	26 36	7 43	10 41
17	20 31	23 41	21 50	19 46	11 42	18 33	6 49	9 47	25 45	24 48	22 52	4 34	3 36	2 38	1 40	12 51	17 43	26 39	16 27	15 28	14 29	13 30	10 32	7 35	5 37	8 44
18	21 32	24 42	22 51	20 47	9 45	12 43	19 34	7 50	10 48	26 46	25 49	23 31	4 35	3 37	2 39	1 41	13 52	18 44	5 40	17 27	16 28	15 29	14 30	11 33	8 36	6 38
19	22 33	25 43	23 52	21 48	7 39	10 46	13 44	20 35	8 51	11 49	5 47	26 50	24 32	4 36	3 38	2 40	1 42	14 31	19 45	6 41	18 27	17 28	16 29	15 30	12 34	9 37
20	23 34	26 44	24 31	22 49	10 38	8 40	11 47	14 45	21 36	9 52	12 50	6 48	5 51	25 33	4 37	3 39	2 41	1 43	15 32	20 46	7 42	19 27	18 28	17 29	16 30	13 35
21	24 35	5 45	25 32	23 50	14 36	11 39	9 41	12 48	15 46	22 37	10 31	13 51	7 49	6 52	26 34	4 38	3 40	2 42	1 44	16 33	21 47	8 43	20 27	19 28	18 29	17 30
22	36 25	46 6	33 26	51 24	30 18	37 15	40 12	42 10	49 13	47 16	38 23	32 11	52 14	50 8	31 7	35 5	39 4	41 3	43 2	45 1	34 17	48 22	44 9	27 21	28 20	29 19
23	26 37	7 47	5 34	25 52	20 29	19 30	16 38	13 41	11 43	14 50	17 48	24 39	12 33	15 31	9 51	8 32	6 36	4 40	3 42	2 44	1 46	18 35	23 49	10 45	22 27	21 28
24	5 38	8 48	6 35	26 31	22 28	21 29	20 30	17 39	14 42	12 44	15 51	18 49	25 40	13 34	16 32	10 52	9 33	7 37	4 41	3 43	2 45	1 47	19 36	24 50	11 46	23 27
25	6 39	9 49	7 36	5 32	24 27	23 28	22 29	21 30	18 40	15 43	13 45	16 52	19 50	26 41	14 35	17 33	11 31	10 34	8 38	4 42	3 44	2 46	1 48	20 37	25 51	12 47
26	7 40	10 50	8 37	6 33	13 48	25 27	24 28	23 29	22 30	19 41	16 44	14 46	17 31	20 51	5 42	15 36	18 34	12 32	11 35	9 39	4 43	3 45	2 47	1 49	21 38	26 52

30 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1 31	3 34	4 32	2 33	18 37	19 38	20 39	21 40	22 41	23 42	24 43	25 44	26 45	27 46	28 47	29 48	30 49	5 50	6 51	7 52	8 53	9 54	10 55	11 56	12 57	13 58	14 59	15 60	16 35	17 36
2	4 33	2 32	1 34	3 31	13 38	14 39	15 40	16 41	17 42	18 43	19 44	20 45	21 46	22 47	23 48	24 49	25 50	26 51	27 52	28 53	29 54	30 55	5 56	6 57	7 58	8 59	9 60	10 35	11 36	12 37
3	2 34	4 31	3 33	1 32	20 41	21 42	22 43	23 44	24 45	25 46	26 47	27 48	28 49	29 50	30 51	5 52	6 53	7 54	8 55	9 56	10 57	11 58	12 59	13 60	14 35	15 36	16 37	17 38	18 39	19 40
4	3 32	1 33	2 31	4 34	23 51	24 52	25 53	26 54	27 55	28 56	29 57	30 58	5 59	6 60	7 35	8 36	9 37	10 38	11 39	12 40	13 41	14 42	15 43	16 44	17 45	18 46	19 47	20 48	21 49	22 50
5	7 48	8 43	11 50	21 53	5 35	12 45	30 31	29 32	28 33	27 34	9 51	16 47	20 60	23 57	26 38	17 40	19 55	6 49	13 37	25 36	24 46	18 44	22 42	10 39	15 59	4 52	3 54	2 56	1 58	14 41
6	8 49	9 44	12 51	22 54	15 42	6 36	13 46	5 31	30 32	29 33	28 34	10 52	17 48	21 35	24 58	27 39	18 41	20 56	7 50	14 38	26 37	25 47	19 45	23 43	11 40	16 60	4 53	3 55	2 57	1 59
7	9 50	10 45	13 52	23 55	1 60	16 43	7 37	14 47	6 31	5 32	30 33	29 34	11 53	18 49	22 36	25 59	28 40	19 42	21 57	8 51	15 39	27 38	26 48	20 46	24 44	12 41	17 35	4 54	3 56	2 58
8	10 51	11 46	14 53	24 56	2 59	1 35	17 44	8 38	15 48	7 31	6 32	5 33	30 34	12 54	19 50	23 37	26 60	29 41	20 43	22 58	9 52	16 40	28 39	27 49	21 47	25 45	13 42	18 36	4 55	3 57
9	11 52	12 47	15 54	25 57	3 58	2 60	1 36	18 45	9 39	16 49	8 31	7 32	6 33	5 34	13 55	20 51	24 38	27 35	30 42	21 44	23 59	10 53	17 41	29 40	28 50	22 48	26 46	14 43	19 37	4 56
10	12 53	13 48	16 55	26 58	4 57	3 59	2 35	1 37	19 46	10 40	17 50	9 31	8 32	7 33	6 34	14 56	21 52	25 39	28 36	5 43	22 45	24 60	11 54	18 42	30 41	29 51	23 49	27 47	15 44	20 38
11	13 54	14 49	17 56	27 59	21 39	4 58	3 60	2 36	1 38	20 47	11 41	18 51	10 31	9 32	8 33	7 34	15 57	22 53	26 40	29 37	6 44	23 46	25 35	12 55	19 43	5 42	30 52	24 50	28 48	16 45
12	14 55	15 50	18 57	28 60	17 46	22 40	4 59	3 35	2 37	1 39	21 48	12 42	19 52	11 31	10 32	9 33	8 34	16 58	23 54	27 41	30 38	7 45	24 47	26 36	13 56	20 44	6 43	5 53	25 51	29 49
13	15 56	16 51	19 58	29 35	30 50	18 47	23 41	4 60	3 36	2 38	1 40	22 49	13 43	20 53	12 31	11 32	10 33	9 34	17 59	24 55	28 42	5 39	8 46	25 48	27 37	14 57	21 45	7 44	6 54	26 52
14	16 57	17 52	20 59	30 36	27 53	5 51	19 48	24 42	4 35	3 37	2 39	1 41	23 50	14 44	21 54	13 31	12 32	11 33	10 34	18 60	25 56	29 43	6 40	9 47	26 49	28 38	15 58	22 46	8 45	7 55
15	17 58	18 53	21 60	5 37	8 56	28 54	6 52	20 49	25 43	4 36	3 38	2 40	1 42	24 51	15 45	22 55	14 31	13 32	12 33	11 34	19 35	26 57	30 44	7 41	10 48	27 50	29 39	16 59	23 47	9 46
16	18 59	19 54	22 35	6 38	10 47	9 57	29 55	7 53	21 50	26 44	4 37	3 39	2 41	1 43	25 52	16 46	23 56	15 31	14 32	13 33	12 34	20 36	27 58	5 45	8 42	11 49	28 51	30 40	17 60	24 48
17	19 60	20 55	23 36	7 39	25 49	11 48	10 58	30 56	8 54	22 51	27 45	4 38	3 40	2 42	1 44	26 53	17 47	24 57	16 31	15 32	14 33	13 34	21 37	28 59	6 46	9 43	12 50	29 52	5 41	18 35
18	20 35	21 56	24 37	8 40	19 36	26 50	12 49	11 59	5 57	9 55	23 52	28 46	4 39	3 41	2 43	1 45	27 54	18 48	25 58	17 31	16 32	15 33	14 34	22 38	29 60	7 47	10 44	13 51	30 53	6 42
19	21 36	22 57	25 38	9 41	7 43	20 37	27 51	13 50	12 60	6 58	10 56	24 53	29 47	4 40	3 42	2 44	1 46	28 55	19 49	26 59	18 31	17 32	16 33	15 34	23 39	30 35	8 48	11 45	14 52	5 54
20	22 37	23 58	26 39	10 42	6 55	8 44	21 38	28 52	14 51	13 35	7 59	11 57	25 54	30 48	4 41	3 43	2 45	1 47	29 56	20 50	27 60	19 31	18 32	17 33	16 34	24 40	5 36	9 49	12 46	15 53
21	23 38	24 59	27 40	11 43	16 54	7 56	9 45	22 39	29 53	15 52	14 36	8 60	12 58	26 55	5 49	4 42	3 44	2 46	1 48	30 57	21 51	28 35	20 31	19 32	18 33	17 34	25 41	6 37	10 50	13 47
22	24 39	25 60	28 41	12 44	14 48	17 55	8 57	10 46	23 40	30 54	16 53	15 37	9 35	13 59	27 56	6 50	4 43	3 45	2 47	1 49	5 58	22 52	29 36	21 31	20 32	19 33	18 34	26 42	7 38	11 51
23	25 40	26 35	29 42	13 45	12 52	15 49	18 56	9 58	11 47	24 41	5 55	17 54	16 38	10 36	14 60	28 57	7 51	4 44	3 46	2 48	1 50	6 59	23 53	30 37	22 31	21 32	20 33	19 34	27 43	8 39
24	26 41	27 36	30 43	14 46	9 40	13 53	16 50	19 57	10 59	12 48	25 42	6 56	18 55	17 39	11 37	15 35	29 58	8 52	4 45	3 47	2 49	1 51	7 60	24 54	5 38	23 31	22 32	21 33	20 34	28 44
25	27 42	28 37	5 44	15 47	29 45	10 41	14 54	17 51	20 58	11 60	13 49	26 43	7 57	19 56	18 40	12 38	16 36	30 59	9 53	4 46	3 48	2 50	1 52	8 35	25 55	6 39	24 31	23 32	22 33	21 34
26	28 43	29 38	6 45	16 48	22 34	30 46	11 42	15 55	18 52	21 59	12 35	14 50	27 44	8 58	20 57	19 41	13 39	17 37	5 60	10 54	4 47	3 49	2 51	1 53	9 36	26 56	7 40	25 31	24 32	23 33
27	29 44	30 39	7 46	17 49	24 33	23 34	5 47	12 43	16 56	19 53	22 60	13 36	15 51	28 45	9 59	21 58	20 42	14 40	18 38	6 35	11 55	4 48	3 50	2 52	1 54	10 37	27 57	8 41	26 31	25 32
28	30 45	5 40	8 47	18 50	26 32	25 33	24 34	6 48	13 44	17 57	20 54	23 35	14 37	16 52	29 46	10 60	22 59	21 43	15 41	19 39	7 36	12 56	4 49	3 51	2 53	1 55	11 38	28 58	9 42	27 31
29	5 46	6 41	9 48	19 51	28 31	27 32	26 33	25 34	7 49	14 45	18 58	21 55	24 36	15 38	17 53	30 47	11 35	23 60	22 44	16 42	20 40	8 37	13 57	4 50	3 52	2 54	1 56	12 39	29 59	10 43
30	6 47	7 42	10 49	20 52	11 44	29 31	28 32	27 33	26 34	8 50	15 46	19 59	22 56	25 37	16 39	18 54	5 48	12 36	24 35	23 45	17 43	21 41	9 38	14 58	4 51	3 53	2 55	1 57	13 40	30 60

Self-orthogonal Mitchells

In the same way that the oddly-even Mitchells are handled with Graeco-Latin squares it is also possible with all other even numbers of tables. The neat thing about these is they are based on *self-orthogonal* Latin squares. This was first discussed in Euler *The headmaster of mathematics.*

In addition the non-prime odd numbers of tables are also dealt with here. The reason for this is the difficulty in organizing rover pairs and rover tables with these numbers (9, 15, 21, 25, 27) as the only solution without these movements is the *trisect modification*, page 118. See *one round short,* page 125, for further details on the rover procedures.

The pair numbers of E-W have the number of tables added for clarity of pairs and the boards are stationary with sets 1 to *N* on tables 1 to *N*.

8 Tables

	1	2	3	4	5	6	7	8
1	1 9	3 13	4 14	5 15	6 16	7 10	8 11	2 12
2	5 11	2 10	7 12	1 16	8 15	4 13	6 9	3 14
3	6 12	4 15	3 11	8 13	1 10	2 16	5 14	7 9
4	7 13	8 9	5 16	4 12	2 14	1 11	3 10	6 15
5	8 14	7 16	2 9	6 10	5 13	3 15	1 12	4 11
6	2 15	5 12	8 10	3 9	7 11	6 14	4 16	1 13
7	3 16	1 14	6 13	2 11	4 9	8 12	7 15	5 10
8	4 10	6 11	1 15	7 14	3 12	5 9	2 13	8 16

9 Tables

	1	2	3	4	5	6	7	8	9
1	1 10	3 15	4 16	5 17	2 18	7 12	8 13	9 14	6 11
2	6 12	2 11	5 15	7 18	9 13	4 14	1 17	3 10	8 16
3	7 13	6 14	3 12	2 16	8 15	9 17	5 11	1 18	4 10
4	8 14	9 16	7 11	4 13	3 17	5 10	6 18	2 12	1 15
5	9 11	4 18	6 17	8 12	5 14	1 16	2 10	7 15	3 13
6	3 16	5 13	8 18	1 14	7 10	6 15	9 12	4 11	2 17
7	4 17	8 10	2 14	9 15	1 11	3 18	7 16	6 13	5 12
8	5 18	1 12	9 10	3 11	6 16	2 13	4 15	8 17	7 14
9	2 15	7 17	1 13	6 10	4 12	8 11	3 14	5 16	9 18

12 Tables

	1	2	3	4	5	6	7	8	9	10	11	12
1	1 13	9 23	4 17	7 22	3 14	10 20	12 18	2 24	11 16	6 21	8 15	5 19
2	11 21	2 14	10 24	5 18	8 23	4 15	6 20	7 13	3 19	12 17	1 22	9 16
3	5 16	12 22	3 15	11 19	6 13	9 24	10 17	1 21	8 14	4 20	7 18	2 23
4	10 19	6 17	7 23	4 16	12 20	1 14	3 24	11 18	2 22	9 15	5 21	8 13
5	2 15	11 20	1 18	8 24	5 17	7 21	9 14	4 19	12 13	3 23	10 16	6 22
6	8 22	3 16	12 21	2 13	9 19	6 18	1 23	10 15	5 20	7 14	4 24	11 17
7	6 24	8 18	5 22	12 15	2 21	11 13	7 19	3 17	10 23	1 16	9 20	4 14
8	12 14	1 19	9 13	6 23	7 16	3 22	5 15	8 20	4 18	11 24	2 17	10 21
9	4 23	7 15	2 20	10 14	1 24	8 17	11 22	6 16	9 21	5 13	12 19	3 18
10	9 18	5 24	8 16	3 21	11 15	2 19	4 13	12 23	1 17	10 22	6 14	7 20
11	3 20	10 13	6 19	9 17	4 22	12 16	8 21	5 14	7 24	2 18	11 23	1 15
12	7 17	4 21	11 14	1 20	10 18	5 23	2 16	9 22	6 15	8 19	3 13	12 24

15 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1 16	12 18	13 20	14 22	15 24	7 26	6 17	5 30	4 29	3 28	2 27	9 25	11 23	8 21	10 19
2	3 27	2 17	12 19	13 21	14 23	15 25	8 16	7 18	6 30	5 29	4 28	10 26	1 24	9 22	11 20
3	5 28	4 27	3 18	12 20	13 22	14 24	15 26	9 17	8 19	7 30	6 29	11 16	2 25	10 23	1 21
4	7 29	6 28	5 27	4 19	12 21	13 23	14 25	15 16	10 18	9 20	8 30	1 17	3 26	11 24	2 22
5	9 30	8 29	7 28	6 27	5 20	12 22	13 24	14 26	15 17	11 19	10 21	2 18	4 16	1 25	3 23
6	11 22	10 30	9 29	8 28	7 27	6 21	12 23	13 25	14 16	15 18	1 20	3 19	5 17	2 26	4 24
7	2 21	1 23	11 30	10 29	9 28	8 27	7 22	12 24	13 26	14 17	15 19	4 20	6 18	3 16	5 25
8	15 20	3 22	2 24	1 30	11 29	10 28	9 27	8 23	12 25	13 16	14 18	5 21	7 19	4 17	6 26
9	14 19	15 21	4 23	3 25	2 30	1 29	11 28	10 27	9 24	12 26	13 17	6 22	8 20	5 18	7 16
10	13 18	14 20	15 22	5 24	4 26	3 30	2 29	1 28	11 27	10 25	12 16	7 23	9 21	6 19	8 17
11	12 17	13 19	14 21	15 23	6 25	5 16	4 30	3 29	2 28	1 27	11 26	8 24	10 22	7 20	9 18
12	10 24	11 25	1 26	2 16	3 17	4 18	5 19	6 20	7 21	8 22	9 23	12 27	15 29	13 30	14 28
13	8 26	9 16	10 17	11 18	1 19	2 20	3 21	4 22	5 23	6 24	7 25	14 30	13 28	15 27	12 29
14	6 23	7 24	8 25	9 26	10 16	11 17	1 18	2 19	3 20	4 21	5 22	15 28	12 30	14 29	13 27
15	4 25	5 26	6 16	7 17	8 18	9 19	10 20	11 21	1 22	2 23	3 24	13 29	14 27	12 28	15 30

16 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1 17	4 19	2 20	3 18	13 25	16 27	14 28	15 26	5 29	8 31	6 32	7 30	9 21	12 23	10 24	11 22
2	3 20	2 18	4 17	1 19	15 28	14 26	16 25	13 27	7 32	6 30	8 29	5 31	11 24	10 22	12 21	9 23
3	4 18	1 20	3 19	2 17	16 26	13 28	15 27	14 25	8 30	5 32	7 31	6 29	12 22	9 24	11 23	10 21
4	2 19	3 17	1 18	4 20	14 27	15 25	13 26	16 28	6 31	7 29	5 30	8 32	10 23	11 21	9 22	12 24
5	9 29	12 31	10 32	11 30	5 21	8 23	6 24	7 22	13 17	16 19	14 20	15 18	1 25	4 27	2 28	3 26
6	11 32	10 30	12 29	9 31	7 24	6 22	8 21	5 23	15 20	14 18	16 17	13 19	3 28	2 26	4 25	1 27
7	12 30	9 32	11 31	10 29	8 22	5 24	7 23	6 21	16 18	13 20	15 19	14 17	4 26	1 28	3 27	2 25
8	10 31	11 29	9 30	12 32	6 23	7 21	5 22	8 24	14 19	15 17	13 18	16 20	2 27	3 25	1 26	4 28
9	13 21	16 23	14 24	15 22	1 29	4 31	2 32	3 30	9 25	12 27	10 28	11 26	5 17	8 19	6 20	7 18
10	15 24	14 22	16 21	13 23	3 32	2 30	4 29	1 31	11 28	10 26	12 25	9 27	7 20	6 18	8 17	5 19
11	16 22	13 24	15 23	14 21	4 30	1 32	3 31	2 29	12 26	9 28	11 27	10 25	8 18	5 20	7 19	6 17
12	14 23	15 21	13 22	16 24	2 31	3 29	1 30	4 32	10 27	11 25	9 26	12 28	6 19	7 17	5 18	8 20
13	5 25	8 27	6 28	7 26	9 17	12 19	10 20	11 18	1 21	4 23	2 24	3 22	13 29	16 31	14 32	15 30
14	7 28	6 26	8 25	5 27	11 20	10 18	12 17	9 19	3 24	2 22	4 21	1 23	15 32	14 30	16 29	13 31
15	8 26	5 28	7 27	6 25	12 18	9 20	11 19	10 17	4 22	1 24	3 23	2 21	16 30	13 32	15 31	14 29
16	6 27	7 25	5 26	8 28	10 19	11 17	9 18	12 20	2 23	3 21	1 22	4 24	14 31	15 29	13 30	16 32

20 Tables

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1 21	4 23	2 24	3 22	17 29	20 31	18 32	19 30	13 37	16 39	14 40	15 38	9 25	12 27	10 28	11 26	5 33	8 35	6 36	7 34
2	3 24	2 22	4 21	1 23	19 32	18 30	20 29	17 31	15 40	14 38	16 37	13 39	11 28	10 26	12 25	9 27	7 36	6 34	8 33	5 35
3	4 22	1 24	3 23	2 21	20 30	17 32	19 31	18 29	16 38	13 40	15 39	14 37	12 26	9 28	11 27	10 25	8 34	5 36	7 35	6 33
4	2 23	3 21	1 22	4 24	18 31	19 29	17 30	20 32	14 39	15 37	13 38	16 40	10 27	11 25	9 26	12 28	6 35	7 33	5 34	8 36
5	9 37	12 39	10 40	11 38	5 25	8 27	6 28	7 26	1 33	4 35	2 36	3 34	17 21	20 23	18 24	19 22	13 29	16 31	14 32	15 30
6	11 40	10 38	12 37	9 39	7 28	6 26	8 25	5 27	3 36	2 34	4 33	1 35	19 24	18 22	20 21	17 23	15 32	14 30	16 29	13 31
7	12 38	9 40	11 39	10 37	8 26	5 28	7 27	6 25	4 34	1 36	3 35	2 33	20 22	17 24	19 23	18 21	16 30	13 32	15 31	14 29
8	10 39	11 37	9 38	12 40	6 27	7 25	5 26	8 28	2 35	3 33	1 34	4 36	18 23	19 21	17 22	20 24	14 31	15 29	13 30	16 32
9	17 33	20 35	18 36	19 34	13 21	16 23	14 24	15 22	9 29	12 31	10 32	11 30	5 37	8 39	6 40	7 38	1 25	4 27	2 28	3 26
10	19 36	18 34	20 33	17 35	15 24	14 22	16 21	13 23	11 32	10 30	12 29	9 31	7 40	6 38	8 37	5 39	3 28	2 26	4 25	1 27
11	20 34	17 36	19 35	18 33	16 22	13 24	15 23	14 21	12 30	9 32	11 31	10 29	8 38	5 40	7 39	6 37	4 26	1 28	3 27	2 25
12	18 35	19 33	17 34	20 36	14 23	15 21	13 22	16 24	10 31	11 29	9 30	12 32	6 39	7 37	5 38	8 40	2 27	3 25	1 26	4 28
13	5 29	8 31	6 32	7 30	1 37	4 39	2 40	3 38	17 25	20 27	18 28	19 26	13 33	16 35	14 36	15 34	9 21	12 23	10 24	11 22
14	7 32	6 30	8 29	5 31	3 40	2 38	4 37	1 39	19 28	18 26	20 25	17 27	15 36	14 34	16 33	13 35	11 24	10 22	12 21	9 23
15	8 30	5 32	7 31	6 29	4 38	1 40	3 39	2 37	20 26	17 28	19 27	18 25	16 34	13 36	15 35	14 33	12 22	9 24	11 23	10 21
16	6 31	7 29	5 30	8 32	2 39	3 37	1 38	4 40	18 27	19 25	17 26	20 28	14 35	15 33	13 34	16 36	10 23	11 21	9 22	12 24
17	13 25	16 27	14 28	15 26	9 33	12 35	10 36	11 34	5 21	8 23	6 24	7 22	1 29	4 31	2 32	3 30	17 37	20 39	18 40	19 38
18	15 28	14 26	16 25	13 27	11 36	10 34	12 33	9 35	7 24	6 22	8 21	5 23	3 32	2 30	4 29	1 31	19 40	18 38	20 37	17 39
19	16 26	13 28	15 27	14 25	12 34	9 36	11 35	10 33	8 22	5 24	7 23	6 21	4 30	1 32	3 31	2 29	20 38	17 40	19 39	18 37
20	14 27	15 25	13 26	16 28	10 35	11 33	9 34	12 36	6 23	7 21	5 22	8 24	2 31	3 29	1 30	4 32	18 39	19 37	17 38	20 40

For 21, 24, 25, 27 28 and 32 tables see *incomplete Mitchell movements*, *one round short*, page 125.

7.3 Switched Mitchells

Chess is a foolish expedient for making idle people believe they are doing something very clever when they are only wasting their time.
George Bernard Shaw

As already pointed out, in a Mitchell movement there are two fields (N-S and E-W) and as such the movement produces two winners. Where one winner is needed and a Howell movement is not suitable because of the number of pairs involved, the solution is to have a switched Mitchell. The switched Mitchell is the same as an ordinary Mitchell; there are still two fields (N-S and E-W) and they play only against each other. The difference arises in the comparison of the scores with the N-S pairs playing as E-W on some of the rounds. This is usually achieved by switching the board through 90 degrees rather than the players changing positions. The comparison of the scores can never be perfect in this movement. (See *Balanced Movements,* page 85).

There are a few methods of conducting switched Mitchells. In the first the director announces each round where boards are to be arrow-switched as it occurs. This means the board is played with the N position of the board pointing E or W on that round so that the E-W pairs are playing as N-S. This also means that the E-W pairs cannot have the same pair number as their original table number, as this would confuse their results with their first N-S opponents. This can be avoided by adding the highest-numbered table number to the pair number of each E-W pair. For example, in a 7-table movement all the E-W pairs would add 7 to their original table number to determine their pair number.

It is important to choose a movement that has the least imbalance and is the simplest to use and organise. Switching single tables throughout the movement on different rounds could be the best balanced movement but in terms of organization and management it is not recommended. Switching on selected rounds is by far the most effective.

The tables below show the imbalance (standard deviation) and calibre in the movement without switching and also after the recommended switching. The calibre before switching illustrates how consistently poor the quality of balance is without switching and the improvement with switching. The calibre before switching is not shown in later tables as it is always less than 50 or generally very poor.

The expression of quality of the movement balance is shown as the calibre and is defined to as follows.

- 90 to 100: excellent
- 80 to 89: very good
- 70 to 79: good
- 60 to 69: acceptable
- 50 to 59: poor
- 30 to 49: very poor
- Less then 30 is undesirable and unacceptable.

Low numbers of tables

Before we look at the list of tables with round switching we must consider small numbers of tables. With a small number of tables (3 to 6 tables) it is most unlikely that a Mitchell movement is the best choice for a one-winner movement, but if you need to use such a movement then switching any round is as bad as any other. With a low number of tables you should switch single tables. This should be done in one round only and the director would announce switch tables 1, 2 etc. Choose round 1 except for 4 and 6 tables 'skip Mitchell' where it is best to use rounds 1 and 5 respectively. Note: With 4 and 6 tables, the 'share-and-relay' movement is the best choice.

Number of tables	Table(s) switched	Calibre	Imbalance with switching
3	1	63	1.0456
4 (skip with revenge)	1,2,3	50	1.8294
4 (skip)	1,2	43	1.1605 (round 1 best)
4 (share & relay)	1,2	69	1.2777
4 (Criss-cross)	1,2	69	1.2777
5	1,2,3	75	1.3806
6 (skip)	1,2,3,4	58	1.6630 (round 5 best)
6 (share & relay)	1,2,3,4,5	84	1.2856

Odd numbers of tables

Of the odd numbers of tables there is one case that is obviously poor, the 11-table Mitchell (see below). It is possible to switch arbitrary tables, on arbitrary rounds, to produce a better balanced movement. For all the other numbers of tables that is not the case, any improvements are marginal. The 11-table Mitchell can be conducted with switching at the tables shown here, for a calibre of 93.

Round:	2	3	3	3	4	4	5	5	5	7	8	8	9	9	9	10	11
Table:	7	1	8	11	5	7	2	4	5	9	1	4	8	10	11	3	6

The recommended rounds for the switches are listed below, but where only one round is involved, any round may be chosen. Switching is given using round 1 as the starting point, but where it applies, as an alternative, those including the last round are shown in brackets.

Tables	Switch on round(s)	Imbalance Not switched	Calibre Not switched	Imbalance Switched	Calibre Switched
3	1	1.4696	44	2.3151	24
5	1	2.4845	46	1.6178	68
7	1	3.4896	47	1.0490	92
9	1	4.4922	48	1.1646	94
11	1	5.4937	48	1.8747	89
13	1,2 or any 2	6.4947	48	1.7726	93
15	1,2 or any 2	7.4955	48	1.6327	95
17	1,2 or any 2	8.4960	49	1.9544	95
19	1,2,5 (15,16,19)	9.4965	49	2.1359	95
21	1,2,5 (17,18,21)	10.4968	49	1.9099	97
23	1,2,5 (19,20,23)	11.4971	49	2.0456	97
25	1,2,6,9 (17,18,22,25)	12.4973	49	2.4455	96
27	1,2,6,9 (19,20,24,27)	13.4975	49	2.1623	98
29	1,2,6,9 (21,22,26,29)	14.4977	49	2.1706	98
31	1,2,6,9 (23,24,28,31)	15.4979	49	2.4936	97
33	4,5,7,13,27	16.4980	49	2.4181	98
35	5,24,31,34,35	17.4982	49	2.2290	98

There are further arrangements of the Mitchell movement where these switching schedules apply equally well. When using stationary boards and standard moves (N-S move is up 1 table, E-W down 1 table) the above switching schedules still apply. On the switched rounds the boards are facing in the E-W direction. In the 13-table example, for rounds 1 and 2 the boards are pointing E-W and for the rest of the movement boards point N-S as usual.

The following seeding should be used to further refine the quality of the movement. The first number applies to the best pair, the second the next best, and so forth. The pairs are numbered 1 to *N* for N-S pairs and *N* + 1 to *2N* for the E-W pairs.

Tables																		
3	1	4	2	5	3	6												
5	1	2	6	3	7	4	8	5	9	10								
7	1	2	3	4	9	8	7	5	10	6	11	12	13	14				
9	1	2	3	4	10	11	5	12	6	13	7	8	14	9	15	16	17	18
11	1	2	3	4	12	13	5	14	6	15	7	16	8	17	9	10	11	18
	19	20	21	22														
13	3	2	1	4	5	6	14	7	15	8	16	9	17	10	11	18	13	12
	19	20	21	22	23	24	26	25										
15	3	2	4	1	5	6	7	16	17	8	18	9	19	10	20	11	21	12
	13	15	14	22	23	24	25	26	27	28	30	29						
17	3	2	4	1	5	6	7	18	19	8	20	9	21	10	22	11	23	12
	24	13	25	14	15	17	16	26	27	28	29	30	31	32	34	33		
19	2	3	5	1	4	6	7	8	9	10	20	21	11	22	12	23	13	24
	14	25	15	16	26	17	19	18	27	28	29	30	31	32	33	34	35	36
	38	37																
21	2	3	5	4	1	6	7	8	9	10	22	23	11	24	12	25	13	26
	27	14	15	28	16	29	17	18	19	21	20	30	31	32	33	34	35	36
	37	38	39	40	42	41												
23	2	3	5	4	1	6	7	8	9	10	24	25	11	26	12	27	13	28
	29	14	15	30	31	16	17	32	18	19	33	20	21	23	22	34	35	36
	37	38	39	40	41	42	43	44	46	45								
25	4	5	6	7	8	9	10	11	12	13	27	26	14	28	29	15	30	18
	17	16	31	19	20	32	22	21	25	23	33	24	34	35	36	37	38	39
	40	41	42	43	44	45	46	47	48	49	2	1	3	50				
27	11	12	13	29	28	14	30	15	31	32	16	33	17	34	20	19	18	35
	21	36	22	37	24	23	27	25	26	38	39	40	41	42	43	44	45	46
	47	48	49	50	51	52	53	2	1	3	4	5	54	6	7	8	9	10
29	32	15	33	34	16	35	17	36	18	37	38	39	19	22	21	20	23	40
	24	26	41	29	25	27	28	42	43	44	45	46	47	48	49	50	51	52
	53	54	55	56	57	2	1	58	3	4	5	6	7	8	9	10	11	12
	13	31	30	14														
31	40	41	19	20	42	21	43	24	23	22	25	44	26	28	31	27	29	45
	30	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
	2	1	3	4	5	6	7	8	9	10	11	12	13	33	32	14	15	34
	35	36	16	37	17	38	18	39										
33	27	26	28	45	29	30	31	32	33	46	47	48	49	50	51	52	53	54
	55	56	57	58	59	60	61	62	63	64	65	66	5	7	6	1	4	8
	2	3	9	10	11	13	12	14	15	16	34	17	35	18	36	37	19	38
	20	39	21	40	41	22	23	42	43	24	25	44						
35	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
	69	70	5	1	2	3	6	4	7	8	9	10	11	12	13	14	15	16
	17	37	36	38	18	39	19	40	41	20	42	43	21	44	22	45	24	23
	25	46	47	26	27	48	28	35	29	31	34	49	30	32	33	50		

Boufos Mitchell

The Boufos Mitchell is named after the town in Sweden from where the switched Mitchell, and in particular this movement, originated. It is a switched Mitchell (for an odd number of tables) where the field is divided into two groups (but *not* a N-S group and an E-W group). With *N* tables in the movement, where *N* is odd, the pairs in group one are numbered from 1 to *N* and in group two from *N* + 1 to 2*N*. Call the first group A and the second B. The above schedule for switching is then used to seat group A. For example, in a 13-table movement, pairs 1, 2 sit N-S at tables 1 and 2 respectively, and pairs 3, 4 and onward sit E-W. Group B fill in the remaining positions. When the change is called, those in group A move *up 1 table,* occupying the position vacated by the pair in group A at that table. Those in group B follow pairs in group B while moving *down 3 tables.* The boards move *down 1 table.* The Boufos Mitchell is particularly useful (with guide-cards for the pairs) in the combined Howell-Mitchell.

Barometer scored Boufos Mitchell

A similar Boufos method (for an odd number of tables) is to use the same boards on all tables for each round (Snow method). Group A move is up 1 table, group B up 2 tables and a new board-set is introduced on each round. As with the Boufos Mitchell the above switching schedule is applied to the tables with those numbers.

Barometer scored switched Mitchell

A similar switched-Mitchell method (for an odd number of tables) is to use the same boards on all tables for each round (Snow method). The N-S pairs are stationary, the E-W pairs move up 1 table on each change and a new board-set is introduced on each round. Unfortunately the switching schedule is not so simple. Using the example of 7 tables (where the recommended switch is for 1 round), on round 1 switch table 7 and only table 7. On round 2 switch table 6 and only table 6. In other words the switched table moves down 1 table on each change, starting at table 7 (or the highest table) and finishes at table 1 on the last round. Where the schedule recommends 2 switches (e.g., 13 tables), table 13 is switched on round 1 and round 2. Table 12 is switched on rounds 2 and 3, and so forth. Ideally table 1 is switched on round 1 also (moving down to 13 on round 2).

Even numbers of tables

These are the switching schedules for ***share-and-relay Mitchell*** movements. For this case, with *N* tables and two rounds switched the rule is switch rounds 1 and ½ *N*. Again, switches including the last round are shown in brackets. For 4 and 6 tables, see *Low numbers of tables* above, page 154.

Tables	Switch on round(s)	Imbalance	Imbalance	Calibre
4	1 (same as none)	1.9794	1.9794	46
6	1	2.9875	1.4621	80
8	1	3.9911	1.2364	91
10	1	4.9930	1.6429	90
12	1,6 (5,12)	5.9943	2.0119	90
14	1,7 (6,14)	6.9951	1.7217	94
16	1,8 (7,16)	7.9958	1.8310	95
18	1,9 (8,18)	8.9963	2.3245	94
20	1,3,11 (10, 12, 20)	9.9967	2.1136	96
22	1,2,5 (18,19,22)	10.9970	2.0074	97
24	1,3,13 (12,14,24)	11.9972	2.3502	96
26	1,3,8,21 (6,8,13,26)	12.9975	2.3775	97
28	1,2,5,19 (10,11,14,28)	13.9976	2.2442	97
30	1,2,5,10 (21,22,25,30)	14.9978	2.3663	97
32	1,2,6,21 (12,13,17,32)	15.9979	2.7972	97

Of the even numbers of tables there are two cases that are poor, the 10 and 12 table Mitchells. It is possible to switch arbitrary tables, on arbitrary rounds, to produce a better balanced movement. For all the other numbers of tables that is not the case, any improvements are marginal. The 10-table Mitchell can be conducted with switching at the tables shown here, for a calibre of 93.

Round:	1	1	1	2	2	3	4	4	4	7	9	9	9	10
Table:	1	7	9	2	9	4	3	7	8	10	1	3	6	5

The 12-table Mitchell can be conducted with switching at the tables shown here, for a calibre of 93.

Round:	1	1	3	3	4	5	5	6	6	7	8	8	8	8	10	12	12	12	12	12
Table:	4	7	3	11	11	4	8	5	7	12	3	6	8	10	2	1	5	6	9	10

The following seeding should be used to further refine the quality of the share-and-relay movements. The seeding is only supplied for the share-and-relay Mitchell as the others shown below based on the skip Mitchell are inferior to start with, and so it is pointless to attempt to improve them with seeding. The first number applies to the best pair, the second the next best, and so forth.

Tables																	
4	1	2	5	6	3	4	7	8									
6	2	1	3	7	8	4	5	9	6	10	12	11					
8	1	2	3	4	9	10	5	11	12	8	6	7	13	14	15	16	
10	1	2	3	4	5	6	11	7	8	12	9	10	13	14	15	16	17
	18	19	20														
12	1	2	3	4	5	6	13	7	14	8	9	15	10	16	11	12	17
	18	19	20	21	22	23	24										
14	1	2	3	4	6	5	7	15	16	8	17	9	18	10	19	11	20
	12	14	13	21	22	23	24	25	26	27	28						
16	1	2	3	4	5	7	6	8	17	18	19	9	20	10	21	11	22
	12	23	13	16	14	24	15	25	26	27	28	29	30	31	32		
18	1	2	3	4	5	6	8	7	19	20	9	21	22	10	23	11	24
	12	25	13	26	14	15	27	18	16	28	17	29	30	31	32	33	34
	35	36															
20	1	2	3	4	5	6	7	8	9	10	21	22	11	12	23	24	13
	14	25	15	26	16	17	27	20	18	19	28	30	29	32	31	33	34
	35	36	37	38	39	40											
22	2	1	3	4	5	6	7	8	9	10	11	24	23	25	26	12	27
	28	13	29	14	30	15	19	16	20	22	31	18	21	17	32	33	34
	35	36	37	38	39	40	41	42	43	44							
24	1	2	3	4	5	6	7	8	9	10	11	25	26	12	27	28	13
	14	29	30	15	16	31	17	32	18	19	33	34	20	21	24	36	35
	22	23	38	37	39	40	41	42	43	44	45	46	47	48			
26	8	9	10	11	12	13	28	27	14	29	30	15	31	32	16	18	19
	17	33	20	34	21	23	35	22	36	24	26	25	37	38	39	40	41
	42	43	44	45	46	47	48	49	50	51	52	2	1	3	4	5	6
	7																
28	29	30	15	31	16	32	17	33	18	34	19	35	20	36	21	37	22
	38	23	24	25	39	26	28	27	40	41	42	43	44	45	46	47	48
	49	50	51	52	53	54	55	56	1	2	3	4	5	6	7	8	10
	11	9	12	13	14												
30	36	17	37	38	18	39	40	19	41	20	23	42	22	21	24	25	27
	28	26	30	43	29	44	45	46	47	48	49	50	51	52	53	54	55
	56	57	58	59	60	2	3	1	4	5	6	7	8	9	10	11	12
	13	14	31	32	15	33	34	35	16								
32	43	20	44	21	22	45	25	24	26	23	46	29	27	30	32	28	31
	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
	64	2	3	1	4	5	6	7	8	9	10	11	12	13	14	34	33
	15	35	16	36	37	38	17	39	40	18	41	42	19				
34	26	28	25	50	31	29	32	34	30	33	51	52	53	54	55	56	57
	58	59	60	61	62	63	64	65	66	67	68	2	3	1	4	5	6
	7	8	9	10	11	12	13	14	15	36	35	16	37	38	17	39	40
	41	18	42	43	44	19	20	45	46	21	47	22	48	23	24	49	27
36	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
	2	3	1	4	5	6	7	8	9	10	11	12	13	14	15	38	37
	16	39	40	17	41	42	18	43	44	45	46	19	20	47	48	21	49
	22	50	23	51	24	52	25	26	29	53	30	28	27	33	31	36	34
	54	32	35	55													

These are the switching schedules for ***skip Mitchell*** movements, playing one round less than the number of tables. Note: Where one round is shown that round is the best and should be used.

Tables	Switch on round(s)	Imbalance	Imbalance	Calibre
4	1	1.3850	1.7496	23
6	5	2.4325	1.7344	56
8	7	3.4525	1.8402	72
10	9	4.4634	2.1827	76
12	1,3	5.4703	2.7047	77
14	1,3 (8,13)	6.4750	2.6943	83
16	3,4 (3,15)	7.4784	2.8155	86
18	2,14 (14,17)	8.4810	3.1959	86
20	2,6,7 (7,16,19)	9.4830	3.3097	88
22	1,16,20 (8,18,21)	10.4847	3.3375	90
24	1,3,19 (9,20,23)	11.4860	3.5751	90
26	2,15,22,23 (10,14,22,25)	12.4872	3.9419	90
28	1,16,23,24 (8,10,24,27)	13.4881	3.8286 (3.8475)	92
30	5,14,16,20 (12,21,26,29)	14.4890	3.9358 (4.0210)	92
32	4,5,22,30(5,21,28,31)	15.4897	4.2533(4.3273)	92

It should be noted the calibre of the skip Mitchell is less than the equivalent share-and-relay movement. This is because one round less is played and generally this will always be the consequences of reducing the number of rounds in a Mitchell movement. Truncating any Mitchell movement will lower the calibre.

Care must be taken when scoring a switched Mitchell and it is particularly important that the pair playing as N-S should enter the score on the score-sheet next to their number.

By playing the ***revenge round in the skip Mitchell*** we find the balance is still not as good as a share-and-relay Mitchell. The switch rounds are different to those shown in the above skip-Mitchell list. For 4 tables, see *Low numbers of tables* above.

Tables	Switch on round(s)	Imbalance Not switched	Imbalance Switched	Calibre Switched
4	1	2.9137	1.9794	46
6	1	3.9333	1.9582	68
8	1	4.9459	2.2939	74
10	1	5.9545	2.9171	73
12	1,12	6.9608	3.0817	78
14	1,14	7.9656	3.1808	82
16	1,16	8.9693	3.5100	83
18	1,18	9.9723	4.0309	83
20	1,3,20	10.9748	4.1306	85
22	4,7,13	11.9769	4.0237	88
24	2,3,5,13	12.9786	4.5498	87
26	2,3,5,14	13.9801	4.5496	89
28	2,3,5,15	14.9814	4.6873 (89.67%)	90
30	1,22,26,28	15.9826	4.9663 (89.89%)	90
32	2,3,5,9,17	16.9836	4.6665 (92%)	92

With the stationary-board Mitchells and even numbers of tables the switch rounds are as follows. The movements used are the double-weave and *oddly-even Mitchells* (see page 145) with the pseudo Graeco-Latin square for 6 tables. The self-orthogonal even-table movements can use the same switching with essentially the same results.

Tables	Switch on round(s)	Imbalance Not switched	Imbalance Switched	Calibre Switched
4	1(same as none)	1.9794	1.9794	46
6	1	2.9875	1.2856	84
8	1	3.9911	0.9977	95
10	1	4.9930	1.4811	92
12	1,3	5.9943	1.9829	90
14	1,2	6.9952	1.6464	95
16	2,9	7.9958	1.8832	95
18	1,5	8.9963	2.2468	94
20	2,3,11	9.9967	2.3612	95
22	5,6,8	10.9970	2.0655	97
24	2,3,13	11.9973	2.5304	96
26	5,6,10,22	12.9975	2.5348	96
28	4,5,15,17	13.9977	2.5025	97
30	5,7,18,28	14.9978	2.5181	97
32	4,5,8,18,20	15.9980	2.8838	97

7.4 Modern Howell movements

One of the advantages bowling has over golf is that you seldom lose a bowling ball.
Don Carter

The objective of a Howell movement is to produce one winner in a bridge tournament. To do this all pairs must meet all other pairs and for complete fairness play all boards. This is called a complete Howell.

The degree of accuracy in the movement can vary dramatically, and the number of variations in the types of Howell movements is quite large. To understand a Howell movement, study the following schedule closely.

Note: all pairs meet all other pairs, all pairs play all boards and all pairs play in all rounds. The number of sets of boards is equal to the number of pairs less 1. There is an even number of pairs. For more details see *Room square* above, page 56.

In this schedule the opposing pairs are shown by numbers connected by a dash and the board-set played by them is shown by a number underneath the pairs.

A set represents an arbitrary number of boards which, together with the number of rounds, dictates the length of the event.

In this example each board-set would normally consist of 4 boards producing a movement of a total of 28 boards. 3-board-sets would make a 21-board movement and 5-board-sets a 35-board movement.

Round	Table 1 NS–EW	Table 2 NS–EW	Table 3 NS–EW	Table 4 NS–EW
1	8–1 1	6–3 4	7–2 6	4–5 7
2	8–2 2	7–4 5	1–3 7	5–6 1
3	8–3 3	1–5 6	2–4 1	6–7 2
4	8–4 4	2–6 7	3–5 2	7–1 3
5	8–5 5	3–7 1	4–6 3	1–2 4
6	8–6 6	4–1 2	5–7 4	2–3 5
7	8–7 7	5–2 3	6–1 5	3–4 6

It is important to notice that after each round, each pair (except 8 or in the general case the highest numbered pair) moves to the spot vacated by the pair with the number immediately below their own. The boards also move in a very orderly fashion, moving to the next lower-numbered table except where there is a relay table. The relays in the above movement are between tables 1 and 2, where there are two board-sets, and between tables 2 and 3, where there is one board-set.

As the movement is orderly after the first round, the above schedule could be shown in a shorthand method such as:

<u>8</u>–1 R R 6–3 R 7–2 4–5 or slightly better like <u>8</u>–1 2R 6–3 R 7–2 4–5.

All the essential information is available. The board-sets are numbered from 1 to 7 from the left. The tables are numbered from 1 to 4 from the left. The sets of boards on the *relays* are shown by R, while 2R shows two relays. Pair <u>8</u> is stationary, shown by the underline. At each table the N-S pair is shown first and E-W pair second separated by a dash.

So that the pairs will know their moves, guide-cards, which can be extracted from the first schedule, need to be supplied. As an example, pair 1 would have a guide-card like

Demonstration guide card

4 Table Howell

Pair number 1

Round	sit	against	boards
1	1E-W	8	1-4
2	3N-S	3	25-28
3	2N-S	5	21-24
4	4E-W	7	9-12
5	4N-S	2	13-16
6	2E-W	4	5-8
7	3E-W	6	17-20

.

As producing one guide card per pair is a tedious process (not to mention what some players do to guide-cards), table guide-cards are usually preferred. These can be constructed from the shorthand description of the movement. As an example, the table 2 guide-card could be as like

2 NORTH 2

WEST EAST

N-S go to E-W table 4

Demonstration guide card

4 Table Howell

Round	N-S	E-W	Boards
1	6	3	13-16
2	7	4	17-20
3	1	5	21-24
4	2	6	25-28
5	3	7	1-4
6	4	1	5-8
7	5	2	9-12

E-W go to E-W table 3

2 SOUTH 2

Comparison of pairs

By expanding the above movement in a different way, it is possible to examine the comparison of pairs with each other throughout the event. The board-sets are shown as 1 to 7 across the top and the rounds are 1 to 7 down the side. The dark border outlines the associated Room square.

Sets:	1	2	3	4	5	6	7
1	8–1			6–3		7–2	4–5
2	5–6	8–2			7–4		1–3
3	2–4	6–7	8–3			1–5	
4		3–5	7–1	8–4			2–6
5	3–7		4–6	1–2	8–5		
6		4–1		5–7	2–3	8–6	
7			5–2		6–1	3–4	8–7

In the case of pair 1 it can be seen that on board-set one, they are compared with pairs 6, 4 and 7. On set 2 with 2, 7 and 5 etc.

Note that each pair is compared with all 7 other pairs and there are 7 sets of boards. Each pair is compared with 3 other pairs on each board-set. Therefore, for the comparisons to be equal, each pair should be compared with every other pair 3 times.

A movement where each pair is compared with every other pair the same number of times is called a *balanced* Howell. This is the case with the above movement and can be checked by the reader quite easily. For example, pair 1 is compared with pair 6 on sets 1, 3 and 4.

For a shorthand method of testing the comparisons, it is necessary to examine the shorthand representation of the Howell.

Eliminating any reference to the placement of the boards, the resultant diagram is called a skeleton.

8–1 6–3 7–2 4–5

or by re-arranging, the result is:

N-S	8	7	6	5
E-W	1	2	3	4

The perfect sequence of numbers should be noted (pairs 6 and 3 are repositioned and pairs 4 and 5 are switched), but it also should be realized that this perfect ordering is not necessary, and in general, is not there. ***Note:*** the switching of 4 and 5 makes the movement unbalanced.

The quality that is essential for this to be a valid skeleton is that the algebraic differences between each pair of numbers are themselves different. The highest numbered pair and pair 1 are ignored for these purposes (i.e., 8 and 1 in this case).

5 minus 4 equals 1
6 minus 3 equals 3
7 minus 2 equals 5

Note that the difference between 7 and 2 is actually 2 (the numbers run in sequence 6, 7, 1, 2, 3, …), so that the three differences are 1, 2 and 3. These are all different.

Examining the expansion of the movement given for the comparisons, it is possible to find a second skeleton under board-set 1.

N-S	5	2	3
E-W	6	4	7
Diff's:	1	2	3

It is this vertical skeleton that has to be examined to determine if in fact the Howell is balanced, not the horizontal skeleton. Notice that the table with pairs 4 and 5 was switched in the diagram of the first (horizontal) skeleton. This means that the 5-6 couple must be also switched (the 5-6 couple is also difference 1) in the vertical skeleton.

The resultant skeleton (eliminating the highest numbered pair as it is not used here) is

N-S		6	2	3
E-W	1	5	4	7

Taking all the algebraic differences of the N-S pairs, and all the algebraic differences of the E-W pairs, it is possible to determine if the movement is balanced or not.

N-S differences: 6 – 2 = 4; 6 – 3 = 3; 3 – 2 = 1
E-W differences: 5 – 1 = 4; 4 – 1 = 3; 7 – 1 = 6; 5 – 4 = 1; 7 – 5 = 2; 7 – 4 = 3

As a difference of 1 is equivalent to a difference of 6 etc., the total number of occurrences of each difference are

Differences:	1&6	2&5	3&4
Totals:	3	1	5

These are not equal; therefore the movement is not balanced. Switching the 6 and 5 back the result is

N-S		5	2	3
E-W	1	6	4	7

N-S differences: 5 – 2 = 3; 5 – 3 = 2; 3 – 2 = 1.
E-W differences: 6 – 1 = 5; 4 – 1 = 3; 7 – 6 = 1; 6 – 4 = 2; 7 – 1 = 6; 7 – 4 = 3.
This gives

Differences:	1&6	2&5	3&4
Totals:	3	3	3

All the totals are equal therefore the movement is balanced. Unfortunately this is only possible for an even number of tables, where these totals will always be equal to $T - 1$ where T = the number of tables.

For an odd number of tables to be balanced, these totals must be equal to T or $T - 2$. Even then, the balance is not perfect, for if it were, it would be necessary for the totals to equal $T - 1$ and this is not possible.

It is possible on the other hand, to find a Howell for an odd number of tables where the switching of the pairs at one table will reverse the comparisons. This means that if half the boards at that table are played N-S on each round, and the other half are played E-W, then the resultant movement will be balanced. For example, switching table 4 in the following 5-table movement will achieve this effect.

10–1 8–9 6–4 4R 7x3 2–5

Deriving Howell movements

Having already observed that the Howell movement has a horizontal skeleton and a vertical skeleton, it may be intuitively reasoned that if all the skeletons can be found, then all the Howell movements can be found. As the horizontal skeleton defines the starting positions of the pairs, this skeleton will be called the super-skeleton (SS). The vertical skeleton must be balanced and is the foundation of the movement, and so will be called the base-skeleton (BS).

Any skeleton can act as an SS or as a BS. That is, the super-imposing of any skeleton upon another produces a Howell movement. The result will not necessarily be a good Howell (meaning a Howell where no tables share boards) and only occasionally will it be a perfect Howell (meaning a Howell with all the relay boards on one relay when the number of tables is odd; and on two relays where the number of tables is even — see the 4 and *5* table examples given above).

It should be noted that any balanced BS is a balanced barometer movement. Typically this is scored using the Butler IMP scoring method. In these movements, all the tables play all the boards for each round at the same time (the Snow method). This is the trivial case of superimposing a BS on itself (or BS = SS).

The next exercise is to derive all the skeletons. This is a relatively simple process with a low number of tables. The first thing to do is to set up a table of all combinations of the different 'differences' of the pairs (ignoring the highest numbered pair and 1).

First write down the trivial situation numbering from left to right and back again.

e.g., 7–2 6–3 5–4

Then add 1 to both the N-S and E-W pairs, reduce modulus $2T-1$ (i.e., divide by $2T-1$ and use the remainder), ignore the pairings where pair 1 occurs, and then enter it as the next entry in that column. The number of rows in each column will always be equal to $2T-3$ (T = number of tables).

e.g., for 4 tables the differences are:

2	3	1
7–2	6–3	5–4
2–4	7–4	6–5
3–5	2–6	7–6
4–6	3–7	3–2
5–7	5–2	4–3

To extract the skeletons from this table, one pair of numbers is taken from each column such that no number is mentioned twice.

Therefore, all the skeletons for 4 tables are

a	8–1	7–2	6–3	5–4
b	8–1	2–4	3–7	6–5
c	8–1	5–7	2–6	4–3

By eliminating the first row in the difference table, and placing an imaginary mirror between the middle two rows like this

2–4	7–4	6–5
3–5	2–6	7–6
4–6	3–7	3–2
5–7	5–2	4–3

It can be seen that the skeleton (c) is in fact a mirror image of skeleton (b). This situation always exists irrespective of the number of tables.

Producing Howell movements

Taking the 4-table example above take the skeleton (a) as a BS. The first step is to balance this base. When doing this, firstly test for balance, then switch one pair of numbers, then test for balance etc., until a balanced case is found. When switching, do not switch the first pair of numbers (8–1), as this is the same as switching all other pairings. The order of switching should follow a pattern so that no one combination will be forgotten.

For simplicity, the recommended pattern is: 1, 2, 1, 3, 1, 2, 1.

For higher numbers of tables just add the next number and repeat the previous pattern:

1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1

So the balanced base is (switching groups 1 and 2): 8–1 2–7 3–6 5–4

The next step in deriving a Howell is to superimpose a skeleton on the base. We will take, for example skeleton (b). The first and second groups of pairs must be switched to be the same as the base, so that now we have: 8–1 4–2 7–3 6–5

Assuming pairs 8–1 are to he seated at table 1, then the tables for the other pairs are found by the calculation

1 + [N-S(SS)] – [N-S(BS)] or 1 + [E-W(SS)] – [E-W(BS)]

If the result is negative it is added to (2*T* – l), which is 7 in the case of 4 tables. As an example take the pairs 4–2 (SS) and 2–7 (BS)

1 + 4(N-S) – 2(N-S) = 3 i.e., 4–2 will be at table 3.

7–3 (SS) and 3–6 (BS) gives: 1 + 3 – 6 = –2

As this is negative, add this to 7 giving 5.

The last set is 1 + 6 – 5 = 2. Therefore the complete Howell is

8–1	4–2	7–3	6–5
1	3	5	2

Or in the shorthand method: 8–1 6–5 4–2 R 7–3 2R.

As an exercise it is suggested that the reader derive all the 4-table Howell movements via this method.

4-Table Howell

The following list has all the 4-table Howell movements, but as a BS can be balanced in more than one way (i.e., by switching different tables) they are not unique, even though the board movements will always be

the same. All have a calibre of 100 and so are called perfectly balanced. It is possible to balance these movements in a number of ways, switching pairs of tables.

BS (a) 8–1 2–7 3–6 5–4
SS (b) 8–1 6–5 4–2 R 7–3 2R
SS (c) 8–1 2R 6–2 R 7–5 4–3
BS (b) 8–1 4–2 3–7 6–5
SS (a) 8–1 2R 6–3 R 2–7 4–5
SS (c) 8–1 2R 7–5 R 4–3 2–6
BS (c) 8–1 5–7 2–6 4–3
SS (a) 8–1 5–4 7–2 R 6–3 2R
SS (b) 8–1 3–7 6–5 R 2–4 2R

Further investigation reveals that (b) on (a) is a mirror of (a) on (b) etc., so that the only unique cases are (b) and (c) on (a), and (c) on (b).

It should be noticed that all the 4-table Howell movements have perfect balance. Generally the balanced even-table Howell movements are perfect and for odd-table movements some are perfect with half-switching at one table.

Note that all the Howell movements shown here were derived by the author in 1974 using a Honeywell mainframe computer, one of the most powerful available at the time. Even so the task took many 100s of hours of computer time. Today with more powerful computers the task would be far less time consuming.

5-Table Howell

Following is a complete treatment of 5-table Howell movements, but as there are Howell movements with shares between tables, they will be shown 5-7/2-6. For the sake of convention the first table shown is thus considered to be the lower-numbered table.

Table of differences for 5 tables:

2	4	3	1
9–2	8–3	7–4	6–5
2–4	9–4	8–5	7–6
3–5	2–6	9–6	8–7
4–6	3–7	2–8	9–8
5–7	4–8	3–9	3–2
6–8	5–9	5–2	4–3
7–9	7–2	6–3	5–4

Table of skeletons:

a	10–1	9–2	8–3	7–4	6–5
b	10–1	2–4	5–9	6–3	8–7
c	10–1	3–5	9–4	2–8	7–6
d	10–1	3–5	2–6	7–4	9–8
e	10–1	4–6	3–7	5–2	9–8
f	10–1	5–7	4–8	9–6	3–2
g	10–1	6–8	5–9	7–4	3–2
h	10–1	6–8	7–2	3–9	5–4
i	10–1	7–9	2–6	8–5	4–3

Now for all the 5-table Howell movements. A balanced version of the BS is shown at the start of each set. Switching half the boards at one or more selected tables will generally improve balance in odd-table Howell movements. At these tables switching all the time will reverse the comparisons. Therefore if the pairs at this table play half the boards N-S and half E-W on each round the calibre is improved. From now on we call that half-switching. In the 5-table Howells shown below, without any switching, the calibre is 83.

Half-switching at one or two tables improves balance on all movements. This is shown with 'x' between the pair numbers at those tables. Where the best is achieved with one half-switch it is shown in preference to a pair of tables. Where table 1 is a possible choice it is shown in preference to other single table options.

The calibre is shown in brackets, the first number with no switching and the second when using half-switching at the indicated table(s).

a • 10–1 2–9 8–3 7–4 6–5

b • 10–1 R 4–2/8x7 3R 5–9 R 6–3 (83, 91)

c • 10–1 9–4/7x6 R 5–3 2–8 4R (83, 91)

d • 10–1/7–4 2R 5–3/2–6/9x8 5R (83, 91)

e • 10–1 2R 9x8 6–4/3–7 2R 5–2 R (83, 91)

f • 10–1 R 9–6 2R 7–5/4–8 3x2 2R (83, 91)

g • 10–1/7–4 5R 5–9/8–6/3x2 2R (83, 91)

h • 10–1 4R 3–9 8–6 R 5x4/7–2 (83, 91)

i • 10–1 8–5 R 2–6 3R 9–7/4x3 R (83, 91)

b • 10–1 4–2 5–9 6–3 8–7

c • 10x1 5–3 2R 9–4 2–8 2R 7–6 (83, 91)

c • 10–1 5–3 2R 9x4 8–2 2R 6–7 (83, 100)

d • 10–1 5–3/4–7/8–9 4R 2x6 2R (83, 100)

e • 10–1 8–9 6–4 4R 7x3 2–5 (83, 100)

f • 10–1 2R 5–7/6–9 3x2 3R 8–4 (83, 100)

g • 10–1/5x9 4–7 2R 8–6/2–3 4R (83, 100)

h • 10–1 R 7x2 R 8–6 R 9–3/4–5 2R (83, 100)

i • 10–1 R 5–8 2R 3–4/9–7 2x6 2R (83, 100)

c • 10–1 3–5 9–4 2–8 7–6

d • 10–1/5–3 R 2x6/8–9 2R 7–4 3R (83, 100)

e • 10–1 4x6 9–8 7–3/5–2 5R (83, 100)

f • 10–1 R 5x7 R 8–4 3–2 R 9–6 R (83, 100)

g • 10–1 2R 6x8 R 9–5/7–4/3–2 3R (83, 100)

h • 10–1 3–9 R 6x8 3R 5–4/2–7 R (83, 100)

i • 10–1 R 6–2 R 7x9 R 8–5/4–3 2R (83, 100)

d • 10–1 3–5 2–6 7–4 9–8

e • 10x1/9–8 4–6/3–7 5R 5–2 R (83, 91)

f • 10x1 R 5–7/4–8/9–6 3–2 5R (83, 91)

g • 10x1/7–4 2R 6–8/5–9/3–2 5R (83, 91)

h • 10x1 2R 6–8 R 7–2/3–9/5–4 3R (83, 91)

i • 10x1/2–6 8–5 2R 7–9/4–3 4R (83, 91)

e • 10–1 4–6 3–7 5–2 9–8

f • 10–1 5x7/4–8 R 2–3 9–6 4R (83, 100)

g • 10–1 R 6x8/5–9/7–4 2–3 5R (83, 100)

h • 10–1 R 6x8 R 7–2 4–5 R 3–9 R (83, 100)

i • 10–1 2R 7–9/8–5 4x3 3R 6–2 (83, 100)

f • 10–1 5–7 4–8 9–6 3–2

g • 10–1/3x2 8–6/5–9 5R 4–7 R (83, 100)

h • 10–1 8x6 5–4 2–7/9–3 5R (83, 100)

i • 10–1 4–3 9x7 4R 6–2 5–8 (83, 100)

g • 10–1 6–8 5–9 7–4 3–2

h • 10x1/6–8 R 7–2/5–4 2R 3–9 3R (83, 91)

i • 10x1 7–9/8–5/4–3 4R 2–6 2R (83, 91)

h • 10–1 6–8 7–2 3–9 5–4

i • 10–1 7–9 2R 2x6 5–8 2R 4–3 (83, 100)

Upon examining these 5-table Howell movements, it becomes clear that there are movements for particular applications.

(e) on (b), for example, is the best starting position for the standard Howell, as all the bye-stand boards are in one group and only one relay table is required. In this movement it is suggested that board-set 1 starts at table 3 so that after 5 rounds it comes out of the movement first and so can be scored. This is the earliest time (5 rounds) that the boards will come out without board sharing in the movement.

I mentioned above how (b) on (e) is a mirror of (e) on (b). Such a movement looks like this:

10–1 6–3 9–5 4R 2–4 8x7 (83, 100)

This also illustrates the uniqueness of a movement (or lack there of) and the ability to convert any movement into another form so as to appear as an entirely different movement. This second form of the movement was published by Olof Hanner in *Movement – a fair approach* (1994), while the first form was published in *Bridge Directing Complete* (1979) by the author. George Beynon in *Tournament and duplicate bridge* (1944) published it as 10x1 R 5–2 R 9–8 7–3 R 4–6 R (83, 91), but by 1955 in *Bridge Director's manual* had improved things with 7–3 5–2 10x1 9–8 4–6 4R (83, 91). This last effort was fairly good having all the relays on one table, but the balance was not the best. Note: Hanner and I agree on the balance because it is a complete movement.

If the scores were required earlier the best choice would be (e) on (d) where most sets (all but 3) would be

scored by the last round. As there is sharing of boards between tables, this movement needs 3 boards per round so as not to slow the movement down.

Any Howell can be played with stationary boards, with one set per table. A movement of this type would expand to a schedule the same as the 'comparison of pairs' schedule shown earlier, with each board-set number the same as the table number. If you need a stationary-board movement for 5 tables, then the choice would be (f) on (c) as the players are spread over the entire length of the movement. This means that it is less likely that the players will overhear each other during the bidding and play.

To summarize the 5-table Howell movements, the following movements are recommended.
Standard movement (83, 100): 10–1 8–9 6–4 4R 7x3 2–5.

Note that table 4 is shown by 7x3. This means that the switching of this table reverses the comparisons. Therefore if the pairs at this table play half the boards N-S and half E-W on each round the calibre is 100. Without any switching the calibre is 83. For checking purposes, the base skeleton is:

10–1 4–2 9x5 3–6 7–8.

It would be more convenient to arrange the switch at table 1 where there is a stationary pair, but this is does not produce a perfect balance for 5 tables (cf. 7 and 9 tables). The calibre of all these 5-table movements does increase to 91 if the switching is at table 1, which is quite acceptable.

An alternative (83, 100): 10–1 4–3 9x7 4R 6–2 5–8.
Early scoring movement (83, 100): 10–1 8x6 5–4 2–7/9–3 5R.
Spread movement (stationary boards) (83, 100): 10–1 R 5x7 R 8–4 3–2 R 9–6 R.

The switching is difficult in this last movement when using stationary boards as it occurs at a different table on each round and there are different pairs at the switching table on each round.

3-Table Howell

As there is only one skeleton possible for 3 tables, it is clear there cannot be a Howell of the desirable form. There are other solutions to this problem.

The first solution is to use the Snow method and is based on the use of the only skeleton available and using it as a round-robin draw with all tables playing the same boards on each round.

Round	Boards *	Table 1	Table 2	Table 3
1	1–6	6–1	5–2	4–3
2	7–12	6–2	1–3	5–4
3	13–18	6–3	2–4	1–5
4	19–24	6–4	3–5	2–1
5	25–30	6–5	4–1	3–2

*** Five-board-sets may be used but half the boards is not an integer for switching**

This movement, as such, is not balanced (calibre 60), but by switching half the boards on each round at table 3 (or 2) gives perfect comparisons (calibre 100). Table 3 is the better choice, as one of the pairs goes into the next round at that same table. The advantage of this method is the boards can be scored after each round (barometer scoring). This movement also can be shortened to 3 or 4 rounds.

This second movement has less board sharing. However, the board movement is not orderly. On the other hand it is not as unordered as those in the third movement. The major objection to this second movement is that the comparison of pairs is not equal, but this can be improved by switching half the boards on each round at table 1 (calibre 79).

Round	Table 1	Table 2	Table 3
1	6–1 1	5–2 4	4–3 2
2	6–2 2	1–3 4	5–4 3
3	6–3 3	2–4 1	1–5 2
4	6–4 4	3–5 1	2–1 3
5	6–5 5	4–1 5	3–2 5

The board-sets are shown under the pairings, these sets usually consisting of 4 to 6 boards.
The third movement has perfect comparison of players, and has no board sharing, but the disadvantage is the board moving.

Round	1	2	3	4	5
Table 1 Boards	6–1 1&2	6–2 3&4	6–3 5&6	6–4 7&8	6–5 9&10
Table 2 Boards	4–3 3&9	5–4 1&5	1–5 3&7	2–1 5&9	3–2 1&7
Table 3 Boards	5–2 6&8	1–3 8&10	2–4 10&2	3–5 2&4	4–1 4&6

The boards shown above refer to sets of 2 or 3 boards.

For the reader interested in constructing Howell movements it is suggested that the aid of a computer is obtained for more than 5 tables. The following table indicates why this is so.

Number Tables	Number of Skeletons	Number of Howells
3	1	0
4	3	6
5	9	72
6	25	600
7	133	17,556
8	631	397,530
9	3,857	14,872,592

6-Table Howell

All these movements have a calibre of 100.

Standard Movement: 12–l 11–4 4R 10–8 5–2 R 9–3 6–7

Note that as with all movements of an even number of tables, the relays are not all on one table, in contrast with an odd number of tables.

Alternative: 12–1/11–7 9–10 6–4 6R 3–8 5–2

This movement eliminates the odd–relay table, but does introduce a share. As the share is at the table with the stationary pair it is easy to manage.

Early scoring: 12–1/8–9 5–7/3–11 8R 2–6/10–4

This movement is particularly unusual with three pairs of shares and one relay table and was first found by the author in 1974. It is useful when playing 3 boards per round so that the sharing of the boards is not a burden. Thus scoring can start after 3 rounds and all but 3 sets can be scored by the last round.

Spread movement (stationary boards): 12–1 3–5 R 11–8 10–4 R 2–9 R 7–6 2R

With an even number of tables the spread movement cannot be perfect (table-relay-table-relay etc.) but involves one extra pair of tables and one pair of relays.

7-Table Howell

As before x is between the pairs indicates half-switching and will produce the best calibre movement. Where the difference in the calibre is minor (less than ½ of one percent) half-switching is still shown as an option that can be used if needed.

Standard Movement: 14x1 4–6 10–13 2–7 6R 3–12 5–11 8–9 (91, 100)

Alternative: 14x1 7–6 10–4 12–3 6R l3–8 5–2 11–9 (91, 100)
Note the table switching, if used, occurs at the table with the stationary pair.

Early scoring:
14x1 8–6 7R 5–13/4–7 3–12 10–11 2–9 (one share one relay) (91, 100)
14x1/8–5 8R 2–6 4–10/13–12 7–9 11–3 (two share one relay) (91, 100)
14x1/8–10 8R 5–9/2–7 13–12 R 6–3/11–4 (three share two relay) (91, 100)

Spread movement (stationary boards):
14–1 R 7–11 R 4–6 R 12–5 2x10 R 13–3 R 8–9 R (91, 100)

Unfortunately it is impossible to switch boards at the table occupied by the highest pair (14) and still retain alternating of the relay tables.

If switching is to be used, this movement is slightly better as pair 14 is always involved in the switching
14x1 R 5–7 R 10–2 13–9 R 3–4 R 8–11 2R 6–12 (91, 100)

8-Table Howell

All these movements are calibre 100.

Standard Movement: 16–1 13–7 9–12 3–2 6–8 14–10 R 15–5 4–11 6R
Alternative: 16–1 7–11 10–13 R 6–8 3–2 9–15 6R 12–5 14–4
Early scores (one relay table): 16–1/13–9 8–15 7–12 6–3 8R 5–14 2–4 10–11
Spread Movement: 16–1 10–11 R 14–2 R 7–9 6–12 R 5–13 R 3–8 R 4–15 2R

9-Table Howell

With 9 or more tables it is assumed that board sharing is not possible and only the standard and spread movements are given. In addition movements are chosen with the property that all the bye board-sets are on one relay table (except in even-table movements, then all but one set) and therefore may not be perfectly

balanced, though they are all without exception extremely well balanced.

18x1 9–10 6–4 16–8 2–14 8R 7–13 3–17 5–12 11–15 (95, 100)
18–1 R 5–13 R 11–6 R 4–10 R 14–7 17–2 R 3–16 R 9–8 R 15–12 R (95)

10-Table Howell

20–1 9–14 17–16 19–10 4–6 15–3 R 12–18 11–7 13–2 8R 5–8 (100)
20–1 7–13 R 6–16 R 15–4 R 14–9 8–5 R 19–2 R 10–17 R 18–3 R 12–11 2R (93)

11-Table Howell

22–1 11–6 15–14 18–9 20–3 12–5 10R 19x4 13–10 16–8 21–2 17–7 (96, 97)
22–1 R 13–5 R 20–3 R 8–15 R 6–16 R 17–14 21–2 R 9–18 R 12–7 R 11–10 R 19–4 R (96)

12-Table Howell

24–1 8–17 14–13 R 4–21 10–15 9–12 20–5 22–3 10R 11–18 6–16 2–23 19–1 (98)
24–1 8–12 20–14 16–23 4–18 11–21 10–15 9–17 7–6 5–3 2–13 3R 19–22 8R (100)
24–1 18–11 R 7–19 R 14–13 R 22–3 R 4–21 9–12 R 10–15 R 5–20 R 6–16 R 2–23 R 17–8 2R (98)

13-Table Howell

26–1 21–12 15–8 11–10 16–6 24–3 5–22 12R 4–23 20–9 18–13 25–2 19–7 17–14 (98)
26–1 R 23–4 R 19–7 R 18–13 R 21–12 R 24–3 R 17–10 22–5 R 15–14 R 11–8 R 2–25 R 6–16 R 20–9 R (98)

14-Table Howell

28–1 17–10 25–4 R 11–14 20–9 5–24 18–13 8–22 26–3 12R 7–19 16–15 2–27 23–6 12–21 (99)
28–1 13–18 R 19–7 R 5–24 R 15–16 R 22–8 R 4–25 6–23 R 10–17 R 14–11 R 9–20 R 26–3 R 2–27 R 21–12 2R (99)

15-Table Howell

30–1 15–14 10–23 21–18 13–24 27–4 25–6 17–12 14R 9–16 20–11 19–7 3–28 8–22 2–29 26–5 (98)
30–1 R 20–11 R 19–7 R 23–10 R 24–13 R 22–8 R 26–5 R 4–27 17–12 R 16–9 R 15–14 R 28–3 R 2–29 R 25–6 R 21–18 R (98)

16-Table Howell

32–1 11–18 25–9 R 7–26 17–20 27–6 15–16 19–14 5–28 3–30 14R 8–22 13–24 12–21 10–23 2–31 29–4 (98)
32–1 13–24 R 18–11 R 9–25 R 23–10 R 7–26 R 22–8 R 27–6 14–19 R 29–4 R 5–28 R 15–16 R 12–21 R 31–2 R 20–17 R 3–30 2R (99)

17-Table Howell

34–1 20–21 30–5 27–14 25–9 18–13 15–24 3–32 8–22 16R 4–31 16–19 11–26 17–10 23–12 7–28 33x2 29–6 (98, 99)
34–1 R 27–14 R 19–10 R 26–11 R 22–8 R 21–20 R 7–28 R 31–4 R 25–9 30–5 R 18–15 R 6–29 R 17–12 R 23–16 R 13–24 R 3–32 R 2–33 R (99)

18-Table Howell

36–1 8–29 27–14 R 22–17 10–28 33–4 18–15 32–5 9–25 24–13 30–7 16R 34–3 23–16 21–12 11–26 6–31 35–2 20–19 (99)
36–1 20–15 R 12–21 R 11–26 R 18–17 R 8–29 R 4–33 R 25–9 R 7–30 5–32 R 31–6 R 27–14 R 24–13 R 19–22 R 16–23 R 28x10 R 34–3 R 35–2 2R (99, 99)

19-Table Howell

38–1 6–33 15–30 12–29 27–16 13–26 35–4 24–17 32–7 9–25 18R 5–34 22–23 20–11 14x19 10–28 36–3 8–31 2–37 21–18 (99, 99)

20-Table Howell

40–1 11–31 20–17 R 13–24 19–26 12–29 36–5 8–33 6–35 18–23 28–10 34–7 18R 25–16 4x37 21–22 27–14 3–38 32–9 39–2 15–30 (99, 99)

Howells and the Snow method

This type of movement is usually called a barometer. All the tables play the same boards on each round so that the results may be calculated at this time. Hence the players can see the 'barometer' readout and know exactly where they are positioned after each round. The event also typically conducted as a round-robin, all pairs playing all other pairs by the end of the session or event
.
Round-robin events (which quite often use Butler scoring) may involve rounds of fairly extensive length. With, for example, two boards per table in a 7-table movement, there are 14 boards in play and the boards are passed around the tables until all the boards have been played. The players then move to the position for the next round and a new board-set is introduced.

Using duplicated boards it is possible to complete the round-robin in one or two sessions. In this event the scoring is often done by scorers in the round following the play, and the scores are posted immediately they are known.

The standard ***balanced*** base skeleton is all that is needed for this movement, with the half-switching of one table necessary for thoroughness in the odd-table movement.

4-table barometer. An expanded movement for 4 tables is:

Round/Table	1	2	3	4	Board-set
1	8–1	5–6	2–4	3–7	1
2	8–2	6–7	3–5	4–1	2
3	8–3	7–1	4–6	5–2	3
4	8–4	1–2	5–7	6–3	4
5	8–5	2–3	6–1	7–4	5
6	8–6	3–4	7–2	1–5	6
7	8–7	4–5	1–3	2–6	7

In a shorthand representation this can be shown as:

8-1 5-6 2-4 3-7 (100)

The origins of such cyclic schedules are discussed in *The first schedules*. There Safford (in 1891) gave us a schedule for 8 players: 1–2 v 5–3, 8–4 v 6–7. This is cyclic and therefore can be modified very simply to

represent pair numbers and the sequence adjusted slightly to have 8 stationary (instead of 1) by subtracting 1 from each number (1 becoming 8) to get 8–1 4–2 7–3 5–6, which with some switching becomes our example of a Howell with the Snow method. It is skeleton (b) for 4 tables as discussed in *Deriving Howell movements*:

a	8–1	7–2	6–3	5–4
b	8–1	2–4	3–7	6–5
c	8–1	5–7	2–6	4–3

The recommended Howell barometer movements are supplied here, with the first number indicating the number of pairs in the movement, and is also the stationary pair.

<u>10</u>–1 4–2 9x5 3–6 7–8 (83, 100)
<u>12</u>–1 4–2 9–6 11–5 10–3 7–8 (100)
<u>14</u>x1 6–5 8–2 9–13 3–11 7–4 12–10 (91, 100)
<u>16</u>–1 15–2 7–3 10–4 13–6 14–9 5–8 11–12 (100)
<u>18</u>x1 4–2 12–16 11–17 13–5 7–14 15–10 6–3 8–9 (95, 100)
<u>20</u>–1 19–2 3–18 4–17 16–5 15–6 14–7 13–8 9–12 11–10 (100)
<u>22</u>–1 2–21 3x20 4–19 13–5 6–16 9–18 8–15 12–7 17–14 11–10 (96, 98)
<u>24</u>–1 2–23 3–22 4–21 5–20 19–6 7–18 17–8 9–16 10–15 14–11 13–12 (100)
<u>26</u>–1 25–2 24–3 23–4 5–22 6–16 7–19 9–20 21–12 10–17 18–13 11–8 15–14 (98)

Also it was mentioned above the simple (a) skeleton is derived by numbering in sequence starting with pair number 1 at table 1 as E-W and then counting in sequence across the E-W seats to the highest table, then placing the next number as N-S there and then counting back again to table 1 for the N-S pairs. This (a) skeleton is a perfectly good barometer movement with table 4 switched for perfect balance.

<u>8</u>–1 7–2 6–3 4–5 (100)

This method of organizing barometer movements for any number of tables is the simplest way for the players and the organizers. I call this an endless schedule and it is also used in the Flower modification.

The original idea was put forward by Mr. A. G. Safford in the 1890s for use in teams' movements. One problem with the endless schedule is that the movements will not always be perfectly balanced, even with optimum table switching. In addition the calibre with no switching in all endless movements is less than 50 and usually about 44. Using these movements without switching is highly undesirable.

After applying the stated switches it is possible to improve the balance with half-switching at one table and for the endless movements this table number is specified. For the other movements the x between N-S and E-W indicates half-switch. The calibre is shown in brackets and the first value produced with no half-switch and the second with half-switch. When these two numbers are the same, the difference is less than one. Half-switching for table numbers greater than 9 does not produce any really meaningful effect and is not considered for more than 23 tables. Also it is possible that alternate switching will produce similar results for balance, but these are not included, particularly as it often needs more than one table to be switched. The minimum switches that produce the best balance are always chosen.

It can be seen that switching more than 13 tables is never required for a calibre of more than 97, except for a few examples where the higher switches does produce a perfect balance. For numbers of tables higher than 45 the simple rule is not to switch the middle one or two tables and then switch the 6 tables either side of the centre table(s). With an odd number of tables leave the centre table un-switched and for an even number the centre two tables. This gives a calibre of 70 or more, which is generally satisfactory, particularly with such a large number of tables.

The recommended endless Howell movements are:

[5] switch table 2 and half-switch 5 (83, 91)
[6] switch table 3 (100)
[7] switch table 5 and half-switch 7 (91, 94)
[8] switch table 4 and half-switch 3 (86, 94)
[9] switch tables 2, 5 and half-switch 3 (90, 94)
[10] switch tables 3, 4, 9 (100)
[11] switch tables 2, 5, 7 and half-switch 3 (94, 95)
[12] switch tables 6, 8, 11, 12 (100)
[13] switch tables 2, 3, 6, 8 and half-switch 3 (96, 97)
[14] switch tables 4, 8, 10 and half-switch 5 (95, 98)
[15] switch tables 2, 8, 9, 11, 13 (98)
[16] switch tables 4, 7, 12, 13, 14, 16 (100)
[17] switch tables 4, 7, 9, 13 and half-switch 12 (96, 96)
[18] switch tables 2, 5, 7, 11, 12 and half-switch 17 (97, 97)
[19] switch tables 5, 7, 8, 12, 18 (97)
[20] switch tables 2, 4, 7, 8, 12, 14 and half-switch 5 (97, 98)
[21] switch tables 3, 5, 9, 10, 11, 19 and half-switch 8 (97, 98)
[22] switch tables 3,4,6,8,9,13,19,20,21 (100)
[23] switch tables 2,7,8,9,12,15,22 and half-switch 20 (97, 98)
[24] switch tables 6,11,12,14,16,20,21,23,24 (100)
[25] switch tables 4,8,13,15,18,19,21,25 (98)
[26] switch tables 2,7,8,9,11,14,16,20,24 (98)
[27] switch tables 1,2,3,4,11,14,16,17,20,21,25,27 (99)
[28] switch tables 4,6,12,14,17,18,21,22,28 (98)
[29] switch tables 8,12,13,14,19,21,23,24,27 (98)
[30] switch tables 3,7,9,11,12,14,15,19,24,25 (100)
[31] switch tables 7,8,11,12,14,17,19,24,28,30 (98)
[32] switch tables 5,6,10,12,13,14,16, 21,24,30 (98)
[33] switch tables 4,6,9,10,14,16,17,22,23,25 (98)
[34] switch tables 3,4,6,8,9,12,13,14,19,21,28,29,31,32,33 (100)
[35] switch tables 7,9,10,1114,16,17,22,28,32 (98)
[36] switch tables 8,12,14,15,18,22,23,24,27,29,32,34,35,36 (100)
[37] switch tables 7,9,11,14,15,16,17,21,27,30,35 (98)
[38] switch tables 4,6,9,10,14,16,17,21,22,25,28,30 (98)
[39] switch tables 8,10,12,13,15,18,19,23, 24,31,33,37 (98)
[40] switch tables 4,7,8,13,15,16,18,25,28,29,30,31,34,35,36,38,40 (100)
[41] switch tables 10,12,16,17,22,24,25,29,32,33,35,38 (98)
[42] switch tables 3,6,7,9,14,15,16,19,20,21,23,25,33,35,36,40 (100)
[43] switch tables 13,15,19,20,22,26,27,30,32,35,36,38 (98)
[44] switch tables 11,14,15,19,22,25,27,28,32,34,37,38,39 (98)
[45] switch tables 10,16,18,21,22,23,29,31,32,34,37,38,41 (98)

Four groups – one session

These cyclic movements are designed for competitions with 4 separate groups where each group needs to play the other groups, but not the pairs in their own group. The shorthand representation shows the start position prefixed by the number of tables and rounds. For 6 tables (12 pairs and 4 groups of 3) playing 9 rounds, it is shown as [6, 9]. Stationary pairs, half-switched tables and calibre are shown in the usual way. The first group is the stationary pairs. The second group is pairs 1, 4, 7… the third 2, 5, 8… and the last 3, 6, 9… All moving pairs follow the next lower pair with pair one following the highest-moving pair who has the same number as the total number of rounds.

The results of the pairs within each group are comparable: all pairs in each group play each board in the same direction. Note: if you use half-switched tables this property disappears.

These cycles are predictable for any number of tables. Start with the stationary pairs sitting N-S at every second table, then pair one at table 1 as E-W numbering in sequence and after two pairs move to N-S after the next stationary pair and so on. At the last table change direction and continue the sequence, filling in the gaps. The calibre and half switching are also predictable.

[6, 9] 10x1 9x2 11–8 3–7 12–4 6–5 (76, 92)

[8, 12] 13x1 12x2 14–11 3–10 15–4 9–5 16–8 6–7 (76, 92)

[10, 15] 16x1 15x2 17x14 3–13 18–4 12–5 19–11 6–10 20–7 9–8 (75, 93)

[12, 18] 19x1 18x2 20x17 3–16 21–4 15–5 22–14 6–13 23–7 12–8 24–11 9–10 (75, 92)

[14, 21] 22x1 21x2 23x20 3x19 24–4 18–5 25–17 6–16 26–7 15–8 27–14 9–13 28–10 12–11 (75, 92)

Four groups – three sessions

These cyclic movements are designed for competitions where there are 4 separate groups, each group needing to play the other groups, but not the pairs in their own group. There are three stanzas using one cycle for all sessions and are easily defined. It uses the simple endless Howell schedule with a small modification.

Consider the simplest example of 6 tables with 3 round stanzas.

The 6-table endless Howell is: 12–1 11–2 10–3 9–4 8–5 7–6, which is defined by numbering from left to right for the N-S pairs and back again for E-W. Consider this as 4 groups of pairs 1–3, 4–6, 7–9 and 10–12. Make 10 to 12 stationary and the other groups follow the pair one less than their number. The lowest follows the highest in their group. This gives a movement for the first 3 rounds, in shorthand is

12–1 11–2 10–3 9–4 8–5 7–6

The expanded schedule for the first stanza is

	1	2	3	4	5	6
1	12 1	11 2	10 3	9 4	8 5	7 6
2	12 2	11 3	10 1	7 5	9 6	8 4
3	12 3	11 1	10 2	8 6	7 4	9 5

For stanzas two and three the stationary pairs remain and the other groups rotate their positions, 1–3 play where 4–6 were, 4–6 to 7–9 and lastly 7–9 to 1–3. This gives

Stanza 2: 12–7 11–8 10–9 6–1 5–2 4–3 and stanza 3: 12–4 11–5 10–6 3–7 2–8 1–9

This gives a complete schedule over three stanzas (76):

	1	2	3	4	5	6
1	12 1	11 2	10 3	9 4	8 5	7 6
2	12 2	11 3	10 1	7 5	9 6	8 4
3	12 3	11 1	10 2	8 6	7 4	9 5
4	12 7	11 8	10 9	6 1	5 2	4 3
5	12 8	11 9	10 7	4 2	6 3	5 1
6	12 9	11 7	10 8	5 3	4 1	6 2
7	12 4	11 5	10 6	3 7	2 8	1 9
8	12 5	11 6	10 4	1 8	3 9	2 7
9	12 6	11 4	10 5	2 9	1 7	3 8

The same principles apply for 8, 10, 12… tables over 12, 15, 18… rounds in three stanzas of 4, 5, 6… rounds. In all cases the calibre is approximately 75.

Double (or interwoven) Howells

There are as many sets of boards in a Howell movement as there are pairs minus one (equal to the number of rounds). There are only as many sets in play as there are tables (using a standard Howell with no shares). There are some (the number of table minus one) sets of boards not in play on each round.

If a second Howell movement is conducted using this same set-of-boards, but using the boards out of play in the first Howell, then only 1 table needs to share boards with the first movement. This gives us 2 Howell movements with the same number of tables in each, playing the same boards.

This movement is valuable in a two-session event where the field is divided into 2 groups of pairs and in the first session a Mitchell movement is used so that the two groups meet each other. In the second session the double Howell is used so that each group meets the pairs in its group. Major Alfred M. Gruenther was the first to describe these in 1933. From that time on they became known as interwoven Howells. The following examples are the double Howells produced by the author in 1974.

Double 4-table Howell

An examination of the list of all the 4-table Howell movements reveals that the first two Howell movements will combine to give a double Howell. These movements were first published in *Bridge Directing Complete*, 1979.

Board-sets	1	2	3	4	5	6	7
Howell 1	8–1	6–5	4–2		7–3		
Howell 2	1–8			2–6		5–7	3–4
Table	1 & 1A	2	3	4	5	6	7

Full expansions:

	1 & 1A	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7
Round 1	8–1	6–5	4–2		7–3		
	1	2	3	4	5	6	7
	1–8			2–6		5–7	3–4
Round 2	8–2	7–6	5–3		1–4		
	2	3	4	5	6	7	1
	2–8			3–7		6–1	4–5
Round 3	8–3	1–7	6–4		2–5		
	3	4	5	6	7	1	2
	3–8			4–1		7–2	5–6
Round 4	8–4	2–1	7–5		3–6		
	4	5	6	7	1	2	3
	4–8			5–2		1–3	6–7
Round 5	8–5	3–2	1–6		4–7		
	5	6	7	1	2	3	4
	5–8			6–3		2–4	7–1
Round 6	8–6	4–3	2–7		5–1		
	6	7	1	2	3	4	5
	6–8			7–4		3–5	1–2
Round 7	8–7	5–4	3–1		6–2		
	7	1	2	3	4	5	6
	7–8			1–5		4–6	2–3

This 8-table movement can be abbreviated to the following, noting 1 follows 7 and 9 follows 15. This is shown as {1-7, 9-15}. The calibre is shown in brackets after each movement and generally the balance is poor because the movements are incomplete with each pair playing only half the field. Any amount of switching does very little to improve the balance and is generally not worth the effort and introduces complications for the players.

8 tables, 7 rounds: 8–1/9–16 6–5 4–2 10–14 7–3 13–15 11–12 (41) {1–7, 9–15}
7 tables and 7 rounds: 8–1 6–5 4–2 9–13 7–3 12–14 10–11 (62) (1–7, 8–14}

Double 5-table Howell

10–1 9–8 6–4 4R 3–7 5–2
3R 3–7 5–2 1–10 9–8 6–4 R

Giving: 10–1 9–8 6–4 13–17 15–12 11–20 19–18 3–7/16–14 5–2

Unfortunately it is impossible for the 2 tables with the stationary pairs to share boards in a 5-table double Howell unless additional shares are introduced.

10–1 4R 9–3 8–6 R 4–5/7–2
1–10 9–4/6–7 R 3–5 2–8 4R

10 tables, 9 rounds: 10–1/11–20 19–14/16–17 R 13–15 12–18 9–3 8–6 R 4–5/7–2 (41) {1–9, 11–19}
9 tables, 9 rounds: 1–10 18–13/16–15 R 12–14 11–17 9–3 8–6 R 4–5/7–2 (45) {1–9, 10–18}

Double 6-table Howell

12–1 R 10–7 3R 5–4 9–11 3–8 R 6–2
1–12 11–7 R 5–10 2–4 9–8 3R 6–3 R

12 Tables, 11 rounds: 12–1/13–24 23–19 10–7 17–22 14–16 21–20 5–4 9–11 3–8 18–15 6–2 (44) {1–11, 13–23}
11 Tables, 11 rounds: 12–1 22–18 10–7 16–21 13–15 20–19 5–4 9–11 3–8 17–14 6–2 (46) {1–11, 12–22}

Double 7-table Howell

14–1 5R 2–8 12–11 5–7 9–4 3–13 R 6–10
1–14 10–7 4–12 8–6 3–2 9–13 5R 11–5 R

14 tables, 13 rounds: 14–1/28–15 24–21 18–26 22–20 17–16 23–27 2–8 12–11 5–7 9–4 3–13 25–19 6–10 (44) {1–13, 15–27}
13 tables, 13 rounds: 14–1 23–20 17–25 21–19 16–15 22–26 2–8 12–11 5–7 9–4 3–13 24–18 6–10 (45) {1–13, 14–26}

Double 8-table Howell

16–1 5–9 7–13 15–10 3–2 14–11 6–8 2R 4–12 5R
1–16 6R 7–9 13–2 R 8–14 6–3 5–15 4–12 11–10

16 tables, 15 rounds: 16–1/17–32 5–9 7–13 15–10 3–2 14–11 6–8 23–25 29–18 4–12 24–30 22–19 21–31 20–28 27–26 (42) {1–15, 17–31}
15 tables, 15 rounds: 16–1 5–9 7–13 15–10 3–2 14–11 6–8 22–24 28–17 4–12 23–29 21–18 20–30 19–27 26–25 (43) {1–15, 16–30}

Double 9-table Howell

18–1 12–7 6–16 3–5 9–17 14–13 5R 4–10 15–2 11–8 3R
1–18 5R 6–8 11–15 7–13 4–16 3–2 3R 17–10 12–9 14–5

18 tables, 17 rounds: 18–1/ 19–36 12–7 6–16 3–5 9–17 14–13 24–26 29–33 25–31 22–34 21–20 4–10 15–2 11–8 35–28 30–27 32–23 (43) {1–17, 19–35}
17 tables, 17 rounds: 18–1 12–7 6–16 3–5 9–17 14–13 23–25 28–32 24–30 21–33 20–19 4–10 15–2 11–8 34–27 29–26 31–22 (43) {1–17, 18–34}

Double 10-table Howell

20–1 9–17 16–11 3–5 2R 19–10 14–13 6–18 8–12 15–2 7–4 7R
1–20 3R 16–11 5–7 6R 17–8 4–3 12–18 9–2 15–19 6–14 13–10

20 tables, 19 rounds: 20–1/21–40 9–17 16–11 3–5 36–31 25–27 19–10 14–13 6–18 8–12 15–2 7–4 37–28 24–23 32–38 29–22 35–39 26–34 33–30 (45) {1–19, 21–39}
19 tables, 19 rounds: 20–1 9–17 16–11 3–5 35–30 24–26 19–10 14–13 6–18 8–12 15–2 7–4 36–27 23–22 31–37 28–21 34–38 25–33 32–29 (45) {1–19, 20–38}

Hanner Double Howells

7 tables, 7 rounds: 8–1 5–4 2–7 10–13 3–6 14–9 11–12 (50) {1–7, 8–14}
8 tables, 7 rounds: 8–1/9–16 5–4 2–7 11–14 3–6 15–10 12–13 (41) {1–7, 9–15}
9 tables, 9 rounds: 10–1 16–17 2–8 R 7–6/15–12 4–9/18–14 R 3–5 11–13 (52) {1–9, 10–18}
10 tables, 9 rounds: 10–1 9–8 4–6 12–16 18–15 11–20 14–13 17–19/7–3 5–2 (43) {1–9, 11–19}
11 tables, 11 rounds: 12–1 22–18 10–2 16–21 13–15 20–19 8–9 11–5 3–7 14–17 6–4 (49) {1–11, 12–22}
12 tables, 11 rounds: 12–1/13–24 23–19 10–2 17–22 14–16 21–20 8–9 11–5 3–7 15–18 6–4 (44) {1–11, 13–23}
13 tables, 13 rounds: 14–1 26–21 17–20 25–19 3–10 12–9 4–8 7–2 11–13 6–5 18–16 15–24 22–23 (50) {1–13, 14–26}
14 tables, 13 rounds: 14–1/15–28 27–22 18–21 26–20 3–10 12–9 4–8 7–2 11–13 6–5 19–17 16–25 23–24 (45) {1–13, 15–27}
15 tables, 15 rounds: 16–1 5–10 4–7 6–15 14–12 11–3 2–13 27–22 29–25 9–8 20–23 19–18 21–30 24–17 28–26 (48) {1–15, 16–30}
16 tables, 15 rounds: 16–1/17–32 5–10 4–7 6–15 14–12 11–3 2–13 28–23 30–26 9–8 21–24 20–19 22–31 25–18 29–27 (45) {1–15, 17–31}
17 tables, 17 rounds: 18–1 13–7 9–12 16–6 15–3 19–31 26–25 30–34 29–32 24–22 21–27 23–33 28–20 11–10 5–14 17–2 4–8 (49) {1–17, 18–34}
18 tables, 17 rounds: 18–1/19–36 13–7 9–12 16–6 15–3 20–32 27–26 31–35 30–33 25–23 22–28 24–34 29–21 11–10 5–14 17–2 4–8 (46) {1–17, 19–35}

Twin Howell

An alternative approach to the double Howell is to place two Howell movements beside each other so that they share boards throughout the movement (also called a twin Howell). Advantage: the scoring can be done during play as the boards come out after the same number of rounds as in a standard Howell.

For example, a 5-Table Howell (doubled up)

10–1	8–9	6–4	4R	7–3	2–5
1–10	3–4	9–7	4R	2–6	8–5

10–1/11–20 8–9/13–14 6–4/19–17 4R 7–3/12–16 2–5/18–15 (41)

By using this in the interwoven movement, the problem of 2 groups of 9 pairs is solved. The Mitchell skips the first round and these pairs meet each other at table 1 in the double Howell.

With the one particular Howell for 6 tables that has 3 shares, it is possible (when playing 33 boards) to have 2 or 3 consecutive Howell movements while still using only one set-of-boards. The arrangement is as follows for 2 Howells

2–6/10–4 12–1/8–9 5–7/3–11 R 14–18/22–16 24–13/20–21 17–19/15–23 4R (44) {1–12, 13–24}

For three Howells

2–6/10–4 12–1/8–9 5–7/3–11 R 14–18/22–16 24–13/20–21 17–19/15–23 R 26–30/34–28 36–25/32–33 29–31/27–35 (29) {1–12, 13–24, 14–36}

Each Howell moves within itself, while the boards move as normal, down one table. The director has to be careful to change the score-sheets (when scoring each section separately), as the boards move from one section to another (especially when there is no relay table where the sheets can be exchanged). The advantage of this movement is the low number of boards needed and the possibility of scoring all three sections after 3 rounds when scoring the sections separately.

A similar, though not as effective, movement for double 5 tables is

5–2 R 10–1/9–8 4–6/3–7 R 12–15 R 11–20/18–19 16–14/17–13 (40) (1–10, 11–20}

7.5 Incomplete Howells

No wife can endure a gambling husband, unless he is a steady winner.
Thomas Robert Dewar

Any Howell where the pairs do not meet all the other pairs is called an incomplete Howell and is also called a shortened or reduced Howell or a ¾-Howell. A simple way to produce an incomplete Howell is to append a table to a complete Howell.

Half table issue

A half table can be treated as a full table in the Howell Movement so that each pair has a bye on one round. The usual procedure with this movement is to treat the stationary pair as the phantom pair so that all pairs sit out at the same table. This also produces the best balanced movements with half-tables, starting with the best available examples the resulting movement will not be perfect, but generally very good.

If the movement is already under way, a pair can be added at virtually any stage during the game by placing the pair at any table where there is not a stationary pair. The pair currently sitting in that direction at the table is displaced. The last pair then remains stationary at that table and as pairs move to that position occupied by the extra pair they sit out for that round and proceed to the next round as though they have played at that table. Essentially this procedure is converting an existing Howell movement into a ¾-Howell. See *appendix pair* below.

When conducting a standard ¾-Howell the half table is not added conveniently as not all the players sit out and the scores have to be factored up for the pairs sitting out.

Appendix table

Any Howell can have one or more tables appended to it without greatly disrupting the movement. An appended table does corrupt the balance of the movement however, and the resulting movement is not a true Howell as all the pairs do not meet all other pairs.

Any table in a Howell movement (which does not have a stationary pair) can act as an appendage table. Selecting one of these tables, an appendix table is placed next to it so that it is possible for the two tables to share boards.

One of the pairs from the appendage table is moved to the appendix table, keeping the same pair number and direction originally assigned. The vacant positions are then filled with two pairs who will be stationary for the whole event. These pairs are assigned pair numbers 1 and 2 more than the highest numbered pair in the movement.

The moving pairs at these tables move as though they have played against each other. The new moving pairs who arrive at the appendage table will either sit in the vacant position, or be directed by the stationary pair who occupies their seat, to move to the appendix table.

Appendix pair

The same principles apply as those used in the appendix table. The additional pair is seated at any table where two pairs are moving and the pairs arriving at that table who would sit in the position occupied by the extra pair sit out for the round. Choosing the table where the pairs pivot (the pair numbers at that table are one apart) is probably the most convenient and generally gives best balance.

¾-Howells

The name ¾-Howell was first used for an incomplete Howell by Sam Gold of Montreal in the early 1950s where they were printed by the ACBL and sold as guide-cards. An appendix table Howell movement is in effect a ¾-Howell movement, as all pairs do not meet all other pairs.

> ***Note:*** In addition to the ¾-Howells supplied here there are bicyclic schedules supplied with scissor schedules. See *pairs' bicyclic schedules* below in *scissor schedules*, page 337.

There are more effective ¾-Howell movements than the appendix table versions; these eliminate the sharing of boards. The method of derivation of the ¾-Howell is the same as for the ordinary Howell with the proviso that the base skeleton and the super skeleton include more than one idle pair. In the case of the standard Howell, pair 1 is the only idle pair in the skeleton (being positioned opposite a stationary pair). Compare the Room square with the *house square* where there are two idle pairs.

For example consider the following 13-round movement.

Base skeleton
(14)–1 5–7 4–3 12–6 10–2 9–13 (l5)–8 (16)–11
Super Skeleton
(14)–1 6–8 3–2 10–4 7–12 5–9 (15)–11 (16)–13

SS – BS give the following starting positions respectively: 1 2 13 12 11 10 4 3

Giving the ¾-Howell <u>14</u>–1 6–8 <u>16</u>–13 <u>15</u>–11 5R 5–9 7–12 10–4 3–2,
or, start one round later for neatness <u>16</u>–l <u>15</u>–12 5R 6–10 8–13 11–5 4–3 <u>14</u>–2 7–9.

In this example pairs 14, 15 and 16 are stationary.

The advantage of the ¾-Howell is that there can be a predetermined number of boards in play irrespective of the number of tables up to a number equal to the number of sets of boards less one. If the number of tables equals the number of sets of boards then the movement could be called a half Howell, but it really is a Mitchell movement. Calibre is shown as usual for all these movements. Half-switch the tables with x shown between the pairs and the second calibre value applies. Also note where there is only one or two tables without stationary pairs, switching the first round will always improve the balance to at least that obtained with half-switching and is much simpler to implement.

It is possible with an odd number of rounds for the event to start as a Howell and, before the move into the second round, change the movement by adding tables without disrupting the players greatly. The number of boards is constant, only the pair numbers need to be changed and one board-set is taken from the relay table and placed on the new table. Where this is possible the 'start Howell' is shown first.

Many of these movements were first published in 1979 by the author (these are shown with '– McKinnon' after the movements) and to this day are the best available in terms of calibre. Other movements produced since then have been examined and none is any better and most are of a poorer quality. Most others were originally produced by Hanner, but have been changed to produce correctly balanced versions. As these are incomplete movements the difference of opinion on the way to balance is clearly illustrated. The remainder are by Schiereck or Gold and have been modified to conform to the presentation methods and improve the balance.

Note: The examples shown with 'start Howell' grow by adding tables after play has commenced, just the player numbers change. The boards in play at each table do not change, except for case of 14 tables playing 15 rounds.

6 rounds

[4] 8–1 R 7x5/4–6 2–3 2R (80, 91) – McKinnon
[5] 10–1 R 9x6 8–5 4–3 7–2 (58, 72) – McKinnon

7 rounds

[4] 8–1 2R 7–5 R 3–4 6–2 (100) (Start Howell) – McKinnon
[5] 10x1 R 6–7 3–5 R 8–4 9–2 (85, 89) – McKinnon
[6] 12–1 R 11x7/10x2 6–5 R 8–4 9–3 (56, 79) – McKinnon

8 rounds

[5] 10–1/9–5 7–4 R 2x3 6–8 3R (85, 93) – McKinnon
[6] 3–2 2R 12–1 11–8 10–6 9x5 4–7 (73, 87) – McKinnon
[7] 14–1 R 13x6 12x2 11x4 7–8 10–3 9–5 (55, 77) – McKinnon

9 rounds

[5] 7x3 2–5 10–1 8–9 6–4 4R (83, 100) (start Howell) – McKinnon
[6] 12–1 11–4 10x8 2–7 9–6 5x3 3R (76, 92) – McKinnon
[7] 12–8 3–2 11x6 10x9 4–7 14–1 R 13–5 R (67, 83) – McKinnon
[8] 16x1 15–6 14x2 13x7 12–3 11–8 R 4–5/10–9 R (54, 74) – McKinnon

10 rounds

[6] 11x1 R 12–10 2–5 R 9–8 R 7x3 R 6x4 (88, 96) – Hanner
[7] 11x1 10x8 6–7 12–5 14–4 13x3 9–2 3R (79, 83) – Hanner
[8] 11x1 12x8 13x2 16–6 14–9 7–4 15–10 3–5 2R (63, 85) – Hanner

11 rounds

[6] 10–8 5–2 R 9–3 6–7 12–1 11–4 4R (100) (start Howell) – McKinnon
[7] 14–1 6–4 9–10 3–7 12–2 13–11 5–8 4R (91) – McKinnon
[8] 16x1 15–4 13–11 14x10 9–8 7–5 12–3 6–2 3R (80, 89) – McKinnon
[9] 18x1 17–5 16x11 10–2 15x9 8–7 14–6 13–4 12–3 2R (61, 82) – McKinnon
[10] 18–1 17x5 16x11 19x10/20–2 15x9 7–8 14x6 13–4 12–3 2R (53, 77) – McKinnon

12 rounds

[7] 13–1 5–7 11–12 3–10 6–9 8–4 14x2 5R (88, 94) – Hanner
[8] 13–1 14–8 11–6 15–9 16x4 3–7 5–2 10–12 4R (86, 90) – Hanner
[9] 13x1 8–4 14–10 15–5 2–9 16–11 17–3 7–6 18x12 3R (69, 81) – Hanner
[10] 13–1 14x7 15x2 16x11 17–8 18–3 9x5 20x12 19–10 6–4 2R (59, 75) – Hanner
[11] 22x1 21x12 20x11 19x10 18–9 R 17–8 16–7 15–6 14–5 13–4 3–2 (53, 78) – Schiereck

13 rounds

[7] 3–12 5–11 8–9 14x1 4–6 10–13 2–7 6R (91, 100) (start Howell) – McKinnon
[8] 6–10 13–8 11–5 4–3 14x2 7–9 16–1 15–12 5R (91, 93) – McKinnon
[9] 18–1 17–12 8–5 10–2 9–13 16x7 15x6 11–4 14–3 4R (84, 92) – Gold
[10] 20x1 19–6 18–8 17x13 3–7 16–12 15x2 5–11 14–10 9x4 3R (69, 84) – Gold
[11] 7–8 22x1 21x3 20–5 19x2 18x10 17–12 16–4 15x11 14–13 6–9 2R (58, 79) – Gold
[12] 7–8 22x1 21–3 20x5 19–2 18–10 17–12 16x4 15–11 14x13 23x6/24–9 2R (52, 77) – Gold

14 rounds

[8] 15–1 R 12–6 2R 14–10 8–3 2R 9–7 2–13 4–5 16–11 R (96) – Hanner
[9] 15–1 4–8 10–2 16–12 3–14 9–7 17–11 18x13 5–6 5R (89, 91) – Hanner
[10] 15x1 16x8 17–6 20x13 10–7 3–4 18–14 12–2 19–5 9–11 4R (77, 85) – Hanner
[11] 15x1 16x5 7–2 17–12 10–14 18x8 19x13 20–4 21–6 3–11 22–9 3R (68, 81) – Hanner
[12] 15–1 16–14 17x8 18x10 19–12 20x5 21x3 11–9 24–6 22–4 23x13 7–2 2R (58, 81) – Hanner

15 rounds

[8] 16–1 13–7 9–12 3–2 6–8 14–10 R 15–5 4–11 6R (100) (start Howell) – McKinnon
[9] 16–1 8–15 2–13 6–12 17–10 7–9 14–11 5x4 18–3 6R (95, 96) – Hanner
[10] 16–1 8–14 17x2 18x7 19–9 11x13 20–15 3–6 5–10 4–12 5R (83, 88) – Hanner
[11] 16–1 11–13 17–6 5x2 18x7 22x10 19–12 14–15 20x4 9–3 21–8 4R (70, 84) – Hanner
[12] 16x1 17x15 18x12 19–5 20x9 11–14 21–10 22–4 8–6 23–2 24–13 3–7 3R (65,80) – Hanner
[13] 16x1 17x8 18x7 11–5 19x10 20–15 21–2 26x4 22–6 13–3 23–9 24–14 25–12 2R (57, 76) – Hanner
[14] 16x1/17x3 24–10 26x12 R 20–6 27x13 28x5 R 22–8 21x7 15–14 18–2 19–4 23–9 25–11 (52, 78) – Schiereck

16 rounds

[9] 17x1 4–8 16–14 11x12 6x9 15–5 3–10 2–13 18–7 7R (89, 97) – Hanner

Incomplete interwoven Howells

In the ¾-Howells the number of tables is always less than the number of rounds. Where the number of tables is greater than the number of rounds it would be usual to conduct an event with a web Mitchell or a similar type of movements.

Here we have reduced interwoven Howells. These were devised by Olof Hanner and are generally only useful in one session events. There are two groups of pairs, shown within the braces {}. These schedules are modified from the originals to give the best balance, though in most cases they are still of very poor quality. Generally, arbitrary switching of tables will not improve the balance and is not worth the effort.

8 rounds

[9] 8-1 7-10 12-6 5-14 16-9 4-11 3-13 2-17/18-15 (51) {1–8, 9–16}
[10] 17–1 9–6 8–16 5–7/ 20–13 4–3/ 18–15 12–14 10–11 2–19 (45) {1–8, 9–16}
[11] 17–1 9–6 21–16/8–22 20–13/5–7 18–15/4–3 12–14 10–11 2–19 (37) {1–8, 9–16}

9 rounds

[11] 19–1/10–17 9–16 15–18 14–20 13–8 21–7 4–6 12–3 2–5/11–22 (47) {1–9, 10–18}
[12] 19–1/10–17 9–16 15–18 14–20 23–8/13–24 21–7 4–6 12–3 11–22/2–5 (42) {1–9, 10–18}

10 rounds

[11] 21-1/22-10 12-9 14-8 16-7 6-18 20-11 13-5 15-4 3-17 19-2 (64) {1–10, 11–20}
[12] 24–1/11–20 17–19 16–21 18–15 22–10 14–9 8–6/13–23 5–4 3–7 2–12 (49) {1–10, 11–20}
[13] 24–1/11–20 17–19 21–16 18–15 22–10 14–9 8–6/23–13 5–4 3–7 25–12/26–2 (41) {1–10, 11–20}
[14] 21–1/11–19 8–22 10–18 20–17/7–23 6–9/28–16 5–4/24–15 25–14 3–26 27–13 12–2 (36) {1–10, 11–20}
[15] 21–1/11–19 8–22 29–18/10–30 7–23/20–17 28–16/6–9 24–15/4–5 25–14 3–26 27–13 12–2 (32) {1–10, 11–20}

11 rounds

[13] 26–1/19–12 22–21 9–11 20–18 17–8 7–10/16–23 6–5 24–4 3–15 14–25 13–2 (51) {1–11, 12–22}
[14] 26–1/19–12 22–21 9–11 18–20 17–27/28–8 16–23/7–10 6–5 24–4 3–15 14–25 13–2 (46) {1–11, 12–22}
[15] 23–1/12–22 24–21 25–20 19–17/26–11 27–10 9–18 5–8/28–16 7–15 6–4/30–14 13–3 29–2 (46) {1–11, 12–22}
[16] 23–1/12–22 21–24 20–25 17–19/26–11 27–10 9–18 5–8/16–28 7–15 6–4/14–30 31–3/13–32 29–2 (36) {1–11, 12–22}

12 rounds

[13] 12-1 23-11 10-17 19-8 6-15 9-14 24-13 4-20 7-22 3-21 5-18 2-25/26-16 (60) {1–12, 13–24}
[14] 28–1/22–13 24–12 21–23 20–11 10–7 19–9 6–8/18–25 5–4 26–3 2–17 16–27 15–14 (50) {1–12, 13–24}
[15] 28–1/13–22 12–24 21–23 29–11/20–30 10–7 9–19 18–25/8–6 5–4 26–3 17–2 16–27 14–15 (44) {1–12, 13–24}
[16] 25–1/13–21 23–24/26–12 22–20 19–27 18–28 17–11 29–10 30–9 8–4 6–7/16–31 3–5/15–32 2–14 (44) {1–12, 13–24}
[17] 25–1/13–21 23–24/26–12 22–20 27–19 18–28 11–17 29–10 30–9 8–4 6–7/16–31 3–5/15–32 34–14/33–2 (36) {1–12, 13–24}
[18] 25–1/13–21 24–23/12–26 22–20/11–27 28–19 29–18 30–17 10–31 9–32 8–33 3–7/36–16 6–5/34–15 2–4/35–14 (32) {1–12, 13–24}
[19] 25–1/13–21 24–23/12–26 22–20/27–11 28–19 29–18/7–2 30–17 31–10 32–9 33–8 37–3/36–16 5–6/34–15 38–4/35–14 (30) {1–12, 13–24}

13 rounds

[15] 27–1/14–26 25–23 22–13 28–12 21–11 24–20 8–10 7–19 18–29 6–17 9–5 3–4/16–30 15–2 (49) {1–13, 14–26}
[16] 27–1/14–26 25–23 22–13 28–12 21–11 24–20 8–10 32–7/19–31 18–29 6–17 9–5 16–30/3–4 15–2 (44) {1–13, 14–26}
[17] 27–1/23–14 26–25 28–13 22–24/29–12 21–11 20–30 10–19 31–9 7–8 4–6/18–32 17–33 3–16 2–5/15–34 (43) {1–13, 14–26}
[18] 27–1/23–14 26–25 28–13 22–24/29–12 21–11 20–30 10–19 31–9 7–8 4–6/18–32 17–33 16–35/36–3 15–34/2–5 (41) {1–13, 14–26}

14 rounds

[15] 14-1 13-16 12-18 11-20 22-10 9-24 8-26 28-15 7-17 6-19 21-5 4-23 3-25 30-27/2-29 (65) {1–14, 15–28}
[16] 32–1/15–25 14–10 12–13 9–11/29–24 8–23 28–7 22–6 5–30 21–20 27–19 31–18 4–17 3–26 2–16 (50) {1–14, 15–28}
[17] 32–1/15–25 14–10 12–13 9–11/24–29 8–23 33–7/28–34 22–6 30–5 20–21 27–19 31–18 4–17 26–3 16–2 (45) {1–14, 15–28}
[18] 36–1/15–25 27–28 26–24/14–29 23–13 10–12/30–22 9–21 8–11/31–20 6–7 5–32 4–33 3–19 34–18 35–17 2–16 (42) {1–14, 15–28}
[19] 36–1/15–25 27–28 26–24/14–29 23–13 12–10/30–22 9–21 11–8/31–20 6–7 5–32 4–33 37–19/3–38 34–18 35–17 2–16 (39) {1–14, 15–28}

15 rounds

[17] 34–1/16–24 30–15 23–14 29–13 31–12 28–22 8–11 10–27 21–32 7–9 20–6 5–26 4–3/19–33 25–2 17–18 (49) {1–15, 16–30}
[18] 34–1/16–24 30–15 23–35/36–14 29–13 31–12 28–22 8–11 10–27 21–32 7–9 20–6 5–26 19–33/4–3 25–2 17–18 (49) {1–15, 16–30}

Incomplete two session Howells

See *Worger two double Howells* below, page 199, for an introduction to this concept, where we are supplied with two Howell sessions with all pairs meeting all other pairs. Using similar principles it is possible to conduct two incomplete Howell sessions of equal length. John Royds of Ireland saw this as a possible solution to the problem of conducting events with 15 to 18 tables over two sessions and still have a manageable number of boards in each session. He suggested that 12 or 13 round sessions are ideal for these numbers of tables, but here I supply two sessions of 7, 9, 10, 12, 13 and 14 rounds. There are other solutions such as appendix Mitchell for one session and a reduced Howell for the second, but it is difficult to organize these and play all boards in each session.

Note it is possible to add one more round to complete all rounds in the one round short cases: 11 tables 10 rounds, 13 tables 12 rounds and 15 tables 14 rounds. In the extra round all the tables share an extra board-set and their opponents are plus or minus $(n - 1) / 2$. For 11 tables 1 plays 6, 2 plays 7 etc. and the two highest pairs play each other, 21 and 22 in the 11 table example.

7 rounds

Let us start with two 7-table movements with 7 rounds. The first is the double Howell for 7 tables and 7 rounds supplied above in *double Howells*, page 178.

[7] 8–1 6–5 4–2 9–13 7–3 12–14 10–11 (62) {1–7, 8–14}

There are no stationary pairs and there are two groups, pairs 1 to 7 and 8 to 14. Each group meets all the pairs in their group plus one pair from the other group. All pairs play all boards.

The second movement is a cut-down version of the 8 table Worger interwoven Howell found in *Worger two double Howells*, page 199.

[7] 7–8 2–13 4–11 6–9 1–14 3–12 10–5 (92) {1–7, 8–14}

This is an unusual Howell movement. There are no stationary pairs and there are two groups, pairs 1 to 7 and 8 to 14. Each group meets all the pairs in the other group. It is essentially a Mitchell movement with all pairs moving as well as the boards. All pairs play all boards. Without the 'switch' at table 7 the movement is poorly balanced.

Then by appending two or more tables to each of these movements (see *appendix table*, page 182) we have two sessions of incomplete Howells. Appending one table arrives at an equivalent to the 'Worger two double-Howells' for 8 tables.

For example the 9 tables and 7 rounds movement has two sessions defined as

[9] 1–8 5–15/6–16 2–4 13–17/9–18 3–7 14–12 11–10
[9] 7–8 2–13 15–11/18–4 9–6 14–1 3–12 5–17/16–10 {1–7, 8–14}

After two sessions the calibre is 89. See the next page for the full expansion.

	1	2	3	4	5	6	7	8	9
1	1–8 (1)	5–15 (2)	6–16 (2)	2–4 (3)	13–17 (4)	9–18 (4)	3–7 (5)	14–12 (6)	11–10 (7)
2	2–9 (2)	6–15 (3)	7–16 (3)	3–5 (4)	14–17 (5)	10–18 (5)	4–1 (6)	8–13 (7)	12–11 (1)
3	3–10 (3)	7–15 (4)	1–16 (4)	4–6 (5)	8–17 (6)	11–18 (6)	5–2 (7)	9–14 (1)	13–12 (2)
4	4–11 (4)	1–15 (5)	2–16 (5)	5–7 (6)	9–17 (7)	12–18 (7)	6–3 (1)	10–8 (2)	14–13 (3)
5	5–12 (5)	2–15 (6)	3–16 (6)	6–1 (7)	10–17 (1)	13–18 (1)	7–4 (2)	11–9 (3)	8–14 (4)
6	6–13 (6)	3–15 (7)	4–16 (7)	7–2 (1)	11–17 (2)	14–18 (2)	1–5 (3)	12–10 (4)	9–8 (5)
7	7–14 (7)	4–15 (1)	5–16 (1)	1–3 (2)	12–17 (3)	8–18 (3)	2–6 (4)	13–11 (5)	10–9 (6)
8	7–8 (8)	2–13 (9)	15–11 (10)	18–4 (10)	9–6 (11)	14–1 (12)	3–12 (13)	5–17 (14)	16–10 (14)
9	1–9 (9)	3–14 (10)	15–12 (11)	18–5 (11)	10–7 (12)	8–2 (13)	4–13 (14)	6–17 (8)	16–11 (8)
10	2–10 (10)	4–8 (11)	15–13 (12)	18–6 (12)	11–1 (13)	9–3 (14)	5–14 (8)	7–17 (9)	16–12 (9)
11	3–11 (11)	5–9 (12)	15–14 (13)	18–7 (13)	12–2 (14)	10–4 (8)	6–8 (9)	1–17 (10)	16–13 (10)
12	4–12 (12)	6–10 (13)	15–8 (14)	18–1 (14)	13–3 (8)	11–5 (9)	7–9 (10)	2–17 (11)	16–14 (11)
13	5–13 (13)	7–11 (14)	15–9 (8)	18–2 (8)	14–4 (9)	12–6 (10)	1–10 (11)	3–17 (12)	16–8 (12)
14	6–14 (14)	1–12 (8)	15–10 (9)	18–3 (9)	8–5 (10)	13–7 (11)	2–11 (12)	4–17 (13)	16–9 (13)

It is possible to use these two movements (sessions 1 and 2 separately) as ¾-Howell movements with 7 rounds. To do so would be extremely unfair as they are very unbalanced. The best quality versions follow but they are still very poor.

[9] 8–1 5–15/6–16 4–2 13–17/9–18 7–3 12–14 10–11 (46) {1–7, 8–14}
[9] 7–8 2–13 15–11/4–18 9–6 1–14 3–12 5–17/16–10 (57) {1–7, 8–14}

Other numbers of tables follow. As always the higher the number of tables and the more pairs not played, the poorer the quality. The movements shown are the best examples without using arbitrary table switching. Arbitrary switching will often improve the calibre, particularly in the higher numbers of tables, but getting the players to adhere to the schedules is generally difficult.

The 7 rounds double schedules for 10 to 13 tables are

[10] 15–1/16–8 18–6/5–17 4–2 20–9/13–19 7–3 12–14 10–11 7R
[10] 8–17/7–19 13–18/2–20 11–15/4–16 9–6 1–14 3–12 10–5 (89) {1–7, 8–14}
[11] 1–8 15–5/16–6 2–17/18–4 19–13/20–9 3–7 14–21/12–22 10–11
[11] 15–8/19–7 17–13/21–2 20–4/16–11 18–9/22–6 14–1 12–3 5–10 (79) {1–7, 8–14}
[12] 1–15/16–8 6–18/17–5 4–2 20–9/19–13 3–21/22–7 14–23/12–24 11–10
[12] 17–8/19–7 18–13/20–2 15–11/16–4 21–9/23–6 24–1/22–14 3–12 10–5 (79) {1–7, 8–14}
[13] 8–1 5–15/6–16 2–17/18–4 19–13/9–20 23–3/24–7 22–12/14–21 25–11/26–10
[13] 15–8/19–7 17–13/21–2 20–4/16–11 18–9/22–6 23–14/25–1 26–3/24–12 10–5 (74) {1–7, 8–14}

9 rounds

The two interwoven Howells used to construct these movements are unusual, the high and low numbers meeting at different times across both movements.

[9] 10–1 9–14 11–4 8–7 12–18 16–17 6–3 13–5 2–15 (52) {1–9, 10–18}
[9] 9–10 13–17 5–16 11–4 1–8 3–7 15–2 6–18 14–12 (57) {1–9, 10–18}

The 9 round double schedules for 11 to 17 tables are

[11] 1–10 9–14 19–4/11–20 8–7 12–18 16–17 6–3 13–5 21–15/2–22
[11] 10–19/9–20 13–17 16–5 4–11 8–1 7–3 2–15 18–22/6–21 14–12 (93) {1–9, 10–18}
[12] 10–1 23–14/9–24 19–4/11–20 7–8 18–12 17–16 6–3 5–13 22–2/21–15

[12] 10–19/9–20 13–17 5–23/16–24 4–11 1–8 3–7 15–2 18–22/6–21 14–12 (87) {1–9, 10–18}
[13] 10–1 14–23/9–24 4–19/11–20 7–8 25–12/26–18 17–16 3–6 5–13 15–21/22–2
[13] 10–19/9–20 13–17 16–24/5–23 4–11 8–25/1–26 7–3 15–2 6–21/18–22 14–12 (83) {1–9, 10–18}
[14] 10–1 23–14/24–9 20–11/19–4 27–7/28–8 25–12/26–18 17–16 6–3 5–13 21–15/22–2
[14] 10–19/20–9 17–13 16–24/5–23 11–4 8–25/26–1 7–3 2–15 21–6/22–18 12–27/28–14 (81) {1–9, 10–18}
[15] 1–10 23–14/24–9 20–11/19–4 27–7/28–8 25–12/26–18 29–16/30–17 6–3 13–5 21–15/22–2
[15] 10–19/20–9 13–17 24–16/5–23 4–11 25–8/26–1 29–7/3–30 15–2 6–21/18–22 12–27/28–14 (77) {1–9, 10–18}
[16] 1–10 14–23/9–24 11–20/19–4 7–27/28–8 25–12/18–26 16–29/30–17 31–3/6–32 13–5 21–15/2–22
[16] 10–19/9–20 13–31/17–32 16–24/5–23 4–11 8–25/1–26 7–29/3–30 15–2 6–21/18–22 12–27/14–28 (75) {1–9, 10–18}
[17] 1–33/34–10 23–14/24–9 20–11/4–19 27–7/8–28 12–25/18–26 16–29/17–30 31–3/6–32 5–13 15–21/22–2
[17] 10–19/9–20 13–31/17–32 16–24/5–23 11–33/4–34 8–25/1–26 7–29/3–30 15–2 6–21/18–22 12–27/14–28 (76) {1–9, 10–18}

10 rounds

[11] 1–12 17–5 8–16 2–11 7–13/9–14 3–20 19–6 18–4 22–10/21–15 R
[11] 21–1/13–22 3–6 20–19 10–9 14–16 4–8 17–11 5–7 12–2 15–18 (89) {1–10, 11–20}
[12] 23–1/10–22 9–12 14–8 16–7 6–18 21–11/24–20 13–5 4–15 3–17 19–2
[12] 21–1/13–22 6–3 20–19 9–10 14–16 4–8 11–17 5–7 12–23/2–24 18–15 (89) {1–10, 11–20}
[13] 23–1/10–22 9–12 8–14 16–7 25–18/26–6 21–11/24–20 5–13 15–4 17–3 2–19
[13] 13–1 3–25/6–21 19–20 10–9 16–14 8–4 26–17/22–11 5–7 24–2/23–12 18–15 (82) {1–10, 11–20}
[14] 23–1/22–10 12–27/28–9 8–14 16–7 18–25/26–6 21–11/24–20 5–13 15–4 17–3 2–19
[14] 27–1/28–13 25–3/21–6 19–20 10–9 14–16 8–4 26–17/22–11 5–7 24–2/23–12 18–15 (77) {1–10, 11–20}
[15] 23–1/22–10 12–27/28–9 14–29/8–30 7–16 25–18/6–26 21–11/24–20 5–13 15–4 17–3 2–19
[15] 13–1 25–3/21–6 30–20/28–19 29–9/27–10 16–14 4–8 26–17/22–11 5–7 24–2/23–12 18–15 (75) {1–10, 11–20}
[16] 23–1/22–10 12–27/28–9 29–14/30–8 31–7/32–16 18–25/6–26 21–11/24–20 5–13 15–4 17–3 2–19
[16] 27–1/28–13 25–3/21–6 31–20/30–19 29–9/32–10 14–16 4–8 26–17/22–11 5–7 24–2/23–12 18–15 (70) {1–10, 11–20}
[17] 23–1/22–10 12–27/28–9 14–29/8–30 31–16/7–32 18–25/6–26 11–21/24–20 33–13/5–34 15–4 17–3 2–19
[17] 13–1 25–3/21–6 30–20/28–19 29–9/27–10 34–14/32–16 33–8/31–4 26–17/22–11 5–7 24–2/23–12 18–15 (71) {1–10, 11–20}
[18] 23–1/22–10 12–27/28–9 29–14/30–8 31–7/32–16 18–25/26–6 21–11/20–24 33–13/5–34 4–35/15–36 17–3 2–19
[18] 27–1/28–13 25–3/21–6 31–20/30–19 29–9/32–10 35–16/34–14 33–8/36–4 26–17/22–11 7–5 24–2/23–12 18–15 (69) {1–10, 11–20}
[19] 23–1/22–10 12–27/28–9 14–29/8–30 31–16/7–32 18–25/6–26 11–21/24–20 33–13/5–34 4–35/15–36 37–3/17–38 19–2
[19] 13–1 25–3/21–6 30–20/28–19 29–9/27–10 34–14/32–16 33–8/31–4 26–17/22–11 38–7/36–5 24–2/23–12 37–15/35–18 (69) {1–10, 11–20}

12 rounds

[13] 14–1 6–22 13–2 19–10 8–23 20–3/17–11 16–9 24–4 7–21 5–15 26–12/25–18 R
[13] 25–1/13–26 9–8 14–24 7–12 21–18 3–5 22–17 10–6 15–23 11–2 16–4 20–19 (93) {1–12, 13–14}
[14] 26–1/12–27 21–11 10–15 8–17 6–13 9–24 22–25/28–23 18–4 7–20 19–3 16–5 2–14
[14] 25–1/13–26 8–7 11–4 2–6 9–12 5–3 27–16/28–10 22–23 14–19 21–17 24–15 18–20 (85) {1–12, 13–24}

[15] 26–1/12–27 11–21 15–10 8–17 13–6 9–24 22–25/28–23 18–4 20–7 19–3 16–5 29–14/2–30
[15] 13–1 25–7/29–8 4–11 2–6 12–9 5–3 27–16/28–10 26–23/30–22 14–19 21–17 24–15 18–20 (85) {1–12, 13–24}
[16] 26–1/12–27 29–11/30–21 31–15/32–10 8–17 13–6 9–24 25–22/28–23 18–4 7–20 19–3 16–5 2–14
[16] 25–1/13–26 31–7/30–8 11–4 6–2 9–12 3–5 27–16/10–28 29–23/22–32 14–19 21–17 24–15 18–20 (77) {1–12, 13–24}
[17] 26–1/12–27 31–21/11–32 10–33/15–34 17–8 13–6 9–24 22–25/28–23 18–4 20–7 3–19 5–16 29–14/2–30
[17] 13–1 25–7/29–8 31–11/34–4 2–6 9–12 5–3 27–16/28–10 26–23/30–22 33–19/32–14 17–21 24–15 18–20 (74) {1–12, 13–24}
[18] 26–1/12–27 29–11/30–21 31–15/32–10 33–17/34–8 35–6/36–13 9–24 25–22/28–23 18–4 20–7 19–3 16–5 2–14
[18] 25–1/13–26 31–7/30–8 33–4/11–36 6–2 12–9 3–5 16–27/10–28 29–23/22–32 35–19/34–14 21–17 24–15 18–20 (69) {1–12, 13–24}
[19] 26–1/12–27 31–21/11–32 10–33/15–34 8–35/17–36 37–6/13–38 9–24 22–25/28–23 18–4 20–7 3–19 16–5 14–29/30–2
[19] 13–1 25–7/29–8 31–11/34–4 38–6/36–2 9–12 5–3 27–16/28–10 26–23/30–22 33–19/32–14 37–21/35–17 15–24 20–18 (70) {1–12, 13–24}
[20] 26–1/12–27 29–11/30–21 31–15/32–10 33–17/34–8 35–6/36–13 37–24/38–9 25–22/28–23 39–4/40–18 20–7 3–19 5–16 14–2
[20] 25–1/13–26 31–7/30–8 33–4/11–36 37–2/6–40 12–9 3–5 16–27/28–10 29–23/22–32 35–19/14–34 39–17/38–21 15–24 20–18 (64) {1–12, 13–24}
[21] 26–1/12–27 31–21/11–32 10–33/15–34 8–35/17–36 6–37/38–13 39–24/9–40 22–25/28–23 41–4/18–42 20–7 3–19 16–5 29–14/2–30
[21] 13–1 25–7/29–8 31–11/34–4 38–6/36–2 39–12/42–9 3–5 27–16/28–10 26–23/30–22 33–19/32–14 37–21/35–17 41–24/40–15 18–20 (66) {1–12, 13–24}
[22] 26–1/12–27 29–11/30–21 31–15/32–10 33–17/34–8 35–6/36–13 37–24/38–9 25–22/28–23 39–4/40–18 41–7/42–20 43–19/44–3 16–5 2–14
[22] 25–1/13–26 31–7/30–8 33–4/11–36 37–2/6–40 43–9/12–42 3–5 16–27/28–10 29–23/22–32 35–19/14–34 17–39/38–21 24–41/44–15 18–20 (62) {1–12, 13–24}
[23] 26–1/12–27 31–21/11–32 10–33/15–34 8–35/17–36 6–37/38–13 24–39/9–40 22–25/28–23 41–4/18–42 43–7/20–44 45–19/3–46 16–5 29–14/30–2
[23] 13–1 25–7/29–8 31–11/34–4 38–6/36–2 39–12/42–9 45–5/44–3 27–16/28–10 26–23/30–22 33–19/32–14 37–21/35–17 41–24/40–15 43–20/46–18 (73) {1–12, 13–24}

13 rounds

The two interwoven movements used to construct these 13 round schedules are

[13] 14–1 23–20 25–17 19–21 15–16 22–26 8–2 12–11 5–7 4–9 3–13 24–18 6–10 (51)
[13] 1–14 25–3 5–23 7–21 9–19 11–17 13–15 2–26 4–24 6–22 8–20 10–18 16–12 (93)

The 13 rounds double schedules for 15 to 25 tables are

[15] 1–14 5–27/10–28 8–7 2–13 24–29/17–30 12–3 6–9 22–19 25–16 4–11 26–15 20–21 18–23
[15] 27–14/1–29 25–28/3–30 23–5 21–7 19–9 11–17 13–15 26–2 24–4 6–22 20–8 10–18 16–12 (93) {1–13, 14–26}
[16] 1–31/14–32 27–5/28–10 8–7 13–2 29–24/30–17 3–12 9–6 22–19 16–25 11–4 26–15 21–20 23–18
[16] 14–27/1–29 25–28/3–30 23–31/5–32 7–21 19–9 11–17 15–13 2–26 4–24 22–6 8–20 18–10 16–12 (92) {1–13, 14–26}
[17] 1–14 27–5/10–28 8–31/32–7 13–2 24–29/30–17 3–12 9–6 19–33/22–34 25–16 11–4 15–26 21–20 23–18
[17] 14–27/1–29 25–28/3–30 5–33/23–32 21–31/7–34 19–9 17–11 15–13 26–2 4–24 6–22 8–20 10–18 12–16 (87) {1–13, 14–26}

[18] 1–31/14–32 5–27/10–28 8–7 2–13 24–29/17–30 3–12 6–33/9–34 19–35/22–36 16–25 4–11 15–26 20–21 23–18
[18] 27–14/29–1 28–25/3–30 23–31/5–32 21–33/35–7 19–34/36–9 17–11 15–13 2–26 24–4 22–6 8–20 10–18 12–16 (86) {1–13, 14–26}
[19] 1–14 5–27/10–28 8–31/7–32 2–35/13–36 24–29/17–30 3–12 9–6 19–33/22–34 16–37/25–38 11–4 26–15 21–20 18–23
[19] 27–14/29–1 28–25/3–30 5–33/23–32 21–31/7–34 35–19/37–9 17–36/38–11 15–13 26–2 24–4 22–6 8–20 10–18 12–16 (81) {1–13, 14–26}
[20] 1–31/14–32 27–5/10–28 8–7 2–13 29–24/30–17 12–3 33–6/34–9 19–35/22–36 16–25 11–37/38–4 39–26/40–15 20–21 23–18
[20] 14–27/1–29 25–28/3–30 23–31/5–32 21–33/7–35 19–34/9–36 17–37/11–39 15–38/13–40 26–2 24–4 6–22 8–20 18–10 12–16 (77) {1–13, 14–26}
[21] 14–1 5–27/10–28 8–31/7–32 2–35/13–36 24–29/17–30 3–12 9–6 19–33/22–34 16–37/25–38 4–39/11–40 15–41/26–42 21–20 18–23
[21] 14–27/29–1 28–25/3–30 5–33/23–32 21–31/7–34 35–19/37–9 36–17/38–11 39–15/13–42 2–41/40–26 4–24 22–6 20–8 10–18 16–12 (76) {1–13, 14–26}
[22] 1–31/14–32 5–27/10–28 8–7 13–2 24–29/17–30 12–41/3–42 6–33/9–34 19–35/22–36 25–43/16–44 11–37/4–38 26–39/15–40 20–21 23–18
[22] 14–27/29–1 28–25/30–3 31–23/5–32 21–33/7–35 34–19/36–9 37–17/39–11 15–38/13–40 2–43/42–26 24–41/4–44 22–6 20–8 18–10 12–16 (77) {1–13, 14–26}
[23] 14–1 5–27/10–28 8–31/7–32 2–35/13–36 24–29/17–30 12–3 6–43/9–44 19–33/22–34 16–37/25–38 4–39/11–40 15–41/26–42 21–45/20–46 23–18
[23] 14–27/29–1 28–25/3–30 5–33/32–23 21–31/7–34 35–19/37–9 36–17/38–11 39–15/13–42 41–2/40–26 4–45/24–44 22–43/6–46 8–20 18–10 16–12 (75) {1–13, 14–26}
[24] 1–31/14–32 5–27/10–28 8–45/7–46 13–2 24–29/17–30 12–41/3–42 6–33/9–34 19–35/22–36 25–43/16–44 11–37/4–38 26–39/15–40 21–47/20–48 23–18
[24] 14–27/29–1 28–25/3–30 23–31/5–32 21–33/35–7 19–34/9–36 17–37/11–39 38–15/13–40 43–2/42–26 24–41/44–4 45–22/47–6 46–20/8–48 10–18 16–12 (78) {1–13, 14–26}
[25] 14–1 5–27/10–28 8–31/7–32 2–35/13–36 24–29/17–30 12–47/3–48 6–43/9–44 19–33/22–34 16–37/25–38 4–39/11–40 15–41/26–42 21–45/20–46 23–49/18–50
[25] 14–27/29–1 25–28/3–30 33–5/23–32 31–21/7–34 35–19/9–37 17–36/11–38 39–15/42–13 41–2/40–26 4–45/24–44 43–22/46–6 49–8/48–20 18–47/50–10 12–16 (78) {1–13, 14–26}

14 rounds

[15] 16–1 22–10 2–15 9–28 3–19 8–25 20–11/13–24 26–4 18–12 17–7 27–6 23–5 30–14/29–21 R
[15] 29–1/30–17 3–4 25–28 18–16 8–13 11–5 22–26 14–12 2–6 27–19 15–24 10–7 9–23 21–20 (95) {1–14, 15–28}
[16] 30–1/14–32 13–16 12–18 20–11 10–22 24–9 26–8 29–15/28–31 7–17 19–6 21–5 23–4 3–25 2–27
[16] 29–1/17–30 3–4 25–28 18–16 8–13 5–11 22–26 14–12 2–6 27–19 24–15 10–7 9–31/23–32 20–21 (92) {1–14, 15–28}
[17] 30–1/32–14 13–16 12–18 20–11 10–22 24–9 26–8 29–15/28–31 7–17 19–6 33–5/34–21 23–4 3–25 2–27
[17] 17–1 4–29/3–34 33–28/25–30 18–16 8–13 5–11 22–26 14–12 2–6 27–19 24–15 7–10 9–31/23–32 20–21 (84) {1–14, 15–28}
[18] 30–1/14–32 33–16/34–13 35–18/36–12 20–11 10–22 24–9 8–26 29–15/31–28 7–17 19–6 21–5 23–4 3–25 2–27
[18] 29–1/17–30 33–4/35–3 34–28/36–25 18–16 8–13 5–11 26–22 14–12 6–2 27–19 24–15 10–7 9–31/23–32 20–21 (80) {1–14, 15–28}
[19] 30–1/32–14 35–16/13–36 37–18/38–12 20–11 22–10 24–9 8–26 29–15/28–31 17–7 6–19 33–5/34–21 23–4 3–25 2–27
[19] 17–1 4–29/3–34 33–28/25–30 16–38/18–36 13–37/8–35 11–5 22–26 14–12 6–2 19–27 24–15 7–10 9–31/23–32 20–21 (77) {1–14, 15–28}

[20] 30–1/14–32 33–16/34–13 35–18/36–12 37–11/38–20 39–22/40–10 9–24 26–8 29–15/31–28 7–17 19–6 21–5 23–4 25–3 2–27
[20] 29–1/17–30 33–4/35–3 34–28/36–25 37–16/40–18 39–13/38–8 11–5 22–26 14–12 6–2 27–19 24–15 10–7 9–31/32–23 20–21 (73) {1–14, 15–28}
[21] 30–1/32–14 35–16/36–13 37–18/38–12 39–11/20–40 41–10/42–22 24–9 8–26 29–15/31–28 17–7 6–19 33–5/34–21 23–4 3–25 2–27
[21] 17–1 4–29/3–34 28–33/25–30 16–38/18–36 13–37/8–35 5–42/11–40 41–26/22–39 12–14 6–2 19–27 24–15 7–10 9–31/23–32 20–21 (73) {1–14, 15–28}
[22] 30–1/14–32 33–16/34–13 35–18/36–12 37–11/38–20 39–22/40–10 41–24/42–9 43–8/44–26 29–15/31–28 17–7 19–6 5–21 23–4 25–3 2–27
[22] 29–1/17–30 33–4/35–3 34–28/36–25 37–16/40–18 39–13/38–8 41–5/44–11 43–26/42–22 14–12 6–2 19–27 24–15 7–10 9–31/32–23 20–21 (69) {1–14, 15–28}
[23] 30–1/32–14 35–16/36–13 37–18/38–12 39–11/40–20 41–10/42–22 43–9/44–24 26–45/46–8 29–15/31–28 17–7 6–19 33–5/34–21 23–4 3–25 2–27
[23] 17–1 4–29/3–34 28–33/25–30 16–38/18–36 37–13/8–35 5–42/11–40 26–41/39–22 14–45/12–44 6–2 43–27/19–46 24–15 7–10 9–31/23–32 20–21 (69) {1–14, 15–28}
[24] 30–1/32–14 33–16/34–13 35–18/36–12 37–11/38–20 39–22/40–10 41–24/42–9 43–8/44–26 29–15/31–28 45–7/46–17 47–6/19–48 5–21 23–4 25–3 2–27
[24] 29–1/30–17 33–4/35–3 34–28/36–25 37–16/40–18 39–13/38–8 41–5/44–11 43–26/42–22 48–12/14–46 6–2 27–47/19–45 24–15 7–10 9–31/32–23 20–21 (67) {1–14, 15–28}
[25] 30–1/32–14 35–16/36–13 37–18/38–12 39–11/40–20 41–10/42–22 43–9/44–24 26–45/46–8 29–15/31–28 47–7/17–48 49–19/50–6 33–5/34–21 23–4 3–25 2–27
[25] 17–1 4–29/3–34 28–33/25–30 16–38/18–36 37–13/8–35 5–42/11–40 26–41/39–22 14–45/12–44 49–2/6–48 43–27/19–46 15–47/24–50 7–10 9–31/23–32 20–21 (66) {1–14, 15–28}
[26] 30–1/32–14 33–16/34–13 35–18/36–12 37–11/38–20 39–22/40–10 41–24/42–9 43–8/44–26 15–29/31–28 45–7/46–17 47–6/48–19 49–21/50–5 51–4/52–23 3–25 27–2
[26] 29–1/30–17 33–4/35–3 34–28/36–25 37–16/40–18 39–13/38–8 41–5/44–11 43–26/42–22 12–48/14–46 49–2/6–52 27–47/45–19 51–15/50–24 7–10 31–9/23–32 21–20 (66) {1–14, 15–28}
[27] 30–1/32–14 35–16/36–13 37–18/38–12 39–11/40–20 41–10/42–22 43–9/44–24 45–26/46–8 29–15/31–28 47–7/17–48 49–19/50–6 33–5/34–21 51–4/52–23 53–25/54–3 27–2
[27] 17–1 4–29/3–34 28–33/25–30 16–38/18–36 37–13/8–35 5–42/11–40 26–41/22–39 14–45/12–44 49–2/6–48 43–27/19–46 15–47/24–50 53–10/7–52 31–9/23–32 21–51/54–20 (67) {1–14, 15–28}

Two twin-Howells

Where the number of rounds in two session events are complete, the use of a Mitchell with a Howell works well. See Hybrid pairs movements for further discussion on this issue, page 204, as well as two sessions in Howell movements, page 199.

Where the number of rounds is less than complete it is possible to use a Mitchell for the first session and Howell for the second. Consider 14 tables, playing 12 rounds in each of two sessions, a typical problem. One good solution is to use a skip web Mitchell over 12 rounds followed by twin 7-table incomplete, 12-round, Howells. The N-S field of the web Mitchell play in one Howell, and the E-W field in the other. It should be noted that this solution requires two sets of boards, as does the following solution.

Similar to the above incomplete movements for odd numbers of rounds, with an even number of rounds a good solution is to use two twin, incomplete, Howell movements for the two sessions. See above, twin Howells, page 181.

To illustrate this alternative two twin-Howells solution, we will continue with the 14 table example. Session 1 uses two 12 round 7 table ¾ Howell schedules supplied above, page 184.

As two separate, parallel, movements they can be shown as
[7] 25–1 5–7 11–12 3–10 6–9 8–4 26–2 5R
[7] 27–13 17–19 23–24 15–22 18–21 20–16 28–14 5R

As one movement it is
[14] 25–1/27–13 5–7/17–19 11–12/23–24 3–10/15–22 6–9/18–21 8–4/20–16 26–2/28–14 5R {1–12, 13–24}

Similarly, session 2 uses the same original reduced Howell with one pair from each table swapped with the pair from the other twin Howell so that the pairs play those of the other group.

[14] 25–13/27–1 5–19/17–7 11–24/23–12 3–22/15–10 6–21/18–9 8–16/20–4 26–14/28–2 5R (85) {1–12, 13–24}

The calibre shown is only valid once both sessions are complete. These two Howells are easy to organize and run, being arranged in two rows and using two sets of boards.

This solution is not that much better than the web Mitchell followed by a Howell, though in some ways it is tidier, but when the total number of tables is odd the two twin-Howells solution is much better.

Consider this problem using the web Mitchell in the first session for 15 tables. The web rover-table Mitchell for 14 tables and 12 rounds is needed, see page 109. The roving E-W pair moves arbitrarily making it very difficult to organize the extra table to meet new pairs in the second session.

With two twin-Howells, session 1 is
[15] 25–1/27–13 5–7/17–19 29–12/11–30/23–24 3–10/15–22 6–9/18–21 8–4/20–16 26–2/28–14 5R {1–12, 13–24}

Session 2
[15] 25–13/27–1 5–19/17–7 11–12/29–24/23–30 3–22/15–10 6–21/18–9 8–16/20–4 26–14/28–2 5R (82) {1–12, 13–24}

It should be noted that the extra table (pairs 29 and 30) is an appendix to table 3 in both sessions. Unless there is an extra set-of-boards available, that extra table must share boards with table 3 throughout the movement.

7.6 Flower modification

If, after the first twenty minutes, you don't know who the sucker at the table is, it's you.
David Levien and Brian Koppelman, Rounders

With the normal Howell the board movement is very orderly and the player movement usually requires the use of a guide card. The endless Howell simply reverses this process, so that the players move in an orderly fashion and each board requires a schedule. This arrangement is named after Mr. Flower.

The tables are numbered from 1 to *N* and the pairs are assigned numbers in some orderly manner. The N-S pair at table 1 remains stationary throughout the movement, while the N-S pairs move up 1 table at each change and the E-W pairs move down 1 table. When the E-W pairs reach table 1 they switch to N-S at table 2 after playing as E-W at table 1. The N-S pairs swivel to E-W at the highest numbered table after playing at that table as N-S (hence it is known as the swivel table). See *Howells and the Snow method*, page 174, for more details on endless Howells.

If the number of boards being played on each round is not less than the number of tables in the movement, then the boards can be shared throughout the movement on each round (for example, 4-table Howell

playing 4 boards per round). Duplication of boards also can be used to overcome this problem.

On the other hand, the boards can be selected from a centre table according to a schedule supplied to each table. When the starting board-set is known for a table the boards can be played in order at that table. For example, in a 4-table Howell, table 2 might start with board-set 3 and on subsequent rounds the boards played are in board-sets 4, 5, 6, 7, 1, 2.

Player starting positions

8-1	7-2	6-3	5-4

Board-sets round 1

1	3	5	2

Generally speaking, these movements are not balanced, though they can be balanced by switching one or more tables so that the players are required to swap directions at that table whenever they play there. This is only perfect for even numbers of tables and is shown like <3> indicating table 3 always to switch.

As the assignment of pair numbers is not important, they are not shown in the following schedules. The board-sets (the starting set at each table) and the table at which the pairs should switch half the boards are shown like {3, 4}. Two values for calibre are shown, the first with any full table switching and the second value the full table switching together with half switching.

[4] 1 3 5 2 (100)
[5] 1 4 2 5 2 {3, 4} (83, 100)
[6] 1 6 2 5 10 4 <3> (100)
[7] 1 11 9 5 13 8 12 <3, 5, 6, 7> {1} (91, 100)
[8] 1 7 11 2 8 12 9 3 <5, 6, 7, 8> (100)
[9] 1 13 9 5 17 15 11 16 7 <2, 7> {6} (95, 97)
[10] 1 8 13 7 17 11 18 15 6 9 <5, 8, 9, 10> {8} (96, 99)
[11] 1 19 15 12 8 11 13 7 16 14 21 <2, 7> {6} (96, 99)
[12] 1 13 8 6 21 23 17 14 12 22 9 4 <3, 4, 7, 9> {5, 8, 11} (96, 98)
[13] 1 7 9 25 20 14 19 3 6 11 2 4 17 <10, 11> {3, 5, 7, 11} (96, 98)

7.7 Extra-complete Howells

I must complain the cards are ill shuffled till I have a good hand.
Jonathan Swift

Standard Howell movements are complete movements. Every pair plays every other pair while playing every board-set. Incomplete Howells as seen in the last chapter are those where these requirements are not met. The number of rounds played is less than the maximum possible. For complete Howells the number of rounds equals the number of pairs less one.

We are now considering the Howells where the pairs play other pairs more than once. Every pair plays every board-set, but the number of sets is equal to or greater than the number of pairs in the movement. This type of movement is incomplete, but it can become complete 'again' by repeating play against all pairs a second time. Therefore I call it an extra-complete movement, it has gone past complete to become incomplete.

Expanded Howell

The first example is a Howell-like cyclic schedule that Olof Hanner calls an expanded Howell. There are no stationary pairs. Compare this with the Double Howell for odd numbers of tables. Pair 1 follows the highest numbered pair and the number of rounds equals the number of pairs. This type of schedule does not exist

for some numbers of tables.

[4] 1–5 4x7 R 2–3 6x8 3R (66, 94) – Hanner
[5] 2–9 6–1 5R 8–10 5x4 3–7 (83, 89) – McKinnon
[8] 1–9 7–5 2R 4–14 12–16 2–3 10–15 13–6 8–11 6R (97) – McKinnon
[9] 10–1 16–5 18–14 12–9 4–6 8–13 11–17 2–3 7–15 9R (96) – McKinnon
[12] 5–8 1–13 9R 2–3 9–18 23–12 7–21 17–24 11–19 4–6 22–16 15–20 14–10 3R (97) – McKinnon
[13] 5–8 22–7 12–24 19–26 23–17 25–16 4–6 21–11 15–20 10–18 9–13 2–3 1–14 13R (96) – McKinnon

Rover-board Howell

This principle can be applied to any Howell movement. It is also called a lengthened Howell, but it is essentially the same idea as that used for a rover-board Mitchell. An extra board-set is introduced into the movement and played on certain rounds instead of the boards that are normally played on that round. For the last and extra round the pairs play the boards they missed when they played the rover set.

Using any standard complete Howell the number of rounds is increased to equal to the number of pairs, each pair playing one other pair twice. A second rover set can also be added in the same way. It is possible to apply this principle to ¾-Howells, but the idea of playing pairs twice in an incomplete movement is hard to explain to players.

Only one table plays the rover set(s) up until the extra rounds. That table is the one having pair numbers that are the greatest number apart (not table 1 where the highest pair is stationary). Consider the 4 table Howell: 8-1 2R 7-5 R 4-3 2-6 where table 2 are two apart (7-5), table 3 one apart (4-3) and table 4 four apart (2-6) and so table 4 is used. Note that table 4 can also be considered to be 3 apart because of the cyclic nature of the numbers (6, 7, 1, 2).

The first extra set is placed on that table (4 in this case) for play instead of the usual board-sets. It remains there for N − 1 rounds, the normal board-sets bypassing that table and placed on a relay for the round. The second extra set (if used) is introduced at the same table on the round just after halfway.

Using the 4-table example, the first rover set (8) is placed on table 4 for 3 rounds. Then board-set 3 or the original board-set played at that table for round 4 is played. The second rover set (9) is placed on table 4 for round 5 onwards. Therefore board-set 8 is played at table 4 for rounds 1 to 3 and then at table 1 for round 8. Board-set 9 is played at table 4 for rounds 5 to 7 and then at table 1 for round 9.

The pairings for the first extra round (8) match those at table 4 for rounds 1 to 3 and they play the sets that were missed on those rounds. Similarly for round 9, the pairings for rounds 5 to 7 at table 4 are repeated. For round 8 the highest (stationary) pair plays against the pair that did not play at table 4 in rounds 1 to 3. Similarly for pair 8 on round 9. In this example they are the pairs that played at table 4 on round 4, pair 5 for round 8 and 2 for round 9. The stationary pair plays rover set 8 on round 8 and set 9 on round 9.

	1	2	3	4
1	8–1 (1)	7–5 (4)	4–3 (6)	2–6 (8)
2	8–2 (2)	1–6 (5)	5–4 (7)	3–7 (8)
3	8–3 (3)	2–7 (6)	6–5 (1)	4–1 (8)
4	8–4 (4)	3–1 (7)	7–6 (2)	5–2 (3)
5	8–5 (5)	4–2 (1)	1–7 (3)	6–3 (9)
6	8–6 (6)	5–3 (2)	2–1 (4)	7–4 (9)
7	8–7 (7)	6–4 (3)	3–2 (5)	1–5 (9)
8	8–5 (8)	2–6 (7)	3–7 (1)	4–1 (2)
9	8–2 (9)	6–3 (4)	7–4 (5)	1–5 (6)

It is possible to use the second rover set without the first, for example:

	1	2	3	4
1	8–1 (1)	7–5 (4)	4–3 (6)	2–6 (7)
2	8–2 (2)	1–6 (5)	5–4 (7)	3–7 (1)
3	8–3 (3)	2–7 (6)	6–5 (1)	4–1 (2)
4	8–4 (4)	3–1 (7)	7–6 (2)	5–2 (3)
5	8–5 (5)	4–2 (1)	1–7 (3)	6–3 (8)
6	8–6 (6)	5–3 (2)	2–1 (4)	7–4 (8)
7	8–7 (7)	6–4 (3)	3–2 (5)	1–5 (8)
8	8–2 (8)	6–3 (4)	7–4 (5)	1–5 (6)

It is possible to add additional rover sets using similar principles, but each additional set adds more complications for the director. For example, rover set 10 can be added to the 4-table movement above by playing it at table 3 on rounds 2, 4 and 6 and round 10 is 8–3 (10) 5–4 (7) 7–6 (2) 2–1 (4).

Extra-board Howell

The rover-board Howell discussed above has a practical limitation to the number of rovers. Here we discuss the possibility of many rover sets, but not having the same ease of use or the orderly movement of the roving boards. The director needs to take charge of the boards or have the players retrieve them from a central location on each round.

In the simplest case (the 4-table Howell) consider the two following movements that have the same starting positions for the pairs but at different tables.

Board-sets	1	2	3	4	5	6	7
Round 1	8–1			6–3		7–2	4–5
Round 2	5–6	8–2			7–4		1–3
Round 3	2–4	6–7	8–3			1–5	
Round 4		3–5	7–1	8–4			2–6
Round 5	3–7		4–6	1–2	8–5		
Round 6		4–1		5–7	2–3	8–6	
Round 7			5–2		6–1	3–4	8–7

Board-sets	1	2	3	4	5	6	7
Round 1	8–1	4–5	7–2		6–3		
Round 2		8–2	5–6	1–3		7–4	
Round 3			8–3	6–7	2–4		1–5
Round 4	2–6			8–4	7–1	3–5	
Round 5		3–7			8–5	1–2	4–6
Round 6	5–7		4–1			8–6	2–3
Round 7	3–4	6–1		5–2			8–7

If each vertical column of this expansion is considered as an extra board-set in the first movement, then it can be readily seen at which tables the extra board-set is to be placed on which rounds. Set 1 goes to table 1 on round 1, displacing board-set 1 of the original set-of-boards (pairs 8-1). Table 2 on round 4 displacing board-set 7 (2-6). Table 3 on round 6 displacing board-set 4 (5-7). Table 4 on round 7 displacing board-set 6 (4-3). Column 2 of the second movement can be used to determine the second roving sets movement, and

so forth. More than 2 roving sets would be unusual. For the 8th round the seating positions would be as under the first column of the second movement. The pairs play the boards that were displaced on the round when they previously met. For the 9th round column 2 is used, and so on.

A shorthand method of showing this is as follows, with the extra round pairs playing the replaced board-sets respectively, using Howell 8–1 2R 6–3 R 7–2 4–5.

Rover set 8

On round:	1	4	6	7
At table:	1	2	3	4
Replace set:	1	7	4	6
Round 8 pairs:	8–1	2–6	5–7	3–4

Rover set 9

On round:	1	2	5	7
At table:	4	1	2	3
Replace set:	7	2	1	5
Round 9 pairs:	4–5	8–2	3–7	6–1

Rover set 10

On round:	1	2	3	6
At table:	3	4	1	2
Replace set:	6	1	3	2
Round 10 pairs:	7–2	5–6	8–3	4–1

Rover set 11

On round:	2	3	4	7
At table:	3	4	1	2
Replace set:	7	2	4	3
Round 10 pairs:	1–3	6–7	8–4	5–2

5-table Howell with extra boards

Using Howell: 10–1 8–9 6–4 4R 7–3 2–5 (with 10–1 R 2–5 R 8–9 7–3 R 6–4 R)

Rover set 10

Round	1	3	5	6	8
Table	1	3	4	2	5
Set	1	5	3	7	7
Pairs	10–1	8–6	2–7	4–5	9–3

Rover set 11

Round	2	4	6	7	9
Table	1	3	4	2	6
Set	2	6	4	8	8
Pairs	10–2	9–7	3–8	5–6	1–4

Note the shares in rounds 10 and 11.

6-table Howell with extra boards

Using Howell: 12–1 11–10 R 9–6 4–8 7–2 3R 3–5 R (with 12–1 R 9–6 3R 3–5 7–2 11–10 R 4–8)

Rover set 12

Round	1	2	4	5	6	10
Table	1	4	2	5	6	3
Set	1	6	5	10	4	2
Pairs	12–1	5–9	3–2	11–6	8–10	7–4

Rover set 13

Round	2	3	5	6	7	11
Table	1	4	2	5	6	3
Set	2	7	6	11	5	3
Pairs	12–2	6–10	4–3	1–7	9–11	8–5

7-table Howell with extra boards

Using Howell: 14–1 10–5 8–7 13–2 R 12–3 9–6 2R 11–4 3R (with 14–1 3R 9–6 2R 13–2 11–4 R 8–7 10–5 12–3)

Rover set 14

Round	1	2	3	4	6	7	10
Table	1	5	2	3	7	4	6
Set	1	7	4	6	2	10	3
Pairs	14–1	13–4	12–7	11–10	3–9	6–8	5–2

Rover set 15

Round	2	3	4	5	7	8	11
Table	1	5	2	3	7	4	6
Set	2	8	5	7	3	11	4
Pairs	14–2	1–5	13–8	12–11	4–10	7–9	6–3

8-table Howell with extra boards

Using Howell: 16–1 R 15–2 10–7 R 14–13 8–3 6–12 11–4 5R 5–9 (with 16–1 R 11–4 8–3 R 15–2 5–9 14–13 6–12 5R 10–7)

Rover set 16

Round	1	2	8	9	10	11	13	14
Table	1	3	6	4	8	2	5	7
Set	1	5	15	14	9	13	4	7
Pairs	16–1	11–8	13–4	7–6	14–3	10–12	5–15	9–2

Rover set 17

Round	2	3	9	10	11	12	14	15
Table	1	3	6	4	8	2	5	7
Set	2	6	1	15	10	14	5	8
Pairs	16–2	12–9	14–5	8–7	15–4	11–13	6–1	10–3

7.8 Two sessions in Howell movements

Old card players never die, they just shuffle away.

Conducting Howell movements over two sessions sometimes simply means the movement is run twice, playing half the boards in each session. Another approach is to use the Snow method so that the session can be stopped at any convenient time and because of the board sharing each set can be scored after each round, barometer style.

Where the number of tables is odd, running the movement twice has the advantage of producing a balanced movement when the switching of one table gives 100. For example, the 5 table Howell

10–1 8–9 6–4 4R 7x3 2–5 (83, 100)

Here table 4 is played as 7v3 in session 1 and 3v7 in session 2 for 100. See more schedules in *producing Howell movements* (page 166) for all examples.

Another non-Howell method is to run the first session using a double Howell and the second with a Mitchell. For example, with 8 tables, run a double 4-table Howell over 7 rounds and then a Mitchell over 8 rounds. With 7 tables eliminate the stationary pairs in the double 8 table and play 7 rounds. Then with the 7-Table Mitchell, skip round 1 so that the pairs do not meet each other a second time. This gives a total of 13 rounds, but the boards in the Mitchell need to be factored to 7 results so all the boards are of equal value.

This latter method does not apply to low numbers of tables. With 4 tables you would need a double 2-table Howell and for 5 or 6 tables, a double 3-table Howell, neither of these existing.

Another type of two-session Howell is found by using the *teams' bicyclic schedules* and particularly those modified for pairs. See *pairs' bicyclic schedules*, page 337, when you need a schedule for one round short with an even number of tables. This gives two sessions with the same number of rounds in each session. The disadvantage with these types of movements is the use of the same board-sets in both sessions so that player discussion of hands between sessions can prove to be a small problem. Even so it is possible to score up after the first session.

Worger two Double Howells

With an even number of tables it is possible to use a double Howell for the first session and then an interwoven Howell with the same two groups meeting each other for the second session. This clever idea originated with Harold Worger, Surry, England.

The following second stanza interwoven Howells can be used with any double Howell for the same number of tables. See *double (or interwoven) Howell* (page 178) for such movements. The number of rounds is the same too, equal to the number of pairs in each group.

[8] 8–9/1–16 3–14 5–12 7–10 2–15 13x4 11x6 (43, 64) {1–7, 9–15}
[10] 10–11/20–1 14–18 6–17 12–5 2–9 8–4 3–16 19–7 15–13 (50) {1–9, 11–19}
[12] 12–13/1–24 8–17 4–21 11–14 18x7 22x3 10–15 6–19 2–23 16–9 5–20 (61, 81) {1–11, 13–23}
[14] 14–15/1–28 26–3 5–24 7–22 9–20 11–18 13–16 2–27 4–25 6–23 8–21 10–19 17–12 (81) {1–13, 15–27}
[16] 16–17/1–32 25–15 4–27 10–24 9–22 5–31 3–21 30–13 11–20 2–23 14–29 6–18 8–28 7–26 12–19 (83) {1–15, 17–31}
[18] 18x19/36–1 30–7 24–13 2–35 29–8 23–14 34–3 28–9 22–15 33–4 27–10 21–16 5–32 26–11 20–17 31–6 25–12 (66, 83) {1–17, 19–35}

Note in the 10 table example the two groups are not kept separate, and this first stanza is required:

[10] 10–1/11–20 9–15 12–4 7–8 19–13 17–18 3–6 14–5 16–2 (51) {1–9, 11–19}

Once these two sessions have been completed one round has not been played, each pair missing one other pair. Each pair has not played the corresponding pair in the other group. For example with 8 tables 1 and 9, 2 and 10, 3 and 11 etc. have not played against each other. A last round can be played with these two pairs meeting each other, playing the same (extra) board-set at all tables. Clearly duplicate copies of these boards will be needed.

Worger and Hanner pointed out that the last round can be played with different sets, eliminating the need to duplicate the extra boards. Even so there is still a need to share boards between two tables. In doing this the player movement in later rounds is irregular at two tables as well as the last round for all tables. This option is not attractive where the director is playing.

These irregularities to the above movements are shown here. In each case the first array shows the irregular pairings at the two tables in the final rounds, and the second array shows the pairs and board-sets (in brackets) played at each table in the final and extra round.

8 Tables play set 15 at tables 2 and 7:

Round:	12	13	14
Table 2:	7–15	1–9	2–10
Table 7:	3–11	4–12	5–13

Round 15, tables 1 to 8:

1	2	3	4	5	6	7	8
8–16 (15)	6–14 (15)	15–3 (11)	9–4 (12)	10–5 (13)	7–11 (13)	1–12 (14)	2–13 (8)

10 Tables play set 19 at tables 2 and 6:

Round:	15	16	17	18
Table 2:	4–14	5–15	6–16	7–17
Table 6:	9–19	1–11	2–12	3–13

Round 19, tables 1 to 10:

1	2	3	4	5	6	7	8	9	10
10–20	8–18	19–14	11–15	12–16	13–17	9–4	1–5	2–6	3–7

12 Tables play set 23 at tables 3 and 10:

Round:	18	19	20	21	22
Table 3:	10–22	11–23	1–13	2–14	3–15
Table 10:	4–16	5–17	6–18	7–19	8–20

Round 23, tables 1 to 12:

1	2	3	4	5	6	7	8	9	10	11	12
12–4 (23)	9–1 (23)	22– (16)	23–5 (17)	13–6 (18)	14–7 (19)	15–8 (20)	10–16 (20)	11–17 (21)	1–18 (22)	2–19 (12)	3–20 (13)

14 Tables play set 27 at tables 6 and 9:

Round	21	22	23	24	25	26
Table	5–9	6–20	7–21	8–22	9–23	10–24
Table	11–15	12–26	13–27	1–15	2–16	3–17

Round 27, tables 1 to 14:

1	2	3	4	5	6	7
14–28	4–18	5–25	6–26	7–27	8–15	11–19
8	9	10	11	12	13	14
9–16	12–20	10–17	13–21	1–22	2–23	3–24

16 Tables play set 31 at tables 6 and 9:

Round:	24	25	26	27	28	29	30
Table	12–28	13–29	14–30	15–31	1–17	2–18	3–19
Table	4–20	5–21	6–22	7–23	8–24	9–25	10–26

Round 31, tables 1 to 16:

1	2	3	4	5	6	7	8
16–32 (31)	11–27 (31)	12–20 (26)	13–21 (27)	14–22 (28)	15–23 (29)	1–24 (30)	2–25 (16)
9	10	11	12	13	14	15	16
3–26 (17)	28–4 (17)	29–5 (18)	30–6 (19)	31–7 (20)	17–8 (21)	18–9 (220	19–10 (23)

18 Tables play set 35 at tables 6 and 13:

Round:	27	28	29	30	31	32	33	34
Table 6:	6–24	7–25	8–26	9–27	10–28	11–29	12–30	13–31
Table 13:	14–32	15–33	16–34	17–35	1–19	2–20	3–21	4–22

Round 35, tables 1 to 18:

1	2	3	4	5	6	7	8	9
18–36 (35)	5–23 (35)	6–32 (32)	7–33 (33)	8–34 (34)	9–35 (18)	10–19 (19)	11–20 (20)	12–21 (21)
10	11	12	13	14	15	16	17	18
13–22 (22)	14–24 (22)	15–25 (23)	16–26 (24)	17–27 (25)	1–28 (26)	2–29 (27)	3–30 (28)	4–31 (29)

7.9 Three sessions in Howell movements

I am always doing that which I cannot do, in order that I may learn how to do it.
Pablo Picasso

10 pairs, 3 sessions

This is the lowest number of pairs that has a number of rounds divisible by 3. Using the Howell movement 10-1/7-4 5R 5-9/8-6/3x2 2R (83, 91) supplied with all the *5-table Howell* movements above we take every third round and play them consecutively for three by 3-round sessions. Tables 1 and 2 play the same board-sets in sequence and tables 3, 4 and 5 also play the same board-sets in sequence, but start with the highest set for the session. It is normal to have 3 sets of boards, though sharing boards can reduce this number. Typically this would be run with one set for each session for, say, 9-board rounds which are easy to share between 2 or 3 tables. The pair movement is regular though not in sequence. Pair 1 follows 7, 7 follows 4 and 4 follows 1 and so forth: 5–2, 9–6, 8–5, 6–3, 3–9 and 2–8 but only within each session. The board-sets move down one group of tables with a relay between tables 2 and 3, boards sets 1 to 3 are played in session one, 4 to 6 in two and 7 to 9 in three.

	1	2	3	4	5
1	10–1 (1)	7–4 (1)	5–9 (3)	8–6 (3)	3–2 (3)
2	10–4 (2)	1–7 (2)	8–3 (1)	2–9 (1)	6–5 (1)
3	10–7 (3)	4–1 (3)	2–6 (2)	5–3 (2)	9–8 (2)
4	10–2 (4)	8–5 (4)	6–1 (6)	9–7 (6)	4–3 (6)
5	10–5 (5)	2–8 (5)	9–4 (4)	3–1 (4)	7–6 (4)
6	10–8 (6)	5–2 (6)	3–7 (5)	6–4 (5)	1–9 (5)
7	10–3 (7)	9–6 (7)	7–2 (9)	1–8 (9)	5–4 (9)
8	10–6 (8)	3–9 (8)	1–5 (7)	4–2 (7)	8–7 (7)
9	10–9 (9)	6–3 (9)	4–8 (8)	7–5 (8)	2–1 (8)

Where you have the ability to change pair numbers in each session then the following movement may be used for all the three sessions:

	1	2	3	4	5
1	10–1 (1)	2–3 (1)	6–8 (3)	5–9 (3)	7–4 (3)
2	10–2 (2)	3–1 (2)	4–9 (1)	6–7 (1)	8–5 (1)
3	10–3 (3)	1–2 (3)	5–7 (2)	4–8 (2)	9–6 (2)

There are three groups of pairs, 1–3; 4–6; 7–9. The pairs stay in the group and follow the lower pair, 1 follows 3, 4 follows 6 and 7 follows 9. Each session the pair numbers of each group are mapped to different numbers. For example the original pairs 1, 2, 3 play in seats 4, 5, 6 in session 2 and seats 7, 8, 9 in session 3. Note each session has poor balance (22), but once complete it is as good as you can get except for switching half the boards at table 5 on each round.

Session Original pairs:	Group 1 1,2,3	Group 2 4,5,6	Group 3 7,8,9
1	1,2,3	4,5,6	7,8,9
2	4,5,6	7,8,9	1,2,3
3	7,8,9	1,2,3	4,5,6

Using previous description methods this can be shown as

10–1/2–3 R 6–8/5–9/7–4 (22, 83) {1–3, 4–6, 7–9} — McKinnon

16 or more pairs over 3 equal sessions

Like 10 pairs movement, those shown here have the number of rounds evenly divisible by 3 for an equal numbers of rounds in each session (Hanner):

16–1/15–14 4–12/7–13 2–10 5–11 8–3/9–6 (25, 100) {1–5, 6–10, 11–15}
22–1/6–9 19–16/3–17 13–15/21–8 7–20 11–12/18–14 10–5 2–4 (29, 96) {1–7, 8–14, 15–21}
28–1/26–4 15–24/19–11 17–14/22–7 10–3 20–25 2–13 8–9 6–16/18–5 21–23/27–12 (39, 99) {1–9, 10–18, 19–27}
34–1/17–12 27–30/21–6 18–20/23–13 16–10 2–14/19–3 9–5 28–8 32–31/15–26 25–22/24–11 4–29 33–7 (38, 99) {1–11, 12–22, 23–33}
40–1/3–12 32–23/35–17 16–9 26–27 30–10/37–8 13–2/5–22 18–19/25–36 29–14 33–6 24–11 15–21 7–38/34–39 20–4/31–28 (35, 99) {1–13, 14–26, 27–39}
46–1/33–23 38–42 27–17/8–30 43–34 22–14 16–4/37–44 10–25/3–35 7–24 11–20/29–15 39–18 26–45 13–32 2–5/31–6 9–36/41–40 12–28/21–19 (32, 99) {1–15, 16–30, 31–45}

One more example is where there are no stationary pairs and 18 rounds.
[9] 1–4/2–17 11–9/16–8 14–15/18–7 12–6 10–3 13–5 (28, 88) {1–6, 7–12, 13–18}

Three sessions and three groups (Snow method)

Typically when conducting barometer events using the Snow method, multiple sessions are organized by simply taking a break in play at any convenient time. Even so, there are events that are a bit more complicated, with several groups of pairs who should not meet each other. For example three clubs wish to compete on a club by club basis, never playing against those pairs from within their own club. This requires special cyclic schedules. Note these do not require three sessions; the break(s) can be made after any round and the number of rounds per session may differ, offering complete flexibility to the organizers.

12 pairs in 3 groups of 4 playing 8 rounds
[6] 9x1/10–8/7–2/11–3/12–6/5–4 7R (73, 87)

Pairs 1, 3, 5, 7 constitute the first group, pairs 2, 4, 6, 8 constitute the second. The pairs 9–12 are stationary and make up the third group. Pair 1 follows pair 8. The remaining moving pairs follow the pair with next lower number.

18 pairs in 3 groups of 6 playing 12 rounds
[9] 13x1/14–12/11–2/15x3/16–10/9–4/17–5/18–8/7–6 11R (74, 88)

The pairs 13–18 are stationary and constitute one group. The remaining pairs move and follow next lower number except pair 1, who follows pair 12. The odd–numbered pairs 1–11 are the second group and the even-numbered pairs 2–12 the third.

24 pairs in 3 groups of 8 playing 16 rounds
[12] 17x1/16–18/2–15/3x19/14–20/4–13/5–21/12–22/6–11/7–23/10–24/8–9 15R (67, 83)

The pairs 17–24 are stationary and constitute one group. The remaining pairs move and follow next lower number except pair 1, who follows pair 16. The odd–numbered pairs 1–15 the second group and the even-numbered pairs 2–16 the third.

30 pairs in 3 groups of 10 pairs playing 20 rounds
[15] 21x1/20–22/2–19/3x23/18–24/4–17/5x25/16–26/6–15/7–27/14–28/8–13/9–29/12–30/10–11 19R (67, 83)

The pairs 21–30 are stationary and constitute one group. The remaining pairs move and follow next lower number except pair 1, who follows pair 20. The odd–numbered pairs 1–19 the second group and the even-numbered pairs 2–20 the third.

24 pairs in 3 groups of 8 playing 12 rounds
This movement covers the need for a shorter contest. The movement is divided into 3 stanzas each with 4 rounds. Within each stanza pair 1 follows pair 8, pair 9 follows pair 16 and pair 17 follows pair 24. The remaining pairs follow the pair with next lower number. The groups consist of the pairs 1–8, 9–16 and 17–24. All pairs are moving and calibre is poor at 40.

Stanza 1 round 1:
[12] 1–9/16–18/17–2/3–15/14–20/23–4/5–13/12–22/21–6/11–7/24–10/8–19 3R
Stanza 2 round 5:
[12] 9–17/24–2/1–10/11–23/22–4/7–12/13–21/20–6/5–14/19–15/8–18/16–3 3R
Stanza 3 round 9:
[12] 17–1/8–10/9–18/19–7/6–12/15–20/21–5/4–14/13–22/3–23/16–2/24–11 3R

7.10 Five sessions in Howell movements

Progress always involves risks. You can't steal second base and keep your foot on first.
Frederick B. Wilcox

The movements shown here have the number of rounds evenly divisible by 5 for an equal numbers of rounds in each session (Hanner). Halve the highest stationary pair for the number of tables.

26–1/2–21/3–10 20–11/5–25/8–17 6–19/14–24 18–16/23–9 12–13/22–15/4–7 (16, 98) {1–5, 6–10, 11–15, 16–20, 21–25}
36–1/9–25/34–19 5–27/15–17 33–12/31–24 2–29/8–3/23–21 6–7/32–28 20–26/18–30/11–35 16–13/4–22/10–14 (17, 100) {1–7, 8–14, 15–21, 22–28, 29–35}
46–1/11–21/31–16 7–33/28–42/35–44 39–9/38–41 5–6/26–19 32–18/34–22/30–13 17–29/4–15 25–12/37–14/2–10 24–40/20–36/45–27 3–8/43–23 (19, 99) {1–9, 10–18, 19–27, 28–36, 37–45}
56–1/24–22/55–46 9–31/54–35/51–36 6–11/7–30 5–26/43–48 4–3/10–29 8–20/41–12/38–52 33–2/18–34 25–28/44–27 21–17/23–14/45–16 19–49/47–40/39–50 37–13/53–15/32–42 (19, 100) {1–11, 12–22, 23–33, 34–44, 45–55}

7.11 Hybrid pairs movements

You'll always miss 100% of the shots you don't take.
Wayne Gretzky

Two-session movements

Howell movements supply two-session events using double Howells but each pair misses playing one pair. To overcome this weakness it is necessary to use non-cyclic moving schedules. These solutions (Schiereck) will lead to one winner, but are not Howell movements.

8 pairs, 2 sessions

The first session is shown here with pairs 8 and 4 stationary and the boards moving down one table at each change. The other players move haphazardly and require guide-cards. In the second session apart from pairs 8 and 1 all the pairs and boards move haphazardly and there is board sharing. Session two plays a different set-of-boards to session 1.

	1	2	3	4
1	8–1 (1)	2–3 (2)	4–5 (3)	6–7 (4)
2	8–7 (2)	6–1 (3)	4–3 (4)	2–5 (1)
3	8–3 (3)	1–2 (4)	4–7 (1)	5–6 (2)
4	8–5 (4)	6–3 (1)	4–1 (2)	2–7 (3)

	1	2	3	4
1	8–2 (1)	1–3 (2)	4–6 (1)	5–7 (2)
2	8–4 (2)	1–5 (3)	6–2 (2)	7–3 (3)
3	8–6 (3)	1–7 (1)	3–5 (1)	2–4 (3)

A simpler first session is to use stationary boards and have all pairs moving. Board-sets 1 to 4 stay at tables 1 to 4. Pair 8 moves up one table and the other pairs move fairly regularly though guide-cards are still needed.

	1	2	3	4
1	8 1	2 3	4 5	6 7
2	2 5	8 7	6 1	4 3
3	4 7	5 6	8 3	1 2
4	6 3	4 1	2 7	8 5

Using session one independent from session two, the calibre is 69. Session two is not so good by itself at 33. When the event is completed over two sessions, the calibre is 61, but that can be improved by switching round 1 in session 1 giving 66.

10 pairs, 2 sessions

The 10-pair movement is very similar to the 8-pair, but the board movement is simpler. The boards are stationary in both sessions, apart from table 5 in session two where the sets are played in the same order as the round number. Session one calibre is 53 and session two 39. With the two sessions played together as one event, the calibre is 68. Switch round 1 in session 1 and rounds 1 and 2 in session 2 for a calibre of 78.

	1	2	3	4	5
1	1 10	9 8	3 2	5 4	7 6
2	2 8	10 6	1 5	3 7	4 9
3	9 3	7 1	10 4	6 2	8 5
4	6 4	2 5	9 7	10 8	1 3
5	7 5	3 4	8 6	9 1	2 10

	1	2	3	4	5
1	4 7	8 3	2 1	9 10	5 6 (1)
2	2 9	10 5	3 6	7 8	4 1 (2)
3	10 3	2 7	5 9	1 6	8 4 (3)
4	1 8	6 9	10 7	4 2	3 5 (4)

12 pairs, 2 sessions

	1	2	3	4	5	6
1	12 1	2 3	4 5	6 7	8 9	10 11
2	2 9	11 12	8 10	1 3	6 5	7 4
3	3 4	7 1	6 12	8 11	10 2	5 9
4	5 11	4 10	1 9	2 12	3 7	8 6
5	8 7	6 9	11 3	5 10	4 12	1 2
6	6 10	5 8	2 7	4 9	1 11	12 3

	1	2	3	4	5	6
1	5 12	10 9	4 2	11 7	6 1	8 3 (1)
2	11 9	7 12	10 1	5 2	8 4	6 3 (2)
3	6 2	11 4	9 7	8 1	10 12	5 3 (3)
4	1 4	2 8	6 11	12 9	5 7	3 10 (4)
5	10 7	5 1	8 12	6 4	11 2	9 3 (5)

Boards are stationary, except table 6 in session 2 where they are played in sequence. Session one calibre 68 and session two 42, combined 72.

14 pairs, 2 sessions

For 14 or more pairs the existing double-Howell used with a Mitchell for the second session works well, but with some qualifications. Consider the 14-pair game where the double 4-table Howell is reduced to 7 tables for the first session, while for the second session, use a 7-table Mitchell, playing 6 rounds. This is not as simple as it first appears. The pairs that meet at table 1 in the double Howell meet again on the first round of the Mitchell. This can be overcome by skipping the play on round 1 of the Mitchell, but you end up with each board-set being played 6 times and each pair plays a different 6 sets. See *incomplete Mitchell movements – one round short* (page 125) for further discussion of this problem.

The best solution is to use the double-Howell as described, with pair-number changes, for the first session together with the 6-table-plus-1 movement for the second, described in *one round short* (page 125). This solution has good qualities, notably stationary boards and all boards being played by all pairs. This is similar to the 12-pair example above and table 7 in the second session plays the boards in ascending order. Session 1 calibre is 62, session 2 is 65 and combined is 80.

	1	2	3	4	5	6	7
1	13 1	6 5	4 2	10 8	7 3	11 12	9 14
2	14 11	10 2	7 6	5 3	9 12	1 4	8 13
3	12 10	11 8	9 3	1 7	6 4	14 13	2 5
4	3 6	13 9	8 12	14 4	2 1	7 5	11 10
5	8 9	4 7	10 14	12 13	11 5	3 2	1 6
6	2 7	12 14	5 1	9 11	13 10	8 6	4 3
7	5 4	3 1	13 11	6 2	14 8	10 9	12 7

	1	2	3	4	5	6	7
1	8 1	9 2	10 3	11 4	12 5	13 6	14 7
2	4 9	6 11	2 12	5 14	1 10	3 8	7 13
3	2 13	5 8	1 9	6 10	3 14	4 12	7 11
4	5 10	3 12	6 14	2 8	4 13	1 11	7 9
5	3 11	4 10	5 13	1 12	6 9	2 14	7 8
6	6 12	1 14	4 8	3 13	2 11	5 9	7 10

Two-sessions, one round short

In *Incomplete two session Howells* (see page 187) we can see two interwoven Howell movements (for two sessions of play) have not been found for some numbers of rounds equal to one less than the number of tables. For example with 9 tables, there are 18 pairs, requiring 17 rounds for all pairs to meet all other pairs. Where this is to be played over two sessions of equal length, each session can only have 8 rounds for a total of 16 rounds.

Here are some examples of how this can be done using a mixture of two schedule types. The important difference between these and the two interwoven Howells is the complete separation of the two groups of pairs. In the first session the first group play as opponents to the second group and in the second session they only play the pairs of the their own group. In the interwoven Howells that is not always the case.

9 tables, 2 sessions

For the first session use the 8 tables plus 1 self-orthogonal Graeco-Latin square (see page 128).

	1	2	3	4	5	6	7	8	9
1	1 18	3 13	4 14	5 15	6 16	7 10	8 11	2 12	17 9
2	5 11	2 18	7 12	1 16	8 15	4 13	6 9	3 14	17 10
3	6 12	4 15	3 18	8 13	1 10	2 16	5 14	7 9	17 11
4	7 13	8 9	5 16	4 18	2 14	1 11	3 10	6 15	17 12
5	8 14	7 16	2 9	6 10	5 18	3 15	1 12	4 11	17 13
6	2 15	5 12	8 10	3 9	7 11	6 18	4 16	1 13	17 14
7	3 16	1 14	6 13	2 11	4 9	8 12	7 18	5 10	17 15
8	4 10	6 11	1 15	7 14	3 12	5 9	2 13	8 18	17 16

The pair numbers are 1–8 N-S and 9–16 E-W. Pairs 17 and 18 are the appendix table pairs. Note the N-S pairs have played all the E-W pairs apart from 1 and 9, 2 and 10 etc. Pair 17 has played 9–16 and pair 18 has played 1–8. The boards are stationary.

For the second session play two 4 table, 8 round expanded Howells of this type:

R 1–5 4–7 R 2–3 6–8 2R

The second Howell is: R 9–13 12–15 R 10–11 14–16 2R

Pairs 17 and 18 are introduced at table 1 in both movements, so that 1 and 9, 2 and 10 etc. are meeting in this session, having missed each other in the first. The complete schedule for session 2 is shown next. Switching rounds 3 and 5 in the first session gives a calibre of 83 after the two sessions are complete.

	1	2	3	4	5	6	7	8	9
1	1–9 (2)	4–7 (3)	2–3 (5)	6–8 (6)	18–13 (2)	12–15 (3)	10–11 (5)	14–16 (6)	17–5 (2)
2	2–10 (3)	5–8 (4)	3–4 (6)	7–1 (7)	18–14 (3)	13–16 (4)	11–12 (6)	15–9 (7)	17–6 (3)
3	3–11 (4)	6–1 (5)	4–5 (7)	8–2 (8)	18–15 (4)	14–9 (5)	12–13 (7)	16–10 (8)	17–7 (4)
4	4–12 (5)	7–2 (6)	5–6 (8)	1–3 (1)	18–16 (5)	15–10 (6)	13–14 (8)	9–11 (1)	17–8 (5)
5	5–13 (6)	8–3 (7)	6–7 (1)	2–4 (2)	18–9 (6)	16–11 (7)	14–15 (1)	10–12 (2)	17–1 (6)
6	6–14 (7)	1–4 (8)	7–8 (2)	3–5 (3)	18–10 (7)	9–12 (8)	15–16 (2)	11–13 (3)	17–2 (7)
7	7–15 (8)	2–5 (1)	8–1 (3)	4–6 (4)	18–11 (8)	10–13 (1)	16–9 (3)	12–14 (4)	17–3 (8)
8	8–16 (1)	3–6 (2)	1–2 (4)	5–7 (5)	18–12 (1)	11–14 (2)	9–10 (4)	13–15 (5)	17–4 (1)

10 tables, 2 sessions

We need two sessions of 9 rounds. For the first session use the 9 tables plus 1 self-orthogonal Graeco-Latin square (see page 129). Use pair numbers 1–9 for N-S and 10–18 for E-W, and pairs 19 (N-S) and 20 (roving E-W) as the appendix table pairs. Once completed the N-S pairs have met the E-W pairs in the base 9 tables, apart from 1 and 10, 2 and 11 etc. Pair 19 has played the E-W pairs 10–18, while pair 20 has played the N-S pairs 1–9.

For the second session use this interwoven Howell:
<u>19</u>–1/10–<u>20</u> 18–13/15–16 R 12–14 11–17 9–3 8–6 R 4–5/7–2 (41) {1–9, 10–18}

Switch rounds 2, 3 and 4 in the first session for an overall calibre of 80.

11 tables, 2 sessions

We need two sessions of 10 rounds. For the first session use the 10 tables plus 1 self-orthogonal Graeco-Latin square (see page 129). Use pair numbers 1–10 for N-S and 11–20 for E-W, and pairs 21 (N-S) and 22 (roving E-W) as the appendix table pairs. Once completed the N-S pairs have met the E-W pairs in the base 10 tables, apart from 1 and 11, 2 and 12 etc. Pair 21 has played the E-W pairs 11–20, while pair 22 has played the N-S pairs 1–10.

For the second session use this interwoven Howell:
12–19 1–11/<u>21</u>–6/16–<u>22</u> 8–10 4–5 3–7 9–2 R 20–18 15–14 17–13 (53) {1–10, 11–20}

Note the share required by tables 2, 3 and 4. This is not difficult to manage as two of the pairs involved, 21 and 22, are stationary.

Switch rounds 3, 6 and 9 in the first session for an overall calibre of 86.

12 and 14 tables, 2 sessions

We need two sessions of 11 and 13 rounds. These are the first examples of prime numbers of tables in the base of the first session. See *one round short* for prime tables on page 125, where you will choose to setup an 11 table plus 1 movement for 24 pairs and similarly 13 plus 1 for 28 pairs.

For the second session use the double Howell for 12 tables with 11 rounds, or the 14 tables with 13 rounds. See page 180.

13 tables, 2 sessions

For 26 pairs we need two sessions of 12 rounds. For the first session we can use the 12 tables plus 1 self-orthogonal Graeco-Latin square (see page 129). The real problem arises for the second session. We need two 6 table expanded Howells interwoven with an additional table. Unfortunately this is impossible as this

is another example of Euler's conjecture. See page 311 for a similar problem. Even so, there is an alternative solution for session 2 shown here. The two interwoven Howells is a much neater solution (see page 189).

In session 1 use pair numbers 1–12 for N-S and 13–24 for E-W, and pairs 25 (N-S) and 26 (roving E-W) as the appendix table pairs.

For session 2, here is a bicyclic schedule based on the scissor schedules for 7 teams (see page 295). The cycles are started on rounds 1 and 7. The only disadvantage is the need for two extra board-sets, 13 and 14, played at tables12 and 13. This is needed to overcome Euler's conjecture. All pairs do not play two of the board-sets.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	13– 1 (1)	5– 6 (3)	17–18 (3)	9–11 (4)	21–23 (4)	14–26 (8)	25– 2 (8)	4–12 (9)	16–24 (9)	7–10 (11)	19–22 (11)	15–20 (13)	3– 8 (13)
2	14– 2 (2)	6– 7 (4)	18–19 (4)	10–12 (5)	22–24 (5)	15–26 (9)	25– 3 (9)	5– 1 (10)	17–13 (10)	8–11 (12)	20–23 (12)	16–21 (14)	4– 9 (14)
3	15– 3 (3)	7– 8 (5)	19–20 (5)	11– 1 (6)	23–13 (6)	16–26 (10)	25– 4 (10)	6– 2 (11)	18–14 (11)	9–12 (7)	21–24 (7)	17–22 (13)	5–10 (13)
4	16– 4 (4)	8– 9 (6)	20–21 (6)	12– 2 (1)	24–14 (1)	17–26 (11)	25– 5 (11)	7– 3 (12)	19–15 (12)	10– 1 (8)	22–13 (8)	18–23 (14)	6–11 (14)
5	17– 5 (5)	9–10 (1)	21–22 (1)	1– 3 (2)	13–15 (2)	18–26 (12)	25– 6 (12)	8– 4 (7)	20–16 (7)	11– 2 (9)	23–14 (9)	19–24 (13)	7–12 (13)
6	18– 6 (6)	10–11 (2)	22–23 (2)	2– 4 (3)	14–16 (3)	19–26 (1)	25– 7 (1)	9– 5 (8)	21–17 (8)	12– 3 (10)	24–15 (10)	20–13 (14)	8– 1 (14)
7	8–25 (2)	26–20 (2)	24–23 (3)	12–11 (3)	17–15 (4)	5– 3 (4)	7–19 (7)	6–10 (9)	18–22 (9)	4– 1 (11)	16–13 (11)	14–21 (13)	2– 9 (13)
8	9–25 (3)	26–21 (3)	13–24 (4)	1–12 (4)	18–16 (5)	6– 4 (5)	8–20 (8)	7–11 (10)	19–23 (10)	5– 2 (12)	17–14 (12)	15–22 (14)	3–10 (14)
9	10–25 (4)	26–22 (4)	14–13 (5)	2– 1 (5)	19–17 (6)	7– 5 (6)	9–21 (9)	8–12 (11)	20–24 (11)	6– 3 (7)	18–15 (7)	16–23 (13)	4–11 (13)
10	11–25 (5)	26–23 (5)	15–14 (6)	3– 2 (6)	20–18 (1)	8– 6 (1)	10–22 (10)	9– 1 (12)	21–13 (12)	7– 4 (8)	19–16 (8)	17–24 (14)	5–12 (14)
11	12–25 (6)	26–24 (6)	16–15 (1)	4– 3 (1)	21–19 (2)	9– 7 (2)	11–23 (11)	10– 2 (7)	22–14 (7)	8– 5 (9)	20–17 (9)	18–13 (13)	6– 1 (13)
12	1–25 (7)	26–13 (7)	17–16 (2)	5– 4 (2)	22–20 (3)	10– 8 (3)	12–24 (12)	11– 3 (8)	23–15 (8)	9– 6 (10)	21–18 (10)	19–14 (14)	7– 2 (14)

15 tables, 2 sessions

For 30 pairs we also have a different problem. We need a 7 table expanded Howell. (See page 194). For the first session of 14 rounds use the 14 table self-orthogonal Graeco-Latin square (see page 147) plus an appendix table with N-S 29 and 30 as the E-W rover.

For the second session use this interwoven Howell:
15–1 R 12–6/26–20 2R 14–10/24–28 8–3/17–22 2R 9–7/21–23 13–2/16–27 4–5/19–18 25–<u>30</u>/11–<u>29</u> R (48) {1–14, 15–28}

16 tables, 2 sessions

We need two sessions of 15 rounds. For the first session use the 15 tables plus 1 self-orthogonal Graeco-Latin square (see page 131). Use pair numbers 1–15 for N-S and 16–30 for E-W, and pairs 31 (N-S) and 32 (roving E-W) as the appendix table pairs.

For the second session use the double Howell for 16 tables with 15 rounds. See page 180.

Three-sessions movements

Howell movements only supply three-session events where the number of rounds is divisible by three. Other than those you need to use combinations of movements or these following examples, which require pair guide-cards and careful organization of the board-sets.

10 pairs, 3 sessions

This number of pairs and rounds has the unique property of supplying an event with 6 rounds per session with a comfortable length of say 24 or 30 boards. As well as that, all pairs play each other twice and the

balance is perfect (100). Note that it is possible to use stationary boards with table 6 having sets 6, 12 and 18 and one table is out of play on each round (starting with table 6, then 5, 4, 3, 2 and 1).

	1	2	3	4	5
1	1–2 (1)	3–4 (2)	5–6 (3)	7–8 (4)	9–10 (5)
2	5–3 (1)	7–2 (2)	10–8 (3)	4–9 (4)	1–6 (6)
3	7–10 (1)	8–6 (2)	1–3 (3)	5–4 (5)	2–9 (6)
4	6–9 (1)	10–5 (2)	3–2 (4)	8–1 (5)	7–4 (6)
5	4–8 (1)	9–7 (3)	5–1 (4)	2–6 (5)	10–3 (6)
6	1–9 (2)	4–2 (3)	10–6 (4)	7–3 (5)	5–8 (6)
7	3–6 (7)	7–5 (8)	8–9 (9)	2–1 (10)	10–4 (11)
8	9–7 (7)	1–6 (8)	10–2 (9)	4–5 (10)	8–3 (12)
9	1–10 (7)	8–2 (8)	7–3 (9)	9–5 (11)	6–4 (12)
10	4–8 (7)	9–10 (8)	7–6 (10)	2–3 (11)	1–5 (12)
11	2–5 (7)	4–1 (9)	9–3 (10)	6–8 (11)	10–7 (12)
12	4–3 (8)	6–5 (9)	10–8 (10)	7–1 (11)	9–2 (12)
13	7–8 (13)	9–1 (14)	2–4 (15)	3–6 (16)	5–10 (17)
14	1–4 (13)	3–8 (14)	10–6 (15)	9–5 (16)	7–2 (18)
15	3–10 (13)	6–2 (14)	1–7 (15)	9–4 (17)	5–8 (18)
16	2–5 (13)	10–4 (14)	1–8 (16)	7–6 (17)	9–3 (18)
17	6–9 (13)	5–3 (15)	7–4 (16)	8–2 (17)	1–10 (18)
18	5–7 (14)	8–9 (15)	10–2 (16)	3–1 (17)	6–4 (18)

12 pairs, 3 sessions

It should be no surprise that with 6 tables the movement is not as regular as others, particularly as the number of rounds is not divisible by 3. The first and second sessions (or stanzas) are 4 rounds in length and the last session is 3 rounds. No more than two tables play the same board-sets at the same time so that it is possible to share boards when playing a few boards per round. The calibre is 87.

	1	2	3	4	5	6
1	12–1 (1)	8–9 (1)	2–3 (2)	10–11 (2)	4–5 (3)	6–7 (4)
2	7–11 (1)	4–10 (1)	5–8 (2)	6–9 (2)	12–3 (3)	2–1 (4)
3	2–5 (1)	12–7 (2)	11–1 (3)	10–6 (3)	8–3 (4)	9–4 (4)
4	6–3 (1)	1–4 (2)	8–7 (3)	2–9 (3)	11–5 (4)	12–10 (4)
5	6–4 (5)	10–1 (5)	12–11 (6)	9–3 (6)	2–8 (7)	5–7 (8)
6	2–11 (5)	9–5 (5)	8–6 (6)	1–7 (7)	12–4 (8)	3–10 (8)
7	3–7 (5)	4–2 (6)	5–1 (6)	12–6 (7)	9–10 (7)	11–8 (8)
8	8–12 (5)	10–7 (6)	4–11 (7)	3–5 (7)	6–2 (8)	1–9 (8)
9	5–12 (9)	6–1 (9)	7–2 (10)	4–3 (10)	8–10 (11)	9–11 (11)
10	11–3 (9)	2–10 (9)	9–12 (10)	1–8 (10)	5–6 (11)	7–4 (11)
11	4–8 (9)	9–7 (9)	10–5 (10)	6–11 (10)	2–12 (11)	1–3 (11)

14 pairs, 3 sessions

The same principles apply as those for 12 pairs, but there are two 4-round sessions and one 5-round session. The calibre is 82.

	1	2	3	4	5	6	7
1	14–1 (1)	8–9 (1)	2–3 (2)	10–11 (2)	4–5 (3)	12–13 (3)	6–7 (4)
2	5–7 (1)	13–10 (1)	12–6 (2)	4–9 (2)	2–14 (3)	3–1 (4)	11–8 (4)
3	4–3 (1)	12–11 (1)	14–7 (2)	6–10 (3)	8–1 (3)	5–9 (4)	2–13 (4)
4	6–2 (1)	13–8 (2)	1–5 (2)	9–7 (3)	3–11 (3)	14–4 (4)	12–10 (4)
5	14–3 (5)	5–12 (5)	6–8 (6)	2–10 (6)	11–13 (7)	9–1 (7)	7–4 (8)
6	6–13 (5)	11–4 (5)	14–5 (6)	9–3 (6)	10–7 (7)	2–8 (8)	12–1 (8)
7	8–10 (5)	1–7 (5)	13–4 (6)	12–2 (7)	3–5 (7)	11–6 (8)	9–14 (8)
8	9–2 (5)	7–12 (6)	1–11 (6)	4–6 (7)	14–8 (7)	13–3 (8)	5–10 (8)
9	14–6 (9)	10–3 (10)	5–11 (11)	4–2 (11)	1–13 (12)	7–8 (12)	9–12 (13)
10	9–10 (9)	11–14 (10)	3–8 (11)	13–7 (11)	5–2 (12)	12–4 (12)	6–1 (13)
11	1–2 (9)	11–7 (9)	13–9 (10)	14–12 (11)	6–3 (12)	5–8 (13)	10–4 (13)
12	13–5 (9)	8–12 (10)	1–4 (10)	9–6 (11)	14–10 (12)	2–11 (13)	3–7 (13)
13	12–3 (9)	4–8 (9)	6–5 (10)	7–2 (10)	1–10 (11)	9–11 (12)	13–14 (13)

7.12 The American Whist League movement for pairs

An original idea. That can't be too hard. The library must be full of them.
Stephen Fry

See the *Barney modification* of the Clay system (page 51) for more details and origins of this movement. The whist movement has the same basic properties as the Mitchell movement: two fields (N-S and E-W), the N-S pairs remaining stationary (by convention) while the E-W pairs move (as do the boards). The balance of comparison of the pairs is perfect when the movement is completed.

The whist movement is useful in pairs' events (but usually only when the number of tables is odd) as it may greatly simplify manual scoring. It is also invaluable in multiple teams' events where the number of tables is odd.
Many of the gadgets used in Mitchell movements can also be used in the whist movement. For example, rover pairs, rover boards, rover tables, board switching, twinning of the movement, etc.

Odd number of tables

As in the Mitchell, this movement has the same four basic components: N-S pairs, E-W pairs, boards and tables. As the movement completely 1oses its character if the components move other than in the conventional manner, only that procedure will be outlined. The N-S pairs remain stationary throughout the event and the E-W pairs move down (or up) two tables at each change while the boards move down (or up) one table. The tables should be arranged in the normal sequential manner, either in one long row or in the horseshoe pattern.

Instruct the E-W pairs to move the boards as they are moving in order to simplify the change procedure. In the Mitchell movement the N-S pairs usually take on this responsibility, and this creates problems in getting the boards from one end of the movement to the other when the tables are arranged in a straight line.

The biggest problem with this movement is the players' habit of discussing deals they have just played while they are changing tables. These discussions will always be about the hands the new opponents will be playing in the very next round The usual problem of overhearing the bidding at the next table is still there, but this can be overcome by using the double-whist move at each change; i.e., boards down (or up) two tables and the E-W pairs down (or up) 4 tables. The E-W pairs should not move to a table that is still in play, for the boards being played there will be those that are to be played by the moving pair on the next round.

Even number of tables

This movement is of no use in teams' events and of limited use in pairs. The disadvantage with this movement is that all the E-W pairs do not meet all the N-S pairs (see the *stagger Mitchell*, page 118, as that does overcome this problem while still having a whist-type move).

The movement for an even number of tables is the same as that for an odd number of tables except that after the half way round, the E-W pairs must move an extra table (down 3 instead of down 2 tables) while the boards move normally.

The last round cannot be played (like the skip Mitchell) because the E-W pairs will have played the boards at the last table. To overcome this problem the move by the E-W pairs will be different on the last round, while the boards move normally. Where the number of tables in the movement is equal to N, and the E-W pairs are moving down two tables, then the move into the last round for the E-W pairs will be down $\frac{1}{2} N - 2$ tables. For example, with 10 tables: $10 / 2 - 2 = 3$; i.e., down 3 tables.

Half tables

This issue only applies to pairs' events, but it can be dealt with much the same way as in the Mitchell movement. The extra pair can be treated as an extra table so that all the E W pairs will have one bye round at that table.

Where this extra table makes the numbers even this method is not particularly good, as one E-W pair would be required to sit out twice; if the movement is curtailed early, then some pairs will not have sat out. The rover pair is the best solution to this problem.

Rover pairs

For more information on rover pairs, see the section on 'rovers' in the Mitchell movement (page 99). Here the procedure is the same, except that the rover moves in a different pattern.

Where the number of tables is a prime, the roving pair starts at table 2 on the second round and moves up one table at each change (this assumes that the boards are moving down one table).

Where the number of tables is even, but not divisible by 3, then the rover starts at table 2 and moves up one table at each change, skipping one table; at the same time the E-W pairs skip a table (i.e. after $\frac{1}{2} N$ rounds).

Where the number of tables is divisible by 3 the movement is irregular so that guide-cards are necessary. It should be noted that the rovers must repeat against some E-W pairs so that there is no repetition of the N-S pairs displaced. As the even-numbered table movements have the revenge round on the last round, this feature of the rover is not as undesirable as it would first appear.

The real advantage of the rover movement is in having all the boards played an equal number of times. In all the schedules given below, the roving pair sits out for the first round and after that, replaces the N-S pair at the table shown in the schedule. The displaced N-S pair returns to their table at the end of that sit-out

round. The whist movement assumed to be in use is: E-W pairs down 2 and the boards down one.

6 Tables:	1, 2, 4, 5, 6 *
9 Tables:	1, 2, 3, 5, 7, 4, 9, 6 *
12 Tables:	1, 2, 3, 4, 5 *, 6, 8, 9, 10, 11, 12 *
15 Tables:	1, 2, 3, 4, 5, 7, 8, 10, 11, 6 *, 13, 14, 9 *, 15
18 Tables:	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16 *, 18

*** Indicates a repeat against an E-W pair.**

In the last movement, for 18 rounds to be played the roving pair must displace an E-W pair at table 15 or table 1, but note that the N-S pairs at these tables have been displaced on previous rounds. Used in conjunction with a switched movement this is not noticeable.

Inverted movement

This movement has the same basic qualities of the whist movement. The only difference is that the movement of the boards and the E-W pairs are reversed. E-W move up (or down) one table and the boards move up (or down) two tables. The E-W pairs should be given the responsibility of pushing the boards up one table further as they move. The only real problem arises when the tables are not arranged in a circular pattern, as the boards must move to the other end of the movement when the E-W pairs do not. The director may need to assist when that is the case.

The advantage is that the boards originate from a different table than the one at which the pairs are waiting for the play to finish, on the previous round. For example, in the standard whist move, the E-W pair moving from table one to table 3 may need to wait for the completion of the last board there before starting the next round. The board being completed there will be played by that pair at table 5 on the next round. The other advantage is that there is less player movement. This does not work as a multiple teams' movement.

Part 8 Teams' schedules

Set of 16 art deco duplicate contract bridge boards, circa 1930.
These were manufactured by Foxcraft, are made from steel and the vulnerability is shown with a 'V'.
North is shown by an arrow with 'N'. The board number is on the back.

Set of 16 boards in a box, made by Little Slam, circa 1933.
Shown in the brochure, they cost $7.50 and $2.00 for the box.

The organisation of multiple teams' tournaments can be a simple process or a very complex one, depending on the requirements of the event. The number of sessions of play is important when considered with the number of teams competing. As there are only a certain number of boards that can be played in one session, it may be found that the number of boards divided by the number of teams is too low to make the event feasible. To overcome this, use an incomplete draw or even better, a 'Swiss' teams draw can be used.

If fewer than four rounds per session are to be played, the best approach is head-to-head matches that are scored as they are completed. This requires a round-robin draw that can be used by the organizers to assign the seating and opponents of the teams. The round-robin origins can be directly attributed to Safford. See *The Safford system* (page 40) for the complete explanation of his system. Also see the *Patton system* (page 241) for similar concepts.

With four or more rounds per session it would be more normal to use a multiple teams' movement. The Clay universal system does cater for all numbers of tables, but the double traversal of the movement by all the E-W pairs is generally considered a weakness. Other than this, the only schedules required for these types of events are:

- American whist league movement;
- New England relay system;
- Hasler system.

Between these, all the possible numbers of tables are handled with ease. Having said this we must remember the original and all-purpose solution in the Clay system, which handles all numbers of tables, but there is one further weakness in the Clay system which we will consider in a moment.

There are a few other movements available for specific numbers of tables, but the above movements will cater for all sizes of sections. The twin Mitchell in the Hasler system can be replaced by the twin whist (making all three of them whist movements), though the first is usually used for the reduced-player movement.

8.1 Round-robin teams

To win you have to risk loss.
Jean-Claude Killy

For an introduction to this subject see part 4, *the round-robin tournament* (page 57), and *the house square* (page 58). There you will see how the draws for round-robin events are derived and particularly how to handle an odd number of teams using the 3-way triangle. There is one problem with such a draw: it is difficult to assign the away and home teams, particularly in the three-way matches. This can be overcome by having the stationary team away to one team on the first half and home to it in the other half. The other two teams are at home and then away to each other (or vice versa) in the two halves.

Most events today are organized with pre-dealt boards and all matches play the same boards on each round. With the 3-way triangle match this is not possible because over the two rounds the teams need three sets. A suggestion by Ed Barnes of Sydney is a scoring method that overcomes this problem.

The same boards are played at each table in the triangle as used in the other matches. At the end of the round the matches are scored in the same way as all the others. The only difference is the pairs from each team do not play the same team at the other table. The scores are still valid and meaningful and after two rounds each pair in each team has played against two other teams though not at the same time as their team mates. Of course these two matches are not strictly teams' matches, but as long as the players understand that there are advantages the players tend to accept the method. The very good reasons for using this method are:

- there are no 'extra' boards in play;
- all teams play the same boards in a session;
- there can be an odd number of rounds and play can stop after any round;
- you can break at lunch with no boards 'hanging', so discussion is possible over lunch;
- teams can score up after each round, knowing their results and outright position immediately.

Definition: *Odd- teams design* (OTD)

Let V be a set of $2n$ symbols, then an Odd teams design $O(n, 2n)$ on symbol set V is an $(n \times 2n)$ array O such that:

- each cell of the array contains an unordered pair from the set of symbols;
- each symbol occurs exactly once in each row of the array;
- the first column contains n ordered pairs twice;
- excluding the ordered pairs in column 1 all other unordered pair of symbols occurs in exactly one cell of the array.

Note: the OTD has less restrictions than a Partitioned Balanced Tournament Design (PBTD). See Appendix E (page 389) for more details.

An example of an OTD of order four ($n = 4$) follows, where the set of symbols are integers from 1 to 8:

1,5	3,4	6,8	2,7
2,6	4,5	7,1	3,8
3,7	5,6	8,2	4,1
4,8	6,7	1,3	5,2
5,1	7,8	2,4	6,3
6,2	8,1	3,5	7,4
7,3	1,2	4,6	8,5
8,4	2,3	5,7	1,6

This OTD defines a draw for 9 teams without a sit-out by selecting pairs of rows 4 apart:

(9)1,5	3,4 7,8	6,8 2,4	2,7 6,3
(9)2,6	4,5 8,1	7,1 3,5	3,8 7,4
(9)3,7	5,6 1,2	8,2 4,6	4,1 8,5
(9)4,8	6,7 2,3	1,3 5,7	5,2 1,6

The Safford-like cyclic nature of this array should be pointed out, so that the definition of the movement can be reduced to the first line of the OTD:

1,5 3,4 6,8 2,7

The differences between each pair of numbers are all different, as found in Howell movements. For more clarity we can show this as two rows with the second lot of values found by adding 4 to the first (modulus 8) and in the general case add n (modulus $2n$). This is called a bicyclic schedule.

(9)1,5	3,4	6,8	2,7
	7,8	2,4	6,3

The other order 4 OTD are:

1,5	2,3	6,8	4,7
1,5	6,7	2,4	3,8
1,5	7,8	2,4	3,6

With 7 Teams (n = 3) the second method of definition is required as the second row is not derived from the first:

(7) 1,4	2,3	5,6
	3,5	6,2

11 teams (n = 5):

(11) 1,6	2,3	7,9	5,8	10,4
	7,8	2,4	10,3	5,9

The other order 5 possibilities are:

1,6	4,5	7,9	3,10	2,8
1,6	7,8	2,4	3,10	5,9
1,6	9,10	2,4	5,8	3,7

Where n is even and ½ N is odd (oddly-even numbers) it is not possible to produce a simple Safford cyclic or bicyclic draw for all the rounds. These are dealt with later as a separate subject in *oddly-even house round-robin schedules* (page 218).

The one exception we need to deal with separately is a draw for 5 teams. There is no house square order 2 and so we cannot produce an OTD order 2 either. There are some simple alternatives including American whist league (AWL) or Clay movements where the teams play the complete movement in one sitting.

There are other possible solutions to this problem, but a simple alternative is to play two 3-way triangles in two rounds, then do it again using the same board-sets and no re-dealing is required. This is not all that different from playing *single* triangles. Arrange the teams at their tables as you would for a normal AWL movement. The board-sets 1 to 5 are placed on tables 1 to 5. The E-W pairs move down 1 table and play commences. At the end of the first round the E-W players move in a normal AWL move, E-W up two and boards up 1. Play for the second half of the 3-way is completed in round 2. The E-W pairs return to their team-mates (leaving the boards where they are) and scoring takes place for the two matches each team has played. The E-W pairs then move up 3 tables from their home table and the boards up 1 table. After that round is played the boards move up 2 and the E-W pairs move down 1. The E-W pairs again return to their teams mates to score.

Round	Table 1	Table 2	Table 3	Table 4	Table 5
start	1–1 (1)	2–2 (2)	3–3 (3)	4–4 (4)	5–5 (5)
1	1–2 (1)	2–3 (2)	3–4 (3)	4–5 (4)	5–1 (5)
2	1–5 (5)	2–1 (1)	3–2 (2)	4–3 (3)	5–4 (4)
3	1–3 (4)	2–4 (5)	3–5 (1)	4–1 (2)	5–2 (3)
4	1–4 (2)	2–5 (3)	3–1 (4)	4–2 (5)	5–3 (1)

Of course each team does not play one board-set but that is no different from the other three way matches for more teams than 5 teams.

An even neater solution is to use the McKinnon modification to the AWL system, see page 237 below.

Bicyclic round-robin schedules

Where n is odd and $(N - 1) / 2$ is odd it is not possible to define the draw with a one-line definition. For bridge organizers the two line definition of the draw is essential for simplicity and therefore all numbers of teams up to 51 are shown here in two lines, as bicyclic draws. The exceptions are those for 13, 21, 29, 37 and 45, which are shown later (page 220).

7	1 4	2 3	5 6									
		2 6	5 3									
9	1 5	2 3	6 8	4 7								
		6 7	2 4	8 3								
11	1 6	2 3	7 9	5 8	4 10							
		7 8	2 4	10 3	9 5							
15	1 8	2 3	9 10	4 6	11 13	5 14	12 7					
		2 13	9 6	3 7	10 14	4 12	11 5					
17	1 9	2 3	4 6	8 11	12 16	10 15	7 13	5 14				
		10 11	12 14	16 3	4 8	2 7	15 5	13 6				
19	1 10	2 3	4 6	8 11	13 17	9 14	12 18	5 16	7 15			
		11 12	13 15	17 2	4 8	18 5	3 9	14 7	16 6			
23	1 12	2 3	13 14	4 6	15 17	5 8	16 19	7 11	18 22	9 21	20 10	
		2 7	13 18	4 10	15 21	5 20	16 9	3 11	14 22	6 19	17 8	
25	1 13	2 3	4 6	5 8	10 14	15 20	16 22	17 24	11 19	9 18	7 21	12 23
		14 15	16 18	17 20	22 2	3 8	4 10	5 12	23 7	21 6	19 9	24 11

27	1 14	2 3	4 6	5 8	7 11	13 18	16 22	19 26	17 25	15 24	10 20	12 23	9 21												
		15 16	17 19	18 21	20 24	26 5	3 9	6 13	4 12	2 11	23 7	25 10	22 8												
31	1 16	2 3	17 18	4 6	19 21	5 8	20 23	7 11	22 26	9 14	24 29	10 30	25 15	12 28	27 13										
		2 8	17 23	3 10	18 25	4 12	19 27	5 14	20 29	7 26	22 11	6 24	21 9	13 30	28 15										
33	1 17	2 3	4 6	5 8	7 11	9 14	15 21	18 25	22 30	23 32	19 29	20 31	16 28	13 26	10 24	12 27									
		18 19	20 22	21 24	23 27	25 30	31 5	2 9	6 14	7 16	3 13	4 15	32 12	29 10	26 8	28 11									
35	1 18	2 3	4 6	5 8	7 11	9 14	13 19	20 27	21 29	23 32	24 34	22 33	16 28	12 25	17 31	15 30	10 26								
		19 20	21 23	22 25	24 28	26 31	30 2	3 10	4 12	6 15	7 17	5 16	33 11	29 8	34 14	32 13	27 9								
39	1 20	2 3	21 22	4 6	23 25	5 8	24 27	7 11	26 30	9 14	28 33	10 16	29 35	12 19	31 38	13 37	32 18	15 36	34 17						
		2 10	21 29	3 12	22 31	4 14	23 33	5 16	24 35	8 34	27 15	13 38	32 19	7 30	26 11	6 28	25 9	17 37	36 18						
41	1 21	2 3	4 6	5 8	7 11	9 14	10 16	15 22	23 31	20 29	26 36	27 38	28 40	24 37	25 39	18 33	19 35	17 34	12 30	13 32					
		22 23	24 26	25 28	27 31	29 34	30 36	35 2	3 11	40 9	6 16	7 18	8 20	4 17	5 19	38 13	39 15	37 14	32 10	33 12					
43	1 22	2 3	4 6	5 8	7 11	9 14	10 16	12 19	23 31	21 30	24 34	27 38	28 40	29 42	25 39	26 41	17 33	20 37	18 36	13 32	15 35				
		23 24	25 27	26 29	28 32	30 35	31 37	33 40	2 10	42 9	3 13	6 17	7 19	8 21	4 18	5 20	38 12	41 16	39 15	34 11	36 14				
47	1 24	2 3	25 26	4 6	27 29	5 8	28 31	7 11	30 34	9 14	32 37	10 16	33 39	12 19	35 42	13 21	36 44	15 46	38 23	17 45	40 22	18 43	41 20		
		2 11	25 34	3 13	26 36	4 15	27 38	5 17	28 40	6 19	29 42	8 22	31 45	7 23	30 46	10 39	33 16	14 41	37 18	9 35	32 12	20 44	43 21		
49	1 25	2 3	4 6	5 8	7 11	9 14	10 16	12 19	13 21	26 35	24 34	27 38	28 40	31 44	32 46	33 48	29 45	30 47	23 41	17 36	22 42	18 39	15 37	20 43	
		26 27	28 30	29 32	31 35	33 38	34 40	36 43	37 45	2 11	48 10	3 14	4 16	7 20	8 22	9 24	5 21	6 23	47 17	41 12	46 18	42 15	39 13	44 19	
51	1 26	2 3	4 6	5 8	7 11	9 14	10 16	12 19	13 21	22 31	27 37	25 36	29 41	32 45	34 48	35 50	33 49	30 47	28 46	23 42	24 44	17 38	18 40	20 43	15 39
		27 28	29 31	30 33	32 36	34 39	35 41	37 44	38 46	47 6	2 12	50 11	4 16	7 20	9 23	10 25	8 24	5 22	3 21	48 17	49 19	42 13	43 15	45 18	40 14

Oddly-even House round-robin schedules

Up until this time the published schedules for 13 teams were untidy at best. No draws existed for numbers of teams 21, 29, 37 etc. In the Stinson and Wallis paper *An even side analogue of Room squares* (1984) we were introduced to the house square. In there they say:

LEMMA 2.4 *If q is a prime power congruent to 1 modulus 4, then there is a starter-adder in $G = G\,F(q) \times \boldsymbol{Z}_2$; and hence there exists a house of order $q + 1$.*

A further proof is then supplied. This is stating there are house squares for those pesky oddly-even orders, but only some of them. It is only for q where it is a prime power. This caters for 13, 21, 29 and 37 teams where q is 5, 9, 13 and 17 respectively which are primes or in the case of 9 a power of a prime (3^2). It does

not cater for 45 teams where q would be 21 (3 × 7). Essentially what this says is to take a known house square of order one less than the required and you then convert this to a valid house of the order needed. Take the first case, 13 teams. Subtract 1 from 13 to get 12 and halve this for 6. We need a house of order 6 to produce the draw for 13 teams. Wallis says we can do this by using a house of order 5, and then add one. The house of order 5 was used in the 11 teams draw.

It is not that simple, we need a house square with very specific qualities. Once we have the required house order 5 we add one to this. A house of order 5 (n = 5) has a side of 10 ($2n$) and so the first thing we do is introduce 2 'stationary' numbers – these are the two extra teams we need. In the house square shown here these two values are shown as 'a' and 'b'. A house of order 6 has side 12 and so we add two rows and two columns and fill out the values.

Below is a house square of order 6 used to produce a 13-team draw. The modified house order 5 is shown in bold, side 10. The order 5 square, with two columns added, is a bicyclic schedule of 10 rounds. The first two rows supply us with the two additional rounds needed for 13 teams. The 13th team is added to the first two columns over two rounds for the three-way matches.

Note the * in row 3 column 4. In the original order 5 square this contained 2,3 (immediately above it in row 2) and then cycled from there in the usual fashion. The entries b,3 and a,2 etc., were not in the original square. The ability to introduce these b,3 and a,2 are the specific qualities of the required house square.

a,b		1,2		3, 4		5, 6		7, 8		9,10	
	a,b		2, 3		4, 5		6, 7		8, 9		10, 1
1, 6		7, 9	*	5, 8	b,3			4,10	a,2		
2, 7			8,10		6, 9	b,4			5, 1	a,3	
3, 8				9, 1		7,10	b,5			6, 2	a,4
4, 9		a,5			10, 2		8, 1	b,6			7, 3
5,10		8, 4	a,6			1, 3		9, 2	b,7		
	6, 1		9, 5	a,7			2, 4		10, 3	b,8	
	7, 2			10, 6	a,8			3, 5		1, 4	b,9
	8, 3	b,10			1, 7	a,9			4, 6		2, 5
	9, 4	3, 6	b,1			2, 8	a,10			5, 7	
	10, 5		4, 7	b,2			3, 9	a,1			6, 8

Round						
1	1v6v13	7v9	5v8	12v3	4v10	11v2
2	1v6v13	9v5	11v7	2v4	10v3	12v8
3	2v7v13	8v10	6v9	12v4	5v1	11v3
4	2v7v13	10v6	11v8	3v5	1v4	12v9
5	3v8v13	9v1	7v10	12v5	6v2	11v4
6	3v8v13	12v10	1v7	11v9	4v6	2v5
7	4v9v13	11v5	10v2	8v1	12v6	7v3
8	4v9v13	3v6	12v1	2v8	11v10	5v7
9	5v10v13	8v4	11v6	1v3	9v2	12v7
10	5v10v13	4v7	12v2	3v9	11v1	5v7
11	11v12v13	1v2	3v4	5v6	7v8	9v10
12	11v12v13	2v3	4v5	6v7	8v9	10v1

The dark border surrounds the bicyclic schedule and the last two rounds finish the draw.

Using a similar approach to that used above for other numbers of teams this schedule can be shown as:

(13) 1v6	7v9	5v8	12v3	4v10	11v2
	9v5	11v7	2v4	10v3	12v8

The underlined 11 and 12 indicate they are not in the cyclic schedule. 1 follows 10 and the others the number one less than theirs. The last two rounds in all cases are very predictable with the three highest teams playing each other in the three-way and the others playing against teams with numbers one more and one less than their own.

(13) 11v12	1v2	3v4	5v6	7v8	9v10
	2v3	4v5	6v7	8v9	10v1

13	1 6	7 9	5 8	12 3	4 10	11 2												
		9 5	11 7	2 4	10 3	12 8												
21	1 10	4 6	5 16	20 3	13 17	12 18	7 15	19 2	8 11	9 14								
		19 11	17 2	18 5	13 15	14 7	20 12	4 8	3 9	16 6								
29	1 14	4 6	17 25	28 3	16 22	15 24	19 26	7 11	9 21	27 2	12 23	10 20	13 18	5 8				
		22 8	27 15	25 10	23 7	26 5	18 21	17 19	4 12	28 16	3 9	2 11	6 13	20 24				
37	1 18	4 6	35 2	8 11	15 23	16 29	5 32	20 26	7 31	30 34	12 28	36 3	10 19	17 22	14 25	9 24	13 27	21 33
		24 14	13 17	29 11	36 20	27 2	34 5	31 8	26 7	30 10	4 16	21 23	35 19	25 28	32 6	33 12	22 15	3 9

45 teams round-robin schedule

At this point I have supplied round-robin draws for odd numbers of teams up to 51 with the one exception, 45. This requires a house square of order 22. We saw above that Lemma 2.4 does not apply. To proceed from here we need to use Lemma 2.9 found in the same paper. This relies on Room frames (commonly called *frames*), a new concept we have not seen in this book, and I am not attempting to deal with this, but the reader may choose to pursue this subject. Put simply, the mathematics gets very messy and complicated, and for the sake of one case, 45 teams, it is not worth the effort. The next such case relying on frames is 69 teams. In addition, if we chose to pursue this type of solution the results are not as 'cyclic' as the rest.

There is another solution to this number of teams which is very manageable, and the technique can be used for higher numbers of teams. We build a draw based on 15, dividing the 45 teams into three groups. This approach will also work for any other number of teams where ⅓rd the number is odd (21, 27, 33…). More generally, this applies where $N \bmod 3 = 0$.

Let the three groups of teams be A, B and C. In each group there are 15 teams numbered 1 to 15. A draw is devised so that all teams in group B play all the teams in group C, and the 15-teams draw (above) is used for group A, where all the teams in group A meet each other. The draw for BvC is the same as using a Mitchell pairs' movement for 15 tables, but instead of N-S 1 to 15 meeting E-W 1 to 15 we have B1 to B15 meeting C1 to C15. The 15 team draw above (for group A) has only 14 rounds, therefore for uniform sessions the first round in the matches BvC are not played. Therefore B1 does not play C1, B2 does not play C2, and so forth. This completes segment one of the draw.

Segment 2 of the draw has group A teams playing group C teams, and the B group play the teams within their group. The last segment similarly has group A playing group B while C play the teams within their group. At this point there have been 42 rounds with two more yet to be played. A1 has not played B1 and C1, A2 has not played B2 and C2, and so forth. Finally, three way matches are played by all these lots of 3 teams, over two rounds, to complete all the remaining matches.

Consider group A as teams 1 to 15, B as 16 to 30 and C as 31 to 45. The first segment then looks like:

Group A draw for rounds 1 to 14:

1	15v1v8	2v3	9v10	4v6	11v13	5v14	12v7
2	15v1v8	2v13	9v6	3v7	10v14	4v12	11v5
3	15v2v9	3v4	10v11	5v7	12v14	6v1	13v8
4	15v2v9	3v14	10v7	4v8	11v1	5v13	12v6
5	15v3v10	4v5	11v12	6v8	13v1	7v2	14v9
6	15v3v10	4v1	11v8	5v9	12v2	6v14	13v7
7	15v4v11	5v6	12v13	7v9	14v2	8v3	1v10
8	15v4v11	5v2	12v9	6v10	13v3	7v1	14v8
9	15v5v12	6v7	13v14	8v10	1v3	9v4	2v11
10	15v5v12	6v3	13v10	7v11	14v4	8v2	1v9
11	15v6v13	7v8	14v1	9v11	2v4	10v5	3v12
12	15v6v13	7v4	14v11	8v12	1v5	9v3	2v10
13	15v7v14	8v9	1v2	10v12	3v5	11v6	4v13
14	15v7v14	8v5	1v12	9v13	2v6	10v4	3v11

Group B verses C draw (ignoring the original round 1):

1	16v45	17v31	18v32	19v33	20v34	21v35	22v36	23v37	24v38	25v39	26v40	27v41	28v42	29v43	30v44
2	16v44	17v45	18v31	19v32	20v33	21v34	22v35	23v36	24v37	25v38	26v39	27v40	28v41	29v42	30v43
3	16v43	17v44	18v45	19v31	20v32	21v33	22v34	23v35	24v36	25v37	26v38	27v39	28v40	29v41	30v42
4	16v42	17v43	18v44	19v45	20v31	21v32	22v33	23v34	24v35	25v36	26v37	27v38	28v39	29v40	30v41
5	16v41	17v42	18v43	19v44	20v45	21v31	22v32	23v33	24v34	25v35	26v36	27v37	28v38	29v39	30v40
6	16v40	17v41	18v42	19v43	20v44	21v45	22v31	23v32	24v33	25v34	26v35	27v36	28v37	29v38	30v39
7	16v39	17v40	18v41	19v42	20v43	21v44	22v45	23v31	24v32	25v33	26v34	27v35	28v36	29v37	30v38
8	16v38	17v39	18v40	19v41	20v42	21v43	22v44	23v45	24v31	25v32	26v33	27v34	28v35	29v36	30v37
9	16v37	17v38	18v39	19v40	20v41	21v42	22v43	23v44	24v45	25v31	26v32	27v33	28v34	29v35	30v36
10	16v36	17v37	18v38	19v39	20v40	21v41	22v42	23v43	24v44	25v45	26v31	27v32	28v33	29v34	30v35
11	16v35	17v36	18v37	19v38	20v39	21v40	22v41	23v42	24v43	25v44	26v45	27v31	28v32	29v33	30v34
12	16v34	17v35	18v36	19v37	20v38	21v39	22v40	23v41	24v42	25v43	26v44	27v45	28v31	29v32	30v33
13	16v33	17v34	18v35	19v36	20v37	21v38	22v39	23v40	24v41	25v42	26v43	27v44	28v45	29v31	30v32
14	16v32	17v33	18v34	19v35	20v36	21v37	22v38	23v39	24v40	25v41	26v42	27v43	28v44	29v45	30v31

Similarly for segments 2 and 3, so that the summary appears like:

Segment	One group	Two groups
1	A (1–15)	B (16–30) v C (31–45)
2	B (16–30)	C (31–45) v A (1–15)
3	C (31–45)	A (1–15) v B (16–30)

The teams in the three-way draws for the last two rounds are:

1v16v31	2v17v32	3v18v33	4v19v34	5v20v35	6v21v36	7v22v37	8v23v38
9v24v39	10v25v40	11v26v41	12v27v42	13v28v43	14v29v44	15v30v45	

A very similar technique can be used with 5 groups. For 45 teams these groups are A(1–9), B(10–18), C(19–27), D(28–36) and E(37–45). When the teams play those in their group the 9-teams draw supplied above is used. This technique applies where N mod 5 = 0. In summary the draw for 45 teams is:

Segment	One group	First two groups	Second two groups
1	A	E v B	D v C
2	E	D v A	C v B
3	D	C v E	B v A
4	C	B v D	A v E
5	B	A v C	E v D

Each segment plays 8 rounds for a total of 40 rounds. The last 4 rounds are played as 5-teams draws (see above page 216) involving the teams that have not played each other. Those teams are A1, B1, C1, D1 and E1 for the first draw. Similarly for A2 to E2, A3 to E3 and so forth.

53 and more odd teams round-robin schedules

It is very unlikely to be a need for round-robin draws for more than 51 teams but to complete the discussion on this subject we can consider the approaches needed.

In dealing with 45 teams (above) we introduced two concepts that deal with numbers of teams (N) where N mod 3 = 0 and N mod 5 = 0. The first approach, N mod 3 = 0, caters for 57, 63, 69 etc. The second, N mod 5 = 0, covers 55, 65, 75 etc.

Now consider the numbers of teams where N mod 3 = 1, catering for 55, 61, 67, 73 etc. Note you should prefer this approach for those numbers where N mod 5 = 0 (e.g. 55).

For 55 teams we use three groups A, B and C where there are 18 teams in each group. In addition we have one more team, say team 55. In each segment the two groups meeting each other play all 18 rounds. Team 55 is added to the one group (when they are playing those in their group) for a 19-teams draw, and 18 rounds, as supplied above. At the end of the three segments all teams have played all other teams over 54 rounds. The only disadvantage is imposed on team 55 who plays in the three-way matches at all times.

Segment	One group	Two groups
1	A (1–18) + 55	B (19–36) v C (37–54)
2	B (19–36) + 55	C (37–54) v A (1–18)
3	C (37–54) + 55	A (1–18) v B (19–36)

Similarly for other numbers of teams, where the group sizes are 20, 22, 24 etc. teams.

This leaves us with one more case, where N mod 3 = 2. This caters for 53, 59, 71 etc. Note you should use the 5 group option where N mod 5 = 0, specifically 65 teams.

Consider 53 teams where we use three groups A, B and C with 17 teams in each group. In addition we need two more teams, say teams 52 and 53. In each segment the two groups meeting each other play 16 rounds, missing the first round where B1 play C1, B2 play C2 etc. In each segment teams 52 and 53 are added to the draw where these two groups (B and C in the first segment) meet each other. The remaining group A use the 17-teams draw (above) playing 16 rounds.

With the draw where the two groups meet each other based on the Mitchell schedule, the two extra teams play the teams at match 2, then 4, 6 and so forth, skipping one match position in each new round. The array on the next page shows the matches (1 to 17) across the top and the rounds (1 to 16) down the side. Team 52 plays the teams in group B and 53 those in group C. For example match 2 in round 1, team 52 plays B2 and team 53 plays team C1. The match B2 v C1 will be played later, as will all the other matches where 52 and 53 are involved.

The complete schedule is shown here with team 52 playing the teams in the first mentioned group of the two and team 53 playing the second:

Segment	One group	Two groups
1	A (1–17)	B (18–34) v C (35–51) + 52 and 53
2	B (18–34)	C (35–51) v A (1–17) + 52 and 53
3	C (35–51)	A (1–17) v B (18–34) + 52 and 53

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	B1C17	52B2 53C1	B3C2	B4C3	B5C4	B6C5	B7C6	B8C7	B9C8	B10C9	B11C10	B12C11	B13C12	B14C13	B15C14	B16C15	B17C16
2	B1C16	B2C17	B3C1	52B4 53C2	B5C3	B6C4	B7C5	B8C6	B9C7	B10C8	B11C9	B12C10	B13C11	B14C12	B15C13	B16C14	B17C15
3	B1C15	B2C16	B3C17	B4C1	B5C2	52B6 53C3	B7C4	B8C5	B9C6	B10C7	B11C8	B12C9	B13C10	B14C11	B15C12	B16C13	B17C14
4	B1C14	B2C15	B3C16	B4C17	B5C1	B6C2	B7C3	52B8 53C4	B9C5	B10C6	B11C7	B12C8	B13C9	B14C10	B15C11	B16C12	B17C13
5	B1C13	B2C14	B3C15	B4C16	B5C17	B6C1	B7C2	B8C3	B9C4	52B10 53C5	B11C6	B12C7	B13C8	B14C9	B15C10	B16C11	B17C12
6	B1C12	B2C13	B3C14	B4C15	B5C16	B6C17	B7C1	B8C2	B9C3	B10C4	B11C5	52B12 53C6	B13C7	B14C8	B15C9	B16C10	B17C11
7	B1C11	B2C12	B3C13	B4C14	B5C15	B6C16	B7C17	B8C1	B9C2	B10C3	B11C4	B12C5	B13C6	52B14 53C7	B15C8	B16C9	B17C10
8	B1C10	B2C11	B3C12	B4C13	B5C14	B6C15	B7C16	B8C17	B9C1	B10C2	B11C3	B12C4	B13C5	B14C6	B15C7	52B16 53C8	B17C9
9	52B1 53C9	B2C10	B3C11	B4C12	B5C13	B6C14	B7C15	B8C16	B9C17	B10C1	B11C2	B12C3	B13C4	B14C5	B15C6	B16C7	B17C8
10	B1C8	B2C9	52B3 53C10	B4C11	B5C12	B6C13	B7C14	B8C15	B9C16	B10C17	B11C1	B12C2	B13C3	B14C4	B15C5	B16C6	B17C7
11	B1C7	B2C8	B3C9	B4C10	52B5 53C11	B6C12	B7C13	B8C14	B9C15	B10C16	B11C17	B12C1	B13C2	B14C3	B15C4	B16C5	B17C6
12	B1C6	B2C7	B3C8	B4C9	B5C10	B6C11	52B7 53C12	B8C13	B9C14	B10C15	B11C16	B12C17	B13C1	B14C2	B15C3	B16C4	B17C5
13	B1C5	B2C6	B3C7	B4C8	B5C9	B6C10	B7C11	B8C12	52B9 53C13	B10C14	B11C15	B12C16	B13C17	B14C1	B15C2	B16C3	B17C4
14	B1C4	B2C5	B3C6	B4C7	B5C8	B6C9	B7C10	B8C11	B9C12	B10C13	52B11 53C14	B12C15	B13C16	B14C17	B15C1	B16C2	B17C3
15	B1C3	B2C4	B3C5	B4C6	B5C7	B6C8	B7C9	B8C10	B9C11	B10C12	B11C13	B12C14	52B13 53C15	B14C16	B15C17	B16C1	B17C2
16	B1C2	B2C3	B3C4	B4C5	B5C6	B6C7	B7C8	B8C9	B9C10	B10C11	B11C12	B12C13	B13C14	B14C15	52B15 53C16	B16C17	B17C1

After 48 rounds the above schedule is complete and there are still 4 rounds to be played to complete the draw. Firstly the teams {A1, B1 and C1} have not played each other. Similarly for {A2, B2 and C2}; {A3, B3 and C3}, and so forth up to {A16, B16 and C16}. All these sets of teams play three-way matches over two rounds. Next the three-way matches involving the teams not played because of the matches involving 52 and 53 (shown above) are played. These are

B2vC1vA4	B4vC2vA8	B6vC3vA12	B8vC4vA16	B10vC5vA3	B12vC6vA7	B14vC7vA11	B16vC8vA15
B1vC9vA2	B3vC10vA6	B5vC11vA10	B7vC12vA14	B9vC13vA1	B11vC14vA5	B13vC15vA9	B15vC16vA13

There is a clear pattern in the order these numbers. It can be seen that teams A17, B17 and C17, in addition to 52 and 53, have not been included in these three-way matches. These five teams play a 5-teams draw over the last 4 rounds to complete all matches.

8.2 The American Whist League system

Only those who dare to fail greatly can ever achieve greatly.
Robert F. Kennedy

See the *Barney modification* of the Clay system, page 51, for more details and origins of this movement. This movement works only for an odd number of tables when used for teams. The N-S pairs remain stationary and the E-W pairs move up (or down) 2 tables and boards up (or down) one table.

The teams are seated at the table with the same number as their team number and the play commences after the first move (each pair not playing its team mates). This means that the number of rounds is equal to one less than the number of tables. Once the event is started, as defined here, the event must be completed in the one session.

Two stanza modification

To play the event in two sittings the E-W pairs are instructed to move a number of tables on the first change and then the movement proceeds normally until approximately half way, when the scoring can be done for the first half. For the second half of the event the E-W pairs return to the last table played and then move normally until the rest of the teams have been played. When the E-W pairs encounter their team-mates they make a second move on that change.

The special initial move for the E-W pairs is a certain number of tables (T) up (down) where $T = ½(N + 1)$ and N = 'the number of teams in the event'. If this resultant value of T is odd, then add one for the starting move. The number of rounds of play is equal to $T\,(T - 1)$ when odd.

For example, with 13 tables, $T = ½(13 + 1) = 7$ (as this is odd, add one for the first move). Therefore 8 is the starting move. To find the number of rounds before the half-way stopping point simply subtract 1 from T (in this example 6).

If it is known at the start of the event that the number of rounds is to be less than the normal $N - 1$ it is possible to cater for this. An example is where there are two sections, one with 13 tables and the other with 15 tables. Suppose, because of time restrictions, it is necessary to play only 12 rounds. The 13-table section is straightforward while it is necessary to reduce the number of rounds for the 15-table section by two.

For every extra change at the start (i.e., for every two tables the E-W pairs move) the event is reduced by two tables. Therefore, with 15 tables playing 12 rounds, the E-W pairs move 4 tables on the first change (rather than 2) and end one round earlier than normal and play 12 teams.

Same-board whist modification

Where there is a large whist movement and the number of rounds is to be reduced to a number less than the maximum number possible, it may be desirable that all the teams play the same boards. This is possible for certain combinations of tables and numbers of rounds desired.

The number of teams involved is T and must be odd. The number of rounds is R and is an even number. The standard whist league movement is used, E-W up (or down) two, boards up (or down) one. With the teams seated at their own table to start with, the E-W pairs move down $R - 1$ tables (or up $T - R + 1$ tables) for the first move and for their first table of play. This will only work where T Modulus $R = 1$.

The boards must be duplicated and the number of sets S required depends on the number of rounds;

$$S = T / R.$$

Given that these board-sets are numbered 1, 2, … R, then the board-sets are distributed, starting at table 1, as

$$1, 2, \ldots, R, 1, 2, \ldots, R, 1, 2, \ldots, R, R + 1.$$

Note one extra board-set ($R + 1$) is required to complete the movement.
For example with 13 tables requiring 4 rounds the board-sets (say, 6 boards per set) would be distributed:

Table:	1	2	3	4	5	6	7	8	9	10	11	12	13
Sets:	1	2	3	4	1	2	3	4	1	2	3	4	5

The initial move by the E-W pairs is down 3 tables.

This movement is useful only for some combinations of tables and rounds. For example:

17 teams and 8 rounds;
21, 31, 41, … teams and 10 rounds;
25, 37, 49, … teams and 12 rounds;
19 teams and 6 rounds.

Every team plays all sets except one at the end of the movement (i.e. after R rounds).

Another similar movement works where T modulus ($R + 1$) = 1. Here the number of board-sets common to each match is one less. The sets are distributed similar to the first method

$$1, 2, \ldots, R + 1, 1, 2, \ldots, R + 1, 1, 2, \ldots, R + 1, R + 2.$$

This movement is useful for:
15 teams and 6 rounds;
19 teams and 8 rounds;
23, 34, 45, … teams and 10 rounds.

This method can be extended as follows:
T Modulus ($R + 2$) = 1;
T Modulus ($R + 3$) = 1 etc.
but each additional board-set in the movement tends to defeat the purpose of the movement.

Split modification

This modification was first documented in *duplicate bridge direction* by Alex Groner in 1961. It is a difficult movement to organize and is supplied here to complete the record.

The split movement applies only to an even number of tables. It is organized in the same way as any American whist league (AWL) movement, with an odd number of tables. There are two special moves: the first called the cross-over and the second the split. Play proceeds in the same way as AWL until the cross-over. This occurs at different times, depending on the number of tables *N*. The cross-over takes place *after* the round *R* found by:

$$R = (N / 4_{\text{ rounded up}}) - 1$$

- For 8 teams, that is 8/4 – 1 = 2 – 1 = 1
- For 10 teams, that is $10/4 - 1 = 2.5_{\text{rounded}} - 1 = 3 - 1 = 2$
- For 12 teams, that is 12/4 – 1 = 3 – 1 = 2 …etc.

The cross-over means that the E-W pairs take the boards they have just played and place it on the table where their N-S team mates are sitting. The E-W pairs return to the table where they have just played and move from there as before, except that if *N* is divisible by 4 (*N* mod 4 = 0) they make two standard moves. That is, the E-W pairs move either two or four tables from the last played table.

Play again proceeds as a normal AWL until the split. The split occurs *after* round 2*R* and involves two things. The E-W pairs move *M* tables in the *opposite* direction to the normal move where *M* is found by either

if ½ *N* is odd, $M = \frac{1}{2} N - 4$

or

if ½ *N* is even, $M = \frac{1}{2} N - 3$.

The boards move in the usual direction *R* + 2 tables.

N	*R*	2*R*	*M*	*R* + 2
8	1	2	1	3
10	2	4	1	4
12	2	4	3	4
14	3	6	3	5
16	3	6	5	5
18	4	8	5	6
20	4	8	7	6

From then on the E-W and board moves are standard. Play can cease after *N* – 2 rounds, but if not, the last round requires a new board-set relayed between the two tables of each match or alternatively multiple copies of the same boards shared across all tables. For example the 8-table movement is

Round	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7	Table 8
1	1–3 (2)	2–4 (3)	3–5 (4)	4–6 (5)	5–7 (6)	6–8 (7)	7–1 (8)	8–2 (1)
2	1–7 (8)	2–8 (1)	3–1 (2)	4–2 (3)	5–3 (4)	6–4 (5)	7–5 (6)	8–6 (7)
3	1–6 (3)	2–7 (4)	3–8 (5)	4–1 (6)	5–2 (7)	6–3 (8)	7–4 (1)	8–5 (2)
4	1–8 (4)	2–1 (5)	3–2 (6)	4–3 (7)	5–4 (8)	6–5 (1)	7–6 (2)	8–7 (3)
5	1–2 (5)	2–3 (6)	3–4 (7)	4–5 (8)	5–6 (1)	6–7 (2)	7–8 (3)	8–1 (4)
6	1–4 (6)	2–5 (7)	3–6 (8)	4–7 (1)	5–8 (2)	6–1 (3)	7–2 (4)	8–3 (5)
7	1–5 (9)	2–6 (9)	3–7 (9)	4–8 (9)	5–1 (9)	6–2 (9)	7–3 (9)	8–4 (9)

Double-skip modification

This is a simpler form of modification than the split, but essentially gives the same outcome. This came about with a suggestion by Richard Freeman in the early 1950s. Again the double-skip movement applies only to an even number of tables. It is organized in the same way as any AWL movement, the same way as for an odd number of tables. There are two special moves. For convenience they are called the *first* and the *second*. Play proceeds in the same way as AWL until the *first* special move when the E-W pairs move an extra table (3 instead of 2), the boards moving normally. This occurs at different times depending on the number of tables N and takes place *after* the round R_1, which is the same value as R in the *split modification*: R_1 = (N / 4 rounded up) – 1. Play proceeds as a normal AWL after this until the *second* special move.

The *second* special move involves the E-W pairs moving an extra table (same as in the *first*), but in addition the boards also move an extra table (2 instead of 1). This occurs at different times depending on the number of tables N and takes place *after* the round R_2 where R_2 = (N / 4 rounded down) – 1 + ½ N.

N	R_1	R_2
8	1	5
10	2	6
12	2	8
14	3	9
16	3	11
18	4	12
20	4	14

From then on the E-W and board moves are standard. Play can cease after N – 2 rounds, but if not, the last round requires a new board-set relayed between the two tables of each match or alternatively multiple copies of the same boards shared across all tables. This is the same as the split modification. For example the 8-table movement is

Round	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7	Table 8
1	1–3 (2)	2–4 (3)	3–5 (4)	4–6 (5)	5–7 (6)	6–8 (7)	7–1 (8)	8–2 (1)
2	1–6 (3)	2–7 (4)	3–8 (5)	4–1 (6)	5–2 (7)	6–3 (8)	7–4 (1)	8–5 (2)
3	1–8 (4)	2–1 (5)	3–2 (6)	4–3 (7)	5–4 (8)	6–5 (1)	7–6 (2)	8–7 (3)
4	1–2 (5)	2–3 (6)	3–4 (7)	4–5 (8)	5–6 (1)	6–7 (2)	7–8 (3)	8–1 (4)
5	1–4 (6)	2–5 (7)	3–6 (8)	4–7 (1)	5–8 (2)	6–1 (3)	7–2 (4)	8–3 (5)
6	1–7 (8)	2–8 (1)	3–1 (2)	4–2 (3)	5–3 (4)	6–4 (5)	7–5 (6)	8–6 (7)
7	1–5 (9)	2–6 (9)	3–7 (9)	4–8 (9)	5–1 (9)	6–2 (9)	7–3 (9)	8–4 (9)

Stationary-board modification

The use of stationary boards in the AWL system does change the nature of the movement significantly. It is directly comparable to the teams Mitchell system with stationary boards, but it is slightly different.

This modification is based on the AWL system, with each team sitting at their home table at the start, teams 1 to 7 sit at tables 1 to 7 and board-sets 1 to 7 are placed on tables 1 to 7.

E-W pairs move down 2 tables at the start and from then on move down 1 table. The N-S pairs are stationary for the first round and move up 1 table after each other round. The 7-table example is shown next.

Round	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7
start	1–1	2–2	3–3	4–4	5–5	6–6	7–7
1	1–3	2–4	3–5	4–6	5–7	6–1	7–2
2	7–4	1–5	2–6	3–7	4–1	5–2	6–3
3	6–5	7–6	1–7	2–1	3–2	4–3	5–4
4	5–6	6–7	7–1	1–2	2–3	3–4	4–5
5	4–7	5–1	6–2	7–3	1–4	2–5	3–6
6	3–1	4–2	5–3	6–4	7–5	1–6	2–7

The movement is the same for even numbers of tables, except for a skip being required by both the E-W and N-S pairs after particular rounds, moving two tables instead of one. The skip rounds occur after different rounds depending on the number of tables. For E-W pairs this round R_1 is found by

$R_1 = ((N / 4_{\text{ rounded up}}) - 1)$ where N is the number of teams.

For N-S their skip occurs after $R_2 = N - 2 - R_1$.

These round numbers are shown here and there is clearly a simple pattern

N	R_1	R_2
6	1	3
8	1	5
10	2	6
12	2	8
14	3	9
16	3	11
18	4	12
20	4	14

With an odd number of tables there is always an even number of rounds and all possible matches are completed by the end of the event. For an even number of teams, that is not the case. The number of rounds is odd. The last round must complete one match in that one round, sharing boards with their team mates. Consider the 8 table movement:

Round	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7	Table 8
start	1–1	2–2	3–3	4–4	5–5	6–6	7–7	8–8
1	1–3	2–4	3–5	4–6	5–7	6–8	7–1	8–2
2	8–5	1–6	2–7	3–8	4–1	5–2	6–3	7–4
3	7–6	8–7	1–8	2–1	3–2	4–3	5–4	6–5
4	6–7	7–8	8–1	1–2	2–3	3–4	4–5	5–6
5	5–8	6–1	7–2	8–3	1–4	2–5	3–6	4–7
6	3–1	4–2	5–3	6–4	7–5	8–6	1–7	2–8
7	1–5	5–1	2–6	6–2	3–7	7–3	4–8	8–4

After round 6 all teams have completed 6 matches and the last match in round 7 needs new boards either shared across all tables or just between the two tables involved in each of the last three matches. In the last round all teams play the team they have not played. Of course the organizers can choose not to play the last round.

To find a team's opponent in the last round either add or subtract ½ N from their team number. If the team number is less than or equal to ½ N, add ½ N, otherwise subtract ½ N.

8.3 New England relay system

An idea whose time has come was waiting there all along.
Carrie Latet

This teams' movement is applicable only for even numbers of tables. It has an advantage over other movements for even numbers of teams, as the movement is regular for the whole event and there is no need for a break in play as in the stagger movement.

The disadvantage is that there is continual board-sharing between pairs of tables (but note that there is no movement for an even number of teams that does not require some boards-sharing or duplication). This is normally not a problem as the number of boards in play on each round is quite often 3 or more. If the number is less than three the movement allows for board duplication by the players on the first round very easily. Pre-duplication of the boards eliminates this problem completely.

The tables are arranged in two equal parallel rows. The tables next to each other share boards for the whole movement. A small relay table or chair should be placed between the tables on each side of the movement. For example, 8 tables are arranged as follows:

1	R	2	R	3	R	4	R
5	R	6	R	7	R	8	R

Note that the arrangement is not the standard horseshoe type; the lowest numbered tables in each row are at the same end. Tables 1 and 5 share boards as do 2 and 6 etc.

The boards are distributed with set 1 on table 1, set 2 on the relay table between tables 1 and 2, set 3 on table 2, set 4 on the next relay, etc. As there is sharing throughout the movement, half the boards can be placed on the other side of the movement. In the above example, with 4 boards per round, boards (l&2) go on table 1, (3&4) on table 5, (5&6) and (7&8) on the relays, (9& 10) on table 2, (11&12) on table 6, and so on.

With each team starting at the table with the same number as their team number, the N-S pairs remain stationary and the E-W pairs move up one table. During the move the E-W pairs are required to move the boards by placing the boards they have just played on the next relay and taking the boards from the relay to be played on the next round.

It should be pointed out to the moving pairs that when they reach the end of a row they are required to go to the *other* end of the *other* row to find their next table. It may help to place a guide card on the end table reading 'East-West go to table …'.

The movement also works with the pairs moving down one table and the boards moving down one station. If duplication of the boards is required (as for example in a 16 table movement where the sharing of boards would slow down the movement) it can be done as follows:

Start with two copies of each board on each table and each relay. For example, table 1 will have two copies of board 1, and the table next to it (sharing) two copies of board 2 (table 9 in the 16-table example). Before the first change each team deals and duplicates the board on their table. The copies are then exchanged between the sharing tables. The first move is made. The boards on the relay tables now move into play and are dealt, played and duplicated and the copies exchanged. All boards are now duplicated and there will be no further sharing.

As with the whist movement, if the number of rounds is to be reduced below the complete movement, the E-W pairs simply move an extra table on the first round for every two fewer rounds. They will, of course, finish one table earlier for every extra table moved at the start. If a break is desired at the halfway point (i.e., a 2-session event) then the stagger movement should be used.

8.4 Hasler system

The best argument against democracy is a five-minute conversation with the average voter.
Winston Churchill

This movement is also known by the names *Twin Mitchell*, *Mirror Mitchell*, *Barclay Movement* and *Rosler movement.* Alexander T. Hasler explains in his 1934 book *Duplicate Bridge Simplified* this movement is known as the *Barclay Movement* in America, though he used this movement in Scotland many years earlier in the very early days of duplicate bridge.

It appears that communication across the Atlantic at that time was not reliable on such issues. In Alfred M. Gruenther's book *Duplicate Contract Complete*, 1933, the Barclay movement is supplied but only the first part of the movement, the *Twin Mitchell* part, is considered as the *Barclay movement.* It can therefore be concluded that Hasler is correct. This complete movement, with its two parts, can be called the Hasler system.

It is applicable only to an even number of teams. Only half the teams are met in this movement, but the rest of the teams can be met if a whist movement is used for the second half of the tournament. Not only must the number of teams be even, but half the number must also be odd to eliminate the share-and-relay problem.

The tables are arranged in two parallel rows of equal numbers of tables. The lowest-numbered tables in each row are at the same end of the movement. The tables next to each other share throughout the movement unless the boards are duplicated. For example, 14 tables are arranged like this:

1	2	3	4	5	6	7
8	9	10	11	12	13	14

Each team is seated at their home table and the E-W pairs in each row exchange places. In the above example E-W pairs 1 and 8 exchange places as do 2 and 9, etc. Play commences and the boards are shared between opposite tables, so that each match is completed in one round. The move is then a standard Mitchell move in each row, with the boards moving down one table. In one row the N-S pairs move up 1 table and in the other row the E-W pairs move up 1 table. This really means that some teams move and the rest remain stationary. The moving pairs move to the other end of the same row when they reach the end of the row.

The whist movement can be used in each row with the same effect as long as the N-S pairs move in one row and the E-W pairs in the other.

When each row has an odd number of tables, the second half of the event can be run as two whist movements (in the example above, tables 1–7 are one movement and 8–14 the other). The boards are usually shared between the two rows in the second half for aesthetic reasons (for discussion of the boards after play), as well as giving all teams an equal chance of having the same swings.

8.5 Clay system

Change is inevitable - except from a vending machine.
Robert C. Gallagher

The Clay system works for any number of tables and as long as the matches involve an even number of boards the procedure is always the same. See *the Clay universal system*, page 49, for more details and the origins of this movement.

Setting up the movement is very simple, with a table for each team numbered from 1 to *N* (the number of teams), placed in order and sequence that is convenient for the room. Teams are asked to sit at their table before play commences. A single relay table is placed between all tables. The number of board-sets involved in the movement is doubled and the number of boards per set halved. The boards are distributed starting at table 1, placing the first half-set on table 1. The second half-set is placed on the relay table between tables 1 and 2. This is illustrated here with 4 teams, playing six rounds and making an extra move when they meet their own team mates.

Round	Table 1	relay	Table 2	relay	Table 3	relay	Table 4	relay
1	1–1 1	2	2–2 3	4	3–3 5	6	4–4 7	8
2	1–4 8	1	2–1 2	3	3–2 4	5	4–3 6	7
3	1–3 7	8	2–4 1	2	3–1 3	4	4–2 5	6
4	1–2 6	7	2–3 8	1	3–4 2	3	4–1 4	5
5	1–1 5	6	2–2 7	8	3–3 1	2	4–4 3	4
6	1–4 4	5	2–1 6	7	3–2 8	1	4–3 2	3
7	1–3 3	4	2–4 5	6	3–1 7	8	4–2 1	2
8	1–2 2	3	2–3 4	5	3–4 6	7	4–1 8	1

The astute observer will quickly notice a weakness in this arrangement. The board-sets are not played in sequence for each match. For example, teams 1 and 4 play board-sets 4 and 8 in their match, teams 1 and 3 play sets 3 and 7, and so forth. This is not a major problem, but it is far from the most desirable.

To overcome this, Clay suggested a procedure. For each different number of tables it is necessary to write down in advance a simple schedule. Write in numerical order the number of rounds to be played. Beneath, in vertical lines, write the numbers of the boards, in regular numerical order, which are to be played each round. For six tables, four boards at a table, the schedule would be written:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1	3	5	7	9	11	13	15	17	19	21	23
2	4	6	8	10	12	14	16	18	20	22	24

Now place on the tables and the relays, beginning with table 1, the boards under the odd numbers first, and afterward those under the even numbers.

That is, place on table 1, boards 1, 2 then relay 5, 6; on table 2, boards 9, 10 and relay 13, 14; on table 3, boards 17, 18 and relay 21, 22; on table 4, boards 3, 4 and relay 7, 8; on table 5, boards 11, 12 and relay 15, 16; on table 6, boards 19, 20 and relay 23, 24.

On the first round, deals 1 and 2, 9 and 10, 17 and 18, 3 and 4, 11 and 12, 19 and 20, are played at the respective tables. This works for any number of tables as long as the number of boards in each set is an even number, divided evenly into two halves.

If the number of boards in a complete match is odd, the formula varies slightly, *but only if the number of tables is also odd*. Thus, for five tables, five boards on a table, the schedule is:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	3	6	9	11	13	16	19	21	23
2	4	7	10	12	14	17	20	22	24
	5	8			15	18			25

Here, since each board-set, being five in number, cannot be divided evenly, it is divided as nearly as possible into halves, the parts consisting of twos and threes. First write down two, then three; then three, then two, and so on, every time alternating the order of division.

The boards are placed on the tables and relays in a similar fashion: Table 1 place 1, 2 relay 6, 7, 8; table 2 place 11, 12 relay16, 17, 18; table 3 place 21, 22 relay 3, 4, 5; table 4 place 9, 10 relay 13, 14, 15; table 5 place 19, 20 relay 23, 24, 25. The full schedule for 5 tables is:

Round	Table 1	relay	Table 2	relay	Table 3	relay	Table 4	relay	Table 5	relay
1	1–1 1,2	6,7,8	2–2 11,12	16,17,18	3–3 21,22	3,4,5	4–4 9,10	13,14,15	5–5 19,20	23,24,25
2	1–5 23,24,25	1,2	2–1 6,7,8	11,12	3–2 16,17,18	21,22	4–3 3,4,5	9,10	5–4 13,14,15	19,20
3	1–4 19,20	23,24,25	2–5 1,2	6,7,8	3–1 11,12	16,17,18	4–2 21,22	3,4,5	5–3 9,10	13,14,15
4	1–3 13,14,15	19,20	2–4 23,24,25	1,2	3–5 6,7,8	11,12	4–1 16,17,18	21,22	5–2 3,4,5	9,10
5	1–2 9,10	13,14,15	2–3 19,20	23,24,25	3–4 1,2	6,7,8	4–5 11,12	16,17,18	5–1 21,22	3,4,5
6	1–1 3,4,5	9,10	2–2 13,14,15	19,20	3–3 23,24,25	1,2	4–4 6,7,8	11,12	5–5 16,17,18	21,22
7	1–5 21,22	3,4,5	2–1 9,10	13,14,15	3–2 19,20	23,24,25	4–3 1,2	6,7,8	5–4 11,12	16,17,18
8	1–4 16,17,18	21,22	2–5 3,4,5	9,10	3–1 13,14,15	19,20	4–2 23,24,25	1,2	5–3 6,7,8	11,12
9	1–3 11,12	16,17,18	2–4 21,22	3,4,5	3–5 9,10	13,14,15	4–1 19,20	23,24,25	5–2 1,2	6,7,8
10	1–2 6,7,8	11,12	2–3 16,17,18	21,22	3–4 3,4,5	9,10	4–5 13,14,15	19,20	5–1 23,24,25	1,2

It can be seen 2 boards are played on the first round, 3 on the second round, 2 on the third, and so forth. In addition each consists of a set of 5 boards in sequence. Teams 1 and 3 play boards 11,12,13,14,15 and teams 2 and 5 play 1,2,3,4,5 and so forth.

This is all very messy for modern organizers and with other options available it is unlikely this procedure would be chosen. It is included here for completeness on this subject, but with one serious difficulty still left the Clay movement is unlikely to be a favourite for organizers except in the form of the *American Whist League movement*. The remaining problem is when we have an even number of tables and an odd number of boards in each match. This only arises in some rare cases such as 8 or 10 teams playing 3-board matches. To solve this use the Barney modification mentioned earlier in the *Clay modifications* where for each additional board in each set an additional circuit of the movement is required for the E-W pairs.

8.6 Team Mitchell system

Time is a dressmaker specializing in alterations.
Faith Baldwin

In the early days of teams' events, particularly whist and auction bridge, the scoring methods adopted were not as sophisticated as those used today. In particular there were no IMP or VP scales or similar scoring methods. Scoring a teams' event was simply be a matter of taking the difference between the N-S and E-W pairs' total point scores for each team. This is a completely different definition of a teams' event. Scoring an event in this way the Mitchell movement could be used for teams. It did not matter that the scores on a particular board were obtained by playing a different team at each table.

This was recognized as a problem by the most pedantic organizers. In particular the Hasler movement was one very good solution because each match was completed and easily scored before moving on to the next.

Because the Mitchell was easy to organize and the players were completely familiar with the procedures it was a movement of choice by many directors. The biggest problem with the Mitchell is that each team plays a different set-of-boards *against other teams* by the end of the movement. Consider a 7-table Mitchell where team 1 N-S and 1 E-W end up playing set 1 against each other (for a net zero for the team) and play sets 2 to 7 against the other teams. Team 2 plays sets 1 and 3 to 7 against the other teams, and so forth.

Worger modification

Recognizing this problem with the Mitchell, Mr. H. O. Worger came up with a modification that applies to a standard *odd*-tables Mitchell.

Tables are numbered the same as in a normal Mitchell, 1 up to the maximum. The teams are numbered in the same way. The seating of the teams is altered slightly. The N-S pairs from each team sit at their same table number, as in any Mitchell. The E-W pairs sit in reverse sequence starting at the highest numbered table. Team 1 E-W sits at the highest table, team 2 E-W at the second highest and so on until the highest numbered team sits E-W at table 1.

The middle team is the only one where the N-S and E-W pairs for that team are seated at the table of their same number.

The boards are distributed in exactly the same way as in a normal Mitchell, set 1 at table 1, set 2 at table 2, and so on. Play proceeds for the first round, with the middle table idle and having a bye, or if the team chooses to play the boards, the result is zero. The schedule below for 7 tables (and 7 rounds) shows E-W pair number and the board-set played in each round and that board-set 4* is played whenever team mates meet.

Table(N-S):	1	2	3	4	5	6	7
1	7 1	6 2	5 3	4 4*	3 5	2 6	1 7
2	5 2	4 3	3 4*	2 5	1 6	7 7	6 1
3	3 3	2 4*	1 5	7 6	6 7	5 1	4 2
4	1 4*	7 5	6 6	5 7	4 1	3 2	2 3
5	6 5	5 6	4 7	3 1	2 2	1 3	7 4*
6	4 6	3 7	2 1	1 2	7 3	6 4*	5 5
7	2 7	1 1	7 2	6 3	5 4*	4 5	3 6

It should be repeated that this is not a true teams' event. Only total points or total-trick scoring can be used. The same boards are not played at each table in a match. It can be seen that the teams do meet each other on the same round, so that by using just one board-set on each round, shared or duplicated as in the Snow

method, this can operate as a round-robin draw, with a bye for one team (though they can play the boards against their team-mates).

Stationary-board modification

The simplest modification to the Mitchell movement for teams is to make the boards stationary. This applies to an odd number of teams and there is no bye round, as found in the Worger modification. With the teams starting at their home table, all the pairs move before play starts, N-S up one table and the E-W pairs down one table. This procedure continues until all teams have played all other teams.

Round:	1	2	3	4	5	6	7
Start:	1 1	2 2	3 3	4 4	5 5	6 6	7 7
1	7 2	1 3	2 4	3 5	4 6	5 7	6 1
2	6 3	7 4	1 5	2 6	3 7	4 1	5 2
3	5 4	6 5	7 6	1 7	2 1	3 2	4 3
4	4 5	5 6	6 7	7 1	1 2	2 3	3 4
5	3 6	4 7	5 1	6 2	7 3	1 4	2 5
6	2 7	3 1	4 2	5 3	6 4	7 5	1 6

This does not have any advantages over (but should be compared with) the stationary-board and McKinnon modifications of the AWL which cover even numbers of teams. The difference between the stationary boards in AWL and Mitchell is very subtle and it could be argued that they are in principle the same movements.

The variations in the movement for teams are extensive. Whether they are called Mitchell or American Whist League is open to debate. Consider the following 9 table teams' movement, with the teams starting as usual at their home table. There are 8 rounds with the E-W pairs moving up one table before the start of play. With stationary boards E-W move up one table at each change and the N-S pairs down one table. On the move after round 4 the team mates of each team will encounter each other and so an extra move is required.

1	1–9	2–1	3–2	4–3	5–4	6–5	7–6	8–7	9–8
2	2–8	3–9	4–1	5–2	6–3	7–4	8–5	9–6	1–7
3	3–7	4–8	5–9	6–1	7–2	8–3	9–4	1–5	2–6
4	4–6	5–7	6–8	7–9	8–1	9–2	1–3	2–4	3–5
5	6–4	7–5	8–6	9–7	1–8	2–9	3–1	4–2	5–3
6	7–3	8–4	9–5	1–6	2–7	3–8	4–9	5–1	6–2
7	8–2	9–3	1–4	2–5	3–6	4–7	5–8	6–9	7–1
8	9–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9

Now consider reorganizing the rounds so that each pair of matches is completed in two rounds. Round 8 is positioned after round 1 and so forth to give this movement

1	1–9	2–1	3–2	4–3	5–4	6–5	7–6	8–7	9–8
2	9–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9
3	2–8	3–9	4–1	5–2	6–3	7–4	8–5	9–6	1–7
4	8–2	9–3	1–4	2–5	3–6	4–7	5–8	6–9	7–1
5	3–7	4–8	5–9	6–1	7–2	8–3	9–4	1–5	2–6
6	7–3	8–4	9–5	1–6	2–7	3–8	4–9	5–1	6–2
7	4–6	5–7	6–8	7–9	8–1	9–2	1–3	2–4	3–5
8	6–4	7–5	8–6	9–7	1–8	2–9	3–1	4–2	5–3

A different movement of the pairs is now required. E-W pairs move up one before the start of play. Round 2 the pairs swap tables but continue to sit in the same direction at the new table. After round 2 the E-W pairs move up 2 tables and the N-S pairs down 2 tables. The pairs swap for round 4. After round 4 E-W move up 4 tables and N-S down 4 tables. They swap again in round 6 and after round 6 up and down 6 tables respectively. After each even numbered round all the teams can compare scores of the matches just completed. Using similar principles as those discussed below in *McKinnon modification* the event is simple enough to conduct with the pairs moving up and down 1 table respectively after the even numbered rounds. See the *same board two stanza modification* below for further discussion of this variation.

Two-stanza modification

This is an extension of the stationary-board modification to give us two stanzas of play, which can be handy for the organizers of such events. This should be compared with the two-stanza modification in the AWL and the principles are the same. This also applies equally well to the stationary-board modification of AWL. The rules are:

- for every extra move at the start of the movement the first stanza is reduced by two rounds;
- second stanza continues from the last round and proceeds as before until all rounds are played;
- when pairs encounter their team-mates a second move is made.

Consider the 7-table movement shown above. At the start the N-S and E-W pairs move twice, getting to round 2, and play starts. The maximum number of rounds is initially 6, but this extra move skips round 1 and stops after round 5. All teams have played 4 rounds (2 to 5) and completed 4 matches. At this point play can stop and the finished matches scored.

In the example above the second stanza starts with round 6 Two moves are required after this because the team-mates meet each other and play finishes after the original round 1. There are in total 2 rounds in the second stanza.

Same board two stanza modification

John Royds of Ireland noticed that the stationary board modification shown above for 9 tables is ideal for a two session event. This applies to numbers of teams where N Mod 4 = 1 (9, 13, 17…). In the 9 team example the two sessions are played over 4 rounds each. The same boards are at all tables in each session, with one set missed by each team in each session. The board-sets are distributed starting at table 1 up to table $(N - 1) / 2$. The same sets are then placed on the remaining tables in order, with the extra set on the last table.

Session 1.

Table:	1	2	3	4	5	6	7	8	9
Sets:	1	2	3	4	1	2	3	4	5
1	1–9	2–1	3–2	4–3	5–4	6–5	7–6	8–7	9–8
2	9–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9
3	2–8	3–9	4–1	5–2	6–3	7–4	8–5	9–6	1–7
4	8–2	9–3	1–4	2–5	3–6	4–7	5–8	6–9	7–1

Session 2.

Sets:	6	7	8	9	6	7	8	9	10
5	3–7	4–8	5–9	6–1	7–2	8–3	9–4	1–5	2–6
6	7–3	8–4	9–5	1–6	2–7	3–8	4–9	5–1	6–2
7	4–6	5–7	6–8	7–9	8–1	9–2	1–3	2–4	3–5
8	6–4	7–5	8–6	9–7	1–8	2–9	3–1	4–2	5–3

This has the advantage that the teams can score up after two rounds as well as all teams playing much the

same sets of boards in each session. The movement can be managed with guide-cards on the home tables or instruction from the director after each round.

All teams start at their home table and proceed as follows.

To play round 1 E-W pairs move up 1.
Round 2 the N-S and E-W pairs swap tables.
Pairs return to the home table and score the two matches.

From their home tables pairs move as follows:

Round 3 E-W pairs move up 2 tables and N-S pairs move down 1 table.
Round 4 the N-S and E-W pairs swap tables.
Pairs return to the home table and score the two matches.

From their home tables pairs move as follows:

Round 5 E-W pairs move up 3 tables and N-S pairs move down 2 tables.
Round 6 the N-S and E-W pairs swap tables.
Pairs return to the home table and score the two matches.

And so forth. This same procedure also applies to 13, 17… tables where the sessions consist of 6, 8… rounds respectively.

For even numbers of tables similar principles can apply. Using the *McKinnon modification* and reorganizing the rounds as done with the 9 table example above the 10 table example is shown here:

Session 1

Table:	1	2	3	4	5	6	7	8	9	10
Sets:	1	2	3	4	5	1	2	3	4	5
1	1–9	2–10	3–1	4–2	5–3	6–4	7–5	8–6	9–7	10–8
2	9–1	10–2	1–3	2–4	3–5	4–6	5–7	6–8	7–9	8–10
3	2–8	3–9	4–10	5–1	6–2	7–3	8–4	9–5	10–6	1–7
4	8–2	9–3	10–4	1–5	2–6	3–7	4–8	5–9	6–10	7–1

Special round 5

Sets:	2	2	3	3	4	4	5	5	1	1
	1–6	6–1	2–7	7–2	3–8	8–3	4–9	9–4	5–10	10–5

Session 2

Sets:	6	7	8	9	10	6	7	8	9	10
5	3–6	4–7	5–8	6–9	7–10	8–1	9–2	10–3	1–4	2–5
6	6–3	7–4	8–5	9–6	10–7	1–8	2–9	3–10	4–1	5–2
7	4–5	5–6	6–7	7–8	8–9	9–10	10–1	1–2	2–3	3–4
8	5–4	6–5	7–6	8–7	9–8	10–9	1–10	2–1	3–2	4–3

Special round 9

Sets:	10	10	6	6	7	7	8	8	9	9
	1–6	6–1	2–7	7–2	3–8	8–3	4–9	9–4	5–10	10–5

Board-sets 1 to 5 are played in session 1 and 6 to 10 in session 2. One of the special rounds 5 or 9 can be played or not depending on the requirements and the time available.

Similar to above for the odd numbers of teams, all teams start at their home table and proceed as follows.

To play round 1 E-W pairs move up 2.
Round 2 the N-S and E-W pairs swap tables.
Pairs return to the home table and score the two matches.

From their home tables pairs move as follows:

Round 3 E-W pairs move up 3 tables and N-S pairs move down 1 table.
Round 4 the N-S and E-W pairs swap tables.
Pairs return to the home table and score the two matches.

From their home tables pairs move as follows:

Round 5 E-W pairs move up 5 tables and N-S pairs move down 2 tables. Note the extra move for E-W after the first half.
Round 6 the N-S and E-W pairs swap tables.
Pairs return to the home table and score the two matches.

And so forth.

8.7 McKinnon modification

If you want to make enemies, try to change something.
Woodrow Wilson

This modification is descended from the AWL system with the stationary-board modification. Also consider the stationary board modification for the *team Mitchell system* above. Each team is sitting at their home table at the start, teams 1 to 7 sit at tables 1 to 7 and board-sets 1 to 7 are placed on tables 1 to 7.

It starts in the same way as AWL and the boards are stationary at all times. The movement is the same for all numbers of tables, odd *and* even:

- E-W pairs moves down 2 tables before play starts;
- after each and every round the N-S and E-W pairs of each team swap tables, the N-S pairs always playing in the N-S seats and similarly for E-W;
- after every even-numbered round is completed (but after the pairs have swapped tables), the E-W pairs move down 1 table and the N-S pairs move up 1 table;
- for even numbers of tables there is a skip round for the E-W pairs.

Let us look at what is happening here. At the start the E-W pairs move down 2 tables. All teams play the first half of the first match. On each round every team is involved in two matches, one where N-S is sitting and one where E-W is sitting. The N-S and E-W pairs from each team swap places at the end of that round and play the other half of those two matches. The pairs again swap, going back to their first table, but, of course, with nothing better to do during that process, the two pairs score up those two matches.

The pairs play two matches, score two matches, play two more matches, score those matches and so forth. Once the players are familiar with the movement it almost runs itself. There is no requirement for a published draw and no board schedule to organize.

For 7 teams the movement looks like this, showing N-S and E-W pairs of the teams separated by a hyphen.

Round	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7
Start	1–1	2–2	3–3	4–4	5–5	6–6	7–7
1	1–3	2–4	3–5	4–6	5–7	6–1	7–2
2	3–1	4–2	5–3	6–4	7–5	1–6	2–7
3	7–4	1–5	2–6	3–7	4–1	5–2	6–3
4	4–7	5–1	6–2	7–3	1–4	2–5	3–6
5	6–5	7–6	1–7	2–1	3–2	4–3	5–4
6	5–6	6–7	7–1	1–2	2–3	3–4	4–5

With an odd number of tables there is always an even number of rounds and all possible matches are completed by the end of the event.

For an even number of teams, that is not the case. The number of rounds is odd. The last round must complete one match in that one round, sharing boards with their team mates. Consider the 6-table movement:

Round	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6
Start	1–1	2–2	3–3	4–4	5–5	6–6
1	1–3	2–4	3–5	4–6	5–1	6–2
2	3–1	4–2	5–3	6–4	1–5	2–6
3	6–5	1–6	2–1	3–2	4–3	5–4
4	5–6	6–1	1–2	2–3	3–4	4–5
5	1–4	4–1	2–5	5–2	3–6	6–3

After round 4 all teams have completed 4 matches and the last match in round 5 needs new boards either shared across all tables or just between the two tables involved in each of the last three matches. In the last round all teams play the team they have not played. Of course the organizers can choose not to play the last round.

As mentioned above, with an even number of teams there is a skip round (sounds familiar) and in the 6-table example shown above it is after round 2. The skip round occurs after different rounds depending on the number of tables. This round R is found by:

$$R = 2 * ((N / 4_{\text{ rounded up}}) - 1) \text{ where } N \text{ is the number of teams.}$$

These round numbers are shown here and there is clearly a simple pattern:

N	*R*
6	2
8	2
10	4
12	4
14	6
16	6
18	8
20	8

To find a team's opponent in the last round either add or subtract ½ N from their team number. If the team number is less than or equal to ½ N add ½ N, otherwise subtract ½ N.

If there is to be a break in play, say, for lunch, clearly it is possible for the players to discuss the hands that others have not played. This is not a new problem and insoluble with the AWL and earlier modifications. With this system the director can introduce new boards after any even numbered round. Likewise when two

(or more) stanzas are required by the organizers it is simple to introduce a break at any time after an even number of rounds.

Hanner-Snow modification

Olof Hanner recognized the need for teams' events where the same boards are played on each round or close to it anyway. In other words, using the Snow method applied to a teams' event. Also see appendix N (page 419) for other options.

Hanner found by playing 3-way triangle matches over two rounds, all teams can complete their two matches in two rounds playing the same boards. With that said, it is difficult to understand why the organizers of such events do not run these types of events as a round-robin with the players scoring up after each match. The Patton system also satisfies this need and is very simple to organize, but only applies to an even number of teams unless there is a bye round.

Unfortunately this modification still requires a published draw, duplicated boards, perhaps shared boards, frequent distribution of boards, a head-to-head match for the last round in even numbers of teams and one board-set in play for the whole event with an odd number of teams. Consider 7 teams.

Round	Table 1	Table 4	Table 2	Table 7	Table 5	Table 3	Table 6
1	7–1 (1)	6–3 (7)	5–4 (1)	4–7 (2)	3–5 (2)	2–6 (1)	1–2 (2)
2	1–7	3–6	4–5	7–4	5–3	6–2	2–1
3	7–2 (3)	1–4 (7)	6–5 (3)	5–7 (4)	4–6 (4)	3–1 (3)	2–3 (4)
4	2–7	4–1	5–6	7–5	6–4	1–3	3–2
5	7–3 (5)	2–5 (7)	1–6 (5)	6–7 (6)	5–1 (6)	4–2 (5)	3–4 (6)
6	3–7	5–2	6–1	7–6	1–5	2–4	4–3

Points to note about this schedule.

1. The highest numbered team N-S remains at its table for the start of each pair of rounds.
2. The team mates swap tables after each round.
3. The board-sets shown in brackets are the same at half the tables.
4. The highest board-set stays at one table (4 in this example).
5. The table numbers are essentially not important, but are numbered to show the tables sharing the same board-sets (1,2,3) and (5,6,7).
6. Team 1 E-W starts against the highest team N-S.
7. The draw for odd rounds is cyclic after round 1, increasing by 1 and ignoring the highest number.
8. N-S numbers with the highest at table 1, decrease by 1 for each next table on round 1.
9. E-W numbers start at 3 on the second table and increase by 1, ignoring 1, on round 1.
10. The N-S team playing E-W of the highest team is $\frac{1}{2}N + 1$ (4 at table 4 in this example).

The full definition of the draw is defined by round 1. A horseshoe arrangement of the tables is useful. Consider 8 teams.

Round	Table 1	Table 5	Table 6	Table 8	Table 2	Table 4	Table 3	Table 7
1	8–1 (1)	7–3 (2)	6–4 (2)	5–8 (2)	4–5 (1)	3–6 (1)	2–7 (1)	1–2 (2)
2	1–8	3–7	4–6	8–5	5–4	6–3	7–2	2–1
3	8–2 (3)	1–4 (4)	7–5 (4)	6–8 (4)	5–6 (3)	4–7 (3)	3–1 (3)	2–3 (4)
4	2–8	4–1	5–7	8–6	6–5	7–4	1–3	3–2
5	8–3 (5)	2–5 (6)	1–6 (6)	7–8 (6)	6–7 (5)	5–1 (5)	4–2 (5)	3–4 (6)
6	3–8	5–2	6–1	8–7	7–6	1–5	2–4	4–3
7	8–4 (7)	3–5	2–6	1–7	7–1	6–2	5–3	4–8

The rules given above also apply to an even number of teams, but point 4 does not apply and is replaced by the requirement that all teams play the same board-set on the last round (set 7 in this example). The last round is a cyclic draw, starting with E-W ½ *N* against team *N* and counting up to *N* − 1 and N-S continuing from 1. The pattern is exposed in the following 5 to 11 team schedules.

5–1 (1)	4–3 (1)	3–5 (2)	2–4 (5)	1–2 (2)						
6–1 (1)	5–3 (2)	4–6 (2)	3–4 (1)	2–5 (1)	1–2 (2)					
7–1 (1)	6–3 (7)	5–4 (1)	4–7 (2)	3–5 (2)	2–6 (1)	1–2 (2)				
8–1 (1)	7–3 (2)	6–4 (2)	5–8 (2)	4–5 (1)	3–6 (1)	2–7 (1)	1–2 (2)			
9–1 (1)	8–3 (2)	7–4 (1)	6–5 (1)	5–9 (2)	4–6 (2)	3–7 (9)	2–8 (1)	1–2 (2)		
10–1 (1)	9–3 (2)	8–4 (2)	7–5 (2)	6–10 (2)	5–6 (1)	4–7 (1)	3–8 (1)	2–9 (1)	1–2 (2)	
11–1 (1)	10–3 (2)	9–4 (11)	8–5 (1)	7–6 (1)	6–11 (2)	5–7 (2)	4–8 (2)	3–9 (1)	2–10 (1)	1–2 (2)

8.8 The stagger system

We spend our time searching for security and hate it when we get it.
John Steinbeck

This movement applies only to an even number of teams and normally would be used only for a 2-session event. The advantage over the previous examples is the standard whist move for most of the event. Boards are shared throughout, with a break in play at approximately halfway. The scoring can be done at this point. The play then continues with the current set-of-boards re-dealt or a new set-of-boards introduced. The tables are arranged in two parallel rows of equal numbers of teams with the lowest-numbered tables at the same end of the movement. The tables next to each other share boards throughout the movement (i.e., tables 1 and 5 in the following example).

For example, 8 tables are arranged:

1	2	3	4
5	6	7	8

The movement consists of two parts (A and B), of which part A may precede part B or vice versa. Which order will depend greatly on the number of boards to be played in the respective sessions. Part A has the *even* numbered teams playing the *odd* numbers and vice versa. Part B has the *odd* numbers meeting the *odd* and the *even* meeting the *even.*

This movement can be useful in three-session events where part A is used with a whist movement (with an odd number of tables). For example, with 25 teams, the field is broken into three groups of equal numbers of teams plus one: 1-8; 9-16; 17 24 and 25.

	Session 1	Session 2	Session 3
Movement 1 (part A stagger)	1–8 v 9–16	17–24 v 1–8	9–16 v 17–24
Movement 2 (9 table whist)	17–24 & 25	9–16 & 25	1–8 & 25

In movement 1, one group of 8 teams sit at odd tables and the other 8 at even tables.

Part A stagger movement

Starting with the teams sitting at their own table, the N-S pairs remain stationary throughout the movement. On the first move the E-W pairs move up 1 table and play commences. It should be pointed out to the moving pairs that when they reach the end of a row that they must go to the other end of the other row to find their next table. In all subsequent moves the E-W pairs move up two tables and the boards up one table (i.e., the standard whist move). Moving in the opposite direction will, of course, work equally well.

When the moving pairs reach the table just before their team-mates, this is the last round in this half of the movement. The E-W pairs then move up one table to their team mates. The scoring can be done at this point. With the teams sitting at their own table, the second half of the movement then begins.

Part B stagger movement

N-S pairs remain stationary. E-W pairs move up (or down) two tables and the boards move up (or down) one table. This move continues for the rest of the event until the E-W pairs return to their team mates. The scoring is done at this point. If this half of the event is used first, then play proceeds into the next half of the event (Part A).

Modification

Part B of this movement can be used effectively as a pairs' movement when it is required that the pairs only meet ¼ of the field. One particular application is with 12 tables over 4 sessions. The field is broken into 4 groups of 6 pairs: 1–6; 7–12; 13–18 and 19–24. The groups are placed at alternating tables and rotated so that they meet all other pairs. Part B of the stagger movement (with play taking place at the first table, of course) has all pairs meeting all other pairs only once.

The starting seating positions for each session are:

Tables:	1 & 7	2 & 8	3 & 9	4 & 10	5 & 11	6 & 12
Session 1:	1–7	13–19	2–8	14–20	3–9	15–21
	4–10	16–22	5–11	17–23	6–12	18–24
Session 2:	1–19	7–13	2–20	8–14	3–21	9–15
	4–22	10–16	5–23	11–17	6–24	12–18
Session 3:	1–13	7–19	2–14	8–20	3–15	9–21
	4–16	10–22	5–17	11–23	6–18	12–24

Session 4: Use four 3-table Howell movements with the boards being shared within each Howell on each round and the boards moving from Howell to Howell on each change. Each Howell has 6 pairs: 1–6; 7–12; 13–18; and 19–24.

8.9 Patton system

If everyone is moving forward together, then success takes care of itself.
Henry Ford

This movement is also known as the Patton schedule, devised by Major C. L. Patton of New York and is based on the standard Howell movement. Even so, an earlier version of this movement was explained in Mitchell's 1897 book *Mitchell's duplicate whist and modern leads*. He also correctly attributes the cyclic nature of round-robin draw to Safford rather than Howell. See *The Safford system*, page 40, for the complete explanation of his system.

It is best for an even number of teams, as with an odd number there will always be one team sitting out. The disadvantages with this movement are that guide-cards are required and boards are shared between both

movements throughout. Even so it is a highly desirable movement for team events as the matches are completed at the end of each round.

The moving of the boards can also be a problem as in any Howell. With all the pairs moving it is difficult to assign this task to any particular pairs and so it is preferable that the director does it. If the matches are sufficiently long, the boards can be re-dealt before each match and can remain stationary throughout. Alternatively, if there is enough space in the playing area, stationary board Howell movements can be used so that only the players are required to move. Note: with stationary boards one table is required for each board-set in play. If you are re-dealing before each round it is better to use the endless Howell movement, as this eliminates the need for guide-cards. A simple solution to board management is to use a new set of pre-dealt duplicated boards on each round.

When arranging a movement for N teams, two Howell movements of ½ *N* tables in each are set up, side by side. The same Howell schedule is used in each movement except that the seating positions are switched at each table.

For example 10 teams:

Howell 1	10–1	4–3	9–7	4R	6–2	5–8
Howell 2	1–10	3–4	7–9	4R	2–6	9–5

The sharing of the boards between the tables next to each other is necessary for each match to be completed on each round.

As in standard Howell movements, the highest-numbered team remains stationary for the entire movement. With an odd number of teams the highest-numbered team is missing and constitutes the sit-out round for each other team.

8.10 Double-Howell system

Coming together is a beginning. Keeping together is progress. Working together is success.
Henry Ford

The double Howell has the same effect as the Patton system, but there are fewer tables sharing boards. For many of the movements, you can be dispense with guide-cards, as the movement of the pairs is quite regular. The disadvantage of having all the pairs (less 2) moving is still there, but the boards can be made stationary, so that this is not a problem.

When stationary boards are used though, the problem of a roving share arises. Even when the regular movement can be defined, the starting position of the teams is not regular, and a starting position schedule needs to be posted at the start of the event.

8 TEAMS

By examining the expansion of the double 4-table Howell movement for pairs it can be seen that the team (consisting of the two pairs with the same number) play the same boards against opponents of the same number.

This movement in the shorthand form is 8–1 / 1–8 6–5 4–2 2–6 7–3 5–7 3–4.

This is the starting position with the tables being numbered from 1 to 7 from the left, with the table sharing with table 1 (pairs 1 & 8) being called 8 (or lA).

The movement after the first round is:

- team 8 stays;
- N-S pairs move down 3 tables (moving from 4 to 8 and then 8 to 5);
- E-W pairs move down 2 tables (ignoring table 8);
- boards down 1 table: (ignoring table 8).

With stationary boards (i.e., with sets 1 to 7 on tables 1 to 7 for the entire movement) the movement after the first round is:

- team 8: E-W table 8 stay; N-S move up 1 table and share boards with their team-mates on each round;
- N-S pairs: move down 2 tables (staying in 1 to 7) and playing at table 8 when they are displaced by N-S 8;
- E-W pairs: move down 1 table (staying in 1 to 7).

12 TEAMS

12–1 / 1–12 11–4 10–7 9–10 8–2 7–5 6–8 5–11 4–3 3–6 2–9

- Team 12 stays.
- N-S pairs: move up 1 table (using 12 or 1A instead of 1).
- E-W pairs: move down 4 tables (ignoring table 12).
- Boards down one table (ignoring table 12).

With stationary boards,

- team 12: E-W table 12 stay; N-S move up 1 table, sharing with 12.
- N-S pairs: move up 2 tables playing at 12 only when displaced by N-S 12.
- EW pairs: move down 3 tables (ignoring table 12).

14 TEAMS

14–1 / 1–14 13–4 12–7 11–10 10–13 9–3 8–6 7–9 6–12 5–2 4–5 3–8 2–11

- Team 14 stays.
- N-S pairs: move up 1 table (using 14 instead of 1).
- E-W pairs: move up 4 tables (ignoring table 14).
- Boards down one table (ignoring table 14).

With stationary boards,

- team 14: E-W table 14 stay; N-S move up one table, sharing with 14.
- N-S pairs: move up 2 tables playing at 14 only when displaced by N-S 14.
- E-W pairs: move up 5 tables (ignoring table 14).

It can be seen that the movements for 10 and 16 teams have not been included in the above schedules. These double-Howell movements do not have a regular movement for the N-S and the E-W pairs. *Note:* one less than the number of teams ($N - 1$) is not a prime number. Standard pairs' movement guide-cards are necessary, as the pairs must play in both N-S and E-W directions. The guide-cards can be extracted from the full expansion of the following starting positions.

One group of pairs stays in the set of tables 1 to 5 and the other in 6 to 10.

10 TEAMS

3–5 1–9 10–8 7–4 6–2 / 9–7 4–5 3–10 8–2 6–1
(team 10 stationary)

16 TEAMS

One group of pairs move on the set of tables 1A, 3 to 5, 9 to 11 and 15, with the other group in the rest.

9–13/13–9 7–2 7–4 10–3 1–14 16–1 14–5 10–11 12–11 8–2 6–16 6–8 12–4 15–3 15–5
(Team 16 stationary).

8.11 Thurner system

Never doubt that a small group of thoughtful, committed people can change the world.
Indeed, it is the only thing that ever has.
Margaret Meade

This teams' movement is for an even number of teams where the number of teams less one is a prime number (i.e. 6, 8, 12, … etc teams). The movement is based on the appendix principle, with one table being appended to American whist league movements of prime numbers.

The advantages are:

- there are only two tables sharing at any one time;
- N-S pairs remain stationary;
- the movement of the E-W pairs is in a regular pattern.

The disadvantages of the movement are:

- the movement does not work for all even numbers of teams;
- the starting positions of the players are not in a regular pattern;
- there are two different tables sharing on each round.

Where there are N teams, the tables are arranged in a horse-shoe pattern of $N - 1$ tables with the highest numbered table (N) in the centre so that it can share boards conveniently with the other tables.

For example, 8 tables.

1		2		3		4
			8			
	7		6		5	

After the pairs have taken their starting positions, all the N-S pairs remain stationary and the E-W pairs at tables 1 to $N - 1$ move up 1 table and the boards (sets 1 to $N - 1$ on tables 1 to $N - 1$) move down one table.

There is a problem with the movement of the highest-numbered team. N-S remains stationary at the highest-numbered table and shares boards with their E-W pair which moves in a particular pattern. This roving E-W pair displaces different pairs on each round, the displaced pairs moving to the highest-numbered table as E-W, playing there for one round and moving normally after that, as though they have played at the table from which they have been displaced.

The starting positions are listed below. It is assumed that the tables are numbered from 1 to N from the left, where table N will be centrally situated.

6 TEAMS

1–6 2–3 3–5 4–2 5–4 6–1

E–W 6 moves up 2 tables, i.e., 1, 3, 5, 2, 4

8 TEAMS

1–8 2–4 3–7 4–3 5–6 6–2 7–5 8–1

E–W 8 moves down 2 tables, i.e., 1, 6, 4, 2, 7, 5, 3

12 TEAMS

1–12 2–6 3–11 4–5 5–10 6–4 7–9 8–3 9–8 10–2 11–7 12–1

E–W 12 moves up 4 tables, i.e., 1, 5, 9, 2, 6, 10, 3, 7, 11, 4, 8

14 TEAMS

1–14 2–7 3–13 4–6 5–12 6–5 7–11 8–4 9–10 10–3 11–9 12–2 13–8 14–1

E-W 14 moves down 4 tables, i.e., 1, 10, 6, 2, 11, 7, 3, 12, 8, 4, 13, 9, 5

From these examples the pattern develops this way. Tables 1 and N are always treated in the same way.

The E-W pairs are positioned as:

- table 2 has E-W pair number ½ N and each subsequent even-numbered table has the next lower-numbered pair;
- table 3 has E-W pair number $N - 1$ and each subsequent odd-numbered table has the next lower-numbered pair.

The roving E-W pair N moves ⅓ N (rounded down) tables – up if ½ N is even – down if ½ N is odd. The N-S pairs are the same as their table numbers. Numbers of teams higher than 16 can be worked out by this method, but $N - 1$ must be prime.

8.12 McKinnon variations

Money is not everything, but it sure helps.
Albert McKinnon

Oddly-even teams

This movement was first published by the author in *Australian Bridge*, June 1973, page 25. It was originally devised to handle 10 teams competing over a single session of play. The other options available for this number of teams are the Patton schedule and the Double Howell. Both these options have most of the pairs moving as well sharing boards.

The oddly-even teams movement, named after the Euler title, only works where half the number of teams is odd. It has an advantage over the stagger movement in that there is no break in play at the halfway point and the movement of the players is the same for the whole event. The disadvantage is that the two rows of tables must share throughout the event but today with pre-duplication of boards this problem is overcome. The tables are arranged in two parallel rows numbered from 1 to ½ N and from ½ $N + 1$ to N.

For example with 10 teams the tables are arranged

1	2	3	4	5
6	7	8	9	10

The tables next to each other share boards throughout the event with the sets 1 to ½ N being placed initially on tables 1 to ½ N. The rest of the sets of boards are placed on a relay table just after table ½ N from where they will move into play on table ½ N. At each change the boards move down one table (and from table 1 to

the relay table and from the relay table to table ½ *N*). The N-S pairs remain stationary and the E-W pairs move up (½ *N* – 2) tables, which is 3 in the ten-table example.

There is one complication. At the start of the event, with each team sitting at its own table, the E-W pairs at every second table in each row are required to exchange places. In the above example E-W pairs 2 and 7, and pairs 4 and 9 would need to exchange places. The first move is then made and the movement proceeds normally to the halfway point, when these mentioned pairs, having returned to their team-mates, must change back for one round. They then *return* and proceed as before.

If this movement is used for pairs' events, the E-W pairs need to move up only 1 table at all times rather than (½ *N* – 2). The exchanging of the E-W pairs is not necessary. If a rover pair is used, they can move up (½ *N* – 2), but must exchange sides of the movement after ½ *N* rounds.

This movement is good for 6 and 10 teams, but 14 and 18 teams require the sharing of two boards between two tables slowing down the movement. This can be overcome by pre-duplication of boards or by having the players duplicate the boards. Sets 1 to *N* are placed on tables 1 to *N*. Sets 1 to ½ *N* can be shuffled, dealt, recorded and duplicated immediately.

Sets ½ *N* + 1 to *N* can be played as though it was round ½ *N*. The E-W pairs next to each other exchange positions for this match (the tables sharing boards). These sets of boards can also be duplicated at this point. The E-W pairs return for the change into the second round. The movement now proceeds normally without any further complications except that there is a double move after round ½ *N* – 1 as round ½ *N* has already been played.

Sixteen teams

This was first published in BDC in 1979. As 16 teams do not constitute an unusually large number of teams, and is a clumsy number for the previous movements, this movement may prove useful.

There is a problem with the New England relay and the Stagger movements. 8 tables are required in two parallel rows and many rooms do not easily accommodate this arrangement. The Thurner and Half-odd movements do not work with this number of teams.

The tables are arranged in three groups of five tables, two of which must be parallel to each other. For example

1	2	3	4	5	(A)
6	7	8	9	10	(B)
11	12	13	14	15	(C)
		16			

Team 16 is placed convenient to the third set of five tables as shown in the arrangement above. On the first stanza the groups (A) and (B) play each other by using a twin-Mitchell movement. Group (C) and team 16 play each other over 5 rounds via (usually) a Thurner movement. The boards must be re-dealt after five rounds.

The move by the teams after they have scored the first five matches is: group (A) moves to the tables 6–10 occupied by group (B) which moves to tables 11–15 and group (C) to tables 1–5. After the next five matches the groups again move up one set of tables for the third stanza.

From a practical point of view, a better move after the first five matches is: Team 16 is moved close to (A) to form a Thurner movement. (B) and (C) form the twin Mitchell.

After the next five matches, groups (A) and (B) swap places. Group (B) and team 16 now form the Thurner movement.

8.13 Swiss system

If only God would give me some clear sign! Like making a large deposit in my name at a Swiss bank.
Woody Allen

Swiss teams are also called elevator teams. It is used where it is impossible to conduct a complete round-robin tournament in the time available and yet necessary to produce a single winner and the result being reasonably reliable. It was first used in a chess tournament in 1895 which was conducted in Zurich Switzerland hence the name Swiss teams.

After each round the draw is determined by the performance of the teams. The leading team plays the team running second, 3rd and 4th play each other, 5th and 6th play each other and so on. The only other restriction on this is that no two teams ever meet twice in the event. The draw is resolved from the top positions such that if first place has played second already, then they would play third place, provided always that they have not met previously. If they have, then first plays fourth place, etc.

The overall effect of this draw is that the good teams (i.e., those performing well) generally play the good teams and the medium teams play the medium and the weakest teams play the weakest. Generally speaking the field is left in one section, even though it is possible to divide it into two or more sections for various purposes.

For the first round, the field is usually seeded with the best teams at the top of the list and the weakest teams at the bottom. The draw is then made according to this order and can be made quite a number of ways.

The first team plays team number $\frac{1}{2}N + 1$, the second $\frac{1}{2}N + 2$ etc., down to $\frac{1}{2}N$ playing N where N is the number of teams (and is also an even number). The effect of this draw is that generally all the top teams have an equally easy match. This is the recommended draw for the first round.

There is another option. The top rated team plays the last, the second plays the second last etc., to the $(\frac{1}{2}N)^{th}$ playing the $(\frac{1}{2}N + 1)^{th}$ team. The reasoning behind this is that the seeding of the centre teams is more difficult and so their results should be determined by play-off.

There are many other methods available, but it is interesting to note that under a mathematical model it has been found that the method used to conduct the first round does not make any significant difference. Therefore the most aesthetically pleasing method should be the one chosen.

As anyone conducting a Swiss teams' event will discover, the determining of the draw becomes more difficult as the number of rounds increases, because the teams have more frequently met their potential opponents. It is therefore recommended that if the number of rounds approaches ½ to ⅔ the number of teams, then a complete round-robin should be preferred (naturally with fewer boards per match).

The minimum number of rounds can be determined more accurately as there is a minimum number of rounds which must be played before it is guaranteed that two teams who have been continually winning will meet each other. This number of rounds $R = \log_2 N$ (rounded up).

In most sports where the Swiss draw is used there are only three possible outcomes for any team: win, draw or loss. That is not the case in bridge where typically there is an IMP or VP result based on a scale. Therefore, from this perspective, the recommended minimum number of rounds is $R = \log_2 N + 2$ (rounded up).

For the following numbers of teams the recommended minimum number of rounds is shown here.

Teams	Rounds
9–16	6
17–32	7
33–64	8
65–128	9

In 2007 there was much debate in Australia about the number of rounds in their premier Swiss teams event that typically had two qualifying sections of about 150 team with the leading 8 teams from each proceeding to the finals. The number of rounds of 20 board matches in this event was set to 14 and over the years it became quite clear to the players that the last day or two did nothing but reorganize the leading qualifiers in each section. If the teams were not in the list of top teams after 9 or 10 rounds then they were very unlikely to be there at the end of the event. After this point most of the leading teams had already played each other and so were simply playing the lesser performing teams and winning most of their matches fairly easily.

On an Internet forum set up for the selection of an Australian team this discussion led to investigation of this subject by Richard Willey (Boston), Steve Willner (Harvard-Smithsonian Center), Gerben Dirkson (Nuremburg), and Alex Ogan (Boston).

Richard Willey said:

> "Alex did [a] simulation yesterday evening. He was specifically looking at the accuracy of a Swiss Team type event in identifying the top eight teams in a large field. Here are a few key parameters.
>
> - The field consisted of 128 teams.
> - The length of a round was fixed at 20 boards.
> - The number of rounds varied between two and twenty.
> - Alex ran 1000 'virtual' tournaments for each round length and then calculated a few key statistics.
> - The first column indicates the number of rounds.
> - The second column indicates the number of times that the strongest team made its way into any of the top eight positions.
> - The third column indicates how many of the top 8 teams made their way into the top eight positions.
> - The last column indicated the average position of the strongest team at the end of the Swiss event.
>
> …"

It should be pointed out that the 128 teams were numbered from 1 to 128 with team 1 the strongest, 2 the next strongest down to team 128. From this array we can see after 12 rounds (of 20 board matches) there is a 95% confidence that the top rated team is in the top 8 places. Also it shows just over 5 of the top 8 teams are in the top 8. Extending this to 20 rounds does not significantly improve the selection process of the event and so 12 rounds are sufficient.

2	545	2.92	13.42
3	706	3.44	7.92
4	788	3.83	6.07
5	815	4.12	5.51
6	862	4.46	4.16
7	879	4.61	4.27
8	917	4.75	3.37
9	914	4.87	2.99
10	932	5	2.9
11	934	5	2.7
12	950	5.16	2.62
13	960	5.23	2.35
14	968	5.38	2.19
15	966	5.47	2.2
16	971	5.5	2.09
17	989	5.67	1.93
18	983	5.79	1.95
19	984	5.83	1.9
20	985	5.9	1.92

Richard goes on to say:

> "Alex [investigated] introducing a simple 'Strength of Schedule' adjustment [to see if it] will significantly improve the accuracy of the Swiss teams format. [What] Alex Ogen is running [is] a whole bunch of *Monte Carlo* simulations. The *Monte Carlo* simulations allow us to have 'virtual teams' with a known strength to compete against one another and observe the results. At the end of the simulation, we can compare the population statistic with the sample statistic.
>
> "The population statistic is our objective data about the relative strength of the different teams. The sample statistic is the result of one specific virtual tournament. We then compare the sample statistic to the population statistic and use this to evaluate the accuracy of the sampling system - in this case, the conditions of contest used for the tournament.
>
> "All the data that we use to generate the Strength of Schedule adjustment comes from the same virtual tournament. We are not using any information about the objective population statistics. Rather, we are making (very substantial) adjustments to the normal scoring system used in a Swiss Teams event. This adjustment essentially states that you want to play as many matches as possible against teams that finish the event with good scores. The fact that you are playing a strong team is actually more important than how well you score against said strong team. Obviously, you still need to win your matches against the strong teams since this means that you will continue to be matched against strong teams.
>
> "This alternative scoring system can be implemented after the tournament without making any changes to the normal Swiss event. Steve and Alex seem to be making a convincing argument that [using] this adjusted scoring system significantly improves the accuracy of the sampling procedure.

Rounds	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
20	985	986	987	987	987	987	987	987	987	987	987
19	984	984	985	985	985	985	986	988	987	987	987
18	983	984	985	988	988	989	990	990	989	989	990
17	989	991	993	992	992	992	991	991	991	991	992
16	971	972	974	977	980	981	983	984	983	983	985
15	966	968	971	973	973	974	973	974	975	977	977
14	968	973	977	979	979	979	981	980	979	981	981
13	960	963	968	969	970	972	974	973	974	977	979
12	950*	963*	966	969	968	968	968	970	969	970	970
11	934	939	945*	949*	951	954	957	961	960	959	960
10	932	934	944	945	948*	947*	946	946	948	951	953
9	914	924	932	933	935	942	946*	947*	948*	949*	948*
8	917	924	927	935	936	939	938	934	939	939	937
7	879	883	890	900	905	905	903	905	904	904	902
6	862	865	878	884	888	886	892	886	880	873	873
5	815	829	834	837	843	843	844	831	831	821	810
4	788	801	808	809	816	809	803	787	780	770	752
3	706	712	721	728	730	718	687	652	603	539	490
2	545	558	566	520	435	316	162	56	30	19	14

…”

Note * is next to each entry that has the first 950 (rounded) out of 1000 events in each column. From this we can see that the 95% confidence in the results starts at 12 rounds without adjusting the scoring method. This drops to 9 rounds if 60% or more of the scores of the opponents of each team are added to their score.

Another way of interpreting this is if the number of boards played in each match is increased, the quality of the score is increased and so the reliability of the results improved. If the number of boards in each match is increased by 60% then only 9 rounds are required to give a 95% confidence in the results. Therefore, a 32 board match is better than 20, but any more than that is not of much value.

John Manning did a similar study where the number of boards per match was considered. He found the average winning margin in a European championship was 0.76 IMPs per board. This means the average edge over an opponent is 0.76 IMPs per board for good quality teams. He then showed the probability of winning a match, charting *the edge over the opponents* to *the length of the match* (using numbers of boards per match).

Length/edge	0 IMPs	.5 IMPs	1 IMPs	1.5 IMPs	2 IMPs
8 boards	.500	.591	.678	.756	.822
16 boards	.500	.628	.743	.836	.904
24 boards	.500	.655	.785	.885	.945*
32 boards	.500	.678	.822	.917	.968
48 boards	.500	.714	.871	.955	.988

* Therefore good teams in a field of average teams (with an expected 2 IMP margin) should expect to win their matches 95% of the time playing about 24 boards per match.

That missing 5% is what makes the game of bridge such a great game. Without that 5% chance of the weak teams winning over the best, there would be very few teams entering events with long matches.

Odd number of teams

As it is almost impossible to have a bye for the odd team in a Swiss team draw, the odd number of teams' event must be handled differently. Ideally it would be best to have a spare team available to cater for an odd number of teams.

It may also be possible to break up the last entered team to make 5th members of other teams or to compile another team from 5th members of other teams, but this is rarely acceptable to the players involved.

The best solution to this problem is to run a three-way match between the three bottom teams. This is similar to the method suggested for round-robins where the bye has been eliminated. The three teams play one half-match on each round so that after two rounds each of the three teams has played two complete matches against the other two teams.

The results of these three teams are not used for compiling the draw except every second round when the matches are complete. An even number of rounds is necessary for this method. If this is not possible then the last two half matches should be scored as half matches on a modified VP scale.

A slight problem with this method is that the draw for the three bottom teams is complicated. Each team should not have met either of its opponents.

A second solution, similar to the previous one, is to have two half-matches on each round scored on a reduced VP scale or scored as one result for each team. The result is included in the next round's draw. The problem is there are many teams playing a different number of teams and the draw can become particularly difficult by the end of the event. This method is recommended only with a large number of teams and a low number of rounds.

Organization suggestions

Each team should be assigned a table with the same number as the team. When two teams are playing each other the E-W pairs should exchange places and the match then commences. The number of boards played in each match will depend on the time available, which will in turn depend on the number of rounds.

It would be normal to have between one and five matches per session so that each match would consist of about 6 to 30 boards. Each match would start with half the boards for the match on each table and after these have been completed the tables would exchange boards.

There is one problem when two opposing teams are situated next to each other where they may overhear the bidding at the next table. This can be overcome by making some spare tables available for use when this situation arises.

The home and away teams are not easily assignable. One solution is for the lower-numbered teams to be the home team on the first round (odd-numbered round) and the higher-numbered teams the home team on the second (even-numbered) rounds.

Another solution is for the home team to be the higher-scoring team or the lower-numbered team in the case of the tie. Other options are toss of a coin or, as in WBF and some other international events 'blind seating' for each round. In WBF events, each captain writes their seating on a sheet (two sheets for each match), without any knowledge of what the other captain has written. Each sheet has the six team members with N / E / S / W beside each name. The captain circles the seating position for the four players in the upcoming match. Thus, unless the two captains have previously agreed, there is in theory no choice available. When both lineups are in, the seating is posted.

As it is imperative to have the results of the matches to determine the draw for the next round, it is

necessary to set time limits on matches. Once the time has expired, play is to cease after the current hand is completed and the result determined immediately (say, 5 minutes to do this). Any hands not completed by both tables should be eliminated from the match.

Because of the nature of the event it is possible for a team that is not leading to win the event in the last round by having a maximum win while all the leaders have drawn matches. It is conceivable that this team may not have played any of the leaders.

It is possible for the leading teams (who cannot be overhauled for qualifying purposes) to manipulate their results in the late rounds to suit their purposes. It is highly desirable for the organisers to eliminate such opportunities.

A method of overcoming this problem is if first place plays second; third place plays fourth, etc., irrespective of whether these teams have played each other or not. This should be automatic for the last 1 to 3 rounds, depending on the total number of rounds.

Where a team is in an unassailable position, it should be drawn against a team near the bottom of the draw so that manipulation of their results cannot affect the outcome of the tournament. In the last few rounds, as far as possible, teams out of contention should not be drawn against teams in contention.

This method of draw by rank (ignoring rematches) is known as Danish Swiss. A disadvantage is that teams may play each other two, three or even more times. Another downside is that by having the team(s) in unassailable positions playing teams totally out of contention, some teams in the last round(s) will have an easier draw as they do not have to meet the unassailable teams.

8.14 Teams of three

Sex at 100 is like playing pool with a rope
George Burns

This is a purely promotional type of event where a club wishes to encourage weaker players to participate in teams' events.

Entries are called for teams of three players. The fourth member of the team is assigned by ballot from a group of volunteer championship players who will play with all three members of the team (say, over three nights, one for each player).

The experienced players will be asked to instruct the weaker players the tactics used in teams' play and to isolate and explain (kindly) how to avoid the mistakes made by the weaker players. A limit on master points should restrict the entries to the people at whom the event is aimed.

Part 9 Individuals' schedules

This is board 'one' of a set of 16 manufactured by Ferd Piatnik in Vienna, Austria, circa 1932.
The cards are very beautiful, but the board design is not very practical. The jutting support for the cards is useful but the box holding the boards is a very strange shape internally.

Individuals are generally fun events, and are not taken particularly seriously by either the players or the organisers. On the other hand this type of event is a particularly sociable one and is usually enjoyed by all those taking part. This aspect was recognized a long time ago and the very first duplicate events in whist were all of this type. As a consequence the first movements were all aimed at individuals, particularly those produced by Safford. See *The first schedules*, page 36.

Several problems arise in the organisation of this event, and the director must be prepared for them. For all the players to meet each other, usually guide-cards must be used. These are either player guide-cards or table guide-cards, depending on the type of movement.

Also see above *The Safford comparative system*, page 64, for early schedules for individuals' play. Individual movements can be divided into three different types and the Snow method can be applied to any movement.

The simplest form is the rainbow movement, which is similar to a Mitchell movement with each player playing at only one compass point throughout the session, and the movement being regular for each player.

The second type is similar to the Howell movement in that the players move to the position occupied by the next lower-numbered player and the boards move down one table. Usually table guide-cards are sufficient to make this movement run smoothly.

The third type is difficult to define. The players generally move haphazardly and quite often the session is divided into several stanzas with one set-of-boards being played in each stanza. Individual player guide-cards are needed with this type of movement.

9.1 Mitchell type

The world's made up of individuals who don't want to be heroes.
Brian Moore

Prime tables Mitchells

The original movement only applies to prime numbers of tables in the simplest form, but can be used as a basis for other numbers of tables by applying the appendix principle for extra tables.

It was originally devised by the late Geoffrey Mott-Smith and was known as the Jacoby movement at that time. It was later to become known as the rainbow movement for the simple reason that the groups of moving players were assigned a colour. Assigning colours is useful where the players change seats on each round, but otherwise they can be nominated by referring to the compass directions.

The major disadvantage with this movement is that all the players do not meet one another. The advantage is the movement is regular and guide-cards are not necessary. Quite a number of the irregular type movements use this type as the basis for the overall movement.

This movement is similar to the pairs Mitchell except that there are 6 components to be considered: North, South, East, West, boards and tables.

With the number of tables prime and greater than 3, one of these components remains stationary together with the tables, while the others move up one table, down one table, up two tables and down two tables. The normal and best component to leave stationary is the boards. The players then move thus: North up one table, South down one table, East up two tables and West down two tables.

The numbering of the players must be given some consideration so that no two players have the same

number. For example, the North payers in a 7-table movement might be numbered 1–7, East 8–14, South 15–21 and West 22–28. Either a special score-sheet with slots for four player numbers for each entry is used or the local pairs' score-sheet can be modified to suit the purpose.

Clearly when using this movement, all the players cannot meet one another, but the movement can be modified so as to double the number of rounds or even treble them. As this movement can be curtailed at any point, there is good control over the number of boards played.

The number of rounds can be doubled by having two players of opposite partnerships exchange places after having played half the boards on the table. For example, with four boards on a table, the first partnership plays two boards and then East and South exchange places and the other two boards are played.

Three partnerships per round can be attained by having three players at each table rotate around the fourth, playing one third of the boards with each partnership. For example, North can remain stationary while East moves to West, West to South and South to East after each of the three partnerships.

The designation of the moves at the start of the event (up one table, etc.) should be made before any rotation of the players so that there will be no confusion as to whom the instructions apply. It should be pointed out to the players that they should always start at the same compass position at each new table. For ease of scoring it is better if the boards are played in numerical order at each table, particularly if the boards are being played in pairs for each partnership.

An alternate approach is to complete the movement without any rotation of players and then commence the movement a second time with two positions exchanged at all tables and new boards introduced. Similarly, a third session is also possible.

To complete all partnerships in these movements it is possible to conduct 4 additional sections where all the North players meet each other in one section and similarly for the other 3 positions E, W and S. Even so it is not always simple because of the number of players in each group. For example a 7-table movement has 7 players in each group requiring 3 players to sit out on each round. On the other hand a 13-table movement can be completed using 13-player 13-round sections; similarly for 17 tables. The best way to solve this problem is to use interwoven Howell movements where all 4 sections play the same movement. See these in *combination type* movements, page 276.

Appendix Mitchell type

This movement was first used by Paul N. Marks of Chicago. As with the appendix Mitchell, this works only for prime numbers of tables. Only one table generally can be appended to the movement. Even where more can be appended, it is not normal to have more than one.

The disadvantage is there can only be as many partnerships as there are rounds, so that it is not possible to rotate the East, South and West players about North. The advantage is that no guide-cards are required to conduct the event.

The basic (prime) movement is set up with the appendix table positioned separately. One board-set is placed on each table including the appendix table.

The players are logically broken up into five groups. North, South, East and West players in the base movement make up four of the groups. The fifth group consists of four roving, displacing players.

Any players trying to occupy the seats of these rovers are sent to the appendix table for that round and continue normally after that as though they have played at the table from which they were displaced.

All the boards remain stationary at all times. The base movement moves normally within itself (i.e., as if

not using the appendix table) and the appendix table is used only by the displaced players.

The displaced players sit at the appendix table one compass position away from the normal. That is: East, West and South players rotate about North one position, but for that round only.

The suggested movements for the players are: North up 2 tables; South up 3 tables; East down 1 table; West down 2 tables. The rovers move up 1 table starting at North table 1, East table 2, South table 3 and West table 4. These 4 displaced players start at the appendix table.

Care must be taken in the numbering of the players so that no numbers are repeated. The rovers are usually assigned the top 4 numbers. For example with 8 tables the arrangement is:

- base movement is 7 tables;
- north numbers 1 to 7;
- east numbers 8 to 14;
- south numbers 15 to21;
- west numbers 22 to 28;
- the four displacing roves are 29 to 32;
- table 8 starts with players 1, 9, 17, 25.

Non-prime tables Mitchells

Of all non-prime number Mitchells it is not possible to have a 6 table individual movement with 4 groups (Euler's conjecture). Note we are not considering the 2MOLS mentioned earlier for pairs but rather a 4MOLS. Despite extensive research by many mathematicians the 4MOLS of order 10 has not been found though it is still possible it does exist. The order 14 was only discovered very recently, see below. Also see below for alternate solutions to 6, 10 and 14 tables. Also there is an alternative for 10 tables supplied by Shomate as well as for 8 and 9 tables.

The rest require special consideration because regular Mitchell-type movements are not possible. There are many possibilities previously published that involve stationary players (such as North), but then all other players and boards need to move and guide-cards for the players essential. The movement of the boards can never be completely regular and so the best solution is to use stationary boards and guide-cards for all the players. The following movements are based on 4MOLS and have been supplied by Wal Wallis. If we used 5MOLS then the boards could be moved with some degree of irregularity, but managing such movements will always be more difficult and so should be rejected.

Consider the 8-table Mitchell. These 4 mutually-orthogonal Latin squares (4MOLS) can be used to produce an 8-table Mitchell individual movement. These represent the 4 groups N, S, E and W and are represented as sets of numbers zero through 7.

0 1 2 3 4 5 6 7	0 1 2 3 4 5 6 7	0 1 2 3 4 5 6 7	0 1 2 3 4 5 6 7
1 0 3 2 5 4 7 6	2 3 0 1 6 7 4 5	3 2 1 0 7 6 5 4	4 5 6 7 0 1 2 3
2 3 0 1 6 7 4 5	4 5 6 7 0 1 2 3	6 7 4 5 2 3 0 1	5 4 7 6 1 0 3 2
3 2 1 0 7 6 5 4	6 7 4 5 2 3 0 1	5 4 7 6 1 0 3 2	1 0 3 2 5 4 7 6
4 5 6 7 0 1 2 3	5 4 7 6 1 0 3 2	1 0 3 2 5 4 7 6	7 6 5 4 3 2 1 0
5 4 7 6 1 0 3 2	7 6 5 4 3 2 1 0	2 3 0 1 6 7 4 5	3 2 1 0 7 6 5 4
6 7 4 5 2 3 0 1	1 0 3 2 5 4 7 6	7 6 5 4 3 2 1 0	2 3 0 1 6 7 4 5
7 6 5 4 3 2 1 0	3 2 1 0 7 6 5 4	4 5 6 7 0 1 2 3	6 7 4 5 2 3 0 1

The following arrays show N, S, E, W in that order and the table numbers across the top. The board-set numbers are the same as the table numbers and usually consists of 3 boards per set when the players rotate. After each board the N, E and S players can rotate about N just in the same way as with the regular Mitchell movements. The boards remain stationary and guide-cards are required for the players. Calibre is 73. This can be completed in another stanza with 4 sections of 8 players over 7 rounds. The 4 groups are 1–8, 9–16,

17–24 and 25–32. In all the following examples N, S, E and W players are in order at tables 1 to *N* on the first round. Also note the North players are in sequential order, starting on round 1, at table 1.

	1	2	3	4	5	6	7	8
1	1 17 9 25	2 18 10 26	3 19 11 27	4 20 12 28	5 21 13 29	6 22 14 30	7 23 15 31	8 24 16 32
2	2 20 11 29	1 19 12 30	4 18 9 31	3 17 10 32	6 24 15 25	5 23 16 26	8 22 13 27	7 21 14 28
3	3 23 13 30	4 24 14 29	1 21 15 32	2 22 16 31	7 19 9 26	8 20 10 25	5 17 11 28	6 18 12 27
4	4 22 15 26	3 21 16 25	2 24 13 28	1 23 14 27	8 18 11 30	7 17 12 29	6 20 9 32	5 19 10 31
5	5 18 14 32	6 17 13 31	7 20 16 30	8 19 15 29	1 22 10 28	2 21 9 27	3 24 12 26	4 23 11 25
6	6 19 16 28	5 20 15 27	8 17 14 26	7 18 13 25	2 23 12 32	1 24 11 31	4 21 10 30	3 22 9 29
7	7 24 10 27	8 23 9 28	5 22 12 25	6 21 11 26	3 20 14 31	4 19 13 32	1 18 16 29	2 17 15 30
8	8 21 12 31	7 22 11 32	6 23 10 29	5 24 9 30	4 17 16 27	3 18 15 28	2 19 14 25	1 20 13 26

Similarly for 9 tables (73) completing in an extra stanza of 4 by 9 players over 9 rounds:

	1	2	3	4	5	6	7	8	9
1	1 19 10 28	2 20 11 29	3 21 12 30	4 22 13 31	5 23 14 32	6 24 15 33	7 25 16 34	8 26 17 35	9 27 18 36
2	2 22 12 32	3 23 10 33	1 24 11 31	5 25 15 35	6 26 13 36	4 27 14 34	8 19 18 29	9 20 16 30	7 21 17 28
3	3 25 11 36	1 26 12 34	2 27 10 35	6 19 14 30	4 20 15 28	5 21 13 29	9 22 17 33	7 23 18 31	8 24 16 32
4	4 23 16 35	5 24 17 36	6 22 18 34	7 26 10 29	8 27 11 30	9 25 12 28	1 20 13 32	2 21 14 33	3 19 15 31
5	5 26 18 30	6 27 16 28	4 25 17 29	8 20 12 33	9 21 10 31	7 19 11 32	2 23 15 36	3 24 13 34	1 22 14 35
6	6 20 17 31	4 21 18 32	5 19 16 33	9 23 11 34	7 24 12 35	8 22 10 36	3 26 14 28	1 27 15 29	2 25 13 30
7	7 27 13 33	8 25 14 31	9 26 15 32	1 21 16 36	2 19 17 34	3 20 18 35	4 24 10 30	5 22 11 28	6 23 12 29
8	8 21 15 34	9 19 13 35	7 20 14 36	2 24 18 28	3 22 16 29	1 23 17 30	5 27 12 31	6 25 10 32	4 26 11 33
9	9 24 14 29	7 22 15 30	8 23 13 28	3 27 17 32	1 25 18 33	2 26 16 31	6 21 11 35	4 19 12 36	5 20 10 34

Similarly for 12 tables (73), switching one round for calibre 80, completing in an extra stanza of 4 by 12 players over 11 rounds. All players follow the next lower number within the 8 groups 1–6, 7–12, 13–18, 19–24, 25–30, 31–36, 37–42 and 43–48. There are two starting positions needed for rounds 1 and 7. This means table guide-cards can be used instead of those for players, but two lots are needed, one for rounds 1 to 6 and the other for rounds 7 to 12. The boards still remain stationary throughout the 12 rounds.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1 25 13 37	2 33 16 41	3 26 19 48	4 36 14 47	5 30 22 39	6 34 24 44	7 28 15 45	8 35 21 43	9 27 18 46	10 32 17 38	11 31 20 40	12 29 23 42
7	7 31 19 43	8 27 22 47	9 32 13 42	10 30 20 41	11 36 16 45	12 28 18 38	1 34 21 39	2 29 15 37	3 33 24 40	4 26 23 44	5 25 14 46	6 35 17 48

Similarly for 15 tables (74), switching one round for calibre 81, but completing with an extra stanza is not as practical as 3 players are sitting out on each round. All players follow the next lower number within the groups 1–15, 16–30, 31–45 and 46–60. Therefore it is possible for the North players to remain stationary and have the board-sets move down one table. Only the first round positions are needed and table guide-cards are sufficient.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1 31 16 46	2 37 18 56	3 34 21 52	4 45 23 47	5 41 25 57	6 38 28 48	7 44 20 53	8 35 17 58	9 42 30 49	10 33 27 54	11 39 19 59	12 36 22 50	13 32 24 60	14 43 26 55	15 40 29 51

Dobromir T. Todorov, a mathematician who has solved many 4MOLS problems, in June 2011 published a solution for 14. The following movement for 14 tables, with the same qualities as for 8 tables shown above, is based on his work. The N and S players are shown above the E and W players. Normally 2 board rounds would be played for a 28 board session(s) and the calibre is 74, switching one round for 81. The last stanza is possible using either movements for 14 players over 14 rounds or 14 players over 13 rounds.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1 29 15 43	2 30 16 44	3 31 17 45	4 32 18 46	5 33 19 47	6 34 20 48	7 35 21 49	8 36 22 50	9 37 23 51	10 38 24 52	11 39 25 53	12 40 26 54	13 41 27 55	14 42 28 56
2	2 33 18 52	1 39 20 47	14 40 23 50	11 35 15 44	13 32 17 51	4 29 27 49	6 30 25 55	7 41 28 53	12 31 24 56	8 34 19 46	5 37 21 54	3 38 16 43	9 42 22 48	10 36 26 45
3	3 34 25 49	12 32 22 45	5 39 18 55	1 42 21 51	11 36 24 54	13 37 16 56	8 33 28 43	4 35 23 47	14 30 17 46	2 41 26 48	6 38 27 44	7 29 19 52	10 40 20 53	9 31 15 50
4	4 37 24 55	13 33 21 50	11 30 22 52	6 40 19 56	1 35 27 46	12 42 18 44	14 38 15 53	3 39 26 51	2 36 28 49	9 29 25 54	8 31 23 48	10 41 17 47	5 34 16 45	7 32 20 43
5	5 41 23 46	8 37 26 49	6 35 16 53	14 29 22 55	10 31 25 43	3 40 21 52	11 42 19 50	2 32 15 54	1 38 18 48	4 39 28 45	12 33 20 51	9 30 27 56	7 36 17 44	13 34 24 47
6	6 39 17 50	4 41 15 52	2 37 27 43	10 33 23 54	7 42 26 55	11 38 28 47	1 40 24 45	13 30 20 49	3 32 19 53	5 31 22 44	9 36 16 46	14 34 18 51	8 29 21 56	12 35 25 48
7	7 38 22 51	5 29 24 53	4 36 20 56	3 37 28 48	14 39 21 44	8 35 17 54	13 31 26 46	9 40 18 43	10 34 27 50	12 30 15 47	1 41 19 49	2 42 25 45	6 32 23 52	11 33 16 55
8	8 42 16 47	14 36 19 48	1 34 26 44	9 39 24 49	2 29 20 50	7 30 23 45	12 37 17 52	5 38 25 56	13 35 22 43	11 40 27 51	10 32 28 55	4 31 21 53	3 33 15 46	6 41 18 54
9	9 35 19 45	6 31 28 51	10 42 15 49	13 36 25 52	8 30 18 53	14 41 24 43	5 32 27 48	12 34 21 55	7 39 16 54	1 33 17 56	2 40 22 47	11 37 20 46	4 38 26 50	3 29 23 44
10	10 30 21 48	7 40 25 46	13 38 19 54	12 41 16 50	9 34 28 52	5 36 15 51	2 39 23 56	14 33 27 45	6 29 26 47	3 35 20 55	4 42 17 43	8 32 24 44	11 31 18 49	1 37 22 53
11	11 32 26 56	3 42 27 54	7 33 24 48	2 34 17 53	12 38 23 49	10 39 22 46	9 41 20 44	1 31 16 52	8 40 15 55	6 36 21 43	13 29 18 45	5 35 28 50	14 37 25 47	4 30 19 51
12	12 36 27 53	11 34 23 43	9 32 21 47	8 38 20 45	3 41 22 56	2 31 19 55	10 29 16 51	6 42 24 46	4 33 25 44	7 37 18 50	14 35 26 52	13 39 15 48	1 30 28 54	5 40 17 49
13	13 40 28 44	10 35 18 56	8 41 25 51	7 31 27 47	6 37 15 45	9 33 26 53	4 34 22 54	11 29 17 48	5 42 20 52	14 32 16 49	3 30 24 50	1 36 23 55	12 39 19 43	2 38 21 46
14	14 31 20 54	9 38 17 55	12 29 28 46	5 30 26 43	4 40 16 48	1 32 25 50	3 36 18 47	10 37 19 44	11 41 21 45	13 42 23 53	7 34 15 56	6 33 22 49	2 35 24 51	8 39 27 52

6, 10 and 14 Tables

One solution for 6 tables is to use the Howell-type 24-player, 12-round movement, but another is to use 2 groups. This is based on the 12-table pairs criss-cross Mitchell. The boards are shared across all tables on each round using the Snow method. The board-set number is the same as the round number and play can be stopped after any round. Completing these movements with a second stanza will produce a well balanced, high calibre event.

	1	2	3	4	5	6
1	1 13 2 14	3 15 4 16	5 17 6 18	7 19 8 20	9 21 10 22	11 23 12 24
2	12 14 3 13	2 16 5 15	4 18 7 17	6 20 9 19	8 22 11 21	10 24 1 23
3	3 23 12 16	5 13 2 18	7 15 4 20	9 17 6 22	11 19 8 24	1 21 10 14
4	10 16 5 23	12 18 7 13	2 20 9 15	4 22 11 17	6 24 1 19	8 14 3 21
5	5 21 10 18	7 23 12 20	9 13 2 22	11 15 4 24	1 17 6 14	3 19 8 16
6	8 18 7 21	10 20 9 23	12 22 11 13	2 24 1 15	4 14 3 17	6 16 5 19
7	7 24 8 15	9 14 10 17	11 16 12 19	1 18 2 21	3 20 4 23	5 22 6 13
8	6 15 9 24	8 17 11 14	10 19 1 16	12 21 3 18	2 23 5 20	4 13 7 22
9	9 22 6 17	11 24 8 19	1 14 10 21	3 16 12 23	5 18 2 13	7 20 4 15
10	4 17 11 22	6 19 1 24	8 21 3 14	10 23 5 16	12 13 7 18	2 15 9 20
11	11 20 4 19	1 22 6 21	3 24 8 23	5 14 10 13	7 16 12 15	9 18 2 17
12	2 19 1 20	4 21 3 22	6 23 5 24	8 13 7 14	10 15 9 16	12 17 11 18

There are two groups of 12 players, 1–12 and 13–24, partnering only those players in the other group. Calibre is 38. For a second stanza these two groups can partner the players within their group using two sections of 12-player 11-round Howell-type movements.

A similar movement for 10 tables has players in two groups, 1–20 and 21–40. Calibre is 34. Use the 20-player 19-round Howell- type for the second stanza in two sections.

	1	2	3	4	5	6	7	8	9	10
1	1 21 2 22	3 23 4 24	5 25 6 26	7 27 8 28	9 29 10 30	11 31 12 32	13 33 14 34	15 35 16 36	17 37 18 38	19 39 20 40
2	20 22 3 21	2 24 5 23	4 26 7 25	6 28 9 27	8 30 11 29	10 32 13 31	12 34 15 33	14 36 17 35	16 38 19 37	18 40 1 39
3	3 39 20 24	5 21 2 26	7 23 4 28	9 25 6 30	11 27 8 32	13 29 10 34	15 31 12 36	17 33 14 38	19 35 16 40	1 37 18 22
4	18 24 5 39	20 26 7 21	2 28 9 23	4 30 11 25	6 32 13 27	8 34 15 29	10 36 17 31	12 38 19 33	14 40 1 35	16 22 3 37
5	5 37 18 26	7 39 20 28	9 21 2 30	11 23 4 32	13 25 6 34	15 27 8 36	17 29 10 38	19 31 12 40	1 33 14 22	3 35 16 24
6	16 26 7 37	18 28 9 39	20 30 11 21	2 32 13 23	4 34 15 25	6 36 17 27	8 38 19 29	10 40 1 31	12 22 3 33	14 24 5 35
7	7 35 16 28	9 37 18 30	11 39 20 32	13 21 2 34	15 23 4 36	17 25 6 38	19 27 8 40	1 29 10 22	3 31 12 24	5 33 14 26
8	14 28 9 35	16 30 11 37	18 32 13 39	20 34 15 21	2 36 17 23	4 38 19 25	6 40 1 27	8 22 3 29	10 24 5 31	12 26 7 33
9	9 33 14 30	11 35 16 32	13 37 18 34	15 39 20 36	17 21 2 38	19 23 4 40	1 25 6 22	3 27 8 24	5 29 10 26	7 31 12 28
10	12 30 11 33	14 32 13 35	16 34 15 37	18 36 17 39	20 38 19 21	2 40 1 23	4 22 3 25	6 24 5 27	8 26 7 29	10 28 9 31
11	11 40 12 23	13 22 14 25	15 24 16 27	17 26 18 29	19 28 20 31	1 30 2 33	3 32 4 35	5 34 6 37	7 36 8 39	9 38 10 21
12	10 23 13 40	12 25 15 22	14 27 17 24	16 29 19 26	18 31 1 28	20 33 3 30	2 35 5 32	4 37 7 34	6 39 9 36	8 21 11 38
13	13 38 10 25	15 40 12 27	17 22 14 29	19 24 16 31	1 26 18 33	3 28 20 35	5 30 2 37	7 32 4 39	9 34 6 21	11 36 8 23
14	8 25 15 38	10 27 17 40	12 29 19 22	14 31 1 24	16 33 3 26	18 35 5 28	20 37 7 30	2 39 9 32	4 21 11 34	6 23 13 36
15	15 36 8 27	17 38 10 29	19 40 12 31	1 22 14 33	3 24 16 35	5 26 18 37	7 28 20 39	9 30 2 21	11 32 4 23	13 34 6 25
16	6 27 17 36	8 29 19 38	10 31 1 40	12 33 3 22	14 35 5 24	16 37 7 26	18 39 9 28	20 21 11 30	2 23 13 32	4 25 15 34
17	17 34 6 29	19 36 8 31	1 38 10 33	3 40 12 35	5 22 14 37	7 24 16 39	9 26 18 21	11 28 20 23	13 30 2 25	15 32 4 27
18	4 29 19 34	6 31 1 36	8 33 3 38	10 35 5 40	12 37 7 22	14 39 9 24	16 21 11 26	18 23 13 28	20 25 15 30	2 27 17 32
19	19 32 4 31	1 34 6 33	3 36 8 35	5 38 10 37	7 40 12 39	9 22 14 21	11 24 16 23	13 26 18 25	15 28 20 27	17 30 2 29
20	2 31 1 32	4 33 3 34	6 35 5 36	8 37 7 38	10 39 9 40	12 21 11 22	14 23 13 24	16 25 15 26	18 27 17 28	20 29 19 30

A similar movement for 14 tables has players in two groups, 1–28 and 29–56. Calibre is 29. Use the 28-player 27-round Snow method for the second stanza in two sections.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1 29 2 30	3 31 4 32	5 33 6 34	7 35 8 36	9 37 10 38	11 39 12 40	13 41 14 42	15 43 16 44	17 45 18 46	19 47 20 48	21 49 22 50	23 51 24 52	25 53 26 54	27 55 28 56
2	28 30 3 29	2 32 5 31	4 34 7 33	6 36 9 35	8 38 11 37	10 40 13 39	12 42 15 41	14 44 17 43	16 46 19 45	18 48 21 47	20 50 23 49	22 52 25 51	24 54 27 53	26 56 1 55
3	3 55 28 32	5 29 2 34	7 31 4 36	9 33 6 38	11 35 8 40	13 37 10 42	15 39 12 44	17 41 14 46	19 43 16 48	21 45 18 50	23 47 20 52	25 49 22 54	27 51 24 56	1 53 26 30
4	26 32 5 55	28 34 7 29	2 36 9 31	4 38 11 33	6 40 13 35	8 42 15 37	10 44 17 39	12 46 19 41	14 48 21 43	16 50 23 45	18 52 25 47	20 54 27 49	22 56 1 51	24 30 3 53
5	5 53 26 34	7 55 28 36	9 29 2 38	11 31 4 40	13 33 6 42	15 35 8 44	17 37 10 46	19 39 12 48	21 41 14 50	23 43 16 52	25 45 18 54	27 47 20 56	1 49 22 30	3 51 24 32
6	24 34 7 53	26 36 9 55	28 38 11 29	2 40 13 31	4 42 15 33	6 44 17 35	8 46 19 37	10 48 21 39	12 50 23 41	14 52 25 43	16 54 27 45	18 56 1 47	20 30 3 49	22 32 5 51
7	7 51 24 36	9 53 26 38	11 55 28 40	13 29 2 42	15 31 4 44	17 33 6 46	19 35 8 48	21 37 10 50	23 39 12 52	25 41 14 54	27 43 16 56	1 45 18 30	3 47 20 32	5 49 22 34
8	22 36 9 51	24 38 11 53	26 40 13 55	28 42 15 29	2 44 17 31	4 46 19 33	6 48 21 35	8 50 23 37	10 52 25 39	12 54 27 41	14 56 1 43	16 30 3 45	18 32 5 47	20 34 7 49
9	9 49 22 38	11 51 24 40	13 53 26 42	15 55 28 44	17 29 2 46	19 31 4 48	21 33 6 50	23 35 8 52	25 37 10 54	27 39 12 56	1 41 14 30	3 43 16 32	5 45 18 34	7 47 20 36
10	20 38 11 49	22 40 13 51	24 42 15 53	26 44 17 55	28 46 19 29	2 48 21 31	4 50 23 33	6 52 25 35	8 54 27 37	10 56 1 39	12 30 3 41	14 32 5 43	16 34 7 45	18 36 9 47
11	11 47 20 40	13 49 22 42	15 51 24 44	17 53 26 46	19 55 28 48	21 29 2 50	23 31 4 52	25 33 6 54	27 35 8 56	1 37 10 30	3 39 12 32	5 41 14 34	7 43 16 36	9 45 18 38
12	18 40 13 47	20 42 15 49	22 44 17 51	24 46 19 53	26 48 21 55	28 50 23 29	2 52 25 31	4 54 27 33	6 56 1 35	8 30 3 37	10 32 5 39	12 34 7 41	14 36 9 43	16 38 11 45
13	13 45 18 42	15 47 20 44	17 49 22 46	19 51 24 48	21 53 26 50	23 55 28 52	25 29 2 54	27 31 4 56	1 33 6 30	3 35 8 32	5 37 10 34	7 39 12 36	9 41 14 38	11 43 16 40
14	16 42 15 45	18 44 17 47	20 46 19 49	22 48 21 51	24 50 23 53	26 52 25 55	28 54 27 29	2 56 1 31	4 30 3 33	6 32 5 35	8 34 7 37	10 36 9 39	12 38 11 41	14 40 13 43
15	15 56 16 31	17 30 18 33	19 32 20 35	21 34 22 37	23 36 24 39	25 38 26 41	27 40 28 43	1 42 2 45	3 44 4 47	5 46 6 49	7 48 8 51	9 50 10 53	11 52 12 55	13 54 14 29
16	14 31 17 56	16 33 19 30	18 35 21 32	20 37 23 34	22 39 25 36	24 41 27 38	26 43 1 40	28 45 3 42	2 47 5 44	4 49 7 46	6 51 9 48	8 53 11 50	10 55 13 52	12 29 15 54
17	17 54 14 33	19 56 16 35	21 30 18 37	23 32 20 39	25 34 22 41	27 36 24 43	1 38 26 45	3 40 28 47	5 42 2 49	7 44 4 51	9 46 6 53	11 48 8 55	13 50 10 29	15 52 12 31
18	12 33 19 54	14 35 21 56	16 37 23 30	18 39 25 32	20 41 27 34	22 43 1 36	24 45 3 38	26 47 5 40	28 49 7 42	2 51 9 44	4 53 11 46	6 55 13 48	8 29 15 50	10 31 17 52
19	19 52 12 35	21 54 14 37	23 56 16 39	25 30 18 41	27 32 20 43	1 34 22 45	3 36 24 47	5 38 26 49	7 40 28 51	9 42 2 53	11 44 4 55	13 46 6 29	15 48 8 31	17 50 10 33
20	10 35 21 52	12 37 23 54	14 39 25 56	16 41 27 30	18 43 1 32	20 45 3 34	22 47 5 36	24 49 7 38	26 51 9 40	28 53 11 42	2 55 13 44	4 29 15 46	6 31 17 48	8 33 19 50
21	21 50 10 37	23 52 12 39	25 54 14 41	27 56 16 43	1 30 18 45	3 32 20 47	5 34 22 49	7 36 24 51	9 38 26 53	11 40 28 55	13 42 2 29	15 44 4 31	17 46 6 33	19 48 8 35
22	8 37 23 50	10 39 25 52	12 41 27 54	14 43 1 56	16 45 3 30	18 47 5 32	20 49 7 34	22 51 9 36	24 53 11 38	26 55 13 40	28 29 15 42	2 31 17 44	4 33 19 46	6 35 21 48
23	23 48 8 39	25 50 10 41	27 52 12 43	1 54 14 45	3 56 16 47	5 30 18 49	7 32 20 51	9 34 22 53	11 36 24 55	13 38 26 29	15 40 28 31	17 42 2 33	19 44 4 35	21 46 6 37
24	6 39 25 48	8 41 27 50	10 43 1 52	12 45 3 54	14 47 5 56	16 49 7 30	18 51 9 32	20 53 11 34	22 55 13 36	24 29 15 38	26 31 17 40	28 33 19 42	2 35 21 44	4 37 23 46
25	25 46 6 41	27 48 8 43	1 50 10 45	3 52 12 47	5 54 14 49	7 56 16 51	9 30 18 53	11 32 20 55	13 34 22 29	15 36 24 31	17 38 26 33	19 40 28 35	21 42 2 37	23 44 4 39
26	4 41 27 46	6 43 1 48	8 45 3 50	10 47 5 52	12 49 7 54	14 51 9 56	16 53 11 30	18 55 13 32	20 29 15 34	22 31 17 36	24 33 19 38	26 35 21 40	28 37 23 42	2 39 25 44
27	27 44 4 43	1 46 6 45	3 48 8 47	5 50 10 49	7 52 12 51	9 54 14 53	11 56 16 55	13 30 18 29	15 32 20 31	17 34 22 33	19 36 24 35	21 38 26 37	23 40 28 39	25 42 2 41
28	2 43 1 44	4 45 3 46	6 47 5 48	8 49 7 50	10 51 9 52	12 53 11 54	14 55 13 56	16 29 15 30	18 31 17 32	20 33 19 34	22 35 21 36	24 37 23 38	26 39 25 40	28 41 27 42

Similar movements can be used for 8, 9, 12, 16, 18 and 20 tables. See the *Snow method* (page 279) and *Two groups in parallel* (page 281).

9.2 Howell type

Nations have their ego, just like individuals.
James Joyce

The advantages of this movement are that the players move regularly, the boards move regularly and all players meet all other players.

It is suggested that the reader study the method of derivation of the Howell movements for a more thorough understanding of the following movements.

When looking for an individual movement we can examine the Howell pairs' movement with an *even-numbered table movement* with double the number of tables and all tables involved in *pairs of shares*. The best example of this is the early scoring movement supplied for a 6-table Howell. This is used as a three table individual movement.

That movement is repeated below for convenience, but it has been modified for an individual movement with the numbers reading from left to right at each table representing North, South, East and West. The same general shorthand method has been retained for describing this movement.

Where the number of players is one less than an even number of tables (for example 3 tables requires 12 players but you need a movement for 11 players) the highest numbered player, the stationary player, is deemed to be the sit-out table for 3 players on each round. In the 3-table example player 12 is the sit-out and players 1, 9 and 8 are sitting out on the first round.

The calibre of the movement is shown in brackets. All movements are shown as N–S–E–W, table 1 first. Where there are groups of players (interwoven or mingled Howells) those groups are shown like {1–13, 14–26} indicating that the players within the group follow the number 1 less than their number, except for the lowest number in the group who follows the highest. In this example 1 follows 13 and 14 follows 26. Where there are sit-out players the first-round sit-out is shown as [1, 10] in this example, showing players 1 and 10 sit out on round 1. Stationary players are underlined.

By combining 2-table movements with other 2-table movements it is possible to have 8, 16, 24, 32, 40, 48 and 56-player movements in which they all play the same boards, and 7 rounds of 3 or 4 boards, for a total of 21 or 28 boards.

A similar approach can be used with three-table movements to produce movements for 12, 24, or 36 players, all playing the same boards and playing 11 rounds for 22 or 33 boards. Call the movements A, B and C and the arrangement is:

Board-set	11	1	2	3	4	5	6	7	8	9	10
Movement	A1	A2	B3	B1	B2	C3	C1	C2	--	--	A3

When this method is used, each group of players must be numbered separately such as 1 to 8, 11 to 18 and 21 to 28 etc. In the case of 12 players it might be 1 to 12, 21 to 32 etc. These are called interwoven Howells.

There are many examples where this interwoven technique is used and can be used in preference to running two movements in parallel or sharing boards. For example, the 32-player 15-round movement is two 16-player movements using one set-of-boards. Many, but not all, examples are included here to give an indication of the possibilities.

1 group

3 Tables, 12 players and 11 rounds

12–1–9–8 5–7–3–11 8R 2–6–4–10 (75) – McKinnon

Player 12 remains stationary, as does the highest-numbered player in all cases. All other players follow the next lower-numbered player. The boards move down one table, with set one usually starting at table 2 and coming out of play after three rounds. All but three sets will have been scored by the end of the last round. Also players play against each other more or less than twice.

10–2–8–6 12–1–4–5 7–11–3–9 8R (71) – Manning. Similar principles apply though the balance is not as good.

Where two relay tables are not a problem use this for better balance:
12–1–5–10 4R 7–11–8–9 2–4–3–6 4R (89) – Groner.

2 Tables, 8 players and 7 rounds

It is quite clear from studying all the 4-table Howell movements available, that there are no 2-table Howell Individual movements of the type explained above for 3 tables. Therefore sharing of the boards between the two tables is required.

8–1–6–2/5–7–3–4 6R (100)

Clearly any 4-table Howell skeleton can be used to derive a 2-table individual movement. The board-sets would consist of either 3 or 4 boards, making 21 or 28 boards in all.

4 Tables, 16 players and 15 rounds

The movements for 4, 5 and 6 tables unfortunately do not have all the relays on one table. Also players play against each other more or less than twice.

16–1–13–6 3–5–15–4 14–11–10–9 9R 2–8–12–7 2R (61) – McKinnon

16–1–15–12 R 6–7–9–11 3R 4–8–14–5 13–3–10–2 7R (87) – Manning

5 Tables, 20 players and 19 rounds

20–1–5–13 7–17–16–9 8R 15–12–8–14 2–18–4–3 4R 15–19–11–6 2R (67) – McKinnon

6 Tables, 24 players and 23 rounds

24–1–21–4 2R 19–16–15–14 3R 13–2–3–17 R 9–11–8–12 3R 10–18–20–7 6–22–5–23 8R (70) – McKinnon

9 players and 9 rounds

Movements similar to Howell movements are possible for 9 or 10 players. There is no stationary player, but the players all follow the player with the number one less than their own. In the 9-player movement everyone sits out once and there are 9 rounds of usually 3 boards per round. Sharing is always carried out between the two tables, and by combining 2 or more movements it is possible to have events for 9, 18, 27, … players.

Some of the possible starting positions for 9 players (all have a calibre 82) are

9–8–4–7/2–6–3–5 player 1 sits out on the first round
6–8–5–9/2–3–4–7 player 1 out
4–8–5–7/3–2–9–6 player 1 out
4–3–5–8/2–6–7–9 player 1 out

10 players and 10 rounds

In the 10-player movement the procedure is the same as that for 9 players, but it would be unusual to have multiple sections for there is a movement available for 20 players.

Each player sits out twice and there are 10 rounds of 2 or 3 boards. The boards are shared throughout the movement between each set of two tables. There are many possible movements for 10 players, therefore those included below are the ones previously published.

1–5–2–4/9–6–8–7 (68) [3, 10] – McKinnon
4–8–3–2/5–10–9–6 (51) [1, 7] – McKinnon
7–8–10–2/5–9–3–6 (82) [1, 4] – Manning

11 players and 11 rounds

9–10–6–11 4–8–5–2 9R (79) [1, 3, 7] – Manning

13 players and 11 rounds

Use the 12-player 11-round movements above and place number 13 stationary at table 2 in the North seat. The displaced player sits out for that round.

<u>12</u>–1–9–8 <u>13</u>–7–3–11 8R 2–6–4–10 (68) – McKinnon
<u>13</u>–2–8–6 <u>12</u>–1–4–5 7–11–3–9 8R (61) – Manning

13 players and 13 rounds

Another very effective movement for an odd number of players is the following one for 13 players. This is derived from the following 7-table Howell:

14–1/13–10/9–8 3–5/12–7 10R 2–6/11–4, giving

2–6–11–4 13–10–9–8 3–5–12–7 10R player 1 out (79) – McKinnon

There are 13 rounds with each player sitting out once. The sit out player can kibitz the table 2 game. With set 1 starting at table 3 it will have been played three times (i.e., finished) after three rounds and so the scoring can start at this point.

This movement can be used in multiples, with each additional movement following the previous one on the same set-of-boards so that the movement is suitable for 13, 26, 39 or 52 players though 52 players is not that useful except in a 4 session event (see *combination* movements, page 276).

If the 4 sections are called A, B, C and D then the arrangement would be

Board-sets:	1	2	3	4	5	6	7	8	9	10	11	12	13
Section:	A	A	A	B	B	B	C	C	C	D	D	D	-

As the same movement is used for each section, they simply start with a different board-set. This is similar to what can be done with the previous 12-player movement except that this can accommodate 4 sections rather than three. By using player guide-cards this movement can be used with stationary boards, which would be clearly better for 52 players, as only one table is not in play.

Where you do not need the above properties, these are better balanced –

2–6–9–11 2R 7–10–8–3 3–4–5–12 8R player 1 out (100) – Manning
12–11–10–3 5–8–2–7 2R 9–13–6–4 8R player 1 out (100) – Hanner
5–7–9–10 4R 11–3–4–8 5R 6–13–12–2 R player 1 out (100) – Groner

14 players and 11 rounds

Use the 12-player 11-round movements above and place number 13 stationary at table 2 and number 14 stationary at table 3 in the North seats. The displaced players sit out for that round.

<u>12</u>–1–9–8 <u>13</u>–7–3–11 8R <u>14</u>–6–4–10 (59) – McKinnon
<u>13</u>–2–8–6 <u>12</u>–1–4–5 <u>14</u>–11–3–9 8R (58) – Manning

14 players and 13 rounds

By using the above 7 table Howell movement it is possible to have a 13-round 14-player movement, the disadvantages being that each player sits out twice, and 14 never sits out. Player 14 is stationary throughout the movement.

2–6–11–4 <u>14</u>–1–9–8 3–5–7–12 10R (87) [13, 10] – McKinnon

14 players and 14 rounds

There is no stationary player and the players all follow the player with the number one less than their own.

10–6–12–7 14–1–9–3 5R 11–8–4–2 6R [13, 5] (91) – McKinnon
12–7–9–5 1–2–14–11 10–4–8–6 11R [13, 3] (58) – Groner
2–6–3–8 11–10–13–5 5R 4–7–12–14 6R [1, 9] (91) – McKinnon

16 players and 12 rounds

These are incomplete Howells. Players 13 through 16 are stationary throughout and 1 follows 12.

<u>13</u>–1–7–5 R 10–2–<u>14</u>–11 R 3–6–8–<u>15</u> R 12–<u>16</u>–4–9 5R (76) – Hanner
7–5–<u>13</u>–1 R 10–2–<u>14</u>–11 R 3–6–8–<u>15</u> R 4–9–12–<u>16</u> 5R (82) – McKinnon

17 players and 13 rounds

These are incomplete Howells. Players 14 through 17 are stationary throughout and 1 follows 13.

<u>14</u>–11–10–5 5R 6–<u>15</u>–2–8 R 7–9–<u>16</u>–12 R 3–4–13–<u>17</u> 2R (77) – Hanner

Switching rounds 10 and 13 improves the balance to 81. The following has fewer relay tables and much the same balance.

17–8–1–3 10–16–6–5 15–12–9–13 2R 7–14–11–4 7R (76) – McKinnon

Switching rounds 1 and 5 improves the calibre to 80.

17 players and 15 rounds

Use either of the 4-table, 16-player 15-round movements above and place number 17 stationary at table 2 in the North seat. The displaced player sits out for that round.

17 players and 17 rounds

Using the following Howell movement it is possible to derive a good 17-player individual movement.

18–1/12–7/13–10 5R 6–8/11–17 5R 16–9/4–3 3R 15–2/14–5

This gives

12–7–13–10 5R 6–8–11–17 5R 16–9–4–3 3R 15–2–14–5 player 1 out (70) – McKinnon

Better balance are

4–7–6–12 2–9–16–3 3R 14–15–5–10 8R 13–11–8–17 2R player 1 out (93) – Hanner
16–14–11–3 2R 7–10–9–15 5–12–2–6 3R 17–1–8–13 8R player 4 out (93) – Manning

There are no stationary players and each player follows the next lower numbered player (1 following 17).

18 players and 15 rounds

Use either of the 16-player 15-round movements above and place number 17 stationary at table 2 and number 18 stationary at table 3 in the North seats. The displaced players sit out for that round.

18 players and 17 rounds

Using the above 9-table Howell the following movement is derived, but the movement is not perfect as player 18 remains stationary throughout and all other players sit out for two rounds. There are only 17 rounds.

18–1–12–7 5R 6–8–11–17 5R 16–9–4–3 3R 15–2–14–5 (63) [13, 10 out] – McKinnon

Any of the 17-player 17-round movements can be used for 18 players by placing player 18 in any seat, the displaced player sitting out for that round.

20 players and 15 rounds

This is an incomplete Howell. Players 16 through 20 are stationary throughout and 1 follows 15.

16–1–6–11 2R 12–17–10–3 2R 18–8–5–4 2R 14–19–13–2 2R 20–15–7–9 2R (84) – Hanner.

Switching 3 rounds, 5 apart (e.g., 1, 6, 11) improves the calibre marginally to 89.

21 players and 21 rounds

20–3–8–18 R 19–16–13–12 R 4–6–9–17 12R 15–21–11–2 R 14–7–10–5 R (79) – McKinnon

There is no stationary player and each player follows the next lower numbered player. Player 1 out on round 1.

22 players and 21 rounds

22–1–20–3 R 19–16–l3–12 R 4–6–9–17 12R 15–21–11–2 R 14–7–10–5 R (74) – McKinnon

Player 22 is stationary throughout with players 8 and 18 out on round 1.

2 Groups interwoven

See *Two groups parallel*, page 281, in the *Snow method* for the possibility for two groups where the groups partner each other, but never within the group. With interwoven Howells the groups only partner and play against those in their group.

24 players and 11 rounds

2–6–4–10 12–1–9–8 5–7–3–11 14–18–16–22 24–13–21–20 17–19–15–23 5R (58) {1–11, 13–23} – McKinnon

26 players and 13 rounds

2–6–11–4 13–10–9–8 3–5–12–7 15–19–24–17 26–23–22–21 16–18–25–20 7R (56) {1–13, 14–26} [1, 14 out] – McKinnon (one relay table)

4–20–16–10 22–8–13–24 15–3–7–2 17–9–12–11 26–6–23–21 2R 19–25–18–5 5R (84) {1–13, 14–26} [1, 14 out] – Hanner (two relay tables)

32 players and 15 rounds

16–1–13–6 3–5–15–4 14–11–10–9 18–24–28–23 2R 17–32–29–22 19–21–31–20 30–27–26–25 3R 2–8–12–7 2R (48) {1–15, 17–35} – McKinnon – switch rounds 1 to 4 for (51)

34 players and 17 rounds

15–2–14–5 12–7–13–10 32–19–31–22 29–24–30–27 3R 6–8–11–17 R 23–25–28–34 3R 16–9–4–3 R 33–26–21–20 R (59) {1–17, 18–24} [1, 18 out] – McKinnon

2 Groups mingled

Here we consider the use of two equal size groups of players in movements where they move within their group, but play with members of both groups. Olof Hanner calls such movements mingled. Unlike those in the Snow method the two groups cannot play a second stanza to complete partnering of all players. These schedules are depicted in the usual way and the groups are shown, for example in the first case, 1 follows 8 and 9 follows 16 is shown as {1–8, 9–16}.

16 players, 8 rounds

1–9–2–4 7–6–13–16 2R 15–3–14–12 11–10–8–5 2R (53) {1–8, 9–16}

18 players, 9 rounds

9–6–2–14 11–5–4–8 15–18–7–17 R 16–3–12–13 4R (73) {1–9, 10–18} [1, 10 out]

20 players, 10 rounds

1–10–3–11 6–16–5–19 9–18–4–8 R 13–17–14–2 3R 12–15–7–20 R (67) {1–10, 11–20}

22 players, 11 rounds

3–18–4–16 2–11–6–20 R 14–9–13–17 5R 21–22–15–5 7–8–10–19 (86) {1–11, 12–22} [1, 12 out]

24 players, 12 rounds

1–13–2–4 7–10–20–17 2R 11–3–9–15 R 23–21–14–6 2R 5–18–19–12 24–16–8–22 R (70) {1–12, 13–24} – Manning

26 players, 13 rounds

2–6–11–4 13–10–9–8 3–5–12–7 15–19–24–17 26–23–22–21 16–18–25–20 7R (56) {1–13, 14–26} [1, 14 out] – McKinnon (one relay table)
4–20–16–10 22–8–13–24 15–3–7–2 17–9–12–11 26–6–23–21 2R 19–25–18–5 5R (84) {1–13, 14–26} [1, 14 out] – Hanner (two relay tables)

28 players, 14 rounds

1–2–15–16 R 9–11–27–6 R 3–25–10–7 R 26–23–14–20 R 5–21–13–4 R 8–19–18–28 R 24–12–17–22 R (68) {1–14, 15–28}

30 players, 15 rounds

3–10–8–20 R 11–17–30–14 21–19–26–7 15–29–28–23 R 6–12–5–27 4R 25–18–9–22 13–24–2–4 2R (84) {1–15, 16–30} [1, 16 out]

24 players, 10 rounds

<u>21</u>–1–4–19 2R 6–18–<u>22</u>–8 16–17–10–11 2–3–15–7 12–5–14–<u>23</u> 2R 20–<u>24</u>–13–9 (66) {1–10, 11–20}

28 players, 12 rounds

<u>25</u>–1–13–7 R 19–12–<u>26</u>–8 5–6–3–24 20–<u>27</u>–22–9 R 16–2–17–<u>28</u> 11–21–4–15 3R 23–18–14–10 (67) {1–12, 13–24}

32 players, 14 rounds

<u>29</u>–1–9–15 26–12–5–28 7–25–<u>30</u>–20 11–<u>31</u>–17–2 6–16–4–23 <u>32</u>–27–21–8 R 24–3–13–14 4R 10–22–18–19 R (73) {1–14, 15–28}

3 Groups

1 Stanza movement

36 players and 11 rounds

2–6–4–10 <u>12</u>–1–9–8 5–7–3–11 14–18–16–22 13–<u>24</u>–21–20 17–19–15–23 26–30–28–34 33–32–<u>36</u>–25 29–31–27–35 2R (50) {1–11, 13–23, 25–35} – McKinnon

39 players and 13 rounds

2–6–11–4 13–10–9–8 3–5–12–7 15–19–24–17 26–23–22–21 16–18–25–20 28–32–37–30 39–36–35–34 29–31–38–33 4R R (45) {1–13, 14–26, 27–39} [1, 14, 27 out] – McKinnon

28 players and 9 rounds

<u>28</u>–1–10–19 16–4–8–14 13–27–22–6 11–12–17–24 2R 2–18–3–5 7–20–9–21 15–23–25–26 (58) {1–9, 10–18, 19–27} – Hanner

2 Stanza movements

The following two schedules consist of 3 groups of players (1 follows 6, 7 follows 12 and 13 follows 18). Board-sets 1 to 6 are in play in the first stanza and 7 to 12 in the second. Stationary players are underlined.

18 players, 12 rounds

6 rounds in each stanza. After 12 rounds calibre is 75

Stanza 1

2–11–13–17 15–5–10–16 14–9–8–18 6–7–4–3 2R [1 and 12 out] (53)

Stanza 2

8–17–1–5 3–11–16–4 2–15–14–6 12–13–10–9 2R [7 and 18 out] (53)

20 players, 12 rounds

6 rounds in each stanza. After 12 rounds calibre is 80

Stanza 1

<u>19</u>–10–2–13 16–4–6–9 8–7–14–18 1–12–17–3 11–<u>20</u>–5–15 R (55)

Stanza 2

<u>19</u>–16–8–1 4–10–12–15 14–13–2–6 7–18–5–9 17–<u>20</u>–11–3 R (55)

The following two schedules consist of 3 groups of players (1 follows 7, 8 follows 14 and 15 follows 21). Board-sets 1 to 7 are in play in the first stanza and 8 to 14 in the second. Stationary players are underlined.

21 players, 14 rounds

7 rounds in each stanza. After 14 rounds calibre is 80

Stanza 1

7–14–8–10 4–15–12–13 11–3–6–18 21–5–9–20 16–19–17–2 2R [1 out] (58)

Stanza 2

14–21–15–17 11–1–19–20 18–10–13–4 7–12–16–6 2–5–3–9 9R [8 out] (58)

24 players, 14 rounds

7 rounds in each stanza. After 14 rounds calibre is 75.

Stanza 1

19–21–10–18 13–9–5–14 1–<u>22</u>–17–11 4–3–6–16 8–<u>23</u>–2–20 <u>24</u>–15–7–12 R (46)

Stanza 2

5–7–17–4 20–16–12–21 8–<u>22</u>–3–18 11–10–13–2 15–<u>23</u>–9–6 <u>24</u>–1–14–19 R (46)

4 Groups

56 players, 14 rounds

1–2–15–16 30–45–50–56 9–11–27–6 41–43–34–35 3–25–10–7 55–44–48–38 26–23–14–20 42–54–52–53 5–21–13–4 51–46–37–32 8–19–18–28 31–49–33–36 24–12–17–22 47–40–39–29 (50) {1–14, 15–28, 29–42, 43–56} – Hanner

60 players, 15 rounds

1–31–16–46 2–37–18–56 3–34–21–52 4–45–23–47 5–41–25–57 6–38–28–48 7–44–20–53 8–35–17–58 9–42–30–49 10–33–27–54 11–39–19–59 12–36–22–50 13–32–24–60 14–43–26–55 15–40–29–51(74) {1–15, 16–30, 31–45, 46–60} 1 to 15 stationary. Note: E, W, S can rotate about N for extra rounds, giving either 30 or 45 rounds – McKinnon

5 Groups

25 players 25 rounds 5 stanzas

5 rounds in each stanza. After 25 rounds calibre is 84. Groups are {1–5, 6–10, 11–15, 16–20, 21–25}

9–6–14–18/10–22–12–11 16–3–8–5 15–25–21–19 23–17–24–13 2–7–4–20 (47) [1 out]
14–11–19–23/15–2–17–16 21–8–13–10 20–5–1–24 3–22–4–18 7–12–9–25 (47) [6 out]
19–16–24–3/20–7–22–21 1–13–18–15 25–10–6–4 8–2–9–23 12–17–14–5 (47) [11 out]
24–21–4–8/25–12–2–1 6–18–23–20 5–15–11–9 13–7–14–3 17–22–19–10 (47) [16 out]
4–1–9–13/5–17–7–6 11–23–3–25 10–20–16–14 18–12–19–8 22–2–24–15 (47) [21 out]

9.3 Irregular type

Biologically the species is the accumulation of the experiments
of all its successful individuals since the beginning.
H. G. Wells

The complete schedules need to be supplied as there are no cyclic movement of the players. The players are shown in the order N, S, E, W.

12 players and 5 rounds

Calibre 71 – Smulders

	1	2	3
1	7 3 9 5 (1)	8 6 11 12 (2)	2 10 4 1 (3)
2	10 6 4 8 (1)	2 1 5 7 (2)	9 12 11 3 (3)
3	7 10 3 6 (4)	4 5 9 8 (5)	2 12 11 1 (1)
4	5 2 11 8 (4)	12 6 7 1 (5)	3 9 4 10 (2)
5	9 4 1 12 (4)	11 2 3 10 (5)	8 5 7 6 (3)

12 players and 6 rounds

Calibre 72 – Smulders

	1	2	3
1	3 4 1 6 (1)	2 10 8 12 (2)	7 9 5 11 (3)
2	11 8 5 2 (1)	9 6 7 4 (2)	1 12 3 10 (3)
3	9 10 7 12 (1)	1 5 3 11 (2)	2 8 4 6 (3)
4	2 4 3 7 (4)	9 8 1 11 (5)	5 6 10 12 (6)
5	5 12 1 8 (4)	7 10 3 6 (5)	9 4 11 2 (6)
6	6 10 9 11 (4)	5 4 2 12 (5)	1 3 7 8 (6)

12 players and 9 rounds

Calibre 86 – Smulders

	1	2	3
1	1 4 3 6 (1)	8 12 2 10 (2)	5 11 7 9 (3)
2	11 8 5 2 (1)	9 6 7 4 (2)	1 3 10 12 (3)
3	7 10 9 12 (1)	3 11 1 5 (2)	4 6 2 8 (3)
4	7 11 3 4 (4)	1 8 9 2 (5)	5 10 6 12 (6)
5	1 12 5 8 (4)	3 10 7 6 (5)	11 2 9 4 (6)
6	2 6 9 10 (4)	5 12 11 4 (5)	3 8 1 7 (6)
7	3 7 5 9 (7)	6 8 11 12 (8)	1 10 2 4 (9)
8	4 8 6 10 (7)	1 2 5 7 (8)	9 11 3 12 (9)
9	1 11 2 12 (7)	3 9 4 10 (8)	5 6 7 8 (9)

12 players and 11 rounds

Calibre 90 – Hanner.

	1	2	3
1	1 3 4 6 (1)	8 10 2 12 (2)	9 5 7 11 (3)
2	8 2 11 5 (1)	7 4 9 6 (2)	1 10 3 12 (3)
3	7 9 10 12 (1)	1 11 3 5 (2)	2 6 8 4 (3)
4	2 4 7 3 (4)	1 9 8 11 (5)	10 5 6 12 (6)
5	1 8 5 12 (4)	10 7 6 3 (5)	4 11 2 9 (6)
6	10 6 9 11 (4)	2 5 4 12 (5)	1 7 8 3 (6)
7	3 9 5 7 (7)	6 11 8 12 (8)	2 10 1 4 (9)
8	6 8 4 10 (7)	1 5 2 7 (8)	3 11 9 12 (9)
9	10 3 7 6 (10)	8 9 5 4 (11)	1 2 12 11 (7)
10	8 5 11 2 (10)	1 6 12 7 (11)	3 4 10 9 (8)
11	9 4 12 1 (10)	10 11 3 2 (11)	5 6 8 7 (9)

13 players and 6 rounds

Calibre 63 – Smulders.

	1	2	3
1	4 1 3 6 (1)	12 2 8 10 (2)	7 5 11 9 (3)
2	5 2 8 11 (1)	9 6 4 13 (2)	10 3 12 1 (3)
3	13 12 10 7 (1)	1 3 11 5 (2)	8 2 6 4 (3)
4	4 3 2 7 (4)	1 11 9 8 (5)	13 6 10 12 (6)
5	12 8 5 1 (4)	10 13 7 6 (5)	4 9 2 11 (6)
6	9 13 6 10 (4)	2 4 5 12 (5)	7 3 8 1 (6)

14 players and 6 rounds

Calibre 58 – Smulders.

	1	2	3
1	6 3 4 1 (1)	10 8 12 2 (2)	7 5 9 11 (3)
2	5 2 8 11 (1)	13 14 9 6 (2)	12 1 10 3 (3)
3	7 14 13 12 (1)	5 11 1 3 (2)	2 8 4 6 (3)
4	4 3 2 7 (4)	14 9 11 1 (5)	12 10 6 13 (6)
5	12 8 5 1 (4)	6 7 10 13 (5)	11 14 4 9 (6)
6	10 14 9 13 (4)	2 4 12 5 (5)	3 7 1 8 (6)

16 players and 5 rounds

Calibre 41 – Smulders. Board-sets numbers equal the round numbers.

	1	2	3	4
1	1 2 3 4	5 6 7 8	9 10 11 12	13 14 15 16
2	1 5 9 13	2 11 15 8	3 16 6 12	4 14 7 10
3	1 8 10 16	6 15 9 4	11 7 13 3	14 2 12 5
4	1 6 11 14	7 16 9 2	12 8 13 4	15 3 10 5
5	1 7 12 15	8 14 9 3	10 6 13 2	16 4 11 5

16 players and 6 rounds

Calibre 79 – Smulders.

	1	2	3	4
1	1 10 13 16 (1)	9 2 15 14 (2)	11 6 7 4 (1)	5 12 3 8 (2)
2	1 15 4 12 (3)	2 16 3 11 (4)	6 8 9 13 (3)	5 7 10 14 (4)
3	1 6 3 14 (5)	13 4 5 2 (6)	15 8 11 10 (5)	9 12 7 16 (6)
4	1 4 7 10 (2)	9 8 3 2 (1)	13 6 11 16 (2)	15 12 5 14 (1)
5	1 12 9 6 (4)	5 10 11 2 (3)	15 4 13 8 (4)	7 14 3 16 (3)
6	1 14 11 8 (6)	13 2 7 12 (5)	15 10 3 6 (6)	9 16 5 4 (5)

16 players and 15 rounds

This movement has advantages when the particular qualities of the movement are needed. The boards come out of play every three rounds. There is restricted player movement with the players rotating around the fourth for three rounds. The disadvantage is that tables 2 and 3 must share boards. It was first published by the author in 1979.

After each round East, South and West rotate around the North player (as in the rainbow movement) and the boards move down one table (from 4 to the share at 2 and 3, and then to table 1 and then 4 etc). After three rounds new boards are introduced or the current boards are re-dealt. Only the starting positions for rounds 1, 4, 7, 10 and 13 are given, for the rotations follow automatically. Calibre 82.

1	1–3–2–4	5–8–7–6	9–10–11–12	13–14–16–15
4	1–9–5–13	2–14–10–6	3–7–15–11	4–8–16–12
7	1–12–6–15	7–9–14–4	10–13–8–3	16–11–2–5
10	1–10–7–16	8–9–15–2	11–13–6–4	14–12–3–5
13	1–11–8–14	6–9–16–3	12–13–7–2	15–10–4–5

There are many other irregular movements for 16 players published over the years, but the Howell type for 4 tables is generally a better choice for simplicity. For best calibre (100) without the Snow method is this one from Peter Smulders and is based on the same principles as above, using 5 triples and board sharing between tables 3 and 4.

	1	2	3	4
1	16 1 6 11 (1)	2 5 9 8 (2)	7 14 13 10 (3)	12 15 4 3 (3)
2	16 11 6 1 (2)	2 8 9 5 (3)	7 10 13 14 (1)	12 3 4 15 (1)
3	16 6 11 1 (3)	2 9 8 5 (1)	7 13 10 14 (2)	12 4 3 15 (2)
4	16 2 7 12 (4)	3 1 10 9 (5)	8 15 14 6 (6)	13 11 5 4 (6)
5	16 12 7 2 (5)	3 9 10 1 (6)	8 6 14 15 (4)	13 4 5 11 (4)
6	16 7 12 2 (6)	3 10 9 1 (4)	8 14 6 15 (5)	13 5 4 11 (5)
7	16 3 8 13 (7)	4 2 6 10 (8)	9 11 15 7 (9)	14 12 1 5 (9)
8	16 13 8 3 (8)	4 10 6 2 (9)	9 7 15 11 (7)	14 5 1 12 (7)
9	16 8 13 3 (9)	4 6 10 2 (7)	9 15 7 11 (8)	14 1 5 12 (8)
10	16 4 9 14 (10)	5 3 7 6 (11)	10 12 11 8 (12)	15 13 2 1 (12)
11	16 14 9 4 (11)	5 6 7 3 (12)	10 8 11 12 (10)	15 1 2 13 (10)
12	16 9 14 4 (12)	5 7 6 3 (10)	10 11 8 12 (11)	15 2 1 13 (11)
13	16 5 10 15 (13)	1 4 8 7 (14)	6 13 12 9 (15)	11 14 3 2 (15)
14	16 15 10 5 (14)	1 7 8 4 (15)	6 9 12 13 (13)	11 2 3 14 (13)
15	16 10 15 5 (15)	1 8 7 4 (13)	6 12 9 13 (14)	11 3 2 14 (14)

The Shomate movement

These movements are similar to the above movement for 16 players where players sitting in the East, South and West seats rotate about the North player, playing one of the 3 boards with each player. All players move and the boards remain stationary throughout the movement.

There are three boards per table and the number of partnerships is equal to the number of boards. The number of changes (i.e., the number of moves from one table to another) depends on the number of tables. Guide-cards are needed for the players to move from one change to the next, but the rotations follow as usual.

The players are divided into four groups in each schedule, while the players within each group do not meet one another. For example, in the 32-player movement the groups are 1–8, 9–16, 17–24 and 25–32. The movement can be completed by having these meet by using four 8-player movements on the same sets of boards. Similarly use 4 × 9 and 4 × 10, respectively, in 36 and 40-player movements.

In these schedules the tables and boards sets are numbered across the top and the round numbers down the side. Only every 3rd round is shown.

32 players Shomate

Calibre (91)

	1	2	3	4	5	6	7	8
1	1 9 17 25	2 10 18 26	3 11 19 27	4 12 20 28	5 13 21 29	6 14 22 30	7 15 23 31	8 16 24 32
4	4 16 22 27	3 15 21 28	2 14 24 25	1 13 23 26	8 12 18 31	7 11 17 32	6 10 20 29	5 9 19 30
7	5 11 18 28	6 12 17 27	7 9 20 26	8 10 19 25	1 15 22 32	2 16 21 31	3 13 24 30	4 14 23 29
10	2 15 20 30	1 16 19 29	4 13 18 32	3 14 17 31	6 11 24 26	5 12 23 25	8 9 22 28	7 10 21 27
13	7 12 24 29	8 11 23 30	5 10 22 31	6 9 21 32	3 16 20 25	4 15 19 26	1 14 18 27	2 13 17 28
16	21 26 14 8	22 25 13 7	23 28 16 6	24 27 15 5	17 30 10 4	18 29 9 3	19 32 12 2	20 31 11 1
19	19 31 13 6	20 32 14 5	17 29 15 8	18 30 16 7	23 27 9 2	24 28 10 1	21 25 11 4	22 26 12 3
22	23 32 10 3	24 31 9 4	21 30 12 1	22 29 11 2	19 28 14 7	20 27 13 8	17 26 16 5	18 25 15 6

36 players Shomate

Calibre (95)

	1	2	3	4	5	6	7	8	9
1	1 10 19 28	2 11 20 29	3 12 21 30	4 13 22 31	5 14 23 32	6 15 24 33	7 16 25 34	8 17 26 35	9 18 27 36
4	2 12 23 36	3 10 24 34	1 11 22 35	5 15 26 30	6 13 27 28	4 14 25 29	8 18 20 33	9 16 21 31	7 17 19 32
7	7 13 21 29	8 14 19 30	9 15 20 28	1 16 24 32	2 17 22 33	3 18 23 31	4 10 27 35	5 11 25 36	6 12 26 34
10	4 16 20 30	5 17 21 28	6 18 19 29	7 10 23 33	8 11 24 31	9 12 22 32	1 13 26 36	2 14 27 34	3 15 25 35
13	9 14 26 33	7 15 27 31	8 13 25 32	3 17 20 36	1 18 21 34	2 16 19 35	6 11 23 30	4 12 24 28	5 10 22 29
16	3 11 27 32	1 12 25 33	2 10 26 31	6 14 21 35	4 15 19 36	5 13 20 34	9 17 24 29	7 18 22 30	8 16 23 28
19	22 34 15 8	23 35 13 9	24 36 14 7	25 28 18 2	26 29 16 3	27 30 17 1	19 31 12 5	20 32 10 6	21 33 11 4
22	24 35 18 5	22 36 16 6	23 34 17 4	27 29 12 8	25 30 10 9	26 28 11 7	21 32 15 2	19 33 13 3	20 31 14 1
25	25 31 17 6	26 32 18 4	27 33 16 5	19 34 11 9	20 35 12 7	21 36 10 8	22 28 14 3	23 29 15 1	24 30 13 2

40 players Shomate

Calibre (90)

	1	2	3	4	5	6	7	8	9	10
1	1 21 11 31	2 22 12 32	3 23 13 33	4 24 14 34	5 25 15 35	6 26 16 36	7 27 17 37	8 28 18 38	9 29 19 39	10 30 20 40
4	6 23 12 34	5 24 11 33	8 21 19 32	7 22 20 31	2 26 17 30	1 25 18 29	4 28 15 39	3 27 16 40	13 37 10 36	14 38 9 35
7	9 36 3 24	10 35 4 23	15 30 1 22	16 29 2 21	13 32 7 28	14 31 8 27	19 40 5 26	20 39 6 25	11 34 17 38	12 33 18 37
10	26 33 7 39	25 34 8 40	16 31 5 37	15 32 6 38	14 29 3 12	13 30 4 11	20 23 1 9	19 24 2 10	27 35 18 21	28 36 17 22
13	22 10 29 8	21 9 30 7	24 17 35 6	23 18 36 5	27 19 33 4	28 20 34 3	25 13 38 2	26 14 37 1	40 12 31 15	39 11 32 16
16	38 27 30 5	37 28 29 6	40 14 36 7	39 13 35 8	10 16 34 1	9 15 33 2	11 18 22 3	12 17 21 4	24 26 32 20	23 25 31 19
19	35 2 40 28	36 1 39 27	29 4 38 20	30 3 37 19	31 6 9 18	32 5 10 17	24 8 12 16	23 7 11 15	33 14 22 25	34 13 21 26
22	32 4 37 25	31 3 38 26	34 2 39 18	33 1 40 17	36 8 11 20	35 7 12 19	21 6 10 14	22 5 9 13	30 16 23 28	29 15 24 27
25	13 19 14 20	17 15 18 16	25 11 26 12	9 27 10 28	37 23 38 24	21 39 22 40	29 35 30 36	33 31 34 32	1 7 2 8	5 3 6 4
28	17 15 18 16	13 19 14 20	9 27 10 28	25 11 26 12	21 39 22 40	37 23 38 24	33 31 34 32	29 35 30 36	5 3 6 4	1 7 2 8

Note players repeat play in round 10 (28) and that round can be left out of the movement.

24/25 players, 5 stanza

George W. Beynon. (92).

	1	2	3	4	5	6
1	25 1 16 6 (1)	24 5 17 7 (2)	8 18 23 4 (3)	3 22 9 19 (4)	20 10 2 21 (5)	13 12 14 11 (1)
2	18 7 3 21 (1)	25 2 19 8 (2)	24 1 20 9 (3)	10 16 23 5 (4)	4 22 6 17 (5)	14 13 15 12 (2)
3	5 22 8 20 (1)	16 9 4 21 (2)	25 3 17 10 (3)	24 2 18 6 (4)	7 19 23 1 (5)	15 14 11 13 (3)
4	9 17 23 2 (1)	1 22 10 18 (2)	19 6 5 21 (3)	25 4 20 7 (4)	24 3 16 8 (5)	11 15 12 14 (4)
5	24 4 19 10 (1)	6 20 23 3 (2)	2 22 7 16 (3)	17 8 1 21 (4)	25 5 18 9 (5)	12 11 13 15 (5)
6	25 6 11 16 (6)	21 10 12 17 (7)	18 13 24 9 (8)	8 23 19 14 (9)	15 20 7 22 (10)	3 2 4 1 (6)
7	13 17 8 22 (6)	25 7 14 18 (7)	21 6 15 19 (8)	20 11 24 10 (9)	9 23 16 12 (10)	4 3 5 2 (7)
8	10 23 18 15 (6)	11 19 9 22 (7)	25 8 12 20 (8)	21 7 13 16 (9)	17 14 24 6 (10)	5 4 1 3 (8)
9	19 12 24 7 (6)	6 23 20 13 (7)	14 16 10 22 (8)	25 9 15 17 (9)	21 8 11 18 (10)	1 5 2 4 (9)
10	21 9 14 20 (6)	16 15 24 8 (7)	7 23 17 11 (8)	12 18 6 22 (9)	25 10 13 19 (10)	2 1 3 5 (10)
11	25 11 6 1 (11)	22 15 7 2 (12)	3 8 21 14 (13)	13 23 4 9 (14)	10 5 12 24 (15)	18 17 19 16 (11)
12	8 2 13 24 (11)	25 12 9 3 (12)	22 11 10 4 (13)	5 6 21 15 (14)	14 23 1 7 (15)	19 18 20 17 (12)
13	15 23 3 10 (11)	6 4 14 24 (12)	25 13 7 5 (13)	22 12 8 1 (14)	2 9 21 11 (15)	20 19 16 18 (13)
14	4 7 21 12 (11)	11 23 5 8 (12)	9 1 15 24 (13)	25 14 10 2 (14)	22 13 6 3 (15)	16 20 17 19 (14)
15	22 14 9 5 (11)	1 10 21 13 (12)	12 23 2 6 (13)	7 3 11 24 (14)	25 15 8 4 (15)	17 16 18 20 (15)
16	25 16 1 11 (16)	23 20 2 12 (17)	13 3 21 19 (18)	18 24 14 4 (19)	5 15 17 22 (20)	8 7 9 6 (16)
17	3 12 18 22 (16)	25 17 4 13 (17)	23 16 5 14 (18)	15 1 21 20 (19)	19 24 11 2 (20)	9 8 10 7 (17)
18	20 24 13 5 (16)	1 14 19 22 (17)	25 18 2 15 (18)	23 17 3 11 (19)	12 4 21 16 (20)	10 9 6 8 (18)
19	21 17 2 14 (16)	3 15 16 24 (17)	20 22 11 4 (18)	25 19 5 12 (19)	13 1 23 18 (20)	6 10 7 9 (19)
20	4 15 19 23 (16)	18 21 11 5 (17)	1 12 17 24 (18)	16 22 13 2 (19)	25 20 3 14 (20)	7 6 8 10 (20)
21	10 12 3 19 (21)	6 11 2 18 (22)	1 17 15 7 (23)	16 5 8 14 (24)	13 9 20 4 (25)	25 21 24 22 (21)
22	18 1 9 14 (21)	17 5 10 13 (22)	6 12 4 16 (23)	7 11 3 20 (24)	8 15 2 19 (25)	25 22 23 24 (22)
23	4 17 11 8 (21)	9 15 3 16 (22)	14 10 20 2 (23)	1 19 13 6 (24)	7 12 5 18 (25)	25 23 22 21 (23)
24	16 2 7 13 (21)	12 8 20 1 (22)	11 9 19 5 (23)	4 18 15 10 (24)	17 3 6 14 (25)	25 24 21 23 (24)
25	20 5 6 15 (21)	14 7 19 4 (22)	18 3 8 13 (23)	2 17 12 9 (24)	10 11 1 16 (25)	24 21 22 23 (25)

28 players, 4 stanza

	1	2	3	4	5	6	7
1	22 1 15 8 (1)	23 2 16 9 (2)	24 3 17 10 (3)	25 4 18 11 (4)	26 5 19 12 (5)	27 6 20 13 (6)	28 7 21 14 (7)
2	28 2 20 11 (1)	22 3 21 12 (2)	23 4 15 13 (3)	24 5 16 14 (4)	25 6 17 8 (5)	26 7 18 9 (6)	27 1 19 10 (7)
3	14 18 27 3 (1)	8 19 28 4 (2)	9 20 22 5 (3)	10 21 23 6 (4)	11 15 24 7 (5)	12 16 25 1 (6)	13 17 26 2 (7)
4	10 16 26 4 (1)	11 17 27 5 (2)	12 18 28 6 (3)	13 19 22 7 (4)	14 20 23 1 (5)	8 21 24 2 (6)	9 15 25 3 (7)
5	13 21 25 5 (1)	14 15 26 6 (2)	8 16 27 7 (3)	9 17 28 1 (4)	10 18 22 2 (5)	11 19 23 3 (6)	12 20 24 4 (7)
6	24 6 19 9 (1)	25 7 20 10 (2)	26 1 21 11 (3)	27 2 15 12 (4)	28 3 16 13 (5)	22 4 17 14 (6)	23 5 18 8 (7)
7	23 7 17 12 (1)	24 1 18 13 (2)	25 2 19 14 (3)	26 3 20 8 (4)	27 4 21 9 (5)	28 5 15 10 (6)	22 6 16 11 (7)
8	22 8 1 15 (8)	23 9 2 16 (9)	24 10 3 17 (10)	25 11 4 18 (11)	26 12 5 19 (12)	27 13 6 20 (13)	28 14 7 21 (14)
9	6 18 9 28 (8)	7 19 10 22 (9)	1 20 11 23 (10)	2 21 12 24 (11)	3 15 13 25 (12)	4 16 14 26 (13)	5 17 8 27 (14)
10	4 21 10 27 (8)	5 15 11 28 (9)	6 16 12 22 (10)	7 17 13 23 (11)	1 18 14 24 (12)	2 19 8 25 (13)	3 20 9 26 (14)
11	2 17 11 26 (8)	3 18 12 27 (9)	4 19 13 28 (10)	5 20 14 22 (11)	6 21 8 23 (12)	7 15 9 24 (13)	1 16 10 25 (14)
12	25 12 7 20 (8)	26 13 1 21 (9)	27 14 2 15 (10)	28 8 3 16 (11)	22 9 4 17 (12)	23 10 5 18 (13)	24 11 6 19 (14)
13	24 13 5 16 (8)	25 14 6 17 (9)	26 8 7 18 (10)	27 9 1 19 (11)	28 10 2 20 (12)	22 11 3 21 (13)	23 12 4 15 (14)
14	3 19 14 23 (8)	4 20 8 24 (9)	5 21 9 25 (10)	6 15 10 26 (11)	7 16 11 27 (12)	1 17 12 28 (13)	2 18 13 22 (14)
15	28 16 13 4 (15)	22 17 14 5 (16)	23 18 8 6 (17)	24 19 9 7 (18)	25 20 10 1 (19)	26 21 11 2 (20)	27 15 12 3 (21)
16	27 17 11 7 (15)	28 18 12 1 (16)	22 19 13 2 (17)	23 20 14 3 (18)	24 21 8 4 (19)	25 15 9 5 (20)	26 16 10 6 (21)
17	26 18 9 3 (15)	27 19 10 4 (16)	28 20 11 5 (17)	22 21 12 6 (18)	23 15 13 7 (19)	24 16 14 1 (20)	25 17 8 2 (21)
18	6 14 25 19 (15)	7 8 26 20 (16)	1 9 27 21 (17)	2 10 28 15 (18)	3 11 22 16 (19)	4 12 23 17 (20)	5 13 24 18 (21)
19	2 12 20 24 (15)	3 13 21 25 (16)	4 14 15 26 (17)	5 8 16 27 (18)	6 9 17 28 (19)	7 10 18 22 (20)	1 11 19 23 (21)
20	23 21 10 5 (15)	24 15 11 6 (16)	25 16 12 7 (17)	26 17 13 1 (18)	27 18 14 2 (19)	28 19 8 3 (20)	22 20 9 4 (21)
21	1 6 22 26 (22)	3 4 8 12 (23)	16 19 25 27 (24)	23 24 11 13 (25)	15 20 9 10 (26)	5 2 17 18 (27)	7 14 21 28 (21) *
22	1 8 15 22 (15) *	2 7 23 27 (23)	4 5 9 13 (24)	17 20 26 28 (25)	24 25 12 14 (26)	16 21 10 11 (27)	6 3 18 19 (28)
23	7 4 19 20 (22)	2 9 16 23 (16) *	3 1 24 28 (24)	5 6 10 14 (25)	18 21 27 22 (26)	25 26 13 8 (27)	17 15 11 12 (28)
24	18 16 12 13 (22)	1 5 20 21 (23)	3 10 17 24 (17) *	4 2 25 22 (25)	6 7 11 8 (26)	19 15 28 23 (27)	26 27 14 9 (28)
25	27 28 8 10 (22)	19 17 13 14 (23)	2 6 21 15 (24)	4 11 18 25 (18)*	5 3 26 23 (26)	7 1 12 9 (27)	20 16 22 24 (28)
26	21 17 23 25 (22)	28 22 9 11 (23)	20 18 14 8 (24)	3 7 15 16 (25)	5 12 19 26 (19)*	6 4 27 24 (27)	1 2 13 10 (28)
27	2 3 14 11 (22)	15 18 24 26 (23)	22 23 10 12 (24)	21 19 8 9 (25)	4 1 16 17 (26)	6 13 20 27 (20) *	7 5 28 25 (28)
28	1 8 15 22 (22)	2 9 16 23 (23)	3 10 17 24 (24)	4 11 18 25 (25)	5 12 19 26 (26)	6 13 20 27 (27)	7 14 21 28 (28)

Note: * marks repeated meetings of players. Either the last round or those meetings are not played, depending on the number of rounds you wish to play.

9.4 Combination type

Groups are grammatical fictions; only individuals exist, and each individual is different.
Robert Anton Wilson

McKenney-Baldwin

The McKenney-Baldwin movement is the first movement in this category. It is similar to the approach used in pairs' movements of the same name. This is a 5-session movement and the field is divided into five sections.

The base is a prime number movement (without player rotation) to which is appended a smaller individual movement. The number of rounds played in each stanza is equal to the number of tables in the prime-number movement. The arrangement is very similar to that used in the combined Howell-Mitchell movement.

36-players McKenney-Baldwin

This movement consists of a 7-table prime-number schedule to which is appended an 8-player individual schedule. The highest-numbered player (number 36 in this case) usually remains in the 8-player schedule throughout as the stationary player. The other players are divided into five groups.

(A) 1 to 7; (B) 8 to 14; (C) 15 to 21; (D) 22 to 28; (E) 29 to 35.

The two tables in the 8-player schedule share boards with table 1 of the prime-number schedule throughout. Therefore at least 3 boards are required in each set, consequently at least 21 boards per session and at least 105 boards overall.

The major advantage with this movement is that no guide-cards are needed with the exception of table guide-cards in the 8-player schedule. The five groups of players occupy the five positions in rotation, a different position on each session.

In the prime-number schedule the positions are North, South, East and West and in the 8-player schedule for the fifth position. If stationary boards are to be used, the 8-player schedule shares boards with the tables in the prime-number schedule, in ascending order.

Session:	1	2	3	4	5
South-prime:	A	E	D	C	B
East-prime:	B	A	E	D	C
8 Player:	C	B	A	E	D
West-prime:	D	C	B	A	E
North-prime:	E	D	C	B	A

56-players McKenney-Baldwin

The same principles apply as in the 36-player movement. A prime table base of 11 tables is used. The 12-player schedule is appended with table 1 sharing with table 1, 2 with 2 and 3 with table 11 of the prime number schedule (assuming moving boards are being used).

The players are divided into 5 groups of 11 and the highest numbered player is stationary in the 12-player schedule. The same rotating of the groups is used as in the 36 player movement. As only two tables ever share boards only two boards per set are needed, i.e., 22 boards per session or overall 110 boards. If the boards are duplicated, then obviously the number of boards per set can be one.

52-players Mitchell-Howell

This is the second type of combination movement, and 52 is the only useful number of players to which it applies. Clearly the number of tables being 13, the prime number movement applies if 13 or 26 rounds are required. Also with 39 boards in one session, 39 partnerships can be played. If 51 rounds are required the following procedure can be applied.

1st session
A prime number movement for 13 tables is set up with the players being assigned numbers North 1–13, East 14–26, South 27–39 and West 40–52. Two boards per table are distributed. Conducting a standard prime-number movement with the South and East players swapping places after each board so that after 26 boards there have been 26 partnerships.

2nd session
The players take their seats as for the original starting positions. Only one board per table is distributed. The South and West players swap places. There is no play at the first table (see the 3rd session) and the players move on to the second round as in the prime-table movement. After 12 rounds of play the session is completed.

3rd session
Using the 13 players (13 rounds) movement given earlier, the four groups of 13 players meet each other within the groups. A new set of 13 boards is used for this session (presumably boards 14 to 26 of the set distributed in session 2, when boards 1 to 13 would have been distributed).

The 4 sit out players in this session are the same players that started as groups of four at tables 1 through 13 in session 2. They can play sets 1 to 13 from session 2 rather than sitting out.

4 Sessions with interwoven Howells

Consider the previous 52-player movement and apply the same principles to all other numbers of tables other than 6, 10 and 14. To be able to do this requires Mitchell-like movements (see *Prime*, page 254, and *non-prime*, page 256 Mitchells above) and interwoven Howell movements that cover the number of players in each of the 4 groups.

Using the Mitchell movement each group plays one of the other groups in each session, switching places for each session for a total of 3 sessions. Then in the fourth session the groups partner the players in their group using the interwoven Howell. See *prime-tables Mitchells* (page 254) above for an introduction.

20 players, 5 rounds

This is constructed from 4 × 5-player 5-round movements

> 2–5–3–4 7–10–8–9 12–15–13–14 17–20–18–19 R (46) {1–5, 6–10, 11–15, 16–20} [1, 6, 11, 16 out] Sit outs can play an extra set.

28 players, 7 rounds

> 2–7–23–27 9–13–4–5 17–20–26–28 24–25–12–14 16–21–10–11 6–3–18–19 R (31) {1–7, 8–14, 15–21, 22–28} [1, 8, 15, 22 out] Sit outs can play an extra set. – Hanner.

32 players, 7 rounds

These are constructed from 4 × 8-player 7-round movements.

> 8–1–3–4/ 5–7–6–2 R 9–16–13–14/ 10–12–15–11 R 19–20–24–17/ 22–18–21–23 R 29–30–25–32/ 31–27–26–28 (62) {1–7, 9–15, 17–23, 25–31}

If a free-standing movement, use this, as the stationary players are 29 to 32.

> 29–1–3–4/ 5–7–6–2 R 8–30–12–13/ 9–11–14–10 R 17–18–31–15/ 20–16–19–21 R 26–27–22–32/ 28–24–23–25 (62) {1–7, 8–14, 15–21, 22–28}

36 players, 9 rounds

This is constructed from 4 × 9-player 9-round movements.

> 2–3–4–7/5–9–6–8 R 12–14–16–13/11–15–18–17 R 23–27–22–25/26–24–21–20 R 36–35–34–31/ 32–30–33–29 2R (54) {1–9, 10–18, 19–27, 28–36} [1, 10, 19, 28] Sit outs can play an extra set. – Hanner

44 players, 11 rounds

> 9–11–8–5 17–22–14–18 3–4–21–20 10–6–7–2 16–19–15–13 31–24–44–38 36–43–41–40 R 33–30–29–28 26–32–25–27 42–39–35–37 (52) {1–11, 12–22, 23–33, 34–44} [1, 12, 23, 34] Sit outs can play an extra set. – Hanner.

48 players, 11 rounds

> 12–1–9–10 33–28–34–35 14–16–31–27 5–8–11–6 13–24–19–20 46–43–15–22 30–32–36–25 7–3–39–40 42–44–37–48 26–29–4–2 17–23–21–18/45–38–41–47 (56) {1–11, 13–23, 25–35, 37–47} – Hanner.

52 players, 13 rounds

> 2–6–11–4 13–10–9–8 3–5–12–7 15–19–24–17 26–23–22–21 16–18–25–20 28–32–37–30 39–36–35–34 29–31–38–33 41–45–50–43 52–49–48–47 42–44–51–46 R (38) {1–13, 14–26, 27–39, 40–52} [1, 14, 27, 40] – McKinnon

For better balance:

> 12–11–10–3 5–8–2–7 17–25–24–20 52–45–41–51 9–13–6–4 46–44–48–49 R 23–21–19–18 32–28–37–35 22–15–16–26 50–42–47–43 33–36–34–39 30–29–31–38 (63) {1–13, 14–26, 27–39, 40–52} [1, 14, 27, 40] Sit outs can play the idle set as table 7. – Hanner.

60 players, 15 rounds

> 3–10–18–21 6–12–15–2 37–42–41–38 53–58–56–60 23–17–30–29 28–20–25–27 33–44–36–35 11–7–14–13 52–59–49–51 55–54–48–57 22–26–19–24 4–9–8–5 34–40–43–45 32–39–47–50 R (57) {1–15, 16–30, 31–45, 46–60} [1, 16, 31, 46] Sit outs can play the idle set as table 15. – Hanner.

64 players, 15 rounds

> 16–1–15–12 34–38–43–45 6–7–9–11 30–19–29–22/57–55–53–52 27–28–24–18 63–60–49–64 4–8–14–5 13–3–10–2 21–26–25–23 44–41–39–47 31–32–20–17 36–35–42–37 46–40–48–33 58–51–59–54 62–56–50–61 (53) {1–15, 17–31, 33–47, 49–63} – Hanner.

9.5 Snow method

Revolutions are the periods of history when individuals count most.
Norman Mailer

Using the Snow method in an individual movement makes a lot of sense. All this requires are pre-duplicated boards and guide-cards for the players. The director needs to introduce new boards on each round. All these movements are cyclic, just as they were in the original Safford schedules. In the 1890s the individual-player events were the most common and the most popular. It is also very difficult to improve upon the original Whitfeld schedules first published in Mitchell's 1897 book. These schedules have all players partnering every other player once and playing against them twice. The wonderful consequence of these requirements is a very good calibre.

The Whitfeld schedules reproduced here have been changed so that the stationary player is the highest number and is underlined. In the originals, player one (1) was stationary as was the custom of the day. The player numbers are shown in the order N-S-E-W, and each player follows the number 1 less than theirs and number 1 follows one less than the highest. The calibre is shown in brackets and the number of rounds is one less than the number of players.

The schedules do not include those for 44 and 60 players. The one for 48 is faulty, which is remarkable given the amount of effort that would have been expended in developing these schedules well before computers. It may well have been Mitchell's mistake in proofreading or transposition, but given these schedules were included in two forms, one the original from Whitfeld and the other from Mitchell, it would appear 48 tables was simply incorrect. The fault did not make the movement unusable, because the players played one person twice, though today we choose a complete movement. At least we can detect this error very quickly today by using a computer. These schedules were checked by computer after manual data entry of the first round description shown here.

Whitfeld individual schedules

<u>8</u>–1–6–2 7–5–4–3 (100)
<u>12</u>–1–11–4 7–5–9–10 8–2–3–6 (87)
<u>16</u>–1–10–8 15–4–5–2 9–3–13–12 7–14–6–11 (100)
<u>20</u>–1–8–9 15–18–14–12 3–10–5–13 17–11–19–4 16–6–2–7 (92)
<u>24</u>–1–18–2 20–7–12–8 22–19–15–10 21–13–11–5 6–4–9–23 17–16–14–3 (93)
<u>28</u>–1–23–24 17–6–16–9 18–21–20–10 15–2–4–8 14–19–26–5 22–3–27–12 25–7–13–11 (80)
<u>32</u>–1–29–21 31–13–24–4 28–18–10–26 20–19–14–2 30–25–11–15 12–9–8–6 23–17–7–16 3–27–5–22 (100)
<u>36</u>–1–31–11 6–16–26–21 5–12–33–19 9–8–35–17 13–4–2–15 14–3–20–32 23–29–7–10 34–18–30–22 28–24–27–25 (86)
<u>40</u>–1–14–27 17–11–3–12 9–6–2–26 5–23–21–33 19–20–22–8 10–30–31–24 25–35–16–32 13–18–28–36 7–29–34–38 4–15–37–39 (91)
<u>52</u>–1–18–35 9–27–11–8 5–14–6–30 3–33–29–41 2–17–15–21 32–22–50–25 42–37–51–13 47–19–26–7 4–10–39–4 38–31–20–28 45–16–36–40 23–34–44–46 12–43–48–49 (90)
<u>56</u>–1–41–46 6–36–11–26 16–51–21–31 17–50–28–39 20–47–34–33 37–30–13–54 55–12–49–18 32–35–3–9 40–27–19–48 42–25–23–44 15–52–24–43 22–45–38–29 10–2–14–53 7–5–8–4 (61)
<u>64</u>–1–22–43 37–28–29–36 19–46–15–50 10–55–8–57 42–12–6–17 53–38–35–9 27–51–18–5 14–26–41–3 39–45–21–2 20–23–11–33 54–25–32–31 59–13–48–16 30–7–56–40 47–4–60–52 24–34–62–58 44–49–63–61 (100)

Hanner schedules

Olof Hanner has done much work in this area and produced these schedules during the 1990s. With an even

numbers of players the highest number is stationary and underlined. The player numbers are shown in the order N-S-E-W, and each player follows the number 1 less than theirs and number 1 follows one less than the highest. The calibre is shown in brackets and the number of rounds is one less than the number of players. Where there are extra players in the movement the number of players and rounds is shown thus [9] and the sit out players on round one thus {1,4}. Player one follows the highest.

8–1–3–4 5–7–6–2 (100)
[9] 2–3–4–7 5–9–6–8 {1} (67)
[10] 7–8–10–2 5–9–3–6 {1,4} (60)
[11] 9–10–6–11 4–8–5–2 {1,3,7} (51)
12–1–6–8 2–9–10–7 4–3–5–11 (82)
[13] 12–11–10–3 5–8–2–7 9–13–6–4 {1} (80)
[14] 2–6–3–8 11–10–13–5 4–7–12–14 {1,9} (65)
16–1–7–9 15–12–13–2 6–11–3–10 8–14–4–5 (100)
[17] 17–2–11–8 16–3–4–15 14–5–7–12 9–10–13–6 {1} (100)
20–1–14–18 9–4–2–19 5–16–8–15 7–6–13–10 11–17–3–12 (100)
[21] 21–9–2–5 7–14–11–13 6–16–15–19 17–18–3–8 20–12–4–10 {1} (98)
24–1–11–17 2–13–5–23 3–12–15–7 4–6–21–20 18–8–19–22 16–9–10–14 (100)
[25] 25–23–8–14 18–9–17–6 22–10–4–11 2–5–7–12 3–13–15–19 16–24–20–21 {1} (99)
28–1–15–23 10–5–25–26 14–4–8–12 16–19–24–18 11–13–20–2 9–21–22–6 3–17–7–27 (98)
[29] 29–26–3–9 14–24–4–13 6–21–20–19 23–2–15–28 5–17–22–27 7–25–18–11 10–8–12–16 {1} (100)
32–1–21–26 5–18–13–12 14–25–27–29 20–10–30–22 9–28–6–23 16–7–17–24 4–19–3–31 8–2–15–11 (100)
[33] 2–18–23–5 10–29–19–25 12–33–7–20 4–28–6–11 13–15–9–16 27–24–26–3 17–21–31–32 14–22–8–30 {1} (99)
36–1–7–23 2–29–6–21 22–28–3–27 10–13–20–33 14–15–16–11 34–8–25–32 5–17–31–35 30–12–9–19 4–18–24–26 (100)
[37] 37–35–23–30 29–11–14–3 31–17–7–19 12–34–18–15 26–2–6–16 4–10–9–25 28–8–36–32 22–27–20–21 5–13–24–33 {1} (100)
40–1–18–31 4–24–37–22 19–3–5–34 30–16–27–39 2–13–6–12 10–9–32–35 25–23–20–15 21–17–36–28 7–14–8–26 29–38–11–33 (99)
[41] 41–2–31–12 40–3–20–23 38–5–39–4 9–34–7–36 17–26–30–13 33–10–25–18 19–24–35–8 6–37–28–15 32–11–14–29 21–22–27–16 {1} (100)
44–1–33–39 10–31–29–18 15–40–34–19 36–9–8–5 41–11–3–27 25–24–22–2 13–17–7–43 21–16–30–28 14–6–32–42 4–38–20–37 23–35–26–12 (100)
[45] 27–4–18–37 32–7–11–24 40–13–41–20 15–31–26–34 16–2–10–3 23–35–8–25 9–19–6–21 29–30–14–5 36–38–42–45 17–12–43–39 22–28–33–44 {1} (99)
48–1–15–43 8–32–35–13 7–27–44–11 3–21–40–10 34–2–47–31 12–25–19–23 17–16–39–46 6–9–30–41 37–42–38–29 18–24–45–33 28–36–22–20 4–14–26–5 (100)
[49] 7–31–8–48 5–28–43–12 16–45–36–40 27–46–44–49 41–25–38–39 19–34–15–29 47–35–22–14 10–21–6–33 11–13–42–3 23–26–32–4 24–18–17–30 9–2–20–37 {1} (99)
52–1–17–35 9–33–38–36 12–40–3–28 46–26–5–27 49–30–20–10 13–48–2–11 8–23–15–29 19–31–51–6 43–32–34–47 22–21–25–4 18–14–7–41 37–44–45–50 24–16–42–39 (99)
[53] 2–12–3–35 17–18–33–15 45–8–36–13 16–7–31–34 29–44–4–52 25–53–49–22 14–38–27–19 50–10–46–24 43–39–32–0 37–26–20–6 47–30–40–28 48–41–42–9 11–5–21–23 {1} (100)
56–1–33–55 30–43–54–2 35–40–22–18 16–23–9–38 49–24–28–14 44–13–20–26 42–8–6–17 45–25–15–51 21–4–12–27 7–46–10–47 36–48–53–52 41–50–39–29 19–11–3–31 34–32–5–37 (99)
[57] 17–48–34–47 14–38–20–35 52–29–13–49 44–24–8–55 43–5–3–33 37–21–31–2 16–30–25–23 41–53–45–54 22–11–9–26 56–7–27–32 57–50–12–15 10–4–18–36 46–42–19–51 39–40–28–6 {1} (99)
60–1–30–47 5–9–23–14 26–51–39–42 22–41–34–40 53–25–31–2 36–4–32–6 12–48–38–33 29–49–10–55 46–3–43–8 35–20–37–24 57–45–56–18 27–16–13–50 7–15–59–58 21–28–11–52 17–19–54–44 (100)
[61] 2–5–3–9 17–4–33–7 13–49–25–36 10–37–19–12 23–28–45–55 48–6–34–11 21–20–41–39 16–61–31–60 58–46–54–30 59–50–56–38 14–53–27–44 26–40–51–18 35–15–8–29 57–42–52–22 43–47–24–32 {1} (100)

Two groups in parallel

The field in any individual event can be divided into two or more groups. Typically the division is only for the organization of the movement and does not make any difference to the players' perception of the event. The groups can be completely separate as found in the interwoven Howells or mixed up as in the mingled Howells.

In this implementation the separation of the groups is complete. The field is divided into two groups of equal numbers of players and the players within each group do not partner the others in their group. The most common type of event is the Professional-Amateur (Pro-Am) where the professionals partner the amateurs and never partner other professionals.

Generally the calibre is poor in these movements and players play against other players an uneven number of times, sometimes up to 5 times. The main reason for this is the incomplete nature of the movement with only half the field meeting each other. Even so, the balance within each group (Pro or Am) is excellent just like any pairs' Mitchell movement. They can be completed in a second stanza by having the two groups play in two separate movements, partnering only those players in their group.

These movements are devised by taking a stationary-board Mitchell movement for double the number of tables and use the tables in pairs to produce an individual movement. Consider a 16-player movement, 8 professionals and 8 amateurs. Using the criss-cross Mitchell for 8 tables (16 pairs), the pair numbers at tables 1 and 2 are used to define the partnerships at table 1 in the pro-am event. Then tables 3 and 4 are used for table 2 and so forth. The boards are always shared on each round.

Players 1 to 8 are one group (Pros) and 9 to 16 the other (Ams). The 4 tables are shown below with the numbers depicting N, S, E, W in that order and the board-set shown in brackets. Calibre is 40.

	1	2	3	4
1	1 9 2 10 (1)	3 11 4 12 (1)	5 13 6 14 (1)	7 15 8 16 (1)
2	8 10 3 9 (2)	2 12 5 11 (2)	4 14 7 13 (2)	6 16 1 15 (2)
3	3 15 8 12 (3)	5 9 2 14 (3)	7 11 4 16 (3)	1 13 6 10 (3)
4	6 12 5 15 (4)	8 14 7 9 (4)	2 16 1 11 (4)	4 10 3 13 (4)
5	5 16 6 11 (5)	7 10 8 13 (5)	1 12 2 15 (5)	3 14 4 9 (5)
6	4 11 7 16 (6)	6 13 1 10 (6)	8 15 3 12 (6)	2 9 5 14 (6)
7	7 14 4 13 (7)	1 16 6 15 (7)	3 10 8 9 (7)	5 12 2 11 (7)
8	2 13 1 14 (8)	4 15 3 16 (8)	6 9 5 10 (8)	8 11 7 12 (8)

The board-set numbers are redundant in this layout and the player movements match those of the criss-cross Mitchell so that it is possible to conduct this movement without guide-cards though it would be difficult to have the players do the right thing. For example, player 1 movement is down 1, from E to N at the same table and from N to E at the next lower table.

Generally guide-cards are needed as the oddly-even Mitchells are not regular. The movements shown here have the round number down the side, equal to the set number. They can be divided into stanzas if needed. The first half of the pairs are the first group and the second half the second group.

Consider 5 tables, based on the 10-table oddly-even pairs movement (calibre 61)

	1	2	3	4	5
1	1 11 4 12	10 13 8 14	9 15 2 16	3 17 6 20	7 19 5 18
2	2 14 7 17	1 16 10 15	3 11 4 19	6 18 5 12	8 20 9 13
3	3 20 6 11	2 12 7 18	10 19 5 17	1 14 8 13	9 16 4 15
4	4 18 5 20	8 17 6 16	2 13 10 14	9 12 7 11	3 15 1 19
5	5 19 1 13	9 20 3 12	8 18 6 15	10 11 4 16	2 17 7 14
6	6 12 9 14	7 15 4 20	5 16 3 13	8 19 10 17	1 18 2 11
7	7 13 8 16	4 11 2 19	1 20 9 18	5 15 3 14	10 12 6 17
8	10 16 2 15	3 18 1 17	6 14 7 20	4 13 9 19	5 11 8 12
9	9 17 10 18	6 19 5 13	7 12 8 11	2 20 1 15	4 14 3 16
10	8 15 3 19	5 14 9 11	4 17 1 12	7 16 2 18	6 13 10 20

The 12-player version is based on the pseudo Graeco-Latin square and has calibre 49.

	1	2	3
1	1 7 2 8	3 9 4 10	5 11 6 12
2	2 9 4 11	5 8 1 12	6 7 3 10
3	6 8 1 10	4 7 5 9	3 12 2 11
4	4 12 6 9	1 11 3 8	2 10 5 7
5	3 11 5 12	6 10 2 7	1 8 4 9
6	5 10 3 7	2 12 6 11	1 9 4 8

The 24-player has calibre 38.

	1	2	3	4	5	6
1	1 13 2 14	3 15 4 16	5 17 6 18	7 19 8 20	9 21 10 22	11 23 12 24
2	12 14 3 13	2 16 5 15	4 18 7 17	6 20 9 19	8 22 11 21	10 24 1 23
3	3 23 12 16	5 13 2 18	7 15 4 20	9 17 6 22	11 19 8 24	1 21 10 14
4	10 16 5 23	12 18 7 13	2 20 9 15	4 22 11 17	6 24 1 19	8 14 3 21
5	5 21 10 18	7 23 12 20	9 13 2 22	11 15 4 24	1 17 6 14	3 19 8 16
6	8 18 7 21	10 20 9 23	12 22 11 13	2 24 1 15	4 14 3 17	6 16 5 19
7	7 24 8 15	9 14 10 17	11 16 12 19	1 18 2 21	3 20 4 23	5 22 6 13
8	6 15 9 24	8 17 11 14	10 19 1 16	12 21 3 18	2 23 5 20	4 13 7 22
9	9 22 6 17	11 24 8 19	1 14 10 21	3 16 12 23	5 18 2 13	7 20 4 15
10	4 17 11 22	6 19 1 24	8 21 3 14	10 23 5 16	12 13 7 18	2 15 9 20
11	11 20 4 19	1 22 6 21	3 24 8 23	5 14 10 13	7 16 12 15	9 18 2 17
12	2 19 1 20	4 21 3 22	6 23 5 24	8 13 7 14	10 15 9 16	12 17 11 18

The 32-player has Calibre 36. 32 players

	1	2	3	4	5	6	7	8
1	1 17 2 18	3 19 4 20	5 21 6 22	7 23 8 24	9 25 10 26	11 27 12 28	13 29 14 30	15 31 16 32
2	16 18 3 17	2 20 5 19	4 22 7 21	6 24 9 23	8 26 11 25	10 28 13 27	12 30 15 29	14 32 1 31
3	3 31 16 20	5 17 2 22	7 19 4 24	9 21 6 26	11 23 8 28	13 25 10 30	15 27 12 32	1 29 14 18
4	14 20 5 31	16 22 7 17	2 24 9 19	4 26 11 21	6 28 13 23	8 30 15 25	10 32 1 27	12 18 3 29
5	5 29 14 22	7 31 16 24	9 17 2 26	11 19 4 28	13 21 6 30	15 23 8 32	1 25 10 18	3 27 12 20
6	12 22 7 29	14 24 9 31	16 26 11 17	2 28 13 19	4 30 15 21	6 32 1 23	8 18 3 25	10 20 5 27
7	7 27 12 24	9 29 14 26	11 31 16 28	13 17 2 30	15 19 4 32	1 21 6 18	3 23 8 20	5 25 10 22
8	10 24 9 27	12 26 11 29	14 28 13 31	16 30 15 17	2 32 1 19	4 18 3 21	6 20 5 23	8 22 7 25
9	9 32 10 19	11 18 12 21	13 20 14 23	15 22 16 25	1 24 2 27	3 26 4 29	5 28 6 31	7 30 8 17
10	8 19 11 32	10 21 13 18	12 23 15 20	14 25 1 22	16 27 3 24	2 29 5 26	4 31 7 28	6 17 9 30
11	11 30 8 21	13 32 10 23	15 18 12 25	1 20 14 27	3 22 16 29	5 24 2 31	7 26 4 17	9 28 6 19
12	6 21 13 30	8 23 15 32	10 25 1 18	12 27 3 20	14 29 5 22	16 31 7 24	2 17 9 26	4 19 11 28
13	13 28 6 23	15 30 8 25	1 32 10 27	3 18 12 29	5 20 14 31	7 22 16 17	9 24 2 19	11 26 4 21
14	4 23 15 28	6 25 1 30	8 27 3 32	10 29 5 18	12 31 7 20	14 17 9 22	16 19 11 24	2 21 13 26
15	15 26 4 25	1 28 6 27	3 30 8 29	5 32 10 31	7 18 12 17	9 20 14 19	11 22 16 21	13 24 2 23
16	2 25 1 26	4 27 3 28	6 29 5 30	8 31 7 32	10 17 9 18	12 19 11 20	14 21 13 22	16 23 15 24

For a similar idea take a look back at *Safford Arrangement for Married Couple Pivot* from the 1920s. In that case we start with 8 married couples and men play with women, but never with their spouse. The movements shown above can be used in this manner by seating the non-playing partnerships (spouses) for round one, then move from there without playing the first round.

9 players and 9 rounds

The following movement is supplied as it has a calibre of 100, but it requires guide-cards – Smulders.

The boards are played in sequence and shared between the tables (set 1 round 1, set 2 round 2 etc). The player movement is generally irregular, but it is regular in each set of three rounds. Each player plays once with and twice against all others.

	1	2
1	2 3 4 7	5 9 6 8
2	3 1 5 8	6 7 4 9
3	1 2 6 9	4 8 5 7
4	5 6 7 1	8 3 9 2
5	6 4 8 2	9 1 7 3
6	4 5 9 3	7 2 8 1
7	8 9 1 4	2 6 3 5
8	9 7 2 5	3 4 1 6
9	7 8 3 6	1 5 2 4

Part 10 Scissor schedules

1932 par point world championship boards from Olympic.
Boards are made from paper and cardboard.
Cards are of the same era..

Art deco boards from ‘Bridgette’.
The cut-out star shows the vulnerability and the arrow the dealer.
The cards are original.

In 1975 Mullins and Wallis published the paper *The existence of Room squa*res. This started all the investigation into production of modern Howell movements and other variations that were used in bridge schedules. My book, *Bridge Directing Complete* (1979), used much of this mathematics.

In 1984 Stinson and Wallis published their paper *An even sided analogue of Room squares,* which defined the existence of the *House square.* On the surface this did not appear to do very much for bridge schedules. In *Bridge Directing Complete* I introduced the concept of the round-robin draw for odd numbers of teams. It was a very brief item and only covered a few cases. In revisiting this subject in the past few years I found the need for the *House Square* to extend that subject to cover all the cases. In the Stinson-Wallis paper they also cover the starter-adders needed for bicyclic schedules. Their paper proved the existence of house squares of order n where $n > 2$.

In 2006 Gennian Ge, Esther Lamken and Alan Ling published a paper called *Scheduling CCRR tournaments*. This paper used the building blocks of Stinson and Wallis to define and prove the existence of schedules for all numbers of teams. As with all pure mathematics this proof verges on the ridiculous when it comes to the real world of bridge schedules. In reality the number of cases we need are very small.

In one sense these new schedules are revolutionary, for they supply bridge players with a new and exciting event. In another way they are evolutionary for they have evolved from an existing concept of cyclic player movement first introduced by Safford in 1891.

The scoring methods are the same as we currently use, so one might ask 'what is so revolutionary?' These schedules supply us with an event that is conducted very much like any other pairs' event, but can be scored as pairs or as teams or as ***both*** pairs and teams. The results can be posted with one winning team and/or one winning pair.

Consider carefully how the American whist league movement works. Each team plays all the other teams in the movement. While doing this each pair meet one, and only one, pair of each of the other teams. Expressed another way, all pairs play only half the pairs in the opposing teams. In addition they do not play one board-set. Whist and bridge players have been happy with this for more than 100 years.

Compare this with a Howell movement for a pairs' event. Each pair plays every other pair in the movement while playing every board-set.

This new event is a combination of these two types of schedules, conducted like any other Howell movement but with one round less than the full quota. Essentially this is a ¾ (reduced) Howell where one pair is missed (their team mates) and for an even number of teams the number of board-sets is the same as the number of rounds. All pairs play against all the pairs in the other teams and play all the boards. Each match consists of two boards-sets, one set played against one pair in the opposing team and the other against the second pair. Each pair plays half the rounds as N-S and half as E-W, though this is not possible for each match.

The big difference in the schedule when compared to any Howell is the use of two cycles for some numbers of tables. These are called bicyclic schedules. Each cycle forms a stanza of the movement, the first cycle operates for the first half of the rounds and the second for the second half. A new set of table guide-cards is needed after half the rounds and the players take up a new starting position. Only one set-of-boards is in play for the whole event and is also divided into two lots. The first half of the boards cycle through the first half of the tables and the second half through the second half of the tables.

Once the pure mathematics was available it was possible to deliver a wide variety of movements to the bridge players. As with all mathematical problems, the pure mathematicians need to vigorously pursue all the possibilities, define all the limitations and deliver the mathematical results in a precise format. For the bridge organizers we simply need the useful cases and the most flexible forms for conducting the

tournament. Even with this pure mathematics available it took more than one year to develop the movements using hundreds of hours of computer time.

These schedules are the first for bridge that required the research of the pure mathematicians before we could run such an event. Previously all movements were born out of need. See *The first duplicate* (page 18).

More importantly these schedules have limitations that prior to the use of computers in managing bridge tournaments, such events would have been almost impossible to conduct. Table and player guide-cards are not that unusual, but are nearly always essential in these cases. Pre-duplicated boards are another essential. Before the advent of dealing machines these movements would be hard to organize apart from the simplest of cases. In addition the use of computers to score these events makes it possible to score both as teams and pairs with the push of a button.

Also it was essential to have a computer to develop these movements. Using computer programs written by me and using many hundreds of hours of computer processing time I was able to tease out the essential movements from the schedules defined by the pure mathematicians.

It is made clear at the outset that this book is about schedules, particularly for bridge. References to scoring methods in bridge are only mentioned to support discussions on schedules and no in-depth study is made on scoring. That said, scoring is an intrinsic part of bridge and the subject is inherent in the discussions on balance in pairs' schedules.

Balance in bridge movements is a pet subject of Ross Moore, therefore when Ross was introduced to these scissor schedules he immediately raised the subject of balance. My attitude towards the pairs scoring in scissor schedules is it is for the purpose of comparing the contributions of the pairs within each team. This gives us a method of measuring the bragging rights. If the primary requirement of the event is a pairs' scored event, then clearly a pairs' movements should be used.

When playing in a teams' event the players adopt an attitude and strategy that is optimum for a teams event, normally IMP scored with a possibility of converting the result to a Victory Point result. On the other hand there are a wide variety of scoring methods available to a pairs' event, the most popular being match-point. The tactics used in a match-point event are quite different from those used in teams. Therefore it is highly recommended that a teams scoring method is adopted in the pairs side of an event where a scissor movement is used.

Whatever the scoring method used in the pairs' event it is important to understand that the balance of the scissor schedules are not so good for pairs. This is quite clear when you realize that the two pairs in each team are always on opposite sides, and that they are never directly compared on any boards. This introduces a significant bias and there is a tendency for both pairs in a team to be doing similarly well.

In practice, if the team-mates are doing well (getting near-top results on most boards) then this is an extra match-point, or some more IMPs, on each of those boards where this favorable comparison occurs. In other words the teams that do well in the teams' scoring are also very likely to do well in the pairs' scoring.

If you want to produce results that are more than bragging-points within each team then Ross Moore recommends you use a method that compensates for this imbalance.

Ross suggests the correct way to score this is to leave out any comparisons with the team-mate's table.

For example for pairs' match-point scoring we would:

1. calculate the match-points (*MP*) in the normal way for each pair;
2. calculate the point-a-board (*PB*) score for each team;
3. calculate the difference *MP* – *PB* as each pair's final match-point score.

Similarly, an IMP score would be:

1. calculate the cross-IMP total (*XI*) for each pair;
2. calculate the IMP score for each team (*TI*);
3. calculate the difference *XI – TI* as each pair's final IMP score.

These methods still gives the bragging-points for pairs in each team, as well as the more balanced pairs' results. For the two pairs in a team, it does not matter whether you look at the match-points as being *MP – PB* or just *MP*, since the subtraction of *PB* is the same for each pair in the team. Similarly when using IMP scoring.

As for the pairs from different teams, *MP – PB* is surely a better score to use, as it removes the contribution from the teammates. *MP – PB*/2 could even be better (averaging the contribution of teammates) but this is probably an unnecessary refinement.

In summary, in all but low numbers of teams, calibre is found by eliminating the comparisons with team mates and the results for pairs' scored events are of a high quality. The calibre shown below in all the scissor schedules are without including team mate comparisons.

10.1 Complete coupling round-robin schedules

Mathematics is as old as Man.
Stefan Banach

In 2006 three pure mathematicians Gennian Ge, Esther Lamken and Alan Ling published a paper called *Scheduling CCRR tournaments*. In the introduction they say *"In this paper, we construct CCRRS, complete coupling round-robin schedules, for n teams each consisting of two pairs. The motivation for these schedules is a problem in scheduling bridge tournaments."*

It was from this acronym 'CCRRS' that I chose the name ***Scissor schedules*** as a way of pronunciation.

This mathematical study is not aimed at the faint-hearted. The paper goes on to say "We construct CCRRS(n) for n a positive integer, $n > 3$, with the possible exceptions of $n \in \{54, 62\}$." The valid claim is they have proved there are movements available for all numbers of teams (maybe two exceptions) with the desirable qualities:

(i) every pair must play against every other pair not on its team exactly once;
(ii) every pair must play one (half) match at every round;
(iii) every pair must play every board exactly once except for odd n each team skips two boards;
(iv) if pair i opposes pair j on a board, then the teammate of j must oppose the teammate of i on the same board;
(v) every board is played in at most one match at a round.

Notes.

- The reference to board is one board though in practice we would use several boards in each round.
- Quality (iv) says pairs of each team must play each pair of every other team but there is no distinction to the direction that the pairs must sit. This is illustrated in the Howell design for 6 teams below.
- Quality (ii) originally did not have 'half', but was referring to the matching of the pairs, not the teams.
- The extent of the paper is complete for all generalities of the problem, far in excess of the needs of bridge players. It does not cover 54 and 63 teams but I am sure a mathematician one day will find the enthusiasm to pursue this problem even though there is no interest from bridge players. These movements would involve 106 and 122 rounds respectively in each session.

The schedules are based on *House squares, Room squares* and *Howell designs*, particularly the work of Wal Wallis. To all bridge players the movements appear very much like any Howell movement with cyclic player-movement, but with a difference. For even numbers of tables the movements are based on bicyclic schedules, the first cycle of the two giving the first half of the session and the second cycle the second half. The total number of rounds in the event is one less than a full Howell with the same number of tables, but the same number of rounds is played in each half session.

Consider the movement for 6 teams. It is based on the *Howell design H*(10, 12) supplied in the CCRRS paper. For the original Howell design see Appendix N, page 419. Also see appendix N for producing barometer teams and pairs event from Howell designs.

The slightly modified version is shown here. Note the symbols shown here represent 6 teams. The stationary-team pairs (or the missing pairs if this schedule is used for 5 teams) are shown as ‘a’ and ‘b’ and the first pairs of each team are 1 through 5 and the second pairs are shown as 1' through 5'. It is important to note this Howell design contains switching of some pairs, and so does not exactly match the original Howell design shown in appendix N.

		a,5	2,4'	4,2'		1,3	5',b		3',1'
5,3'			a,1	3,5'	4',2'		2,4	1',b	
4,1'	1,4'			a,2		5',3'		3,5	2',b
a,3	5,2'	2,5'			3',b		1',4'		4,1
	a,4	1,3'	3,1'		5,2	4',b		2',5'	
4',5'	2,3'			b,1	1',a	5,4	3,2'		
b,2	5',1'	3,4'				2',a	1,5	4,3'	
	b,3	1',2'	4,5'				3',a	2,1	5,4'
		b,4	2',3'	5,1'	1,5'			4',a	3,2
1,2'			b,5	3',4'	4,3	2,1'			5',a

This transforms into a movement for 6 teams with tables 1 to 6 shown across the top and rounds 1 to 10 down the side:

	1	2	3	4	5	6
1	5–8 (2)	4–6 (3)	12–3 (4)	9–10 (6)	11–2 (7)	1–7 (10)
2	1–9 (3)	5–7 (4)	12–4 (5)	10–6 (7)	11–3 (8)	2–8 (6)
3	2–10 (4)	1–8 (5)	12–5 (1)	6–7 (8)	11–4 (9)	3–9 (7)
4	3–6 (5)	2–9 (1)	12–1 (2)	7–8 (9)	11–5 (10)	4–10 (8)
5	4–7 (1)	3–10 (2)	12–2 (3)	8–9 (10)	11–1 (6)	5–6 (9)
6	9–7 (2)	8–11 (4)	5–2 (5)	6–12 (6)	1–10 (9)	4–3 (10)
7	10–8 (3)	9–11 (5)	1–3 (1)	7–12 (7)	2–6 (10)	5–4 (6)
8	6–9 (4)	10–11 (1)	2–4 (2)	8–12 (8)	3–7 (6)	1–5 (7)
9	7–10 (5)	6–11 (2)	3–5 (3)	9–12 (9)	4–8 (7)	2–1 (8)
10	8–6 (1)	7–11 (3)	4–1 (4)	10–12 (10)	5–9 (8)	3–2 (9)

The field is divided into three groups, pairs 1 to 5 (originally 1 to 5), 6 to 10 (1' to 5') and 11 and 12 (‘a’ and ‘b’). Pairs 11 and 12 form one team and are stationary throughout. Pair 1 follows 5, pair 6 follows 10 and

all other pairs follow the pair one less than their number. The teams are made up from two pairs, one pair from 1 to 5 and the other from 6 to 10. Pairs 1 and 6 are team mates, as are 2 and 7, 3 and 8, and so forth. For example, pairs 5 and 8 meet on round 1 playing board-set 2, while their team mates 3 and 10 meet on round 5 at table 2. Later on in round 7 pairs 8 and 10 meet at table 1 playing set 3 while their team mates 3 and 5 meet at table 3 on round 9 also playing set 3. This complete match is scored using the results of their encounters from sets 2 and 3.

The boards are also divided into two groups and remain in their half of the movement unlike the pairs who can and do play at any table. Boards 1 to 5 are cycled in tables 1 to 3 and boards 6 to 10 cycle through tables 4 to 6. Note that the board groups have the same numbers as the pair groups. No pair will play against their team mates. The number of rounds equals the highest moving pair number (10 in this example).

After the halfway round (5 in this case) the director must establish a new starting position for all pairs and boards. We call this stanza two. Once set up the second cycle proceeds in the same way as the first.

This 6-team movement also works for 5 teams. The problem is each pair must sit out for two rounds, missing 2 board-sets. The stationary pairs (11 and 12 in this example) are the bye rounds in this 6-table example. For the first half, tables 3 and 5 are byes and in the second half, tables 2 and 4 are byes. There are better solutions for odd numbers of teams based on cyclic schedules, see page 295.

10.2 Even numbers of teams

Our whole life is solving puzzles
Erno Rubik

Two teams

First we must consider the simplest case of 2 teams. This type of event is trivial and each board-set is involved in only one match, therefore scoring it as a pairs' event is practically pointless. The teams' event winners are the same as the pairs' event winners. Even so, Ross Moore correctly felt we need to consider this case to complete the subject.

The 2 teams play each other normally, but in 2 sessions they just swap opponent pairs for the 2nd session. Socially, each pair faces each opposing pair equally, hence compares with each opponent pair equally.

The result of the pairs comparisons, as match-points, is just counting the number of boards each team wins; hence the point-a-board score. Both pairs in the winning team for this way of counting become equal winners. Alternatively, using IMP scoring the pairs winners are just the same as the teams winners.

To study the quality of this movement, each pair will have comparisons: 0 with oneself (not relevant); –2 with team-mates; 2 with opposing pair; 2 with other opposing pair.

The "quality" is $Q = 300/7$, approximately 43. Since this is the only possible movement of this kind, Ross suggests the calibre must be, by definition, 100. This claim is further supported by the elimination of the comparison with their team mates, where the balance is then seen to be perfect.

Note it is critical to the balance of these schedules that the teams' matches are played over equal numbers of boards. If for some reason the second session plays a different number of boards from the first session, then the balance will be effected. This also applies to all other scissor schedules.

Four teams

Now consider the simplest useful case, 4 teams. The schedule for 4 teams has similar problems to three teams. This does not come as a bicyclic schedule like other even numbers of teams so the full array must be supplied.

The best we can do is keep the boards stationary for the most part and if you choose to use 6 tables with the same board-set number as the table number then the movement can be simple enough to conduct with player guide-cards. The two pairs making up the four teams are 1&4, 2&5, 3&6 and 7&8. Calibre 70.

	1	2	3	4
1	1–2 (1)	6–7 (2)	5–8 (3)	4–3 (4)
2	5–4 (1)	8–3 (2)	7–2 (3)	6–1 (4)
3	7–3 (1)	2–4 (2)	6–5 (5)	8–1 (6)
4	6–8 (1)	1–5 (2)	2–3 (5)	4–7 (6)
5	6–4 (3)	8–2 (4)	7–1 (5)	3–5 (6)
6	1–3 (3)	5–7 (4)	4–8 (5)	2–6 (6)

As four teams cannot be conducted with a bicyclic schedule the following example uses a different team/pair map:

Team/pair map

1	2	3	4
1,5	2,6	3,7	4,8

	1	2	3	4
1	1–2 (1)	7–4 (2)	6–8 (3)	5–3 (4)
2	6–5 (1)	8–3 (2)	4–2 (3)	7–1 (4)
3	4–3 (1)	2–5 (2)	7–6 (5)	8–1 (6)
4	7–8 (1)	1–6 (2)	2–3 (5)	5–4 (6)
5	7–5 (3)	8–2 (4)	4–1 (5)	3–6 (6)
6	1–3 (3)	6–4 (4)	5–8 (5)	2–7 (6)

6 teams

The rest of the bicyclic movements for even numbers of teams are described in brief using the usual Howell method, but does require two starting schedules, the second marked *. The number of teams are shown as a pair like [5,6], indicating that it is possible to use these schedules for either 5 or 6 teams, though the odd numbers can be handled better – see page 295. For the odd numbers the stationary pairs are the sit out. The calibre is supplied as usual, but these movements are not balanced for pairs using switching, for to do so would alter the coupling of the pairs in the teams. Even so these can be scored both as teams and pairs and the results published with both the rank of the teams and the rank of the pairs. For best quality pairs' movements see *pair's bicyclic schedules* below, page 337.

> **Note:** Some numbers of tables cannot be accommodated with bicyclic schedules. The first example is for 15/16 teams. These numbers of teams require movement schedules based on the 4-teams non-cyclic schedule above. This only applies to the odd numbers divisible by 3 and 5, namely 15/16, 21/22, 25/26, 27/28 etc.

This first example for 6 tables is shown above in 10.1 (page 288) fully expanded.

[5,6] R 5–8 4–6 <u>12</u>–3 R 9–10 <u>11</u>–2 2R 1–7
* R 9–7 R 8–<u>11</u> 5–2 6–<u>12</u> 2R 1–10 4–3 (67) {1–5, 6–10}

At this point I also introduce the way these schedules are generally shown from now on. The half-board-set cycles are 1 to 5 and 6 to 10.

The following map shows the 6 teams and their pair numbers.

1	2	3	4	5	6
1,6	2,7	3,8	4,9	5,10	11,12

Rounds 1 to 5

1	2	3	4	5	6
5–8 (2)	4–6 (3)	12–3 (4)	9–10 (6)	11–2 (7)	1–7 (10)

Rounds 6 to 10

1	2	3	4	5	6
9–7 (2)	8–11 (4)	5–2 (5)	6–12 (6)	1–10 (9)	4–3 (10)

8 teams

[7,8] 16–1 2R 12–7 R 10–4 2–6 9–11 R 14–8 13–3 15–5 2R
* 8–15 2R 14–5 R 11–3 13–9 4–2 R 1–7 10–6 12–16 2R (79) {1–7, 8–14}

The half-board-sets cycles are 1 to 7 and 8 to 14.

The following map shows the 8 teams and their pair numbers.

1	2	3	4	5	6	7	8
1,8	2,9	3,10	4,11	5,12	6,13	7,14	15,16

Rounds 1 to 7

1	2	3	4	5	6	7	8
16–1 (1)	12–7 (4)	10–4 (6)	2–6 (7)	9–11 (8)	14–8 (10)	13–3 (11)	15–5 (12)

Rounds 8 to 14

1	2	3	4	5	6	7	8
8–15 (1)	14–5 (4)	11–3 (6)	13–9 (7)	4–2 (8)	1–7 (10)	10–6 (11)	12–16 (12)

10 teams

[9,10] 2R 16–18 6–14 10–4 7–17 R 20–8 R 3–2 12–13 4R 5–1 11–15 19–9
* 17–4 2R 12–9 R 3–1 R 11–6 18–19 14–7 10–16 3R 15–20 13–2 R 5–8 (87) {1–9, 10–18}

The half-board-sets cycles are 1 to 9 and 10 to 18.

The following map shows the 10 teams and their pair numbers.

1	2	3	4	5	6	7	8	9	10
1,10	2,11	3,12	4,13	5,14	6,15	7,16	8,17	9,18	19,20

Rounds 1 to 9

1	2	3	4	5	6	7	8	9	10
16–18 (3)	6–14 (4)	10–4 (5)	7–17 (6)	20–8 (8)	3–2 (10)	12–13 (11)	5–1 (16)	11–15 (17)	19–9 (18)

Rounds 10 to 18

1	2	3	4	5	6	7	8	9	10
17–4 (1)	12–9 (4)	3–1 (6)	11–6 (8)	18–19 (9)	14–7 (10)	10–16 (11)	15–20 (15)	13–2 (16)	5–8 (18)

12 teams

[11,12] 24–1 9–16 R 3–13 6–11 8–21 3R 7–15 2R 12–22 4–20 17–19 3R 5–2 R 23–10 18–14
* 12–23 5–20 R 2–14 22–17 10–19 3R 4–18 2R 11–1 9–15 8–6 3R 13–16 R 21–24 3–7 (79) {1–11, 12–22}

The half-board-sets cycles are 1 to 11 and 12 to 22.

The following map shows the 12 teams and their pair numbers.

1	2	3	4	5	6	7	8	9	10	11	12
1,12	2,13	3,14	4,15	5,16	6,17	7,18	8,19	9,20	10,21	11,22	23,24

Rounds 1 to 11

1	2	3	4	5	6	7	8	9	10	11	12
24–1 (1)	9–16 (2)	3–13 (4)	6–11 (5)	8–21 (6)	7–15 (10)	12–22 (13)	4–20 (14)	17–19 (15)	5–2 (19)	23–10 (21)	18–14 (22)

Rounds 12 to 22

1	2	3	4	5	6	7	8	9	10	11	12
12–23 (1)	5–20 (2)	2–14 (4)	22–17 (5)	10–19 (6)	4–18 (10)	11– 1 (13)	9–15 (14)	8– 6 (15)	13–16 (19)	21–24 (21)	3– 7 (22)

14 teams

[13,14] R 16–6 19–2 28–4 14–15 2R 25–9 R 22–13 2R 5–7 17–26 24–12 R 21–18 2R 8–11 23–3 R 27–10 R 20–1 R
* R 4–10 R 1–13 11–16 R 5–26 R 22–27 18–25 R 19–17 R 14–28 R 2–6 24–9 23–12 3R 20–15 3–8 21–7 2R (86) {1–13, 14–26}

The half-board-sets cycles are 1 to 13 and 14 to 26.

The following map shows the 14 teams and their pair numbers.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1,14	2,15	3,16	4,17	5,18	6,19	7,20	8,21	9,22	10,23	11,24	12,25	13,26	27,28

Rounds 1 to 13

1	2	3	4	5	6	7	8	9	10	11	12	13	14
16–6 (2)	19–2 (3)	28–4 (4)	14–15 (5)	25–9 (8)	22–13 (10)	5–7 (13)	17–26 (14)	24–12 (15)	21–18 (17)	8–11 (20)	23–3 (21)	27–10 (23)	20–1 (25)

Rounds 14 to 26

1	2	3	4	5	6	7	8	9	10	11	12	13	14
4–10 (2)	1–13 (4)	11–16 (5)	5–26 (7)	22–27 (9)	18–25 (10)	19–17 (12)	14–28 (14)	2–6 (16)	24–9 (17)	23–12 (18)	20–15 (22)	3–8 (23)	21–7 (24)

18 teams

[17,18] 36–1 24–32 7–9 33–20 R 26–15 14–4 2R 18–21 R 27–5 2R 25–13 3R 35–2 R 31–19 2R 29–3 11–17 R 12–28 23–10 2R 22–8 16–34 6–30 R
* 18–35 33–26 16–8 2R 5–1 2R 28–27 R 12–13 6–3 R 23–17 R 22–20 2R 19–36 31–11 25–10 4R 7–2 R 34–15 21–32 R 30–9 R 29–4 14–24 (93) {1–17, 18–34}

The half-board-sets cycles are 1 to 17 and 18 to 34.

The following map shows the 18 teams and their pair numbers.

1	2	3	4	5	6	7	8	9
1,18	2,19	3,20	4,21	5,22	6,23	7,24	8,25	9,26

10	11	12	13	14	15	16	17	18
10,27	11,28	12,29	13,30	14,31	15,32	16,33	17,34	35,36

Rounds 1 to 17

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
36–1 (1)	24–32 (2)	7–9 (3)	33–20 (4)	26–15 (6)	14–4 (7)	18–21 (10)	27–5 (12)	25–13 (15)	35–2 (19)	31–19 (21)	29–3 (24)	11–17 (25)	12–28 (27)	23–10 (28)	22–8 (31)	16–34 (32)	6–30 (33)

Rounds 18 to 34

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
18–35 (1)	33–26 (2)	16– 8 (3)	5– 1 (6)	28–27 (9)	12–13 (11)	6– 3 (12)	23–17 (14)	22–20 (16)	19–36 (19)	31–11 (20)	25–10 (21)	7– 2 (26)	34–15 (28)	21–32 (29)	30– 9 (31)	29– 4 (33)	14–24 (34)

20 teams

[19,20] 40–1 R 12–30 8–35 4R 7–22 R 18–13 R 29–4 25–9 21–33 36–19 2R 5–15 24–32 4R 23–38 R 34–10 R 26–20 3–6 37–11 14–16 2R 2–31 39–17 R 28–27
* 20–39 R 11–31 16–27 4R 3–26 R 32–37 R 23–10 28–6 14–2 38–17 2R 34–24 13–5 4R 19–4 R 29–15 R 1–7 25–22 30–18 35–33 2R 12–21 36–40 R 8–9 (90) {1–19, 20–38}

The half-board-sets cycles are 1 to 19 and 20 to 38.

The following map shows the 20 teams and their pair numbers.

1	2	3	4	5	6	7	8	9	10
1,20	2,21	3,22	4,23	5,24	6,25	7,26	8,27	9,28	10,29

11	12	13	14	15	16	17	18	19	20
11,30	12,31	13,32	14,33	15,34	16,35	17,36	18,37	19,38	39,40

Rounds 1 to 19

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
40–1 (1)	12–30 (3)	8–35 (4)	7–22 (9)	18–13 (11)	29–4 (13)	25–9 (14)	21–33 (15)	36–19 (16)	5–15 (19)	24–32 (20)	23–38 (25)	34–10 (27)	26–20 (29)	3–6 (30)	37–11 (31)	14–16 (32)	2–31 (35)	39–17 (36)	28–27 (38)

Rounds 20 to 38

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20–39 (1)	11–31 (3)	16–27 (4)	3–26 (9)	32–37 (11)	23–10 (13)	28–6 (14)	14–2 (15)	38–17 (16)	34–24 (19)	13–5 (20)	19–4 (25)	29–15 (27)	1–7 (29)	25–22 (30)	30–18 (31)	35–33 (32)	12–21 (35)	36–40 (36)	8–9 (38)

24 teams

[23,24] 48–1 4R 9–44 R 26–29 2R 40–18 37–45 2R 5–34 25–15 R 42–46 R 13–31 10–12 30–16 4–20 27–39 R 21–24 2R 35–36 32–17 2R 23–6 43–33 R 14–41 R 8–3 28–7 2–11 22–38 47–19 4R
* 24–47 4R 21–32 R 6–3 2R 41–17 22–14 2R 11–28 38–2 R 23–19 R 8–36 35–33 39–7 43–27 16–4 R 1–44 2R 13–12 40–9 2R 29–46 10–20 R 18–37 R 26–31 30–5 34–25 15–45 42–48 4R (92) {1–23, 24–46}

The half-board-sets cycles are 1 to 23 and 24 to 46.

The following map shows the 24 teams and their pair numbers.

1	2	3	4	5	6	7	8	9	10	11	12
1,24	2,25	3,26	4,27	5,28	6,29	7,30	8,31	9,32	10,33	11,34	12,35

13	14	15	16	17	18	19	20	21	22	23	24
13,36	14,37	15,38	16,39	17,40	18,41	19,42	20,43	21,44	22,45	23,46	47,48

Rounds 1 to 23

1	2	3	4	5	6	7	8	9	10	11	12
48–1 (1)	9–44 (6)	26–29 (8)	40–18 (11)	37–45 (12)	5–34 (15)	25–15 (16)	42–46 (18)	13–31 (20)	10–12 (21)	30–16 (22)	4–20 (23)

13	14	15	16	17	18	19	20	21	22	23	24
27–39 (24)	21–24 (26)	35–36 (29)	32–17 (30)	23–6 (33)	43–33 (34)	14–41 (36)	8–3 (38)	28–7 (39)	2–11 (40)	22–38 (41)	47–19 (42)

Rounds 24 to 46

1	2	3	4	5	6	7	8	9	10	11	12
24–47 (1)	21–32 (6)	6–3 (8)	41–17 (11)	22–14 (12)	11–28 (15)	38–2 (16)	23–19 (18)	8–36 (20)	35–33 (21)	39–7 (22)	43–27 (23)

13	14	15	16	17	18	19	20	21	22	23	24
16–4 (24)	1–44 (26)	13–12 (29)	40–9 (30)	29–46 (33)	10–20 (34)	18–37 (36)	26–31 (38)	30–5 (39)	34–25 (40)	15–45 (41)	42–48 (42)

10.3 Odd numbers of teams

Pure mathematics is, in its way, the poetry of logical ideas.
Albert Einstein

The bicyclic schedules supplied above for even numbers of teams do not work well when used for odd numbers of teams. The pairs need to sit out for two rounds as well as missing two board-sets.

Apart from 3 teams the movements for odd numbers of teams are constructed from cyclic schedules that were proved to exist in one brief paragraph by Stinson and Wallis in 1984. Mostly these schedules are cyclic for the pairs and bicyclic for some of the boards. Generally more than one set-of-boards will make these movements easier to organize.

Three teams

First we will consider the case for 3 teams. There are 6 pairs involved and each pair plays 4 other pairs from the other two teams. Play is over 4 rounds and involves 6 board-sets. Each pair misses two sets and plays a total of 4 sets each usually consisting of 6, 7 or 8 boards and each match completed over 12 to 16 boards. Though not cyclic it is very easy to organize.

This type of event is trivial. Each board-set is involved in only one match, therefore scoring it as a pairs' event is practically pointless. The teams' event winners are the same as the pairs' event winners when using the same scoring method. Calibre 88.

Team/pair map

1	2	3
1,2	3,4	5,6

	1	2	3
1	3–5 (1)	1–6 (3)	2–4 (6)
2	6–4 (1)	1–5 (4)	2–3 (5)
3	4–5 (2)	6–2 (4)	3–1 (6)
4	6–3 (2)	5–2 (3)	4–1 (5)

5 teams

These movements are for 5 teams playing 3 or 4 boards per round for a 24 or 32 board sessions. A total of 30 or 40 boards are in play. The movements are cyclic for the players apart from the pairs switching sides after round 4. Pair 1 follows 8, 9 and 10 are stationary and all others follow the number one less than theirs. Tables 1 and 2 share board-sets and are played in sequence. Table 5 plays the two highest board-sets (9 and 10) and alternate after each round. Pairs are grouped 1 to 4 and 5 to 8 with the teams made up from 1 and 5, 2 and 6 etc with 9 and 10 the last team. Each pair misses two board-sets out of the 10.

In the first example, board-sets are shared at 3 tables on each round or use 3 sets of boards. Board-sets are played in sequence at tables 3 and 4, but need restarting after round 4 because they are only half-sets. These board cycles are called half-set cyclic because they use only half the boards at each of tables 3 and 4. All pairs swap sides after round 4 but continue with the original cyclic pattern. This is called a semi-cyclic schedule. Calibre for pairs is 76.

	1	2	3	4	5
1	10–1 (1)	5–9 (1)	6–7 (1)	2–4 (5)	3–8 (9)
2	10–2 (2)	6–9 (2)	7–8 (2)	3–5 (6)	4–1 (10)
3	10–3 (3)	7–9 (3)	8–1 (3)	4–6 (7)	5–2 (9)
4	10–4 (4)	8–9 (4)	1–2 (4)	5–7 (8)	6–3 (10)
5	5–10 (5)	9–1 (5)	3–2 (1)	8–6 (5)	4–7 (9)
6	6–10 (6)	9–2 (6)	4–3 (2)	1–7 (6)	5–8 (10)
7	7–10 (7)	9–3 (7)	5–4 (3)	2–8 (7)	6–1 (9)
8	8–10 (8)	9–4 (8)	6–5 (4)	3–1 (8)	7–2 (10)

This schedule is called a 3 board-sets semi-cyclic and can be described in two lines as

	1	2	3	4	5
1	10–1 (1)	5–9 (1)	6–7 (1)	2–4 (5)	3–8 (9)

Another approach to this schedule is to swap the starting positions at tables 3 and 4 in the second half. This allows easy board-sharing at tables 1 to 3 throughout the movement.

	1	2	3	4	5
1	10–1 (1)	5–9 (1)	6–7 (1)	2–4 (5)	3–8 (9)
2	10–2 (2)	6–9 (2)	7–8 (2)	3–5 (6)	4–1 (10)
3	10–3 (3)	7–9 (3)	8–1 (3)	4–6 (7)	5–2 (9)
4	10–4 (4)	8–9 (4)	1–2 (4)	5–7 (8)	6–3 (10)
5	5–10 (5)	9–1 (5)	8–6 (5)	3–2 (1)	4–7 (9)
6	6–10 (6)	9–2 (6)	1–7 (6)	4–3 (2)	5–8 (10)
7	7–10 (7)	9–3 (7)	2–8 (7)	5–4 (3)	6–1 (9)
8	8–10 (8)	9–4 (8)	3–1 (8)	6–5 (4)	7–2 (10)

In this second example there is no sharing of board-sets apart from tables 1 and 2. Tables 3 and 4 have a slightly more irregular board sequence, though in practice easily managed as they are still semi-cyclic. Calibre for pairs is 73.

	1	2	3	4	5
1	10–1 (1)	5–9 (1)	3–4 (2)	6–8 (7)	2–7 (9)
2	10–2 (2)	6–9 (2)	4–5 (3)	7–1 (8)	3–8 (10)
3	10–3 (3)	7–9 (3)	5–6 (4)	8–2 (5)	4–1 (9)
4	10–4 (4)	8–9 (4)	6–7 (1)	1–3 (6)	5–2 (10)
5	5–10 (5)	9–1 (5)	8–7 (2)	4–2 (7)	3–6 (9)
6	6–10 (6)	9–2 (6)	1–8 (3)	5–3 (8)	4–7 (10)
7	7–10 (7)	9–3 (7)	2–1 (4)	6–4 (5)	5–8 (9)
8	8–10 (8)	9–4 (8)	3–2 (1)	7–5 (6)	6–1 (10)

7 teams

This movement is for 7 teams playing 2 or 3 boards per round for a 24 or 36-board session. A total of 28 or 42 boards are in play. Similar principles apply to the schedule for 5 teams above, but no tables ever share boards and so you require only 1 set-of-boards in play. Pairs are grouped 1 to 6 and 7 to 12 plus 13 and 14 for the last team. The pairs move in a cycle and change sides after round 6. The boards also cycle in an orderly fashion with two tables playing all sets (tables 1 and 4) and others are semi-cyclic with the first half (1 to 6) or second half (7 to 12) of the board-sets. Table 7 plays only sets 13 and 14, alternately.

Using four sets of boards improves the organization of the event. Two sets are used, one each at tables 1 and 4 (or share one set) and half-sets at tables 2, 3, 5 and 6. Calibre for pairs is 77.

	1	2	3	4	5	6	7
1	14–1 (1)	5–6 (3)	9–11 (4)	13–2 (8)	4–12 (9)	7–10 (11)	3–8 (13)
2	14–2 (2)	6–7 (4)	10–12 (5)	13–3 (9)	5–1 (10)	8–11 (12)	4–9 (14)
3	14–3 (3)	7–8 (5)	11–1 (6)	13–4 (10)	6–2 (11)	9–12 (7)	5–10 (13)
4	14–4 (4)	8–9 (6)	12–2 (1)	13–5 (11)	7–3 (12)	10–1 (8)	6–11 (14)
5	14–5 (5)	9–10 (1)	1–3 (2)	13–6 (12)	8–4 (7)	11–2 (9)	7–12 (13)
6	14–6 (6)	10–11 (2)	2–4 (3)	7–13 (1)	9–5 (8)	12–3 (10)	8–1 (14)
7	7–14 (7)	12–11 (3)	5–3 (4)	8–13 (2)	6–10 (9)	4–1 (11)	2–9 (13)
8	8–14 (8)	1–12 (4)	6–4 (5)	9–13 (3)	7–11 (10)	5–2 (12)	3–10 (14)
9	9–14 (9)	2–1 (5)	7–5 (6)	10–13 (4)	8–12 (11)	6–3 (7)	4–11 (13)
10	10–14 (10)	3–2 (6)	8–6 (1)	11–13 (5)	9–1 (12)	7–4 (8)	5–12 (14)
11	11–14 (11)	4–3 (1)	9–7 (2)	12–13 (6)	10–2 (7)	8–5 (9)	6–1 (13)
12	12–14 (12)	5–4 (2)	10–8 (3)	13–1 (7)	11–3 (8)	9–6 (10)	7–2 (14)

Alternatively the 4 board-sets semi-cyclic schedule for 7 teams (83) is

	1	2	3	4	5	6	7
1	14–1 (1)	13–2 (8)	9–10 (1)	4–6 (7)	5–8 (1)	3–11 (7)	7–12 (13)

Note tables 1 and 2 play all board-sets in sequence, table 2 starting with board-set 8. Tables 3 to 6 are semi-cyclic and play half-sets and table 7 plays only sets 13 and 14. Pair 13 swaps sides after round 5 and swaps back on the last round, 12. All other pairs swap from round 7 onwards. Expanded, this looks like:

	1	2	3	4	5	6	7
1	14–1 (1)	13–2 (8)	9–10 (1)	4–6 (7)	5–8 (1)	3–11 (7)	7–12 (13)
2	14–2 (2)	13–3 (9)	10–11 (2)	5–7 (8)	6–9 (2)	4–12 (8)	8–1 (14)
3	14–3 (3)	13–4 (10)	11–12 (3)	6–8 (9)	7–10 (3)	5–1 (9)	9–2 (13)
4	14–4 (4)	13–5 (11)	12–1 (4)	7–9 (10)	8–11 (4)	6–2 (10)	10–3 (14)
5	14–5 (5)	13–6 (12)	1–2 (5)	8–10 (11)	9–12 (5)	7–3 (11)	11–4 (13)
6	14–6 (6)	7–13 (1)	2–3 (6)	9–11 (12)	10–1 (6)	8–4 (12)	12–5 (14)
7	7–14 (7)	8–13 (2)	4–3 (1)	12–10 (7)	2–11 (1)	5–9 (7)	6–1 (13)
8	8–14 (8)	9–13 (3)	5–4 (2)	1–11 (8)	3–12 (2)	6–10 (8)	7–2 (14)
9	9–14 (9)	10–13 (4)	6–5 (3)	2–12 (9)	4–1 (3)	7–11 (9)	8–3 (13)
10	10–14 (10)	11–13 (5)	7–6 (4)	3–1 (10)	5–2 (4)	8–12 (10)	9–4 (14)
11	11–14 (11)	12–13 (6)	8–7 (5)	4–2 (11)	6–3 (5)	9–1 (11)	10–5 (13)
12	12–14 (12)	13–1 (7)	9–8 (6)	5–3 (12)	7–4 (6)	10–2 (12)	11–6 (14)

9 teams

This movement is for 9 teams playing 2 boards per round for a 32-board session. A total of 36 boards are in play. Similar principles apply to the schedule for 5 teams above with a share between tables 1 and 2, 4 and 5 and lastly 7 and 8, but one set is never played at more than two tables. Pairs are grouped 1 to 8 and 9 to 16 plus 17 and 18 for the last team.

Two, three or four sets of boards may be used to improve the organization of the event. Two sets are used, one each at tables 1 and 2 and half-sets at tables 3 to 8. All player movement is cyclic and the boards in the dark boxes are semi-cyclic. Pairs swap sides after round 8. Calibre for the pairs is 82.

	1	2	3	4	5	6	7	8	9
1	18–1 (1)	9–17 (1)	12–15 (3)	5–16 (3)	10–11 (4)	3–7 (13)	2–4 (15)	8–14 (15)	6–13 (17)
2	18–2 (2)	10–17 (2)	13–16 (4)	6–1 (4)	11–12 (5)	4–8 (14)	3–5 (16)	9–15 (16)	7–14 (18)
3	18–3 (3)	11–17 (3)	14–1 (5)	7–2 (5)	12–13 (6)	5–9 (15)	4–6 (9)	10–16 (9)	8–15 (17)
4	18–4 (4)	12–17 (4)	15–2 (6)	8–3 (6)	13–14 (7)	6–10 (16)	5–7 (10)	11–1 (10)	9–16 (18)
5	18–5 (5)	13–17 (5)	16–3 (7)	9–4 (7)	14–15 (8)	7–11 (9)	6–8 (11)	12–2 (11)	10–1 (17)
6	18–6 (6)	14–17 (6)	1–4 (8)	10–5 (8)	15–16 (1)	8–12 (10)	7–9 (12)	13–3 (12)	11–2 (18)
7	18–7 (7)	15–17 (7)	2–5 (1)	11–6 (1)	16–1 (2)	9–13 (11)	8–10 (13)	14–4 (13)	12–3 (17)
8	18–8 (8)	16–17 (8)	3–6 (2)	12–7 (2)	1–2 (3)	10–14 (12)	9–11 (14)	15–5 (14)	13–4 (18)
9	9–18 (9)	17–1 (9)	7–4 (3)	8–13 (3)	3–2 (4)	15–11 (13)	12–10 (15)	6–16 (15)	5–14 (17)
10	10–18 (10)	17–2 (10)	8–5 (4)	9–14 (4)	4–3 (5)	16–12 (14)	13–11 (16)	7–1 (16)	6–15 (18)
11	11–18 (11)	17–3 (11)	9–6 (5)	10–15 (5)	5–4 (6)	1–13 (15)	14–12 (9)	8–2 (9)	7–16 (17)
12	12–18 (12)	17–4 (12)	10–7 (6)	11–16 (6)	6–5 (7)	2–14 (16)	15–13 (10)	9–3 (10)	8–1 (18)
13	13–18 (13)	17–5 (13)	11–8 (7)	12–1 (7)	7–6 (8)	3–15 (9)	16–14 (11)	10–4 (11)	9–2 (17)
14	14–18 (14)	17–6 (14)	12–9 (8)	13–2 (8)	8–7 (1)	4–16 (10)	1–15 (12)	11–5 (12)	10–3 (18)
15	15–18 (15)	17–7 (15)	13–10 (1)	14–3 (1)	9–8 (2)	5–1 (11)	2–16 (13)	12–6 (13)	11–4 (17)
16	16–18 (16)	17–8 (16)	14–11 (2)	15–4 (2)	10–9 (3)	6–2 (12)	3–1 (14)	13–7 (14)	12–5 (18)

The 5 board-sets semi-cyclic schedule for 9 teams (84) is

	1	2	3	4	5	6	7	8	9
1	18–1 (1)	9–17 (1)	2–3 (1)	6–8 (9)	13–16 (1)	11–15 (9)	7–12 (1)	4–10 (9)	5–14 (17)

Note: Tables 1 and 2 play all board-sets in sequence. Tables 3 to 8 play half-sets and table 9 only plays board-sets 17 and 18. All pairs swap sides after round 8.

11 teams

This schedule is for 11 teams playing 1 or 2 boards per round for a 20 or 40-board session.

	1	2	3	4	5	6	7	8	9	10	11
1	22–1 (1)	10–17 (3)	8–13 (5)	16–19 (7)	4–5 (9)	21–2 (12)	14–18 (13)	7–9 (15)	3–15 (17)	6–12 (19)	11–20 (21)
2	22–2 (2)	11–18 (4)	9–14 (6)	17–20 (8)	5–6 (10)	21–3 (13)	15–19 (14)	8–10 (16)	4–16 (18)	7–13 (20)	12–1 (22)
3	22–3 (3)	12–19 (5)	10–15 (7)	18–1 (9)	6–7 (1)	21–4 (14)	16–20 (15)	9–11 (17)	5–17 (19)	8–14 (11)	13–2 (21)
4	22–4 (4)	13–20 (6)	11–16 (8)	19–2 (10)	7–8 (2)	21–5 (15)	17–1 (16)	10–12 (18)	6–18 (20)	9–15 (12)	14–3 (22)
5	22–5 (5)	14–1 (7)	12–17 (9)	20–3 (1)	8–9 (3)	21–6 (16)	18–2 (17)	11–13 (19)	7–19 (11)	10–16 (13)	15–4 (21)
6	22–6 (6)	15–2 (8)	13–18 (10)	1–4 (2)	9–10 (4)	21–7 (17)	19–3 (18)	12–14 (20)	8–20 (12)	11–17 (14)	16–5 (22)
7	22–7 (7)	16–3 (9)	14–19 (1)	2–5 (3)	10–11 (5)	21–8 (18)	20–4 (19)	13–15 (11)	9–1 (13)	12–18 (15)	17–6 (21)
8	22–8 (8)	17–4 (10)	15–20 (2)	3–6 (4)	11–12 (6)	21–9 (19)	1–5 (20)	14–16 (12)	10–2 (14)	13–19 (16)	18–7 (22)
9	22–9 (9)	18–5 (1)	16–1 (3)	4–7 (5)	12–13 (7)	21–10 (20)	2–6 (11)	15–17 (13)	11–3 (15)	14–20 (17)	19–8 (21)
10	22–10 (10)	19–6 (2)	17–2 (4)	5–8 (6)	13–14 (8)	11–21 (1)	3–7 (12)	16–18 (14)	12–4 (16)	15–1 (18)	20–9 (22)
11	11–22 (11)	7–20 (3)	3–18 (5)	9–6 (7)	15–14 (9)	12–21 (2)	8–4 (13)	19–17 (15)	5–13 (17)	2–16 (19)	10–1 (21)
12	12–22 (12)	8–1 (4)	4–19 (6)	10–7 (8)	16–15 (10)	13–21 (3)	9–5 (14)	20–18 (16)	6–14 (18)	3–17 (20)	11–2 (22)
13	13–22 (13)	9–2 (5)	5–20 (7)	11–8 (9)	17–16 (1)	14–21 (4)	10–6 (15)	1–19 (17)	7–15 (19)	4–18 (11)	12–3 (21)
14	14–22 (14)	10–3 (6)	6–1 (8)	12–9 (10)	18–17 (2)	15–21 (5)	11–7 (16)	2–20 (18)	8–16 (20)	5–19 (12)	13–4 (22)
15	15–22 (15)	11–4 (7)	7–2 (9)	13–10 (1)	19–18 (3)	16–21 (6)	12–8 (17)	3–1 (19)	9–17 (11)	6–20 (13)	14–5 (21)
16	16–22 (16)	12–5 (8)	8–3 (10)	14–11 (2)	20–19 (4)	17–21 (7)	13–9 (18)	4–2 (20)	10–18 (12)	7–1 (14)	15–6 (22)
17	17–22 (17)	13–6 (9)	9–4 (1)	15–12 (3)	1–20 (5)	18–21 (8)	14–10 (19)	5–3 (11)	11–19 (13)	8–2 (15)	16–7 (21)
18	18–22 (18)	14–7 (10)	10–5 (2)	16–13 (4)	2–1 (6)	19–21 (9)	15–11 (20)	6–4 (12)	12–20 (14)	9–3 (16)	17–8 (22)
19	19–22 (19)	15–8 (1)	11–6 (3)	17–14 (5)	3–2 (7)	20–21 (10)	16–12 (11)	7–5 (13)	13–1 (15)	10–4 (17)	18–9 (21)
20	20–22 (20)	16–9 (2)	12–7 (4)	18–15 (6)	4–3 (8)	21–1 (11)	17–13 (12)	8–6 (14)	14–2 (16)	11–5 (18)	19–10 (22)

A total of 24 or 44 boards are in play. Similar principles apply to the schedule for 7 teams above with no sharing required. Pairs are grouped 1 to 10 and 11 to 20 plus 21 and 22 for the last team. All pairs and boards are cyclic throughout with half-sets in the area with the dark border. Tables 1 and 6 play all boards in sequence. All pairs swap their role at half way, but pair 21 swaps sides after round 9 and swap back on the last round, 20. Board-sets 21 and 22 are only played at table 11. Calibre for pairs is 81.

The second schedule requires 5 or 6 sets of boards but cyclic schedules apply to both pairs and boards. Tables 1 and 2 play a set each or can share one as they do not play the same boards on each round. The other 4 sets are used by the next 4 pairs of tables, board-sets 1 to 10 on the odd numbered tables and 11 to 20 on the even. Calibre for pairs is 88.

This is the 6 board-sets semi-cyclic schedule for 11 teams is

	1	2	3	4	5	6	7	8	9	10	11
1	22–1 (1)	21–2 (12)	3–4 (1)	6–8 (11)	15–18 (1)	10–14 (11)	7–12 (1)	13–19 (11)	9–16 (1)	5–17 (11)	11–20 (21)

Note: Tables 1 and 2 play all board-sets in sequence, table 2 starting with set 12. Tables 3 to 10 play half-sets and table 11 plays only sets 21 and 22. Pair 21 swaps sides after round 9 and swap back on the last round, 20. All other pairs swap from round 11 onwards.

13 teams

This movement for 13 teams requires 24 rounds. It is unlikely to be suitable for two-board rounds as the length of the game may be unmanageable. Therefore only the semi-cyclic movement is supplied, requiring 7 sets of boards.

Only one copy of boards 25 and 26 is required and they remain at table 13. Tables 1 and 2 use one set each, played in order. The odd-numbered tables (3, 5, 7, 9 and 11) use boards 1 to 12 from the other 5 sets. The 5 other even-numbered tables use the remaining 5 copies of boards numbered 13 to 24. All board-sets are played in order at each table once the first board is established on round 1. All pairs swap sides after round 12. Pairs are grouped 1 to 12 and 13 to 24 plus 25 and 26 for the last team. Calibre for pairs is 83.

This 7 board-sets semi-cyclic schedule for 13 teams is

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	26–1 (1)	13–25 (1)	2–3 (1)	4–6 (13)	5–8 (1)	10–14 (13)	19–24 (1)	15–21 (13)	16–23 (1)	12–20 (13)	9–18 (1)	7–17 (13)	11–22 (25)

15 teams

The movement for 15 teams requires 28 rounds which is an awkward number. In addition a schedule for 15 teams with 28 rounds is highly irregular and for all practical purposes unusable. The solution is to use a three-stanza movement. Each stanza has 10 rounds for a total of 30 rounds, with each team not playing for two of those rounds. See *Three sessions for odd numbers of teams,* page 320.

17 teams

17 teams is similar to 13 teams. Pairs are grouped 1 to 16 and 17 to 32 plus 33 and 34 for the last team. Calibre for pairs is 87. Pairs swap sides after round 16, perhaps with a break in play.

The 7 board-sets semi-cyclic schedule for 17 teams is

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	34– 1 (1)	17–33 (1)	2– 3 (1)	4– 6 (17)	5– 8 (1)	7–11 (17)	9–14 (1)	19–25 (17)	15–22 (1)	24–32 (17)	20–29 (1)	21–31 (17)	16–27 (1)	18–30 (17)	10–23 (1)	12–26 (17)	13–28 (33)

19 teams

19 teams is similar to 11 teams. Pairs are grouped 1 to 18 and 19 to 36 plus 37 and 38 for the last team. There are 36 one-board rounds. Calibre for pairs is 77. Pairs swap sides after round 18 apart for pair 37 that swaps after 17 and then for the last round.

The 10 board-sets semi-cyclic schedule for 19 teams is

	1	2	3	4	5	6	7	8	9	10
1	38–1 (1)	37–2 (20)	3–4 (1)	5–7 (19)	6–9 (1)	8–12 (19)	10–15 (1)	22–28 (19)	23–30 (1)	24–32 (19)

11	12	13	14	15	16	17	18	19
16–25 (1)	17–27 (19)	18–29 (1)	21–33 (19)	13–26 (1)	20–34 (19)	14–35 (1)	11–31 (19)	19–36 (37)

21 teams

Use the same method that is used for 15 teams. The schedule for 7 teams supplied above is used 3 times with the three groups 1 to 14 (A), 15 to 28 (B) and 29 to 42 (C). Each of the 7 tables requires only one set-of-boards.

Alternatively use this 21-teams fully cyclic. It is similar to 13 teams. Pairs are grouped 1 to 20 and 21 to 40 plus 41 and 42 for the last team. There are 40 one-board rounds. Calibre for pairs is 83. Pairs swap sides after round 20, perhaps with a break in play.

The 11 board-sets semi-cyclic schedule for 21 teams is

	1	2	3	4	5	6	7	8	9	10	11
1	42–1 (1)	21–41 (1)	2–3 (1)	4–6 (21)	5–8 (1)	7–11 (21)	9–14 (1)	10–16 (21)	24–31 (1)	17–25 (21)	27–36 (1)

12	13	14	15	16	17	18	19	20	21
23–33 (21)	26–37 (1)	28–40 (21)	19–32 (1)	15–29 (21)	20–35 (1)	22–38 (21)	13–30 (1)	12–34 (21)	18–39 (41)

23 teams

23 teams is similar to 11 teams. Pairs are grouped 1 to 22 and 23 to 44 plus 45 and 46 for the last team. There are 44 one-board rounds. Calibre for pairs is 85. Pairs swap sides after round 22 apart for pair 45 that swaps after 21 and then for the last round.

The 12 board-sets semi-cyclic schedule for 23 teams is

	1	2	3	4	5	6	7	8	9	10	11	12
1	46–1 (1)	45–2 (24)	3–4 (1)	5–7 (23)	6–9 (1)	8–12 (23)	10–15 (1)	11–17 (23)	20–27 (1)	18–26 (23)	30–39 (1)	31–41 (23)

13	14	15	16	17	18	19	20	21	22	23
29–40 (1)	24–36 (23)	22–35 (1)	28–42 (23)	19–34 (1)	16–32 (23)	21–38 (1)	25–43 (23)	14–33 (1)	13–37 (23)	23–44 (45)

25 teams

25 teams is similar to 13 teams. Pairs are grouped 1 to 24 and 25 to 48 plus 49 and 50 for the last team. There are 48 one-board rounds. Calibre for pairs is 88. Pairs swap sides after round 24, perhaps with a break in play.

The 13 board-sets semi-cyclic schedule for 25 teams is

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	50–1 (1)	25–49 (1)	2–3 (1)	4–6 (25)	5–8 (1)	7–11 (25)	9–14 (1)	10–16 (25)	12–19 (1)	13–21 (25)	28–37 (1)	33–43 (25)	31–42 (1)

14	15	16	17	18	19	20	21	22	23	24	25
27–39 (25)	22–35 (1)	32–46 (25)	30–45 (1)	20–36 (25)	24–41 (1)	29–47 (25)	15–34 (1)	18–38 (25)	23–44 (1)	26–48 (25)	17–40 (49)

10.4 Two sessions, half even

If people do not believe that mathematics is simple,
it is only because they do not realize how complicated life is.
John Louis von Neumann

Examination of the list of schedules above will quickly show that the larger numbers of teams require unmanageable numbers of rounds for one session of bridge. To overcome this weakness it is possible to use the smaller-numbered schedules, doubled up. This technique applies where half the number of teams is an even number (12, 16, 20 …).

Consider the 6-teams schedule above. If we use this twice we have 12 teams playing 20 rounds. The schedules are identical to those used for 6 teams, but the groups of pairs are extended. The pairs following each other in the schedules are 1–5 (A), 6–10 (B), 13–17 (C), 18–22 (D) with 11, 12, 23 and 24 (E) stationary. New boards (sets 11–20) are introduced after 10 rounds when a new session is commenced. Each session is a bicyclic schedule. Groups A and B form one lot of teams, C and D a second lot with E the last.

This is very useful because the 12-teams draw shown above has 22 rounds playing 22 or 44 board sessions. With this doubled up schedule we have two equal length sessions of 30 boards playing 6-board matches. The only problem is that each team misses play against one other. This can be addressed with a 6-board session of two rounds where the remaining teams meet.

	1	2	3	4	5	6	7	8	9	10	11	12
1	5– 8 (2)	4– 6 (3)	12– 3 (4)	9–10 (6)	11– 2 (7)	1– 7 (10)	17–20 (2)	16–18 (3)	24–15 (4)	21–22 (6)	23–14 (7)	13–19 (10)
2	1– 9 (3)	5– 7 (4)	12– 4 (5)	10– 6 (7)	11– 3 (8)	2– 8 (6)	13–21 (3)	17–19 (4)	24–16 (5)	22–18 (7)	23–15 (8)	14–20 (6)
3	2–10 (4)	1– 8 (5)	12– 5 (1)	6– 7 (8)	11– 4 (9)	3– 9 (7)	14–22 (4)	13–20 (5)	24–17 (1)	18–19 (8)	23–16 (9)	15–21 (7)
4	3– 6 (5)	2– 9 (1)	12– 1 (2)	7– 8 (9)	11– 5 (10)	4–10 (8)	15–18 (5)	14–21 (1)	24–13 (2)	19–20 (9)	23–17 (10)	16–22 (8)
5	4– 7 (1)	3–10 (2)	12– 2 (3)	8– 9 (10)	11– 1 (6)	5– 6 (9)	16–19 (1)	15–22 (2)	24–14 (3)	20–21 (10)	23–13 (6)	17–18 (9)
6	9– 7 (2)	8–11 (4)	5– 2 (5)	6–12 (6)	1–10 (9)	4– 3 (10)	21–19 (2)	20–23 (4)	17–14 (5)	18–24 (6)	13–22 (9)	16–15 (10)
7	10– 8 (3)	9–11 (5)	1– 3 (1)	7–12 (7)	2– 6 (10)	5– 4 (6)	22–20 (3)	21–23 (5)	13–15 (1)	19–24 (7)	14–18 (10)	17–16 (6)
8	6– 9 (4)	10–11 (1)	2– 4 (2)	8–12 (8)	3– 7 (6)	1– 5 (7)	18–21 (4)	22–23 (1)	14–16 (2)	20–24 (8)	15–19 (6)	13–17 (7)
9	7–10 (5)	6–11 (2)	3– 5 (3)	9–12 (9)	4– 8 (7)	2– 1 (8)	19–22 (5)	18–23 (2)	15–17 (3)	21–24 (9)	16–20 (7)	14–13 (8)
10	8– 6 (1)	7–11 (3)	4– 1 (4)	10–12 (10)	5– 9 (8)	3– 2 (9)	20–18 (1)	19–23 (3)	16–13 (4)	22–24 (10)	17–21 (8)	15–14 (9)

	1	2	3	4	5	6	7	8	9	10	11	12
11	5–20 (12)	4–18 (13)	12–15 (14)	9–22 (16)	11–14 (17)	1–19 (20)	17– 8 (12)	16– 6 (13)	24– 3 (14)	21–10 (16)	23– 2 (17)	13– 7 (20)
12	1–21 (13)	5–19 (14)	12–16 (15)	10–18 (17)	11–15 (18)	2–20 (16)	13– 9 (13)	17– 7 (14)	24– 4 (15)	22– 6 (17)	23– 3 (18)	14– 8 (16)
13	2–22 (14)	1–20 (15)	12–17 (11)	6–19 (18)	11–16 (19)	3–21 (17)	14–10 (14)	13– 8 (15)	24– 5 (11)	18– 7 (18)	23– 4 (19)	15– 9 (17)
14	3–18 (15)	2–21 (11)	12–13 (12)	7–20 (19)	11–17 (20)	4–22 (18)	15– 6 (15)	14– 9 (11)	24– 1 (12)	19– 8 (19)	23– 5 (20)	16–10 (18)
15	4–19 (11)	3–22 (12)	12–14 (13)	8–21 (20)	11–13 (16)	5–18 (19)	16– 7 (11)	15–10 (12)	24– 2 (13)	20– 9 (20)	23– 1 (16)	17– 6 (19)
16	9–19 (12)	20–11 (14)	17– 2 (15)	18–12 (16)	1–22 (19)	16– 3 (20)	21– 7 (12)	8–23 (14)	5–14 (15)	6–24 (16)	13–10 (19)	4–15 (20)
17	10–20 (13)	21–11 (15)	13– 3 (11)	19–12 (17)	2–18 (20)	17– 4 (16)	22– 8 (13)	9–23 (15)	1–15 (11)	7–24 (17)	14– 6 (20)	5–16 (16)
18	6–21 (14)	22–11 (11)	14– 4 (12)	20–12 (18)	3–19 (16)	13– 5 (17)	18– 9 (14)	10–23 (11)	2–16 (12)	8–24 (18)	15– 7 (16)	1–17 (17)
19	7–22 (15)	18–11 (12)	15– 5 (13)	21–12 (19)	4–20 (17)	14– 1 (18)	19–10 (15)	6–23 (12)	3–17 (13)	9–24 (19)	16– 8 (17)	2–13 (18)
20	8–18 (11)	19–11 (13)	16– 1 (14)	22–12 (20)	5–21 (18)	15– 2 (19)	20– 6 (11)	7–23 (13)	4–13 (14)	10–24 (20)	17– 9 (18)	3–14 (19)

21	1–13 (21)	2–14 (21)	3–15 (21)	4–16 (21)	5–17 (21)	11–23 (21)	18– 6 (21)	19– 7 (21)	20– 8 (21)	21– 9 (21)	22–10 (21)	24–12 (21)
22	13– 6 (22)	14– 7 (22)	15– 8 (22)	16– 9 (22)	17–10 (22)	24–11 (22)	1–18 (22)	2–19 (22)	3–20 (22)	4–21 (22)	5–22 (22)	12–23 (22)

This schedule can be represented using the following shorthand method.

12 teams, two sessions of 10 rounds

The two schedules for the first session are:
Rounds 1 to 5

5–8 (2)	4–6 (3)	12–3 (4)	9–10 (6)	11–2 (7)	1–7 (10)
17–20 (2)	16–18 (3)	24–15 (4)	21–22 (6)	23–14 (7)	13–19 (10)

Rounds 6 to 10

9–7 (2)	8–11 (4)	5–2 (5)	6–12 (6)	1–10 (9)	4–3 (10)
21–19 (2)	20–23 (4)	17–14 (5)	18–24 (6)	13–22 (9)	16–15 (10)

The two schedules for the second session are:
Rounds 11 to 15

5–20 (12)	4–18 (13)	12–15 (14)	9–22 (16)	11–14 (17)	1–19 (20)
17–8 (12)	16–6 (13)	24–3 (14)	21–10 (16)	23–2 (17)	13–7 (20)

Rounds 16 to 20

9–19 (12)	20–11 (14)	17–2 (15)	18–12 (16)	1–22 (19)	16–3 (20)
21–7 (12)	8–23 (14)	5–14 (15)	6–24 (16)	13–10 (19)	4–15 (20)

For round 21, play group A (N-S) against C (E-W) and D (N-S) against B (E-W).
For round 22, play group C (N-S) against B (E-W) and A (N-S) against D (E-W).
Teams 11/12 and 23/24 form the last match in these rounds. Pairs' calibre 61.

16 teams, two sessions of 14 rounds

Session 1, rounds 1 to 7

16–1 (1)	12–7 (4)	10–4 (6)	2–6 (7)	9–11 (8)	14–8 (10)	13–3 (11)	15–5 (12)
32–17 (1)	28–23 (4)	26–20 (6)	18–22 (7)	25–27 (8)	30–24 (10)	29–19 (11)	31–21 (12)

Session 1, rounds 8 to 14

8–15 (1)	14–5 (4)	11–3 (6)	13–9 (7)	4–2 (8)	1–7 (10)	10–6 (11)	12–16 (12)
24–31 (1)	30–21 (4)	27–19 (6)	29–25 (7)	20–18 (8)	17–23 (10)	26–22 (11)	28–32 (12)

Session 2, rounds 1 to 7

16–17 (15)	28–7 (18)	26–4 (20)	18–6 (21)	9–27 (22)	14–24 (24)	29–3 (25)	15–21 (26)
32–1 (15)	12–23 (18)	10–20 (20)	2–22 (21)	25–11 (22)	30–8 (24)	13–19 (25)	31–5 (26)

Session 2, rounds 8 to 14

24–15 (15)	30–5 (18)	27–3 (20)	13–25 (21)	20–2 (22)	17–7 (24)	26–6 (25)	28–16 (26)
8–31 (15)	14–21 (18)	11–19 (20)	29–9 (21)	4–18 (22)	1–23 (24)	10–22 (25)	12–32 (26)

Groups 1–7 (A), 8–14 (B), 17–23 (C), 24–30 (D), 15,16/31,32 (E). Pairs' calibre 68.

20 teams, two sessions of 18 rounds

Session 1, rounds 1 to 9

16–18 (3)	6–14 (4)	10– 4 (5)	7–17 (6)	20– 8 (8)	3– 2 (10)	12–13 (11)	5– 1 (16)	11–15 (17)	19– 9 (18)
36–38 (3)	26–34 (4)	30–24 (5)	27–37 (6)	40–28 (8)	23–22 (10)	32–33 (11)	25–21 (16)	31–35 (17)	39–29 (18)

Session 1, rounds 10 to 18

17– 4 (1)	12– 9 (4)	3– 1 (6)	11– 6 (8)	18–19 (9)	14– 7 (10)	10–16 (11)	15–20 (15)	13– 2 (16)	5– 8 (18)
37–24 (1)	32–29 (4)	23–21 (6)	31–26 (8)	38–39 (9)	34–27 (10)	30–36 (11)	35–40 (15)	33–22 (16)	25–28 (18)

Session 2, rounds 1 to 9

16–38 (21)	6–34 (22)	30– 4 (23)	7–37 (24)	20–28 (26)	23– 2 (28)	12–33 (29)	25– 1 (34)	11–35 (35)	19–29 (36)
36–18 (21)	26–14 (22)	10–24 (23)	27–17 (24)	40– 8 (26)	3–22 (28)	32–13 (29)	5–21 (34)	31–15 (35)	39– 9 (36)

Session 2, rounds 10 to 18

37– 4 (19)	32– 9 (22)	23– 1 (24)	31– 6 (26)	38–19 (27)	34– 7 (28)	10–36 (29)	35–20 (33)	33– 2 (34)	25– 8 (36)
17–24 (19)	12–29 (22)	3–21 (24)	11–26 (26)	18–39 (27)	14–27 (28)	30–16 (29)	15–40 (33)	13–22 (34)	5–28 (36)

Groups 1–9 (A), 10–18 (B), 21–29 (C), 30–38 (D), 19,20/39,40 (E). Pairs' calibre 77.

24 teams, two sessions of 22 rounds

Session 1, rounds 1 to 11

24– 1 (1)	9–16 (2)	3–13 (4)	6–11 (5)	8–21 (6)	7–15 (10)	12–22 (13)	4–20 (14)	17–19 (15)	5– 2 (19)	23–10 (21)	18–14 (22)
48–25 (1)	33–40 (2)	27–37 (4)	30–35 (5)	32–45 (6)	31–39 (10)	36–46 (13)	28–44 (14)	41–43 (15)	29–26 (19)	47–34 (21)	42–38 (22)

Session 1, rounds 12 to 22

12–23 (1)	5–20 (2)	2–14 (4)	22–17 (5)	10–19 (6)	4–18 (10)	11– 1 (13)	9–15 (14)	8– 6 (15)	13–16 (19)	21–24 (21)	3– 7 (22)
36–47 (1)	29–44 (2)	26–38 (4)	46–41 (5)	34–43 (6)	28–42 (10)	35–25 (13)	33–39 (14)	32–30 (15)	37–40 (19)	45–48 (21)	27–31 (22)

Session 2, rounds 1 to 11

24–25 (23)	9–40 (24)	3–37 (26)	30–11 (27)	8–45 (28)	7–39 (32)	12–46 (35)	4–44 (36)	17–43 (37)	29– 2 (41)	23–34 (43)	18–38 (44)
48– 1 (23)	33–16 (24)	27–13 (26)	6–35 (27)	32–21 (28)	31–15 (32)	36–22 (35)	28–20 (36)	41–19 (37)	5–26 (41)	47–10 (43)	42–14 (44)

Session 2, rounds 12 to 22

36–23 (23)	5–44 (24)	2–38 (26)	22–41 (27)	10–43 (28)	4–42 (32)	35– 1 (35)	9–39 (36)	32– 6 (37)	13–40 (41)	45–24 (43)	27– 7 (44)
12–47 (23)	29–20 (24)	26–14 (26)	46–17 (27)	34–19 (28)	28–18 (32)	11–25 (35)	33–15 (36)	8–30 (37)	37–16 (41)	21–48 (43)	3–31 (44)

Groups 1–11 (A), 12–22 (B), 25–35 (C), 36–46 (D), 23,24/47,48 (E). Pairs' calibre 72.

28 teams, two sessions of 26 rounds

Session 1, rounds 1 to 13

16–6 (2)	19–2 (3)	28–4 (4)	14–15 (5)	25–9 (8)	22–13 (10)	5–7 (13)	17–26 (14)	24–12 (15)	21–18 (17)	8–11 (20)	23–3 (21)	27–10 (23)	20–1 (25)
44–34 (2)	47–30 (3)	56–32 (4)	42–43 (5)	53–37 (8)	50–41 (10)	33–35 (13)	45–54 (14)	52–40 (15)	49–46 (17)	36–39 (20)	51–31 (21)	55–38 (23)	48–29 (25)

Session 1, rounds 14 to 26

4–10 (2)	1–13 (4)	11–16 (5)	5–26 (7)	22–27 (9)	18–25 (10)	19–17 (12)	14–28 (14)	2– 6 (16)	24–9 (17)	23–12 (18)	20–15 (22)	3–8 (23)	21–7 (24)
32–38 (2)	29–41 (4)	39–44 (5)	33–54 (7)	50–55 (9)	46–53 (10)	47–45 (12)	42–56 (14)	30–34 (16)	52–37 (17)	51–40 (18)	48–43 (22)	31–36 (23)	49–35 (24)

Session 2, rounds 1 to 13 (add 26 to the board-sets)

44–6 (2)	47–2 (3)	28–32 (4)	14–43 (5)	53–9 (8)	50–13 (10)	33–7 (13)	17–54 (14)	52–12 (15)	21–46 (17)	36–11 (20)	51–3 (21)	27–38 (23)	48–1 (25)
16–34 (2)	19–30 (3)	56–4 (4)	42–15 (5)	25–37 (8)	22–41 (10)	5–35 (13)	45–26 (14)	24–40 (15)	49–18 (17)	8–39 (20)	23–31 (21)	55–10 (23)	20–29 (25)

Session 2, rounds 14 to 26 (add 26 to the board-sets)

32–10 (2)	29–13 (4)	11–44 (5)	5–54 (7)	50–27 (9)	18–53 (10)	19–45 (12)	42–28 (14)	30–6 (16)	52–9 (17)	51–12 (18)	20–43 (22)	31–8 (23)	49–7 (24)
4–38 (2)	1–41 (4)	39–16 (5)	33–26 (7)	22–55 (9)	46–25 (10)	47–17 (12)	14–56 (14)	2–34 (16)	24–37 (17)	23–40 (18)	48–15 (22)	3–36 (23)	21–35 (24)

Groups 1–13 (A), 14–26 (B), 29–41 (C), 42–54 (D), 27,28/55,56 (E). Pairs calibre 78.

36 teams, two sessions of 34 rounds

Session 1, rounds 1 to 17

36– 1 (1)	24–32 (2)	7– 9 (3)	33–20 (4)	26–15 (6)	14– 4 (7)	18–21 (10)	27– 5 (12)	25–13 (15)	35– 2 (19)	31–19 (21)	29– 3 (24)	11–17 (25)	12–28 (27)	23–10 (28)	22– 8 (31)	16–34 (32)	6–30 (33)
72–37 (1)	60–68 (2)	43–45 (3)	69–56 (4)	62–51 (6)	50–40 (7)	54–57 (10)	63–41 (12)	61–49 (15)	71–38 (19)	67–55 (21)	65–39 (24)	47–53 (25)	48–64 (27)	59–46 (28)	58–44 (31)	52–70 (32)	42–66 (33)

Session 1, rounds 18 to 34

18–35 (1)	33–26 (2)	16– 8 (3)	5– 1 (6)	28–27 (9)	12–13 (11)	6– 3 (12)	23–17 (14)	22–20 (16)	19–36 (19)	31–11 (20)	25–10 (21)	7– 2 (26)	34–15 (28)	21–32 (29)	30– 9 (31)	29– 4 (33)	14–24 (34)
54–71 (1)	69–62 (2)	52–44 (3)	41–37 (6)	64–63 (9)	48–49 (11)	42–39 (12)	59–53 (14)	58–56 (16)	55–72 (19)	67–47 (20)	61–46 (21)	43–38 (26)	70–51 (28)	57–68 (29)	66–45 (31)	65–40 (33)	50–60 (34)

Session 2, rounds 1 to 17 (add 34 to the board-sets)

36–37 (1)	24–68 (2)	43– 9 (3)	33–56 (4)	62–15 (6)	50– 4 (7)	18–57 (10)	63– 5 (12)	61–13 (15)	35–38 (19)	31–55 (21)	65– 3 (24)	47–17 (25)	12–64 (27)	59–10 (28)	58– 8 (31)	16–70 (32)	6–66 (33)
72– 1 (1)	60–32 (2)	7–45 (3)	69–20 (4)	26–51 (6)	14–40 (7)	54–21 (10)	27–41 (12)	25–49 (15)	71– 2 (19)	67–19 (21)	29–39 (24)	11–53 (25)	48–28 (27)	23–46 (28)	22–44 (31)	52–34 (32)	42–30 (33)

Session 2, rounds 18 to 34 (add 34 to the board-sets)

54–35 (1)	33–62 (2)	52– 8 (3)	41– 1 (6)	28–63 (9)	48–13 (11)	42– 3 (12)	59–17 (14)	22–56 (16)	55–36 (19)	67–11 (20)	61–10 (21)	43– 2 (26)	70–15 (28)	21–68 (29)	66– 9 (31)	65– 4 (33)	14–60 (34)
18–71 (1)	69–26 (2)	16–44 (3)	5–37 (6)	64–27 (9)	12–49 (11)	6–39 (12)	23–53 (14)	58–20 (16)	19–72 (19)	31–47 (20)	25–46 (21)	7–38 (26)	34–51 (28)	57–32 (29)	30–45 (31)	29–40 (33)	50–24 (34)

Groups 1–17 (A), 18–34 (B), 37–53 (C), 54–70 (D), 35,36/71,72 (E). Pairs' calibre 85.

40 teams, two sessions of 38 rounds

Session 1, rounds 1 to 19

40–1 (1)	12–30 (3)	8–35 (4)	7–22 (9)	18–13 (11)	29–4 (13)	25–9 (14)	21–33 (15)	36–19 (16)	5–15 (19)	24–32 (20)	23–38 (25)	34–10 (27)	26–20 (29)	3–6 (30)	37–11 (31)	14–16 (32)	2–31 (35)	39–17 (36)	28–27 (38)
80–41 (1)	52–70 (3)	48–75 (4)	47–62 (9)	58–53 (11)	69–44 (13)	65–49 (14)	61–73 (15)	76–59 (16)	45–55 (19)	64–72 (20)	63–78 (25)	74–50 (27)	66–60 (29)	43–46 (30)	77–51 (31)	54–56 (32)	42–71 (35)	79–57 (36)	68–67 (38)

Session 1, rounds 20 to 38

20–39 (1)	11–31 (3)	16–27 (4)	3–26 (9)	32–37 (11)	23–10 (13)	28–6 (14)	14–2 (15)	38–17 (16)	34–24 (19)	13–5 (20)	19–4 (25)	29–15 (27)	1–7 (29)	25–22 (30)	30–18 (31)	35–33 (32)	12–21 (35)	36–40 (36)	8–9 (38)
60–79 (1)	51–71 (3)	56–67 (4)	43–66 (9)	72–77 (11)	63–50 (13)	68–46 (14)	54–42 (15)	78–57 (16)	74–64 (19)	53–45 (20)	59–44 (25)	69–55 (27)	41–47 (29)	65–62 (30)	70–58 (31)	75–73 (32)	52–61 (35)	76–80 (36)	48–49 (38)

Session 2, rounds 1 to 19 (add 38 to the board-sets)

40–41 (1)	12–70 (3)	8–75 (4)	7–62 (9)	58–13 (11)	69–4 (13)	65–9 (14)	21–73 (15)	76–19 (16)	45–15 (19)	24–72 (20)	23–78 (25)	74–10 (27)	26–60 (29)	43–6 (30)	77–11 (31)	54–16 (32)	2–71 (35)	39–57 (36)	28–67 (38)
80–1 (1)	52–30 (3)	48–35 (4)	47–22 (9)	18–53 (11)	29–44 (13)	25–49 (14)	61–33 (15)	36–59 (16)	5–55 (19)	64–32 (20)	63–38 (25)	34–50 (27)	66–20 (29)	3–46 (30)	37–51 (31)	14–56 (32)	42–31 (35)	79–17 (36)	68–27 (38)

Session 2, rounds 20 to 38 (add 38 to the board-sets)

60–39 (1)	11–71 (3)	16–67 (4)	3–66 (9)	32–77 (11)	63–10 (13)	68–6 (14)	54–2 (15)	78–17 (16)	34–64 (19)	53–5 (20)	59–4 (25)	69–15 (27)	41–7 (29)	25–62 (30)	70–18 (31)	35–73 (32)	12–61 (35)	76–40 (36)	48–9 (38)
20–79 (1)	51–31 (3)	56–27 (4)	43–26 (9)	72–37 (11)	23–50 (13)	28–46 (14)	14–42 (15)	38–57 (16)	74–24 (19)	13–45 (20)	19–44 (25)	29–55 (27)	1–47 (29)	65–22 (30)	30–58 (31)	75–33 (32)	52–21 (35)	36–80 (36)	8–49 (38)

Groups 1–19 (A), 20–38 (B), 41–59 (C), 60–78 (D), 39,40/79,80 (E). Pairs' calibre 84.

10.5 Two sessions, half odd

Mathematics are well and good but nature keeps dragging us around by the nose.
Albert Einstein

This technique applies where half the number of teams is an odd number (10, 14, 18 …). In principle the method is similar to the half-even numbers of teams, doubling up the schedule for half the number of teams.

Consider 10 teams where we use the 5-teams semi-cyclic schedule twice, once in session one and then again in session two.

Session one.
For teams 1 to 5 the original 5-team schedule applies. Pair 1 follows 8, 9 and 10 are stationary and all others follow the number one less than theirs. Tables 1 and 2 share board-sets and are played in sequence. Table 5 plays the two highest board-sets (9 and 10) alternately. Pairs are grouped 1 to 4 and 5 to 8 with the teams made up from 1 and 5, 2 and 6, etc., with 9 and 10 the last team. Each pair misses two board-sets out of the 10.

The same applies for teams 6 to 10. To achieve this add 10 to the pair numbers in the second instance of the schedule. Tables 6 and 7 share board-sets and are played in sequence. Table 10 plays the two highest board-sets (9 and 10) alternately. Pairs swap sides after round 4, but the cycle is consistent. Pairs are grouped 11 to 14 and 15 to 18, with the teams made up from 11 and 15, 12 and 16, etc., with 19 and 20 the last team.

Session two.
The schedule is the same, consisting of the same two 5-team schedules. The difference here is that half the pairs swap places between the two schedules so that teams 1 to 5 meet 6 to 10. The make up of the teams is the same and the pairs follow each other as before. In the first session the pairs remained in one side of the movement. In the second session they may move about all 10 tables. A new set-of-boards is introduced for the second session.

The last two rounds have the teams meeting the last teams and pairs they have not played. They also play the remaining board-sets. All 10 board-sets are played in this session. The complete schedule for 10 teams follows.

10 teams, two sessions

Session one.

	1	2	3	4	5	6	7	8	9	10
1	10–1 (1)	5–9 (1)	3–4 (2)	6–8 (7)	2–7 (9)	20–11 (1)	15–19 (1)	13–14 (2)	16–18 (7)	12–17 (9)
2	10–2 (2)	6–9 (2)	4–5 (3)	7–1 (8)	3–8 (10)	20–12 (2)	16–19 (2)	14–15 (3)	17–11 (8)	13–18 (10)
3	10–3 (3)	7–9 (3)	5–6 (4)	8–2 (5)	4–1 (9)	20–13 (3)	17–19 (3)	15–16 (4)	18–12 (5)	14–11 (9)
4	10–4 (4)	8–9 (4)	6–7 (1)	1–3 (6)	5–2 (10)	20–14 (4)	18–19 (4)	16–17 (1)	11–13 (6)	15–12 (10)
5	5–10 (5)	9–1 (5)	8–7 (2)	4–2 (7)	3–6 (9)	15–20 (5)	19–11 (5)	18–17 (2)	14–12 (7)	13–16 (9)
6	6–10 (6)	9–2 (6)	1–8 (3)	5–3 (8)	4–7 (10)	16–20 (6)	19–12 (6)	11–18 (3)	15–13 (8)	14–17 (10)
7	7–10 (7)	9–3 (7)	2–1 (4)	6–4 (5)	5–8 (9)	17–20 (7)	19–13 (7)	12–11 (4)	16–14 (5)	15–18 (9)
8	8–10 (8)	9–4 (8)	3–2 (1)	7–5 (6)	6–1 (10)	18–20 (8)	19–14 (8)	13–12 (1)	17–15 (6)	16–11 (10)

Session two.

	1	2	3	4	5	6	7	8	9	10
1	10–11 (1)	5–19 (1)	3–14 (2)	6–18 (7)	2–17 (9)	20–1 (1)	15–9 (1)	13–4 (2)	16–8 (7)	12–7 (9)
2	10–12 (2)	6–19 (2)	4–15 (3)	7–11 (8)	3–18 (10)	20–2 (2)	16–9 (2)	14–5 (3)	17–1 (8)	13–8 (10)
3	10–13 (3)	7–19 (3)	5–16 (4)	8–12 (5)	4–11 (9)	20–3 (3)	17–9 (3)	15–6 (4)	18–2 (5)	14–1 (9)
4	10–14 (4)	8–19 (4)	6–17 (1)	1–13 (6)	5–12 (10)	20–4 (4)	18–9 (4)	16–7 (1)	11–3 (6)	15–2 (10)
5	15–10 (5)	19–1 (5)	18–7 (2)	14–2 (7)	13–6 (9)	5–20 (5)	9–11 (5)	8–17 (2)	4–12 (7)	3–16 (9)
6	16–10 (6)	19–2 (6)	11–8 (3)	15–3 (8)	14–7 (10)	6–20 (6)	9–12 (6)	1–18 (3)	5–13 (8)	4–17 (10)
7	17–10 (7)	19–3 (7)	12–1 (4)	16–4 (5)	15–8 (9)	7–20 (7)	9–13 (7)	2–11 (4)	6–14 (5)	5–18 (9)
8	18–10 (8)	19–4 (8)	13–2 (1)	17–5 (6)	16–1 (10)	8–20 (8)	9–14 (8)	3–12 (1)	7–15 (6)	6–11 (10)
9	1–11 (2)	2–12 (3)	3–13 (4)	4–14 (1)	15–5 (2)	16–6 (3)	17–7 (4)	18–8 (1)	9–19 (9)	20–10 (9)
10	11–5 (7)	12–6 (8)	13–7 (5)	14–8 (6)	1–15 (7)	2–16 (8)	3–17 (5)	4–18 (6)	19–10 (10)	9–20 (10)

This schedule can be depicted as follows.
8 and 10 rounds. The pairs switch after round 4 in both sessions. Pairs' calibre 64.

Teams	1	2	3	4	5	6	7	8	9	10
Pairs	1, 5	2, 6	3, 7	4, 8	9, 10	11, 15	12, 16	13, 17	14, 18	19, 20

Session 1

10–1 (1)	5–9 (1)	3–4 (2)	6–8 (7)	2–7 (9)	20–11 (1)	15–19 (1)	13–14 (2)	16–18 (7)	12–17 (9)

Session 2

10–11 (1)	5–19 (1)	3–14 (2)	6–18 (7)	2–17 (9)	20–1 (1)	15–9 (1)	13–4 (2)	16–8 (7)	12–7 (9)

Last two rounds

1–11 (2)	2–12 (3)	3–13 (4)	4–14 (1)	15–5 (2)	16–6 (3)	17–7 (4)	18–8 (1)	9–19 (9)	20–10 (9)
11–5 (7)	12–6 (8)	13–7 (5)	14–8 (6)	1–15 (7)	2–16 (8)	3–17 (5)	4–18 (6)	19–10 (10)	9–20 (10)

14 teams, two sessions

12 and 14 rounds, switch after round 6 except tables 4 and 11 switch after rounds 5 and 11. Pairs' calibre 70.

Teams	1	2	3	4	5	6	7
Pairs	1, 7	2, 8	3, 9	4, 10	5, 11	6, 12	13, 14

Teams	8	9	10	11	12	13	14
Pairs	15, 21	16, 22	17, 23	18, 24	19, 25	20, 26	27, 28

Session 1

1	2	3	4	5	6	7
14–1 (1)	5–6 (3)	9–11 (4)	13–2 (8)	4–12 (9)	7–10 (11)	3–8 (13)

8	9	10	11	12	13	14
28–15 (1)	19–20 (3)	23–25 (4)	27–16 (8)	18–26 (9)	21–24 (11)	17–22 (13)

Session 2

1	2	3	4	5	6	7
14–15 (1)	5–20 (3)	9–25 (4)	13–16 (8)	4–26 (9)	7–24 (11)	3–22 (13)

8	9	10	11	12	13	14
28–1 (1)	19–6 (3)	23–11 (4)	27–2 (8)	18–12 (9)	21–10 (11)	17–8 (13)

Last two rounds

1–15 (3)	2–16 (4)	3–17 (5)	4–18 (6)	5–19 (1)	6–20 (2)	21–7 (3)
15–7 (9)	16–8 (10)	17–9 (11)	18–10 (12)	19–11 (7)	20–12 (8)	1–21 (9)

22–8 (4)	23–9 (5)	24–10 (6)	25–11 (1)	26–12 (2)	13–27 (13)	28–14 (13)
2–22 (10)	3–23 (11)	4–24 (12)	5–25 (7)	6–26 (8)	27–14 (14)	13–28 (14)

18 teams, two sessions

16 and 18 rounds, switch after round 8. Pairs' calibre 74.

Teams	1	2	3	4	5	6	7	8	9
Pairs	1, 9	2, 10	3, 11	4, 12	5, 13	6, 14	7, 15	8, 16	17, 18

Teams	10	11	12	13	14	15	16	17	18
Pairs	19, 27	20, 28	21, 29	22, 30	23, 31	24, 32	25, 33	26, 34	35, 36

Session 1

1	2	3	4	5	6	7	8	9
18–1 (1)	9–17 (1)	12–15 (3)	5–16 (3)	10–11 (4)	3–7 (13)	2–4 (15)	8–14 (15)	6–13 (17)

10	11	12	13	14	15	16	17	18
36–19 (1)	27–35 (1)	30–33 (3)	23–34 (3)	28–29 (4)	21–25 (13)	20–22 (15)	26–32 (15)	24–31 (17)

Session 2

1	2	3	4	5	6	7	8	9
18–19 (1)	9–35 (1)	12–33 (3)	5–34 (3)	10–29 (4)	3–25 (13)	2–22 (15)	8–32 (15)	6–31 (17)

10	11	12	13	14	15	16	17	18
36–1 (1)	27–17 (1)	30–15 (3)	23–16 (3)	28–11 (4)	21–7 (13)	20–4 (15)	26–14 (15)	24–13 (17)

Last two rounds

1–19 (6)	2–20 (7)	3–21 (8)	4–22 (1)	5–23 (2)	6–24 (3)	7–25 (4)	8–26 (5)
19–9 (13)	20–10 (14)	21–11 (15)	22–12 (16)	23–13 (9)	24–14 (10)	25–15 (11)	26–16 (12)

27–9 (6)	28–10 (7)	29–11 (8)	30–12 (1)	31–13 (2)	32–14 (3)	33–15 (4)	34–16 (5)	17–35 (17)	36–18 (17)
1–27 (13)	2–28 (14)	3–29 (15)	4–30 (16)	5–31 (9)	6–32 (10)	7–33 (11)	8–34 (12)	35–18 (18)	17–36 (18)

22 teams, two sessions

20 and 22 rounds, switch after round 10 except tables 6 and 17 switch after 9 and 19. Pairs' calibre 75.

Teams	1	2	3	4	5	6	7	8	9	10	11
Pairs	1, 11	2, 12	3, 13	4, 14	5, 15	6, 16	7, 17	8, 18	9, 19	10, 20	21, 22

Teams	12	13	14	15	16	17	18	19	20	21	22
Pairs	23, 33	24, 34	25, 35	26, 36	27, 37	28, 38	29, 39	30, 40	31, 41	32, 42	43, 44

Session 1

1	2	3	4	5	6	7	8	9	10	11
22–1 (1)	10–17 (3)	8–13 (5)	16–19 (7)	4–5 (9)	21–2 (12)	14–18 (13)	7–9 (15)	3–15 (17)	6–12 (19)	11–20 (21)

12	13	14	15	16	17	18	19	20	21	22
44–23 (1)	32–39 (3)	30–35 (5)	38–41 (7)	26–27 (9)	43–24 (12)	36–40 (13)	29–31 (15)	25–37 (17)	28–34 (19)	33–42 (21)

Session 2

1	2	3	4	5	6	7	8	9	10	11
22–23 (1)	10–39 (3)	8–35 (5)	16–41 (7)	4–27 (9)	21–24 (12)	14–40 (13)	7–31 (15)	3–37 (17)	6–34 (19)	11–42 (21)

12	13	14	15	16	17	18	19	20	21	22
44–1 (1)	32–17 (3)	30–13 (5)	38–19 (7)	26–5 (9)	43–2 (12)	36–18 (13)	29–9 (15)	25–15 (17)	28–12 (19)	33–20 (21)

Last two rounds

1–23 (10)	2–24 (1)	3–25 (2)	4–26 (3)	5–27 (4)	6–28 (5)	7–29 (6)	8–30 (7)	9–31 (8)	10–32 (9)
23–11 (12)	24–12 (13)	25–13 (14)	26–14 (15)	27–15 (16)	28–16 (17)	29–17 (18)	30–18 (19)	31–19 (20)	32–20 (11)

33–11 (10)	34–12 (1)	35–13 (2)	36–14 (3)	37–15 (4)	38–16 (5)	39–17 (6)	40–18 (7)	41–19 (8)	42–20 (9)	21–43 (21)	44–22 (21)
1–33 (12)	2–34 (13)	3–35 (14)	4–36 (15)	5–37 (16)	6–38 (17)	7–39 (18)	8–40 (19)	9–41 (20)	10–42 (11)	43–22 (22)	21–44 (22)

10.6 Two sessions, odd numbers of teams

I never did very well in math –
I could never seem to persuade the teacher that I hadn't meant my answers literally.
Calvin Trillin

Two sessions, technique one

The first type of movement is the two-session or two-stanza event where the number of teams is 1 modulus 4 (5, 9, 13…).

5 teams, two sessions of 4 rounds

Each session is over 4 rounds, completing 4 half matches and playing half the pairs. New board-sets are introduced for the second session. In the schedule below sets 1 to 5 are used in session 1 and 6 to 10 in session 2. Unfortunately the schedules for the pairs and boards are not cyclic. In fact this is a bicyclic

schedule, but the significance is lost because of the two round cycles. Compare this with the 9-team two-session schedule below.

Pairs are grouped 1 to 4 and 5 to 8 (1 and 5 are team mates, etc.,) plus 9 and 10 for the last team. A new set-of-boards is introduced for session 2. There are 5 board-sets played in each session, with each team not playing one set in each session. Pairs' calibre 79.

Session 1, rounds 1 to 4.

	1	2	3	4	5
1	10–1 (1)	5–9 (1)	7–4 (2)	2–3 (3)	6–8 (4)
2	10–2 (2)	6–9 (2)	8–5 (3)	3–4 (1)	7–1 (5)
3	10–5 (4)	1–9 (4)	8–3 (2)	7–6 (3)	4–2 (4)
4	10–6 (5)	2–9 (5)	1–4 (3)	8–7 (1)	5–3 (5)

Session 2, rounds 5 to 8.

	1	2	3	4	5
5	10–3 (8)	7–9 (8)	1–6 (7)	4–5 (8)	8–2 (10)
6	10–4 (9)	8–9 (9)	2–7 (6)	5–6 (9)	1–3 (10)
7	10–7 (7)	3–9 (7)	2–5 (7)	1–8 (8)	6–4 (10)
8	10–8 (6)	4–9 (6)	3–6 (6)	2–1 (9)	7–5 (10)

This is another example. Here the pairs are grouped in a different way and only two copies of each board are required (or only two tables share a set). The disadvantage is the pair and board schedules are haphazard though the boards are very predictable at any one table. Clearly guide-cards are needed, but that is really no different to the movement above. This movement will be revisited for higher numbers of teams and was originally developed by Wal Wallis for that specific purpose. Calibre 73.

Teams	1	2	3	4	5
Pairs	1,6	2,7	3,8	4,9	5,10

Session 1, rounds 1 to 4

	1	2	3	4	5
1	1–7 (1)	2–8 (3)	3–9 (5)	4–10 (1)	5–6 (3)
2	2–6 (1)	7–9 (4)	4–3 (2)	8–5 (4)	1–10 (3)
3	2–1 (2)	3–7 (3)	4–8 (5)	5–9 (1)	6–10 (5)
4	6–7 (2)	4–2 (4)	8–9 (2)	10–3 (4)	5–1 (5)

Session 2, rounds 5 to 8

	1	2	3	4	5
5	1–8 (6)	3–5 (8)	10–7 (10)	2–9 (6)	4–6 (8)
6	3–6 (6)	5–4 (9)	10–2 (7)	7–8 (9)	1–9 (8)
7	3–1 (7)	10–8 (8)	2–5 (10)	4–7 (6)	6–9 (10)
8	6–8 (7)	9–10 (9)	7–5 (7)	3–2 (9)	4–1 (10)

9 teams, two sessions of 8 rounds

The pairs' schedule for 9 teams is bicyclic, but the board-sets are not. Even so, this movement is fairly friendly, particularly if tables 1 and 2 share boards. Typically this would be conducted with 3 boards per round so that sharing is not an issue. Using 4 copies of the boards, tables 1 and 2 can share one copy and play the same boards on each round. The other tables do not need to share on each round, but the second copy is used at tables 3 and 4, the third at tables 5 and 6 while the last on 7 and 8. All tables are given their board-sets at start of play where they remain for the whole event. Tables 1 and 2 receive 8 board-sets (shared), the last table (9) uses only 3 board-sets though they can have 4 (two copies of 5) and the rest of the tables 4 board-sets.

Pairs are grouped 1 to 8 and 9 to 16 (1 and 9 are team mates, etc.,) plus 17 and 18 for the last team. Pair movement is bicyclic with each cycle shown within the dark boarder. New starting positions are needed on rounds 1, 5, 9 and 13. A new set-of-boards is introduced for session 2. There are 9 board-sets played in each session with each team not playing one set in each session. Calibre for pairs is 79.

Session one

	1	2	3	4	5	6	7	8	9
1	18–1 (1)	9–17 (1)	6–7 (2)	12–14 (6)	2–5 (1)	11–15 (6)	3–8 (5)	10–16 (9)	4–13 (5)
2	18–2 (2)	10–17 (2)	7–8 (3)	13–15 (7)	3–6 (7)	12–16 (2)	4–9 (7)	11–1 (2)	5–14 (9)
3	18–3 (3)	11–17 (3)	8–9 (4)	14–16 (8)	4–7 (8)	13–1 (3)	5–10 (8)	12–2 (3)	6–15 (5)
4	18–4 (4)	12–17 (4)	9–10 (5)	15–1 (9)	5–8 (6)	14–2 (4)	6–11 (1)	13–3 (4)	7–16 (1)
5	18–9 (6)	1–17 (6)	15–14 (2)	6–4 (6)	13–10 (1)	7–3 (6)	16–11 (5)	8–2 (9)	5–12 (5)
6	18–10 (7)	2–17 (7)	16–15 (3)	7–5 (7)	14–11 (7)	8–4 (2)	1–12 (7)	9–3 (2)	6–13 (9)
7	18–11 (8)	3–17 (8)	1–16 (4)	8–6 (8)	15–12 (8)	9–5 (3)	2–13 (8)	10–4 (3)	7–14 (5)
8	18–12 (9)	4–17 (9)	2–1 (5)	9–7 (9)	16–13 (6)	10–6 (4)	3–14 (1)	11–5 (4)	8–15 (1)

Session two

	1	2	3	4	5	6	7	8	9
9	18–5 (10)	13–17 (10)	10–11 (13)	16–2 (16)	6–9 (10)	15–3 (18)	7–12 (14)	14–4 (13)	8–1 (14)
10	18–6 (11)	14–17 (11)	11–12 (15)	1–3 (17)	7–10 (15)	16–4 (12)	8–13 (17)	15–5 (16)	9–2 (18)
11	18–7 (12)	15–17 (12)	12–13 (18)	2–4 (11)	8–11 (11)	1–5 (13)	9–14 (12)	16–6 (15)	10–3 (14)
12	18–8 (13)	16–17 (13)	13–14 (14)	3–5 (12)	9–12 (16)	2–6 (17)	10–15 (10)	1–7 (11)	11–4 (10)
13	18–13 (15)	5–17 (15)	3–2 (13)	10–8 (16)	1–14 (10)	11–7 (18)	4–15 (14)	12–6 (13)	9–16 (14)
14	18–14 (16)	6–17 (16)	4–3 (15)	11–9 (17)	2–15 (15)	12–8 (12)	5–16 (17)	13–7 (16)	10–1 (18)
15	18–15 (17)	7–17 (17)	5–4 (18)	12–10 (11)	3–16 (11)	13–9 (13)	6–1 (12)	14–8 (15)	11–2 (14)
16	18–16 (18)	8–17 (18)	6–5 (14)	13–11 (12)	4–1 (16)	14–10 (17)	7–2 (10)	15–9 (11)	12–3 (10)

13 teams, two sessions of 12 rounds

This is a particularly difficult problem for the pure mathematicians. With two sessions there are 2 order 6 arrays plus one team. The number 6 immediately raises the red flag. I presented this problem to Wal Wallis early in 2011 and he came up with a solution to 17 teams (see next) with the following comments: 'I realize that this is not the most useful number. However, I hope this will generalize to numbers of the form $4n + 1$. However, some modifications will be needed for 13, I haven't got it to work yet'. No success in this quest as we went to the printing of this book.

17 teams, two sessions of 16 rounds

This is based on the Wal Wallis 5-teams schedule above. See *Constructing 17 teams CCRS over 2 sessions* in appendix C, page 381. Unfortunately pairs and boards move haphazardly. Board-sets 1–22 are played in session 1 and 23–44 in session 2. Each team misses 6 board-sets per session. Board-sets 21, 22, 43 and 44 are played 32 times, while all others are played 24 times. Calibre 79 with each session alone 42.

Teams:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Pairs:	1,6	2,7	3,8	4,9	5,10	11,15	12,16	13,17	14,18	19,23	20,24	21,25	22,26	27,31	28,32	29,33	30,34

Session one

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1–7 (1)	2–8 (3)	3–9 (5)	4–10 (1)	5–6 (3)	11–27 (1)	13–30 (2)	21–14 (6)	22–12 (7)	28–19 (9)	29–20 (10)	31–15 (1)	34–17 (2)	18–25 (6)	16–26 (7)	23–32 (9)	24–33 (10)
2	2–6 (1)	7–9 (4)	4–3 (2)	8–5 (4)	1–10 (3)	11–31 (2)	13–34 (1)	21–18 (7)	22–16 (6)	28–23 (21)	29–24 (22)	27–15 (2)	30–17 (1)	14–25 (7)	12–26 (6)	19–32 (21)	20–33 (22)
3	2–1 (2)	3–7 (3)	4–8 (5)	5–9 (1)	6–10 (5)	14–29 (1)	12–28 (2)	22–13 (5)	21–11 (8)	30–20 (9)	27–19 (10)	33–18 (1)	32–16 (2)	17–26 (5)	15–25 (8)	24–34 (9)	23–31 (10)
4	6–7 (2)	4–2 (4)	8–9 (2)	10–3 (4)	5–1 (5)	14–33 (2)	12–32 (1)	22–17 (8)	21–15 (5)	30–24 (21)	27–23 (22)	29–18 (2)	28–16 (1)	13–26 (8)	11–25 (5)	20–34 (21)	19–31 (22)
5	1–8 (6)	3–5 (8)	10–7 (10)	2–9 (6)	4–6 (8)	13–29 (3)	11–28 (4)	20–14 (5)	19–12 (8)	27–21 (9)	30–22 (10)	33–17 (3)	32–15 (4)	18–24 (5)	16–23 (8)	25–31 (9)	26–34 (10)
6	3–6 (6)	5–4 (9)	10–2 (7)	7–8 (9)	1–9 (8)	13–33 (4)	11–32 (3)	20–18 (8)	19–16 (5)	27–25 (21)	30–26 (22)	29–17 (4)	28–15 (3)	14–24 (8)	12–23 (5)	21–31 (21)	22–34 (22)
7	3–1 (7)	10–8 (8)	2–5 (10)	4–7 (6)	6–9 (10)	12–27 (3)	14–30 (4)	19–13 (6)	20–11 (7)	29–22 (9)	28–21 (10)	31–16 (3)	34–18 (4)	17–23 (6)	15–24 (7)	26–33 (9)	25–32 (10)
8	6–8 (7)	9–10 (9)	7–5 (7)	3–2 (9)	4–1 (10)	12–31 (4)	14–34 (3)	19–17 (7)	20–15 (6)	29–26 (21)	28–25 (22)	27–16 (4)	30–18 (3)	13–23 (7)	11–24 (6)	22–33 (21)	21–32 (22)
9	1–31 (11)	27–32 (13)	28–33 (15)	29–34 (11)	30–6 (13)	20–5 (13)	21–3 (14)	11–19 (15)	14–22 (17)	4–12 (20)	2–13 (22)	10–24 (13)	8–25 (14)	23–15 (15)	26–18 (17)	16–9 (20)	17–7 (22)
10	27–6 (11)	31–33 (14)	29–28 (12)	32–30 (14)	1–34 (13)	20–10 (14)	21–8 (13)	11–23 (17)	14–26 (15)	4–16 (21)	2–17 (19)	5–24 (14)	3–25 (13)	19–15 (17)	22–18 (15)	12–9 (21)	13–7 (19)
11	27–1 (12)	28–31 (13)	29–32 (15)	30–33 (11)	6–34 (15)	22–2 (13)	19–4 (14)	12–20 (16)	13–21 (17)	5–11 (21)	3–14 (22)	7–26 (13)	9–23 (14)	24–16 (16)	25–17 (17)	15–10 (21)	18–8 (22)
12	6–31 (12)	29–27 (14)	32–33 (12)	34–28 (14)	30–1 (15)	22–7 (14)	19–9 (13)	12–24 (17)	13–25 (16)	5–15 (20)	3–18 (19)	2–26 (14)	4–23 (13)	20–16 (17)	21–17 (16)	11–10 (20)	14–8 (19)
13	1–32 (16)	28–30 (18)	34–31 (20)	27–33 (16)	29–6 (18)	22–3 (11)	19–5 (12)	12–21 (15)	13–20 (18)	2–14 (21)	4–11 (22)	8–26 (11)	10–23 (12)	25–16 (15)	24–17 (18)	18–7 (21)	15–9 (22)
14	28–6 (16)	30–29 (19)	34–27 (17)	31–32 (19)	1–33 (18)	22–8 (12)	19–10 (11)	12–25 (18)	13–24 (15)	2–18 (20)	4–15 (19)	3–26 (12)	5–23 (11)	21–16 (18)	20–17 (15)	14–7 (20)	11–9 (19)
15	28–1 (17)	34–32 (18)	27–30 (20)	29–31 (16)	6–33 (20)	20–4 (11)	21–2 (12)	11–22 (16)	14–19 (18)	3–13 (20)	5–12 (22)	9–24 (11)	7–25 (12)	26–15 (16)	23–18 (18)	17–8 (20)	16–10 (22)
16	6–32 (17)	33–34 (19)	31–30 (17)	28–27 (19)	29–1 (20)	20–9 (12)	21–7 (11)	11–26 (18)	14–23 (16)	3–17 (21)	5–16 (19)	4–24 (12)	2–25 (11)	22–15 (18)	19–18 (16)	13–8 (21)	12–10 (19)

Session two

17	1–15 (23)	11–16 (25)	12–17 (27)	13–18 (23)	14–6 (25)	2–19 (23)	4–22 (24)	29–5 (28)	27–3 (29)	20–28 (31)	21–30 (32)	23–7 (23)	26–9 (24)	10–33 (28)	8–31 (29)	32–24 (31)	34–25 (32)
18	11–6 (23)	15–17 (26)	13–12 (24)	16–14 (26)	1–18 (25)	2–23 (24)	4–26 (23)	29–10 (29)	27–8 (28)	20–32 (43)	21–34 (44)	19–7 (24)	22–9 (23)	5–33 (29)	3–31 (28)	28–24 (43)	30–25 (44)
19	11–1 (24)	12–15 (25)	13–16 (27)	14–17 (23)	6–18 (27)	5–21 (23)	3–20 (24)	27–4 (27)	29–2 (30)	19–30 (31)	22–28 (32)	25–10 (23)	24–8 (24)	9–31 (27)	7–33 (30)	34–23 (31)	32–26 (32)
20	6–15 (24)	13–11 (26)	16–17 (24)	18–12 (26)	14–1 (27)	5–25 (24)	3–24 (23)	27–9 (30)	29–7 (27)	19–34 (43)	22–32 (44)	21–10 (24)	20–8 (23)	4–31 (30)	2–33 (27)	30–23 (43)	28–26 (44)
21	1–16 (28)	12–14 (30)	18–15 (32)	11–17 (28)	13–6 (30)	4–21 (25)	2–20 (26)	28–5 (27)	30–3 (30)	22–27 (31)	19–29 (32)	25–9 (25)	24–7 (26)	10–32 (27)	8–34 (30)	31–26 (31)	33–23 (32)
22	12–6 (28)	14–13 (31)	18–11 (29)	15–16 (31)	1–17 (30)	4–25 (26)	2–24 (25)	28–10 (30)	30–8 (27)	22–31 (43)	19–33 (44)	21–9 (26)	20–7 (25)	5–32 (30)	3–34 (27)	27–26 (43)	29–23 (44)
23	12–1 (29)	18–16 (30)	11–14 (32)	13–15 (28)	6–17 (32)	3–19 (25)	5–22 (26)	30–4 (28)	28–2 (29)	21–29 (31)	20–27 (32)	23–8 (25)	26–10 (26)	9–34 (28)	7–32 (29)	33–25 (31)	31–24 (32)
24	6–16 (29)	17–18 (31)	15–14 (29)	12–11 (31)	13–1 (32)	3–23 (26)	5–26 (25)	30–9 (29)	28–7 (28)	21–33 (43)	20–31 (44)	19–8 (26)	22–10 (25)	4–34 (29)	2–32 (28)	29–25 (43)	27–24 (44)
25	1–23 (33)	19–24 (35)	20–25 (37)	21–26 (33)	22–6 (35)	14–28 (35)	12–29 (36)	2–27 (37)	5–30 (39)	11–3 (41)	13–4 (43)	32–18 (35)	33–16 (36)	31–7 (37)	34–10 (39)	8–15 (41)	9–17 (43)
26	19–6 (33)	23–25 (36)	21–20 (34)	24–22 (36)	1–26 (35)	14–32 (36)	12–33 (35)	2–31 (39)	5–34 (37)	11–8 (44)	13–9 (42)	28–18 (36)	29–16 (35)	27–7 (39)	30–10 (37)	3–15 (44)	4–17 (42)
27	19–1 (34)	20–23 (35)	21–24 (37)	22–25 (33)	6–26 (37)	11–30 (35)	13–27 (36)	3–28 (38)	4–29 (40)	12–2 (41)	14–5 (42)	34–15 (35)	31–17 (36)	32–8 (38)	33–9 (40)	7–16 (41)	10–18 (42)
28	6–23 (34)	21–19 (36)	24–25 (34)	26–20 (36)	22–1 (37)	11–34 (36)	13–31 (35)	3–32 (40)	4–33 (38)	12–7 (44)	14–10 (43)	30–15 (36)	27–17 (35)	28–8 (40)	29–9 (38)	2–16 (44)	5–18 (43)
29	1–24 (38)	20–22 (40)	26–23 (42)	19–25 (38)	21–6 (40)	12–30 (33)	14–27 (34)	3–29 (37)	4–28 (39)	13–5 (41)	11–2 (42)	34–16 (33)	31–18 (34)	33–8 (37)	32–9 (39)	10–17 (41)	7–15 (42)
30	20–6 (38)	22–21 (41)	26–19 (39)	23–24 (41)	1–25 (40)	12–34 (34)	14–31 (33)	3–33 (39)	4–32 (37)	13–10 (44)	11–7 (43)	30–16 (34)	27–18 (33)	29–8 (39)	28–9 (37)	5–17 (44)	2–15 (43)
31	20–1 (39)	26–24 (40)	19–22 (42)	21–23 (38)	6–25 (42)	13–28 (33)	11–29 (34)	2–30 (38)	5–27 (40)	14–4 (41)	12–3 (43)	32–17 (33)	33–15 (34)	34–7 (38)	31–10 (40)	9–18 (41)	8–16 (43)
32	6–24 (39)	25–26 (41)	23–22 (39)	20–19 (41)	21–1 (42)	13–32 (34)	11–33 (33)	2–34 (40)	5–31 (38)	14–9 (44)	12–8 (42)	28–17 (34)	29–15 (33)	30–7 (40)	27–10 (38)	4–18 (44)	3–16 (42)

Two sessions, technique two

The following address the same issue of the larger numbers of teams requiring unmanageable numbers of rounds for one session of bridge. Similar to the even numbers of teams this weakness can be overcome by using the smaller-numbered schedules, doubled up. Once an even-numbered movement is used twice, we need to eliminate one team so that the resulting number of teams is odd. This applies where $(n + 1)/2$ is an even number (7, 11, 15…).

Consider the 6-teams schedule. Using this twice we have 12 teams, but by eliminating one team we are left with 11 teams and 20 rounds, playing all the other teams in the event over 20 rounds. The schedules are identical to those used for 6 teams but the groups of pairs are extended. The pairs following each other in the schedules are 1–5 (A), 6–10 (B), 13–17 (C), 18–22 (D) with 11, 12 (E) stationary. Pairs 23 and 24 are eliminated from the original 12-team schedule.

One problem with this approach is the need to introduce an extra board-set that is not played by all teams. This set is used at one table occupied by the stationary pairs. In practice board-sets 1 to 15 are used in the first stanza and 16 to 30 in the second. Note also that in all cases you can use the same board numbers in each stanza, but of course the boards must be re-dealt. For 19 teams or more the schedules are shown that way.

Groups A and B form one lot of teams, C and D a second lot with E the last. One minor disadvantage is that each pair plays against one other pair twice. It is not possible to have such a movement where this match is played with the other pair in the rematch. Pairs 11 and 12 (the stationary team) do not suffer this problem. The other weakness, mentioned above, is that all teams do not play all the boards, consequently the pairs' calibre is very poor.

11 teams, interwoven two session, based on H(10, 12)

Each row represents a board-set and each column a round of play or half-match. After round 10 a new session commences and new boards are introduced. See appendix N for more information (page 419).

		β,5	2,4' 7,9'	4,2' 9,7'		1,3 6,8	5',α		3',1' 8',6'										
5,3' 10,8'			β,1	3,5' 8,10'	4',2' 9',7'		2,4 7,9	1',α											
4,1' 9,6'	1,4' 6,9'			β,2		5',3' 10',8'		3,5 8,10	2',α										
β,3	5,2' 10,7'	2,5' 7,10'			3',α		1',4' 6',9'		4,1 9,6										
	β,4	1,3' 6,8'	3,1' 8,6'		5,2 10,7	4',α		2',5' 7',10'											
4',5' 9',10'	2,3' 7,8'			6,1	1',6'	5,4 10,9	3,2' 8,7'												
7,2	5',1' 10',6'	3,4' 8,9'				2',7'	1,5 6,10	4,3' 9,8'											
	8,3	1',2' 6',7'	4,5' 9,10'				3',8'	2,1 7,6	5,4' 10,9'										
		9,4	2',3' 7',8'	5,1' 10,6'	1,5' 6,10'			4',9'	3,2 8,7										
1,2' 6,7'			10,5	3',4' 8',9'	4,3 9,8	2,1' 7,6'			5',10'										
		α,10					10',β												
			α,6					6',β											
				α,7					7',β										
α,8					8',β														
	α,9					9',β													

												10,5	2,9' 7,4'	4,7' 9,2'		6,3 1,8	5',10'		3',6' 8',1'
										5,8' 10,3'			6,1	3,10' 8,5'	4',7' 9',2'		7,4 2,9	1',6'	
										4,6' 9,1'	1,9' 6,4'			7,2		5',8' 10',3'		8,5 3,10	2',7'
										8,3	5,7' 10,2'	2,10' 7,5'			3',8'		1',9' 6',4'		9,1 4,6
											9,4	1,8' 6,3'	3,6' 8,1'		10,2 5,7	4',9'		2',10' 7',5'	
										4',10' 9',5'	2,8' 7,3'			α,1	1',β	10,4 5,9	3,7' 8,2'		
										α,2	5',6' 10',1'	3,9' 8,4'				2',β	6,5 1,10	4,8' 9,3'	
											α,3	1',7' 6',2'	4,10' 9,5'				3',β	7,1 2,6	5,9' 10,4'
												α,4	2',8' 7',3'	5,6' 10,1'	1,10' 6,5'			4',β	8,2 3,7
										1,7' 6,2'			α,5	3',9' 8',4'	9,3 4,8	2,6' 7,1'			5',β
														β,6	6',α				
										β,7						7',α			
											β,8						8',α		
												β,9						9',α	
													β,10						10',α

7 teams, two sessions of 6 rounds

It is not possible to conduct 7-teams events with a cyclic schedule and so the full schedule is given here. The teams consist of 1&4, 2&5, 3&6, 9&12, 10&13, 11&14 with the stationary team 7&8. In the first stanza of 6 rounds, board-sets 1 to 6 are played by all teams apart from the '3-way' match involving the stationary team at table 6 where boards 7 to 9 are played in order.

Similarly in the second stanza, boards 10 to 15 are played by all except table 6 where 16 to 18 are played in order.

To avoid board sharing, 2 copies of board-sets 1 to 6 and 10 to 15 are required. It is not possible to conduct this with table guide-cards and so each pair requires a guide card. Table numbers are shown across the top and rounds to the left. The number of boards per set would be typically 4 or 5 and so playing a total of 24 or 30 boards in each session. Pairs' calibre 36.

Session 1

	1	2	3	4	5	6	7
1	2–3 (3)	4–12 (6)	6–8 (1)	1–5 (4)	10–11 (3)	14–7 (7)	9–13 (4)
2	1–2 (1)	6–14 (4)	5–8 (2)	4–3 (5)	9–10 (1)	13–7 (8)	12–11 (5)
3	1–3 (2)	5–13 (5)	4–8 (3)	2–6 (6)	9–11 (2)	12–7 (9)	10–14 (6)
4	6–5 (3)	9–1 (6)	7–3 (1)	2–4 (4)	14–13 (3)	8–11 (7)	10–12 (4)
5	5–4 (1)	11–3 (4)	7–2 (2)	6–1 (5)	13–12 (1)	8–10 (8)	14–9 (5)
6	6–4 (2)	10–2 (5)	7–1 (3)	3–5 (6)	14–12 (2)	8–9 (9)	11–13 (6)

Session 2

	1	2	3	4	5	6	7
7	9–2 (10)	6–7 (13)	13–5 (11)	12–3 (14)	1–10 (10)	14–8 (16)	4–11 (14)
8	9–3 (11)	5–7 (14)	12–4 (12)	10–6 (15)	1–11 (11)	13–8 (17)	2–14 (15)
9	10–3 (12)	4–7 (15)	14–6 (10)	9–5 (13)	2–11 (12)	12–8 (18)	1–13 (13)
10	13–4 (10)	8–3 (13)	2–10 (11)	14–1 (14)	5–12 (10)	7–11 (16)	6–9 (14)
11	14–4 (11)	8–2 (14)	1–9 (12)	11–5 (15)	6–12 (11)	7–10 (17)	3–13 (15)
12	14–5 (12)	8–1 (15)	3–11 (10)	10–4 (13)	6–13 (12)	7–9 (18)	2–12 (13)

One match for each team has a pair playing a second time against the same pair for the 'reverse' meeting. This occurs at table 3 in the second stanza and involves pairs 1 and 9, 2 and 10 etc. This does not happen for the stationary team, pairs 7&8.

11 teams two sessions of 10 rounds

The following schedule for 11 teams is based on the double *H*(10,12) shown above. This is using the same bicyclic methods used for even numbers of teams above, page 290. The full schedule is shown to illustrate the method and the end result is a quadcyclic schedule. Each session is played in two halves, a new cycle for each half. The table with the extra board-sets is shaded and there the boards are played in sequence though not necessarily starting with the lowest number.

Session 1

	1	2	3	4	5	6	7	8	9	10	11
1	5- 8 (2)	4- 6 (3)	12- 3 (4)	9-10 (6)	14- 2 (7)	1- 7 (10)	17-20 (2)	16-18 (3)	11-15 (14)	21-22 (6)	13-19 (10)
2	1- 9 (3)	5- 7 (4)	12- 4 (5)	10- 6 (7)	15- 3 (8)	2- 8 (6)	13-21 (3)	17-19 (4)	11-16 (15)	22-18 (7)	14-20 (6)
3	2-10 (4)	1- 8 (5)	12- 5 (1)	6- 7 (8)	16- 4 (9)	3- 9 (7)	14-22 (4)	13-20 (5)	11-17 (11)	18-19 (8)	15-21 (7)
4	3- 6 (5)	2- 9 (1)	12- 1 (2)	7- 8 (9)	17- 5 (10)	4-10 (8)	15-18 (5)	14-21 (1)	11-13 (12)	19-20 (9)	16-22 (8)
5	4- 7 (1)	3-10 (2)	12- 2 (3)	8- 9 (10)	13- 1 (6)	5- 6 (9)	16-19 (1)	15-22 (2)	11-14 (13)	20-21 (10)	17-18 (9)
6	9- 7 (2)	8-11 (4)	5- 2 (5)	6-18 (6)	1-10 (9)	4- 3 (10)	21-19 (2)	20-12 (14)	17-14 (5)	13-22 (9)	16-15 (10)
7	10- 8 (3)	9-11 (5)	1- 3 (1)	7-19 (7)	2- 6 (10)	5- 4 (6)	22-20 (3)	21-12 (15)	13-15 (1)	14-18 (10)	17-16 (6)
8	6- 9 (4)	10-11 (1)	2- 4 (2)	8-20 (8)	3- 7 (6)	1- 5 (7)	18-21 (4)	22-12 (11)	14-16 (2)	15-19 (6)	13-17 (7)
9	7-10 (5)	6-11 (2)	3- 5 (3)	9-21 (9)	4- 8 (7)	2- 1 (8)	19-22 (5)	18-12 (12)	15-17 (3)	16-20 (7)	14-13 (8)
10	8- 6 (1)	7-11 (3)	4- 1 (4)	10-22 (10)	5- 9 (8)	3- 2 (9)	20-18 (1)	19-12 (13)	16-13 (4)	17-21 (8)	15-14 (9)

Session 2

11	5-20 (17)	4-18 (18)	3-15 (19)	9-22 (21)	11- 2 (22)	1-19 (25)	17- 8 (17)	16- 6 (18)	21-10 (21)	12-14 (27)	13- 7 (25)
12	1-21 (18)	5-19 (19)	4-16 (20)	10-18 (22)	11- 3 (23)	2-20 (21)	13- 9 (18)	17- 7 (19)	22- 6 (22)	12-15 (28)	14- 8 (21)
13	2-22 (19)	1-20 (20)	5-17 (16)	6-19 (23)	11- 4 (24)	3-21 (22)	14-10 (19)	13- 8 (20)	18- 7 (23)	12-16 (29)	15- 9 (22)
14	3-18 (20)	2-21 (16)	1-13 (17)	7-20 (24)	11- 5 (25)	4-22 (23)	15- 6 (20)	14- 9 (16)	19- 8 (24)	12-17 (30)	16-10 (23)
15	4-19 (16)	3-22 (17)	2-14 (18)	8-21 (25)	11- 1 (21)	5-18 (24)	16- 7 (16)	15-10 (17)	20- 9 (25)	12-13 (26)	17- 6 (24)
16	9-19 (17)	20- 8 (19)	17- 2 (20)	6-12 (21)	1-22 (24)	16- 3 (25)	21- 7 (17)	5-14 (20)	18-11 (26)	13-10 (24)	4-15 (25)
17	10-20 (18)	21- 9 (20)	13- 3 (16)	7-12 (22)	2-18 (25)	17- 4 (21)	22- 8 (18)	1-15 (16)	19-11 (27)	14- 6 (25)	5-16 (21)
18	6-21 (19)	22-10 (16)	14- 4 (17)	8-12 (23)	3-19 (21)	13- 5 (22)	18- 9 (19)	2-16 (17)	20-11 (28)	15- 7 (21)	1-17 (22)
19	7-22 (20)	18- 6 (17)	15- 5 (18)	9-12 (24)	4-20 (22)	14- 1 (23)	19-10 (20)	3-17 (18)	21-11 (29)	16- 8 (22)	2-13 (23)
20	8-18 (16)	19- 7 (18)	16- 1 (19)	10-12 (25)	5-21 (23)	15- 2 (24)	20- 6 (16)	4-13 (19)	22-11 (30)	17- 9 (23)	3-14 (24)

This full schedule can be described using the following cycle starters.

Session 1

Stanza 1 rounds 1 to 5

5–8 (2)	4–6 (3)	12–3 (4)	9–10 (6)	14–2 (7)	1–7 (10)
17–20 (2)	16–18 (3)	11–15 (14)*	21–22 (6)	13–19 (10)	

Stanza 2 rounds 6 to 10

9–7 (2)	8–11 (4)	5–2 (5)	6–18 (6)	1–10 (9)	4–3 (10)
21–19 (2)	20–12 (14)*	17–14 (5)	13–22 (9)	16–15 (10)	

Session 2

Stanza 3 rounds 11 to 15

5–20 (17)	4–18 (18)	3–15 (19)	9–22 (21)	11–2 (22)	1–19 (25)
17–8 (17)	16–6 (18)	21–10 (21)	12–14 (27)*	13–7 (25)	

Stanza 4 rounds 11 to 15

9–19 (17)	20–8 (19)	17–2 (20)	6–12 (21)	1–22 (24)	16–3 (25)
21–7 (17)	5–14 (20)	18–11 (26)*	13–10 (24)	4–15 (25)	

Team groups are 1–5 with 6–10 and 13–17 with 18–22. 11 and 12 are stationary.
Board groups are 1–5 with 6–10 and 16–20 with 21–25.
*extra boards 11–15 and 26–30.
Pairs' calibre 26.

15 teams, two sessions of 14 rounds

Session 1

Stanza 1 rounds 1 to 7

16–1 (1)	12–7	10–4	2–6 (7)	9–11	14–8	13–3	21–5 (12)
15–17	28–23	26–20	18–22	25–27	30–24	29–19	

Stanza 2 rounds 8 to 14

8–15 (1)	14–5 (4)	11–3 (6)	13–9 (7)	4–2 (8)	1–7 (10)	10–6 (11)	12–28 (12)
24–16 (15)*	30–21 (4)	27–19 (6)	29–25 (7)	20–18 (8)	17–23 (10)	26–22 (11)	

Session 2

Stanza 3 rounds 15 to 21

1–17 (22)	28–7 (25)	26–4 (27)	18–6 (28)	9–27 (29)	14–24 (31)	29–3 (32)	15–5 (33)
12–23 (25)	10–20 (27)	2–22 (28)	25–11 (29)	30–8 (31)	13–19 (32)	16–21 (40)*	

Stanza 4 rounds 22 to 28

24–8 (22)	30–5 (25)	27–3 (27)	13–25 (28)	20–2 (29)	17–7 (31)	26–6 (32)	12–16 (33)
14–21 (25)	11–19 (27)	29–9 (28)	4–18 (29)	1–23 (31)	10–22 (32)	28–15 (40)*	

Team groups are 1–7 with 8–14 and 17–23 with 24–30. 15 and 16 are stationary.
Board groups are 1–7 with 8–14 and 22–28 with 29–35.
*extra boards 15–21 and 36–42.
Pairs' calibre 22.

19 teams, two sessions of 18 rounds

Session 1

Stanza 1 rounds 1 to 9

16–18 (3)	6–14 (4)	10–4 (5)	7–17 (6)	20–8 (8)	3–2 (10)	12–13 (11)	5–1 (16)	11–15 (17)	29–9 (18)
36–38 (3)	26–34 (4)	30–24 (5)	27–37 (6)	19–28 (26)*	23–22 (10)	32–33 (11)	25–21 (16)	31–35 (17)	

Stanza 2 rounds 10 to 18

17–4 (1)	12–9 (4)	3–1 (6)	11–6 (8)	18–19 (9)	14–7 (10)	10–16 (11)	15–35 (15)	13–2 (16)	5–8 (18)
37–24 (1)	32–29 (4)	23–21 (6)	31–26 (8)	38–20 (27)*	34–27 (10)	30–36 (11)	33–22 (16)	25–28 (18)	

Session 2

Stanza 3 rounds 19 to 27

16–38 (3)	6–34 (4)	30–4 (5)	7–37 (6)	8–28 (8)	23–2 (10)	12–33 (11)	25–1 (16)	11–35 (17)	19–9 (18)
36–18 (3)	26–14 (4)	10–24 (5)	27–17 (6)	3–22 (10)	32–13 (11)	5–21 (16)	31–15 (17)	20–29 (27)*	

Stanza 4 rounds 28 to 36

37–4 (1)	32–9 (4)	23–1 (6)	31–6 (8)	38–18 (9)	34–7 (10)	10–36 (11)	15–20 (15)	33–2 (16)	25–8 (18)
17–24 (1)	12–29 (4)	3–21 (6)	11–26 (8)	14–27 (10)	30–16 (11)	35–19 (24)*	13–22 (16)	5–28 (18)	

Team groups are 1–9 with 10–18 and 21–29 with 30–38. 19 and 20 are stationary.
Board groups are 1–9 with 10–18. These board-set numbers are repeated in session 2.
*extra boards 19–27.
Pairs' calibre 15.

23 teams, two sessions of 22 rounds

Session 1

Stanza 1 rounds 1 to 11

24–1 (1)	9–16 (2)	3–13 (4)	6–11 (5)	8–21 (6)	7–15 (10)	12–22 (13)	4–20 (14)	17–19 (15)	5–2 (19)	34–10 (21)	18–14 (22)
23–25 (23)	33–40 (2)	27–37 (4)	30–35 (5)	32–45 (6)	31–39 (10)	36–46 (13)	28–44 (14)	41–43 (15)	29–26 (19)	42–38 (22)	

Stanza 2 rounds 12 to 22

12–23 (1)	5–20 (2)	2–14 (4)	22–17 (5)	10–19 (6)	4–18 (10)	11– 1 (13)	9–15 (14)	8– 6 (15)	13–16 (19)	21–45 (21)	3– 7 (22)
36–24 (23)	29–44 (2)	26–38 (4)	46–41 (5)	34–43 (6)	28–42 (10)	35–25 (13)	33–39 (14)	32–30 (15)	37–40 (19)	27–31 (22)	

Session 2

Stanza 3 rounds 23 to 33

1–25 (1)	9–40 (2)	3–37 (4)	30–11 (5)	8–45 (6)	7–39 (10)	12–46 (13)	4–44 (14)	17–43 (15)	29–2 (19)	23–10 (21)	18–38 (22)
33–16 (2)	27–13 (4)	6–35 (5)	32–21 (6)	31–15 (10)	36–22 (13)	28–20 (14)	41–19 (15)	5–26 (19)	24–34 (32)	42–14 (22)	

Stanza 4 rounds 34 to 44

36–12 (1)	5–44 (2)	2–38 (4)	22–41 (5)	10–43 (6)	4–42 (10)	35– 1 (13)	9–39 (14)	32– 6 (15)	13–40 (19)	21–24 (21)	27– 7 (22)
29–20 (2)	26–14 (4)	46–17 (5)	34–19 (6)	28–18 (10)	11–25 (13)	33–15 (14)	8–30 (15)	37–16 (19)	45–23 (32)	3–31 (22)	

Team groups are 1–11 with 12–22 and 25–35 with 36–46. 23 and 24 are stationary.
Board groups are 1–11 with 12–22. These board–set numbers are repeated in session 2.
*extra boards 23–33. Pairs calibre 12,

27 teams two sessions of 26 rounds

Session 1

Stanza 1 rounds 1 to 13

16–6 (2)	19–2 (3)	28–4 (4)	14–15 (5)	25–9 (8)	22–13 (10)	5–7 (13)	17–26 (14)	24–12 (15)	21–18 (17)	8–11 (20)	23–3 (21)	38–10 (23)	20–1 (25)
44–34 (2)	47–30 (3)	27–32 (30)	42–43 (5)	53–37 (8)	50–41 (10)	33–35 (13)	45–54 (14)	52–40 (15)	49–46 (17)	36–39 (20)	51–31 (21)	48–29 (25)	

Stanza 2 rounds 14 to 26

4–10 (2)	1–13 (4)	11–16 (5)	5–26 (7)	22–27 (9)	18–25 (10)	19–17 (12)	14–42 (14)	2–6 (16)	24–9 (17)	23–12 (18)	20–15 (22)	3–8 (23)	21–7 (24)
32–38 (2)	29–41 (4)	39–44 (5)	33–54 (7)	50–28 (35)	46–53 (10)	47–45 (12)	30–34 (16)	52–37 (17)	51–40 (18)	48–43 (22)	31–36 (23)	49–35 (24)	

Session 2

Stanza 3 rounds 27 to 39

44–6 (2)	47–2 (3)	4–32 (4)	14–43 (5)	53–9 (8)	50–13 (10)	33–7 (13)	17–54 (14)	52–12 (15)	21–46 (17)	36–11 (20)	51–3 (21)	27–10 (23)	48–1 (25)
16–34 (2)	19–30 (3)	42–15 (5)	25–37 (8)	22–41 (10)	5–35 (13)	45–26 (14)	24–40 (15)	49–18 (17)	8–39 (20)	23–31 (21)	28–38 (36)	20–29 (25)	

Stanza 4 rounds 40 to 52

32–10 (2)	29–13 (4)	11–44 (5)	5–54 (7)	50–22 (9)	18–53 (10)	19–45 (12)	14–28 (14)	30–6 (16)	52–9 (17)	51–12 (18)	20–43 (22)	31–8 (23)	49–7 (24)
4–38 (2)	1–41 (4)	39–16 (5)	33–26 (7)	46–25 (10)	47–17 (12)	42–27 (27)	2–34 (16)	24–37 (17)	23–40 (18)	48–15 (22)	3–36 (23)	21–35 (24)	

Team groups are 1–13 with 14–26 and 29–41 with 42–54. 27 and 28 are stationary.
Board groups are 1–13 with 14–26. These board-set numbers are repeated in session 2.
*extra boards 27–39.
Pairs' calibre 11.

Two sessions, technique three

In technique two for odd numbers of teams over two sessions we used two schedules of half the number of teams, rounded up. So for 11 teams we used two 6-table schedules minus one team. The consequence was a movement where additional board-sets were needed. This is unattractive and many organizers would choose not to use this method.

This third technique uses a similar basic principle where two half schedules are used. The difference is that we assign a bye team from the even-team schedule for one more than the number required.

Therefore, for 11 teams we use the 12-teams two-sessions movement. Then deem the last team (pairs 23 and 24) are sit-out rounds for the pairs that play against them. This means we have a movement where all pairs sit out for two rounds and do not play two board-sets.

Similarly for 13 teams use the 14 team two-session movement, 15 teams use the 16 and so forth.

10.7 Three sessions

If you think dogs can't count, try putting three dog biscuits in your pocket
and then giving Fido only two of them.
Phil Pastoret

13 teams, three sessions of 8 rounds

The schedule supplied here is for three sessions, each of 8 rounds, usually playing 3-board rounds for a total of 24 boards per session. There are 10 board-sets in play for each session and each team fails to play 2 sets in each session. The schedule is not cyclic and requires several copies of each board, 5 copies being the best.

This is based on the 4-team and 5-team schedules supplied earlier. The field is divided into three groups, A, B and C with an additional, stationary team D, made up of pairs 25 and 26. The groups are 1 to 8 (A), 9 to 16 (B), 17 to 24 (C). Team-mates are 1 and 5, 2 and 6… 9 and 13, etc.

In session 1, group A play group B, while group C play within their own group with the stationary team D. Then these groups are reorganized after each session:

Session	Two groups meet	One group + D
1	A v B	C + D
2	B v C	A + D
3	C v A	B + D

Below the dark borders outline the different intra-movements for 4 or 5 tables. Depending on the number of boards per set, sharing can be used between tables. For example, 1 and 5, 2 and 6, etc., can share. The player movement in the 4-table groups are not cyclic and require guide-cards. Pairs' calibre 68.

Session 1

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1–10 (1)	7–12 (2)	6–16 (3)	5–11 (4)	9–2 (1)	15–4 (2)	14–8 (3)	13–3 (4)	26–17 (1)	21–25 (1)	19–20 (2)	22–24 (7)	18–23 (9)
2	6–13 (1)	8–11 (2)	4–10 (3)	7–9 (4)	14–5 (1)	16–3 (2)	12–2 (3)	15–1 (4)	26–18 (2)	22–25 (2)	20–21 (3)	23–17 (8)	19–24 (10)
3	4–11 (1)	2–13 (2)	7–14 (5)	8–9 (6)	12–3 (1)	10–5 (2)	15–6 (5)	16–1 (6)	26–19 (3)	23–25 (3)	21–22 (4)	24–18 (5)	20–17 (9)
4	7–16 (1)	1–14 (2)	2–11 (5)	5–12 (6)	15–8 (1)	9–6 (2)	10–3 (5)	13–4 (6)	26–20 (4)	24–25 (4)	22–23 (1)	17–19 (6)	21–18 (10)
5	7–13 (3)	8–10 (4)	4–9 (5)	3–14 (6)	15–5 (3)	16–2 (4)	12–1 (5)	11–6 (6)	21–26 (5)	25–17 (5)	24–23 (2)	20–18 (7)	19–22 (9)
6	1–11 (3)	6–12 (4)	5–16 (5)	2–15 (6)	9–3 (3)	14–4 (4)	13–8 (5)	10–7 (6)	22–26 (6)	25–18 (6)	17–24 (3)	21–19 (8)	20–23 (10)
7	1–9 (7)	2–10 (8)	3–11 (9)	4–12 (10)	13–5 (7)	14–6 (8)	15–7 (9)	16–8 (10)	23–26 (7)	25–19 (7)	18–17 (4)	22–20 (5)	21–24 (9)
8	13–1 (8)	14–2 (9)	15–3 (10)	16–4 (7)	5–9 (8)	6–10 (9)	7–11 (10)	8–12 (7)	24–26 (8)	25–20 (8)	19–18 (1)	23–21 (6)	22–17 (10)

Session 2

	1	2	3	4	5	6	7	8	9	10	11	12	13
9	9–18 (11)	15–20 (12)	14–24 (13)	13–19 (14)	17–10 (11)	23–12 (12)	22–16 (13)	21–11 (14)	26–1 (11)	5–25 (11)	3–4 (12)	6–8 (17)	2–7 (19)
10	14–21 (11)	16–19 (12)	12–18 (13)	15–17 (14)	22–13 (11)	24–11 (12)	20–10 (13)	23–9 (14)	26–2 (12)	6–25 (12)	4–5 (13)	7–1 (18)	3–8 (20)
11	12–19 (11)	10–21 (12)	15–22 (15)	16–17 (16)	20–11 (11)	18–13 (12)	23–14 (15)	24–9 (16)	26–3 (13)	7–25 (13)	5–6 (14)	8–2 (15)	4–1 (19)
12	15–24 (11)	9–22 (12)	10–19 (15)	13–20 (16)	23–16 (11)	17–14 (12)	18–11 (15)	21–12 (16)	26–4 (14)	8–25 (14)	6–7 (11)	1–3 (16)	5–2 (20)
13	15–21 (13)	16–18 (14)	12–17 (15)	11–22 (16)	23–13 (13)	24–10 (14)	20–9 (15)	19–14 (16)	5–26 (15)	25–1 (15)	8–7 (12)	4–2 (17)	3–6 (19)
14	9–19 (13)	14–20 (14)	13–24 (15)	10–23 (16)	17–11 (13)	22–12 (14)	21–16 (15)	18–15 (16)	6–26 (16)	25–2 (16)	1–8 (13)	5–3 (18)	4–7 (20)
15	9–17 (17)	10–18 (18)	11–19 (19)	12–20 (20)	21–13 (17)	22–14 (18)	23–15 (19)	24–16 (20)	7–26 (17)	25–3 (17)	2–1 (14)	6–4 (15)	5–8 (19)
16	21–9 (18)	22–10 (19)	23–11 (20)	24–12 (17)	13–17 (18)	14–18 (19)	15–19 (20)	16–20 (17)	8–26 (18)	25–4 (18)	3–2 (11)	7–5 (16)	6–1 (20)

Session 3

	1	2	3	4	5	6	7	8	9	10	11	12	13
17	1–18 (21)	7–20 (22)	6–24 (23)	5–19 (24)	17–2 (21)	23–4 (22)	22–8 (23)	21–3 (24)	26–9 (21)	13–25 (21)	11–12 (22)	14–16 (27)	10–15 (29)
18	6–21 (21)	8–19 (22)	4–18 (23)	7–17 (24)	22–5 (21)	24–3 (22)	20–2 (23)	23–1 (24)	26–10 (22)	14–25 (22)	12–13 (23)	15–9 (28)	11–16 (30)
19	4–19 (21)	2–21 (22)	7–22 (25)	8–17 (26)	20–3 (21)	18–5 (22)	23–6 (25)	24–1 (26)	26–11 (23)	15–25 (23)	13–14 (24)	16–10 (25)	12–9 (29)
20	7–24 (21)	1–22 (22)	2–19 (25)	5–20 (26)	23–8 (21)	17–6 (22)	18–3 (25)	21–4 (26)	26–12 (24)	16–25 (24)	14–15 (21)	9–11 (26)	13–10 (30)
21	7–21 (23)	8–18 (24)	4–17 (25)	3–22 (26)	23–5 (23)	24–2 (24)	20–1 (25)	19–6 (26)	13–26 (25)	25–9 (25)	16–15 (22)	12–10 (27)	11–14 (29)
22	1–19 (23)	6–20 (24)	5–24 (25)	2–23 (26)	17–3 (23)	22–4 (24)	21–8 (25)	18–7 (26)	14–26 (26)	25–10 (26)	9–16 (23)	13–11 (28)	12–15 (30)
23	1–17 (27)	2–18 (28)	3–19 (29)	4–20 (30)	21–5 (27)	22–6 (28)	23–7 (29)	24–8 (30)	15–26 (27)	25–11 (27)	10–9 (24)	14–12 (25)	13–16 (29)
24	21–1 (28)	22–2 (29)	23–3 (30)	24–4 (27)	5–17 (28)	6–18 (29)	7–19 (30)	8–20 (27)	16–26 (28)	25–12 (28)	11–10 (21)	15–13 (26)	14–9 (30)

A similar technique to that supplied with 15 tables below can be used with 13, remapping the pair numbers for each team in each of the three sessions.

15 teams, three sessions of 10 rounds

The movement for 15 teams playing one session requires 28 rounds, which is an awkward number. In addition a schedule for 15 teams with 28 rounds is highly irregular and for all practical purposes unusable. The solution is to supply a three-session movement. Each session has 10 rounds for a total of 30 rounds, with each team not playing for two of those rounds.

We use one of the 5-team schedules supplied above. The field of pairs is divided into 3 groups, pairs 1 to 10 (group A), 11 to 20 (group B) and 21 to 30 (group C).

The schedule shown here has groups A and B playing each other and group C playing those within its group. Pairs 9 and 10 are stationary apart from the switch after round 4. Similarly, pairs 19 and 20 in group B, and pairs 29 and 30 in group C.

The movement is regular, each pair following the lower-numbered pair. Pair 1 follows 8, pair 11 follows 18 and pair 21 follows 28. For rounds 9 and 10, groups A and B play the teams they have not played, as well as using the boards they have not played. Pairs ‘11 and 15’ play ‘1 and 5’, using sets ‘2 and 7’. Then ‘12 and 16’ play ‘2 and 6’, using sets ‘3 and 8’, and so forth. See the expanded schedules below for full details. Group C play only 8 rounds, missing two board-sets.

This schedule can apply to all three stanzas of the event. One of the unusual features of these movements is the distinction between teams and pairs. There are 15 teams that are made up of two pairs from each group. By remapping the pair numbers in each stanza the teams use different pair numbers in each session or stanza so that the one set of guide-cards apply to each session. The following chart shows the team numbers with their allocated pair numbers for each session.

Team	Session 1	Session 2	Session 3
1	1,5	11,15	21,25
2	2,6	12,16	22,26
3	3,7	13,17	23,27
4	4,8	14,18	24,28
5	9,10	19,20	29,30
6	21,25	1, 5	11,15
7	22,26	2,6	12,16
8	23,27	3,7	13,17
9	24,28	4,8	14,18
10	29,30	9,10	19,20
11	11,15	21,25	1, 5
12	12,16	22,26	2,6
13	13,17	23,27	3,7
14	14,18	24,28	4,8
15	19,20	29,30	9,10

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	10–11 (1)	15–9 (1)	3–14 (2)	6–18 (7)	2–17 (9)	20–1 (1)	5–19 (1)	13–4 (2)	16–8 (7)	12–7 (9)	30–21 (1)	25–29 (1)	23–24 (2)	26–28 (7)	22–27 (9)
2	10–12 (2)	16–9 (2)	4–15 (3)	7–11 (8)	3–18 (10)	20–2 (2)	6–19 (2)	14–5 (3)	17–1 (8)	13–8 (10)	30–22 (2)	26–29 (2)	24–25 (3)	27–21 (8)	23–28 (10)
3	10–13 (3)	17–9 (3)	5–16 (4)	8–12 (5)	4–11 (9)	20–3 (3)	7–19 (3)	15–6 (4)	18–2 (5)	14–1 (9)	30–23 (3)	27–29 (3)	25–26 (4)	28–22 (5)	24–21 (9)
4	10–14 (4)	18–9 (4)	6–17 (1)	1–13 (6)	5–12 (10)	20–4 (4)	8–19 (4)	16–7 (1)	11–3 (6)	15–2 (10)	30–24 (4)	28–29 (4)	26–27 (1)	21–23 (6)	25–22 (10)
5	15–10 (5)	9–11 (5)	18–7 (2)	14–2 (7)	13–6 (9)	5–20 (5)	19–1 (5)	8–17 (2)	4–12 (7)	3–16 (9)	25–30 (5)	29–21 (5)	28–27 (2)	24–22 (7)	23–26 (9)
6	16–10 (6)	9–12 (6)	11–8 (3)	15–3 (8)	14–7 (10)	6–20 (6)	19–2 (6)	1–18 (3)	5–13 (8)	4–17 (10)	26–30 (6)	29–22 (6)	21–28 (3)	25–23 (8)	24–27 (10)
7	17–10 (7)	9–13 (7)	12–1 (4)	16–4 (5)	15–8 (9)	7–20 (7)	19–3 (7)	2–11 (4)	6–14 (5)	5–18 (9)	27–30 (7)	29–23 (7)	22–21 (4)	26–24 (5)	25–28 (9)
8	18–10 (8)	9–14 (8)	13–2 (1)	17–5 (6)	16–1 (10)	8–20 (8)	19–4 (8)	3–12 (1)	7–15 (6)	6–11 (10)	28–30 (8)	29–24 (8)	23–22 (1)	27–25 (6)	26–21 (10)
9	1–11 (2)	2–12 (3)	3–13 (4)	4–14 (1)	15–5 (2)	16–6 (3)	17–7 (4)	18–8 (1)	9–19 (9)	20–10 (9)					
10	11–5 (7)	12–6 (8)	13–7 (5)	14–8 (6)	1–15 (7)	2–16 (8)	3–17 (5)	4–18 (6)	19–10 (10)	9–20 (10)					

16 teams, three sessions of 10 rounds

The same principles apply to 16 teams as those used with 15 teams above. The schedule used in all three sessions is constructed from two five-team movements with one six-team movement appended. . The field of pairs is divided into 3 groups, pairs 1 to 10 (group A), 11 to 20 (group B) and 21 to 30 (group C), but there is one additional team 31 and 32.

The schedule shown here has groups A and B playing each other and group C playing those within its group. The pairs 9 and 10 are stationary apart from the switch after round 4. Similarly, pairs 19 and 20 in group B. Pairs 29 and 30 in group C are moving in their group and so the teams are made up from 21 to 25 with 26 to 30. Pairs 21 and 26 are team-mates, as are 22 and 27, etc. Pairs 31 and 32 are stationary. Every pair plays all the boards.

The schedules are cyclic or bicyclic and require 5 copies of the boards for best organization, while sharing boards between tables 1 and 2 in the first 5-table movement and between tables 6 and 7 in the second.

Team to pair mapping

Team	Session 1	Session 2	Session 3
1	1,5	11,15	21,26
2	2,6	12,16	22,27
3	3,7	13,17	23,28
4	4,8	14,18	24,29
5	9,10	19,20	25,30
6	21,26	1, 5	11,15
7	22,27	2,6	12,16
8	23,28	3,7	13,17
9	24,29	4,8	14,18
10	25,30	9,10	19,20
11	11,15	21,26	1, 5
12	12,16	22,27	2,6
13	13,17	23,28	3,7
14	14,18	24,29	4,8
15	19,20	25,30	9,10
16	31,32	31,32	31,32

Session schedule

	1	2	3	4	5	6	7	8	9	10
1	10–11 (1)	15–9 (1)	3–14 (2)	6–18 (7)	2–17 (9)	20–1 (1)	5–19 (1)	13–4 (2)	16–8 (7)	12–7 (9)
2	10–12 (2)	16–9 (2)	4–15 (3)	7–11 (8)	3–18 (10)	20–2 (2)	6–19 (2)	14–5 (3)	17–1 (8)	13–8 (10)
3	10–13 (3)	17–9 (3)	5–16 (4)	8–12 (5)	4–11 (9)	20–3 (3)	7–19 (3)	15–6 (4)	18–2 (5)	14–1 (9)
4	10–14 (4)	18–9 (4)	6–17 (1)	1–13 (6)	5–12 (10)	20–4 (4)	8–19 (4)	16–7 (1)	11–3 (6)	15–2 (10)
5	15–10 (5)	9–11 (5)	18–7 (2)	14–2 (7)	13–6 (9)	5–20 (5)	19–1 (5)	8–17 (2)	4–12 (7)	3–16 (9)
6	16–10 (6)	9–12 (6)	11–8 (3)	15–3 (8)	14–7 (10)	6–20 (6)	19–2 (6)	1–18 (3)	5–13 (8)	4–17 (10)
7	17–10 (7)	9–13 (7)	12–1 (4)	16–4 (5)	15–8 (9)	7–20 (7)	19–3 (7)	2–11 (4)	6–14 (5)	5–18 (9)
8	18–10 (8)	9–14 (8)	13–2 (1)	17–5 (6)	16–1 (10)	8–20 (8)	19–4 (8)	3–12 (1)	7–15 (6)	6–11 (10)
9	1–11 (2)	2–12 (3)	3–13 (4)	4–14 (1)	15–5 (2)	16–6 (3)	17–7 (4)	18–8 (1)	9–19 (9)	20–10 (9)
10	11–5 (7)	12–6 (8)	13–7 (5)	14–8 (6)	1–15 (7)	2–16 (8)	3–17 (5)	4–18 (6)	19–10 (10)	9–20 (10)

Tables 11–16 are continued over.

	11	12	13	14	15	16
1	25–28 (2)	24–26 (3)	32–23 (4)	29–30 (6)	31–22 (7)	21–27 (10)
2	21–29 (3)	25–27 (4)	32–24 (5)	30–26 (7)	31–23 (8)	22–28 (6)
3	22–30 (4)	21–28 (5)	32–25 (1)	26–27 (8)	31–24 (9)	23–29 (7)
4	23–26 (5)	22–29 (1)	32–21 (2)	27–28 (9)	31–25 (10)	24–30 (8)
5	24–27 (1)	23–30 (2)	32–22 (3)	28–29 (10)	31–21 (6)	25–26 (9)
6	29–27 (2)	28–31 (4)	25–22 (5)	26–32 (6)	21–30 (9)	24–23 (10)
7	30–28 (3)	29–31 (5)	21–23 (1)	27–32 (7)	22–26 (10)	25–24 (6)
8	26–29 (4)	30–31 (1)	22–24 (2)	28–32 (8)	23–27 (6)	21–25 (7)
9	27–30 (5)	26–31 (2)	23–25 (3)	29–32 (9)	24–28 (7)	22–21 (8)
10	28–26 (1)	27–31 (3)	24–21 (4)	30–32 (10)	25–29 (8)	23–22 (9)

18 teams, three sessions of 12 rounds

The same principles apply to 18 teams as those used with 15 teams above. The movement is based on three 6-table bicyclic schedules. There are a total of 36 rounds, with each team not playing 2.

Team to pair mapping

Team	Session 1	Session 2	Session 3
1	1,6	13,18	25,30
2	2,7	14,19	26,31
3	3,8	15,20	27,32
4	4,9	16,21	28,33
5	5,10	17,22	29,34
6	11,12	23,24	35,36
7	25,30	1,6	13,18
8	26,31	2,7	14,19
9	27,32	3,8	15,20
10	28,33	4,9	16,21
11	29,34	5,10	17,22
12	35,36	11,12	23,24
13	13,18	25,30	1,6
14	14,19	26,31	2,7
15	15,20	27,32	3,8
16	16,21	28,33	4,9
17	17,22	29,34	5,10
18	23,24	35,36	11,12

Session schedule

	1	2	3	4	5	6	7	8	9	10	11	12
1	5–20 (2)	4–18 (3)	12–15 (4)	9–22 (6)	11–14 (7)	1–19 (10)	17–8 (2)	16–6 (3)	24–3 (4)	21–10 (6)	23–2 (7)	13–7 (10)
2	1–21 (3)	5–19 (4)	12–16 (5)	10–18 (7)	11–15 (8)	2–20 (6)	13–9 (3)	17–7 (4)	24–4 (5)	22–6 (7)	23–3 (8)	14–8 (6)
3	2–22 (4)	1–20 (5)	12–17 (1)	6–19 (8)	11–16 (9)	3–21 (7)	14–10 (4)	13–8 (5)	24–5 (1)	18–7 (8)	23–4 (9)	15–9 (7)
4	3–18 (5)	2–21 (1)	12–13 (2)	7–20 (9)	11–17 (10)	4–22 (8)	15–6 (5)	14–9 (1)	24–1 (2)	19–8 (9)	23–5 (10)	16–10 (8)
5	4–19 (1)	3–22 (2)	12–14 (3)	8–21 (10)	11–13 (6)	5–18 (9)	16–7 (1)	15–10 (2)	24–2 (3)	20–9 (10)	23–1 (6)	17–6 (9)
6	21–7 (2)	20–11 (4)	17–2 (5)	18–12 (6)	13–10 (9)	16–3 (10)	9–19 (2)	8–23 (4)	5–14 (5)	6–24 (6)	1–22 (9)	4–15 (10)
7	22–8 (3)	21–11 (5)	13–3 (1)	19–12 (7)	14–6 (10)	17–4 (6)	10–20 (3)	9–23 (5)	1–15 (1)	7–24 (7)	2–18 (10)	5–16 (6)
8	18–9 (4)	22–11 (1)	14–4 (2)	20–12 (8)	15–7 (6)	13–5 (7)	6–21 (4)	10–23 (1)	2–16 (2)	8–24 (8)	3–19 (6)	1–17 (7)
9	19–10 (5)	18–11 (2)	15–5 (3)	21–12 (9)	16–8 (7)	14–1 (8)	7–22 (5)	6–23 (2)	3–17 (3)	9–24 (9)	4–20 (7)	2–13 (8)
10	20–6 (1)	19–11 (3)	16–1 (4)	22–12 (10)	17–9 (8)	15–2 (9)	8–18 (1)	7–23 (3)	4–13 (4)	10–24 (10)	5–21 (8)	3–14 (9)
11	1–13 (11)	2–14 (11)	3–15 (11)	4–16 (11)	5–17 (11)	18–6 (11)	19–7 (11)	20–8 (11)	21–9 (11)	22–10 (11)	11–23 (11)	24–12 (11)
12	13–6 (12)	14–7 (12)	15–8 (12)	16–9 (12)	17–10 (12)	1–18 (12)	2–19 (12)	3–20 (12)	4–21 (12)	5–22 (12)	23–12 (12)	11–24 (12)

	13	14	15	16	17	18
1	29–32 (2)	28–30 (3)	36–27 (4)	33–34 (6)	35–26 (7)	25–31 (10)
2	25–33 (3)	29–31 (4)	36–28 (5)	34–30 (7)	35–27 (8)	26–32 (6)
3	26–34 (4)	25–32 (5)	36–29 (1)	30–31 (8)	35–28 (9)	27–33 (7)
4	27–30 (5)	26–33 (1)	36–25 (2)	31–32 (9)	35–29 (10)	28–34 (8)
5	28–31 (1)	27–34 (2)	36–26 (3)	32–33 (10)	35–25 (6)	29–30 (9)
6	33–31 (2)	32–35 (4)	29–26 (5)	30–36 (6)	25–34 (9)	28–27 (10)
7	34–32 (3)	33–35 (5)	25–27 (1)	31–36 (7)	26–30 (10)	29–28 (6)
8	30–33 (4)	34–35 (1)	26–28 (2)	32–36 (8)	27–31 (6)	25–29 (7)
9	31–34 (5)	30–35 (2)	27–29 (3)	33–36 (9)	28–32 (7)	26–25 (8)
10	32–30 (1)	31–35 (3)	28–25 (4)	34–36 (10)	29–33 (8)	27–26 (9)
11						
12						

19 teams, three sessions of 12 rounds

The same principles apply to 19 teams as those used with 16 teams above. The movement is based on two 6-table bicyclic schedules plus one 7-table schedule. There are 36 rounds, with each team not playing 2 board-sets in each session.

Team to pair mapping

Team	Session 1	Session 2	Session 3
1	1,6	13,18	25,31
2	2,7	14,19	26,32
3	3,8	15,20	27,33
4	4,9	16,21	28,34
5	5,10	17,22	29,35
6	11,12	23,24	30,36
7	25,31	1,6	13,18
8	26,32	2,7	14,19
9	27,33	3,8	15,20
10	28,34	4,9	16,21
11	29,35	5,10	17,22
12	30,36	11,12	23,24
13	13,18	25,31	1,6
14	14,19	26,32	2,7
15	15,20	27,33	3,8
16	16,21	28,34	4,9
17	17,22	29,35	5,10
18	23,24	30,36	11,12
19	37,38	37,38	37,38

Session schedule

	1	2	3	4	5	6	7	8	9	10	11	12
1	5–20 (2)	4–18 (3)	12–15 (4)	9–22 (6)	11–14 (7)	1–19 (10)	17–8 (2)	16–6 (3)	24–3 (4)	21–10 (6)	23–2 (7)	13–7 (10)
2	1–21 (3)	5–19 (4)	12–16 (5)	10–18 (7)	11–15 (8)	2–20 (6)	13–9 (3)	17–7 (4)	24–4 (5)	22–6 (7)	23–3 (8)	14–8 (6)
3	2–22 (4)	1–20 (5)	12–17 (1)	6–19 (8)	11–16 (9)	3–21 (7)	14–10 (4)	13–8 (5)	24–5 (1)	18–7 (8)	23–4 (9)	15–9 (7)
4	3–18 (5)	2–21 (1)	12–13 (2)	7–20 (9)	11–17 (10)	4–22 (8)	15–6 (5)	14–9 (1)	24–1 (2)	19–8 (9)	23–5 (10)	16–10 (8)
5	4–19 (1)	3–22 (2)	12–14 (3)	8–21 (10)	11–13 (6)	5–18 (9)	16–7 (1)	15–10 (2)	24–2 (3)	20–9 (10)	23–1 (6)	17–6 (9)
6	21–7 (2)	20–11 (4)	17–2 (5)	18–12 (6)	13–10 (9)	16–3 (10)	9–19 (2)	8–23 (4)	5–14 (5)	6–24 (6)	1–22 (9)	4–15 (10)
7	22–8 (3)	21–11 (5)	13–3 (1)	19–12 (7)	14–6 (10)	17–4 (6)	10–20 (3)	9–23 (5)	1–15 (1)	7–24 (7)	2–18 (10)	5–16 (6)
8	18–9 (4)	22–11 (1)	14–4 (2)	20–12 (8)	15–7 (6)	13–5 (7)	6–21 (4)	10–23 (1)	2–16 (2)	8–24 (8)	3–19 (6)	1–17 (7)
9	19–10 (5)	18–11 (2)	15–5 (3)	21–12 (9)	16–8 (7)	14–1 (8)	7–22 (5)	6–23 (2)	3–17 (3)	9–24 (9)	4–20 (7)	2–13 (8)
10	20–6 (1)	19–11 (3)	16–1 (4)	22–12 (10)	17–9 (8)	15–2 (9)	8–18 (1)	7–23 (3)	4–13 (4)	10–24 (10)	5–21 (8)	3–14 (9)
11	1–13 (11)	2–14 (11)	3–15 (11)	4–16 (11)	5–17 (11)	18–6 (11)	19–7 (11)	20–8 (11)	21–9 (11)	22–10 (11)	11–23 (11)	24–12 (11)
12	13–6 (12)	14–7 (12)	15–8 (12)	16–9 (12)	17–10 (12)	1–18 (12)	2–19 (12)	3–20 (12)	4–21 (12)	5–22 (12)	23–12 (12)	11–24 (12)

	13	14	15	16	17	18	19
1	38–25 (1)	29–30 (3)	33–35 (4)	37–26 (8)	28–36 (9)	31–34 (11)	27–32 (13)
2	38–26 (2)	30–31 (4)	34–36 (5)	37–27 (9)	29–25 (10)	32–35 (12)	28–33 (14)
3	38–27 (3)	31–32 (5)	35–25 (6)	37–28 (10)	30–26 (11)	33–36 (7)	29–34 (13)
4	38–28 (4)	32–33 (6)	36–26 (1)	37–29 (11)	31–27 (12)	34–25 (8)	30–35 (14)
5	38–29 (5)	33–34 (1)	25–27 (2)	37–30 (12)	32–28 (7)	35–26 (9)	31–36 (13)
6	38–30 (6)	34–35 (2)	26–28 (3)	31–37 (1)	33–29 (8)	36–27 (10)	32–25 (14)
7	31–38 (7)	36–35 (3)	29–27 (4)	32–37 (2)	30–34 (9)	28–25 (11)	26–33 (13)
8	32–38 (8)	25–36 (4)	30–28 (5)	33–37 (3)	31–35 (10)	29–26 (12)	27–34 (14)
9	33–38 (9)	26–25 (5)	31–29 (6)	34–37 (4)	32–36 (11)	30–27 (7)	28–35 (13)
10	34–38 (10)	27–26 (6)	32–30 (1)	35–37 (5)	33–25 (12)	31–28 (8)	29–36 (14)
11	35–38 (11)	28–27 (1)	33–31 (2)	36–37 (6)	34–26 (7)	32–29 (9)	30–25 (13)
12	36–38 (12)	29–28 (2)	34–32 (3)	37–25 (7)	35–27 (8)	33–30 (10)	31–26 (14)

21 teams, three sessions of 14 rounds

The same principles apply to 21 teams as those used with 15 teams above. The movement is based on three 7-table bicyclic schedules. There are 42 rounds with each team not playing 2 rounds and board-sets.

Team to pair mapping

Team	Session 1	Session 2	Session 3
1	1,7	15,21	29,35
2	2,8	16,22	30,36
3	3,9	17,23	31,37
4	4,10	18,24	32,38
5	5,11	19,25	33,34
6	6,12	20,26	34,40
7	13,14	27,28	41,42
8	29,35	1,7	15,21
9	30,36	2,8	16,22
10	31,37	3,9	17,23
11	32,38	4,10	18,24
12	33,34	5,11	19,25
13	34,40	6,12	20,26
14	41,42	13,14	27,28
15	15,21	29,35	1,7
16	16,22	30,36	2,8
17	17,23	31,37	3,9
18	18,24	32,38	4,10
19	19,25	33,34	5,11
20	20,26	34,40	6,12
21	27,28	41,42	13,14

Session schedule

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	14–15 (1)	5–20 (3)	9–25 (4)	13–16 (8)	4–26 (9)	7–24 (11)	3–22 (13)	28–1 (1)	19–6 (3)	23–11 (4)	27–2 (8)	18–12 (9)	21–10 (11)	17–8 (13)
2	14–16 (2)	6–21 (4)	10–26 (5)	13–17 (9)	5–15 (10)	8–25 (12)	4–23 (14)	28–2 (2)	20–7 (4)	24–12 (5)	27–3 (9)	19–1 (10)	22–11 (12)	18–9 (14)
3	14–17 (3)	7–22 (5)	11–15 (6)	13–18 (10)	6–16 (11)	9–26 (7)	5–24 (13)	28–3 (3)	21–8 (5)	25–1 (6)	27–4 (10)	20–2 (11)	23–12 (7)	19–10 (13)
4	14–18 (4)	8–23 (6)	12–16 (1)	13–19 (11)	7–17 (12)	10–15 (8)	6–25 (14)	28–4 (4)	22–9 (6)	26–2 (1)	27–5 (11)	21–3 (12)	24–1 (8)	20–11 (14)
5	14–19 (5)	9–24 (1)	1–17 (2)	13–20 (12)	8–18 (7)	11–16 (9)	7–26 (13)	28–5 (5)	23–10 (1)	15–3 (2)	27–6 (12)	22–4 (7)	25–2 (9)	21–12 (13)
6	14–20 (6)	10–25 (2)	2–18 (3)	21–13 (1)	9–19 (8)	12–17 (10)	8–15 (14)	28–6 (6)	24–11 (2)	16–4 (3)	7–27 (1)	23–5 (8)	26–3 (10)	22–1 (14)
7	21–14 (7)	26–11 (3)	19–3 (4)	22–13 (2)	20–10 (9)	18–1 (11)	16–9 (13)	7–28 (7)	12–25 (3)	5–17 (4)	8–27 (2)	6–24 (9)	4–15 (11)	2–23 (13)
8	22–14 (8)	15–12 (4)	20–4 (5)	23–13 (3)	21–11 (10)	19–2 (12)	17–10 (14)	8–28 (8)	1–26 (4)	6–18 (5)	9–27 (3)	7–25 (10)	5–16 (12)	3–24 (14)
9	23–14 (9)	16–1 (5)	21–5 (6)	24–13 (4)	22–12 (11)	20–3 (7)	18–11 (13)	9–28 (9)	2–15 (5)	7–19 (6)	10–27 (4)	8–26 (11)	6–17 (7)	4–25 (13)
10	24–14 (10)	17–2 (6)	22–6 (1)	25–13 (5)	23–1 (12)	21–4 (8)	19–12 (14)	10–28 (10)	3–16 (6)	8–20 (1)	11–27 (5)	9–15 (12)	7–18 (8)	5–26 (14)
11	25–14 (11)	18–3 (1)	23–7 (2)	26–13 (6)	24–2 (7)	22–5 (9)	20–1 (13)	11–28 (11)	4–17 (1)	9–21 (2)	12–27 (6)	10–16 (7)	8–19 (9)	6–15 (13)
12	26–14 (12)	19–4 (2)	24–8 (3)	13–15 (7)	25–3 (8)	23–6 (10)	21–2 (14)	12–28 (12)	5–18 (2)	10–22 (3)	27–1 (7)	11–17 (8)	9–20 (10)	7–16 (14)
13	1–15 (3)	2–16 (4)	3–17 (5)	4–18 (6)	5–19 (1)	6–20 (2)	21–7 (3)	22–8 (4)	23–9 (5)	24–10 (6)	25–11 (1)	26–12 (2)	13–27 (13)	28–14 (13)
14	15–7 (9)	16–8 (10)	17–9 (11)	18–10 (12)	19–11 (7)	20–12 (8)	1–21 (9)	2–22 (10)	3–23 (11)	4–24 (12)	5–25 (7)	6–26 (8)	27–14 (14)	13–28 (14)

	15	16	17	18	19	20	21
1	42–29 (1)	33–34 (3)	37–39 (4)	41–30 (8)	32–40 (9)	35–38 (11)	31–36 (13)
2	42–30 (2)	34–35 (4)	38–40 (5)	41–31 (9)	33–29 (10)	36–39 (12)	32–37 (14)
3	42–31 (3)	35–36 (5)	39–29 (6)	41–32 (10)	34–30 (11)	37–40 (7)	33–38 (13)
4	42–32 (4)	36–37 (6)	40–30 (1)	41–33 (11)	35–31 (12)	38–29 (8)	34–39 (14)
5	42–33 (5)	37–38 (1)	29–31 (2)	41–34 (12)	36–32 (7)	39–30 (9)	35–40 (13)
6	42–34 (6)	38–39 (2)	30–32 (3)	35–41 (1)	37–33 (8)	40–31 (10)	36–29 (14)
7	35–42 (7)	40–39 (3)	33–31 (4)	36–41 (2)	34–38 (9)	32–29 (11)	30–37 (13)
8	36–42 (8)	29–40 (4)	34–32 (5)	37–41 (3)	35–39 (10)	33–30 (12)	31–38 (14)
9	37–42 (9)	30–29 (5)	35–33 (6)	38–41 (4)	36–40 (11)	34–31 (7)	32–39 (13)
10	38–42 (10)	31–30 (6)	36–34 (1)	39–41 (5)	37–29 (12)	35–32 (8)	33–40 (14)
11	39–42 (11)	32–31 (1)	37–35 (2)	40–41 (6)	38–30 (7)	36–33 (9)	34–29 (13)
12	40–42 (12)	33–32 (2)	38–36 (3)	41–29 (7)	39–31 (8)	37–34 (10)	35–30 (14)
13							
14							

10.8 Four sessions

The essence of mathematics is not to make simple things complicated,
but to make complicated things simple.
S. Gudder

16 teams, four sessions

The schedules for 16 teams are not cyclic because they are made from the 4-teams non-cyclic schedule. There are 4 groups of teams, 1 to 4, 5 to 8, 9 to 12 and 13 to 16. Two schedules are used, the first for session 1 when the teams meet those in their group. The second schedule is for the last three sessions where they meet the teams from the other groups. The complete schedules are given below. The pairs are in 4 groups, 1 to 8, 9 to 16, 17 to 24 and 25 to 32. Each team has pair numbers assigned for each of the sessions. Some organizers may prefer to play session 1 last so that players can get away earlier. This approach can also apply to any of the following schedules for four sessions.

The following map shows the 16 teams and their pair numbers for the 4 sessions.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1,5	2,6	3,7	4,8	9,13	10,14	11,15	12,16	17,21	18,22	19,23	20,24	25,29	26,30	27,31	28,32
2	1,5	2,6	3,7	4,8	9,13	10,14	11,15	12,16	17,21	18,22	19,23	20,24	25,29	26,30	27,31	28,32
3	1,5	2,6	3,7	4,8	17,21	18,22	19,23	20,24	25,29	26,30	27,31	28,32	9,13	10,14	11,15	12,16
4	1,5	2,6	3,7	4,8	25,29	26,30	27,31	28,32	9,13	10,14	11,15	12,16	17,21	18,22	19,23	20,24

Session 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1–2 (1)	7–4 (2)	6–8 (3)	5–3 (4)	9–10 (1)	15–12 (2)	14–16 (3)	13–11 (4)	17–18 (1)	23–20 (2)	22–24 (3)	21–19 (4)	25–26 (1)	31–28 (2)	30–32 (3)	29–27 (4)
2	6–5 (1)	8–3 (2)	4–2 (3)	7–1 (4)	14–13 (1)	16–11 (2)	12–10 (3)	15–9 (4)	22–21 (1)	24–19 (2)	20–18 (3)	23–17 (4)	30–29 (1)	32–27 (2)	28–26 (3)	31–25 (4)
3	4–3 (1)	2–5 (2)	7–6 (5)	8–1 (6)	12–11 (1)	10–13 (2)	15–14 (5)	16–9 (6)	20–19 (1)	18–21 (2)	23–22 (5)	24–17 (6)	28–27 (1)	26–29 (2)	31–30 (5)	32–25 (6)
4	7–8 (1)	1–6 (2)	2–3 (5)	5–4 (6)	15–16 (1)	9–14 (2)	10–11 (5)	13–12 (6)	23–24 (1)	17–22 (2)	18–19 (5)	21–20 (6)	31–32 (1)	25–30 (2)	26–27 (5)	29–28 (6)
5	7–5 (3)	8–2 (4)	4–1 (5)	3–6 (6)	15–13 (3)	16–10 (4)	12–9 (5)	11–14 (6)	23–21 (3)	24–18 (4)	20–17 (5)	19–22 (6)	31–29 (3)	32–26 (4)	28–25 (5)	27–30 (6)
6	1–3 (3)	6–4 (4)	5–8 (5)	2–7 (6)	9–11 (3)	14–12 (4)	13–16 (5)	10–15 (6)	17–19 (3)	22–20 (4)	21–24 (5)	18–23 (6)	25–27 (3)	30–28 (4)	29–32 (5)	26–31 (6)

Sessions 2, 3 and 4

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1–10 (1)	7–12 (2)	6–16 (3)	5–11 (4)	9–2 (1)	15–4 (2)	14–8 (3)	13–3 (4)	17–26 (1)	23–28 (2)	22–32 (3)	21–27 (4)	25–18 (1)	31–20 (2)	30–24 (3)	29–19 (4)
2	6–13 (1)	8–11 (2)	4–10 (3)	7–9 (4)	14–5 (1)	16–3 (2)	12–2 (3)	15–1 (4)	22–29 (1)	24–27 (2)	20–26 (3)	23–25 (4)	30–21 (1)	32–19 (2)	28–18 (3)	31–17 (4)
3	4–11 (1)	2–13 (2)	7–14 (5)	8–9 (6)	12–3 (1)	10–5 (2)	15–6 (5)	16–1 (6)	20–27 (1)	18–29 (2)	23–30 (5)	24–25 (6)	28–19 (1)	26–21 (2)	31–22 (5)	32–17 (6)
4	7–16 (1)	1–14 (2)	2–11 (5)	5–12 (6)	15–8 (1)	9–6 (2)	10–3 (5)	13–4 (6)	23–32 (1)	17–30 (2)	18–27 (5)	21–28 (6)	31–24 (1)	25–22 (2)	26–19 (5)	29–20 (6)
5	7–13 (3)	8–10 (4)	4–9 (5)	3–14 (6)	15–5 (3)	16–2 (4)	12–1 (5)	11–6 (6)	23–29 (3)	24–26 (4)	20–25 (5)	19–30 (6)	31–21 (3)	32–18 (4)	28–17 (5)	27–22 (6)
6	1–11 (3)	6–12 (4)	5–16 (5)	2–15 (6)	9–3 (3)	14–4 (4)	13–8 (5)	10–7 (6)	17–27 (3)	22–28 (4)	21–32 (5)	18–31 (6)	25–19 (3)	30–20 (4)	29–24 (5)	26–23 (6)
7	1–9 (7)	2–10 (7)	3–11 (7)	4–12 (7)	13–5 (7)	14–6 (7)	15–7 (7)	16–8 (7)	17–25 (8)	18–26 (8)	19–27 (8)	20–28 (8)	29–21 (8)	30–22 (8)	31–23 (8)	32–24 (8)
8	13–1 (8)	14–2 (8)	15–3 (8)	16–4 (8)	5–9 (8)	6–10 (8)	7–11 (8)	8–12 (8)	29–17 (7)	30–18 (7)	31–19 (7)	32–20 (7)	21–25 (7)	22–26 (7)	23–27 (7)	24–28 (7)

20 teams, four sessions

The schedules for 20 teams are cyclic because they are made from the 5 teams cyclic schedule. There are 4 groups of teams, 1 to 5, 6 to 10, 11 to 15 and 16 to 20. There are two schedules, the first for session 1 when the teams meet those in their group and 8 rounds are played, each team missing two board-sets. The second schedule is for the last three sessions where they meet the teams from the other groups playing 10 rounds per session and all board-sets. The pairs are in 4 groups, 1 to 10, 11 to 20, 21 to 30 and 31 to 40. Each team has pair numbers assigned for each of the sessions.

The following map shows the 20 teams and their pair numbers for the 4 sessions.

	1	2	3	4	5	6	7	8	9	10
1	1,5	2,6	3,7	4,8	9,10	11,15	12,16	13,17	14,18	19,20
2	1,5	2,6	3,7	4,8	9,10	11,15	12,16	13,17	14,18	19,20
3	1,5	2,6	3,7	4,8	9,10	21,25	22,26	23,27	24,28	29,30
4	1,5	2,6	3,7	4,8	9,10	31,35	32,36	33,37	34,38	39,40

	11	12	13	14	15	16	17	18	19	20
1	21,25	22,26	23,27	24,28	29,30	31,35	32,36	33,37	34,38	39,40
2	21,25	22,26	23,27	24,28	29,30	31,35	32,36	33,37	34,38	39,40
3	31,35	32,36	33,37	34,38	39,40	11,15	12,16	13,17	14,18	19,20
4	11,15	12,16	13,17	14,18	19,20	21,25	22,26	23,27	24,28	29,30

The pairs are cyclic with a switch after 4 rounds. Tables 5, 10, 15 and 20 alternate sets 9 and 10. The first two tables play full sets (1 to 8) and the middle tables play half-sets (2,3,4,1 and 7,8,5,6).

Session 1

1	2	3	4	5	6	7	8	9	10
10–1 (1)	5–9 (1)	3–4 (2)	6–8 (7)	2–7 (9)	20–11 (1)	15–19 (1)	13–14 (2)	16–18 (7)	12–17 (9)

11	12	13	14	15	16	17	18	19	20
30–21 (1)	25–29 (1)	23–24 (2)	26–28 (7)	22–27 (9)	40–31 (1)	35–39 (1)	33–34 (2)	36–38 (7)	32–37 (9)

Sessions 2 to 4

1	2	3	4	5	6	7	8	9	10
10–11 (1)	5–19 (1)	3–14 (2)	6–18 (7)	2–17 (9)	20–1 (1)	15–9 (1)	13–4 (2)	16–8 (7)	12–7 (9)

11	12	13	14	15	16	17	18	19	20
30–31 (1)	25–39 (1)	23–34 (2)	26–38 (7)	22–37 (9)	40–21 (1)	35–29 (1)	33–24 (2)	36–28 (7)	32–27 (9)

Last two rounds

1–11 (2)	2–12 (3)	3–13 (4)	4–14 (1)	15–5 (2)	16–6 (3)	17–7 (4)	18–8 (1)	9–19 (9)	20–10 (9)
11–5 (7)	12–6 (8)	13–7 (5)	14–8 (6)	1–15 (7)	2–16 (8)	3–17 (5)	4–18 (6)	19–10 (10)	9–20 (10)

21–31 (2)	22–32 (3)	23–33 (4)	24–34 (1)	35–25 (2)	36–26 (3)	37–27 (4)	38–28 (1)	29–39 (9)	40–30 (9)
31–25 (7)	32–26 (8)	33–27 (5)	34–28 (6)	21–35 (7)	22–36 (8)	23–37 (5)	24–38 (6)	39–30 (10)	29–40 (10)

24 teams, four sessions

The schedules for 24 teams are bicyclic because they are made from the 6-teams bicyclic schedule. There are 4 groups of teams, 1 to 6, 7 to 12, 13 to 18 and 19 to 24. There are four schedules, the first two for session 1 (both sessions are bicyclic) when the teams meet those in their group and 10 rounds are played. The second two schedules are for the last three sessions where they meet the teams from the other groups playing 12 rounds per session and all board-sets. The pairs are in 4 groups, 1 to 12, 13 to 24, 25 to 36 and 37 to 48. Each team has pair numbers assigned for each of the sessions.

The following map shows the 24 teams and their pair numbers for the 4 sessions.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1,6	2,7	3,8	4,9	5,10	11,12	13,18	14,19	15,20	16,21	17,22	23,24
2	1,6	2,7	3,8	4,9	5,10	11,12	13,18	14,19	15,20	16,21	17,22	23,24
3	1,6	2,7	3,8	4,9	5,10	11,12	25,30	26,31	27,32	28,33	29,34	35,36
4	1,6	2,7	3,8	4,9	5,10	11,12	37,42	38,43	39,44	40,45	41,46	47,48

	13	14	15	16	17	18	19	20	21	22	23	24
1	25,30	26,31	27,32	28,33	29,34	35,36	37,42	38,43	39,44	40,45	41,46	47,48
2	25,30	26,31	27,32	28,33	29,34	35,36	37,42	38,43	39,44	40,45	41,46	47,48
3	37,42	38,43	39,44	40,45	41,46	47,48	13,18	14,19	15,20	16,21	17,22	23,24
4	13,18	14,19	15,20	16,21	17,22	23,24	25,30	26,31	27,32	28,33	29,34	35,36

Session 1 rounds 1 to 5

1	2	3	4	5	6	7	8	9	10	11	12
5–8 (2)	4–6 (3)	12–3 (4)	9–10 (6)	11–2 (7)	1–7 (10)	17–20 (2)	16–18 (3)	24–15 (4)	21–22 (6)	23–14 (7)	13–19 (10)

13	14	15	16	17	18	19	20	21	22	23	24
29–32 (2)	28–30 (3)	36–27 (4)	33–34 (6)	35–26 (7)	25–31 (10)	41–44 (2)	40–42 (3)	48–39 (4)	45–46 (6)	47–38 (7)	37–43 (10)

Session 1 rounds 6 to 10

1	2	3	4	5	6	7	8	9	10	11	12
4–7 (1)	3–10 (2)	12–2 (3)	8–9 (10)	11–1 (6)	5–6 (9)	16–19 (1)	15–22 (2)	24–14 (3)	20–21 (10)	23–13 (6)	17–18 (9)

13	14	15	16	17	18	19	20	21	22	23	24
28–31 (1)	27–34 (2)	36–26 (3)	32–33 (10)	35–25 (6)	29–30 (9)	40–43 (1)	39–46 (2)	48–38 (3)	44–45 (10)	47–37 (6)	41–42 (9)

Sessions 2 to 4 rounds 1 to 5

1	2	3	4	5	6	7	8	9	10	11	12
5–20 (2)	4–18 (3)	12–15 (4)	9–22 (6)	11–14 (7)	1–19 (10)	17–8 (2)	16–6 (3)	24–3 (4)	21–10 (6)	23–2 (7)	13–7 (10)

13	14	15	16	17	18	19	20	21	22	23	24
29–44 (2)	28–42 (3)	36–39 (4)	33–46 (6)	35–38 (7)	25–43 (10)	41–32 (2)	40–30 (3)	48–27 (4)	45–34 (6)	47–26 (7)	37–31 (10)

Sessions 2 to 4 rounds 6 to 10

1	2	3	4	5	6	7	8	9	10	11	12
21–7 (2)	20–11 (4)	17–2 (5)	18–12 (6)	13–10 (9)	16–3 (10)	9–19 (2)	8–23 (4)	5–14 (5)	6–24 (6)	1–22 (9)	4–15 (10)

13	14	15	16	17	18	19	20	21	22	23	24
45–31 (2)	44–35 (4)	41–26 (5)	42–36 (6)	37–34 (9)	40–27 (10)	33–43 (2)	32–47 (4)	29–38 (5)	30–48 (6)	25–46 (9)	28–39 (10)

Last two rounds

1–13 (11)	2–14 (11)	3–15 (11)	4–16 (11)	5–17 (11)	18–6 (11)	19–7 (11)	20–8 (11)	21–9 (11)	22–10 (11)	11–23 (11)	24–12 (11)
13–6 (12)	14–7 (12)	15–8 (12)	16–9 (12)	17–10 (12)	1–18 (12)	2–19 (12)	3–20 (12)	4–21 (12)	5–22 (12)	23–12 (12)	11–24 (12)

25–37 (12)	26–38 (12)	27–39 (12)	28–40 (12)	29–41 (12)	42–30 (12)	43–31 (12)	44–32 (12)	45–33 (12)	46–34 (12)	35–47 (12)	48–36 (12)
37–30 (11)	38–31 (11)	39–32 (11)	40–33 (11)	41–34 (11)	25–42 (11)	26–43 (11)	27–44 (11)	28–45 (11)	29–46 (11)	47–36 (11)	35–48 (11)

28 teams, four sessions

The schedules for 28 teams are cyclic because they are made from the 7-teams cyclic schedule. There are 4 groups of teams, 1 to 7, 8 to 14, 15 to 21 and 22 to 28. There are two schedules, the first for session 1 when the teams meet those in their group and 12 rounds are played, each team missing two board-sets. The second schedule is for the last three sessions where they meet the teams from the other groups playing 14 rounds per session and all board-sets. The pairs are in 4 groups, 1 to 14, 15 to 28, 29 to 42 and 43 to 56. Each team has pair numbers assigned for each of the sessions.

The following map shows the 28 teams and their pair numbers for the 4 sessions.

	1	2	3	4	5	6	7
1 to 4	1,7	2,8	3,9	4,10	5,11	6,12	13,14

	8	9	10	11	12	13	14
1 and 2	15,21	16,22	17,23	18,24	19,25	20,26	27,28
3	29,35	30,36	31,37	32,38	33,39	34,40	41,42
4	43,49	44,50	45,51	46,52	47,53	48,54	55,56

	15	16	17	18	19	20	21
1 and 2	29,35	30,36	31,37	32,38	33,39	34,40	41,42
3	43,49	44,50	45,51	46,52	47,53	48,54	55,56
4	15,21	16,22	17,23	18,24	19,25	20,26	27,28

	22	23	24	25	26	27	28
1 and 2	43,49	44,50	45,51	46,52	47,53	48,54	55,56
3	15,21	16,22	17,23	18,24	19,25	20,26	27,28
4	29,35	30,36	31,37	32,38	33,39	34,40	41,42

The pairs are cyclic with a switch after 6 rounds. Tables 7, 14, 21 and 28 alternate sets 13 and 14. Tables 1 and 4, 8 and 11, 15 and 18, 22 and 25 play full sets (1 to 12) and the other tables cycle half-sets (1 to 6 or 7 to 12). Tables 4, 11, 18 and 25 switch after rounds 5 and 11.

Session 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14
14– 1 (1)	5– 6 (3)	9–11 (4)	13– 2 (8)	4–12 (9)	7–10 (11)	3– 8 (13)	28–15 (1)	19–20 (3)	23–25 (4)	27–16 (8)	18–26 (9)	21–24 (11)	17–22 (13)

15	16	17	18	19	20	21	22	23	24	25	26	27	28
42–29 (1)	33–34 (3)	37–39 (4)	41–30 (8)	32–40 (9)	35–38 (11)	31–36 (13)	56–43 (1)	47–48 (3)	51–53 (4)	55–44 (8)	46–54 (9)	49–52 (11)	45–50 (13)

Sessions 2 to 4

1	2	3	4	5	6	7	8	9	10	11	12	13	14
14–15 (1)	5–20 (3)	9–25 (4)	13–16 (8)	4–26 (9)	7–24 (11)	3–22 (13)	28– 1 (1)	19– 6 (3)	23–11 (4)	27– 2 (8)	18–12 (9)	21–10 (11)	17– 8 (13)

15	16	17	18	19	20	21	22	23	24	25	26	27	28
42–43 (1)	33–48 (3)	37–53 (4)	41–44 (8)	32–54 (9)	35–52 (11)	31–50 (13)	56–29 (1)	47–34 (3)	51–39 (4)	55–30 (8)	46–40 (9)	49–38 (11)	45–36 (13)

Last two rounds

1–15 (3)	2–16 (4)	3–17 (5)	4–18 (6)	5–19 (1)	6–20 (2)	21– 7 (3)	22– 8 (4)	23– 9 (5)	24–10 (6)	25–11 (1)	26–12 (2)	13–27 (13)	28–14 (13)
15– 7 (9)	16– 8 (10)	17– 9 (11)	18–10 (12)	19–11 (7)	20–12 (8)	1–21 (9)	2–22 (10)	3–23 (11)	4–24 (12)	5–25 (7)	6–26 (8)	27–14 (14)	13–28 (14)

29–43 (3)	30–44 (4)	31–45 (5)	32–46 (6)	33–47 (1)	34–48 (2)	49–35 (3)	50–36 (4)	51–37 (5)	52–38 (6)	53–39 (1)	54–40 (2)	41–55 (13)	56–42 (13)
43–35 (9)	44–36 (10)	45–37 (11)	46–38 (12)	47–39 (7)	48–40 (8)	29–49 (9)	30–50 (10)	31–51 (11)	32–52 (12)	33–53 (7)	34–54 (8)	55–42 (14)	41–56 (14)

32 teams, four sessions

The schedules for 32 teams are bicyclic because they are made from the 8-teams bicyclic schedule. There are 4 groups of teams, 1 to 8, 9 to 16, 17 to 24 and 25 to 32. There are four schedules, the first two for session 1 (both sessions are bicyclic) when the teams meet those in their group and 14 rounds are played. The second two schedules are for the last three sessions where they meet the teams from the other groups playing 16 rounds per session and all board-sets. The pairs are in 4 groups, 1 to 16, 17 to 32, 33 to 48 and 49 to 64. Each team has pair numbers assigned for each of the sessions.

The following map shows the 32 teams and their pair numbers for the 4 sessions.

Sessions	1	2	3	4	5	6	7	8
1 to 4	1,8	2,9	3,10	4,11	5,12	6,13	7,14	15,16

Sessions	9	10	11	12	13	14	15	16
1 and 2	17,24	18,25	19,26	20,27	21,28	22,29	23,30	31,32
3	33,40	34,41	35,42	36,43	37,44	38,45	39,46	47,48
4	49,56	50,57	51,58	52,59	53,60	54,61	55,62	63,64

Sessions	17	18	19	20	21	22	23	24
1 and 2	33,40	34,41	35,42	36,43	37,44	38,45	39,46	47,48
3	49,56	50,57	51,58	52,59	53,60	54,61	55,62	63,64
4	17,24	18,25	19,26	20,27	21,28	22,29	23,30	31,32

Sessions	25	26	27	28	29	30	31	32
1 and 2	49,56	50,57	51,58	52,59	53,60	54,61	55,62	63,64
3	17,24	18,25	19,26	20,27	21,28	22,29	23,30	31,32
4	33,40	34,41	35,42	36,43	37,44	38,45	39,46	47,48

Session 1 rounds 1 to 7

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
16–1 (1)	12–7 (4)	10–4 (6)	2–6 (7)	9–11 (8)	14–8 (10)	13–3 (11)	15–5 (12)	32–17 (1)	28–23 (4)	26–20 (6)	18–22 (7)	25–27 (8)	30–24 (10)	29–19 (11)	31–21 (12)

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
48–33 (1)	44–39 (4)	42–36 (6)	34–38 (7)	41–43 (8)	46–40 (10)	45–35 (11)	47–37 (12)	64–49 (1)	60–55 (4)	58–52 (6)	50–54 (7)	57–59 (8)	62–56 (10)	61–51 (11)	63–53 (12)

Session 1 rounds 8 to 14

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
8–15 (1)	14–5 (4)	11–3 (6)	13–9 (7)	4–2 (8)	1–7 (10)	10–6 (11)	12–16 (12)	24–31 (1)	30–21 (4)	27–19 (6)	29–25 (7)	20–18 (8)	17–23 (10)	26–22 (11)	28–32 (12)

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
40–47 (1)	46–37 (4)	43–35 (6)	45–41 (7)	36–34 (8)	33–39 (10)	42–38 (11)	44–48 (12)	56–63 (1)	62–53 (4)	59–51 (6)	61–57 (7)	52–50 (8)	49–55 (10)	58–54 (11)	60–64 (12)

Sessions 2 to 4 rounds 1 to 7

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
16–17 (1)	12–23 (4)	10–20 (6)	2–22 (7)	9–27 (8)	14–24 (10)	13–19 (11)	15–21 (12)	32–1 (1)	28–7 (4)	26–4 (6)	18–6 (7)	25–11 (8)	30–8 (10)	29–3 (11)	31–5 (12)

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
48–49 (1)	44–55 (4)	42–52 (6)	34–54 (7)	41–59 (8)	46–56 (10)	45–51 (11)	47–53 (12)	64–33 (1)	60–39 (4)	58–36 (6)	50–38 (7)	57–43 (8)	62–40 (10)	61–35 (11)	63–37 (12)

Sessions 2 to 4 rounds 8 to 14

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
24–15 (1)	30–5 (4)	27–3 (6)	29–9 (7)	20–2 (8)	17–7 (10)	26–6 (11)	28–16 (12)	8–31 (1)	14–21 (4)	11–19 (6)	13–25 (7)	4–18 (8)	1–23 (10)	10–22 (11)	12–32 (12)

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
56–47 (1)	62–37 (4)	59–35 (6)	61–41 (7)	52–34 (8)	49–39 (10)	58–38 (11)	60–48 (12)	40–63 (1)	46–53 (4)	43–51 (6)	45–57 (7)	36–50 (8)	33–55 (10)	42–54 (11)	44–64 (12)

Last two rounds

1–17 (15)	2–18 (15)	3–19 (15)	4–20 (15)	5–21 (15)	6–22 (15)	7–23 (15)	24–8 (15)	25–9 (15)	26–10 (15)	27–11 (15)	28–12 (15)	29–13 (15)	30–14 (15)	15–31 (15)	32–16 (15)
17–8 (16)	18–9 (16)	19–10 (16)	20–11 (16)	21–12 (16)	22–13 (16)	23–14 (16)	1–24 (16)	2–25 (16)	3–26 (16)	4–27 (16)	5–28 (16)	6–29 (16)	7–30 (16)	31–16 (16)	15–32 (16)

33–49 (16)	34–50 (16)	35–51 (16)	36–52 (16)	37–53 (16)	38–54 (16)	39–55 (16)	56–40 (16)	57–41 (16)	58–42 (16)	59–43 (16)	60–44 (16)	61–45 (16)	62–46 (16)	47–63 (16)	64–48 (16)
49–40 (15)	50–41 (15)	51–42 (15)	52–43 (15)	53–44 (15)	54–45 (15)	55–46 (15)	33–56 (15)	34–57 (15)	35–58 (15)	36–59 (15)	37–60 (15)	38–61 (15)	39–62 (15)	63–48 (15)	47–64 (15)

36 teams, four sessions

The schedules for 36 teams are cyclic because they are made from the 9-teams cyclic schedule. There are 4 groups of teams, 1 to 9, 10 to 18, 19 to 27 and 28 to 36. There are two schedules, the first for session 1 when the teams meet those in their group and 16 rounds are played, each team missing two board-sets. The second schedule is for the last three sessions where they meet the teams from the other groups playing 18 rounds per session and all board-sets. The pairs are in 4 groups, 1 to 18, 19 to 36, 37 to 54 and 55 to 72. Each team has pair numbers assigned for each of the sessions.

The following map shows the 36 teams and their pair numbers for the 4 sessions.

Sessions	1	2	3	4	5	6	7	8	9
1 to 4	1,9	2,10	3,11	4,12	5,13	6,14	7,15	8,16	17,18

Sessions	10	11	12	13	14	15	16	17	18
1 and 2	19,27	20,28	21,29	22,30	23,31	24,32	25,33	26,34	35,36
3	37,45	38,46	39,47	40,48	41,49	42,50	43,51	44,52	53,54
4	55,63	56,64	57,65	58,66	59,67	60,68	61,69	62,70	71,72

Sessions	19	20	21	22	23	24	25	26	27
1 and 2	37,45	38,46	39,47	40,48	41,49	42,50	43,51	44,52	53,54
3	55,63	56,64	57,65	58,66	59,67	60,68	61,69	62,70	71,72
4	19,27	20,28	21,29	22,30	23,31	24,32	25,33	26,34	35,36

Sessions	28	29	30	31	32	33	34	35	36
1 and 2	55,63	56,64	57,65	58,66	59,67	60,68	61,69	62,70	71,72
3	19,27	20,28	21,29	22,30	23,31	24,32	25,33	26,34	35,36
4	37,45	38,46	39,47	40,48	41,49	42,50	43,51	44,52	53,54

Session 1 - 16 rounds

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
18–1 (1)	9–17 (1)	12–15 (3)	5–16 (3)	10–11 (4)	3–7 (13)	2–4 (15)	8–14 (15)	6–13 (17)	36–19 (1)	27–35 (1)	30–33 (3)	23–34 (3)	28–29 (4)	21–25 (13)	20–22 (15)	26–32 (15)	24–31 (17)

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
54–37 (1)	45–53 (1)	48–51 (3)	41–52 (3)	46–47 (4)	39–43 (13)	38–40 (15)	44–50 (15)	42–49 (17)	72–55 (1)	63–71 (1)	66–69 (3)	59–70 (3)	64–65 (4)	57–61 (13)	56–58 (15)	62–68 (15)	60–67 (17)

Sessions 2 to 4 –16 rounds

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
18–19 (1)	9–35 (1)	12–33 (3)	5–34 (3)	10–29 (4)	3–25 (13)	2–22 (15)	8–32 (15)	6–31 (17)	36–1 (1)	27–17 (1)	30–15 (3)	23–16 (3)	28–11 (4)	21–7 (13)	20–4 (15)	26–14 (15)	24–13 (17)

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
54–55 (1)	45–71 (1)	48–69 (3)	41–70 (3)	46–65 (4)	39–61 (13)	38–58 (15)	44–68 (15)	42–67 (17)	72–37 (1)	63–53 (1)	66–51 (3)	59–52 (3)	64–47 (4)	57–43 (13)	56–40 (15)	62–50 (15)	60–49 (17)

Last two rounds

1–19 (6)	2–20 (7)	3–21 (8)	4–22 (1)	5–23 (2)	6–24 (3)	7–25 (4)	8–26 (5)	27–9 (6)	28–10 (7)	29–11 (8)	30–12 (1)	31–13 (2)	32–14 (3)	33–15 (4)	34–16 (5)	17–35 (17)	36–18 (17)
19–9 (13)	20–10 (14)	21–11 (15)	22–12 (16)	23–13 (9)	24–14 (10)	25–15 (11)	26–16 (12)	1–27 (13)	2–28 (14)	3–29 (15)	4–30 (16)	5–31 (9)	6–32 (10)	7–33 (11)	8–34 (12)	35–18 (18)	17–36 (18)

37–55 (6)	38–56 (7)	39–57 (8)	40–58 (1)	41–59 (2)	42–60 (3)	43–61 (4)	44–62 (5)	63–45 (6)	64–46 (7)	65–47 (8)	66–48 (1)	67–49 (2)	68–50 (3)	69–51 (4)	70–52 (5)	53–71 (17)	72–54 (17)
55–45 (13)	56–46 (14)	57–47 (15)	58–48 (16)	59–49 (9)	60–50 (10)	61–51 (11)	62–52 (12)	37–63 (13)	38–64 (14)	39–65 (15)	40–66 (16)	41–67 (9)	42–68 (10)	43–69 (11)	44–70 (12)	71–54 (18)	53–72 (18)

40 teams, four sessions

The schedules for 40 teams are bicyclic because they are made from the 10-teams bicyclic schedule. There are 4 groups of teams, 1 to 10, 11 to 20, 21 to 30 and 31 to 40. There are four schedules, the first two for session 1 (both sessions are bicyclic) when the teams meet those in their group and 18 rounds are played. The second two schedules are for the last three sessions where they meet the teams from the other groups playing 20 rounds per session and all board-sets. The pairs are in 4 groups, 1 to 20, 21 to 40, 41 to 60 and 61 to 80. Each team has pair numbers assigned for each of the sessions.

The following map shows the 40 teams and their pair numbers for the 4 sessions.

Sessions	1	2	3	4	5	6	7	8	9	10
1 to 4	1,10	2,11	3,12	4,13	5,14	6,15	7,16	8,17	9,18	19,20

Sessions	11	12	13	14	15	16	17	18	19	20
1 and 2	21,30	22,31	23,32	24,33	25,34	26,35	27,36	28,37	29,38	39,40
3	41,50	42,51	43,52	44,63	45,54	46,55	47,56	48,57	49,58	59,60
4	61,70	62,71	63,72	64,73	65,74	66,75	67,76	68,77	69,78	79,80

Sessions	21	22	23	24	25	26	27	28	29	30
1 and 2	41,50	42,51	43,52	44,63	45,54	46,55	47,56	48,57	49,58	59,60
3	61,70	62,71	63,72	64,73	65,74	66,75	67,76	68,77	69,78	79,80
4	21,30	22,31	23,32	24,33	25,34	26,35	27,36	28,37	29,38	39,40

Sessions	11	12	13	14	15	16	17	18	19	20
1 and 2	61,70	62,71	63,72	64,73	65,74	66,75	67,76	68,77	69,78	79,80
3	21,30	22,31	23,32	24,33	25,34	26,35	27,36	28,37	29,38	39,40
4	41,50	42,51	43,52	44,63	45,54	46,55	47,56	48,57	49,58	59,60

Session 1 rounds 1 to 9

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
16–18 (3)	6–14 (4)	10–4 (5)	7–17 (6)	20–8 (8)	3–2 (10)	12–13 (11)	5–1 (16)	11–15 (17)	19–9 (18)	36–38 (3)	26–34 (4)	30–24 (5)	27–37 (6)	40–28 (8)	23–22 (10)	32–33 (11)	25–21 (16)	31–35 (17)	39–29 (18)

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
56–58 (3)	46–54 (4)	50–44 (5)	47–57 (6)	60–48 (8)	43–42 (10)	52–53 (11)	45–41 (16)	51–55 (17)	59–49 (18)	76–78 (3)	66–74 (4)	70–64 (5)	67–77 (6)	80–68 (8)	63–62 (10)	72–73 (11)	65–61 (16)	71–75 (17)	79–69 (18)

Session 1 rounds 10 to 18

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
17–4 (1)	12–9 (4)	3–1 (6)	11–6 (8)	18–19 (9)	14–7 (10)	10–16 (11)	15–20 (15)	13–2 (16)	5–8 (18)	37–24 (1)	32–29 (4)	23–21 (6)	31–26 (8)	38–39 (9)	34–27 (10)	30–36 (11)	35–40 (15)	33–22 (16)	25–28 (18)

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
57–44 (1)	52–49 (4)	43–41 (6)	51–46 (8)	58–59 (9)	54–47 (10)	50–56 (11)	55–60 (15)	53–42 (16)	45–48 (18)	77–64 (1)	72–69 (4)	63–61 (6)	71–66 (8)	78–79 (9)	74–67 (10)	70–76 (11)	75–80 (15)	73–62 (16)	65–68 (18)

Sessions 2 to 4 rounds 1 to 9

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
16–38 (3)	6–34 (4)	10–24 (5)	7–37 (6)	20–28 (8)	3–22 (10)	12–33 (11)	5–21 (16)	11–35 (17)	19–29 (18)	36–18 (3)	26–14 (4)	30–4 (5)	27–17 (6)	40–8 (8)	23–2 (10)	32–13 (11)	25–1 (16)	31–15 (17)	39–9 (18)

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
56–78 (3)	46–74 (4)	50–64 (5)	47–77 (6)	60–68 (8)	43–62 (10)	52–73 (11)	45–61 (16)	51–75 (17)	59–69 (18)	76–58 (3)	66–54 (4)	70–44 (5)	67–57 (6)	80–48 (8)	63–42 (10)	72–53 (11)	65–41 (16)	71–55 (17)	79–49 (18)

Sessions 2 to 4 rounds 10 to 18

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
37–4 (1)	32–9 (4)	23–1 (6)	31–6 (8)	38–19 (9)	34–7 (10)	30–16 (11)	35–20 (15)	33–2 (16)	25–8 (18)	17–24 (1)	12–29 (4)	3–21 (6)	11–26 (8)	18–39 (9)	14–27 (10)	10–36 (11)	15–40 (15)	13–22 (16)	5–28 (18)

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
77–44 (1)	72–49 (4)	63–41 (6)	71–46 (8)	78–59 (9)	74–47 (10)	70–56 (11)	75–60 (15)	73–42 (16)	65–48 (18)	57–64 (1)	52–69 (4)	43–61 (6)	51–66 (8)	58–79 (9)	54–67 (10)	50–76 (11)	55–80 (15)	53–62 (16)	45–68 (18)

Last two rounds

1–21 (19)	2–22 (19)	3–23 (19)	4–24 (19)	5–25 (19)	6–26 (19)	7–27 (19)	8–28 (19)	9–29 (19)	30–10 (19)	31–11 (19)	32–12 (19)	33–13 (19)	34–14 (19)	35–15 (19)	36–16 (19)	37–17 (19)	38–18 (19)	19–39 (19)	40–20 (19)
21–10 (20)	22–11 (20)	23–12 (20)	24–13 (20)	25–14 (20)	26–15 (20)	27–16 (20)	28–17 (20)	29–18 (20)	1–30 (20)	2–31 (20)	3–32 (20)	4–33 (20)	5–34 (20)	6–35 (20)	7–36 (20)	8–37 (20)	9–38 (20)	39–20 (20)	19–40 (20)

41–61 (20)	42–62 (20)	43–63 (20)	44–64 (20)	45–65 (20)	46–66 (20)	47–67 (20)	48–68 (20)	49–69 (20)	70–50 (20)	71–51 (20)	72–52 (20)	73–53 (20)	74–54 (20)	75–55 (20)	76–56 (20)	77–57 (20)	78–58 (20)	59–79 (20)	80–60 (20)
61–50 (19)	62–51 (19)	63–52 (19)	64–53 (19)	65–54 (19)	66–55 (19)	67–56 (19)	68–57 (19)	69–58 (19)	41–70 (19)	42–71 (19)	43–72 (19)	44–73 (19)	45–74 (19)	46–75 (19)	47–76 (19)	48–77 (19)	49–78 (19)	79–60 (19)	59–80 (19)

10.9 Pairs' bicyclic schedules

When I see an adult on a bicycle, I do not despair for the future of the human race.
H.G. Wells

These movements are based on bicyclic schedules, the first cycle of the two giving the first half of the session and the second cycle the second half. The first stanza can be used separately but the quality of such a movement is poor. The total number of rounds in the event is one less than a full Howell for the same number of tables, effectively supplying and alternative to the standard ¾-Howell pairs' movements. The total number of rounds is an even number and the same number of rounds (half total) is played in each stanza. Each pair does not play one other pair but all pairs play all boards. Compare these with the scissor schedules for teams given above. Those supplied here are not suitable for teams and have been optimized for use in pairs, and the calibre is always excellent.

The pairs are divided into two groups. The boards are also divided into two groups and remain in their half of the movement unlike the pairs who can and do play at any table. For example in 6 tables boards 1 to 5 are cycled through tables 1 to 3 and boards 6 to 10 cycle through tables 4 to 6.

Note: the board groups have the same numbers as the pair groups in each movement and the half-sets are used in the cycles.

The six-table example is shown here to illustrate the principles. Rounds 1 to 5 are played in the first stanza with 6 to 10 for the second.

	1	2	3	4	5	6
1	12–5 (3)	9–2 (4)	4–7 (5)	1–3 (7)	11–10 (8)	8–6 (10)
2	12–1 (4)	10–3 (5)	5–8 (1)	2–4 (8)	11–6 (9)	9–7 (6)
3	12–2 (5)	6–4 (1)	1–9 (2)	3–5 (9)	11–7 (10)	10–8 (7)
4	12–3 (1)	7–5 (2)	2–10 (3)	4–1 (10)	11–8 (6)	6–9 (8)
5	12–4 (2)	8–1 (3)	3–6 (4)	5–2 (6)	11–9 (7)	7–10 (9)
6	9–10 (1)	2–8 (2)	11–1 (5)	12–6 (6)	4–5 (7)	7–3 (8)
7	10–6 (2)	3–9 (3)	11–2 (1)	12–7 (7)	5–1 (8)	8–4 (9)
8	6–7 (3)	4–10 (4)	11–3 (2)	12–8 (8)	1–2 (9)	9–5 (10)
9	7–8 (4)	5–6 (5)	11–4 (3)	12–9 (9)	2–3 (10)	10–1 (6)
10	8–9 (5)	1–7 (1)	11–5 (4)	12–10 (10)	3–4 (6)	6–2 (7)

All these schedules are described in brief form using the usual Howell method but does require two starting positions, the second marked *. Conducting these movements in two stanzas with a significant break (for example a lunch break) does introduce the chance of security breaches where the players can discuss the hands that others have not yet played. The organizers need to be aware of this issue.

6 tables, 10 rounds

Pair 1 follows 5, and 6 follows 10, and all other pairs follow the pair one less than their number. Pairs 11 and 12 are stationary in each stanza.

[6] 2R 12-5 9-2 4-7 R 1-3 11-10 R 8-6
* 9-10 2-8 2R 11-1 12-6 4-5 7-3 2R (88) {1-5, 6-10}

1	12–5 (3)	9–2 (4)	4–7 (5)	1–3 (7)	11–10 (8)	8–6 (10)
6	9–10 (1)	2–8 (2)	11–1 (5)	12–6 (6)	4–5 (7)	7–3 (8)

8 tables, 14 rounds

[8] 16–1 3–7 12–6 R 4–9 2R 15–8 10–14 5–13 R 11–2 2R
* 9–11 2R 15–1 10–7 12–13 R 2–4 2R 16–8 3–14 5–6 R (96) {1–7, 8–14}

1	16–1 (1)	3–7 (2)	12–6 (3)	4–9 (5)	15–8 (8)	10–14 (9)	5–13 (10)	11–2 (12)
8	9–11 (1)	15–1 (4)	10–7 (5)	12–13 (6)	2–4 (8)	16–8 (11)	3–14 (12)	5–6 (13)

10 tables, 18 rounds

[10] 2R 20–1 R 2–12 15–9 11–3 14–16 R 17–4 19–10 13–8 R 5–7 R 6–18 2R
* 2–3 19–1 13–17 4–8 4R 11–12 7–14 6–9 R 5–16 20–10 3R 15–18 (95) {1–9, 10–18}

1	20–1 (3)	2–12 (5)	15–9 (6)	11–3 (7)	14–16 (8)	17–4 (10)	19–10 (11)	13–8 (12)	5–7 (14)	6–18 (16)
10	2–3 (1)	19–1 (2)	13–17 (3)	4–8 (4)	11–12 (9)	7–14 (10)	6–9 (11)	5–16 (13)	20–10 (14)	15–18 (18)

12 tables, 22 rounds

[12] 24–1 R 9–17 3R 3–16 2–7 21–11 R 8–15 23–12 R 20–6 3R 14–5 13–18 10–22 R 19–4
* R 19–15 23–1 R 9–6 3R 14–16 18–2 22–21 R 8–4 24–12 R 20–17 3R 3–5 7–13 11–10 (91) {1–11, 12–22}

1	24–1 (1)	9–17 (3)	3–16 (7)	2–7 (8)	21–11 (9)	8–15 (11)	23–12 (12)	20–6 (14)	14–5 (18)	13–18 (19)	10–22 (20)	19–4 (22)
12	19–15 (2)	23–1 (3)	9–6 (5)	14–16 (9)	18–2 (10)	22–21 (11)	8–4 (13)	24–12 (14)	20–17 (16)	3–5 (20)	7–13 (21)	11–10 (22)

14 tables, 26 rounds

[14] R 6–8 2R 4–26 R 2–18 2R 24–23 1–28 13–17 15–5 2R 19–21 R 16–22 27–14 R 20–12 R 25–7 10–11 R 3–9
* 17–26 R 3–22 R 27–1 R 9–16 5–2 2R 18–15 R 23–11 28–14 2R 10–24 7–12 20–25 3R 8–19 6–21 4–13 R (93) {1–13, 14–26}

1	6–8 (2)	4–26 (5)	2–18 (7)	24–23 (10)	1–28 (11)	13–17 (12)	15–5 (13)	19–21 (16)	16–22 (18)	27–14 (19)	20–12 (21)	25–7 (23)	10–11 (24)	3–9 (26)
14	17–26 (1)	3–22 (3)	27–1 (5)	9–16 (7)	5–2 (8)	18–15 (11)	23–11 (13)	28–14 (14)	10–24 (17)	7–12 (18)	20–25 (19)	8–19 (23)	6–21 (24)	4–13 (25)

18 tables, 34 rounds

[18] 36–1 24–32 7–9 33–20 R 26–15 4–14 2R 21–18 R 27–5 2R 13–25 3R 35–2 R 31–19 2R 29–3 17–11 R 28–12 23–10 2R 8–22 16–34 6–30 R
* 18–35 33–26 16–8 2R 5–1 2R 28–27 R 13–12 3–6 R 23–17 R 20–22 2R 19–36 31–11 25–10 4R 2–7 R 15–34 21–32 R 9–30 R 29–4 14–24 (92) {1–17, 18–34}

1	36– 1 (1)	24–32 (2)	7– 9 (3)	33–20 (4)	26–15 (6)	4–14 (7)	21–18 (10)	27– 5 (12)	13–25 (15)	35– 2 (19)	31–19 (21)	29– 3 (24)	17–11 (25)	28–12 (27)	23–10 (28)	8–22 (31)	16–34 (32)	6–30 (33)
18	18–35 (1)	33–26 (2)	16– 8 (3)	5– 1 (6)	28–27 (9)	13–12 (11)	3– 6 (12)	23–17 (14)	20–22 (16)	19–36 (19)	31–11 (20)	25–10 (21)	2– 7 (26)	15–34 (28)	21–32 (29)	9–30 (31)	29– 4 (33)	14–24 (34)

20 tables, 38 rounds

[20] 40–1 6–16 2R 21–4 38–26 15–31 11–36 R 8–3 R 18–33 4R 5–32 10–28 R 39–20 25–35 2R 2–23 19–7 34–12 30–17 R 27–22 R 37–14 4R 24–13 29–9 R
* 24–32 29–28 R 39–1 35–6 2R 4–2 19–26 12–15 36–30 R 27–3 R 37–33 4R 5–13 10–9 R 40–20 16–25 2R 23–21 38–7 31–34 17–11 R 8–22 R 18–14 4R (97) {1–19, 20–38}

1	40–1 (1)	6–16 (2)	21–4 (5)	38–26 (6)	15–31 (7)	11–36 (8)	8–3 (10)	18–33 (12)	5–32 (17)	10–28 (18)	39–20 (20)	25–35 (21)	2–23 (24)	19–7 (25)	34–12 (26)	30–17 (27)	27–22 (29)	37–14 (31)	24–13 (36)	29–9 (37)
20	24–32 (1)	29–28 (2)	39–1 (4)	35–6 (5)	4–2 (8)	19–26 (9)	12–15 (10)	36–30 (11)	27–3 (13)	37–33 (15)	5–13 (20)	10–9 (21)	40–20 (23)	16–25 (24)	23–21 (27)	38–7 (28)	31–34 (29)	17–11 (30)	8–22 (32)	18–14 (34)

24 tables, 46 rounds

[23,24] 48–1 4R 9–44 R 26–29 2R 40–18 37–45 2R 5–34 25–15 R 42–46 R 13–31 10–12 30–16 4–20 27–39 R 21–24 2R 35–36 32–17 2R 23–6 43–33 R 14–41 R 8–3 28–7 2–11 22–38 47–19 4R
* 47–24 4R 32–21 R 3–6 2R 17–41 14–22 2R 28–11 2–38 R 19–23 R 36–8 33–35 7–39 27–43 4–16 R 44–1 2R 12–13 9–40 2R 46–29 20–10 R 37–18 R 31–26 5–30 25–34 45–15 48–42 4R (98) {1–23, 24–46}

1	48–1 (1)	9–44 (6)	26–29 (8)	40–18 (11)	37–45 (12)	5–34 (15)	25–15 (16)	42–46 (18)	13–31 (20)	10–12 (21)	30–16 (22)	4–20 (23)
	27–39 (24)	21–24 (26)	35–36 (29)	32–17 (30)	23–6 (33)	43–33 (34)	14–41 (36)	8–3 (38)	28–7 (39)	2–11 (40)	22–38 (41)	47–19 (42)

24	47–24 (1)	32–21 (6)	3–6 (8)	17–41 (11)	14–22 (12)	28–11 (15)	2–38 (16)	19–23 (18)	36–8 (20)	33–35 (21)	7–39 (22)	27–43 (23)
	4–16 (24)	44–1 (26)	12–13 (29)	9–40 (30)	46–29 (33)	20–10 (34)	37–18 (36)	31–26 (38)	5–30 (39)	25–34 (40)	45–15 (41)	48–42 (42)

Part 11 Teams of many pairs

Set of 8 boards manufactured by Index sales corporation, circa 1950.
They are light, compact and durable, but the insertion of the cards into the slots is not so simple, as explained by the note on top of the box.
They were made in a variety of colours and styles.

Set of 16 deluxe boards in box, from Taylor, circa 1950.
Cloth covered, they were sold over a long period.

This type of movement is usually used to match two or more teams (usually representing their club) against each other without meeting the members of their own team. As well as the inter-club events, it is possible to conduct events within a club as novelty events or in conjunction with standard pairs' events simply by having the scoring done in a second way.

There are a number of ways to deal with two teams of pairs. The Mitchell type movements do allow for this in various ways and in particular the *switched Mitchell*, *Scheveningen system* and the *Snow system* should be considered. In particular see *Howells and the Snow method*, page 174, and the *Snow modification* of the Mitchell movement, page 120.

The outline of the movements given below is broken into two major types; those for two teams and those for multiple numbers of teams. Special considerations have to be given to odd or even numbers of pairs in the teams, and odd or even numbers of teams.

The usual scoring methods apply to this type of event, with match point, total point and Butler method being most popular.

Note: even though these events are called 'teams' they are pairs contests, scored as a pairs' event.

11.1 Two teams of pairs

Go down deep enough into anything and you will find mathematics.
Dean Schlicter

In the U.S.A. competitions between two teams of pairs started to come into popularity in the early 1930's when matches between two cities, or two business firms, were arranged. To cater for the needs of the day the Gruenther relay-movement was devised by Major General A.M. Gruenther with the help of Major L. R. Gracy (U.S.A. Army). This movement is both simple and effective.

Two teams of an even number of pairs

The tables are set up as two rows of equal numbers, parallel to each other so as to facilitate the sharing of the boards between the two rows. The number of tables in each row should be equal to half the number of pairs in each team.

Half the number is odd

Half the pairs in the first team sit as N-S at the tables in the first row and the other half sit as E-W in the second row. The second team sits in the vacant places opposite the first team. The movement then proceeds as a normal twin Mitchell with the boards being shared between the two rows of tables throughout the event.

One team remains stationary throughout the event with the other team moving up one table at each change. The pairs stay in the same row throughout the event and the boards move down 1 table. After N rounds the first half of the event is finished (where N = the number of tables in each row). The home (moving) team then swaps places, the N-S pairs moving to the positions occupied by the E-W pairs and vice-versa. New boards are introduced (or the old ones re-dealt) and the second half proceeds just as the first half did.

Half the number is even

Where there is an even number of tables in each row the twin Mitchell movement introduces new problems. The skip Mitchell may be used, with the players repeating the play against their first pair and failing to play against the pair they skip at half way.

The Gruenther method was to have share-and-relay movements in use, but the share and relays were placed at different points in each row. This would need only 3 boards per round rather than 4. Clearly it is best to have the share of one row opposite the relay of the other to reduce the number of sets of boards needed to only two.

A simpler version of this movement was devised in England by Mr. R. G. Doig (in 1952). The skip movement is used and proceeds normally until the round when the pairs meet the same opponents for the second time (the last round of the first half). The N-S and E-W pairs of the moving team then swap places on this round. The last round of the first half is then played with these new opponents.

For play in the second half, the players no longer have to swap sides, but the moving players have to move to the table opposite the table they skipped in the first half of the event (i.e., up ½ N tables where N = the number of tables in each row). The play then proceeds normally (with the boards having been re-dealt), with a skip after ½ N rounds.

For the last round the moving pairs again have to swap sides to meet the one remaining pair they have not met.

The stagger movement

Part A of the stagger movement (see multiple teams' movements, page 240) can be used when half the number of pairs is *odd or even* with no disadvantages over the previous two methods. One team sits at the even-numbered tables and the other at the odd-numbered tables.

As explained in the stagger movement this movement is used for the even-numbered tables to meet the odd numbers. The E-W pairs start by moving up (or down) one table and then move up (or down) two tables, changing sides of the movement as needed.

If it is required that all the pairs of one team meet all the pairs of the other team, then for the second session have one team swap its N-S and E-W pairs about at each table and then repeat the movement.

Two teams of an odd number of pairs

This movement was suggested by Col R. L. Telfer (England). The problem is that each board played has to be played the same number of times N-S as E-W. With an odd number of pairs, the result of the nth pair has no result with which to compare. The following method overcomes this problem. Unfortunately each pair misses one pair from the opposing team.

The number of tables in the movement is equal to the number of pairs in each team. The away team (visiting team) is asked to sit as N-S at the odd-numbered tables, and E-W at the even-numbered tables. The home team then occupies the vacant places. The boards may be dealt at this stage, but no play takes place before the change into the first round.

The away team remains stationary for the whole event while the home team moves up one table at each change with the boards moving down one table. The moving pairs occupy the seats vacated by their team-mates.

The only real complication exists at table 1 where the stationary pair from the away team is required to switch after each round, including the first round change. This means that this pair plays as E-W on the first round of play, N-S on the second, E-W on the third etc. The event can be improved by having the event divided into two halves, with the home team moving in the first half and the away team moving in the second half.

11.2 Multiple teams of pairs

The most savage controversies are those about matters as to which there is no good evidence either way. Persecution is used in theology, not in arithmetic.
Bertrand Russell

The organisation of this type of event is only difficult if the number of pairs in the teams is odd. Where such movements are possible they are shown below and are handled with interwoven or incomplete Howells. The calibre is shown for these, but it is only applicable if the movements are used for pair's events.

First we will deal with the movements based on the Mitchell and Clay systems. At the start of the event the director assigns a team letter to each team and the pairs in the team are then assigned numbers according to the table at which they start play, with the E-W pairs taking the same number as the N-S pair. Alternatively, have the highest-numbered table number added to their number for distinction. Pairs might have numbers such as A2 or B6, etc., the letter indicating the team and the number of the pair.

The pairs are then asked to take their seats with two pairs from the same team sitting at one table. The tables are assigned to the teams in rotating order until all tables are used. For example, if there are three teams of 12 players (i.e., 6 pairs or 3 tables of four), and the first is called Al, the second A2, and the third A3, then the 9 tables are filled as follows:

Al, B1, C1, A2, B2, C2, A3, B3, C3.

There is no play on the first round. The movement proceeds as a standard whist teams' movement from there on, with the exception that no play takes place on the rounds when the members of the same team are sitting opposite to each other. This situation arises once every N rounds (where N = the number of teams). For example if there are 3 teams, then two rounds are played between each double move.

The above movement (i.e., the whist movement) only applies where the number of teams times the number of fours is odd (i.e., where the number of tables is odd). Where this is even, the stagger movement is used, still making the extra move on the same rounds.

The Mitchell movement can also be utilised for the event in a similar fashion to the whist movement. This was first pointed out by A. Groner. The seating of the players is assigned in exactly the same way as for the whist movement, with the exception that with an even number of tables, the tables are arranged in one row rather than two as in the stagger movement.

The movement is a standard Mitchell move throughout (E-W up 1, boards down 1) with the exception that the E-W pairs have to make an extra move on the occasions when they meet their team-mates (which is at the same frequency as that for the whist movement). The total number of rounds of play is equal to the number of tables less the number of fours in each team.

Where the number of pairs in a team is odd, and the number of teams is also odd, then there are many problems, the most obvious one being the half-table. Where the number of teams is even then the following can be done.

By assigning special starting positions the Mitchell movement can be used, but with special moves. For teams of 3 pairs this method is reasonable, but with more than that, the movement becomes unbalanced and only the first round move is a standard move.

Three teams of four pairs

This movement is an incomplete Howell, playing 8 rounds. The teams are organized in three groups, the odd numbers (1,3,5,7), the even numbers (2,4,6,8) and the high numbers (9,10,11,12), who are stationary.

After 8 rounds each team has played the other two teams.

9–1 6–10 4–11 12–7 2–3 8–5 2R (40) – Hanner

Three teams of six pairs

This movement is an incomplete Howell, playing 12 rounds. The teams are organized in three groups, the odd numbers, even numbers and the high numbers (13–18), who are stationary. After 12 rounds each team has played the other two teams.

13–1 14–11 10–15 8–3 7–6 9–12 5–16 17–4 2–18 3R (44) – Hanner

Three teams of eight pairs

This movement is an incomplete Howell, playing 16 rounds. The teams are organized in three groups, the odd numbers, even numbers and the high numbers (17–24), who are stationary. After 16 rounds each team has played the other two teams.

17–1 12–18 8–19 20–7 21–6 22–2 15–14 5–10 4–13 16–3 9–23 11–24 4R (46) – Hanner

Four teams of two pairs

This movement is an incomplete Howell, playing 6 rounds. The teams are organized in four groups, 1,4; 2,5; 3,6; 7,8, who are stationary. After 6 rounds each team has played the other three teams.

7–1/3–8 5–6 4–2 3R (33) – Hanner

Four teams of three pairs

This movement is an incomplete Howell, playing 9 rounds. The teams are organized in four groups, 1,4,7; 2,5,8; 3,6,9; 10–12, who are stationary. After 9 rounds each team has played the other three teams.

10–1 5–11 9–12 8–3 7–6 4–2 3R (51) – Hanner

The alternative is to use the Mitchell system with starting positions
A1–D3 Bl–A2 C1–B3 Dl–C2 A3–D2 B2–C3

Movement:

- N-S remain stationary;
- E-W up 1 for first two moves;
- then after 3rd round skip one table;
- then after 4th round skip one table;
- then after 5th round skip two tables.

Four teams of four pairs

This movement is an incomplete Howell, playing 12 rounds. The teams are organized in four groups, 1,4,7,10; 2,5,8,11; 3,6,9,12; 13–16, who are stationary. After 12 rounds each team has played the other three teams.

13–1 5–14 3R 15–8 12–16 9–2 6–4 R 7–3 10–11 (49) – Hanner

Four teams of five pairs

This movement is an incomplete Howell, playing 15 rounds. The teams are organized in four groups, 1,4,7,10,13; 2,5,8,11,14; 3,6,9,12,15; 16–20, who are stationary. After 15 rounds each team has played the other three teams.

16–1 6–7 12–5 15–11 2–4 14–9 13–17 3–18 10–19 20–8 5R (56) – Hanner

Alternatively use the Mitchell system with starting positions
Al–D5 Bl–A4 Cl–B3 D1–C5 A2–D4 B2–A5 C2–B4 D2–C3 A3–D3 C4–B5

Movement:

- N-S pairs remain stationary throughout;
- E-W pairs move down after each of the first 9 rounds respectively: UP 1, 3, 4, 2, 4 DOWN 3, 3, UP 2, 4;
- boards always move down 1 table.

By using the Telfer movement (see page 343) for two teams of an odd number of pairs, it is possible to have a reasonably balanced result. A round-robin type of event is conducted, with each head-to-head match using the Telfer movement. For example, with four teams of 5 pairs, assign the teams the letter A, B, C, and D. The tables are arranged in two rows of 5 tables. Two boards a round would be played, with the two rows of tables sharing boards.

- Session 1: In the first row A v B and in the second row C v D. There are four rounds in each match, i.e., 8 boards are played.
- Session 2: A v C and D v B.
- Session 3: A v D and B v C.

The total length of this event would be 24 boards.

Four teams of seven pairs

This movement is an interwoven Howell, playing 14 rounds. The teams are organized in four groups: 1,3,5,7,9,11,13; 2,4,6,8,10,12,14; 15,17,19,21,23,25,27; 16,18,20,22,24,26,28. After 14 rounds each team has played half the pairs of the other three teams. 1 follows 14 and 15 follows 28.

1–6 22–5 7–26 16–3 2–13 20–25 18–15 14–21 10–24 12–28 4–27 19–8 23–11 17–9 (44) – Hanner

Five teams of two pairs

This movement is an incomplete Howell, playing 8 rounds. The teams are organized in five groups: 1,5; 2,6; 3,7; 4,8; 9,10, who are stationary. After 8 rounds each team has played the other four teams.

9–1/ 5–10 4–7 R 3–2 6–8 3R (37) – Hanner

Five teams of four pairs

This movement is an incomplete Howell, playing 16 rounds. The teams are organized in five groups: 1,5,9,13; 2,6,10,14; 3,711,15; 4,8,12,16; 17–20, who are stationary. After 16 rounds each team has played the other four teams.

17–1 16–18 14–5 11–9 19–15 10–20 13–8 4–7 6–12 3–2 6R (57) – Hanner

Six teams of two pairs

This movement is an incomplete Howell, playing 10 rounds. The teams are organized in six groups: 1,6; 2,7; 3,8; 4,9; 5,10; 11,12, who are stationary. After 10 rounds each team has played the other five teams.

11–1 R 10–12 5–2 R 9–8 R 3–7 R 4–6 (47) – Hanner

Six teams of three pairs

This movement is an incomplete Howell, playing 15 rounds. The teams are organized in six groups: 1,6,11; 2,7,12; 3,8,13; 4,9,14; 5,10,15; 16–18, who are stationary. After 15 rounds each team has played the other five teams.

16–1 15–8 13–2 6–12 17–10 7–9 14–11 4–5 3–18 6R (67) – Hanner

Six teams of five pairs

This movement is an incomplete Howell, playing 15 rounds. The teams are organized in six groups: 1,4,7,10,13; 2,5,8,11,14; 3,6,9,12,15; 16,19,22,25,28; 17,20,23,26,29; 18,21,24,27,30. After 15 rounds each team has played half of the pairs from each of the other five teams.

1–8 4–5 29–14 19–26 30–11 7–25 15–28 3–24 21–13 23–9 20–16 6–2 10–27 12–22 17–18 (40) – Hanner

11.3 Teams of eight players

How many times can you subtract 7 from 83, and what is left afterwards?
You can subtract it as many times as you want, and it leaves 76 every time.
Author Unknown

This number of players is being treated separately for the simple reason that this number is frequently used, and when there are only two teams, the movement does not follow normal procedures.

Two teams of eight

Using the Mitchell system the two teams will be assigned four pair numbers such as: Al, A2, A3, and A4. Opposing B1, B2, B3, and B4.

Round	Table 1	Table 2	Table 3	Table 4	Boards
1	A1–B1	B2–A2	A3–B3	B4–A4	1–8
2	A1–B2	B1–A2	A3–B4	B3–A4	9–16
3	A1–B3	B4–A2	A3–B1	B2–A4	17–24
4	A1–B4	B3–A2	A3–B2	B1–A4	25–32

Team A is stationary. The eight boards played on each round are shared between the four tables. The N-S pair is shown first in the above schedule. More or fewer boards can be used than those shown.

Following is an alternative board movement that makes the use of fewer boards and so is easier to accommodate. On round 1 tables 1 and 2 share boards 1–8 (or 1–6 etc.) and tables 3 and 4 share 9–16. For the second round these two sets are swapped over so that 1 and 2 share 9–16 and 3 and 4 share 1–8. Similarly for rounds 3 and 4.

If stationary boards are desirable then the following movement is to be used:

Round	Table 1	Table 2	Table 3	Table 4
	(Set 1)	(Set 2)	(Set 3)	(Set 4)
1	A1–B1	A2–B3	B2–A4	B4–A3
2	A2–B2	A1–B4	B1–A3	B3–A4
3	B3–A3	B1–A4	A2–B4	A1–B2
4	B4–A4	B2–A3	A1–B3	A2–B1

This idea originating from Dr. H. G. Hudson, England

Multiple teams of eight

See three, four and five teams of 4 pairs above for these numbers of teams of eight players, page 344. Quite often the easiest way to handle this type of event is to run the movement in two or three stages. The field can be treated as two movements of multiple teams of four. This can be best illustrated by an example, with say 7, teams of eight. Two parallel rows of 7 tables each are set up. Each team occupies the two sharing tables in both rows. For example, team 1 sits at table 1 next to each other.

Two separate multiple teams-of-four whist movements are run (one on each row of tables), with two boards per round. The two rows share boards throughout (or the boards can be duplicated to eliminate the sharing). After 6 rounds this stage is complete.

With the teams back at their home tables, the E-W pairs in the two rows swap places. The above movement is then repeated.

For a stage three, the E-W pairs in one row swap with their N-S teammates in the other row and the movement repeated yet again. By varying the number of boards per round and playing a number of sessions best suited to the event's needs, the director has good control over the length of the event.

Where the number of tables in each movement is even, the same principles apply; only a stagger movement is used in both sections. In section 1, Part A of the stagger movement is used first, then part B. In section 2 it is vice versa. This means the boards used in part A of section 1 can be used in part A of section 2. Similarly with the boards for part B (see the *stagger movement*, page 240, for further details).

Part 12 Large pairs' movements

1960s 'J-R' aluminium boards. Strong, light and durable.
The package on the right is marked $4.00 for the 4 boards.

1970s plastic boards. Cheap to manufacture.
The package on the right is marked $13.75 for the 4 boards.

Large pairs' movements are those with around 18 tables and more. Many of the movements in previous sections cover large pairs' movements as well as the medium to small, particularly the Mitchell based movements such as the appendix Mitchell, rover Mitchell, curtailed Mitchell, twin Mitchell, and the web Mitchell.

Large pairs' movements arise for several different reasons. The local club may simply have a large number of tables on each night. The event may be a multi-night tournament. The event could be a congress of many and varied groups of players.

The type of movement used will depend on the criteria such as: 1 or 2 winners; number of sessions in the event; are players required to play all the boards?; must each pair play all other pairs?; is the scoring to be done over the whole field or can it be scored section by section?; are the boards pre-duplicated or can the players deal them?

It is up to the tournament director to determine all the requirements and choose the movement accordingly.

Where the session is made up from a number of sections and the scores combined and scored across the field the balance is directly influenced by the balance of each section. Where each of the sections are balanced then the combined sections will also be balanced or at least to an equivalent level. Clearly all pairs have not met all pairs in the combined sections for any session and the balance will not be complete until all sessions are played.

For example, with 5 sessions, for practical reasons it is very difficult to optimize the balance over all the sessions. Choosing the highest-quality balanced movements for each section in each session is the best an organizer can do and in addition the balance will be very close to the best achievable.

12.1 Multi-section congress events

We could use up two Eternities in learning all that is to be learned about our own world and the thousands of nations that have arisen and flourished and vanished from it. Mathematics alone would occupy me eight million years.
Mark Twain

In this type of event the tables are divided into multiple sections of the same size. This means that all the pairs in one section play only the pairs in that section. If the sections are Mitchell movements, then the pairs meet only half the pairs in the section. If the sections are Howell movements, then they usually meet all the pairs in the section. If the boards have been pre-duplicated, it is possible to score the event over the entire field, but it would still be preferable to score the sections independently, as this would be much faster with a scorer being assigned to each section. Computers will, of course, simplify scoring across the field.

The pre-duplication of the boards would therefore appear to be unnecessary, except where the scoring is to be done over the entire field. However, the playing of the same boards over the whole field is appreciated, as the players can discuss the hands after play. Dealing machines have made this task simple.

Congress movements

For large events where there are many copies of each board available the best method is to use the Snow modification with the boards moving from section to section on each change. The movements in each section can be standard Mitchell, ring-dance Mitchell or endless-Howell-cyclic movements. With Mitchell movements there are usually many same-size sections.

With the endless Howell the number of tables can be even or odd, no guide-cards are needed, and the movement can be curtailed whenever needed. All the tables in each section play the same boards on each round (duplicate copies in play) and the boards move down (or up) one *section* at each change.

Consider 50 tables. These would be arranged in 10 rows of 5 tables, giving 10 sections A, B, C, …, J. Each table in section A would initially be given boards 1 through 3; section 2 for boards 4, 5, and 6; and so on to boards 28, 29 and 30 in section J. After each round the boards are passed down one section and the players move normally as in the endless Howell. After 9 rounds all the players in each section will have met each other and everybody will have played all the boards.

Of course the number of tables in the event will not always be a convenient number such as 50. Because the number of tables in the endless Howell does not change the movement, a table can be added on to any section and that table plus the one next to it may have to share boards. The sections with the extra table will still play the same number of rounds as the other sections.

12.2 Block Mitchell

Can you do Division? Divide a loaf by a knife - what's the answer to that?
Lewis Carroll, Through the Looking Glass

As an introduction to this section, the chapter on tri-sect Mitchell movements is recommended (page 118). This movement is useful for premises where there is not one large room. There may be several floors or several large rooms and these are not always the same size. This movement allows for three groups of tables ranging from 3:4:4 up to 8:8:8 except there cannot be more than one three-table section.

Each section plays 4 rounds (as a separate movement) before there is any major movement between sections. In each section there are only 4 migrating pairs irrespective of the number of tables in it. When the first four rounds are complete, the boards move down one section or block, and the migrating pairs move up one section. This again occurs after the next 4 rounds. With this number of migrating pairs and a number of tables greater than 12, it can be seen that the amount of inter-room movement is less than there would be with other movements.

The next problem is movements for 3 through 8 tables which have 4 rounds and 4 migrating pairs. These are dealt with systematically below as was first explained by E. E. Blandon in the *Bridge Magazine*, April 1973.

Five to eight tables

At the time Blandon wrote the article he was not aware of the power of the self orthogonal Latin square. Today for 5 or more tables we can use the *four tables plus 1* presented in *incomplete Mitchell movements*, *one round short* (page 125). That movement is based on the self orthogonal Latin Square and supplies a 4-table Mitchell with one to four appendix tables. Consider the following square where the pairs 1 to 4 are the resident pairs in the 4-table stationary-board Mitchell and pairs 5 and 6 are the two pairs at the rover table. Pairs m1 through m4 are the migrating pairs. The E-W rover visits tables 1 to 4 in rounds 1 to 4, sharing boards with table 5 and displacing the E-W pair who plays at table 5. The only change from the usual rover table is the roving E-W pair displaces N-S on round 2. After each 4-round stanza the migrating pairs change rooms or sections while in this movement pairs 1 and 2 take the seats of 3 and 4. Similarly, 3 and 4 replace 5 and 6 and finally 5 and 6 replace 1 and 2. With a new lot of migrating pairs the movement proceeds in the same way. The cycle is the same for the 3rd stanza.

	1	2	3	4	5
1	1 6	2 m3	3 m4	4 m2	5 m1
2	3 m2	6 m4	1 m3	2 m1	5 4
3	4 m3	3 m1	2 6	1 m4	5 m2
4	2 m4	1 m2	4 m1	3 6	5 m3

For 6, 7 or 8 tables use the same movement as that used in the 5-table case and append another table. Table 6 would have resident pairs 7 and 8 with N-S 7 stationary throughout the three stanzas, always playing the same boards as E-W rover 8. E-W rover 8 starts at table 2 displacing the E-W pair to play against N-S pair 7. Pair 8 then moves up one table at each change, moving from table 4 to 1. E-W 8 displaces the E-W pair on round 1, N-S on round 2, E-W on round 3 and N-S on round 4. Appendix tables 7 and 8 are the same as appendix table 6 only the E-W rovers start at tables 3 and 4 respectively. Note it is possible for the first rover pair (6) to displace N-S on round 4 too, making all the displacements the same, E-W pairs on rounds 1 and 3 and N-S pairs on rounds 2 and 4.

Three tables

Use the hesitation Mitchell for 3 tables, with the N-S pairs at tables 1 and 2 remaining stationary for the event. The other 4 pairs are the migrating pairs. Note R = Relay and M = Migrating pair.

Board-sets:	1	2	3	4
Pairs:	1–M	R	2–M	M–M

Four tables

Use the standard share-and-relay Mitchell for 4 tables or the self orthogonal stationary-board Mitchell. The N-S pairs are the stationary pairs and the E-W pairs are the migrating pairs.

Board-sets:	1	2	3	4
Pairs:	1–M	R	2–M	3–M
				4–M

Five tables

The above four-table movement is used with a rover table added to it in the following fashion. Two additional resident pairs are placed at table 5 (resident pairs 5 and 6). They play each other on round 1 sharing boards with table 1.

- On round 2 pair 6 goes to table 2, displacing the E-W pair which moves to the roving table and shares boards with table 2.
- On round 3 pair 6 goes to table 1, displacing the E-W pair as in round 2.
- On round 4 pair 6 displaces N-S at table 2 similarly. Therefore, table 5 shares boards alternatively with tables 1 and 2.

For the second lap the resident pairs rotate about each other in order not to play each other a second time. Pairs 1 and 2 move to N-S at tables 3 and 4; 3 and 4 move to the rover table; 5 and 6 move to N-S at tables 1 and 2. For the last lap they rotate again: 5 and 6 to tables 3 and 4; 1 and 2 to rover; lastly 3 and 4 to tables 1 and 2.

Board-sets:	1	2	3	4
Pairs:	1–M	R	2–M	3–M
	5–6			4–M

Six tables

To the 5-table movement is added yet another rover table with the resident pairs 7 and 8. On the first round they play each other sharing boards with table 2. For the rest of the movement pair 7 stays at table 6 as N-S and pair 8 displace the pairs at tables 1 and 2 alternatively, displacing the moving pairs on the first visit in each lap, and the stationary pairs on the second visit. Table 6 always shares boards with the table at which pair 8 is sitting.

Board-sets:	1	2	3	4
Pairs:	1–M	R	2–M	3–M
	5–6		7–8	4–M

Seven tables

Twin Mitchells are set up, one of them a hesitation 3-table movement and the other a 4-table movement.

Board-sets:	1	2	3	4
Pairs:	1–M	R	2–M	3–M
				5–M
	4–7	F–8	6–9	R

If one mentally exchanges the pairs represented above as F and 5, it can be readily seen that there is a pattern. There are three groups of pairs: 1, 2, 3; 4, 5, 6; and 7, 8, 9. At the end of each lap these groups rotate about each other, i.e. 1, 2, 3 replace 7, 8, 9; 7, 8, 9 replace 4, 5, 6; and 4, 5, 6 replace 1, 2, 3.

Similarly for the next lap. So that pairs do not meet each other twice it is necessary to introduce a type of fixed resident (F). This pair always starts a lap at N-S table 2 in the hesitation Mitchell, the pair so displaced moving to N-S at table 4 in the Mitchell and staying there for this lap. As table 2 in the hesitation Mitchell is the table at which the E-W pairs switch directions (or hesitate for one round), the fixed resident is only fixed in the sense that this pair always start at that position, but move as a normal E-W pair on each lap.

Eight tables

This is run similarly to the 7-table movement except that the twinned movements are standard 4-table Mitchells: Pairs 1–4; 5–8; 9–12 are the rotating groups of resident pairs.

Board-sets:	1	2	3	4
Pairs:	1–M	R	2–M	3–M
				4–M
	5–9	6–10	8–12	R
		7–11		

12.3 Bowman for large pairs movements

Mathematics is the supreme judge; from its decisions there is no appeal.
Tobias Dantzig

This movement has been derived from the basic Bowman-Ewing Mitchell movement. This is also known as the Super-Bowman movement. Pre-duplication of the boards, while not necessary, is desirable. The movement is similar to the web Mitchell in that the boards move in two directions. There is always a base set of tables equal to the number of rounds in the event, and tables are appended in pairs to this base. Therefore an odd base gives an odd number of rounds and will only work for an odd number of tables up to three times the number in the base. An even number in the base gives an even number of rounds and will only work for an even number of tables up to three times the number in the base.

The movement of the players is basically simple and Mitchell-like, with no player displacement. No sharing of boards is required except where the boards have not been duplicated. Any number of tables greater than the base and less than three times the base is catered for, particularly if the choice between an even or odd number of rounds is flexible (for example 12 or 13 rounds).

As an example, the following 6-table base movement expanded to 18 tables is illustrated. It is assumed that there are three sets of boards, having been pre-duplicated.

Boards A; set nos:	1	2	3	4	5	6
Tables:	lA	2A	3A	4A	5A	6A
Boards B; set nos:	6	1	2	3	4	5
Tables:	lB	2B	3B	4B	5B	6B
Boards C; set nos:	1	2	3	4	5	6
Tables:	lC	2C	3C	4C	5C	6C

The base is the set of A tables. Tables may be deleted from this movement in pairs, taking one from row B and one from row C, starting at the higher end. Therefore 6B and 6C would be the first pair of tables to come out. The board-sets on these tables must remain in play, however, but now they will only be on relay tables.

All N-S pairs remain stationary. Sets-of-boards A and C move down one table (from 2A to 1A to 6A etc.). Set-of-boards B moves up one table from 5B to 6B to lB, etc.). All E-W pairs move up one table at each change, but the word 'up' needs some qualification.

E-W moves: 1A, 1B**,** lC, 2C, 2B, 2A, 3A, etc., until all B and C pairs are eliminated, and then they move one table at a time through the base movement. Note that as this movement contains an even number of rounds, a skip is required after half the rounds. If tables lB and lC were the only appended tables then the movement would be lA, 1B, 1C, 2A, 3A, etc.

The movement of the E-W pairs can be lA, 1B, 1C, 2A, 2B, 2C, 3A, 3B, 3C etc., if so desired. If only two sets-of-boards are available then sharing of boards between sections A and C is possible.

For example:

Set No:	1	2	3	4	5	6
A set:	1A	2A	3A	4A	5A	6A
	lC	2C				
B set:	2B	R	R	R	R	lB

Pre-duplication may not be necessary with the above arrangement, as the tables in row A can make duplicate copies as they play the first round. Play would then start late in row B, particularly at 2B where there may not be a spare board to use. lB can share with table 6A.

Of course, this movement is more likely to be used when the number of tables in the base is higher and the number of boards being played on each round is only 2. Sharing is then not so convenient. Where the number of tables in the B and C rows is large, it is possible for the players to set the hands and to skip the first round as far as play is concerned. Instead of a 13-table base, the 14-table base is used and one round is not played.

Most useful for

- 12-table base (up to 36 tables).
- 13-table base (up to 39 tables).
- 14-table base (up to 42 tables).

12.4 Hesitation super-Bowman

There is no disputing that Christians and all heathens and Mormons;
all, no matter what their faith agree in Mathematics.
William John Wills, Australian explorer and navigator

The research into movements by both Bowman and Ewing came up with some of the most interesting variations that have been applied to the Mitchell movement.

The first was the Ewing modification, where we can start with a Mitchell movement, add one extra table without adding more board-sets. This is also called the Hesitation Bowman. See Ewing modification on page 103.

Then there is the Bowman-Ewing modification where you can append two tables to any Mitchell movement, again without adding any extra boards. See the Bowman-Ewing modification on page 108.

They then extended this to the super-Bowman (above, page 353) where we are able to add pairs of tables to a Mitchell movement, one pair of tables added to each table in the base Mitchell. Still no more board-sets are added to the movement, though duplication of the boards is desirable, but that is not difficult today.

The original Ewing modification, commonly called a hesitation Mitchell, only applies to one winner events because the E-W pairs also play as N-S on one round. This is not a problem for the organizers requiring that feature.

Where you have a large number of tables, and only a very limited time for one session of play, and the need to produce one winner, then these modifications can be combined to produce a very simple solution. This only applies where the number of board-sets (and tables in the base) are an odd number and the total number of tables is even, or vice-versa. Where they are both odd or both even you simply use a switched version of the super-Bowman. Also consider the web Mitchell hesitation modification, see page 104.

Setup a standard super-Bowman for the number of board-sets you wish to play, and add the required pairs-of-tables as normally. Then add an extra table using the hesitation Mitchell rules defined in the Ewing modification.

For example with 15 tables and the need for 12 rounds, and one winner in one session, setup a 14 table super-Bowman based on the 12 table base. Add the last table using the same procedures that are used for the normal hesitation Mitchell.

The resulting expanded movement is shown here, noting tables 1 and 3 can share, and tables 2 and 15 use a second set-of-boards. Alternatively the boards for table 1 or 3 can be taken from that second set, eliminating all sharing.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1–15 (1)	2–16 (12)	3–17 (1)	4–18 (2)	5–19 (3)	6–20 (4)	7–21 (5)	8–22 (6)	9–23 (7)	10–24 (8)	11–25 (9)	12–26 (10)	13–27 (11)	30–28 (12)	14–29 (11)
2	1–30 (2)	2–15 (11)	3–16 (2)	4–17 (3)	5–18 (4)	6–19 (5)	7–20 (6)	8–21 (7)	9–22 (8)	10–23 (9)	11–24 (10)	12–25 (11)	13–26 (12)	29–27 (1)	14–28 (10)
3	29– 1 (3)	30– 2 (10)	15– 3 (3)	16– 4 (4)	17– 5 (5)	18– 6 (6)	19– 7 (7)	20– 8 (8)	21– 9 (9)	22–10 (10)	23–11 (11)	24–12 (12)	25–13 (1)	26–28 (2)	27–14 (9)
4	1–28 (4)	2–29 (9)	3–30 (4)	4–15 (5)	5–16 (6)	6–17 (7)	7–18 (8)	8–19 (9)	9–20 (10)	10–21 (11)	11–22 (12)	12–23 (1)	13–24 (2)	27–25 (3)	14–26 (8)
5	1–27 (5)	2–28 (8)	3–29 (5)	4–30 (6)	5–15 (7)	6–16 (8)	7–17 (9)	8–18 (10)	9–19 (11)	10–20 (12)	11–21 (1)	12–22 (2)	13–23 (3)	26–24 (4)	14–25 (7)
6	1–26 (6)	2–27 (7)	3–28 (6)	4–29 (7)	5–30 (8)	6–15 (9)	7–16 (10)	8–17 (11)	9–18 (12)	10–19 (1)	11–20 (2)	12–21 (3)	13–22 (4)	25–23 (5)	14–24 (6)
7	1–24 (7)	2–25 (6)	3–26 (7)	4–27 (8)	5–28 (9)	6–29 (10)	7–30 (11)	8–15 (12)	9–16 (1)	10–17 (2)	11–18 (3)	12–19 (4)	13–20 (5)	23–21 (6)	14–22 (5)
8	1–23 (8)	2–24 (5)	3–25 (8)	4–26 (9)	5–27 (10)	6–28 (11)	7–29 (12)	8–30 (1)	9–15 (2)	10–16 (3)	11–17 (4)	12–18 (5)	13–19 (6)	22–20 (7)	14–21 (4)
9	1–22 (9)	2–23 (4)	3–24 (9)	4–25 (10)	5–26 (11)	6–27 (12)	7–28 (1)	8–29 (2)	9–30 (3)	10–15 (4)	11–16 (5)	12–17 (6)	13–18 (7)	21–19 (8)	14–20 (3)
10	21– 1 (10)	22– 2 (3)	23– 3 (10)	24– 4 (11)	25– 5 (12)	26– 6 (1)	27– 7 (2)	28– 8 (3)	29– 9 (4)	30–10 (5)	15–11 (6)	16–12 (7)	17–13 (8)	18–20 (9)	19–14 (2)
11	1–20 (11)	2–21 (2)	3–22 (11)	4–23 (12)	5–24 (1)	6–25 (2)	7–26 (3)	8–27 (4)	9–28 (5)	10–29 (6)	11–30 (7)	12–15 (8)	13–16 (9)	19–17 (10)	14–18 (1)
12	1–19 (12)	2–20 (1)	3–21 (12)	4–22 (1)	5–23 (2)	6–24 (3)	7–25 (4)	8–26 (5)	9–27 (6)	10–28 (7)	11–29 (8)	12–30 (9)	13–15 (10)	18–16 (11)	14–17 (12)

12.5 Combined Howell-Mitchell movement

God does not care about our mathematical difficulties;
He integrates empirically.
Albert Einstein

This movement is particularly useful for obtaining one winner from a large field, especially if the event is conducted over 3 sessions.

The principle is to append a Howell movement to a Mitchell movement. There are two basic forms of the movement. In the perfect form all pairs play all other pairs and play all the boards in the event. In the imperfect forms, one or both of the above qualities is missing, but other numbers of tables are accommodated.

Perfect Howell-Mitchell

This applies to numbers of tables where T (the total number of tables) is restricted by T mod 3 = 2 and the number of tables is greater than 7, i.e., 8, 11, 14, ….

The event consists of two distinct movements, one a Mitchell and the other a Howell. These two movements play the same boards; either through shared or duplicated boards.

The number of tables in the Mitchell $= (2T - 1) / 3$
$=$ the number of rounds

The number of tables in the Howell $= ((T - 2) / 3) + 1$
Examples:

- for an 8-table movement: 5-table Mitchell + 3-table (pseudo) Howell;
- for an 11-table movement: 7-table Mitchell + 4-table Howell;
- for a 14-table movement: 9-table Mitchell + 5-table Howell.

The field is divided into three distinct groups plus one other pair. The number of pairs in each group is equal to the number of tables in the Mitchell movement. The one odd pair always plays in the Howell movement as the stationary pair. If you are short one pair use this spot for the sit-out.

As the event is conducted over three sessions, the three groups play once each in the positions:

- N-S field in the Mitchell;
- E-W field in the Mitchell;
- Howell movement.

For example, with the three groups A, B, and C.

	N-S field	E-W field	Howell
Session 1:	A	B	C+1
Session 2:	C	A	B+1
Session 3:	B	C	A+1

As there are as many tables in the Mitchell movement as there are board-sets in the Howell movement, then by positioning the Howell movement tables correctly along-side the Mitchell, the board movement in the Howell will be done automatically by the Mitchell movement as long as the boards are always passed back to the Mitchell after each round.

For example, with 11 tables:

7-table Mitchell:	1	2	3	4	5	6	7
4–table Howell:	8–1	6–5	4–2	R	7–3	R	R

The Mitchell is a standard movement with N-S stationary and E-W moving up one while the boards move down one. The Howell movement is standard with the exception that the boards are moved down one by the players in the Mitchell movement. For better comparisons of the results it is preferable to use a switch-Mitchell.

Note: with the 8-table movement, the 3-table Howell is used and therefore sharing between these three tables and one of the tables in the Mitchell is necessary. Therefore the minimum number of boards in each set is 4, preferably higher. With duplicated boards (one set-of-boards for each movement) this problem does not arise.

With an extra pair, use the appendix pair method in the Howell movement. This pair remains stationary throughout and all moving pairs sit out for one round and a score adjustment is necessary.

Stationary-Boards Howell-Mitchell

This is the most desirable form of the Howell-Mitchell movement. Even though the procedure just described of moving the boards appears to be straightforward, boards are still misplaced and sometimes not moved at all. This movement overcomes that problem.

Two rows of tables of equal length are set up parallel to each other. One row is the Mitchell and the other a stationary-board Howell. The boards are distributed on the Mitchell tables and the Howell shares boards with these tables, as they are needed.

After each round the boards are returned to the Mitchell tables so that they will be available for the next round in the Mitchell. Even if they are not returned there is no real problem, for when the pairs arrive for the next round and find they are lacking some boards they simply retrieve them from the Howell table next to them.

In this movement the N-S pairs from the Mitchell move up one table and the E-W pairs move down one table. All pairs move in the Howell (see stationary-board Howell movements, page 170).

The disadvantage with this movement is that more physical tables are required than are being used, but this usually does not present a problem.

Another advantage with this form is in the use of board-switching in the Mitchell. The recommended method is for the N-S and E-W pairs to swap directions for the last round(s) of the event. This eliminates many of the problems associated with board-switching. Be sure to advise the pairs to move in the same direction as they were originally moving. If the N-S pairs were moving up one table, then after the switch they still will be moving up one table as E-W.

Imperfect Howell-Mitchell

There are several methods of adding tables to the combined Howell-Mitchell movement in order to cater for numbers of tables that the perfect form does not.

Adding one table to the perfect form of the movement, the number of tables catered for is where
T mod 3 = 0 (9, 12, 15…).
Adding two tables caters for T mod 3 = 1 (10, 13, 16…)

Method 1

The first method adds one table to the Mitchell movement, so that there will be an even number of tables. The main issue is the increased number of rounds in the Mitchell movement. This is solved by using the *Incomplete Mitchell movements*, *One round short* methods (page 125). Essentially this is an appendix table added to the Mitchell movement, though for 9, 15 and 21 tables the movement is not as simple as the others. Not all pairs meet in this method.

Method 2

Similar to method one: Add one table to the Mitchell movement, so there will be an even number of tables. This means there is one more round in the Mitchell than in the Howell.

This can be handled in one of three ways.

1. The Mitchell is played as a skip Mitchell, skipping after half way. This way all pairs play the same number of boards. One board-set is played only in the Mitchell and must be factored up to the 'Top' on all other boards. Not all pairs meet in this method.
2. The Mitchell movement used is the share-and-relay type. In this method the Mitchell plays one more set-of-boards than the Howell movement and the Howell pairs' scores are factored up accordingly.
3. The Mitchell movement used is the share-and-relay type. The Howell movement used is an *extra-complete* type (page 194). The same number of rounds is played in both the Howell and Mitchell, but each pair meets one other pair twice. The advantage over the first 2 methods is that all pairs play all the boards and all the boards are played the same number of times.

The three groups of pairs are all the same size and equal to the number of tables in the Mitchell. Therefore there is no resident pair in the Howell movement.

Method 3

The third method is to add one table to the Howell via the standard appendix method or use a standard ¾-Howell movement. See *incomplete Howells*, page 182, for full details. When using this method there is no need to factor scores, for all pairs play the same number of boards.

The two extra pairs in the Howell remain there for the whole event occupying the same seats at all times. This means that the three groups of pairs that rotate after each session remain the same as in the perfect form. Not all pairs meet in this method.

Method 4

When you have an extra pair, as well as the extra table, use a technique similar to method two. Add one table to the Mitchell movement and use the share-and-relay type. Also add one table to the Howell movement and use an *incomplete* type playing the same number of rounds as the Mitchell. One of the stationary pairs in the Howell is a ghost and all pairs sit out there. The boards in the Howell are not played the same number of times as the Mitchell and will need to be factored to the same top.

Method 5

Add two tables. Add one table to the Mitchell and append one table to the Howell. The three groups of pairs are equal to the number of tables in the Mitchell. The stationary pairs in the Howell (those at the appendix table) are the only ones that play at the same positions throughout the event. Alternatively, use a ¾-Howell with 2 stationary pairs. The share-and-relay Mitchell movement would be used. Not all pairs meet in this method.

When you add two extra tables as well as one more pair, use the next higher perfect Howell-Mitchell with one pair missing.

12.6 Double Howell plus Mitchell movement

If there is a God, he's a great mathematician.
Paul Dirac

This movement is designed for two-session events. It was first developed by Major Alfred M. Gruenther in the early 1930s and is sometimes called the interwoven movement, though the term 'interwoven' actually applies only to the double-Howell. The advantage is that all players meet all others, all play the same boards, and the comparisons are at least adequate.

The first session of the event is a standard Mitchell movement. Two slightly different methods are used depending on whether the number of tables in the Mitchell is odd or even.

Even-tables Mitchell

The standard share-and-relay Mitchell is used with the N-S field meeting the whole E-W field. A switched Mitchell should be used for better comparisons of scores. If the number of rounds is too high for the comfortable completion of the movement, the skip Mitchell should be used and curtailed at a point suitable for the event. This does mean that not all pairs will meet.

The second session of the event will then be a double Howell, with the pairs from the N-S field making up one of the Howells and the pairs from the E-W field the other.

The number of rounds in the double Howell will be one less than the number in the Mitchell movement. For example, with 10 tables, the first session would be a share-and-relay Mitchell for 10 tables with 10 rounds. The second session would be a double 5-table Howell, playing 9 rounds.

Odd-tables Mitchell

A standard odd-table Mitchell is set up, but no play takes place on the first round. The pairs who do not play each other on this round will meet at table 1 of the double Howell (or the table with the non-existent stationary pairs). Therefore the number of rounds played in the Howell will be one more than that played in the Mitchell. Therefore the results on the boards played in the Mitchell will need to be factored up to the same top as the Howell so that all boards are of the same value. It is generally considered better to play a pair twice than factor up the scores and so playing the first round in the Mitchell has merit.

Consider 9 tables in the Mitchell: The first round is not played. The double Howell would be a double 5-table Howell with 9 rounds. In the case of 5 tables the standard Double Howells are not applicable because the tables with the stationary pairs are not sharing boards (see *double Howell* movements, page 178). Any one of the following six Howells can be used with another Howell from a different group.

Group A								
10–1	5–3	2R	9–4	2–8	2R	7–6		
10–1	7–9	2R	2–6	8–5	2R	4–3		
Group B								
10–1	9–8	6–4	4R	3–7	5–2			
10–1	4–3	7–9	4R	2–6	8–5			
Group C								
10–1	R	5–7	R	4–8	3–2	R	9–6	R
10–1	R	6–8	R	7–2	5–4	R	3–9	R

For example

10–1	5–3	R	R	9–4	2–8	R	R	7–6
1–10	9–8	6–4	R	R	R	R	3–7	5–2

Note in the example there is board sharing at board-sets 2 and 9. As pairs 10 in both Howells are non-existent, there is only one pair at table 1 in both movements so that the two pairs '1' play against each other on the first round at table 1 (this is the round skipped at the start of the Mitchell movement). The pairs originally numbered 2 meet on the second round and 3 on the third etc. See *double Howells* for an alternative solution, page 178.

Snow modification

This type of event can be conducted with barometer scoring by using the Snow modification for both the interwoven Howell and the Michell movement. See *further Mitchell modifications - Snow modification* (page 120) for details of the ring-dance Mitchell and use the standard *double (or interwoven) Howell* (page 178) movements with shared boards on each round.

For an even number of tables you divide the pairs into two groups of equal size and the same principles apply as those used in the standard double-Howell and Mitchell movements. The ring-dance Mitchell plays the same number of rounds as there are tables and the interwoven Howell one round less.

It is possible to have two pairs stationary in the ring-dance, the two highest pairs in the groups. This way the number of rounds per session are equal, but another one round session or stanza is required to complete the full round-robin.

Consider 8 tables:
Stanza 1. Round 1.

Table	1	2	3	4	5	6	7	8
NS	1	2	3	4	5	6	7	8
EW	9	10	11	12	13	14	15	16

Stanza 2. Rounds 2-8. Round 2.

Table	1	2	3	4	5	6	7	8
NS	8	7	6	4	9	10	11	13
EW	1	2	3	5	16	15	14	12

Pairs 8 and 16 are stationary. Pair 1 follows 7, 9 follows 15.

Stanza 3. Rounds 9-15. Round 9.

Table	1	2	3	4	5	6	7	8
NS	16	15	14	4	12	6	7	8
EW	1	2	3	13	5	11	10	9

Pairs 8 and 16 are stationary. Pair 1 follows 7, 9 follows 15.

With an odd number of tables each group gets an odd number of pairs. In the double Howell there are no stationary pairs and the number of rounds played is equal to the number of tables. In the Mitchell stanza you avoid these pairs meeting again by using a ring-dance Mitchell with stationary pairs.

Thus the pair with highest number in each group is moving in the first stanza but is stationary in the second. Notice that the pairs with the lowest number in the groups follow different pairs in the two stanzas. The remaining moving-pairs follow the pair with next lower number.

A 5-table example is shown here because a double-Howell for 5 tables does not exist, but when the boards are shared on each round the following movement works.

Stanza 1. Rounds 1-5. Round 1.

Table	1	2	3	4	5
NS	3	2	1	7	9
EW	4	5	6	10	8

Pair 1 follows 5, 6 follows 10.

Stanza 2. Rounds 6-9. Round 6.

Table	1	2	3	4	5
NS	6	7	3	2	1
EW	5	4	10	8	9

Pairs 5 and 10 are now stationary. Pair 1 follows 4, 6 follows 9.

12.7 Double Mitchell-Howell movement

There was a young man from Trinity,
Who solved the square root of infinity.
While counting the digits,
He was seized by the fidgets,
Dropped science, and took up divinity.
Author Unknown

This movement is an extension of the combined Howell-Mitchell movement. That movement is designed for three-session events, but this is used in five-session events. It provides an orderly arrangement and movement of the pairs so that all pairs play each other and they play all the boards.

The standard *combined Howell-Mitchell movement* is set up and to this is added one more Mitchell movement. The normal table arrangement would be one row of Mitchell tables, one row of Howell tables, and then another Mitchell. The use of the stationary-board Howell-Mitchell is recommended so that the number of tables in the Howell row is the same as the number in the Mitchell rows.

The same principles apply to this movement as those of the combined Howell-Mitchell, but there are three rows sharing boards instead of two. If the boards have been duplicated then sharing is eliminated. Where the sharing is between the three rows then obviously at least three boards must be played on each round and preferably more than three.

Therefore when there is board sharing the movement is useful only for 36 or 46 pairs. Another possibility is 26 pairs, but at least 5 boards per round are required, as the Howell for 3 tables needs to share boards, so that a 5-way share is required.

There are 5 groups in this movement. Each group has as many pairs as there are tables in the Mitchell movements. There is one stationary pair in the Howell. One group is placed in the Howell and each of the other groups is placed as either N-S or E-W in a Mitchell.

The 5 groups can be shown diagrammatically as

	Mitchell 1	Mitchell 2	Howell
N-S	G1	G2	G3+1
E-W	G5	G4	

After each session the groups are rotated about each other; G1 moves to the position occupied by G2, G2 to that of group G3 etc. After 5 sessions all pairs will have met each other.
The scrambled Mitchell or Boufos Mitchell should be used for all the Mitchells in the event.

12.8 Double combined Howell-Mitchell movement

The laws of nature are but the mathematical thoughts of God.
Euclid

This movement has very few uses, but it is included here as an example of what can be done with the previous movements with a little bit of thought. It can be used when two combined Howell-Mitchell movements are being used in two separate sections when there is a restriction on the amount of space available in the playing room.

A standard stationary-board Howell-Mitchell is set up. Obviously there are several tables not in use in the Howell. By placing another Howell-Mitchell on the other side of the movement, these spare tables can be used in the Howell of the second movement. Unfortunately there is one table short.

For example, a 7-table Mitchell plus a 4-table Howell has only 7 tables in the stationary-board Howell. Therefore if the two Howells are using the same tables, one more table must be added to give a situation similar to the Double Howell.

Continuing with the 7+4 table example, the tables would be arranged as

Mitchell A:		1	2	3	4	5	6	7
Howells:	1	8	2	3	4	5	6	7
Mitchell B:		1	2	3	4	5	6	7

Table 8 is the additional table 1 (it could be called table 1A).

Consider the following two 4-table Howells

8–1	5–6	4–2	R	7–3	R	R
8–1*	R	R	6–2	R	7–5	3–4

*** Table 8 on the first round.**

The tables in Mitchell ‘A’ are in section 1 and those in Mitchell ‘B’ are in section 2. Board-sets 1 through 7 are on tables 1 through 7 of the Mitchells and the two sets of boards are not necessarily duplicated. The boards remain stationary throughout the event, with the Mitchell players moving N-S up 1 and E-W down 1.

Board switching is desirable, therefore the pairs in the Mitchell movement would change directions for the last round to give adequate comparisons. Guide-cards are necessary for the Howell movements. They share boards with the respective Mitchell movement with which they are associated.

Tables of the same number are sharing, so that if section 1 pairs are occupying the Howell table they share with the table of the same number in the Mitchell movement A. Similarly for section 2.

The only problem arises with table 8. On round 1 table 8 shares with lB. The table numbers 2 and 8 are then exchanged. The N-S pair on table 8 moves up one table (staying with table 8) and then share with table 2B. After the second round, table numbers 3 and 8 are exchanged, and so on. The duty of exchanging the table numbers can be assigned to the N-S pair at table 8, for they remain at that table. The N-S pair in Howell movement 1, at table 1, is simply required to move up one table at each change.

Part 13 Additional information

Recent plastic boards used in dealing machines.

Epilogue

Any intelligent fool can make things bigger and more complex..
It takes a touch of genius - and a lot of courage to move in the opposite direction.
Albert Einstein

The discoveries made during the compilation of this book were far more extensive than I ever expected. The historical items were often difficult to locate, but the big surprises to me included the Allison improvement, Safford's contributions to the first Howell movements and the Clay universal movement.

Mathematically, the biggest discovery was the paper produced in 2006 by Gennian Ge, Esther Lamken and Alan Ling that gave us the scissor schedules. Finding this paper certainly slowed progress on the book. The applied mathematics required for that subject took the best part of a year to define and produce the movements that bridge players can now use in this new type of bridge event. Before leaving this subject I would like to make a special mention of thanks to Esther Lamken who supplied a correct solution to one particular array, $H(34, 36)$, that I found to be faulty in the original CCRRS paper.

My own research produced two very significant outcomes. The first was the definition of all the round-robin teams' draws for odd numbers of teams. This idea germinated in my mind around the mid-1970s, and was published in my first book in 1979. At that time all I was able to produce were draws for 7, 9 and 11 teams. Fortunately for us the pure mathematics research paper on House squares written by Wal Wallis and Doug Stinson in 1984, was the major breakthrough on this subject. Applying that mathematics we are now able to produce the draw for any number of odd-numbered teams.

The second was the extensions to the web Mitchell. This was extremely satisfying. To find that the rover-table modification could be applied to the web Mitchell was an extremely useful breakthrough. This then lead to the application of the Ewing modification to the web Mitchell, or the hesitation movement. It worked almost the same way as with any Mitchell movement, only slightly better. With the use of the web Mitchell in the appending-movements modification (previously this idea was known in limited forms, but never written up) it means the web Mitchell is one of the most significant movements used in pairs' events around the world today.

One final and special thank you goes to Olof Hanner. His contributions to bridge movements throughout his career have been very extensive. At the age of 90, just before going to print, he supplied 3 interwoven Howell movements that completed the subject on two-session incomplete-Howells for 10, 12 and 14 rounds (see page 189).

I do hope I have broadened the scope of bridge movements for tournament directors all around the world, as well as laying down the foundations for any future mathematical research on this subject.

As for my future on this subject the reader can keep in touch using the forum on my web site. There you will find the latest on Jeanie, the movement wizard, the program used to develop all the schedules in this book.

Jeanie is available for purchase from the publisher on their web site at:
http://masterpointpress.com/pages/bookinfo.php?prod=502&cat=125

A special discount is offered to owners of this book. Use this discount code: **Jeanie2012IMcK**

My forum is at: http://www.asecomputing.com/forums/

Appendix A

In choosing a particular movement the director has four major factors to consider.

- The number of players.
- The type of event (pairs, teams, individuals).
- The number of winners.
- The number of boards to be played (which a time limit directly affects).

Other influences on making your choice of movement include

- Does the movement need to be balanced?
- Can the movement be incomplete?
- Are shared boards allowed?
- Is a complex movement acceptable?
- Do you have duplicated boards?

Broadly speaking, pairs' movements can be divided into two basic types: Mitchell movements which generally produce two winners and Howell movements which produce one winner. The web Mitchell generally remains as a choice for any number of tables and rounds, particularly with rover-table, hesitation and appendix movement modifications.

Teams' movements can be broken down into three basic types: round-robin, multiple teams and Swiss (elevator) movements.

Individual movements do not appear to conform to any particular pattern with each number of tables requiring separate treatment, except for the Mitchell and Howell like movements.

Pairs' movements ready reckoner

In the following list each number of tables is shown with a list of movements that should be considered for use. The list tends to be in the order of preference, but any two or more entries next to each other may be of equal merit. The selection of a movement will tend to be based on the number of rounds to be played, and the number of winners desired.

The number of rounds needed to complete the movement is shown in brackets next to each entry. If there is more than one possibility, they are separated by a comma. A two-session event is shown as 10+11. An event with three sessions of equal size is shown as 7x3.

The length of an event (i.e., total number of boards) is found by multiplying the number of rounds by the number of boards in each set.

3 tables

Howell (5); hesitation switch-Mitchell (4); whist (3); Mitchell (3).

3½ tables

Howell (7); extra-board Mitchell rover (7).

4 tables

Howell (7); extra-board Mitchell (8); share-and-relay Mitchell (4); skip Mitchell (3).

4½ tables

Howell (9); extra-board Mitchell rover (8); switch-Mitchell rover (4).

5 tables

Howell (9); whist (5); Mitchell (5); hesitation switch-Mitchell (6); appendix Mitchell (3); extra-board Mitchell (7, 9).

5½ tables

Howell (11); extra-board Mitchell rover (11); share-and-relay Mitchell (6); whist rover (5); Mitchell rover (5); 1½-table Mitchell (4).

6 tables

Howell (11); share-and-relay Mitchell (6); half-odd movement (6); twin Mitchell (6); ¾-Howell (9); skip Mitchell (5); twin Mitchell (3); extra-board Mitchell (8, 12).

6½ tables

Howell (13); share-and-relay Mitchell rover (6); whist (7); Mitchell (7); 1½-table Mitchell (5); appendix Howell (9, 11).

7 tables

Whist (7); Howell (13); Mitchell (7); hesitation switch-Mitchell (8); interwoven (6+7) appendix Mitchell (5); appendix Howell (9, 11); ¾-Howell (8, 9, 11); extra-board Mitchell (9, 11).

7½ tables

Share-and-relay Mitchell (8); Howell (15); whist rover (7); Mitchell rover (7); 1½-table Mitchell (6); appendix Mitchell (5, 7); appendix Howell (11, 13).

8 tables

Share-and-relay Mitchell (8); Howell (15); interwoven (8+7); combined Howell-Mitchell (5x3); extra-board Mitchell (10, 14); ¾-Howell (9, 11, 13); appendix Mitchell (5, 7); appendix Howell (11, 13); bowman Mitchell (6); skip Mitchell (7); twin Mitchell (4, 8).

8½ tables

Whist (9); Mitchell (9); share-and-relay Mitchell rover (8); Howell (17); 1½-table Mitchell (7); appendix Mitchell (5, 7); appendix Howell (13, 15); appendix Howell (9, 11); ¾-Howell (11, 13).

9 tables

Whist (9); Mitchell (9); Howell (17); interwoven (8+9); extra-board Mitchell (11, 13, 15); appendix Howell (9, 11); ¾-Howell (11, 13); hesitation switch-Mitchell (10); appendix Mitchell (5, 7); bowman Mitchell (7).

9½ tables

Whist rover (9); Mitchell rover (9); share-and-relay Mitchell (10); Howell (19); 1½-table Mitchell (8); appendix Howell (13, 15); appendix Mitchell (5, 7).

10 tables

Share-and-relay Mitchell (10); Howell (19); appendix Mitchell (7); half-odd movement (10); twin Mitchell (10); extra-board Mitchell (12, 16); appendix Howell (13, 15); interwoven (10+9); ¾-Howell (11, 13); appendix Howell (11, 17); bowman Mitchell (8).

10½ tables

Share-and-relay Mitchell rover (10); Mitchell (21); 1½-table Howell (9); half-odd movement rover (10); appendix Mitchell (7); appendix Howell (13, 15); appendix Howell (17, 19).

11 tables

Whist (11); Mitchell (11); Howell (21); extra-board Mitchell (13); hesitation switch-Mitchell (12); combined Howell-Mitchell (7); share-and-relay (3); interwoven (10+11); bowman Mitchell (9); appendix Mitchell (7); ¾-Howell (13); appendix Howell (13, 15); appendix Howell (17, 19).

11½ tables

Share-and-relay Mitchell (12); Howell (23); whist rover (11); Mitchell rover (11); 1½-table Mitchell (10); appendix Mitchell (7); ¾-Howell (13); appendix Howell (13, 15); appendix Howell (17, 19); appendix Howell (21).

12 tables

Share-and-relay Mitchell (12); Howell (23); extra-board Mitchell (14); interwoven (12+11); skip Mitchell (11); bowman Mitchell (10); appendix Mitchell (7,11); stagger Mitchell (6+6); double Howell (11); ¾-Howell (13); appendix Howell (13, 15); appendix Howell (17, 19); appendix Howell (21).

12½ tables

Whist (13); Mitchell (13); share-and-relay rover Mitchell (12); Howell (25); 1½-table Mitchell (11); appendix Mitchell (7, 11); appendix Howell (23).

13 tables

Whist (13); Mitchell (13); Howell (25); interwoven (12+13); hesitation switch-Mitchell (14); extra-board Mitchell (15); curtailed Mitchell.

13½ tables

Rover whist (13); rover Mitchell (13); Howell (27); 1½-table Mitchell (12); share-and-relay Mitchell (14); appendix Mitchell (11, 13); appendix Howell (25).

14 tables

Share-and-relay Mitchell (14); Howell (27); interwoven (14+13); combined Howell-Mitchell (9x3); appendix Mitchell (11, 13); bowman Mitchell (12); skip Mitchell (13); extra-board Mitchell (16); curtailed skip Mitchell; stagger Mitchell (7+7); double Howell (13); half-odd movement (14); twin Mitchell (7).

14½ tables

Share-and-relay Mitchell rover (14); 1½-table Mitchell (13); whist (15); Mitchell (15); Howell (29); appendix Mitchell (11, 13).

15 tables

Whist (15); Mitchell (15); interwoven (14+15); curtailed Mitchell; appendix Mitchell (11,13); Howell (29); hesitation switch-Mitchell (16); extra-board Mitchell (17).

15½ tables

Rover whist (15); rover Mitchell (15); share-and-relay Mitchell (16); appendix Mitchell (13); 1½-table Mitchell (14); Howell (31); appendix Howell (25, 27).

16 tables

Share-and-relay Mitche1l (16); interwoven (16+17); double Howell (15); stagger Mitchell (8+8); curtailed skip Mitchell; half-odd movement (16); twin Mitchell (8); extra-board Mitchell (18, 22); Howell (31); appendix Howell (25, 27).

16½ tables

Share-and-relay rover Mitchell (16); appendix Mitchell (11, 13); 1½-table Mitchell (15); whist (17); Mitchell (17); Howell (33).

17 tables

Whist (17); Mitchell (17); combined Howell-Mitchell (11x3); interwoven (16+17); appendix Mitchell (11, 13); Howell (33); hesitation switch-Mitchell (18); appendix Mitchell (19, 23); curtailed Mitchell.

17½ tables

Rover whist (17); rover Mitchell (17); appendix Mitchell (13); 1½-table Mitchell (16); share-and-relay Mitchell (18); Howell (35); curtailed rover Mitchell.

18 tables

Share-and-relay Mitchell (18); interwoven (18+17); curtailed skip Mitchell; half-odd movement (18); stagger Mitchell (9+9); double Howell (17); Howell (35); twin Mitchell (9); appendix Mitchell (13, 17); extra-board Mitchell (20, 24).

18½ tables

Share-and-relay rover Mitchell (18); 1½-table Mitchell (17); curtailed Mitchell; whist (19); Mitchell (19); appendix Mitchell (13, 17); Howell (37).

19 tables

Whist (19); Mitchell (19); curtailed Mitchell; extra-board Mitchell (21, 25); hesitation switch-Mitchell (20); appendix Mitchell (13, 17); interwoven (18+15); Howell (37).

19½ tables

Share-and-relay Mitchell (20); curtailed skip Mitchell; rover whist (19); rover Mitchell (19); 1½ table Mitchell (18); appendix Mitchell (13, 17); Howell (39).

20 tables

Share-and-relay Mitchell (20); combined Howell-Mitchell (13x3); interwoven (20+19); extra-board Mitchell (22, 26); curtailed skip Mitchell; appendix Howell (13, 17); stagger Mitchell (10+11); double Howell (19); twin Mitchell (10); Howell (39).

Appendix B

John Manning's 1979 paper

This mathematics paper was produced by John Manning in 1979 and was the first on the research into the balance of movements. A copy was sent to me by John in the early 1980s and is reproduced here as accurately as possible to record this ground-breaking publication. Unfortunately John died in September 1998.

Mathematics of Duplicate Bridge Tournaments

J.R. Manning, FIMA
Shoe and Allied Trades Research Association, Kettering

(Reproduced from the Bulletin of Institute of Mathematics and its Applications Vol. 15, August/September 1979, pp201 - 206)

Introduction
The commonest form of bridge competition is the pairs contest, where the unit is the partnership, and each partnership competes against other partnerships. In order that the contest should be fair, and not dependent on the random element of the deal, the same hands are played several times - after playing to a trick, players do not mix their cards together, but place them face down in front of them. At the end of the hand, the cards are replaced in slots (labelled North, South, East, West) in a 'board', to be replayed elsewhere. At the end of a round, which usually consists of two, three or four boards, boards are moved to another table; players likewise move so that they encounter fresh opponents on the next round.

A typical session in a bridge club lasts 3 to 3½ hours, during which there is time to play 24 to 28 boards. The competition is controlled by a tournament director, who is frequently himself a competitor, and who has to ensure that players and boards move correctly after each round. His task is eased and mistakes are less likely to occur if the movements are devised to be as simple as possible - cyclic and preferably one table at a time.

Part I: The Mitchell movement and its derivatives

The simplest form of movement is the Mitchell movement, as exemplified in Table I for the case of 7 tables, 14 pairs.

The tables are numbered 1 to 7, and the pairs are number 1 to 14. The boards are designated by the letters *A* to *G*, each letter representing a set of *k* boards, the value of *k* being usually 2, 3 or 4. Pairs 1 to 7 remain stationary at their own tables and play NS (North-South). Pairs 8 to 14 play EW, and move cyclically up one table each round. The boards are moved cyclically down one table each round.

Table I indicates the NS pair number, the board letter and the EW pair number on each table each round. (The NS pair numbers are repeated for the sake of uniformity - in other types of movement the NS pairs may change their positions.) For example, at table 2 in the first round, boards B are played by pair 2 sitting NS against pair 9 sitting EW.

If there are only 13 pairs competing, the final column of Table I is omitted; there is no pair 7, and the moving pairs have to sit out for one round.

With an even number of tables (e.g. 8) the Mitchell movement is different in structure (Table IIa). Here tables 1 and 8 *share* boards, possible because the symbols *A, B* etc each represent a set of *k* boards, and *k* is normally greater than 1. Between tables 4 and 5 there is a board-set (*A* relay*)* which is out of play for one round.

Table I
7-Table Mitchell

Round	Table Number																				
	1			2			3			4			5			6			7		
1	1	*A*	8	2	B	9	3	C	10	4	*D*	11	5	*E*	12	6	*F*	13	7	G	14
2	1	B	14	2	C	8	3	*D*	9	4	*E*	10	5	*F*	11	6	G	12	7	*A*	13
3	1	C	13	2	*D*	14	3	*E*	8	4	*F*	9	5	G	10	6	*A*	11	7	B	12
4	1	*D*	12	2	*E*	13	3	*F*	14	4	G	8	5	*A*	9	6	B	10	7	C	11
5	1	*E*	11	2	*F*	12	3	G	13	4	*A*	14	5	B	8	6	C	9	7	*D*	10
6	1	*F*	10	2	G	11	3	*A*	12	4	B	13	5	C	14	6	*D*	8	7	*E*	9
7	1	G	9	2	*A*	10	3	B	11	4	C	12	5	*D*	13	6	*E*	14	7	*F*	8

Key: 1 *A* 8, where 1 is the NS pair number, *A* is the board and 8 is the EW pair number.

Table IIa
8 table Mitchell

Round	Table Number																				
	1			2			3			4			5			6			7		
1	1	*A*	9	2	B	10	3	C	11	4	*D*	12	5	F	13	6	G	14	7	*H*	15	8	*A*	16
2	1	B	16	2	C	9	3	*D*	10	4	*E*	11	5	G	12	6	*H*	13	7	*A*	14	8	B	15
3	1	C	15	2	*D*	16	3	*E*	9	4	F	10	5	*H*	11	6	*A*	12	7	B	13	8	C	14
4	1	*D*	14	2	*E*	15	3	F	16	4	G	9	5	*A*	10	6	B	11	7	C	12	8	*D*	13
5	1	*E*	13	2	F	14	3	G	15	4	*H*	16	5	B	9	6	C	10	7	*D*	11	8	*E*	12
6	1	F	12	2	G	13	3	*H*	14	4	*A*	15	5	C	16	6	*D*	9	7	*E*	10	8	F	11
7	1	G	11	2	*H*	12	3	*A*	13	4	B	14	5	*D*	15	6	*E*	16	7	F	9	8	G	10
8	1	*H*	10	2	*A*	11	3	B	12	4	C	13	5	*E*	14	6	F	15	7	G	16	8	*H*	9

Rules for the construction of tables

The numbers representing moving pairs and the letters representing boards form a Graeco-Latin square in Table I, and indeed the rules governing the construction of Mitchell-type movements are similar to, but not identical with, those governing the construction of Graeco-Latin squares. The rules are as follows.

(1A)	No letter may appear more than once in the same column. (A stationary pair may not play the same boards more than once.)
(1 B)	No number may appear more than once in the same row. (A moving pair cannot play at more than one table at a time.)
(2)	No combination of letter and number may appear more than once. (A moving pair may not play the same boards more than once.) A *complete* movement is defined as one in which all the pairs play all the boards. A complete movement must also obey rules (3A) and (3B).
(3A)	Each column must contain a complete permutation of the letters.
(3B)	Each row must contain a complete permutation of the numbers. Although complete movements are preferred to incomplete ones, there are many occasions when incomplete movements must be played, e.g. when there is an odd number of pairs or when time runs short and the movement has to be curtailed. There are two further rules which are in the nature of mild recommendations, but may, however, be broken.
(4A)	A number should not appear more than once in the same column. (Pairs should not encounter the same opponents more than once.)
(4B)	A letter should not appear more than once in the same row. (Boards should not have to be shared between tables.)

Table I obeys all seven rules; Table IIa obeys all except (4B).

Standard form

These rules permit any permutations of rows and columns, and any re-labelling of EW pair numbers and boards to be made. In order to allow the movement to be easily controlled, tables are wherever possible expressed in standard form thus:

The first row contains the sets of boards *A, B*, *C*, etc in order, with allowance for sharing tables and relays. The first row contain the EW pair numbers in ascending order.

In each subsequent row, the board symbols are displaced one column to the left, and the EW pair numbers are displaced one column to the right (cyclically).

The advantages of the standard form are as follows.

- The first round is easy to set out.
- Boards move cyclically down one table after each round.
- Moving pairs move cyclically up one table after each round.
- Stationary pairs play the boards in sequence.

Duality

We can establish a duality by exchanging rows with columns, and letters with numbers of moving pairs. Evidently Rules (1A) and (1 B), (3A) and (3B), (4A) and (4B) are dual pairs, while Rule (2) is self-dual. Hence corresponding to any Mitchell-type movement, there exists a dual movement obtained by interchanging rows with columns and letters with numbers. Moreover the dual of a complete movement is a complete movement.

Table IIb
Skip movement for 8 tables

	Table Number																							
Round	1			2			3			4			5			6			7			8		
1	1	*A*	9	2	B	10	3	C	11	4	*D*	12	5	*E*	13	6	*F*	14	7	G	15	8	*H*	16
2	1	B	16	2	C	9	3	*D*	10	4	*E*	11	5	*F*	12	6	G	13	7	*H*	14	8	*A*	15
3	1	C	15	2	*D*	16	3	*E*	9	4	*F*	10	5	G	11	6	*H*	12	7	*A*	13	8	B	14
4	1	*D*	14	2	*E*	15	3	*F*	16	4	G	9	5	*H*	10	6	*A*	11	7	B	12	8	C	13
5	1	*E*	12	2	*F*	13	3	G	14	4	*H*	15	5	*A*	16	6	B	9	7	C	10	8	*D*	11
6	1	*F*	11	2	G	12	3	*H*	13	4	*A*	14	5	B	15	6	C	16	7	*D*	9	8	*E*	10
7	1	G	10	2	*H*	11	3	*A*	12	4	B	13	5	C	14	6	*D*	15	7	*E*	16	8	*F*	9
8	1	*H*	9	2	*A*	10	3	B	11	4	C	12	5	*D*	13	6	*E*	14	7	*F*	15	8	G	16

Table I is self-dual. Table IIb is the dual of Table IIa.
Note: Table IIa may be converted into a valid skip movement by the following transformations:

1. Interchange 9, 10, 11, 12, 13, 14, 15, 16 with *A, B, C, D, E, F,* G, *H,* respectively.
2. Transpose rows and columns. In order to convert to Table IIb, which is in standard form, two additional transformations must be made:
3. Reverse the order of the rows.
4. Reletter the boards *A, B, C, D, E, F, G,* as *H, G, F, E, D, C, B, A, respectively.* The duality between Tables IIa and IIb shows up in the following features:
    - Whereas in Table IIa there is a relay between tables 4 and 5, so in Table IIb the EW pairs make a double move after round 4.
    - Whereas in Table IIa, boards are shared between tables 1 and 8, so in Table IIb pairs have

the same opponents in rounds 1 and 8.

Rectangular movements

Tables need not be square. Table IIIa shows a 'rover' movement for 7 rounds and 8 tables, and is typical of movements for $T - 1$ rounds and T tables, where T is of the form $6m$ or $6m + 2$. Pair 16 is the rover pair and moves up two tables round the circuit 1 to 7. All other EW pairs move up one table at a time, except when they are displaced by the rover, when they go to table 8 for one round before resuming their normal progress. Table 8 shares boards with the table occupied by the rover.

The dual of the rover is the *Worger* movement, displayed in Table IIIb. Here there is a roving set-of-boards *(X),* which move two tables at a time, displacing the 'ordinary' boards. On the final round, moving pairs return to the table where they played X, and play the ordinary boards which they missed. Note that the rover movement violates Rule (4B), whereas the Worger Movement violates Rule (4A).

Table IIIa
Rover Movement

Round	Table Number																							
	1			2			3			4			5			6			7			8		
1	1	*A*	16	2	B	10	3	C	11	4	*D*	12	5	*E*	13	6	*F*	14	7	G	15	8	*A*	9
2	1	B	15	2	C	9	3	*D*	16	4	*E*	11	5	*F*	12	6	G	13	7	*A*	14	8	*D*	10
3	1	C	14	2	*D*	15	3	*E*	9	4	*F*	10	5	G	16	6	*A*	12	7	B	13	8	G	11
4	1	*D*	13	2	*E*	14	3	*F*	15	4	G	9	5	*A*	10	6	B	11	7	C	16	8	C	12
5	1	*E*	12	2	*F*	16	3	G	14	4	*A*	15	5	B	9	6	C	10	7	*D*	11	8	*F*	13
6	1	*F*	11	2	G	12	3	*A*	13	4	B	16	5	C	15	6	*D*	9	7	*E*	10	8	B	14
7	1	G	10	2	*A*	11	3	B	12	4	C	13	5	*D*	14	6	*E*	16	7	*F*	9	8	*E*	15

Table IIIb
Worger Movement

Round	Table Number																				
	1			2			3			4			5			6			7		
1	1	*A*	8	2	B	9	3	C	10	4	*D*	11	5	*E*	12	6	*F*	13	7	*X*	14
2	1	B	14	2	C	8	3	*D*	9	4	*E*	10	5	*X*	11	6	G	12	7	*A*	13
3	1	C	13	2	*D*	14	3	*X*	8	4	*F*	9	5	G	10	6	*A*	11	7	B	12
4	1	*X*	12	2	*E*	13	3	*F*	14	4	G	8	5	*A*	9	6	B	10	7	C	11
5	1	*E*	11	2	*F*	12	3	G	13	4	*A*	14	5	B	8	6	*X*	9	7	*D*	10
6	1	*F*	10	2	G	11	3	*A*	12	4	*X*	13	5	C	14	6	*D*	8	7	*E*	9
7	1	G	9	2	*X*	10	3	B	11	4	C	12	5	*D*	13	6	*E*	14	7	*F*	8
8	1	*D*	12	2	*A*	10	3	*E*	8	4	B	13	5	*F*	11	6	C	9	7	G	14

Extended Mitchell

It may be convenient for the number of rounds to be two greater than the number of tables *T;* for example, when $T = 10$ and 24 boards are to be played. On such occasions an extended Mitchell may be used. There are two cases, depending on whether T is odd or even. Tables IV and V show the movements when $T = 7$ and $T = 8$ respectively.

Note that in Table IV pair 4 encounters pair 11 on three occasions; this constitutes a particularly flagrant violation of Rule (4A). For this reason the two final rounds are usually modified as shown in Table IVa.

Note that in Table V there is a relay between tables 4 and 5, as well as between tables 8 and 1.

Table IV
Extended Mitchell – Odd number of tables

	Table Number																				
Round	1			2			3			4			5			6			7		
1	1	*A*	8	2	B	9	3	C	10	4	*D*	11	5	*E*	12	6	*F*	13	7	G	14
2	1	B	14	2	C	8	3	*D*	9	4	*E*	10	5	*F*	11	6	G	12	7	*H*	13
3	1	C	13	2	*D*	14	3	*E*	8	4	*F*	9	5	G	10	6	*H*	11	7	*I*	12
4	1	*X*	12	2	*E*	13	3	*F*	14	4	G	8	5	*H*	9	6	*I*	10	7	*A*	11
5	1	*E*	11	2	*F*	12	3	G	13	4	*H*	14	5	*I*	8	6	*A*	9	7	B	10
6	1	*F*	10	2	G	11	3	*H*	12	4	*I*	13	5	*A*	14	6	B	8	7	C	9
7	1	G	9	2	*H*	10	3	*I*	11	4	*A*	12	5	B	13	6	C	14	7	*D*	8
8	1	*H*	8	2	*I*	9	3	*A*	10	4	B	11	5	C	12	6	*D*	13	7	*E*	14
9	1	*I*	14	2	*A*	13	3	B	12	4	C	11	5	*D*	10	6	*E*	9	7	*F*	8

Table IVa

Round																					
8	1	*H*	8	2	*I*	9	3	*A*	10	4	C	11	5	C	12	6	*D*	13	7	*E*	14
9	1	*I*	14	2	*A*	13	3	B	11	4	B	12	5	*D*	10	6	*E*	9	7	*F*	8

Table V
Extended Mitchell – Even Number of Tables

	Table Number																							
Round	1			2			3			4			5			6			7			8		
1	1	*A*	9	2	B	10	3	C	11	4	*D*	12	5	*F*	13	6	G	14	7	*H*	15	8	*I*	16
2	1	B	16	2	C	9	3	*D*	10	4	*E*	11	5	G	12	6	*H*	13	7	*I*	14	8	*J*	15
3	1	C	15	2	*D*	16	3	*E*	9	4	*F*	10	5	*H*	11	6	*I*	12	7	*J*	13	8	*A*	14
4	1	*D*	14	2	*E*	15	3	*F*	16	4	G	9	5	*I*	10	6	*J*	11	7	*A*	12	8	B	13
5	1	*E*	13	2	*F*	14	3	G	15	4	*H*	16	5	*J*	9	6	*A*	10	7	B	11	8	C	12
6	1	*F*	12	2	G	13	3	*H*	14	4	*I*	15	5	*A*	16	6	B	9	7	C	10	8	*D*	11
7	1	G	11	2	*H*	12	3	*I*	13	4	*J*	14	5	B	15	6	C	16	7	*D*	9	8	*E*	10
8	1	*H*	10	2	*I*	11	3	*J*	12	4	*A*	13	5	C	14	6	*D*	15	7	*E*	16	8	*F*	9
9	1	*I*	9	2	*J*	10	3	*A*	11	4	B	12	5	*D*	13	6	*E*	14	7	*F*	15	8	G	16
10	1	*J*	16	2	*A*	15	3	B	14	4	C	13	5	*E*	12	6	*F*	11	7	G	10	8	*H*	9

Bowman movements
The duals of the extended Mitchells may be used when the number of rounds is two fewer than the number of tables. These are known as Bowman movements. The dual of Table IV involves sharing boards amongst three tables on the fourth round; which can be avoided by means of a dual modification.

Part 11: Scoring at duplicate bridge
The result of each hand is scored as in rubber bridge, with the following exceptions.

- There is no distinction between 'above the line' and 'below the line'.
- Honours do not count.
- There is a bonus of 50 points for fulfilling a part-score contract, 300 for a non-vulnerable game and 500 for a vulnerable game.
- Part-scores are not carried over from one hand to another. Vulnerability is printed on the board.

The scores are subsequently converted to *match points,* which are equivalent to ranks. Each board carries with it a travelling score slip, on which the result is entered every time the board is played. At the end of the competition, each score slip is match pointed. For example, the score slip for the first board of set *A* played according to Table I might be as shown below.

						Match Points	
NS	EW	Contract	By	Tricks	Score to NS	NS	EW
1	8	2D	N	7	-50	7	5
2	10	2D	N	6	-100	2	10
3	12	2C	E	8	-90	4	8
4	14	2D	N	8	+90	12	0
5	9	3C	E	9	-110	0	12
6	11	2D	N	7	-50	7	5
7	13	2S	W	7	+50	10	2

The scores are ranked: the NS pair with the lowest score (-110) is awarded no match points; the next lowest, two match points, and so on until the NS pair with the highest score (+90) receives $2(n - 1)$ match points (a 'top') where n is the number of times the board has been played. The factor of 2 is included to avoid fractions in the case of ties, for example the two scores of -50 as above. The EW pairs score the complementary number of match points to their opponents: i.e. the difference from $2(n - 1)$.

Each pair, whether playing NS or EW, can be said to receive two match points in respect of each pair playing the same way with a worse score, and one match point in respect of each pair with an equal score.

Measure of competition

Because scoring is by ranks, it is possible to quantify the amount of competition between two pairs. There are four cases.

(i) Pairs encounter one another

Consider pairs 1 to 8 in the above example. There are $2(n - 1)$ (=12) match points at stake, to be divided between pairs 1 and 8 according to the outcome of the encounter. Pair 8 can influence pair 1's match point score to the extent of ± 6.

Hence the amount of competition between pair 1 and pair 8 on the board is 6, or in general $(n - 1)$.

(ii) Pairs play in the same *direction*

Consider pairs 1 and 2 in the above example. Pair 1's match point score includes a contribution of 2 match points because they achieved a better score than pair 2. The influence of pair 2 on pair 1's score is ± 1 match point.

Hence the amount of competition between pair 1 and pair 2 on the board is 1.

(iii) Pairs play in opposite directions

Consider pairs 1 and 10 in the above example. By achieving a good score (+100) pair 10 have thereby enhanced the match point score of pair 1, who receive a contribution of two match points for scoring more than pair 10's opponents.

As in case (*ii*), the performance of pair 10 can affect the score of pair 1 by ± 1 match point. The difference is that pairs 1 and 10 are not competing on the board, on the contrary their interests temporarily coincide.

Hence the amount of competition between pair 1 and pair 10 on the board is –1.

(iv) Board not played by both pairs

Evidently if a board is not played by a pair (e.g. because there is a half table) the amount of competition is zero.

Notation – let:

the number of pairs in a competition be	P
the number of boards in play be	N
the number of times each board is played be	n
the number of tables in play be	T
the number of boards in each set be	k
the number of rounds be	r

$P = 2T$ *or* $P = 2T + 1$, according to whether the number of pairs P is even or odd.
The total number of hands played is

$$T\,r\,k = N\,n$$

A movement is *complete* if all the pairs play all the boards, when

$$P = 2\,T,\ n = T,\ N = k\,r.$$

All the tables in Part I represent *complete* movements.

Competition analysis

Consider two pairs, numbered i and j, with $i < j$. Let:

a_{ij} be the number of boards where i and j encounter one another;
b_{ij} be the number of boards which i and j play in the same direction;
c_{ij} be the number of boards which i and j play in opposite directions, but not as a direct encounter;
d_{ij} be the number of boards not played by i and j (dij is always zero in a complete movement);
s_{ij} be the amount of competition between i and j.

$$N = a_{ij} + b_{ij} + c_{ij} + d_{ij}$$
$$s_{ij} = (n - 1)\,a_{ij} + b_{ij} - c_{ij}$$

Mitchell Movement (Tables I and IIa)

Tables I and IIa are square as well as complete, and therefore $r = n = T;\ N = k\,n.$
If i and j are both stationary pairs, or both moving pairs, $a_{ij} = c_{ij} = d_{ij} = 0$; $b_{ij} = N,\ s_{ij} = N.$
If i is stationary and j is moving,

$$a_{ij} = k;\ b_{ij}, = d_{ij} = 0;\ c_{ij} = N - k;$$
$$s_{ij} = (n - 1)\,k - (N - k) = n\,k - N = 0, \text{ since } N = n\,k.$$

Hence there is no net comparison between a NS pair and an EW pair; the two sets of players are decoupled from each other; there are effectively two separate competitions proceeding simultaneously, and it is appropriate to issue two results lists and to reward an EW winner and a NS winner.

Skip Movement (Table IIb)

The situation is more complicated for the skip movement of Table IIb.

As before $s_{ij} = N$ if i and j are both moving or both stationary.

If i is stationary and j is moving, there are three cases, depending on whether i and j encounter one another 0, 1 or 2 times, and the value of s_{ij} is $-N$, 0 or $+N$. There is now competition between some NS pairs and some EW pairs, and the two sets of players are no longer decoupled from each other.

For example, referring to Table IIb, if pair 9 happen to be strong players and pair 13 happen to be weak players, pair 1 are at a disadvantage because they have to encounter pair 9 twice and pair 13 not at all. Pair 5 are at a corresponding advantage and might be able to ride to victory on the backs of pair 9.

Balanced movements

In all the rectangular movements described in Part I, it will be found that the amount of comparison between a NS pair and an EW pair is never zero, but is sometimes positive and sometimes negative. Hence separate results lists for the two sets of players would not be fair, just as they would not be fair for the skip Mitchell of Table IIb. Moreover, most bridge players would prefer to think of themselves as participating in one large competition, and would prefer to see a single results list covering the whole field. Hence movements are required wherein the amount of competition between two pairs is independent as far as possible of where they happen to sit at the start.

There are P pairs, and thus there are ½$P(P-1)$ values of s_{ij}. We require these values of s_{ij} to be all equal, or failing that, their dispersion as measured by their standard deviation ought to be as small as possible. If the s_{ij} are all equal and the movement is also complete, the movement is said to be *perfect.*

By summing over all pairs of pairs, we arrive at the following relations, valid for incomplete as well as complete movements:

$$\sum_{i<j} a_{ij} = nN,$$

$$\sum_{i<j} b_{ij} = \sum_{i<j} c_{ij} = n(n-1)N,$$

$$\sum_{i<j} s_{ij} = (n-1)\sum_{i<j} a_{ij} + \sum_{i<j} b_{ij} - \sum_{i<j} c_{ij} = n(n-1)N,$$

$\overline{s_{ij}} = n(n-1)N \;/\; ½P(P-1)$ where $\overline{s_{ij}}$ is the average amount of competition between two pairs.

For a *complete* movement in which $P = 2T$ and $n = T$, the expression for $\overline{s_{ij}}$ simplifies to

$$\overline{s_{ij}} = N(T-1) / (2T-1).$$

For a *perfect* movement, all the values of s_{ij} are equal, so that s_{ij} must be integral, and therefore N must be a multiple of $(2T-1)$. Perfect movements do exist, but are not discussed in this paper.

Switching

The movements of Part I can be modified so that a single list of results can be produced which is not grossly unfair. Certain boards at certain tables are switched so that the stationary pairs play EW and the moving pairs play NS. The process is often called 'arrow switching' because boards have a large arrow painted on them to indicate the north. For example, Table VI is derived from Table I by switching the last round.

Table VI

Round	Table Number																				
	1			2			3			4			5			6			7		
1	1	*A*	8	2	B	9	3	C	10	4	*0*	11	5	*E*	12	6	*F*	13	7	G	14
2	1	B	14	2	C	8	3	*0*	9	4	*E*	10	5	*F*	11	6	G	12	7	*A*	13
3	1	C	13	2	*0*	14	3	*E*	8	4	*F*	9	5	G	10	6	*A*	11	7	B	12
4	1	*0*	12	2	*E*	13	3	*F*	14	4	G	8	5	*A*	9	6	B	10	7	C	11
5	1	*E*	11	2	*F*	12	3	G	13	4	*A*	14	5	B	8	6	C	9	7	*0*	10
6	1	*F*	10	2	G	11	3	*A*	12	4	B	13	5	C	14	6	*0*	8	7	*E*	9
7	9	G	1	10	*A*	2	11	B	3	12	C	4	13	*0*	5	14	*E*	6	8	*F*	7

The values of s_{ij} instead of being either 0 or 7k, as they are for Table I, are now 0 or 3k or 4k. The distribution table for the s_{ij} is:

Value of s_{ij}	Example pairs	Number of pairs of pairs
0	1,9	7
3k	1,2	42
4k	1,8	42

Standard deviation = 1.05k

This is reasonably well balanced and is regarded as satisfactory. Any further switching becomes counter productive; for example, if Table I is switched on round 6 as well as round 7, the distribution table for the s_{ij} becomes:

Value of s_{ij}	Example pairs	Number of pairs of pairs
$-k$	1,3	28
$3k$	1,2	14
$4k$	1,9	28
$8k$	1,12	21

Standard deviation = $3.3k$

This is clearly ill balanced, and inferior to Table VI. One way of improving Table VI still further is to switch one board of each set on each of the last k rounds. For example if $k = 4$,

switch the first board of each set in round number 4,
switch the second board of each set in round number 5,
switch the third board of each set in round number 6,
switch the fourth board of each set in round number 7.

The distribution table for the s_{ij} is then:

Value of s_{ij}	Number of pairs of pairs
12	70
16	21

Standard deviation = 1.70

The movement is approaching perfection, but it is seldom used, because of the difficulty in making sure that all the players switch the right boards at the right time.

How much to switch?

A rough and ready rule is to switch about one eighth of the boards in a Mitchell-type movement. Suppose that q out of N boards are switched, the switch always being made simultaneously at all tables. We assume also, as has been indicated by the above examples, that q is small compared to N.

Then, considering two stationary pairs i and j, it is likely that $a_{ij} = 0$; $b_{ij} = N - 2q$; $c_{ij} = 2q$; $s_{ij} = N - 2q$. The $2q$ boards which i and j play in opposite directions comprise the q boards which i play switched and the q boards which j play switched. In most cases the two sets of q boards are distinct.

Since s_{ij} is a little less than ½ N (cf. equation (1)), it follows that q should be a little greater than $N / 8$, so that the majority of values of s_{ij} should be close to the mean. Since all values of s_{ij} are integral, and switching does not alter the parity of s_{ij}, a first order approximation is as much as can reasonably be expected.

In movements such as the extended Mitchell (Table IV) and larger movements not considered here, the situation is less easily analysed, and a computer program is needed to calculate the optimum amount of switching. In a bridge club where there is little at stake, the rough justice which is offered by switching on the last round, or the last two rounds if there are 12 or more rounds, is acceptable. In some international competitions, however, movements have been prepared by computer, which involve irregular switching – each competing pair being given individual itineraries. No accusations of unfairness can then be levelled at the organisers.

Publications

The standard manual on bridge movements is 'Duplicate Bridge Movements' by Frank Farrington and published by the English Bridge Union, although not on sale to the public. Farrington's manual lists dozens of different types of movement, many of which raise interesting mathematical questions. *Bridge Magazine* and other bridge periodicals carry articles on movements from time to time. Bridge club secretaries usually purchase sets of cards which indicate the movements for different numbers of tables – many of the movement cards which are commercially offered are seriously unbalanced.

I have been unable to trace any serious mathematical study of the subject, and I hope that this introductory article may stimulate others to build a corpus of theory. The ideas of duality in Part I, and of quantifying and attempting to equalise the amount of competition in Part II, have not previously been aired in print, although they have been floating round the bridge fraternity for several years.

Appendix C

Constructing 17 teams CCRRS over 2 sessions

Wal Wallis

Start with 5 teams.
There are five teams, labeled 0, 1, 2, 3, 4. Team *X* consists of pairs *x* and *x*'.

They will play a teams tournament in which teams *X* and *Y* play two sets of boards: in one set, *x* plays *y* and *x*' plays *y*' (with *x* and *y*' sitting in the same direction), while in the other set *x* plays *y*' and *x*' plays *y*.

Here is the layout showing who plays whom in each round. The board-sets are *S1* through *S5*.

S1	*S2*	*S3*	*S4*	*S5*	*S2*	*S1*	*S4*	*S3*	*S5*
01'		12'		23'		34'		40'	
0'1			1'3'		32		2'4	4'0	
	10	21'		32'		43'			0'4'
	1'0'		31		2'3'		4'2		04

** Shuffle **

02'		24		4'1'		13'		30'	
0'2			43		4'1		1'2'	3'0	
	20	4'2'		14		31'			0'3'
	2'0'		3'4'		1'4		21		03

Now the symbol
---
0 1 2
3 4
---
represents the 5 team tournament array above.

Similarly
---
0 5 6
7 8
---
refers to the same tournament with 1 replaced by 5, 2 by 6, and so on.

Then using the symbols 0 plus 1 through 8 and A through H for the 17 teams we have:

----------	5E	7H				C8	D6		FA	GB		
0 1 2	8G	6F			D7			C5	HB	EA		
3 4		7G	5F	B8			A6	EC	HD			
----------			6E	8H		A7	B5		GD	FC		
----------	1A	3D				G4	E2				BF	CH
0 5 6	4C	2B			E3			G1			AH	DF
7 8		3C	1B	F4			H2			DE	AG	
----------			2A	4D		H3	F1				CG	BE
----------			8F	6G	1E		4H		52		73	
0 A B			5H	7E		2F		3G	61	84		
C D	6H	8E			2G		3F		74	51		
----------	7F	5G				1H		4E	83		62	
----------			B4	C2	5A		8D			36		17
0 E F			D1	A3		6B	7C				45	28
G H	D2	A4			6C			7B			18	35
----------	B3	C1				5D		8A		27		46

5E means 5'E' with E5 and 5'E with E'5, two complete matches.

Appendix D

Constructing self orthogonal latin squares (SOLS)

Wal Wallis

1 Definitions and Notation

We will be dealing with arrays. If an array has r rows and s columns, we say it is an $r \times s$ array. We shall denote arrays by capital letters. The element at the intersection of row i with column j of an array A is written a_{ij} (or $a_{i,j}$ if any confusion is possible — for example, if i or j is replaced by an expression of more than one symbol).

For example, if
$A =$

1	3	5	2
5	9	4	2
1	1	6	3

then A is 3×4, row 2 of A is [5 9 4 2], and $a_{23} = 4$.

The *transpose* of an $r \times s$ array A is an $s \times r$ array with a_{ji} at the intersection of row i with column j, and is written AT .

A *Latin square* of order (or side) n, based on $\{1, 2, \ldots , n\}$, is an $n \times n$ array where the symbols of any row are the numbers $1, 2, \ldots , n$, once each, in some order, and the same for any column. A simple example of side 4 is

1	2	3	4
2	3	4	1
4	1	2	3
3	4	1	2

(It is sometimes more convenient to use other symbols for the entries, but we'll usually use $\{1, 2, \ldots , n\}$.) Latin squares A and B of the same order n are *orthogonal* if the n^2 ordered pairs(a_{ij} , b_{ij}) are all different (and this means that every possible pair occurs exactly once). In particular, if A and A^T are orthogonal, A is called a *self-orthogonal Latin square*. Here is a self-orthogonal Latin square of side 4:

$L_4 =$

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

A Latin square A is called *diagonal* if all its diagonal elements — that is, the elements $a_{11}, a_{22}, \ldots , a_{nn}$ — are different. It follows that every possible symbol will appear exactly once on the diagonal. It is easy to see that any self-orthogonal Latin square must be diagonal.

2 Direct Construction

Here is a very simple construction for a self-orthogonal Latin square of side n, provided n is not divisible by 2 or 3. The first row is

$$1\ n\ n-1\ \ldots\ 3\ 2,$$

and subsequent rows are constructed by the rule

$$a_{i+1,j+1} = a_{ij} + 1$$

where the numbers are interpreted modulo n: that is, if $a_{ij} = n$ then $a_{i+1,j+1} = 1$;

$$a_{i+1,1} = a_{in} + 1.$$

As an example, when $n = 5$, the array is

$L_5 =$

1	5	4	3	2
3	2	1	5	4
5	4	3	2	1
2	1	5	4	3
4	3	2	1	5

When n is even but prime to 3, the orthogonality property holds, but there are repeats in the columns. When 3 divides n and n is odd, the array is Latin, but there are repeated pairs.

There is no self-orthogonal Latin square of order 2, 3 or 6. We have already given an example of order 4. Orders 5 and 7 are covered by the above construction. For completeness, here are examples of orders 8 and 9:

$L_8 =$

1	6	8	7	3	5	2	4
3	2	7	8	1	4	6	5
7	4	3	1	8	2	5	6
6	1	5	4	2	8	3	7
4	7	2	6	5	3	8	1
8	5	1	3	7	6	4	2
5	8	6	2	4	1	7	3
2	3	4	5	6	7	1	8

$L_9 =$

1	4	6	8	3	9	2	7	5
5	2	1	7	8	4	9	3	6
8	6	3	2	4	5	1	9	7
3	5	7	4	9	1	6	2	8
4	7	9	6	5	8	3	1	2
7	1	8	9	2	6	5	4	3
9	8	2	5	1	3	7	6	4
6	9	5	3	7	2	4	8	1
2	3	4	1	6	7	8	5	9

3 Product Construction

Suppose $A = (a_{ij})$ and $B = (b_{ij})$ are square arrays: A is $r \times r$ and B is $s \times s$. We shall assume A is based on the symbols $\{1, 2, \ldots, r\}$ and B is based on $\{1, 2, \ldots, s\}$.

We define a set of r arrays $B_1, B_2, \ldots, B_r$ as follows: B_i is derived from B by adding $(i - 1)\, s$ to each element. Thus $B_1 = B$, and is based on $\{1, 2, \ldots, s\}$, B_2 is based on $\{s + 1, s + 2, \ldots, 2s\}$, and so on.

By $A \otimes B$ we mean the following $rs \times rs$ array:

$$A \otimes B =$$

$$\begin{matrix} B_{a11} & B_{a12} & \cdots & B_{a1s} \\ B_{a21} & B_{a22} & \cdots & B_{a1s} \\ & \cdots & \cdots & \\ B_{ar1} & B_{ar2} & \cdots & B_{ars} \end{matrix}$$

.

If A and B are Latin squares, then $A \otimes B$ is a Latin square. If A and A^* are orthogonal Latin squares, and B and B^* are orthogonal Latin squares, then $A \otimes B$ is orthogonal to $A^* \otimes B^*$. And if A and B are self-orthogonal Latin squares, then $A \otimes B$ is a self-orthogonal Latin square.

An example with $r = s = 4$. We can take $A = B = B1 = L_4$, the self-orthogonal latin square shown in Section 1. Then

$B_2 =$

5	7	8	6
8	6	5	7
6	8	7	5
7	5	6	8

$B_3 =$

9	11	12	10
12	10	9	11
10	12	11	9
11	9	10	12

$B_4 =$

13	15	16	14
16	14	13	15
14	16	15	13
15	13	14	16

The result is
L_{16} =

1	3	4	2	9	11	12	10	13	15	16	14	5	7	8	6
4	2	1	3	12	10	9	11	16	14	13	15	8	6	5	7
2	4	3	1	10	12	11	9	14	16	15	13	6	8	7	5
3	1	2	4	11	9	10	12	15	13	14	16	7	5	6	8
13	15	16	14	5	7	8	6	1	3	4	2	9	11	12	10
16	14	13	15	8	6	5	7	4	2	1	3	12	10	9	11
14	16	15	13	6	8	7	5	2	4	3	1	10	12	11	9
15	13	14	16	7	5	6	8	3	1	2	4	11	9	10	12
5	7	8	6	13	15	16	14	9	11	12	10	1	3	4	2
8	6	5	7	16	14	13	15	12	10	9	11	4	2	1	3
6	8	7	5	14	16	15	13	10	12	11	9	2	4	3	1
7	5	6	8	15	13	14	16	11	9	10	12	3	1	2	4
9	11	12	10	1	3	4	2	5	7	8	6	13	15	16	14
12	10	9	11	4	2	1	3	8	6	5	7	16	14	13	15
10	12	11	9	2	4	3	1	6	8	7	5	14	16	15	13
11	9	10	12	3	1	2	4	7	5	6	8	15	13	14	16

Examples: for order 25, use
$A = B =$

1	4	5	2	3
3	2	1	5	4
4	5	3	1	2
5	3	2	4	1
2	1	4	3	5

for order 28, use
$A =$

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

,B =

1	7	6	5	4	3	2
3	2	1	7	6	5	4
5	4	3	2	1	7	6
7	6	5	4	3	2	1
2	1	7	6	5	4	3
4	3	2	1	7	6	5
6	5	4	3	2	1	7

.

4 PBD Construction

A *pairwise balanced design* of index λ on a set S is a collection of subsets of S, called *blocks*, with the property that, given any two members of S, there are exactly λ blocks that contain both of them.

Suppose we have a pairwise balanced design of index 1 on a set of n elements. For each block $B = \{b_1, b_2, \ldots, b_k\}$ of size k, select a diagonal Latin square $L(B)$ of order k, whose elements are the members of B. In particular, the diagonal of the square should be $(b_1, b_2, \ldots, b_k)$. Then use these squares to construct an $n \times n$ square A as follows. The (b_i, b_j) element of A is the (i, j) element of $L(B)$. Since there is exactly one block containing both bi and bj , there will be exactly one candidate for position (b_i, b_j). Then A will be a diagonal Latin square. If, further, all the squares $L(B)$ are self-orthogonal, the resulting square A will be self-orthogonal.

To illustrate the method, we use the pairwise balanced design with blocks 123, 145, 167, 246, 257, 347, 356 to construct a diagonal Latin square of side 7. The arrays $L(B)$ are derived from

1	3	2
3	2	1
2	1	3

For example, the array shown corresponds to the block 123; the blocks corresponding to 145 and 356 are

1	5	4
5	4	1
4	1	5

and

3	6	5
6	5	3
5	3	6

The result of using block 356, and the final result, are

		3		6	5	
		6		5	3	
		5		3	6	

and

1	3	2	5	4	7	6
3	2	1	6	7	4	5
2	1	3	7	6	5	4
5	6	7	4	1	2	3
4	7	6	1	5	3	2
7	4	5	2	3	6	1
6	5	4	3	2	1	7

The small squares all need to be diagonal, so designs with blocks of size 2 are not useful. Similarly, for self-orthogonal squares, block sizes 2, 3 and 6 cannot be used. Here are pairwise balanced designs of order 21 and 24, with all blocks of size 4 and 5. They are both derived from a finite affine plane of order 25.

24 :

2	7	12	17	22	3	8	13	18	23	4	9	14	19	24
5	10	15	20		1	7	13	19		2	8	14	20	21
3	9	15	16	22	4	10	11	17	23	5	6	12	18	24
1	8	15	17	24	2	9	11	18		3	10	12	19	21
4	6	13	20	22	5	7	14	16	23	1	9	12	20	23
2	10	13	16	24	3	6	14	17		4	7	15	18	21
5	8	11	19	22	1	10	14	18	22	2	6	15	19	23
3	7	11	20	24	4	8	12	16		5	9	13	17	21
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24		1	6	11	16	21

21 :

2	7	12	17		3	8	13	18		4	9	14	19	
5	10	15	20		1	7	13	19		2	8	14	20	21
3	9	15	16		4	10	11	17		5	6	12	18	
1	8	15	17		2	9	11	18		3	10	12	19	21
4	6	13	20		5	7	14	16		1	9	12	20	
2	10	13	16		3	6	14	17		4	7	15	18	21
5	8	11	19		1	10	14	18		2	6	15	19	
3	7	11	20		4	8	12	16		5	9	13	17	21
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20						1	6	11	16	21

Appendix E

Balanced tournament design

Esther Lamken

A balanced tournament design, BTD(n), defined on a $2n$-set V is an arrangement of the

$$\binom{2n}{n}$$

distinct unordered pairs of the elements of V into an $n \times (2n - 1)$ array such that

(1) every element of V is contained in precisely one cell of each column, and
(2) every element of V is contained in at most two cells of each row.

An element which is contained only once in row i of a BTD(n) is called a deficient element of row i. The two deficient elements of row i are referred to as the deficient pair of row i. An example of a balanced tournament design, a BTD(4), is displayed in Figure 1. The deficient pairs are 56,78, 12, and 34 respectively.

12	13	58	37	47	28	46
34	57	14	26	25	36	18
56	24	67	48	38	17	35
78	68	23	15	16	45	27

Fig. 1. A BTD(4).

The existence of BTD(n) was first established in [6]. We note that a simpler proof of this result can be found in [5].

Theorem 0.1 [*6*] *For n a positive integer, $n \neq 2$, there exists a* BTD(n).

Balanced tournament designs can be used to schedule round-robin tournaments. We think of the rows of the design as representing the locations, the columns as rounds of play, and the elements as players or teams. The properties of the array give us a schedule of play which has each team playing each other team precisely once, each team plays each round, and no team plays more than twice at any location. (For more information on scheduling tournaments see [1].)

Let V be a set of $2n$ elements. A Howell design of side s and order $2n$, or more briefly an $H(s, 2n)$, is an $s \times s$ array in which each cell is either empty or contains an unordered pair of elements from V such that

(1) each row and each column is Latin (that is, every element of V is in precisely one cell of each row and column) and
(2) every unordered pair of elements of V is in at most one cell of the array.

It follows immediately from the definition of an $H(s, 2n)$ that $n \leq s \leq 2n - 1$.

If we can partition the columns of a BTD(n) defined on V into three sets $C1$, $C2$, $C3$ of sizes 1, $n - 1$, $n - 1$ respectively so that the columns in $C1 \cup C2$ form an $H(n, 2n)$ and the columns in $C1 \cup C3$ form an $H(n, 2n)$, then the BTD(n) is called a partitioned balanced tournament design or a PBTD(n). An example of a partitioned balanced tournament design is displayed in Figure 3. $C1$ is the first column, $C2$ consists of columns 2-5, and $C3$ consists of the last 4 columns.

06	α4	∞2	13	57	23	45	∞7	α1
17	∞3	α5	46	02	∞4	α2	05	63
42	56	03	α7	∞1	67	01	α3	∞5
53	12	47	∞0	α6	α0	∞6	14	72
$\alpha\infty$	07	16	25	43	15	37	26	04

Fig. 3. A PBTD(5).

The existence of PBTD(n) has been established with at most 3 possible exceptions.

Theorem 0.2 [*3*] *There exists a* PBTD(n) *for n a positive integer, $n \geq 5$, except possibly for $n \in \{9, 11, 15\}$.*

An *Odd team design* can be constructed from a PBTD(n). The array

$$\begin{pmatrix} \text{C1} & \text{C2} \\ \text{C1} & \text{C3} \end{pmatrix}$$

is an odd team design of order n.

There are several different types of balanced tournament designs. In particular, there is also an analogue for an odd number of teams or players where one team (player) sits out each round (odd balanced tournament designs). See [2] or the older survey [4].

References

[1] J.H. Dinitz, D.Froncek, E.R. Lamken, and W.D.Wallis, *Scheduling a tournament* in The CRC Handbook of Combinatorial Designs (C.J. Colbourn, J.H. Dinitz, eds), CRC Press, Boca Raton, FL (2006) 591–606.
[2] E.R. Lamken, *Balanced tournament designs* in The CRC Handbook of Combinatorial Designs (C.J. Colbourn, J.H. Dinitz, eds), CRC Press, Boca Raton, FL (2006) 591–606.
[3] E.R. Lamken, A fewmore partitioned balanced tournament designs, *Ars Combin.* **43** (1996) 121–134.
[4] E.R. Lamken and S.A. Vanstone, Balanced tournament designs and related topics, *Discrete Math.* **77** (1989) 159–176.
[5] E.R. Lamken and S.A. Vanstone, The existence of factored balanced tournament designs, *Ars Combin.* **19** (1985) 157–160.
[6] P.J. Schellenberg, G.H.J. van Rees, and S.A. Vanstone, The existence of balanced tournament designs, *Ars Combin.* **3** (1977) 303–318.

Appendix F

4 Table appendix modification

This is based on the 3-table appendix modification with an additional table appended. The additional table is appended using similar principles to those used in the 3-table appendix. The last table is added with the N-S pair stationary and the E-W pair displaces a pair in the other tables on each round, the displaced pair playing against the N-S pair at the highest table. They play the same boards as those at the displaced table.

The rover in the 3-table appendix is still required and uses the schedule given in the 3-table appendix modification (page 111) for the same number of rounds.

Note the use of a web Mitchell appended to an odd-tables Mitchell will always work where the 4-table appendix works and with less complications. See *appending movements modification* (page 123) for more details. Even so, the 4-table appendix is needed so that the 5-table appendix can work (see page 112).

Tables	Rounds	Rover replaces EW at
8	4	2, 4, 1, 5
9	5	2, 4, 1, 3, 5
10	6	4, 6, 1, 2, 8, 7
11	7	2, 4, 6, 1, 3, 5, 7
12	8	8, 3, 7, 2, 6, 9, 1, 4
13	9	2, 5, 10, 6, 3, 11, 4, 7, 9
14	10	3, 6, 12, 9, 11, 5, 8, 4, 1, 7
15	11	2, 4, 6, 8, 10, 1, 3, 5, 7, 9, 11
16	12	3, 11, 9, 1, 6, 2, 8, 14, 13, 4, 10, 7
17	13	2, 4, 1, 8, 10, 12, 6, 3, 5, 13, 11, 9, 7
18	14	2, 4, 1, 7, 9, 13, 16, 5, 15, 6, 11, 3, 8, 12
19	15	3, 5, 8, 17, 9, 15, 1, 16, 7, 10, 6, 13, 2, 4, 12
20	16	2, 4, 1, 7, 11, 17, 14, 16, 6, 8, 3, 5, 13, 15, 10, 12
21	17	2, 4, 1, 3, 12, 10, 15, 19, 9, 5, 16, 18, 6, 17, 11, 13, 8
22	18	3, 5, 7, 14, 10, 18, 16, 8, 2, 17, 6, 9, 1, 12, 4, 15, 13, 20

Appendix G

The Clay universal movement

The original description of this movement was given in Butler's *The whist reference book* (1898). The complete article is supplied here. The astute reader will notice something intriguing about the use of the words 'board' and 'tray'. This is the earliest article I have seen that uses the two terms interchangeably.

Schedules of Play for Pairs, Teams of Four, etc. - A most important system is described in the July, 1897, number of *Whist*, by Charles M. Clay (q. v.}, its originator. It is a universal system for compass matches between pairs, for multiple teams of four, or for two teams of any number, and with any number of tables whatever, though its practical limit is probably sixteen tables, on account of the time required.

When the match is between fours, each four seat themselves together at a table, then the east-west pairs move up one table before beginning play.

After each round the east-west pairs move up one table.

When the match is between two teams of more than four on a side, the visiting team seat themselves at tables 1, 3, 5, etc., and the home team at tables 2, 4, 6, etc., then the east-west pairs move up one table before beginning play. After each round the home pairs move, the visitors sitting still throughout the play. When the match is between pairs, no preliminary arrangement is necessary.

The system of play consists in placing upon each table more than one board, and playing, each round, case of five boards, say, playing only two the first round, and the remaining three the next round. This system requires that the moving players shall make the circuit of the tables twice, so that there are always twice as many rounds as tables.

For each different number of tables it is necessary to write down in advance a simple schedule, as follows. Write in numerical order the number of rounds to be played. Beneath, in vertical lines, write the numbers of the trays, in regular numerical order, which are to be played each round. For six tables, four trays at a table, the schedule would be written thus:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1	3	5	7	9	11	13	15	17	19	21	23
2	4	6	8	10	12	14	16	18	20	22	24

Now place on the tables, beginning with table 1, the trays under the odd numbers first, and afterward these under the even numbers.

That is, place on table 1 trays 1, 2, 5, 6; on table 2, trays 9, 10, 13, 14; on table 3, trays 17, 18, 21, 22; on table 4, trays 3, 4, 7, 8; on table 5, trays 11, 12, 15, 16; on table 6, trays 19, 20, 23, 24.

The deals are to be played in regular numerical order, except, of course, that when the last numbers occur on the same table with the first, they are to be played before the first. That is, deals 23, 24, are to be played before deals 1, 2, if they come on the same table. On the first round, deals 1 and 2, 9 and 10, 17 and 18, 3 and 4, 11 and 12, 19 and 20, are to be played at the respective tables.

If the number of trays on a table is odd, the formula varies slightly. Thus, for five tables, five trays on a table, the schedule is:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	3	6	9	11	13	16	19	21	23
2	4	7	10	12	14	17	20	22	24
	5	8			15	18			25

Here, since each set of trays, being five in number, cannot be divided evenly, it is divided as nearly as possible into halves, the parts consisting of twos and threes.

First write down two, then three; then three, then two, and so on, every time alternating the order of division. The trays are placed on the several tables thus: Table 1, trays 1, 2, 6, 7, 8; table 2, trays 11, 12, 16, 17, 18; table 3, trays 21, 22, 3, 4, 5; table 4, trays 9, 10, 13, 14, 15; table 5, trays 19, 20, 23, 24, 25.

On the first round, the first two deals at each table are to be played, then the next three, and so on. On table 3, of course deals 21, 22, are to be played first.

After each round the players move up one table, always taking with them the deals just played, and placing them beneath the others which they find there. Of course, they are to play those which they have not played before.

In matches between fours the east-west players will meet, at the end of the first circuit, with their north-south partners. They should seat themselves at the table with them and note what boards should be played according to the schedule. These boards they may play against their north-south partners, if they wish to, but, evidently, they will not affect the result. Therefore it is better to consider these boards as played that is, to omit playing them and to pass on to the next table, playing there the next boards in order. When the second circuit is completed, it will be found that each team plays and overplays against each other team the same boards; also, that the same set-of-boards has been played by more than two teams.

Unfortunately, there is one exception to the universal application of this system. That is, in matches between multiple teams of four, it will not work with an odd number of boards upon an even number of tables. The only cases where that would be likely to occur are eight or ten tables of three boards each.

The defect is that it makes team 1 play only two boards against team 2, but four boards against team 3. This can be obviated in either of two ways: first, by using a dummy table, as in the Howell system; second, by a device of Walter H. Barney, of putting only two boards on each table except the last, and placing upon that all the remaining boards. This necessitates, however, going round three times.

It is not necessary to place the trays according to the formulas given. They can be placed in their normal order, if preferred. The advantage of the formula is that each team plays against each other every board in consecutive order. The advantage of placing the boards on the tables in their normal order is, that the pairs sitting still play all the boards in regular numerical order, while those moving play them in reverse order, hence it is very easy to detect any mistake.

Mr. Barney has given the system much study, and has done much to perfect it, and is fully convinced of its superiority.

After the first circuit, all north players should exchange places with their south partners.

Appendix H

Olof Hanner semi-barometer

For a semi-barometer you may also consider a three-rounds Mitchell. This is a pair contest with barometer, but where the barometer is updated only every third round. When changing for a new round the pairs move as in Mitchell; the NS-pairs are stationary and the EW-pairs move to the next higher table and from the last table to table 1. This movement is used also when the number of tables is even.

In every period of three rounds you let three board-sets circulate so that all tables get to play all three board-sets. The boards have to be pre-duplicated to a number of a little more than a third of the number of tables. In the first round you start with board-set 1 on tables 1, 4. 7, …, board-set 2 on tables 2, 5, 8, … and board-set 3 on tables 3, 6, 9, … The only exception will be the two last tables for which special rules apply. If the number of tables is a multiple of 3, for instance 12 or 15 tables, also the two last tables should start regularly so that the last three tables will start with board-set 1, 2 and 3 respectively. If, on the other hand, the number of tables is not a multiple of 3, for instance 13 or 14 tables, you should in the first round let the last table but one start with board-set 1 and the last table with board-set 3.

This start and the Mitchell movement tell what boards that has to be played at each table in rounds 2 and 3. In the second round there are at each table two pairs which have played different boards in the first round. Hence there will be only one board-set of the three which neither has played. In the third round, when each pair has played two of the three board-sets, it turns out that there is at each table a board-set neither of the pairs have played.

For regular tables, i.e. all tables except maybe the last two tables, this means that you will play the board-sets in numerical order cyclically, so that board-set 3 is followed by board-set 1. You can simply organize this as in a regular Mitchell: you ask the North players to send the played boards to the next lower table. If also the last tables are regular, when the number of tables is a multiple of 3, this movement of the boards also refer to these tables and you ask North at table 1 to send their played boards to the last table.

But if the two last tables start irregularly you may use small guide-cards for these tables. No pair numbers are needed on these guide-cards, only which boards should be played in the different rounds. You need therefore only two sets of guide-cards for these tables: one for the case that the number of tables is 1 more than a multipel of 3, one for the case that the number of tables is 2 more than such a number. You may find it convenient to have a little board table where the last two tables can place their played boards and where the last three tables may fetch their boards for the next round. You ask North at table 1 to place their played boards on this table. In the first case you should start the board table with one board-set 2 and one board-set 3, in the second case with two copies of board-set 2.

After the 3 first rounds you can start the barometer. You continue in the same way in 3-round periods. With 2 boards per set you may for instance play 24 or 30 boards with 3 boards per set you can get a 27-round contest.

Appendix I

Solutions

Sudoku

La France on July 6 1895. In the *Daily entertainment* section a square appeared under the title *Carré Magique Diabolique* (diabolical magic square) by M. B. Meyniel.

7	8	9	1	2	3	4	5	6
3	1	2	6	4	5	9	7	8
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	5	6	4
1	2	3	4	5	6	7	8	9
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	6	4	5
2	3	1	5	6	4	8	9	7
4	5	6	7	8	9	1	2	3

Ken ken.

36× 1	3	2 2	2÷ 4
3	4	2− 1	2
8+ 4	2	3	12× 1
2	1 1	4	3

Appendix J

Golf problems

Mathematicians frequently are approached by their golf playing friends to come up with schedules to suit their needs. Here we have two examples, one from Wal Wallis and the other from Ross Moore.

24 Players, 6 groups and 7 rounds

One problem posed to Wal Wallis was the following. He was asked for a golf schedule for 24 players, playing in six groups of four, for six rounds, in such a way that no two players are in the same group of four more than once. In fact he was able to produce 7 rounds.

The schedule in detail, with people numbered 1, 2, … , 24, the people who never play together are: 1, 2, 3; 4, 5, 6; 7, 8, 9; 10, 11, 12; 13, 14, 15; 16, 17, 18; 19, 20, 21; and 22, 23, 24. Then the seven golf rounds are:

1	13	20	24	1	4	11	15
2	19	23	15	2	10	14	6
3	22	14	21	3	13	5	12
7	10	16	5	19	22	7	17
8	11	17	6	20	23	8	18
9	12	18	4	21	24	9	16
1	16	23	6	1	7	14	18
2	22	5	18	2	13	17	9
3	4	17	24	3	16	8	15
10	13	19	8	22	4	10	20
11	14	20	9	23	5	11	21
12	15	21	7	24	6	12	19
1	19	5	9	1	10	17	21
2	4	8	21	2	16	20	12
3	7	20	6	3	19	11	18
13	16	22	11	4	7	13	23
14	17	23	12	5	8	14	24
15	18	24	10	6	9	15	22
1	22	8	12				
2	7	11	24				
3	10	23	9				
16	19	4	14				
17	20	5	15				
18	21	6	13				

Happy golfing!

'Couples', 'Captains' and 'Swingers' with 12 Social Golfers

Ross Moore produced this paper in January 2005. In this case the foursomes can be considered to be the four players at each table in an individuals event for bridge. Ross wrote as follows.

Recently on an email news list the problem was posed on how to schedule a 'social' golf tournament for 12 players over 5 days, with golf rounds played in foursomes, such that

(G1) each player plays exactly once each day;

(G2) each player meets (i.e., plays in a foursome with) every other player at least once—this is known as 'full socialisation';

(G3) no player meets any other player more than twice.

In fact it is quite likely (but not yet proved) that these conditions are not consistent, and no such tournament schedule can be devised. Since it is G3 that is least important in the social context, then it can be replaced by the following weaker conditions:

(G4) no player meets any other player more than three times;

(G5) minimize the number of pairs of players who meet thrice.

Before presenting a schedule which meets all the requirements G1, G2, G4 & G5 (see Table 5), some other useful schedules which satisfy subsets of G1–G3 are first derived. These schedules (in Tables 2, 3, 4) have nice properties that might be useful in other kinds of social (or non-social) tournaments, and their structure helps in understanding why the one given in Table 5 has its desirable properties.

Firstly, assess the simple requirements of a 5-round tournament among 12 players, played in foursomes, with property G1 satisfied. Each player plays with three others in each round, so meeting $5 \times 3 = 15$ others. But there are only 11 other players, so it ought to be possible to meet all the other players, but necessarily there will be some repeat meetings. Indeed, since $12 = 4 \times 3$, meaning that there are three foursomes played each day, it is clear that for the 2nd round of play it is not possible to have 4 players all having played none of the others. In each foursome of the 2nd and later rounds, there must be at least two players who were in the same foursome on the 1st round. Furthermore, since 11 does not divide 15, the overall schedule does not form a *balanced* incomplete block-design (BIBD), as not all pairs of players can meet an equal number of times. This means that it is unlikely to find good schedules described already in the academic literature.

Secondly, schedules in which all players meet everyone else (property G2) certainly exist. For example, if the 12 players come as couples (husbands and wives, say), then a complete round-robin amongst the 6 couples will have everyone meeting everyone else. This is what happens in Table 1. To understand this schedule it is sufficient to see how the complete schedule is developed cyclically from just the first round. That is, to get round B from round A, leave player 6 fixed, and permute numbers 1 to 5 according to the cyclic permutation (1 2 3 4 5); that is, add 1 to each of the numbers 1 through 4, with 5 becoming 1. Repeat this operation for each successive round.

The pairs of player numbers {5, 2}, {4, 3} from round A constitute a *difference family* modulo 5. That is, the set of differences $5 - 2 = 3$, $2 - 5 = -3 \equiv 2 \pmod 5$, $4 - 3 = 1$, $3 - 4 = -1 \equiv 2 \pmod 5$ comprises the complete set of non-zero residues ± 1, ± 2 modulo 5. This means that as the schedule is developed cyclically, every pair (i, j) with $i, j \in \{1, 2, 3, 4, 5\}$ and $i - j$ being one of these differences, must occur. Since all possible differences are covered, then all (unordered) pairs (i, j) with $i \neq j$ must occur. Together with the $(6, i)$ pairings, all possible player meetings are covered in the schedule of Table 1.

A:	6 1	5 2	4 3
B:	6 2	1 3	5 4
C:	6 3	2 4	1 5
D:	6 4	3 5	2 1
E:	6 5	4 1	3 2

Table 1: 5-cyclic Round-Robin

A:	6 1 12 7	5 2 11 8	4 3 10 9
B:	6 2 12 8	1 3 7 9	5 4 11 10
C:	6 3 12 9	2 4 8 10	1 5 7 11
D:	6 4 12 10	3 5 9 11	2 1 8 7
E:	6 5 12 11	4 1 10 7	3 2 9 8

Table 2: The ‘couples’ schedule.

To make a 12-player schedule from the 6-player schedule of Table 1, duplicate the round-robin by extending each pairing to become a foursome. Do this using players numbered 7–12, by simply adding 6 to each number in the 6-player round-robin schedule. The result appears in Table 2. Of course this schedule does not satisfy condition G3, nor G4, since every player is in the same foursome as their spouse, in each round; they always play as couples, hence the name attached to the schedule in Table 2.

Next generate a new schedule by changing the first round as follows:

A : 6 1 12 7 5 2 11 8 4 3 10 9 (1)

then developing cyclicly as before. This splits all of the ‘couples’, except (6, 12), from the schedule in Table 2, giving the schedule in Table 3. In this schedule each player, except for 6 and 12, meets each other either once or twice. Since 6 meets 12 in every round, still conditions G3 and G4 are not satisfied. Yet this schedule might be used in practice where players 1–6 form a single team, with players 7–12 forming another team. Players 6 and 12 are the ‘captains’ of their respective teams-of-6. They meet each other every round, along with one other team member. Other players meet all five of their team-mates and the opposing captain exactly once, and four of the other team’s players twice each. This is because the mixed differences in the generating 1st-round foursomes (e.g., for 4 3 11 8 we have $11 - 4 - 1 = 6 \equiv 2 \pmod 5$, $11 - 3 - 1 = 7 \equiv 2 \pmod 5$, $8 - 4 - 1 = 3$, $8 - 3 - 1 = 4$) both cover all non-zero residues modulo 5. Thus pairs $(i, j + 6)$ for $i, j \in \{1, \ldots, 6\}$ with $i \neq j$, occur exactly twice in the full schedule.

A:	6 1 12 7	5 2 10 9	4 3 11 8
B:	6 2 12 8	1 3 11 10	5 4 7 9
C:	6 3 12 9	2 4 7 11	1 5 8 10
D:	6 4 12 10	3 5 8 7	2 1 9 11
E:	6 5 12 11	4 1 9 8	3 2 10 7

Table 3: The ‘captains’ schedule.

A:	6 1 10 9	5 2 12 7	4 3 11 8
B:	6 2 11 10	1 3 12 8	5 4 7 9
C:	6 3 7 11	2 4 12 9	1 5 8 10
D:	6 4 8 7	3 5 12 10	2 1 9 11
E:	6 5 9 8	4 1 12 11	3 2 10 7

Table 4: The ‘swingers’ schedule.

In a similar fashion, generate a new schedule by changing the first round of Table 2 as follows:

A : 6 1 12 7 5 2 11 8 4 3 10 9 (2)

and develop cyclicly. The result is given in Table 4. This schedule has social aspects of both the previous schedules. The pairs (j, $6 + j$), for $j = 1 \ldots 6$, can be regarded as couples, with the teams-of-6 being all the husbands/wives respectively. Now each player meets all of the their team-mate's spouses—twice each, in fact—but does not meet their own spouse. Hence the name 'swingers'. This schedule satisfies conditions G1 and G3, but not G2.

'full socialisation' schedule.

A:	6	1	10	9	5	2	12	7	4	3	11	8
B:	6	2	11	10	1	3	12	8	5	4	7	9
C:	6	3	7	11	2	4	12	9	1	5	8	10
D:	6	4	8	7	3	5	12	10	2	1	9	11
E:	6	5	12	11	4	1	10	7	3	2	9	8

Table 5: To get full socialisation, simply adjust one round from the 'swingers' schedule; e.g., the last round, as done here.

Now it is easy to see how to modify the 'swingers' schedule so as to recover full socialisation, but with G4 and G5, rather than with G3. Simply replace any round of that schedule with the corresponding round from the 'couples' schedule. This is just undoing the effect of the cycling in (2) on that round. Table 5 shows this, for undoing in the final round. For the 10 meetings with players from the other team-of-6, distributed as (0, 2, 2, 2, 2, 2) for 'swingers', becomes redistributed as (1, 1, 1, 2, 2, 3). Thus each player meets one other (from the opposite team-of-6) three times, two of them twice, and everyone else once. These facts can be seen in Table 6, which shows when each player meets each other.

	1	2	3	4	5	6	7	8	9	10	11	12
1		D	B	E	C	A	E	B,C	A,D	A,C,E	D	B
2	D		E	C	A	B	A	E	C,D,E	B	B,D	A,C
3	B	E		A	D	C	C	A,B,E	E	D	A,C	B,D
4	E	C	A		B	D	B,D,E	A,D	B,C	E	A	C
5	C	A	D	B		E	A,B	C	B	C,D,E	E	A,D
6	A	B	C	D	E		C,D	D	A	A,B	B,C,E	E
7	E	A	C	B,D,E	A,B	C,D		D	B	E	C	A
8	B,C	E	A,B,E	A,D	C	D	D		E	C	A	B
9	A,D	C,D,E	E	B,C	B	A	B	E		A	D	C
10	A,C,E	B	D	E	C,D	A,B	E	C	A		B	D
11	D	B,D	A,C	A	E	B,C,E	C	A	D	B		E
12	B	A,C	B,D	C	A,D,E	E	A	B	C	D	E	

Table 6: Recap matrix: expanded form of the 'full socialisation' schedule, indicating the round(s) in which players meet each other.

It is recommended to undo the cycling in the final round, as then there will have been no triple meetings within the first four rounds of the schedule — all triples in Table 6 include 'E'. Indeed, each player will have had 6 single meetings and only 3 double meetings within the first 4 rounds[1]. Furthermore, the tournament organiser can choose to play the 5th round as either a 'swingers' to avoid any triple meetings, or as given in Table 5 for full socialisation. This decision can be delayed until the last possible moment.

20-player 5 × 5 affine plane

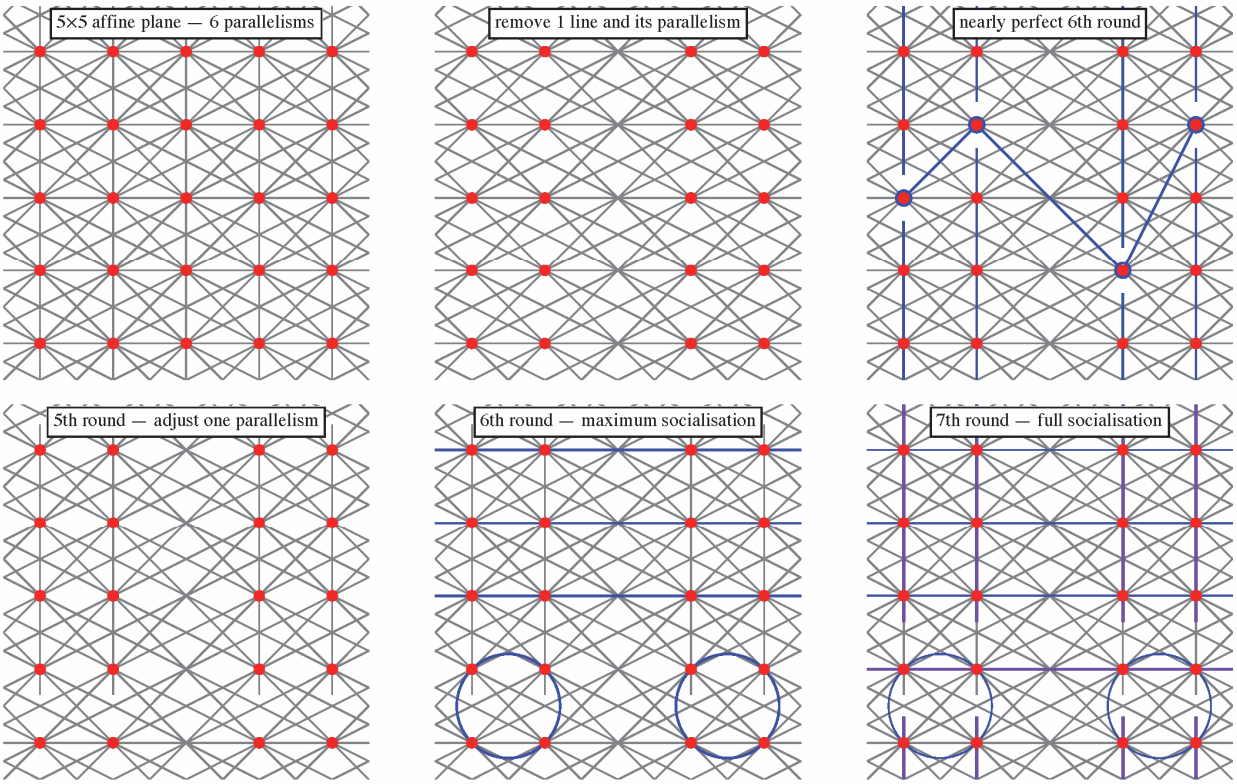
The following diagram shows some line graphics that I used to explain the 20-player social golf solution. It includes a 7th round, ensuring that everyone has met everyone at least once and no more than twice.

Then the scheduled rounds are:

top-middle image (4 rounds)	A,B,C,D say
bottom-left	E
bottom-middle	F
bottom-right	G

Thus everyone plays exactly 2 others twice. However, some pairs of 4-somes have 3 players in common. To make those repeat meetings as far separated as possible, play the rounds as:

E, A, B, C, D, F, G



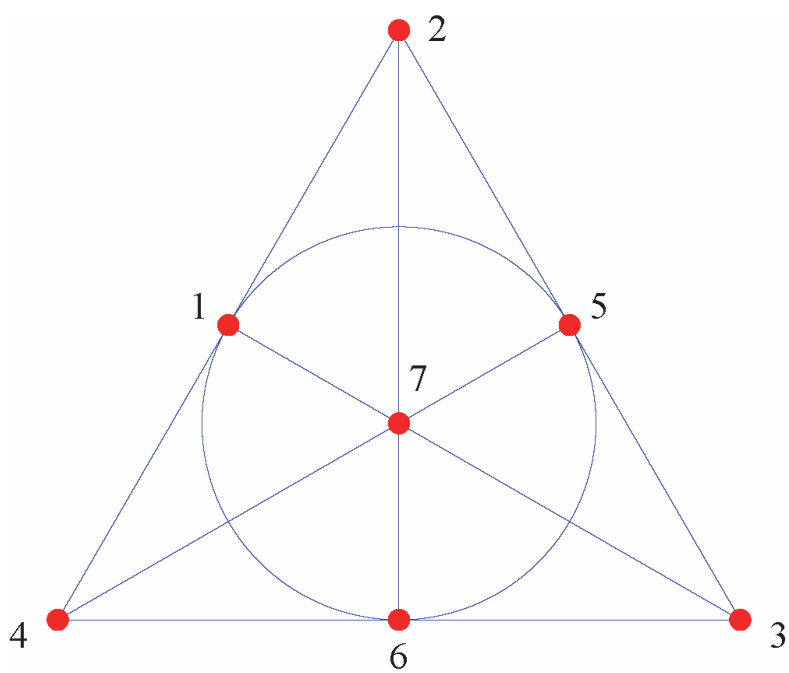
8-player Fano plane

The 8-player individual defined by Safford was the first schedules ever produced. It was shown as a Latin square in section 3.2 (see page 38). That square is reproduced here for convenience.

1	4	2	7	3	5	6
2	5	3	1	4	6	7
3	6	4	2	5	7	1
4	7	5	3	6	1	2
5	1	6	4	7	2	3
6	2	7	5	1	3	4
7	3	1	6	2	4	5

This is also usable in golf ‘fours’. The eighth player is not shown here, and is the ‘stationary’ player. Taking the first three columns, together with the eighth player, we have the table one in the whist schedule. Today we can define this with a Fano plane. This is named after the work of the Italian mathematician Gino Fano (1871–1952).

Each of the 7 groups of numbers in the first three columns are found by one of the lines joining three numbers in this diagram.



Appendix K

Kirkman's 'Schoolgirl Problem'

Kirkman's original solution to his 'Schoolgirl Problem' used a Room square, although he didn't define those objects. He started with a Fano plane — a (7, 7, 3, 3, 1)-BIBD. For convenience we'll use symbols A,B,C,D,E, F,G; everybody's favorite example is the cyclic design

ABD	*BCE*	*CDF*	*DEG*	*EFA*	*FGB*	*GAC*

although any example will do. He constructed an array with these elements in one column and the remaining elements in columns of their own:

ABD	C	E	F	G
BCE	D	F	G	A
CDF	E	G	A	B
DEG	F	A	B	C
EFA	G	B	C	D
FGB	A	C	D	E
GAC	B	D	E	F

and glued on a Room square:

ABD	C81	E36	F27	G45
BCE	D82	F47	G31	A56
CDF	E83	G51	A42	B67
DEG	F84	A62	B53	C71
EFA	G85	B73	C64	D12
FGB	A86	C14	D75	E23
GAC	B87	D25	E16	F34

Kirkman didn't use a nice cyclic example, but another Room square. In that case, you need to exercise a little care to make sure the balance and resolvability properties work out.

Appendix L

Ross Moore's 1992 paper

Too many arrow-switches spoil the balance.

by Ross Moore, Sydney, Australia

The currently recommended systems of "arrow-switching" to improve the fairness of unbalanced ¾-Howell, Scrambled Mitchell — and similar movements in which pairs do not play every other — are based upon an inappropriate criterion, so are invalid. A better criterion leads to fairer movements, which in many cases are also easier to implement. This criterion is applicable to all Duplicate Bridge movements, is closely related to matchpoint scoring, and for complete Howell movements is equivalent to the usual comparison criterion. Failure to meet a pair head-to-head is compensated by having more comparisons in the same direction. A head-to-head encounter has a greater effect on the score, so fewer comparisons are required for overall fairness. As a "rule-of-thumb", arrow-switching in a single round is best for Scrambled Mitchell movements of up to 11 rounds, while two arrow-switches suffice for up to 18 rounds.

In considering the fairness of Howell movements the concept of comparisons has been well-recognized since the work of Ach & Kennedy [1, 7] in the 1930s. One counts the number of times that each partnership plays boards in the same direction as each other partnership, thereby allowing a direct comparison of the scores obtained. The movement is said to be completely balanced when these are the same for every pair of partnerships. As it is not always possible to achieve this ideal, the term balance is used as a qualitative measure of how closely it is approached. A movement being 'more balanced' than another is taken to mean that the movement is a fairer test of ability — it is less subject to correlations between the overall performances of different pairs due to their positions within the movement.

The validity of this use of comparisons as a measure of 'fairness' is due to the fact that with Howell movements two other important criteria are met; namely that …

- each partnership plays against each other partnership, for an equal number of boards; and
- each partnership plays every board within the movement, from one side or other.

Frequently movements are used in which these criteria are not met; for example ¾-Howell and Scrambled Mitchell or Bofors-Mitchell movements. Current directors' manuals[2, 5, 8] and the Encyclopaedia of Bridge[3, 4] recognise that these types of movement do not automatically have good balance, but nevertheless recommend a schedule for arrow-switching board-sets in certain rounds in an attempt to improve the comparisons. That these schedules achieve their aims is not in question. It is the notion that balance of the number of comparisons provides an appropriate measure of fairness which is invalid.

For movements in which not every partnership plays each other the measure of comparison must be altered as follows. For two partnerships A and B define the match-point comparison, shortened to *MPcomp*, by the formula

$$\begin{aligned} \mathit{MPcomp}(A, B) = {} & {}^{\#}\text{(board-sets where } A, B \text{ sit in the same direction)} \\ & - {}^{\#}\text{(board-sets where } A, B \text{ sit in opposite directions)} \\ & + (N - 1) \times {}^{\#}\text{(board-sets on which } A, B \text{ sit as opponents)} \end{aligned}$$

where N is the number of times that each board-set is played, i.e the number of scores on the traveling score-sheets from which the matchpointing is calculated. For a fair movement, it is the numbers *MPcomp(A, B)* which ideally should be the same for all pairs of partnerships (A, B).

Notice that, provided each partnership plays every board, each board-set contributes to one or other of the three terms in *MPcomp(A, B)*. Furthermore, if every partnerships plays equally often against each other partnership then the 3rd term contributes equally to every possible *MPcomp(A, B)*. When both these criteria are satisfied, as in a complete Howell movement, then having good balance among the matchpoint comparisons is equivalent to having good balance among the usual comparisons.

To understand the above formula, consider how matchpointing is calculated. On each board a partnership scores a matchpoint for each other partnership, sitting the same direction, who they outscore. Thus if A v B at one table and on the same board C v D in corresponding directions at another table, then whichever of A and C scores higher wins the matchpoint. Since B and D score the negative of A and C respectively, then D wins a matchpoint whenever A does; similarly B and C win or lose together. For the purpose of matchpointing this board A and D are effectively team-mates, with B and C teamed-up as opponents. If the board is played a total of N times, there are $(N - 1)$ such comparisons of scores involving A v B at one table with encounters C' v D' at other tables. Thus B opposes A in the comparisons for $(N - 1)$ matchpoints, whereas each partnership sitting in the same direction as A is an opponent for just 1 matchpoint and each other partnership sitting the opposite way is effectively a team-mate.

The formula for *MPcomp(A, B)* simply counts the difference in the numbers of times A, B are either on opposing sides or are team-mates for matchpointing purposes — hence the name matchpoint comparisons. It is a direct measure of the extent to which two different partnerships in the movement are in competition with each other. It is clear that good balance among matchpoint comparisons is the correct criterion for fairness.

Intuitively one expects to gain a greater advantage by avoiding playing against a strong partnership (for whatever reason) than by simply having fewer comparisons with them. This is manifest in the above formula. The means to counter this, thereby achieving fairness, is to have more comparisons with those opponents which are not met head-to-head, and fewer with those that are.

Standard Formulae.

For the rest of this article the term 'comparison' will refer to matchpoint comparison and the 'balance' of a movement will be with respect to the matchpoint comparisons. The term 'partnership' will be replaced by the more commonly used 'pair'; though *(A,B)* may still refer to a pair of partnerships.

The ideal value for *MPcomp(A, B)* can be calculated either as the average over all comparisons for a fixed pair A, or by using global properties of the movement:

$$\textbf{\textit{Average comparison}} = \sum_{B \neq A} \textbf{\textit{MPcomp}}(A,B) \,/\, (\#(\textbf{\textit{pairs in the movement}}) - 1)$$

$$= \frac{\#(\textbf{\textit{boardsets}}) \times (N - 1)}{\#(\textbf{pairs in the movement}) - 1}$$

assuming that every pair plays every board exactly once. As previously, N denotes the number of times each board is played, which is usually once at each table.

Summing all comparisons with a fixed pair *A*, and for all pairs of partnerships, give useful matchpoint totals:

$$\sum_{B \neq A} \textbf{\textit{MPcomp}}(A,B) = \textbf{\textit{maximum total matchpoints winnable by a single pair}}$$

$$\tfrac{1}{2} \sum_{\substack{A,B \\ B \neq A}} \textbf{\textit{MPcomp}}(A,B) = \textbf{\textit{total matchpoints available to all pairs in the movement}}$$

Achieving the ideal balance is only possible in a small number of Howell movements. For other movements the following provides a quantitative measure of the lack of balance.

$$\textbf{\textit{Gross imbalance}} = \tfrac{1}{2} \sum_{\substack{A,B \\ B \neq A}} [\textbf{\textit{MPcomp}}(A, B) - \textbf{\textit{average comparison}}]^2$$

$$\textbf{Average imbalance} = \sqrt{(\textbf{\textit{gross imbalance}}) \,/\, (\tfrac{1}{2}\,(^{\#}\textbf{\textit{pairs}}) \times (^{\#}\textbf{\textit{pairs}} - 1))}$$

This (average) imbalance mimics the definition of statistical standard deviation of the *MPcomp(A, B)*s for all possible pairs of partnerships. Lower values of the imbalance correspond to fairer movements. An imbalance of 1 or less means an extremely fair movement indeed.

Mitchell Movements.

The standard Mitchell movement, for an odd number *N* of tables, has *N* board-sets each played *N* times - once at each table. There are *2N* pairs, half of which always sit *NS*, the others sitting *EW*. The *EW* pairs move to different tables each round, cycling in one direction while the board-sets cycle in the opposite direction, with *NS* pairs remaining stationary. Thus each *NS* pair meets each *EW* pair exactly once, and everybody plays every board. The comparisons are *MPcomp(A, B) = N* for pairs sitting the same direction, and *MPcomp(A, B) = 0* when *A, B* sit in opposite directions. This latter is true since the encounter on a given round contributes $N - 1$ to the comparison with the opposing pair; but this pair is effectively a team-mate on each of the other $N - 1$ rounds.

It is usual to treat the pairs sitting in opposite directions independently, as if in separate movements. The comparisons within each sub-movement are then perfectly balanced; however there must be two winners – one for each direction.

Table 1. Imbalance for Scrambled Mitchell movements — odd tables.

no. tables	Ave. comp.	Mitchell no switch	McKinnon Farrington Rosler	Scrambled Mitchell (number of switches) 1	2	3	4
5	2.222	2.4845	3.3921	1.6178	3.3921		
7	3.231	3.4896	4.4229	1.0491	3.3086		
				4.6929			
				3.3086			
9	4.235	4.4922	5.6102	1.1647	2.8186	switches in	
			5.6102		3.1348	rounds	
			5.6102			1, 2, 5	
11	5.238	5.4938	6.4506	1.8748	2.2447		
			6.4506				
			5.7645				
13	6.240	6.4948	7.6277	2.7609	1.7727	3.6582	switches in
			7.6277				rounds
			7.4580				1, 2, 6, 9
15	7.241	7.4955	8.4635	3.7014	1.6327	3.1366	
			7.0008		1.9415		
17	8.242	8.4961	9.4678	4.6647	1.9545	2.5941	
19	9.243	9.4965	10.5534	5.6397	2.5932	2.1360	
21	10.244	10.4969	10.2439	6.6214	3.3842	1.9099	
					3.4976		
23	11.244	11.4972	13.0336	7.6075	4.2487	2.0457	
25	12.245	12.4974	13.7893	8.5965	5.1529	2.5114	2.4456
							(2.3082)
27	13.245	13.4976	14.6660	9.5875	6.0809	3.1796	2.1623
						6.1304	(2.0179)
29	14.246	14.4978	15.8571	10.5802	7.0245	3.9571	2.1706
							(2.0372)
31	15.246	15.4979	16.6720	11.5739	7.9788	4.7964	2.4936
							(2.3861)
33	16.246	16.4980		12.5686	8.9409	5.6732	3.0485
						8.9684	(2.9666)

If a single winner is required this movement is badly unbalanced. The imbalance is close to ½N but just a little less, with exact value given by:

$$\textbf{\textit{Imbalance for N-table Mitchell}} = N / (2N - 1)\sqrt{N(N-1)} \approx (N / 2) - (1 / 16\,N) - (1 / 16\,N^2).$$

For an even number N of tables the movement is structured a little differently. To avoid the moving *EW* pairs encountering the same board-sets after ½N rounds, there is an extra 'relay' table located between tables ½N and ½N + 1 to which boards move, but *EW* pairs do not. Also, tables 1 and N share their board-set each round. Thus both the board-sets and the moving pairs cycle among N tables, but not the same N tables; each round only $N - 1$ different board-sets are used, but one is used twice. Overall every pair plays every board exactly once, with each *NS* pair meeting each *EW* pair exactly once. The formula for the imbalance is the same as above, only now with N even.

Table 2. Imbalance for Scrambled Relay-Mitchell movements even tables

no. tables	av. comp.	Mitchell no switch	McKinnon Farrington Rosler	Scrambled Mitchells (number of switches) 1	2	3	4
4	1.714	1.9795		1.4846			
				2.1189			
6	2.727	2.2065	3.8081	1.2856	switches in		
			3.8712	1.5428	rounds 1, ½N		
			3.8398		unless shown		
8	3.733	2.9147	4.9728	1.3888			
			5.2341	1.2893			switches in
			4.6399				stated rounds.
10	4.737	3.6224	6.0319	1.9261	2.2674		
			6.0458	1.6684	2.4114		
			4.3630		2.6286		
12	5.739	4.3298	7.0560	2.6900	1.9386	switches in	
			7.0478	2.4176	1.9829	rounds 1,2,5	
			6.9775		2.0119	unless shown	
14	6.741	5.0371	8.1183	3.5518	1.8291 {1,5}		
			7.6402	3.2985	1.7822		
			6.9100		1.8175		
16	7.742	5.7444	9.1192	4.4611	2.1170	2.7588	
				4.2310	1.9670	2.9889 {1,3,9}	
					1.8832 {1,9}		
18	8.743	6.4516	10.1177	5.3967	2.6656 {1,6}	2.3409	
				5.1880	2.4882	2.4882 {1,3,10}	
					2.3463 {1,10}		
20	9.744	7.1588	11.0818	6.3485	3.3796	2.1711	
				6.1582	3.0443 {1,11}	2.1378 {1,3,11}	
22	10.744	7.8660		7.3110	4.1804 {1,7}	2.2796 {1,3,12}	
				7.1365	3.8585 {1,12}	2.0778 {1,2,6}	
24	10.244	8.5731		8.2809	5.0422	2.6775 {1,2,6}	2.7916 {1,2,8,21}
				8.1200	4.7332 {1,13}	2.3653 {1,3,13}	2.9449 {1,2,6,19}
26	12.745	9.2803		9.2562	5.9312 {1,8}	3.2724 {1,2,6}	2.5252 {1,2,6,9}
				9.1070	5.6424 {1,14}	2.9177 {1,3,14}	2.4964 {1,2,5,18}
28	13.745	9.9874		10.2355		3.9880 {1,2,6}	2.4606 {1,2,5,16}
				10.0966		3.6264 {1,3,15}	2.3921 {1,2,6,19
30	14.746	10.6946		11.2180		4.7733 {1,3,10}	2.8132 {1,2,6,9}
				11.0879		4.4233 {1,3,16}	2.5360 {1,2,6,9}
32	15.746	11.4017		12.2029		5.6131 {1,2,6}	3.2267 {1,2,5,12}
				12.0807		5.2802 {1,5,18}	2.9058 {1,2,6,9}

Scrambled Mitchells.

To obtain a single winner, but retain the simplicity of the mechanics of the Mitchell movement, a small variation is introduced. On a particular round all board-sets are played arrow-switched; that is, the *NS* pairs and *EW* pairs swap directions for that round, reverting to their normal positions for subsequent rounds. Since different tables are playing different board-sets, the effect is to alter the comparisons quite dramatically. This arrow-switching may happen on several rounds. The problem is to determine which set of arrow-switchings give the most balanced movements.

Recommended schedules of arrow-switchings — e.g. the so-called Bofors chart — appear in the Encyclopaedia of Bridge[4, p.386], and are reproduced in directors' manuals [2, p.l3],[5, p.35],[8, p.30]. However these are based on consideration only of the Howell-type comparisons; that is, trying to get all pairs sitting the same direction on equal numbers of board-sets. Since not every pair meets every other pair this cannot be valid. For balanced matchpoint comparisons, pairs who do not meet each other should be sitting the same direction on more board-sets. Those that do meet get a large comparison from this encounter, so to compensate they should be sitting in opposite directions more frequently.

The manuals recommend frequent arrow-switching. This is wrong; fewer arrow-switches produce movements with better balance. Table 1 lists the imbalances obtained from different movements. It can be seen that the "recommended" movements have very poor balance. Indeed in almost all cases a lower imbalance is obtained using the straight Mitchell with no arrow-switching at all; yet no serious competition player would accept that as a valid means of comparison.

Much lower imbalance figures are obtained by arrow-switching in few rounds. When only 1 round is arrow-switched it matters not which round this is. For 2 arrow-switches again it does not matter when N is odd, with the following exception. If N is a multiple of 3 then a higher imbalance is produced if the difference in the numbers of the switching rounds is $^1/_3 N$ or $^2/_3 N$ than if not.

These results are understood by considering the effect that arrow-switching a single board has on the matchpoint comparisons. If *A v B* is arrow-switched to become *B v A* then *MPcomp(A, B)* is unchanged but *MPcomp (A, C)* and *MPcomp (B, D)* are altered by ±2 for all other pairs *C* and *D* according to which way they are sitting when they play the arrow-switched board.

When a single complete round is arrow-switched each (stationary) *NS* pair is switching a different boardset, so that *MPcomp (NS, NS′)* decreases by 4; namely 2 for the set switched by pair *NS* and another 2 for the set switched by *NS′*. Similarly each *MPcomp(EW, EW′)* decreases by 4 to $N - 4$. On the other hand each *MPcomp (NS, EW)* increases by 4, except for the pairs playing each other on the arrow-switch round for whom the comparison remains unchanged at 0. All comparisons are known, so the imbalance can be calculated using the formula given below. Furthermore the value of the imbalance is unaffected by which round it is that is arrow-switched, hence the single entry in the 1-switch column of Table 1.

When a second round is arrow-switched the comparisons between stationary pairs decreases further. Now *MPcomp(NS, NS′)* becomes $N - 8$ unless *NS* and *NS′* switch on the same board-set, in which case it becomes $N - 4$. Similarly *MPcomp(NS, EW)* increases to 8 unless *NS* and *EW* both arrow-switch on the same board-set in which case it becomes 4. Normally there are 4 different *EW*-pairs having arrow-switches on the same two board-sets as a given *NS*-pair. However when N is a multiple of 3 it is possible for one *EW*-pair to arrow-switch on both board-sets; this requires the arrow-switch rounds to be separated by $^1/_3 N$ or $^2/_3 N$. Apart from this special case the average imbalance is independent of which two rounds are arrow-switched. Table 1 lists this value, as well as the extra value possible when N is a multiple of 3.

In general, for k arrow-switch rounds *MPcomp(NS, NS′)* and *MPcomp (EW, EW′)* may take values $N - 4k$ and $N - 4k + 4$, while *MPcomp(NS, EW)* may take values $4k$ and $4k - 4$. For given N one chooses k so that these numbers are as close together as possible.

This occurs when $4k \approx N - 4k + 4$, giving as a rule-of-thumb …

(number of arrow-switches) ≈ (N+4) / 8
producing movements having imbalance ≈ 2.0 or less.

A precise formulae for the imbalance with k arrow-switch rounds is as follows:

$$\textit{average} = N\,(N-1)\,/\,(2\,N-1)$$

$$\textbf{gross imbalance} = N((N-(k^2-k)-1)\,(N-4k-\text{average})^2 + (k^2-k)\,(N-4k+4-\textit{average}) + (N-k^2)\,(4k-\textit{average})^2 + k^2\,(4k-4-\textit{average})^2)$$

$$\textit{average imbalance} = \left(\frac{\textit{gross imbalance}}{N(2N-1)}\right)^{1/2}$$

$$= \sqrt{(16k(1-3k+4k^2) - 8k\,(4-7k+16k^2)N + 8k(1+10k)N^2 - (1+16k)\,N^3 + N^4)}\,/\,(2N-1)$$

The above formula assumes that it is possible to choose the arrow-switch rounds so that k^2 different *EW*-pairs take part in the arrow-switches on the k different board-sets on which any given *NS*-pair arrow-switches. It also requires that $k^2 - k$ different *NS*-pairs each have a single arrow-switch among these k board-sets. These conditions also impose restrictions on just which sets of rounds may be switched for best balance. As already seen there is no restriction at all when $k = 1$, while for $k = 2$ there is a restriction only when N is a multiple of 3. When $k = 3$ there are many triples of rounds which give the listed imbalance; in particular arrow-switching in rounds {1,2,5} works for all odd $N \geq 13$. For higher k it becomes increasingly harder to satisfy the conditions for N in the range where k is optimal according to the rule-of-thumb. The parenthesized imbalance values given in Table 1 for 4 arrow-switches should be treated as ideal minimum values only; actual movements which achieve these do not exist. The un-parenthesized values are the best possible by switching complete rounds. It may well be possible to improve on these by arrow-switching specific tables in specific rounds.

Choice of arrow-switch rounds.

Cyclicly permuting the arrow-switch rounds gives a new set of switches which result in the same imbalance. For example switching in rounds {2,3,6} or {5,6,9} give the same imbalance as using {1,2,5}. This cyclic symmetry is also applicable to the relay scrambled Mitchell and ¾-Howell movements.

For Mitchells with odd number of tables a further way of obtaining sets of switching rounds giving the same imbalance is to reverse the order of the differences in round numbers, taken pairwise. For example, {1,4,5} is derived in this way from {1,2,5}. More generally, multiply the differences by a fixed amount provided this amount has no prime factor in common with the number N of tables. Thus {1,4,13} = 3 × {1,2,5} − 2 × {1,1,1} has the same imbalance as {1,2,5} in most movements of 13+ tables, but not in a 15-table movement since 3 divides 15. Of course if the numbers get too high, subtract multiples of N. Thus {1,5,17} = 4 × {1,2,5} − 3 × {1,1,1} has the same imbalance as {1,2,5}; but for $N = 13$ this is just {1,5,4}, equivalently {1,4,5}. Reversing the order is the same as having a multiplying factor of $N - 1$.

Relay Scrambled Mitchells.

When the number of tables N is even, the same general analysis applies as for N odd. However the details are a more complicated, due to the presence of a bye-stand (relay table) and board-sharing, making for a less symmetrical movement. To arrow-switch a complete round in a relay-Mitchell produces a single arrow-switch on $N - 2$ boardsets: no arrow-switch on the set at the relay table, and two arrow-switches on the set being shared between tables 1 and N.

A useful suggestion, due to Olof Hanner[6] is to not switch at table 1, thus reducing to a single arrow-

switch on that boardset. Although intended to improve just the original type of balance, this also reduces the imbalance used here. The figures given in Table 2 have been calculated assuming no switch at table 1, except in the Farrington & Rosier published movements.

The missing arrow-switch due to the relay table can be recovered by a "delayed" switch only at table 1 occurring ½*N* rounds earlier or later. The effect of this is shown by the second entry in the 1-switch column of Table 2, for each value of *N*. For the lowest values there is no gain, but this changes as *N* increases. Similar behaviour is observed with two arrow-switches — the upper and lower of each group of three imbalance values refer to having respectively none and two delayed switches, the middle entry is for having only 1 delayed switch. Normally the best imbalance with a single delayed switch lies between the best possible with zero and two, however for 14 tables a single delayed switch is best. Where only two entries appear these are the best imbalances for zero and two delayed switches respectively.

For a single arrow-switch round without the delayed switch there is both a stationary pair and a moving pair who do not arrow-switch. With the delayed switch the price for having all stationary pairs switching once, thereby having equal comparisons amongst themselves, is to have one moving pair switching twice while another still has no switches. While it is relatively easy to write down a formula for the imbalance in these two cases, the complexity of such formulae increases drastically with the introduction of more arrow-switch rounds — with or without delayed switches.

The two values in Table 2 for 3 and 4 arrow-switches give imbalances respectively without and with delayed switches. These are the best possible for switching complete rounds; which rounds are as indicated. Again lower imbalances may be achievable by arrow-switching specific tables in specific rounds.

Table 3. Imbalance for ¾-Howell movements.

number of tables	av. comp.	Groner Farrington	switches 0	1	2	round 1 (ignoring switches)
6 tables	4.091		2.3142	2.4167	2.6096	12 v l 11 v 4 10 v 8 2 v 7
9 rounds		2.6245			{1,5}	6 v 9 5 v 3
7 tables	5.077	3.9620	1.5845	1.9792	2.3823	14 v l 6 v 4 9 v l0 3 v 7
11 rounds					{1,4}	12 v 2 13 v 11 5 v 8
8 tables	5.133	4.1129	2.5785	2.3343	2.6297	16 v l 15 v 4 13 v 11 14 v l0
11 rounds					{1,2}	9 v 8 7 v 5 12 v 3 6 v 2
9 tables	5.176	4.0634	4.1651	3.0403	3.2239	18 v l 17 v 5 16 v 11 2 v l0
11 rounds					{1,3}	15 v 9 8 v 7 14 v 6 13 v 4 12 v 3
10 tables	5.211	3.9356	4.9373	3.0776	2.8943	18 v l 17 v 5 16 v 11 19 v l0
11 rounds				2.8797	{1,2}	15 v 9 8 v 7 14 v 6 13 v 4
					2.8650	12 v 3 20 v 2
8 tables	6.067	5.2341	1.9821	2.0806	2.2647	16 v l 15 v 4 11 v l3 14 v 7
13 rounds	3.2958				{1,6}	10 v 9 12 v 5 3 v 8 6 v 2
9 tables	6.118	5.4772	2.6641	2.3402	2.3844	18 v l 17 v l2 8 v 5 10 v 2
13 rounds	5.3075				{1,5}	9 v l3 16 v 7 15 v 6 11 v 4 14 v 3
10 tables	6.158	5.6982	4.1506	3.2519	3.2519	20 v l 19 v 6 18 v 8 17 v l3
13 rounds		5.9158			{1,5}	3 v 7 16 v l215 v 25 v 1114v10 9v4
11 tables	6.190	5.8877	5.2569	3.6796	3.4714	7 v 8 22 v l 21 v 320 v 5
13 rounds		6.1241			{1,2}	19 v 2 18 v l0 17 v l2 16 v 4
						15 v 11 14 v l3 6 v 9
12 tables	6.217	7.1685	5.9487	3.7178	3.1062	8 v 7 22 v l 21 v 3 20 v 5
13 rounds		7.4850		3.5585	{1,3}	19 v 2 18 v l0 17 v l2 16 v 4
					3.0686	15 v 11 14 v l3 23 v 6 24 v 9

There are many possible ¾-Howell movements catering for different numbers of pairs and board-sets. Only a sample of the more commonly-used ones appear here, in Table 3 — a forthcoming paper will deal with other combinations of numbers of pairs and rounds. The movements for 13 board-sets were originally devised by Sam Gold; minor variations, supposedly to improve the balance, appear in directors' manuals

e.g. [5,p.166-174], [2, pp.48-52], [8, pp.69-72] and [4, p.413 & p.517]. As the entries in Table 3 show, these attempts come nowhere near the best possible, being based upon the wrong criterion. Again only 1 or 2 arrow-switch rounds suffice to give movements which have much lower unbalance. Indeed for those movements having only three stationary pairs it is best to not arrow-switch at all.

When the number of tables is one less than the number of rounds, board-sharing is required. The second entry for a single arrow-switching round refers to having a delayed switch at the shared table. Similarly the second entry for two arrow-switches refers to having a single delayed switch, corresponding to the first arrow-switch round — two delayed switches gives a worse imbalance.

In the right-hand column is listed the starting positions of all pairs in the movements. Pairs with high numbers — greater than the number of rounds — remain stationary; others move by following the pair with next lower number. Thus at any table the pair numbers at the moving position(s) cycle, increasing by one with each new round. Careful comparison will reveal that some of the non-stationary tables are arrow-switched with respect to what occurs in other published movements. This is intentional, as it can have a large effect on the balance. The movements here have been selected for lowest imbalance before consideration of arrow-switching at the stationary tables; investigation of the best rounds in which to arrow-switch, then gives even better balance.

The imbalance values for these movements are relatively high, compared to what could be achieved with Scrambled Mitchells — though significantly better than movements currently in use. This seems to be an inherent property of ¾-Howells, especially those for which there are very few encounters among the moving pairs. The strategy of switching complete rounds, though convenient to implement, may not be best for considerations of balance. Also, not all the possible ¾-Howells have yet been considered. It is possible that a more exhaustive analysis will uncover movements having even better balance than those presented here.

Acknowledgements.

Thanks are due to bridge director Tom Goodyer, who brought this problem to my attention by noticing the poor balance in the published ¾-Howell movements. He observed that in some there are pairs that are never compared, so could conceivably both score 100. Thanks also to lan McKinnon for replacing a defective copy of his book, and allowing access to his correspondence with Hanner[6]. The various movements studied here were modeled on a computer using the programming language Mathematica®[9]. Combinations of arrow-switches could then be easily tested, allowing the imbalance values to be calculated for Tables 1-3.

References.

[1] J. L. Ach. Testing pair schedules. The Bridge World, pages 12-15, January 1935.
[2] F. Farrington. Duplicate Bridge Movements. English Bridge Union, Oxford, England, 1980(7). revised & expanded from the original.
[3] H. Francis & D. Hayward, editor. The Official Encyclopedia of Bridge. American Contract Bridge League, New York, 4th edition, 1984. ISBN 0-517-55373-0.
[4] R. Frey, A. Truscott & A. Kearse, editor. The Official Encyclopedia of Bridge. Crown Publishers Inc., New York, 3rd edition, 1976. ISBN 0-517-52724-3.
[5] A. Groner. Duplicate Bridge Direction, A Complete Handbook. Barclay Bridge Supplies, Inc., Port Chester, NY 10573, 10th edition, 1987. Library of Congress Catalog Card No. 67-29818.
[6] 0. Hanner. Goteborg, Sweden, 1984-1986. correspondence with lan McKinnon.
[7] C. T. Kennedy. Why the best man loses. The Bridge World, pages 5-11, December 1934.
[8] I. McKinnon. Bridge Directing Complete. Computer Accounting Services Pty. Ltd., Sydney, Australia, 1979. ISBN 0-9595931-0-1.
[9] S. Wolfram. Mathematica: A System for Doing Mathematics by Computer. Addison-Wesley Publ. Co., New York, 1988. ISBN 0-201-19334-5.

Appendix M

Why the best man loses

By Charles T. Kennedy
The Bridge World, December 1934, pages 5 to 11.
Copyright 1934 by The Bridge World, Inc.

If you were an owner of real estate, would you be interested in proof that most of the titles to property in your neighbourhood were defective? If so, perhaps, as a patron of Bridge tournaments, you will be interested in proof that most of the titles to Bridge tournaments are defective.

Duplicate is designed to minimize the element of luck and to make the game of bridge a true test of skill. Yes in most tournaments the choice of movements is such as to make the winners of the tournament almost as a matter of luck as if the director had drawn the names out of a hat.

Suppose you and your partner arrive at a Mitchell tournament just as play is about to begin. Suppose the tournament director gives you your choice of seats. Suppose you find the ten strongest pairs of your acquaintance sitting North and South while the East and West seats are occupied by ten of the poorest pairs you have ever met. If you really want to win that tournament, which way will you sit, North and South, or East and West?

Most players protest against the use of a Mitchell movement for the final rounds of a championship event, particularly if the division between the two sides is left largely to chance. But by the use of complicated cards and direction slips, these same players can be hood-winked into playing in movements which are little better than un-seeded Mitchell movements, and which are so unsound as to be practically worthless as a test of the contestants' ability.

Consider a team-of-four match. If you play North-South at one table, your team-mates play East-West at the other table, and the better they play, the better your score. Furthermore, the pair that plays the same way with you at that other table are your opponents. The same essential relation exists in any movement. On any given board, your competitors are the pair against whom you actually play that board, together with all the other pairs who play the same way as you do when that board is played at the other tables. The pairs that play the opposite way from you at the other tables are actually, as far as the board goes, your team-mates.

This should be evident to every player who has ever seen a tournament recapitulation sheet. Suppose you play North-South on a given board against an equally strong East-West pair. Suppose that by very careful play against the best defence, you make three spades. Suppose that every time that board is played at the other tables, the North-South pairs are strong players who are likely to pick up a trick or two, while their East-West opponents are weak players who are equally likely to drop a trick or two. Under these conditions, wouldn't you expect the other North-South pairs to make more than three spades on that hand? You would probably get a bottom on the board, without having done anything to deserve it. If this happened on only one hand, you would dismiss it as an isolated example of bad luck, but if you found that you were consistently playing the same way as the strong pairs and the opposite way from the weak pairs, wouldn't you begin to wonder if it were a system? .

Every time a board is played at another table, a team-of-four match is completed which was begun at your table, in which your team-mates are the pair playing the opposite way of the board and your opponents the pair playing the same way. Your match point score is affected just as much by what occurs at these other tables as by what you do at your table. Would you like to play all these team-of-four matches with a weak pair for team-mates, particularly when these weak sisters were playing against a strong pair of opponents?

The arrangement of the other pairs at the other tables is clearly a matter of vital importance; obviously it should be such that you will have the good pairs playing the same way with you the same number of times

as the poor pairs.

Every pair must play the same way of the board the same number of times with every other pair. In no other way can a fair competition be assured.

There are at present two sets of Howell movements on the market the original – the original Howell movements invented some thirty years ago by the late Professor E. C. Howell, and the 'revised and perfected' McKenney-Baldwin Howell movement which are in official use by the American Bridge League.

Of the original Howell movements, more than half are unsound and many of the others are open to improvement. Of the McKenney-Baldwin movements, all that the writer has analyzed show such a complete ignorance of the fundamentals of tournament competition as to totally unfit them for use.

Let us consider the twelve-pair McKenney-Baldwin movement. Let the letters A to L, inclusive, designate the twelve pairs. Assume that these twelve pairs are of varying degrees of ability, A being the best and L the poorest. Let pair G be an average pair, and assume that the other pairs have an average playing ability expressed in points per hand, in accordance with the following table, plus indicating points better than average and minus points below average, thus:

Pair	Average playing ability
A	plus 120
B	plus 100
C	plus 80
D	plus 60
E	plus 40
F	plus 20
G	0
H	minus 20
I	minus 40
J	minus 60
K	minus 80
L	minus 100

Now, assume that in the play of the match each pair plays each board strictly in accordance with its average ability as given in this table, so that when A plays B, A wins by 20 points, while when B plays L B, wins by 200 points, the scores being the difference between the respective playing abilities and the winning pair being the one ranking higher in the table.

Take the McKenney-Baldwin Howell movement cards, and assume that, either by assignment, pairs A to L take the following pair numbers: Pair A, 1; Pair B, 5; Pair C 8; Pair D; Pair E, 4; Pair F, 12; Pair G, 3; Pair H, 2; Pair I, 11; Pair J, 6; Pair K, 7; Pair L, 10. Now take a standard Howell recapitulation sheet and fill it out for eleven sets of one board each, the pairs playing each other in the manner called for by the movement and the scores being the differences between their average playing abilities as given in the table above. For example, on Board 1, Pair 12 (F) plays North and South against Pair 1 (A) East and West. The rating of Pair F is plus 20; that of Pair A is plus 120. Pair 12 (F) therefore scores minus 100 in the North and South column; Pair 1 (A) scores plus 100 in the East and West column. On this same board, Pair 2 (H) plays North and South against Pair 11 (I) East and West. The rating of Pair H is minus 20; that of Pair I is minus 40. Pair 2 (H) therefore scores plus 20 in the North and South column; Pair 11 (I) scores minus 20 in the East and West column. Enter the scores for each pair in this way for each of the eleven boards; then match point in the usual manner. Add up the match points for each pair, and the score sheet will be as shown in diagram 1.

The outstanding feature of this tabulation is that Pair G has won the tournament, although it is only the average pair and has had a bad score every time it has played against the six better pairs. Pair A, which has a better total point score on the eleven boards than any other pair, finishes second.

Suppose, however, that at the next meeting of the same twelve pairs, the seating arrangements are different, Pair A being number 5; Pair B, 8; Pair C, 9; Pair D, 4; Pair E, 12; Pair F, 2; Pair G, 1; Pair H, 11; Pair I, 6; Pair J, 7; Pair K, 10; Pair L, 3. Now repeat the process of tabulating scores, using the same ratings as given in the original table and again entering the difference in the average playing abilities in favor of the better pair. The result will be as shown in diagram 2.

.

The striking feature of the second tabulation is that Pair G is now last.

Diagram no. 1 — good seats give the 7th best pair top score in a McKenney-Baldwin 12-pair Howell tournament*.

No.	Pair	Ability	Rank	Score	1	2	3	4	5	6	7	8	9	10	11
					EW	EW	NS	NS	EW	EW	NS	NS	EW	EW	NS
1	A	120	2	38	4.5	2.5	2	5	1.5	4	4	3	4.5	2	5
					100	120	20	200	60	160	140	80	180	40	220
					NS	EW	EW	NS	NS	EW	EW	NS	NS	EW	EW
2	H	-20	8.5	19	3.5	0	0	1	0.5	4	1	1	2	3.5	3.5
					20	-40	-60	40	-100	80	-140	-20	-120	60	-80
					EW	NS	EW	EW	NS	NS	EW	EW	NS	NS	EW
3	G	0	1	41.5	4.5	2.5	3	3	5	3	5	4	4	4	3.5
					100	-120	-20	-100	80	-60	40	20	-40	60	-80
					EW	EW	NS	EW	EW	NS	NS	EW	EW	NS	NS
4	E	40	6	32.5	1.5	1	5	5	4.5	4	4	2	1	1.5	3
					-20	80	60	20	100	-40	140	-80	40	-60	120
					NS	EW	EW	NS	EW	EW	NS	NS	EW	EW	NS
5	B	100	3	37.5	3.5	5	3	2	3	5	1	4.5	3	3.5	4
					20	200	-20	100	80	180	40	140	120	60	160
					NS	NS	EW	EW	NS	EW	EW	NS	NS	EW	EW
6	J	-60	8.5	19	3.5	2.5	3	4	0.5	0	1	2	0.5	1	1
					20	-120	-20	-40	-100	-80	-140	40	-180	-60	-160
					EW	NS	NS	EW	EW	NS	EW	EW	NS	NS	EW
7	K	-80	11	15.5	1.5	1	2	0	0	0	3	0.5	4	1.5	2
					-20	-160	20	-200	-80	-180	-100	-140	-40	-60	-120
					EW	EW	NS	NS	EW	EW	NS	EW	EW	NS	NS
8	C	80	5	34	1.5	4	2	3	4.5	1	4	5	4.5	3	1.5
					-20	160	20	120	100	40	140	60	180	-40	80
					NS	EW	EW	NS	NS	EW	EW	NS	EW	EW	NS
9	D	60	4	34.5	3.5	2.5	3	4	3.5	2	4	4.5	1	5	1.5
					20	120	-20	160	-60	60	-40	140	40	100	80
					NS	NS	EW	EW	NS	NS	EW	EW	NS	EW	EW
10	L	-100	12	14.5	0.5	0	3	1	3.5	2	1	3	0.5	0	0
					-100	-200	-20	-160	-60	-80	-140	-40	-180	-120	-220
					EW	NS	NS	EW	EW	NS	NS	EW	EW	NS	EW
11	I	-40	10	18.5	1.5	4	2	2	1.5	1	0	0.5	1	0	5
					-20	-80	20	-120	60	-160	-40	-140	40	-100	-60
					NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
12	F	20	7	25.5	0.5	5	2	0	2	5	2	0	4	5	0
					-100	40	20	-20	-80	80	100	-60	-40	120	60

*** All factors except seating are identical in Diagrams 1, 2, 3 and 4. The first line of figures indicates match-point scores and the second, total point scores.**

Diagram no. 2 — bad seats give the 7th best pair bottom score in a McKenney-Baldwin 12-pair Howell tournament*.

No	Pair	Ability	Rank	Score	1	2	3	4	5	6	7	8	9	10	11
					EW	EW	NS	NS	EW	EW	NS	NS	EW	EW	NS
1	G	0	12	9.5	0.5	2	0	1.5	0	1	0	0.5	1	1	2
					-40	100	-120	60	-80	20	-20	-60	40	-100	80
					NS	EW	EW	NS	NS	EW	EW	NS	NS	EW	EW
2	F	20	6	30	4.5	0	1	1.5	1.5	3	5	3	2	4	4.5
					40	-20	-40	60	-80	100	20	120	-100	80	-60
					EW	NS	EW	EW	NS	NS	EW	EW	NS	NS	EW
3	L	-100	10	19.5	3.5	3	0	0	4	0.5	3	2	1	2.5	0
					-20	-100	-140	-220	-40	-180	-80	-120	-160	-60	-20
					EW	EW	NS	EW	EW	NS	NS	EW	EW	NS	NS
4	D	60	2	40	3.5	1	4	5	5	3	4.5	4.5	4	2.5	3
					-20	80	40	20	100	-40	140	60	160	-60	120
					NS	EW	EW	NS	EW	EW	NS	NS	EW	EW	NS
5	A	120	3	39.5	1.5	5	5	5	3.5	4.5	1	4.5	3	2.5	4
					20	200	120	220	80	180	40	140	100	60	160
					NS	NS	EW	EW	NS	EW	EW	NS	NS	EW	EW
6	I	-40	9	20	1.5	2	3	3.5	0	0	0.5	2	4	2.5	1
					20	-120	-20	-60	-100	-80	-140	40	-40	60	-160
					EW	NS	NS	EW	EW	NS	EW	EW	NS	NS	EW
7	J	-60	8	21	3.5	1	2	3.5	1	0.5	2	0.5	4	1	2
					-20	-160	20	-60	40	-180	-100	-140	-40	-80	-120
					EW	EW	NS	NS	EW	EW	NS	EW	EW	NS	NS
8	B	100	1	41	3.5	4	2	3	3.5	2	4.5	4.5	5	4	5
					-20	160	20	120	80	40	140	60	180	100	200
					NS	EW	EW	NS	NS	EW	EW	NS	EW	EW	NS
9	C	80	4	36	1.5	3	3	4	5	4.5	4	4.5	1	5	0.5
					20	120	-20	160	80	180	-40	140	40	100	60
					NS	NS	EW	EW	NS	NS	EW	EW	NS	EW	EW
10	K	-80	11	17	1.5	0	3	1	3	2	0.5	3	0	0	3
					20	-200	-20	-160	-60	-100	-140	-40	-180	-120	-80
					EW	NS	NS	EW	EW	NS	NS	EW	EW	NS	EW
11	H	-20	7	22.5	0.5	4	2	2	2	4	2	0.5	1	0	4.5
					-40	-80	20	-120	60	-20	80	-140	40	-100	-60
					NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
12	E	40	5	34	4.5	5	5	0	1.5	5	3	0.5	4	5	0.5
					40	20	140	-20	-80	80	100	-60	-40	120	60

*** All factors except seating are identical in Diagrams 1, 2, 3 and 4. The first line of figures indicates match-point scores and the second, total point scores.**

Compare the results of these two hypothetical tournaments. In the first, Pair G is high with 41½ match points, a 75 per cent game. In the second, Pair G is last with 9½ match points, a 17 per cent game. In a full 22-board movement, this would be equivalent to a difference of 64 match points, where an average score is 55.

Remember that in each tournament every pair has played strictly in accordance with its rating, which has been exactly the same for both matches; the playing ability of each pair has been assumed to be the same on every board in both tournaments, and the results have been determined by the playing abilities of the opposing pairs. In each case, for example, when Pair G has played Pair A, the former has lost by 100 points; in each case where Pair G has played Pair L the former has won by 100 points. Similarly, all the other pairs have played exactly the same in both tournaments. The only difference has been a change in the seating arrangements in the first round, and the consequent change in the pair numbers in the movement.

Any intermediate result is of course possible; the average pair, by suitable changes in the pair numbers, may be made to finish anywhere from first to twelfth inclusive. Similarly, the score of any other pair, from

the best to poorest, may be radically increased or lessened by suitable changes in the pair numbers.

The reason for these astonishing variations is the lack of proper comparisons among the twelve pairs. In a perfect movement, each pair should play five times in the same way of the board and five times the opposite way with each other pair. In the movement under discussion, each pair (except the anchor pair) plays nine times the same way and once the opposite way with two pairs, seven times the same way and three times the opposite way with two other pairs, five times each way with three pairs, three times the same way and seven times the opposite way with two pairs, and once the same way and nine times the opposite way with the remaining two pairs.

The results given in the analysis of the McKenney-Baldwin twelve-pair Howell movement are typical of all Howell movements where the basis of comparisons is unsound. They apply, in greater or less degree to all the other McKenney-Baldwin movements which the writer has analyzed, and to more than of the original Howell movements. Consider, for example, the original fourteen-pair Howell movement. While this schedule is more nearly correct than the movement just analyzed, it is yet so unsound that a difference of twenty-six match points may be produced in the score of a given pair by applying the same method used in this article and simply varying the pair numbers without changing the relative abilities of the contestants. As the average match point score for this movement is 78, a variation of 26 points is of the first order of magnitude.

It is important to note that these results are by no means exceptions to the general rule. The defects in the comparisons in the twelve-pair McKenney-Baldwin Howell movement are merely illustrative of the complete disregard of fundamentals which characterizes this entire series. Of the original Howell movements, the fourteen-pair is better than some and worse than others.

With these considerations in mind, it is impossible to escape the conclusion that many Bridge championships, decided by the use of unsound Howell movements, have been won under false pretences, by pairs whose victories have been due as much to defects in the movements as to the superiority of their game.

The widespread use of the defective movements discussed in this article leads to an interesting speculation. In a conversation with the writer some years ago, Ely Culbertson expressed his dissatisfaction with the accepted methods of tournament procedure. Probably realizing that the results of many tournaments were inconsistent with the known abilities of the contestants, he has placed the blame on the match-point method of scoring. Perhaps, after all, this system of scoring, imperfect though it may be, is not mainly responsible for the unsatisfactory results for which it has been blamed; perhaps the choice of movement was at least equally at fault.

The remedy lies in the use of sound movements. Strange as it may seem, sound Howell-type movements are not in general impossible. The writer, in conjunction with Jacques L. Ach, has devised a complete set of circulatory movements, for from four to thirty pairs inclusive. From four to twenty pairs, inclusive, these movements are sound for both odd and even numbers of pairs, except for the nine- and fifteen-pair movements; sound movements are mathematically impossible for these two numbers, except by the use of too complex a movement of the boards.

These movements are entirely circulatory in character, being directed by the use of the familiar type of table card. They are sound for every case where soundness is mathematically possible, and the defective movements are so noted on the master sheets. Except for the ten- and sixteen-pair movements, all of the sound even-numbered movements are also sound for the lower odd number. This is a point of importance in club tournaments and qualifying rounds, where the number where the number of entrants is as likely to be an odd as an even number, and where it is as necessary to have a sound eleven-pair movement, for example, as a sound twelve-pair movement.

This new series will be found in practically every case to present a definite advantage over the

corresponding original Howell or McKenney-Baldwin Howell movements. By way of illustration, the method of analysis applied in this article to the twelve-pair McKenney-Baldwin Howell movement has also been applied to the twelve-pair movement of this new series, using exactly the same values for the different pairs and making the same changes in the seating arrangements. The results, shown in diagrams 3 and 4, are substantially the same regardless of the pair numbers of the contestants; the minor changes are due, as a matter of fact, not to the movement or seating, but to inherent defects in the match-point method of scoring.

Diagram no. 3 — The best pair is top and the worst pair bottom in an Ach-Kennedy 12-pair Howell tournament*.

No.	Pair	Ability	Rank	Score	1	2	3	4	5	6	7	8	9	10	11
					EW	NS	EW	EW	EW	NS	NS	NS	EW	NS	EW
1	A	120	1	45	5	3	4	5	3.5	4.5	5	5	5	1	4
					180	60	20	220	80	120	200	160	140	40	100
					EW	EW	NS	EW	EW	EW	NS	NS	NS	EW	NS
2	H	-20	8	24	2	4	3.5	3	1.5	0.5	1	1.5	0	5	2
					-40	60	80	40	20	-120	-60	-100	-140	-20	-80
					NS	EW	EW	NS	EW	EW	EW	NS	NS	NS	EW
3	G	0	7	25	5	3	1.5	4	3.5	0.5	4	1.5	1	0	1
					100	-20	-80	40	80	-120	60	-100	-60	20	-40
					EW	NS	EW	EW	NS	EW	EW	EW	NS	NS	NS
4	E	40	5	36.5	1	4	4	2	1.5	3	4	5	4	4	4
					-60	80	20	-20	-80	-40	60	120	100	140	40
					NS	EW	NS	EW	EW	NS	EW	EW	EW	NS	NS
5	B	100	2	42	4	5	1	4	5	4.5	2	3.5	3	5	5
					60	160	-20	80	200	120	40	100	20	180	140
					NS	NS	EW	NS	EW	EW	NS	EW	EW	EW	NS
6	J	-60	10	15.5	0	0	4	2	0	4	1	1.5	1	2	0
					-180	-160	20	-40	-80	-20	-60	40	-100	-120	-140
					NS	NS	NS	EW	NS	EW	EW	NS	EW	EW	EW
7	K	-80	11	10.5	1	1	1	0	1.5	2	0	0	2	0	2
					-140	-60	-20	-160	-80	-100	-200	-120	-40	-180	20
					EW	NS	NS	NS	EW	NS	EW	EW	NS	EW	EW
8	C	80	3	38.5	3	5	3.5	5	1.5	2	4	3.5	2	4	5
					120	180	80	160	20	40	60	100	-20	-40	140
					EW	EW	NS	NS	NS	EW	NS	EW	EW	NS	EW
9	D	60	4	37	4	2	5	3	3.5	5	3	1.5	4	3	3
					140	-60	100	20	-20	160	-40	40	60	120	80
					EW	EW	EW	NS	NS	NS	EW	NS	EW	EW	NS
10	L	-100	12	10	0	0	1.5	0	0	0	1	3.5	0	1	3
					-100	-180	-80	-220	-200	-160	-60	-40	-120	-140	-30
					NS	EW	EW	EW	NS	NS	NS	EW	NS	EW	EW
11	I	-40	9	18.5	2	1	0	1	3.5	1	4	0	3	3	0
					-120	-80	-100	-40	-20	20	60	-160	40	-60	-140
					NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
12	F	20	6	27.5	3	2	1	1	5	3	1	3.5	5	2	1
					40	20	-20	-80	80	100	-60	-40	120	60	-100

*** All factors except seating are identical in Diagrams 1, 2, 3 and 4. The first line of figures indicates match-point scores and the second, total point scores.**

Diagram no. 4 — The result is the same regardless of seating in an Ach-Kennedy 12-pair Howell tournament*.

No.	Pair	Ability	Rank	Score	1	2	3	4	5	6	7	8	9	10	11
					EW	NS	EW	EW	EW	NS	NS	NS	EW	NS	EW
1	G	0	6	29.5	3	1.5	1	4	1	4	4	5	3	1	2
					40	-80	-120	80	-60	100	60	20	-20	-100	-40
					EW	EW	NS	EW	EW	EW	NS	NS	NS	EW	NS
2	F	20	7	25.5	1	3.5	2.5	2	4	1	1.5	2	2	5	1
					-20	80	100	60	40	-100	-40	-80	20	120	-60
					NS	EW	EW	NS	EW	EW	EW	NS	NS	NS	EW
3	L	-100	12	9	3	1	0	1	2	1	1	0	0	0	0
					-20	-140	-200	-80	-40	-100	-60	-220	-180	-120	-160
					EW	NS	EW	EW	NS	EW	EW	EW	NS	NS	NS
4	D	60	4	36	0	3	4.5	1	4	3	3.5	4	4	4	5
					-60	80	20	-20	60	-40	40	120	100	140	160
					NS	EW	NS	EW	EW	NS	EW	EW	EW	NS	NS
5	A	120	1	48.5	5	5	4	4	5	4	3.5	5	4	5	4
					60	160	120	80	200	100	40	220	20	180	140
					NS	NS	EW	NS	EW	EW	NS	EW	EW	EW	NS
6	I	-40	9	22	2	0	4.5	3	0	4	4	1.5	1	2	0
					-40	-160	20	-60	-80	-20	60	40	-100	-120	-140
					NS	NS	NS	EW	NS	EW	EW	NS	EW	EW	EW
7	J	-60	10.5	13	0	1.5	0.5	0	3	1	1	1	2	0	3
					-140	-80	-20	-160	40	-100	-60	-120	-40	-180	20
					EW	NS	NS	NS	EW	NS	EW	EW	NS	EW	EW
8	B	100	2	42	4	5	5	5	3	2	5	3	1	4	5
					120	180	200	160	20	40	60	80	-20	100	140
					EW	EW	NS	NS	NS	EW	NS	EW	EW	NS	EW
9	C	80	3	37	5	3.5	2.5	4	2	5	1.5	1.5	5	3	4
					140	80	100	20	-20	160	-40	40	180	120	60
					EW	EW	EW	NS	NS	NS	EW	NS	EW	EW	NS
10	K	-80	10.5	13	2	0	2.5	1	0	0	1	3.5	0	1	2
					20	-180	-100	-80	-200	-160	-60	-40	-120	-140	-20
					NS	EW	EW	EW	NS	NS	NS	EW	NS	EW	EW
11	H	-20	8	22.5	1	2	2.5	4	1	1	4	0	3	3	1
					-120	-80	-100	80	-40	20	60	-20	40	-60	-140
					NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
12	E	40	5	32	4	4	0.5	1	5	4	0	3.5	5	2	3
					20	140	-20	-80	80	100	-60	-40	120	60	40

*** All factors except seating are identical in Diagrams 1, 2, 3 and 4. The first line of figures indicates match-point scores and the second, total point scores.**

Appendix N

CCRRS Howell designs

This is the original Howell design supplied in the CCRRS paper by Gennian Ge, Esther Lamken and Alan Ling. Note the symbols shown here represent 6 teams. The stationary-team pairs (or the missing pairs if this schedule is used for an odd number of teams) are shown as α and β and the first pairs of each team are 0 through 4 and the second pairs are shown as 0' through 4'. It is important to note this Howell design will not operate as a movement for bridge without switching some of the pairings. This Howell design is $H(10, 12)$, or more generally $H(2m, 2m + 2)$. A CCRRS(n) is produced from a Howell design where $m = n - 1$.

		α,4	1,3'	3,1'		0,2	β,4'		0', 2'
4, 2'			α,0	2,4'	1',3'		1,3	β,0'	
3,0'	0,3'			α,1		2',4'		2,4	β,1'
α,2	4,1'	1,4'			β,2'		3',0'		3,0
	α,3	0,2'	2,0'		4,1	β,3'		4',1'	
3',4'	1,2'			β,0	α,0'	4,3	1',2		
β,1	4',0'	2,3'				α,1'	4,0	2',3	
	β,2	0',1'	3,4'				α,2'	0,1	3',4
		β,3	1',2'	4,0'	4',0			α,3'	1,2
0,1'			β,4	2',3'	2,3	0',1			α,4'

A better representation of this is discussed in the introduction to Scissor schedules, page 288. It is reproduced here for convenience.

		a,5	2,4'	4,2'		1,3	5',b		3',1'
5,3'			a,1	3,5'	4',2'		2,4	1',b	
4,1'	1,4'			a,2		5',3'		3,5	2',b
a,3	5,2'	2,5'			3',b		1',4'		4,1
	a,4	1,3'	3,1'		5,2	4',b		2',5'	
4',5'	2,3'			b,1	1',a	5,4	3,2'		
b,2	5',1'	3,4'				2',a	1,5	4,3'	
	b,3	1',2'	4,5'				3',a	2,1	5,4'
		b,4	2',3'	5,1'	1,5'			4',a	3,2
1,2'			b,5	3',4'	4,3	2,1'			5',a

It is possible to use this schedule for a teams event that is to be scored as pairs, but where the same boards are played by all teams on each round. That is, a barometer teams *and* pairs event. Each row is a round in the event, and each match is completed in the round. Each round can also be scored as a round in a pairs event. All of these rounds consist of two schedules, and can be described with 4 starting points, on rounds 1 and 6. See below for 6 teams.

The following Howell designs are those used in Part 10.2 for even numbers of teams (page 290). The two rows show the starters for rounds 1 and n where n is the number of teams in CCRRS(n).

6 Teams barometer

		a,5	2,4'	4,2'		1,3	5',b		3',1'
4',5'	2,3'			b,1	1',a	5,4	3,2'		

8 Teams barometer

a,1	3,7	5',6		2',4			1',b	7',3'	6',5		4',2		
2',4'			b,1	3',7	5',6'		4,2			1',a	7',3	6,5	

10 Teams barometer

		a,1		2,3'	6',9	3,2'	5',7'		8',4	1',b	4',8		7,5		9',6		
3,2	b,1	4',8'	8,4					2',3'	5',7	6,9		7',5	1',a				9',6'

12 Teams barometer

a,1		9,6'				3,5'	2,7	11,10'		8,4'
1',b		6,9'				5,3'	7',2'	10,11'		4,8'
	8',4'	b,1		9,6				3',5'	2,7'	11',10'
4,8	1',a		6',9'				5,3	7,2'	10,11	

14 Teams barometer

	6,8			13',4		5',2			10',11'	a,1	4',13	2',5
		8',6'		9',3'	1',b		12,7'		7,12'	11,10		3,9
4',13'		9',3		b,1		3',9	2,5			5',2'		10',11
1',a			11',10	7,12	12',7'				6',8	8',6	13,4	

18 Teams barometer

	a,1		6,10'	12,11	3,9					8',17			17',8	2,5'	7',14'	11',12'
				16',4'	5,2'		9',3'	13,15'	15,13'	1',b			10,6'	4,16		14,7
5',2'	12,11'			b,1		2,5			11,12'		8',17'		7',14	14',7	16',4	
3',9		13,15	1',a	15',13'	10',6'		6,10			9',3			17,8		4',16	

20 Teams barometer

a,1	6,16			2',4	7',19'	12',15	17',11		8,3		18,14'					5,13'	10,9'	
1',b	16',6'			4',2	19,7	15',12	11',17		3',8'		14,18'					13,5'	9,10'	
5',13'	10',9'		b,1	6,16'			2,4	7',19	12,15	17',11'		8',3		18',14'				
13,5	9,10		1',a	16,6'			4',2'	19',7	15',12'	11,17		3',8		14,18				

24 Teams barometer

a,1	5,21	9',18	13,15	17,12'		2',6'		10',23	14,20'			3',11'	7',8			19',22'		4,16'				
1',b	21',5'	18',9	15',13'	12,17'		6,2		23',10	20,14'			11,3	8',7			22,19		16,4'				
4',16'					b,1	5,21'	9,18	13',15	17,12		2,6'		10',23'	14,20			3',11	7',8'			19,22'	
16,4					1',a	21,5'	18',9'	15',13	12',17'		6,2'		23,10	20',14'			11',3	8,7			22,19'	

Glossary

Balance
The balance is a reference to the fairness of a movement. The balance is quantified by the calibre.
Board
A board (or tray) is a container for storage of playing cards. There are 4 slots, each intended to contain 13 cards.
Board-set
A board-set is a group of (usually) contiguous boards. In a schedule the boards played are defined as a set with a set number. When the schedule is used in a movement the number of boards in each set is specified by the organizers, but typically in a pairs' game it would be between 2 and 4 boards.
Duplicate
It is a form of bridge game where the same deal is played more than once, allowing for the comparison of results between two or more tables.
Calibre
This is a measure of the quality of a movement and is a number between 0 and 100. It is expressed as a percentage of the best possible, with 100 being perfect. Also called balance.
Event
An event is a game of bridge played under the auspices of the organizing body. It can be played over many days, and involves at least one session of bridge, producing one or more winners.
Modification
A modification refers to a change made to an existing schedule, producing a different outcome, but essentially based on the original schedule.
Move
A move refers to the place where a pair of bridge players (or individual) proceed to according to the schedule. A move is made after each round.
Movement
A bridge movement is based on one or more schedules. It is the instance of the schedule(s) in use in a section (or sections) of an event. It places limitations on the boards played on each round, defines board numbers and pair numbers.
Pair
A pair is the smallest moving unit in a pairs' movement and consists of two players. The pair is required to stay in play for the whole movement.
Round
A round of play in a movement or schedule played against one opponent, be that an individual, pair or team. At the completion of a round a move is made to the next round, where different boards are played against different opponents.
Schedule
A schedule defines the opponents of the individuals, pairs or teams in a bridge event. These opponents differ on each round, playing different board-sets.
Section
A section of a bridge event is based on (usually) one schedule and is organized with a movement. The individuals and pairs in the section remain in that section for the session of play.
Session
A session of play defines the time limits on the game. A session will have one or more sections of players involved, and the result usually produces one or two winners. A session of play is typically over 25 to 32 boards, though it is not uncommon for it to be outside this range.
Set-of-boards
A set-of-boards consists of several board-sets, making up the whole collection of boards played in a movement. The number of board-sets multiplied by the number of boards per set gives the total number of boards in the set-of-boards.
Team
A team is the smallest unit in a teams' event, and consists of four to six players. Only four players participate in each session of play and the team is required to stay in play for the whole session.

Bibliography

Those people who have influenced this document include the following but are certainly not limited to this list:

Professor Emeritus Walter Wallis, Dr. Ross Moore; John R. Manning; Professor Olof Hanner; Hans Olof Hallen; Per Jannersten; George Coffin; George W. Beynon; Frank Farrington; Peter Smulders; Joop van Wijk; Paul Vermaseren; Catherine Jourdan; Alex Groner; Pieter Bussemaker; Frans C. Schiereck; John Probst; John Royds.

A special note for Tim Bourke who was most helpful in locating reference books, particularly those found in the A.B.F. library. Also David Levy and Wolf Klewe for access to their extensive libraries, particularly copies of magazines and photo-copies of missing pages.

References

R. C. Mullin and W. D. Wallis The existence of Room squares Aequationes Mathematicae 13 (1975), 1–7.

Combinatorics: Room Squares, Sum-Free Sets, Hadamard Matrices BY: W D Wallis; Anne Penfold Street; Jennifer Seberry

Handbook of Combinatorial Designs, Second Edition (Discrete Mathematics and Its Applications)
by Editor-Charles J. Colbourn; Editor-Jeffrey H. Dinitz

Gennian Ge, Esther Lamken and Alan Ling *Scheduling CCRR tournaments* journal of combinatorial theory, series A (2006)

D. R. Stinson and W. D. Wallis An even side analogue of Room squares Aequationes Mathematicae 27 (1984) 201-213

G. Ge and L. Zhu. On the existence of Room frames of type t^u for u = 4 and 5. J. Combin. Designs 1 (1993), 183–191.

Jeffrey H. Dinitz and Gregory S. Warrington The Spectra of Certain Classes of Room Frames: The Last Cases May 20, 2010 Mathematics Subject Classification: 05B15

The Whist Reference Book …Wherein Information is presented concerning the NOBLE GAME, in all its aspects, after the Manner of…CYCLOPEDIA, DICTIONARY, AND DIGEST ALL COMBINED IN ONE by William Mill Butler (1898)

Whist Sketches, review and sketches of the First American Whist Congress, held in Milwaukee, April 1891. Personal of distinguished players. Biographies of 'Cavendish' and Trist. Reminiscences of James Clay, etc., etc. By C. S. B. (Charles Boutcher).

Duplicate Whist, its rules and methods of play. By John T. Mitchell Chicago A C McClure and company (1891).

Duplicate Whist, by John T. Mitchell KALAMAZOO, MICH. Ihling Bros. & Everard (1897)

Foster's Duplicate Whist. A complete system of instruction in Whist strategy. By R. F. Foster. Brentano's New York, Chicago, Washington. (1894).

Foster's Bridge manual, a complete system of instruction in the game to which is added Dummy Bridge and Duplicate bridge by R. F. Foster Brentano's New York (1901)

Foster's complete bridge by R. F. Foster McClure, Philips & Co. New York (1907)

Bose, R. C.; Shrikhande, S. S.; and Parker, E. T. 'Further Results on the Construction of Mutually Orthogonal Latin Squares and the Falsity of Euler's Conjecture.' *Canad. J. Math.* 12, 189, (1960).

Duplicate Bridge Guide (Bruelheide, Minneapolis) comb-bound, 220x140mm, pp64. ed1 F. E. Bruelheide (1938)

Duplicate Bridge (John Waddington Ltd, London) stapled, pp8. ed1 Alexander T. Hasler (circa 1930)

Movements - a fair approach Hans-Olof Hallen, Olof Hanner & Per Jannersten (1994)

The EBU manual of duplicate bridge movements Edited by J.R. Manning (1992, 2002)

New Mitchells for everyone George S. Coffin (1971)

Duplicate bridge movements F. Farrington (1979)

The History of Australian bridge 1930-1990 Cathy Chua (1993)

Bridge Magazine 1970-2010

Duplicate bridge direction Alex Groner (1967 – 2004)

Tournament and duplicate bridge George W. Beynon (1944)

Bridge director's manual for duplicate games George W. Beynon (1943-1961)

Whist and duplicate whist The U.S. Playing card company (1900)

Whist – a monthly Journal 1891 – 1899

Duplicate contract complete Alfred M. Gruenther (1933)

Duplicate bridge simplified Alexander T. Hasler (1933)

Index

¾

¾-Howell, 60, 82, 86, 182, 183, 188, 337, 359, 403, 409, 410, 411

1

10 ½ tables 9 rounds, 101
10 players, 76, 262, 263
10 tables, 72, 75, 101, 104, 118, 122, 123, 129, 131, 144, 146, 173, 179, 181, 200, 211, 256, 259, 306, 338, 360, 369, 410
10 teams, 226, 232, 242, 245, 246, 289, 292, 306, 419
1½ table, 101, 108
1½ table appendix, 101, 105
11 players, 261, 263
11 tables, 49, 72, 75, 173, 180
11 teams, 216, 219, 299, 300, 301, 313, 314, 316, 319
12 players, 37, 257, 259, 261, 262, 269, 270, 344
12 tables, 72, 75, 100, 101, 109, 118, 119, 121, 131, 132, 143, 151, 173, 180, 181, 200, 212, 241, 257, 338, 369
12 teams, 224, 226, 293, 302, 313
13 players, 263, 271, 277
13 tables, 49, 72, 75, 173, 180
13 teams, 218, 219, 300, 301, 311, 320
14 players, 258, 264, 271
14 tables, 72, 75, 96, 101, 117, 123, 131, 147, 173, 180, 181, 201, 230, 256, 258, 260, 338, 370, 394, 410
14 teams, 293, 307
15 tables, 72, 75, 100, 105, 118, 120, 126, 152, 173, 180, 181, 193, 212, 220, 224, 257, 321, 355, 370, 394
15 teams, 225, 300, 317, 321, 322, 324, 326
16 players, 31, 36, 37, 262, 264, 266, 271, 272
16 tables, 72, 101, 152, 173, 180, 201
16 teams, 243, 246, 291, 303, 322, 325, 328
17 players, 264, 265
17 tables, 72, 173
17 teams, 225, 300, 312, 381, 382
18 players, 265, 266, 268
18 tables, 72, 100, 101, 107, 114, 123, 174, 180, 181, 187, 201, 212, 339, 350, 354, 371
18 teams, 246, 293, 294, 308, 324
19 tables, 174
19 teams, 225, 300, 313, 318, 325

2

20 players, 262, 263, 265, 267, 268, 277
20 tables, 134, 174, 180, 260, 339, 371
20 teams, 294, 303, 329
21 players, 265, 268
21 teams, 300, 301, 326
22 players, 266, 267
22 teams, 309
23 teams, 301, 318
24 players, 15, 262, 266, 267, 268, 396
24 tables, 117, 137, 190, 339
24 teams, 294, 295, 304, 330
24/25 players, 274
25 players, 269
25 teams, 240, 301
26 players, 266, 267
27 teams, 218, 319
28 players, 267, 268, 275, 277
28 teams, 304, 331
2MOLS, 13, 14, 256
2-table appendix, 102, 111, 112, 124, 142

3

3 tables, 170
3 teams, 295, 344
3 to 6 tables, 154
30 players, 267
3½ tables, 106, 107, 114, 367
31 teams, 218
32 players, 73, 266, 267, 273, 278, 283
32 teams, 332
33 teams, 218
34 players, 266
35 teams, 218
36 players, 261, 267, 273, 278
36 teams, 305, 334
37 teams, 220
39 teams, 218

4

4 tables, 75, 101, 107, 262
4 teams, 52, 231, 290
40 players, 274
40 teams, 305, 335
41 teams, 218
43 teams, 218
44 players, 278
45 teams, 219
47 teams, 218
48 players, 278
49 teams, 218

5

5 tables, 49, 72, 75, 107, 262
5 teams, 216, 296
51 teams, 218
52 players, 263, 264, 278
56 players, 269, 276

6

6 tables, 72, 75, 92, 100, 101, 102, 106, 107, 115, 127, 146, 154, 157, 160, 171, 175, 176, 177, 181, 199, 209, 212, 235, 258, 262, 291, 337, 338, 368, 410
6 teams, 288, 289, 291, 292, 302, 313

60 players, 269, 278, 279
64 players, 278

7

7 tables, 49, 72, 75, 103, 172
7 teams, 57, 58, 59, 217, 237, 239, 297, 299, 300, 315
7-Table, 199, 373

8

8 players, 37, 174, 256, 262
8 tables, 72, 75, 101, 103, 109, 111, 128, 143, 151, 172, 179, 181, 187, 199, 200, 229, 240, 244, 246, 256, 281, 338, 351, 352, 361, 368, 374, 375, 410
8 teams, 57, 226, 239, 240, 248, 292

9

9 players, 257, 262, 263, 283
9 tables, 49, 72, 75, 100, 101, 118, 119, 129, 130, 151, 170, 172, 179, 181, 187, 212, 235, 256, 257, 344, 360, 369
9 teams, 215, 217, 298, 311

A

Ach-Kennedy, 82, 83, 417, 418
Allison, 23, 24, 25, 27, 29
appended movement, 123, 124
appendix, 15, 57, 85, 88, 101, 102, 104, 106, 107, 108, 109, 111, 112, 124, 126, 127, 130, 142, 143, 182, 183, 187, 215, 244, 254, 255, 256, 312, 350, 351, 352, 357, 358, 359, 391
AWL, 216, 226, 227, 234, 235, 237, 238

B

Bachet, 8
Baker, 47, 96, 97
balance, xx, 20, 40, 82, 83, 84, 85, 86, 87, 88, 89, 91, 103, 106, 121, 122, 154, 159, 164, 166, 167, 168, 169, 170, 175, 179, 182, 183, 202, 209, 210, 262, 265, 278, 281, 350, 402
Barney, xv, 51, 52, 64, 77, 79, 210, 224, 232, 393
barometer teams, 419
Beynon, 107, 113, 117, 124, 169, 274, 422, 423
bicyclic, 79, 183, 199, 215, 216, 217, 219, 286, 289, 290, 291, 295, 302, 309, 311, 316, 323, 324, 325, 326, 330, 332, 335, 337
Blackpool, 114
Blandon, 103, 108, 116, 118, 351
Block design, 83
Block Design, 83, 84, 85
Block Mitchell, 118
boards, 29, 31, 37, 47, 49, 60, 70, 90, 97, 99, 101, 102, 109, 115, 116, 120, 123, 125, 127, 129, 143, 151, 155, 157, 160, 169, 170, 178, 194, 197, 198, 225, 228, 233, 237, 242, 243, 245, 254, 257, 258, 261, 272, 276, 279, 281, 283, 286, 295, 296, 297, 309, 310, 319, 320, 322, 337, 342, 344, 346, 350, 356, 360, 361, 362, 367, 381, 391
Boufos, 121, 157, 362
Bowman, 101, 103, 108, 124, 142, 143, 353, 355
Bridge World, 23, 114, 117, 411
Bussemaker, 93, 94, 422
Butler, xiv, 27, 29, 38, 53, 54, 73, 74, 165, 174, 342, 392, 422

C

calibre, 72, 75, 87, 89, 90, 91, 94, 143, 154, 159, 167, 168, 170, 171, 172, 175, 176, 177, 178, 179, 183, 194, 205, 209, 210, 261, 263, 265, 268, 269, 272, 279, 280, 281, 282, 283, 291, 303, 304, 305, 307, 308, 309, 310, 315, 317, 318, 319, 320, 344
Cavendish, 19
Cayley, 2, 38, 56
Clay, xv, 49, 51, 52, 77, 118, 210, 214, 216, 224, 231, 232, 344, 366, 392, 422
Coffin, 113, 118, 126, 128, 130, 422, 423
comparison, 1, 18, 20, 57, 65, 82, 86, 87, 96, 154, 163, 170, 171, 210

D

Depaulis, 18
design, v, xvii, 46, 59, 60, 83, 84, 215, 253, 288, 289, 387, 389, 390, 402, 419
Double-skip, 227
double-weave, 113, 123, 127, 132, 134, 144, 160
Duplicate bridge, iii, xix, xx, xxi, 18, 97, 103, 423

E

Einstein, 1, 7, 15, 18, 76, 85, 295, 306, 356
Elaikim Hastings Moore, 56
Elaikim Hastings Moore, 38
endless, 7, 74, 116, 175, 176, 177, 193, 242, 350, 351
endless Howell, 74, 193, 242
Euler, 11, 12, 13, 14, 15, 39, 45, 56, 57, 126, 127, 145, 146, 151, 245, 423
expanded groups, 121
expanded Howell, 194

F

Fano plane, 401
Farrington, xx, 103, 422, 423
Fifteen Schoolgirls, 56
Foster, 18, 19, 36, 38, 41, 44, 53, 422, 423
frames, 220
Freeman, 227

G

golf, 396
Groner, 226, 262, 264, 344, 422, 423
Gruenther, 76, 97, 178, 230, 342, 343, 359, 423

H

Hadlock, 46
Half-switching, 168, 175
Hallen, 85, 422, 423
Hanner, xx, 76, 85, 87, 106, 120, 121, 169, 181, 183, 194,

195, 200, 202, 204, 239, 264, 265, 266, 267, 268, 269, 270, 277, 278, 279, 345, 346, 347, 394, 422, 423
Hasler, 97, 214, 230, 233, 423
hesitation, 28, 103, 104, 105, 117, 352, 353, 355, 366, 367
house square, 58, 59, 183, 214, 216, 218, 219, 220
Howell, 36, 38, 41, 44, 48, 51, 53, 54, 57, 58, 59, 60, 61, 64, 69, 70, 71, 72, 74, 75, 77, 78, 79, 82, 83, 84, 86, 91, 92, 94, 106, 116, 120, 121, 154, 157, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 193, 194, 195, 196, 197, 198, 199, 201, 204, 205, 208, 215, 241, 242, 243, 245, 254, 255, 258, 259, 261, 262, 263, 264, 265, 276, 277, 286, 288, 289, 291, 337, 344, 345, 346, 347, 350, 351, 356, 357, 358, 359, 360, 361, 362, 363, 367, 371, 393
Howell sessions, 187

I

imbalance, xx, 85, 86, 87, 89, 91, 154, 405, 406, 408, 409, 410, 411
interwoven, 178, 181, 185, 187, 188, 190, 199, 255, 261, 266, 277, 281, 314, 344, 346, 359, 360, 370

J

James Allison, 23
Jannersten, 85, 86, 422, 423
John T. Mitchell, 26

K

Kennedy, 85
Kirkman, 56, 57, 402

L

Latin, 11, 12, 13, 14, 15, 39, 45, 48, 54, 57, 59, 60, 61, 126, 129, 130, 131, 145, 146, 151, 160, 256, 282, 351, 383, 384, 385, 387, 423

M

magic square, 6, 7, 8, 9, 10, 12, 45, 395
Manning, xx, 85, 86, 87, 88, 89, 103, 250, 262, 263, 264, 265, 267, 422, 423
McKenney-Baldwin, 83, 276, 413, 414, 415, 416, 417
McKinnon, xxi, 77, 79, 195, 202, 216, 234, 237, 245, 262, 263, 264, 265, 266, 267, 269, 278
Milton C. Work, 22, 23
Mitchell, xv, 18, 19, 22, 26, 27, 28, 29, 30, 31, 36, 38, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 58, 59, 60, 61, 73, 74, 76, 77, 78, 79, 82, 84, 85, 86, 87, 94, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 127, 129, 131, 132, 142, 143, 144, 145, 153, 154, 155, 157, 159, 178, 181, 183, 195, 199, 205, 210, 211, 214, 227, 230, 233, 234, 241, 246, 254, 255, 256, 258, 276, 277, 279, 281, 342, 344, 345, 346, 347, 350, 351, 352, 353, 356, 357, 358, 359, 360, 361, 362, 363, 367, 391, 394, 422
MOLS, 146

O

odd number of teams, 251
Ollerenshaw, 9
Orthogonal, 15, 423
Oszy, 115
Ozanam, 9, 10, 11

P

Paine, xv, 29, 30, 70
Parker, 114, 124, 145, 423
pivot, 42, 65, 67, 99, 106, 182

R

reduced rounds, 121
Relay, 60, 97, 98, 109, 352
Riddle, 10
Room square, 56, 57, 58, 59, 71, 160, 163, 183, 402
Room squares, 57, 58, 59, 60, 146, 218, 286, 289
Rosler, 85
Ross Moore, xx, 85, 86, 422
round-robin, 15, 40, 57, 58, 214, 217, 218, 220, 222, 239, 247
rover, 99, 100, 101, 104, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 126, 127, 128, 129, 131, 132, 134, 142, 143, 151, 193, 195, 196, 210, 211, 246, 350, 351, 352, 366, 367, 375, 391
rover-table, 109, 111, 112, 123, 126
Royds, 187, 235, 422

S

Safford, xv, 36, 37, 38, 39, 40, 53, 54, 57, 64, 65, 67, 69, 70, 73, 74, 75, 77, 78, 120, 174, 175, 214, 215, 216, 241, 254, 279, 283, 286
Same-board whist, 225
Scheveningen, 76, 77, 143, 342
Schiereck, 183, 184, 185, 204, 422
scissor schedules, 59, 183, 287, 288, 290, 337, 366
self-orthogonal, 126, 128, 129, 130, 132, 133, 135, 136, 137, 138, 139, 140, 141, 151, 160, 383, 384, 385, 387
skeletons, 165, 167
skip, 96, 99, 100, 101, 144
Smulders, xx, 87, 88, 91, 269, 270, 271, 272, 283, 422
Snow, 57, 60, 74, 76, 77, 116, 120, 122, 125, 143, 170, 174, 175, 193, 199, 203, 233, 239, 254, 258, 260, 266, 272, 279, 342, 350, 360
Stagger, 118, 144, 145, 246
Stinson, xvii, 58, 218, 286, 295, 366, 422
Sudoku, xxi, 10, 39, 395
switched, 39, 45, 64, 70, 74, 82, 83, 86, 94, 97, 104, 116, 121, 143, 144, 154, 155, 157, 159, 160, 163, 164, 166, 175, 176, 177, 212, 242, 342, 355, 359, 379, 380, 408, 409, 411
switched Mitchells, 45, 86, 97, 121, 144, 154

T

Telfer, 343, 346
trisect, 118, 119, 120, 126, 129, 130, 131, 132, 136, 151
Trisect, 118

Trist, xv, 24, 25, 422
two rounds short, 142
Two stanza, 224

U

US Playing Card, 64, 76

V

van der Velde, 87
van Wijk, 91, 422
Vermaseren, 87, 422

W

Wallis, xx, 2, 15, 56, 57, 58, 84, 146, 218, 256, 286, 289, 295, 310, 312, 381, 383, 396, 422
Web, 101, 103, 104, 109, 110, 111, 112, 123, 124, 127, 142, 143, 353, 391
Whist, xv, 18, 19, 20, 21, 22, 23, 24, 26, 27, 32, 35, 36, 37, 38, 44, 50, 51, 52, 53, 69, 73, 74, 77, 79, 96, 210, 224, 232, 234, 286, 392, 422, 423
Whitfeld, xvi, 36, 38, 53, 73, 74, 279
Worger, 116, 187, 199, 200, 233, 234, 375

ADVANCED

ISBN 978-1-55494-525-2

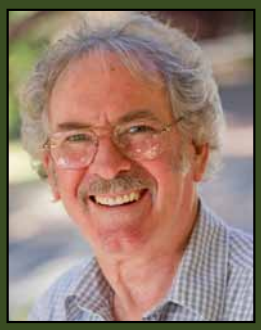
THE BRIDGE MOVEMENTS ENCYCLOPEDIA

Duplicate Bridge Schedules, History and Mathematics is an essential book for tournament directors as well as bridge players curious about the history of the game of duplicate bridge. This comprehensive volume supplies you with all the movements ever thought of and many hundreds of new ones. Included for each movement are the variations, modifications, origins, authors and history of its development. Each movement is then assessed for its measure of quality, called calibre.

The author presents a brand new event type, the Scissor movement — run like any Howell movement. In this type of event the players play as pairs as usual, but also have their teammates as another pair, never meeting each other. This allows the event to be scored both as teams and pairs, producing a winning team and a winning pair.

Duplicate bridge players will find the history of their favorite game most intriguing. The book delves into the lives of well-known figures such as John T. Mitchell and Edwin C. Howell. When did they live, what did they contribute to bridge, and what were the politics of their time? In addition, many lesser-known historical figures are examined for their contributions to the development of duplicate movements.

For the mathematically inclined there are plenty of interesting oddities. The mathematics of balance of movements, giving the measure of quality, is thoroughly discussed. The controversial debate over movement quality, along with its history, is presented through the ideas and opinions of players and mathematicians.



IAN McKINNON is a mathematician, expert bridge player, tournament director, author and computer professional. Through circumstance, around 1970, he started tournament directing at a major bridge club in Sydney which eventually led to him being the senior Tournament Director within the Australian Bridge Federation during the 1970s. He produced his first book, *Bridge Directing Complete*, in 1979. All those years of experience, and the last ten years of intense research and computer programming, have resulted in this book.

AN HONORS eBOOK FROM MASTER POINT PRESS