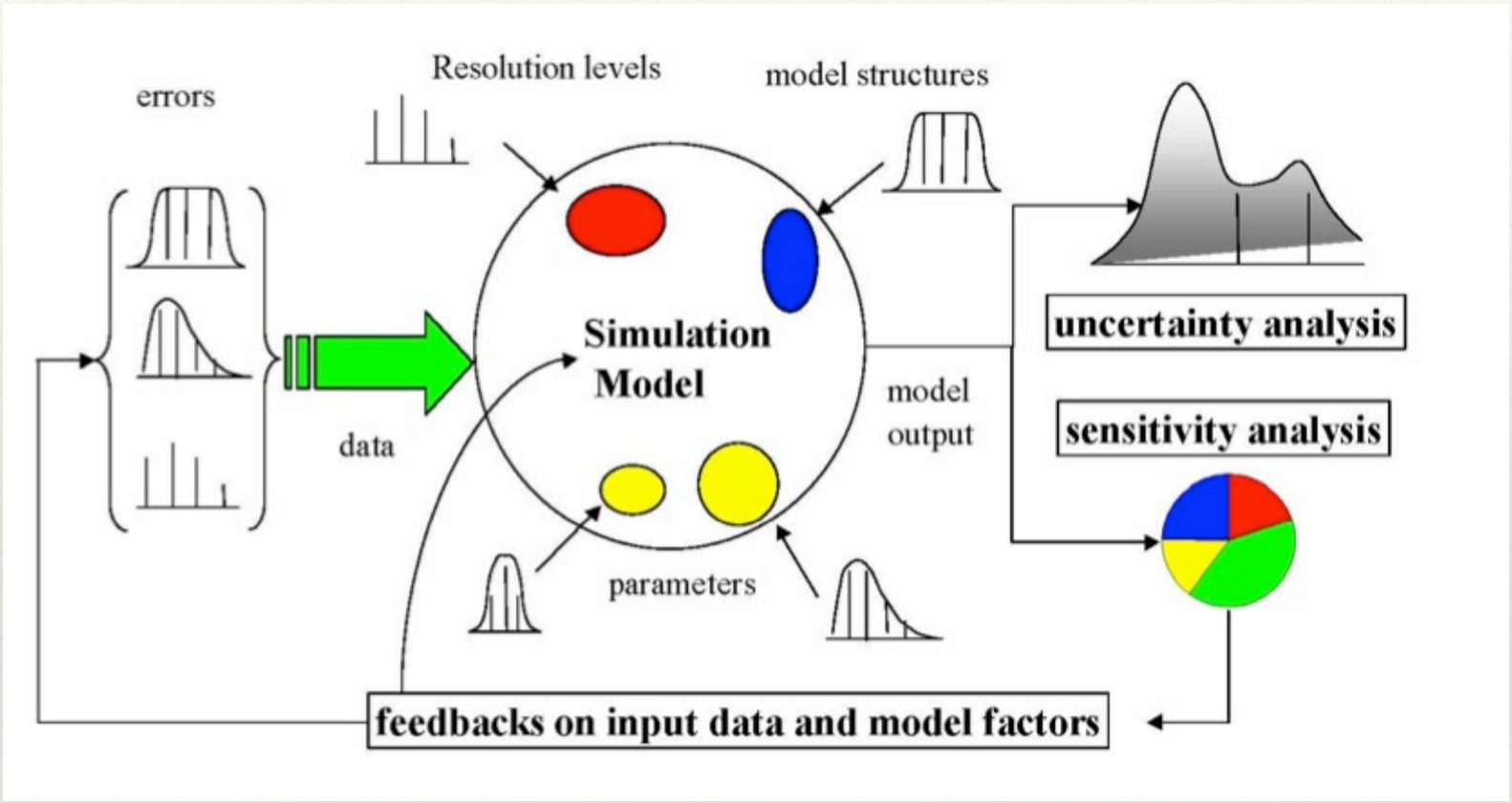
Sensitivity Analysis

- Parameter uncertainty
  - measurement error
  - estimation error
  - future conditions
  - spatial-temporal variation
- How do the results of our model change if we account for the uncertainty in our parameter estimates
- Domain of statics (think of the p-value in the estimation of a mean)
- (optimization and calibration are particular subsets of sensitivity analysis

Sensitivity/Uncertainty Analysis Analysis



Saltelli 2009 http://sensitivity-analysis.jrc.ec.europa.eu

Sensitivity Analysis

- Many approaches (entire text books)
- Two main classes
    - global simultaneous: vary all parameters over possible ranges
        - local parameter specific: hold all other parameters content and then vary
- Challenge is sampling parameter uncertainty space and balancing this against computational limits

Single Parameter: hold others constant

- sensitivity to a parameter may change as a function of other parameters
- consider the sensitivity of a seasonal snow accumulation and melt model to both temperature and radiation
  - for high temperatures, radiation may not impact rates substantially since there is less snow
  - for lower temperature, radiation may matter much more - so temperatures related to radiation absorption (e.g albedo) will have a greater impact on output

Variance based Sensitivity Analysis

- Methods that break the variance of the output into components related to inputs and parameters
- How much output variance is related to each parameter/input on its own (first-order indices)
- How much output variance is related to interaction effects and parameter itself (total effect indices)
- **Sobol**

Steps in Sensitivity Analysis

- Define the range (pdf) of input parameters
- Define the outputs to be considered (e.g if you had streamflow are you looking at daily, max, min, annual)
- Sample the pdf of input parameters and use this sample to run the model - repeat many times
- Graph the results
- Quantify sensitivity

Sensitivity Analysis - Key questions

- What ranges should I vary my parameters over!
  - range (min, max, middle values)
  - pdf
- If parameters are physically based this can come from literature (e.g range of snow albedo's, range of hydraulic conductivity in a soil)
- If parameter's are estimated (e.g a regression slope, coefficients from another model) then statistics such as confidence bounds can help you to define the ranges

Sensitivity Analysis - Key questions

- What output should I use to examine sensitivity
- You can look at several - but not many…too complicated

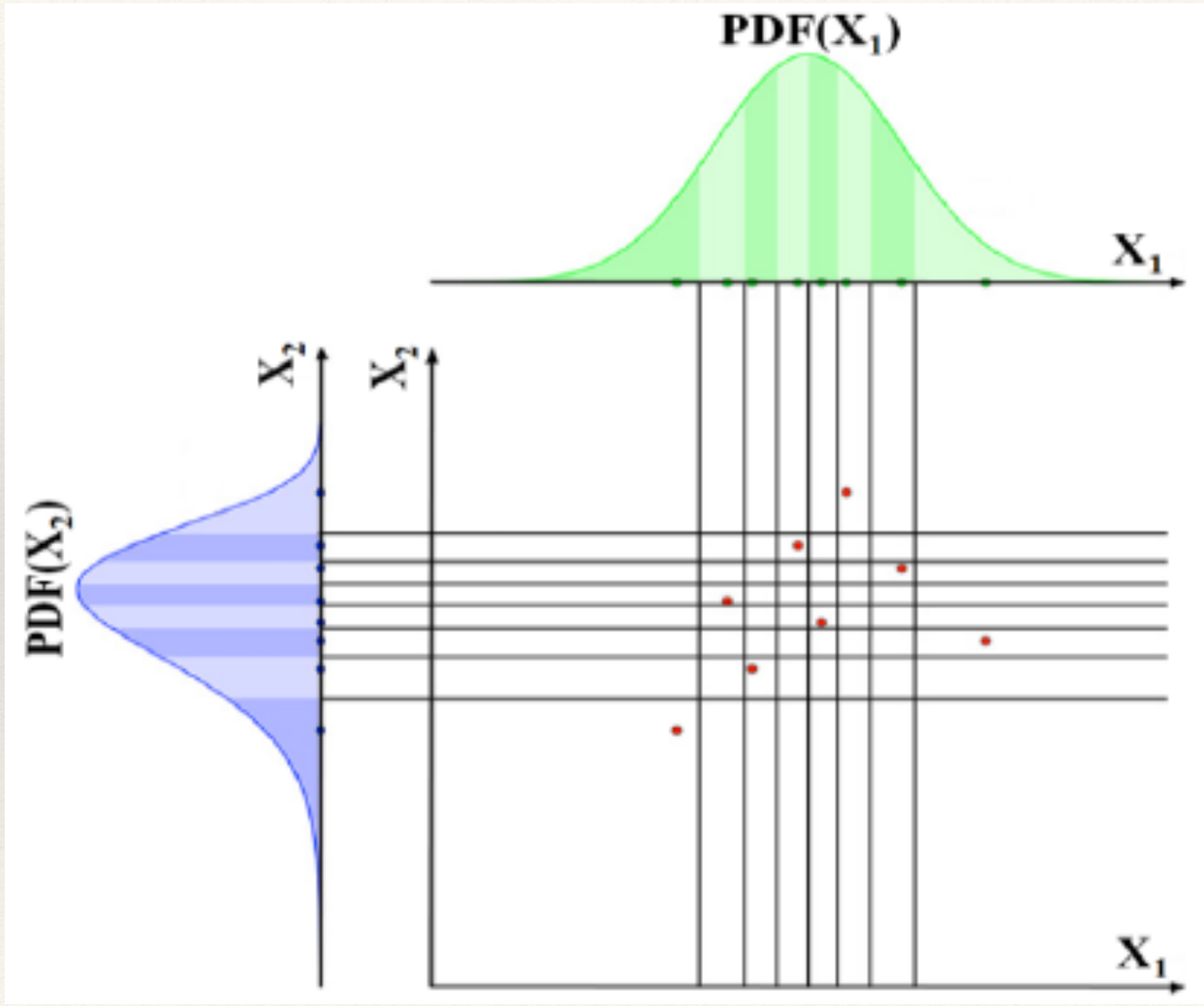
- Model output - daily streamflow (1000s of values)
  - Develop summary statistics
  - Mean daily streamflow, max streamflow, …..
- Choose outputs for sensitivity analysis based on what you ultimately care about!!!

Approaches to sampling parameter space

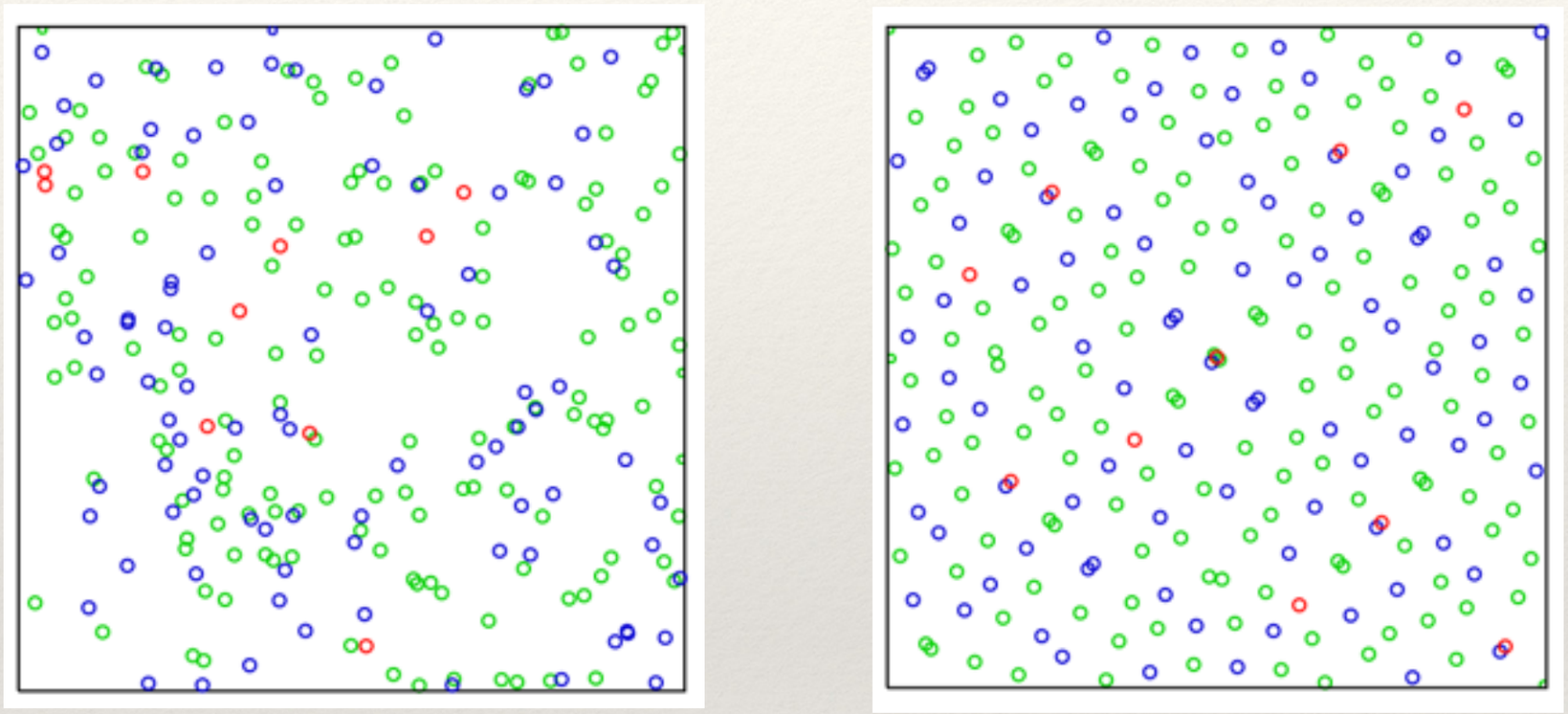
- random - Monte Carlo
- Sampling that better represents that ‘parameter space’ (quasi-random)
- Allow you to cover ‘parameter space’ quickly (and in a sense evenly)
  - Latin Hyper Cube Sample
  - Sobol Sequences
-

Latin Hyper Cube

- generating random samples from equally probability intervals



Sobol' sequences of quasi-random points sample spaces very evenly



256 points from a pseudorandom number source (left); compared with the first 256 points from the 2,3 Sobol sequence (right). The Sobol sequence covers the space more evenly (red=1,..,10, blue=11,..,100, green=101,..,256)

University of Bristol: Sensitivity Analysis - University of Bristol

Tools in R

- PSE (Parameter Space Exploration with Latin Hypercubes)
- Library for Latin Hypercube Sampling and Sensitivity Analysis
- Install

- This approach works when you want to run model offline (outside of R)
- Use LHS to generate samples
- Run model for samples
- Add to LHS object to use plotscatter, plotprcc (and other functions in “pse” package

Sensitivity Analysis in R - workflow

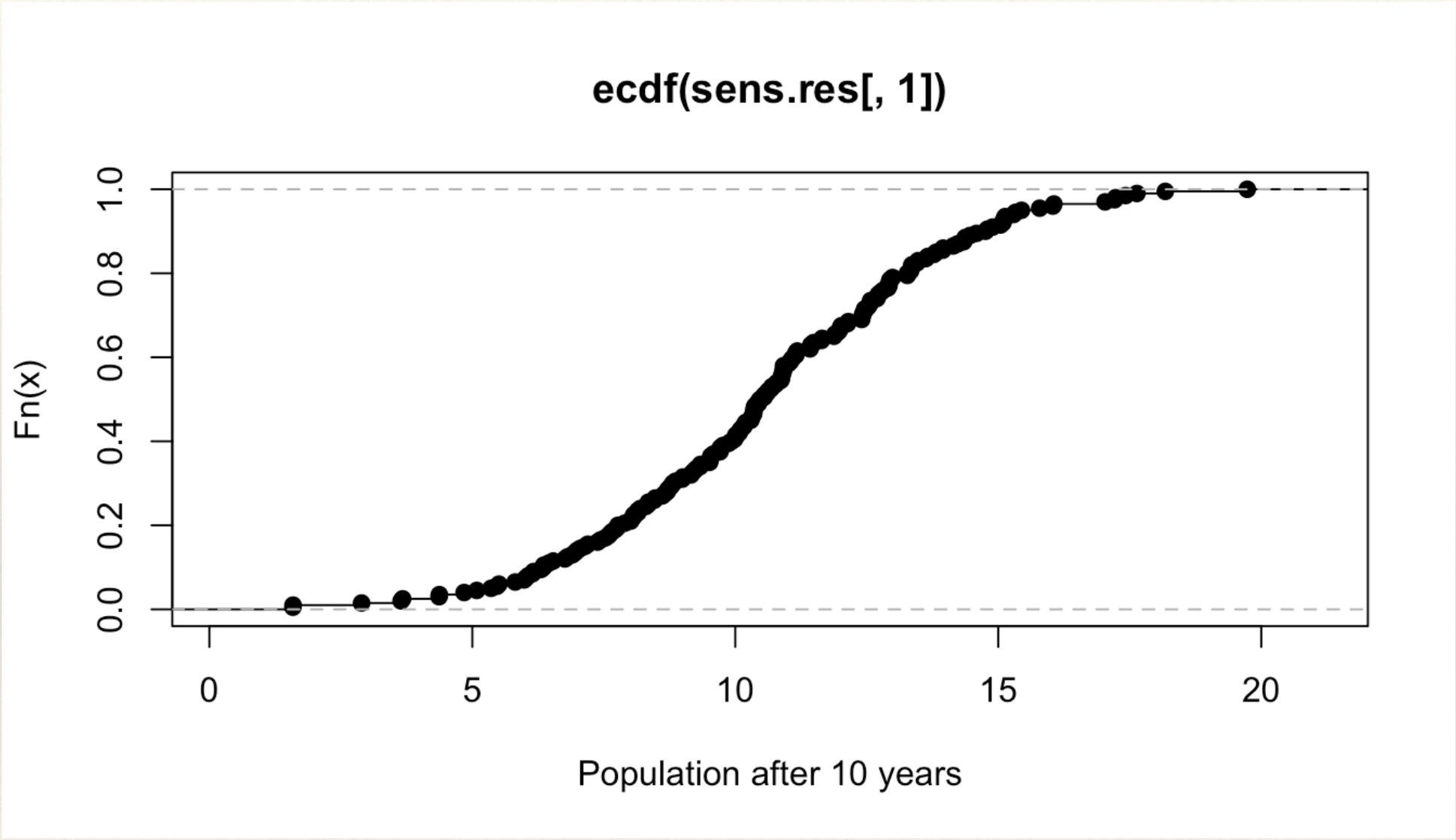
- Create parameters sets using Latin Hypercube Sampling,
- Run the model for these parameter sets
- Graph and Evaluate the results

- Use LHS function
  - parameter names
  - parameter distributions
  - number of parameter sets

Plotting

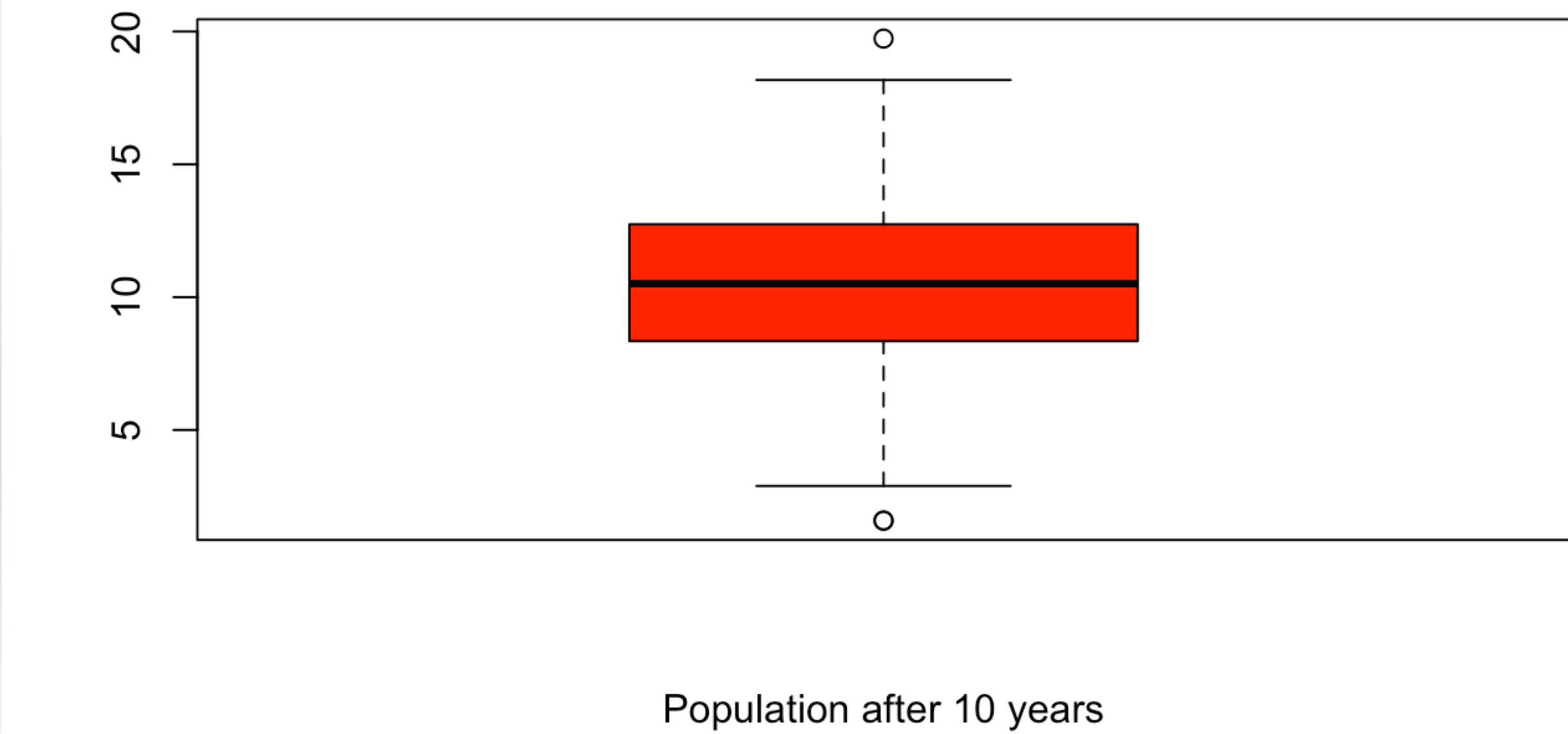
- Plot relationship between parameter and output
  - to understand how uncertainty in parameter impacts the output
  - to determine over what ranges of the parameter uncertainty is most important (biggest effect)
    - can guide sampling
      - can guide use and interpretation of model outputs
- Use a box plot (of output)
  - to graphically show the impact of uncertainty on output of interest

Cumulative Distribution of Output

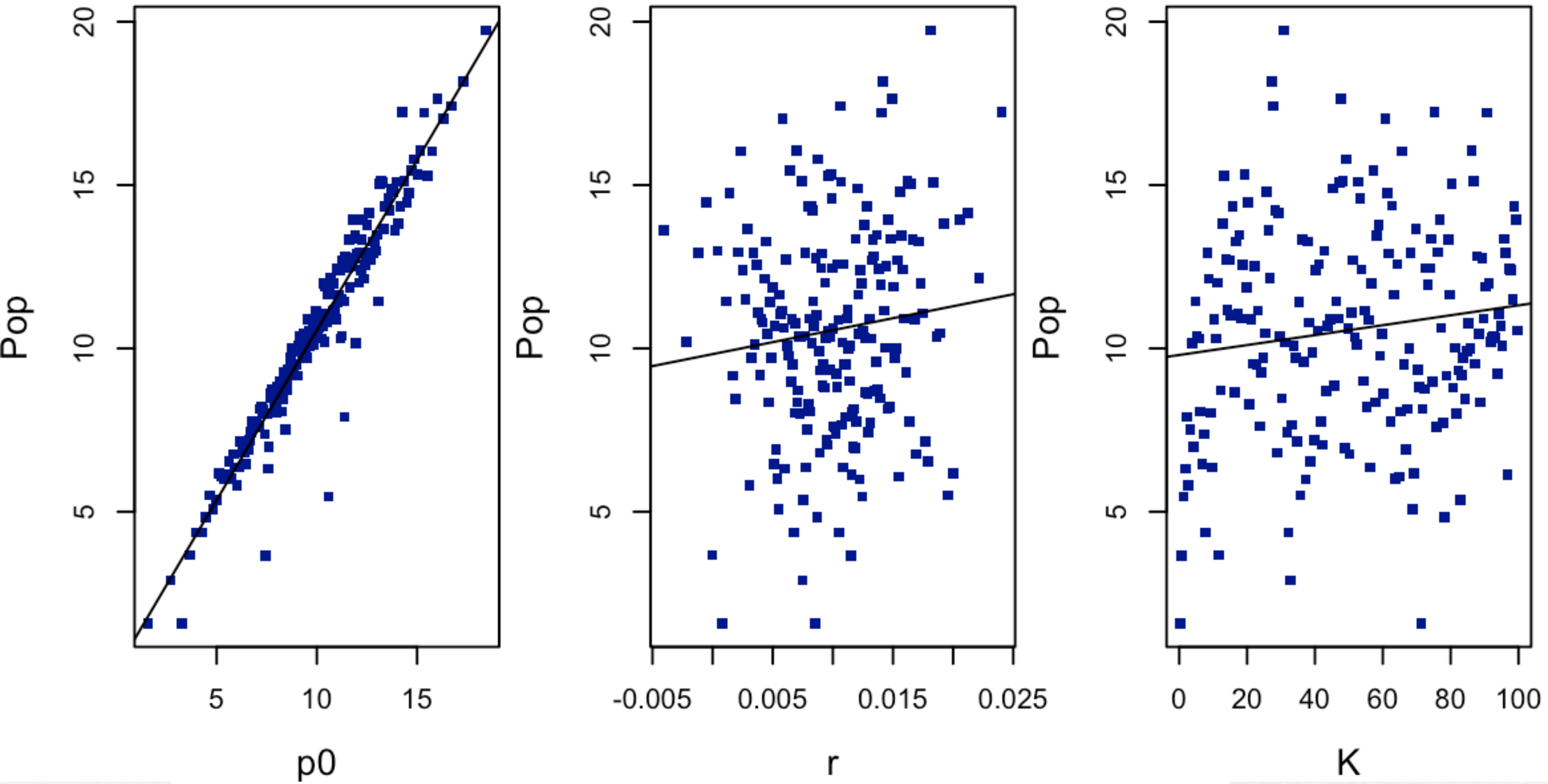



```
plot(ecdf(sens.res[,1]))
```

- Results: Uncertainty of Estimate



- Parameter effects



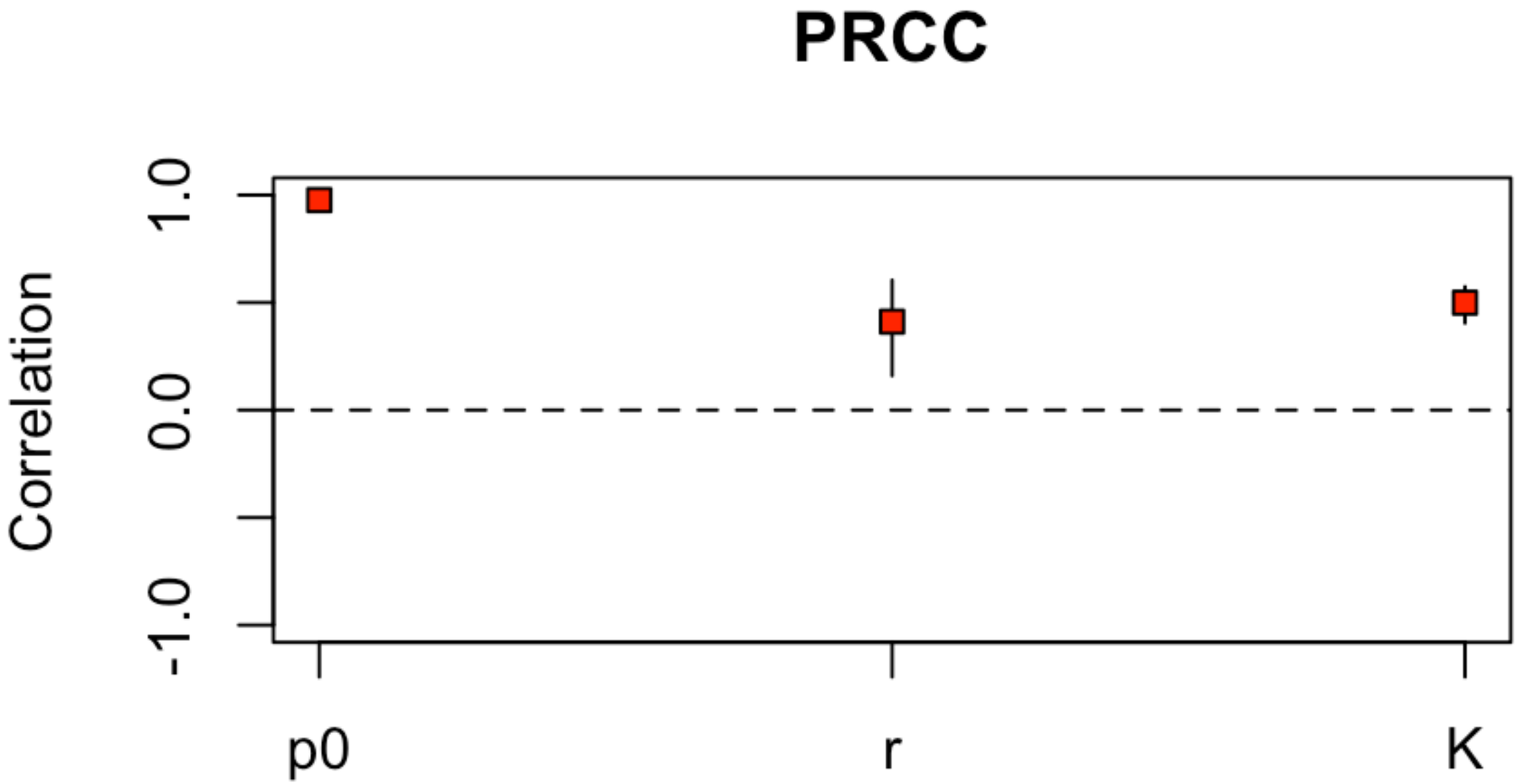

```
plotscatter(sens, ylab="Pop", cex=6, col="darkblue", cex.lab=1.3)
>
```

Quantifying Sensitivity

- If relationships between output and input are close to linear,
  - correlation coefficient between output and input
    - (Partial correlation coefficient for multiple parameters - correlation I-O after other effects accounted for)
  - we can essentially fit a regression relationship slope is a measure of sensitivity (standardized regression coefficients)

Quantifying Sensitivity

- If relationships are not linear
  - spearman rank correlation coefficient
  - (Partial rank correlation coefficient for multiple parameters, PRCC)
  - Sobol method
  - Fourier Amplitude Sensitivity Test




```
plotprcc(sens, cex=2, col="red", ylab="Correlation")
```

- https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2570191/figure/F2/. :Published Example for a predictor prey model
- Detailed sensitivity analysis summary: Pianaso et al. (in gauchospace)
- A shorter simpler review: http://dpannell.fnas.uwa.edu.au/dpap971f.htm

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