Types of Models: WHAT's in the BOX

Conceptual.............Mathematical

Static………….Dynamic :*TIME*

Lumped.............Spatially Distributed: *SPACE*

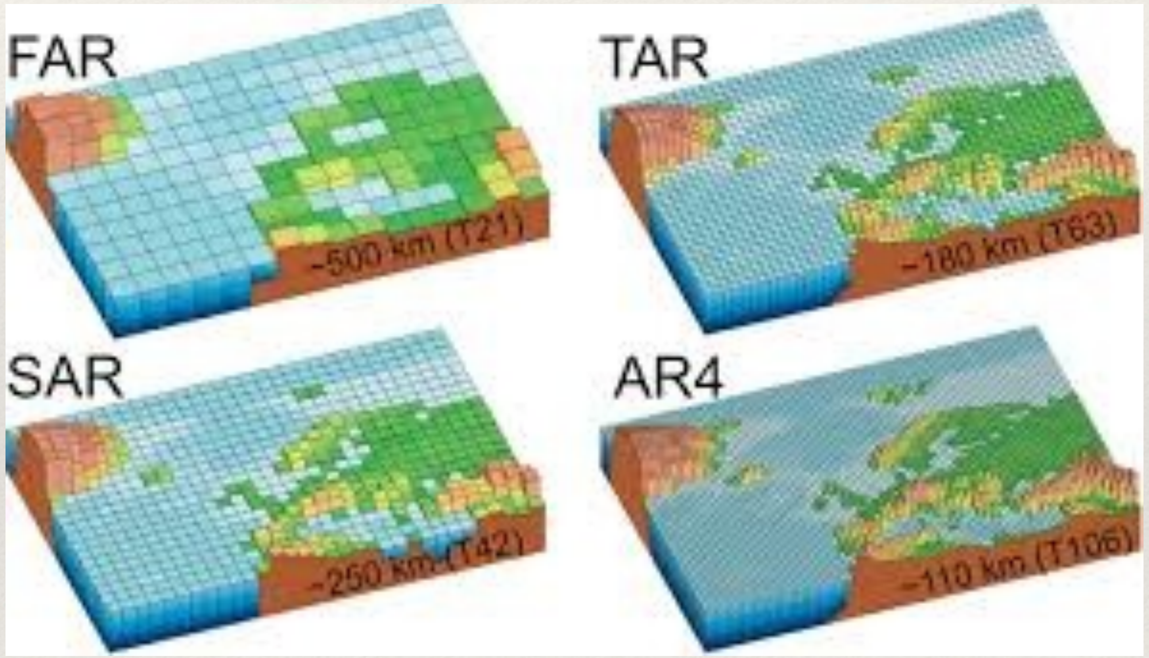
Stochastic.............Deterministic

Abstract………….Physically/Process Based

but biggest differences may often be the degree specific processes/parameters are accounted for

Lumped ...Spatially distributed

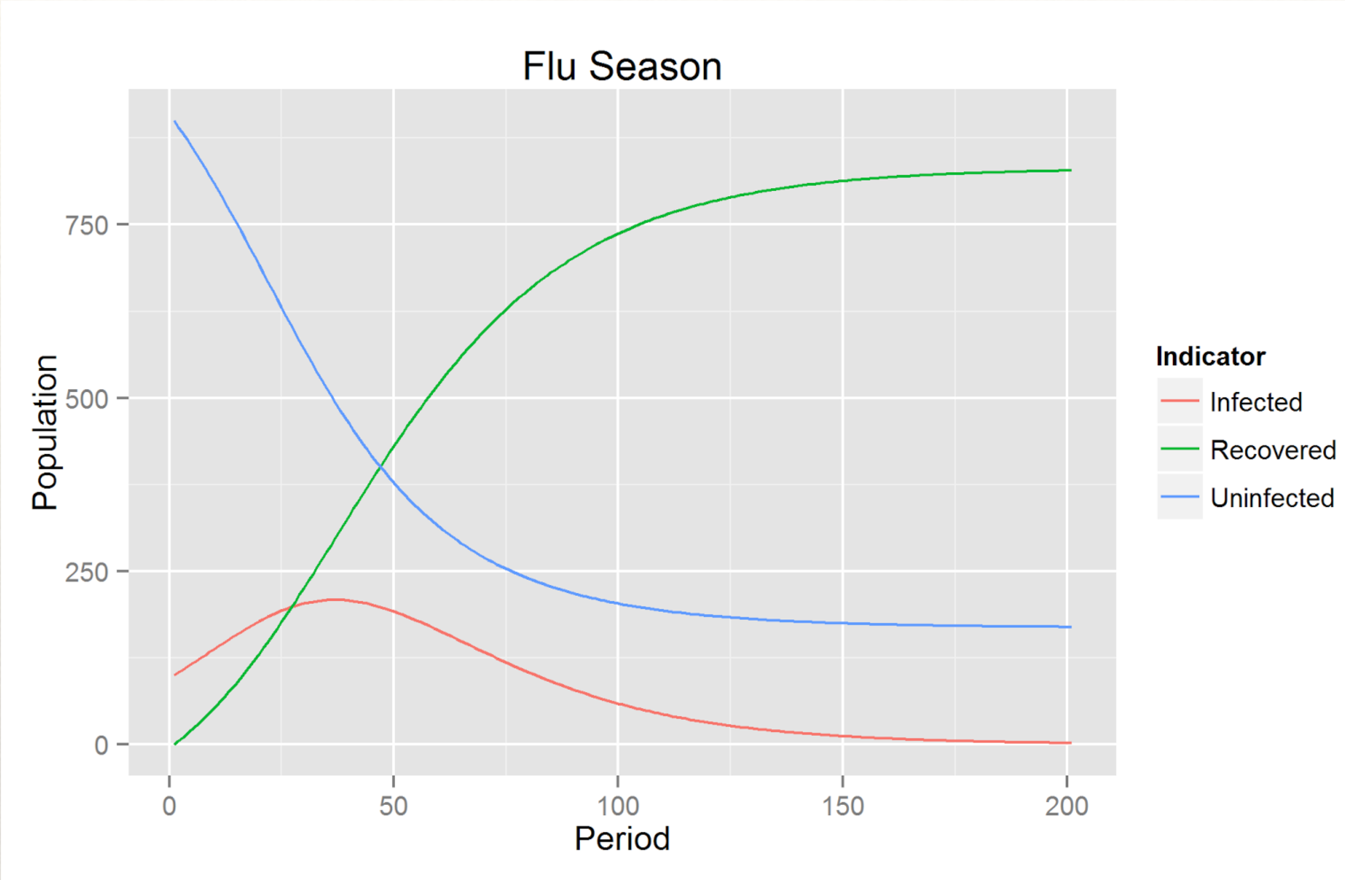
- Lumped - single point in space, or space doesn't matter
- Spatially distributed - model is applied to different "patches" in space
  - spatial units are independent
  - **spatial units interact with each other**



http://eo.ucar.edu/staff/rrussell/climate/modeling/climate_model_resolution.html

Static- Dynamic Time Varying

- Static - Processes or Variables modeled do not evolve with time
- Dynamic - model elements evolve through time - and variables/results at one time step typically depends on previous time step



http://www.econometricsbysimulation.com/2013/05/sir-model-flue-season-dynamic.html

Dynamics - connection in space and time

- Dynamic modeling is common in environmental problems solving
- Similar issues: what happens at one place, depends on neighbors; what happens at one time; depends on previous time
- Space - two way; Time is usually one-way
- Dynamic system modeling - quickly becomes complex (Engineering degrees spend a lot of time on this; there are books, entire journals etc on this topic)

Dynamics models

- Many environmental problems and questions can be related to
    - Diffusion
    - Population
- Both often require dynamics models; and both often require thinking about dynamics in space and in time

Dynamic Systems

Some useful terminology

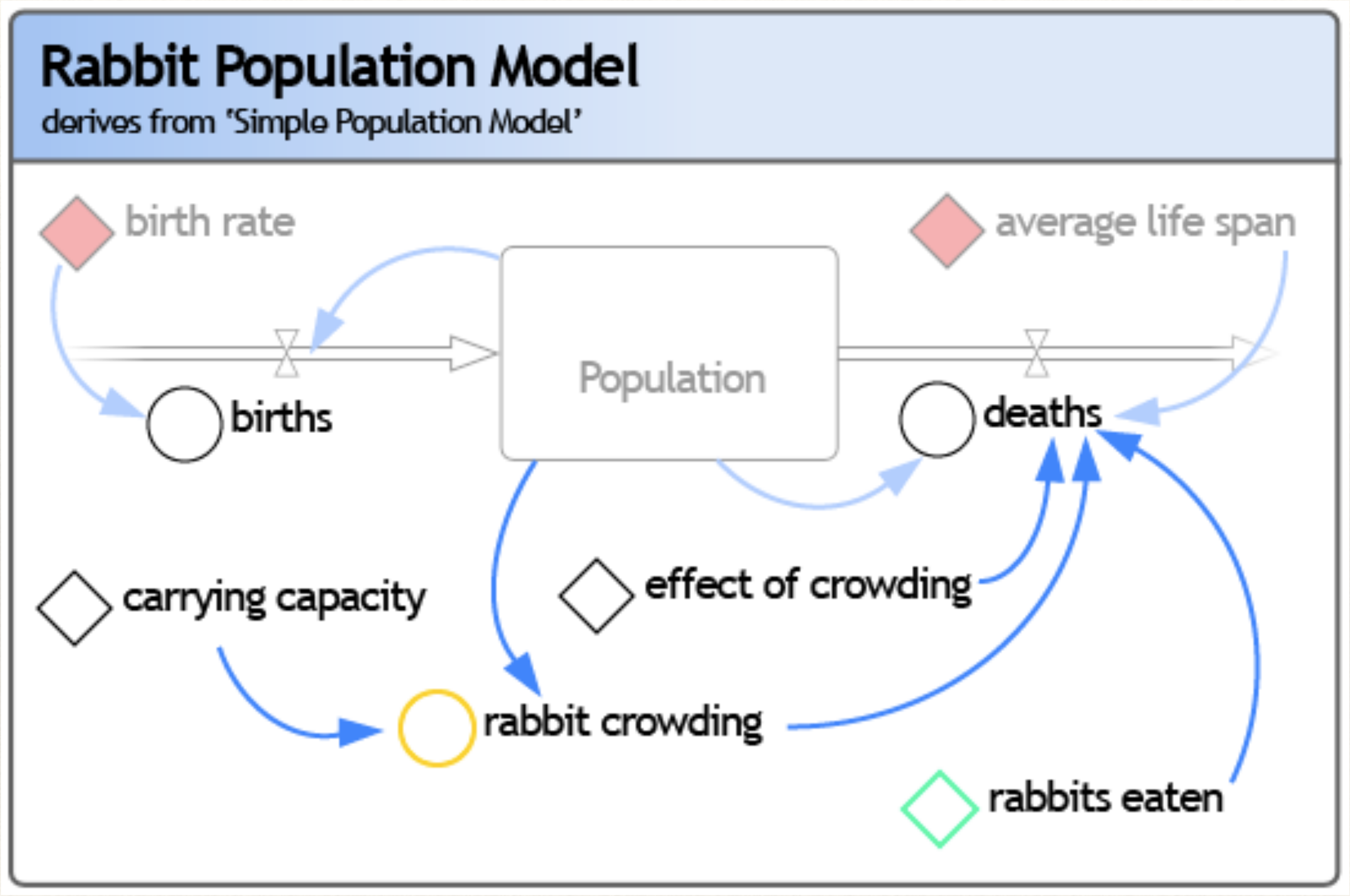
- *stocks* - variables that evolve over time
- *flows* - transfers between variables or from the system
- *parameters* - values that controls the relationship between stocks and flows
- *sink* - something that absorbs flows
- *source* - something that generates flows

Dynamic Systems

- *System state*: value of all variables need to describe the "entity that evolves through time" at a particular point in time
  - usually think of these as stores (soil moisture, bank account balance, number of individuals in a particular age class)
- *State-space*: description of the entity may require multiple variables - for a watershed this could be soil moisture, water currently in dam and water stored in trees, and for each "grid" in a watershed)
- *State-space trajectories*: how the system state evolves through time, often involving transfers/flows between different stocks
- *Initial conditions*: values to describe the system state at the beginning
- *Dynamic systems modeling*: there at least one feedback loop

Dynamic Systems

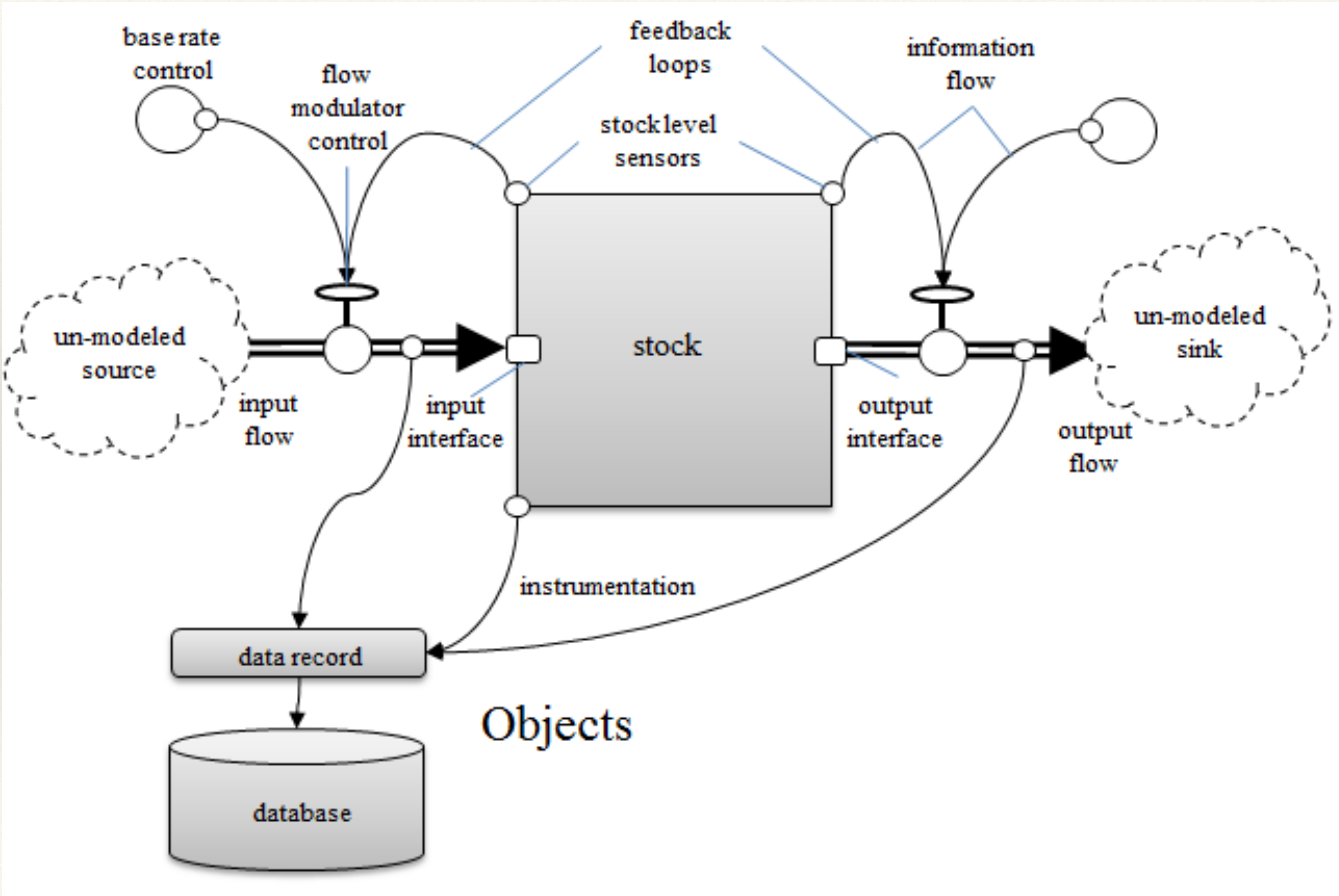
Nature



Notice how the change (flow in/out) of stock (population) depends on current value of stock
Feedback loop!

Dynamic Systems

Human Engineered



Dynamic Systems

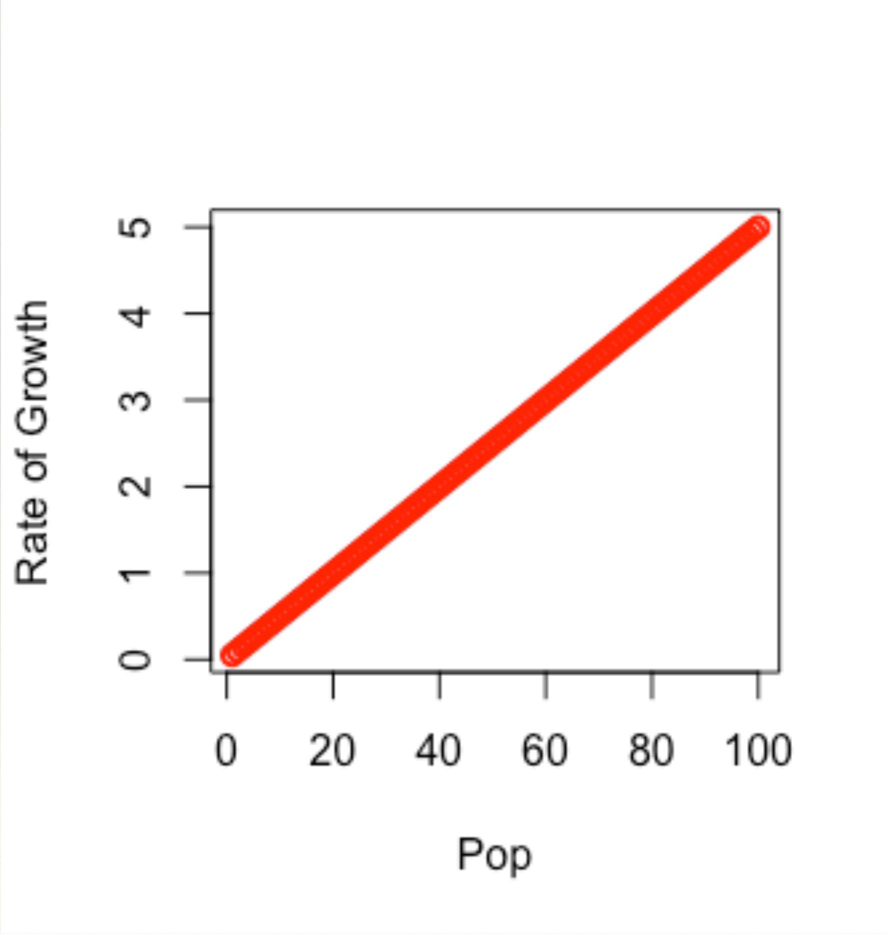
- dynamic systems have **feedback** loops
  - positive feedback
  - negative feedback
- feedback loops often lead to highly non-linear responses
- IF you have a feedback loop you need:
  - *difference* and *differential* equations: basically describe how the state evolves through time

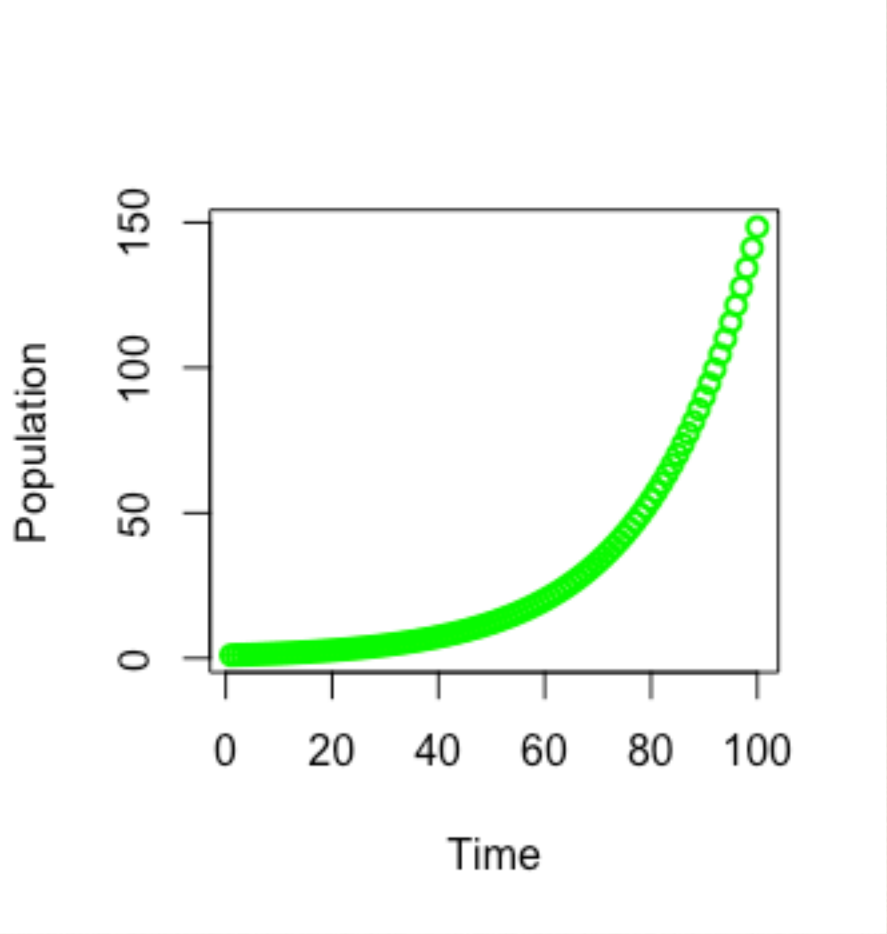
Dynamic Systems

- Dynamic system may lead to stable or unstable states over time
    - stable …converge over time to a set of values or a repeated pattern
    - unstable…grow to infinity
    - chaotic - high sensitivity to initial conditions
    - for the same dynamic system (same set of equations), whether you are stable or unstable can depend on initial conditions and parameters

Exponential Growth - Simple Dynamic System

- rate of growth(change) = r * population(density)
- differential equation
- $dP/dt = rP$



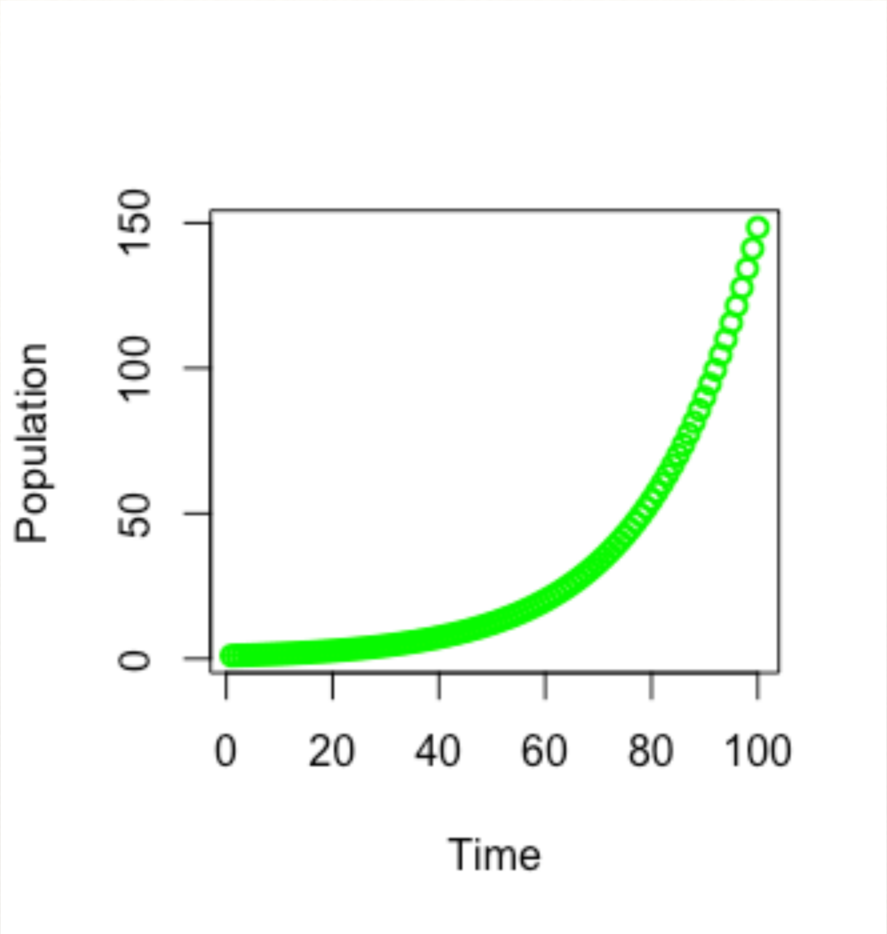


Exponential Growth - Simple Dynamic System

- differential equation
- $dP/dt = rP$
- an analytic solution exists so we can write Population as a function of time (integrating both sides)
    - $P = P0 * \exp(rt)$

Exponential Growth - Simple Dynamic System

```
#'  Simple population growth
#' @param T  period of growth
#' @param P initial population
#' @param r intrinsic growth rate
#' @return population at time T
#'
exppop = function(T,P0,r) {
   P = P0 * exp(r*T)
   return(P)
}
```



See Rmarkdown *SimpleDynamicModels*.Rmd

Exponential Growth - Simple Dynamic System

But what if we couldn’t ‘solve’ it analytically ????

Integrate the differential equation step by step

Also called numerical integration!

R has tools to help you do this!

First you need to code your differential equation as a function

Exponential Growth - Simple Dynamic System

Use R's ODE solver in the de**S**olve package - note the S

ODE works with ordinary differential equations…

Ordinary differential equations have derivatives of only one variable ((population) with respect to (time).,..it can have multiple order derivatives

dp/d**t** + d2p/d**t** + c = 0

Partial differential equations have derivatives with respect to more than one variable (think of spatial issues 3du/d**x** + 4du/d**y** = 0

Numerical Integration

Use R's ODE solver in the de**S**olve package

ODE requires initial conditions

Values of independent variable that you want results for

The function itself and parameters (as a single list)

Example: Dynamic Systems

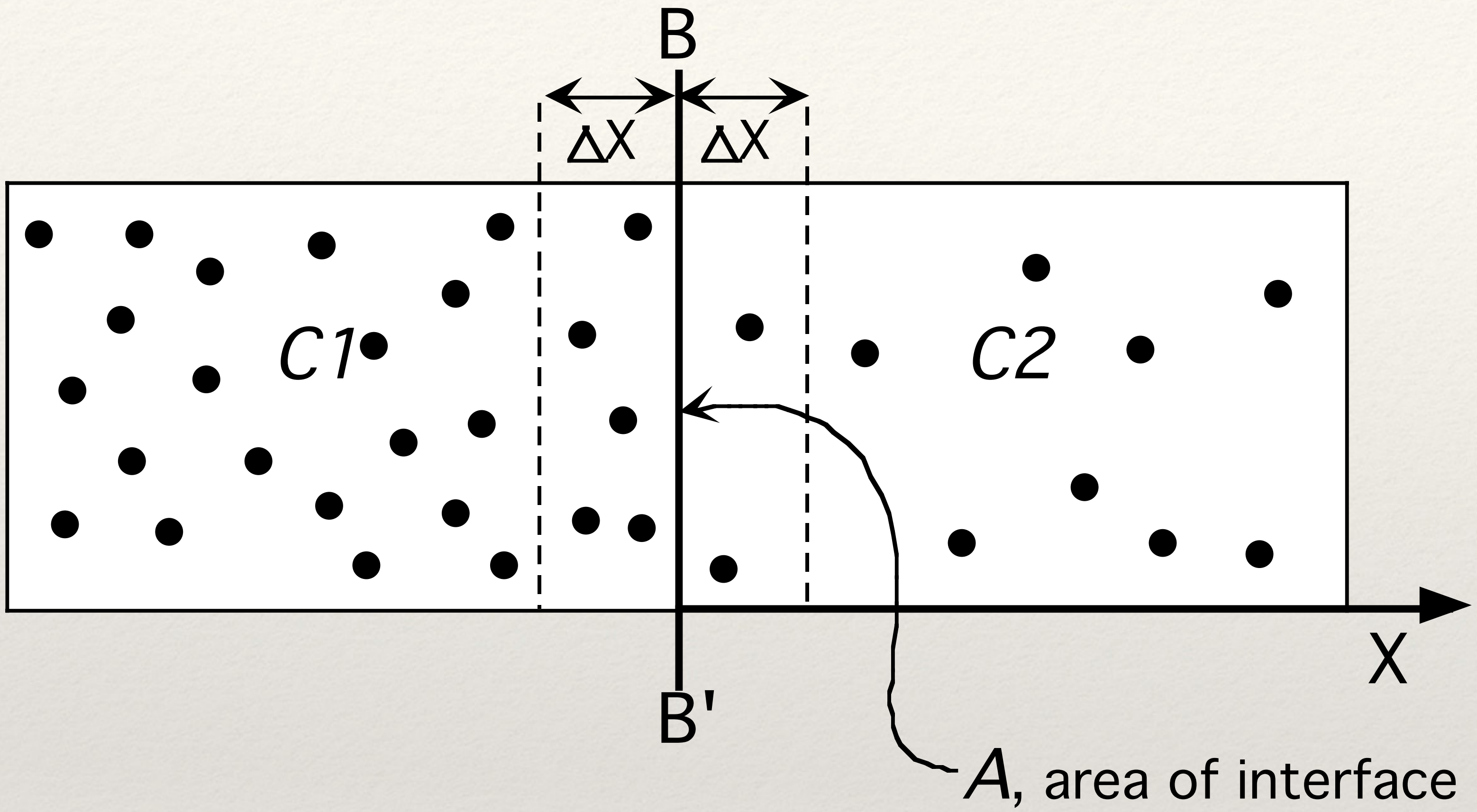
- Diffusion - Ficks Law
- flux (mass/time) =

    -Diffusivity * concentration gradient * area

    a concentration gradient is C2-C1 / length

Example: Dynamic Systems

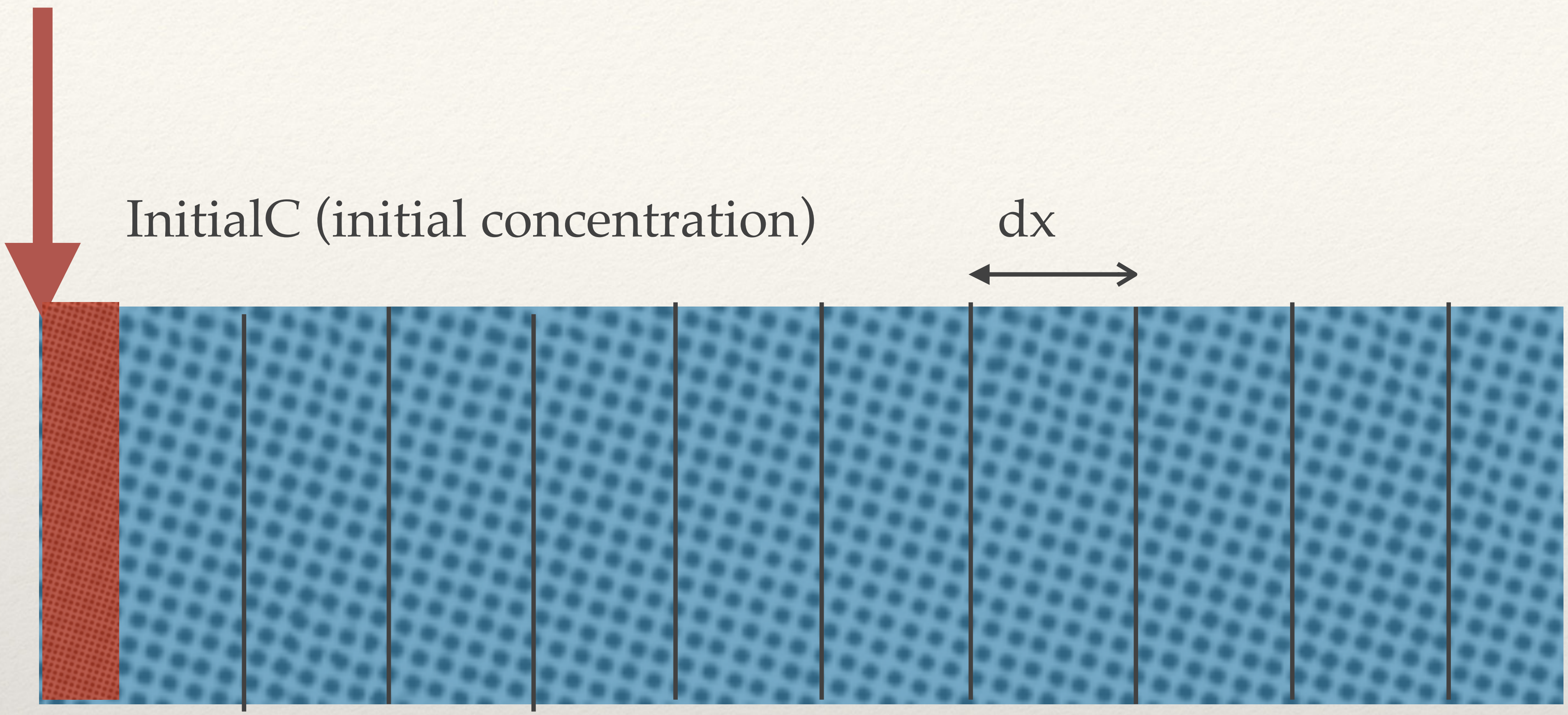
- Diffusion



How big that delta X is, depends on characteristics of the particles - often represented as a diffusivity (D) term

$$(5) \qquad q_x = \frac{0.5\Delta XA(C1 - C2)}{\Delta t}.$$

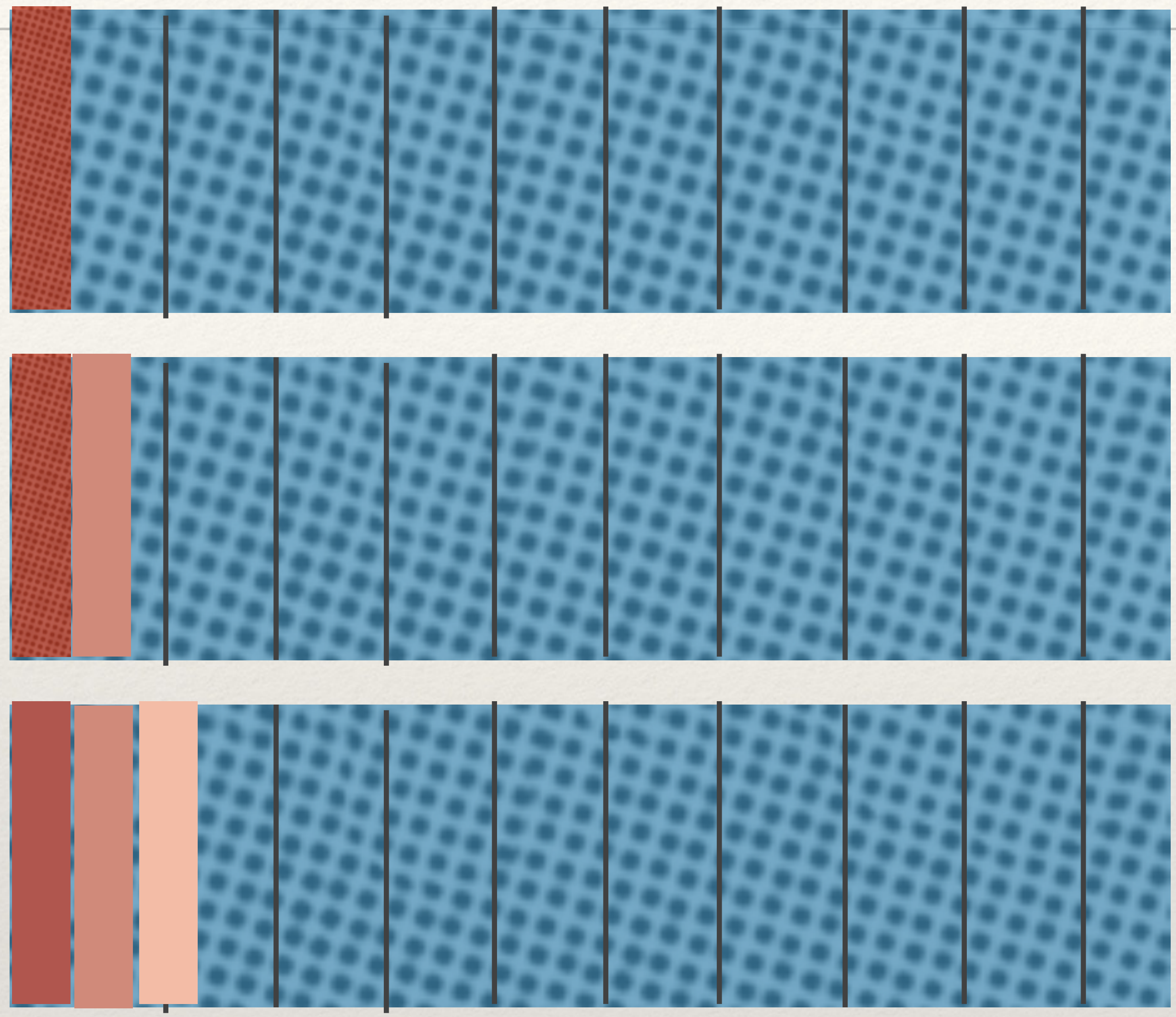
Diffusion Example: Difference equation



Length = nx * dx

Nx: number of "boxes"
dx: discretization of space

Diffusion Example



Timestep 1

Timestep 2

Timestep 3

State evolves through time (state space trajectory)

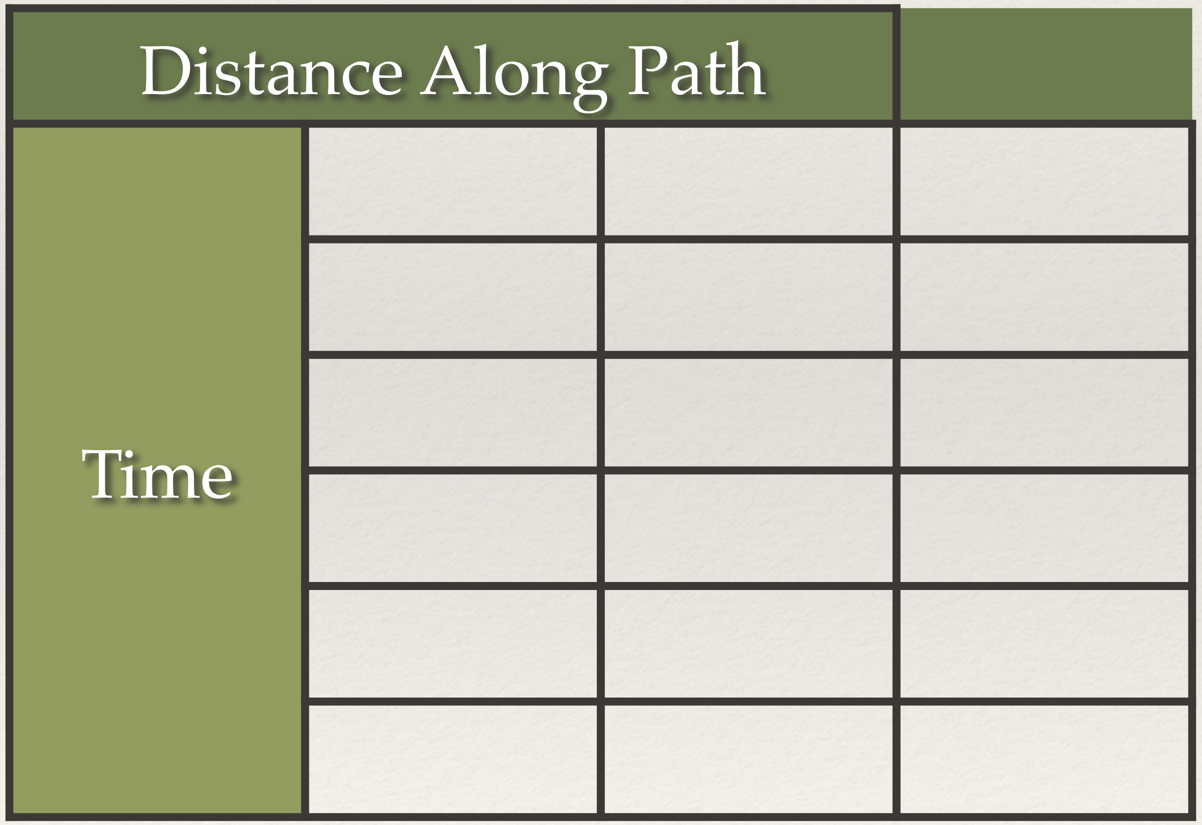
…

Total Simulation time = nt*dt

nt : number of time steps
dt - discretization of time

Dynamic Modeling

- data structure to store the state (conc)
- 2-d array (rows are time, columns are distance along path)
- use it to track concentration through time



Dynamic - Diffusion modeling

- Choosing the appropriate time and space step is important - if either are too large then it is easy to overshoot and create unstable oscillations
- These are a due to using a discrete model (dividing things into units) to model what is actually a continuous process
- Trade-off - computational efficiency vs stability
- Differential equations help us to think through this problem - but often implemented in discrete ways