OLS Classification on Randomly Generated Data

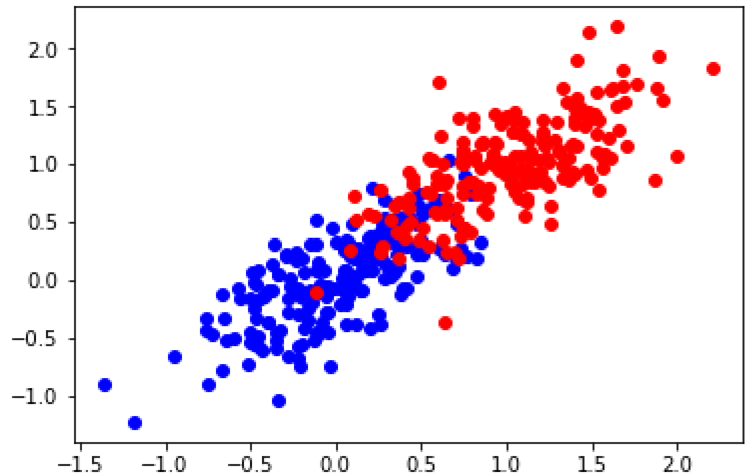
In this notebook, we use Ordinary Least Squares to carry out classification on a randomly generated data set.

In [1]:
```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
from scipy import linalg
from scipy import io
```

In [2]:
```
### Ordinary Least Squares
### SOLVES 2-CLASS LEAST SQUARES PROBLEM

### LOAD DATA ###
### IF LoadClasses IS True, THEN LOAD DATA FROM FILES ###
### OTHERSIE, RANDOMLY GENERATE DATA ###
LoadClasses    = False
TrainOutliers  = False
TestOutliers   = False
NOut           = 20
NSampsClass    = 200
NSamps         = 2*NSampsClass

if LoadClasses:

    ### GET FILENAMES %%%
    ### THESE ARE THE OPTIONS ###
    ### LinSepC1, LinSepC2,LinSepC2Outlier (Still Linearly Separable) ###
    ### NonLinSepC1, NonLinSepC2, NonLinSepC22 ###

    InFile1          = '../data/PaulGader/LinSepC1.mat'
    InFile2          = '../data/PaulGader/LinSepC2.mat' #Change to LinSepC2Outlie
    C1Dict           = io.loadmat(InFile1)
    C2Dict           = io.loadmat(InFile2)
    C1               = C1Dict['LinSepC1']
    C2               = C2Dict['LinSepC2']

    if TrainOutliers:
        ### Let's Make Some Noise ###
        Out1        = 2*np.random.rand(NOut,2)-0.5
        Out2        = 2*np.random.rand(NOut,2)-0.5
        C1          = np.concatenate((C1,Out1),axis=0)
        C2          = np.concatenate((C2,Out2),axis=0)
        NSampsClass = NSampsClass+NOut
        NSamps      = 2*NSampsClass
else:
    ### Randomly Generate Some Data
    ### Make a covariance using a diagonal array and rotation matrix
    pi      = 3.141592653589793
    Lambda1 = 0.25
    Lambda2 = 0.05
    DiagMat = np.array([[Lambda1, 0.0],[0.0, Lambda2]])
    RotMat  = np.array([[np.sin(pi/4), np.cos(pi/4)], [-np.cos(pi/4), np.sin(pi/4
    mu1     = np.array([0,0])
    mu2     = np.array([1,1])
    Sigma   = np.dot(np.dot(RotMat.T, DiagMat), RotMat)
    C1      = np.random.multivariate_normal(mu1, Sigma, NSampsClass) #generate tr
    C2      = np.random.multivariate_normal(mu2, Sigma, NSampsClass)
    #to generate a test set, repeat lines defining C1, C2 many times from same di
    #generate AllSamps
    #

    print(Sigma)
    print(C1.shape)
    print(C2.shape)
```

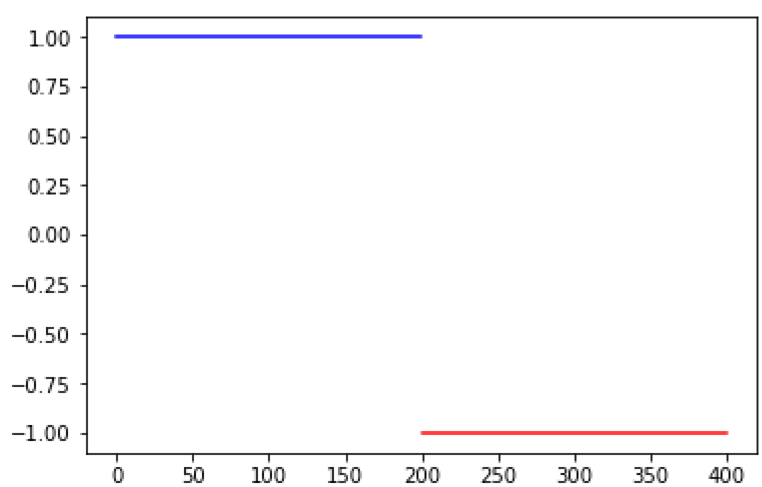
```
### PLOT DATA ###
plt.figure(1)
plt.plot(C1[:NSampsClass, 0], C1[:NSampsClass, 1], 'bo')
plt.plot(C2[:NSampsClass, 0], C2[:NSampsClass, 1], 'ro')
plt.show()
```

```
[[ 0.15  0.1 ]
 [ 0.1   0.15]]
(200, 2)
(200, 2)
```



Set up target outputs and plot:

In [3]:
```
### SET UP TARGET OUTPUTS ###
TargetOutputs          = np.ones((NSamps,1)) #Set 1st half (400 values) to 1s
TargetOutputs[NSampsClass:NSamps] = -TargetOutputs[NSampsClass:NSamps]
#Set 2nd half to -1s (now 1st 200 are 1, 2nd 200 are -1)

### PLOT TARGET OUTPUTS ###
plt.figure(2)
plt.plot(range(NSampsClass),         TargetOutputs[range(NSampsClass)],   'b-')
plt.plot(range(NSampsClass, NSamps), TargetOutputs[range(NSampsClass, NSamps)], '
plt.show()
```



Calculate the least squares solutions using Ordinary Least Squares Solution:

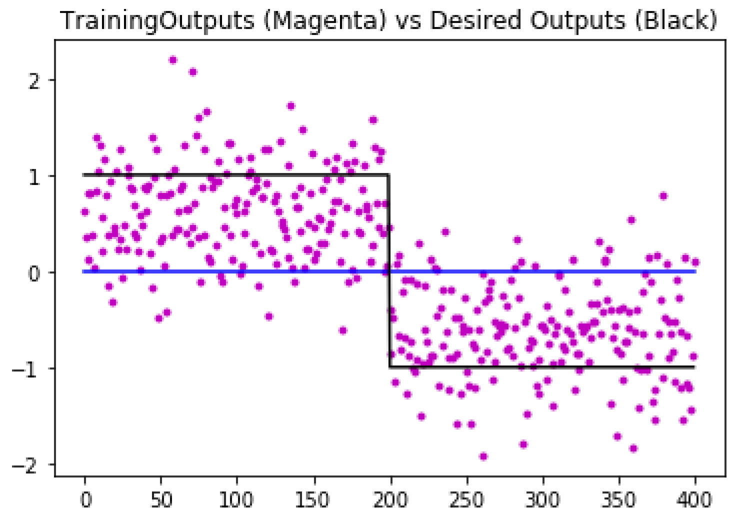
$$w = (X^T X)X - Ty$$

In [4]:
```
### FIND LEAST SQUARES SOLUTION ###
AllSamps     = np.concatenate((C1,C2),axis=0) #concatenate two classes into a sin
#(before classes were separated into two arrays; axis tells you the way you are co

#add ones to last column -- this adds the y-intercept (i.e. shifts line up and dow
AllSampsBias = np.concatenate((AllSamps, np.ones((NSamps,1))), axis=1)

#calculate pseudo-inverse
Pseudo       = linalg.pinv2(AllSampsBias)

#multipy pseudo-inverse by the target outputs to get the weighting factors
w            = Pseudo.dot(TargetOutputs) #weights (parameters you are trying to e
```

In [5]:
```
# Display the shapes of the original samples and the concatenated ones:
print('C1 shape:',C1.shape)
print('C2 shape:',C2.shape)
print('AllSamps shape:',AllSamps.shape)
print('weighting factors:\n',w)
```

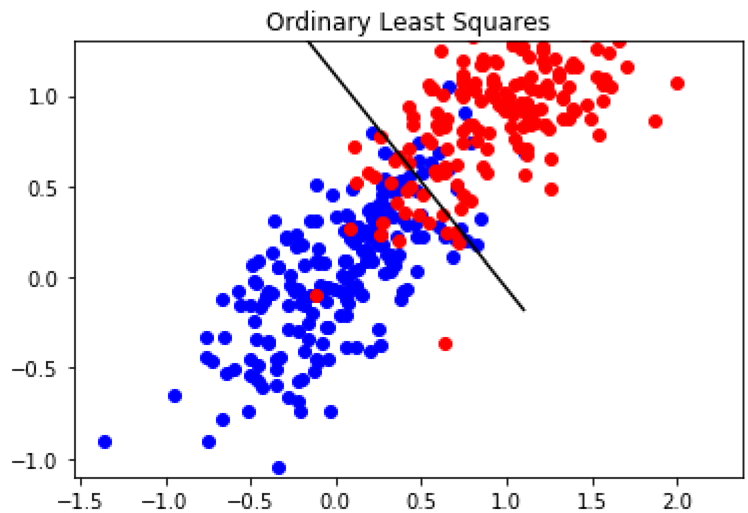
```
C1 shape: (200, 2)
C2 shape: (200, 2)
AllSamps shape: (400, 2)
weighting factors:
 [[-0.59095569]
 [-0.69307886]
 [ 0.65632439]]
```

In [15]:
```
### COMPUTE OUTPUTS ON TRAINING DATA ###
y = AllSampsBias.dot(w) #calculate y from the least squares

### PLOT OUTPUTS FROM TRAINING DATA ###
plt.figure(3)
plt.plot(range(NSamps), y, 'm.')
plt.plot(range(NSamps),np.zeros((NSamps,1)), 'b') #Zero is the classification "th
plt.plot(range(NSamps), TargetOutputs, 'k')
plt.title('TrainingOutputs (Magenta) vs Desired Outputs (Black)')
plt.show()
```



The line is the decision criteria for classification

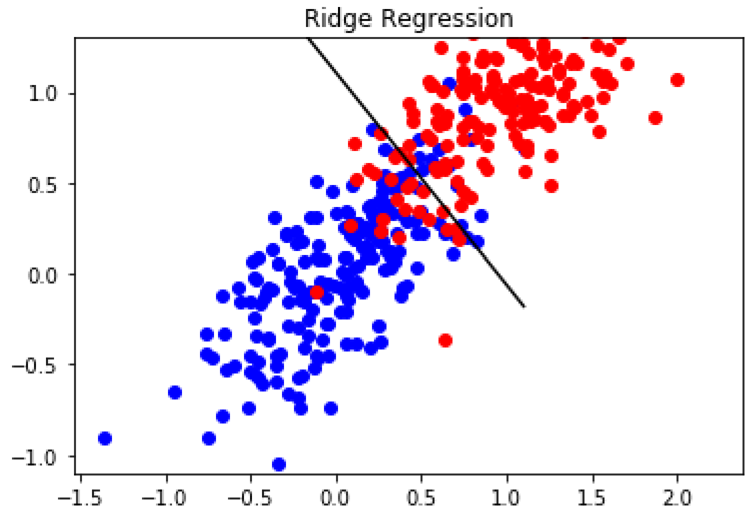
In [7]:
```
### CALCULATE AND PLOT LINEAR DISCRIMINANT ###
Slope     = -w[1]/w[0]
Intercept = -w[2]/w[0]
Domain    = np.linspace(-1.1, 1.1, 60) #Tile the decision surface over the range
Disc      = Slope*Domain+Intercept

plt.figure(4)
plt.plot(C1[:NSampsClass, 0], C1[:NSampsClass, 1], 'bo')
plt.plot(C2[:NSampsClass, 0], C2[:NSampsClass, 1], 'ro')
plt.plot(Domain, Disc, 'k-')
plt.ylim([-1.1,1.3])
plt.title('Ordinary Least Squares')
plt.show()
```



Ridge Regression -- Diagonally Load by adding $$\lambda

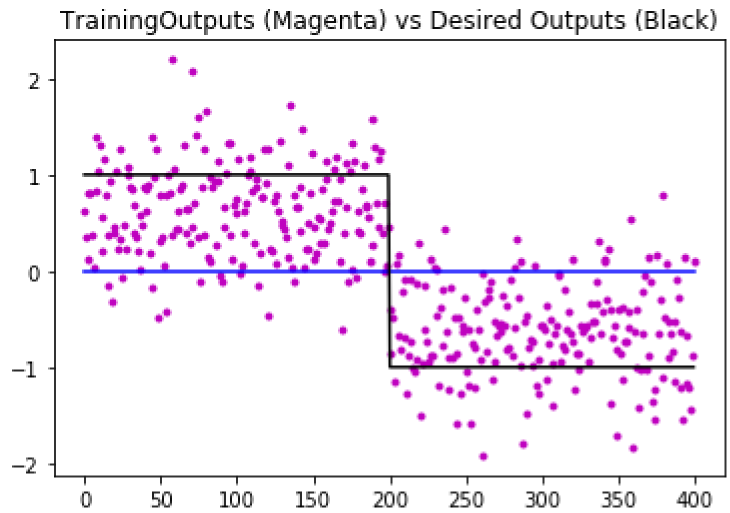
In [8]:
```
RegConst      = 0.1 #set lambda to 0.1
AllSampsBias  = np.concatenate((AllSamps, np.ones((NSamps,1))), axis=1)

#calculate transpose of matrix
AllSampsBiasT = AllSampsBias.T

#multiply the transpose by the original matrix (XTX)
XtX           = AllSampsBiasT.dot(AllSampsBias)

#Add regularization constant term (lambda)
AllSampsReg   = XtX + RegConst*np.eye(3) #np.eye(3) is the 3x3 identity matrix
Pseudo        = linalg.pinv2(AllSampsReg) #take the pseudo-inverse
wr            = Pseudo.dot(AllSampsBiasT.dot(TargetOutputs)) #calculate ridge reg
```

In [9]:
```
Slope     = -wr[1]/wr[0]
Intercept = -wr[2]/wr[0]
Domain    = np.linspace(-1.1, 1.1, 60)
Disc      = Slope*Domain+Intercept

matplotlib.pyplot.figure(5)
matplotlib.pyplot.plot(C1[:NSampsClass, 0], C1[:NSampsClass, 1], 'bo')
matplotlib.pyplot.plot(C2[:NSampsClass, 0], C2[:NSampsClass, 1], 'ro')
matplotlib.pyplot.plot(Domain, Disc, 'k-')
matplotlib.pyplot.ylim([-1.1,1.3])
matplotlib.pyplot.title('Ridge Regression')
matplotlib.pyplot.show()
```



Save this project with the name: OLSandRidgeRegress2DPGader. Make a New Project for Spectra.

In [14]:
```
### COMPUTE OUTPUTS ON TRAINING DATA ###
yr = AllSampsBias.dot(wr)

### PLOT OUTPUTS FROM TRAINING DATA ###
matplotlib.pyplot.figure(6)
matplotlib.pyplot.plot(range(NSamps), yr, 'm.')
matplotlib.pyplot.plot(range(NSamps),np.zeros((NSamps,1)), 'b')
matplotlib.pyplot.plot(range(NSamps), TargetOutputs, 'k')
matplotlib.pyplot.title('TrainingOutputs (Magenta) vs Desired Outputs (Black)')
matplotlib.pyplot.show()
```



In [11]:
```
#Ordinary Least Squares
y1 = y[range(NSampsClass)]
y2 = y[range(NSampsClass, NSamps)]
Corr1 = np.sum([y1>0]) #of correctly classified points in first class
Corr2 = np.sum([y2<0]) #of correctly classified points in second class

#Ridge Regression
y1r = yr[range(NSampsClass)]
y2r = yr[range(NSampsClass, NSamps)]
Corr1r = np.sum([y1r>0])
Corr2r = np.sum([y2r<0])
```

In [12]:
```
print('Result for Ordinary Least Squares')
CorrClassRate=(Corr1+Corr2)/NSamps
print(Corr1 + Corr2, 'Correctly Classified for a', round(100*CorrClassRate), '% C

print('Result for Ridge Regression')
CorrClassRater=(Corr1r+Corr2r)/NSamps
print(Corr1r + Corr2r, 'Correctly Classified for a', round(100*CorrClassRater), '
```

```
Result for Ordinary Least Squares
360 Correctly Classified for a 90.0 % Correct Classification

Result for Ridge Regression
360 Correctly Classified for a 90.0 % Correct Classification
```

In [13]:
```
### Make Confusion Matrices ###

# A confusion matrix tells you how many you got right v. wrong
# If all are right, should be 100% along the diagonal
NumClasses = 2;
Cm         = np.zeros((NumClasses,NumClasses))
Cm[(0,0)]  = Corr1/NSampsClass
Cm[(0,1)]  = (NSampsClass-Corr1)/NSampsClass
Cm[(1,0)]  = (NSampsClass-Corr2)/NSampsClass
Cm[(1,1)]  = Corr2/NSampsClass
Cm           = np.round(100*Cm)
print('Confusion Matrix for OLS Regression \n', Cm, '\n')

Cm           = np.zeros((NumClasses,NumClasses))
Cm[(0,0)]    = Corr1r/NSampsClass
Cm[(0,1)]    = (NSampsClass-Corr1r)/NSampsClass
Cm[(1,0)]    = (NSampsClass-Corr2r)/NSampsClass
Cm[(1,1)]    = Corr2r/NSampsClass
Cm           = np.round(100*Cm)
print('Confusion Matrix for Ridge Regression \n', Cm, '\n')
```

```
Confusion Matrix for OLS Regression 
 [[ 92.   8.]
 [ 12.  88.]] 

Confusion Matrix for Ridge Regression 
 [[ 92.   8.]
 [ 12.  88.]] 
```

EXERCISE: Run Ordinary Least Squares and Ridge Regression on spectra and plot the weights.