Utility and implicatures of imperatives

Maria Aloni
ILLC/Department of Philosophy
University of Amsterdam
M.D.Aloni@uva.nl

Abstract

The article defines the relevance or utility of an imperative in terms of how far it can help in increasing the probability of the occurrence of a desirable future world. In terms of this notion, we account for (i) the potential of imperatives to license free choice *any* in their scope; and (ii) the free choice effects of disjunctive and *any*-imperatives.

1 Choice-offering imperatives

1.1 *Or* in imperatives

It is a well known fact that *or* in imperatives can give rise to a free choice effect, see (Ross, 1941; Åquist, 1965; Hamblin, 1987) and more recently (Aloni, 2003).

(1) $!(A \vee B) \;\Rightarrow\; \Diamond A \wedge \Diamond B$

As an illustration of (1), consider the following example:

(2) SMITH: Take her to Knightsbridge or Bond Street!

JONES STARTS TO LEAVE.

SMITH: (?) Don't you dare take her to Bond Street!

Intuitively the most natural interpretation of Smith's first imperative is as one presenting a choice between two different actions. Smith's subsequent imperative can be regarded as negating this choice, and, therefore, strikes us as out of place here.

The free choice inference in (1), however, is not always warranted as illustrated by the following example from Rescher and Robinson (1964):

(3) TEACHER: John, stop that foolishness or leave the room!

JOHN STARTS TO LEAVE.

TEACHER: Don't you dare leave this room!

Examples like (3) suggest to treat free choice effects as pragmatic implicatures, rather than semantic entailments. In the classical literature (notably (Åquist, 1965)), examples like (3) has been presented as evidence in favor of an ambiguity between choice-offering and alternative-presenting disjunctive imperatives. On a pragmatic approach, the failure of the free choice inference in example (3) can be explained as an implicature cancelation without multiplying the senses of imperative sentences.

A further indication that free choice effects of disjunctive imperatives are conversational implicatures is the fact that they disappear in negative environments (e.g. Gazdar 1979).

(4) Don't post this letter or burn it!

If free choice inferences had the status of logical entailment, then (4) could be used in a situation in which one wants the letter to be posted or burnt, but doesn't want to leave the choice to the hearer. This is clearly not so.

1.2 *Any* in imperatives

Another example of a 'choice-offering' imperative is (5) with an occurrence of free choice *any* which is licensed in this context.

(5) Take any card!

Like disjunctive imperatives, *any*-imperatives should be interpreted as carrying with them the inference that a choice is being offered.

(6) $!(any\ x\ \phi)\ \Rightarrow\ \forall x \Diamond \phi$

As in the case of disjunctive imperatives, the free choice effect in (6) disappears under negation. One needs a special stress to retain it, as in (8).[1]

(7) Don't take any card!

(8) Don't take just ANY card!

Contrary to disjunctive imperatives, however, in a positive environment, the inference in (6) is hard to cancel. Contrast (9) with (10).

(9) MARIA: Take any card!

YOU START TO TAKE A CARD.

MARIA: # Don't you dare take the ace!

(10) MARIA: Take a card!

YOU START TO TAKE A CARD.

MARIA: (?) Don't you dare take the ace!

Imagine a context in which it is well known that aces cannot be taken. In such a context, Maria's second imperative in (10) would be natural. In (9), however, it would be still out of place. By using *any*, in (9), rather than *a*, Maria conveys that no exceptions apply to her prescription: even aces must be permissible options.

This reduced tolerance of exceptions typical of uses of *any* has been discussed in (Kadmon and Landman, 1993). On their account, *any* has the effect of WIDENING the domain of quantification compared to a standard use of an indefinite noun phrase. Furthermore, domain widening should be for a reason. *Any* is licensed only in those cases where widening the domain is functional, i.e., leads to a STRENGTHENING of the statement made.

Domain widening and strengthening (defined in terms of *entailment*) explain the following distribution facts:

(11) a. John did not take any card.

$\neg\exists x\phi$

b. # John took any card.

$\exists x\phi$

Enlarging the domain of an existential in the scope of negation does create a stronger statement (example (11a)). In an episodic sentence, it doesn't (example (11b)).

It is easy to see, however, that this sort of explanation does not extend directly to non-declarative cases. Let us assume Groenendijk and Stokhof's (1984) notion of entailment for interrogatives, and the standard notion of entailment for imperatives defined in terms of inclusion of their compliance conditions.[2] Then, widening the domain of an existential in an interrogative or an imperative does not create a stronger sentence, still *any* is licensed in (12) and (13).

[1] The use of *any* illustrated in (8) have been called anti-indiscriminative in (Horn, 2000) and anti-depreciative in (Haspelmath, 1997). On the present account, sentences like (8) must be taken to involve a metalinguistic use of negation.

[2] Imperative I entails I' iff each way of complying with I is a way of complying with I'. See e.g. (Hamblin, 1987).

(12) Did John take any card? $?\exists x\phi$

(13) Take any card! $!\exists x\phi$

To explain (12), (van Rooij, 2003) proposed to interpret strength in terms of *relevance* rather than entailment, and provided a perspicuous characterization of the relevance of a question in terms of the decision theoretic notion of *expected utility*.

In this article I would like to extend van Rooij's (2003) proposal to imperatives. In order to do this, I will define a notion of the relevance or utility of an imperative in a context as a function of the probability of its compliance and its desirability. According to this notion, in example (13), domain widening can lead to an interpretation with a higher expected utility because it can increase the probability of a positive response from the hearer. In this sense, I would like to suggest, imperatives meet Kadmon and Landman's requirement that domain widening should be functional. Intuitively, by enlarging the domain of an existential quantifier in an imperative the speaker indicates that she will be pleased by more ways of complying with her wishes. This increases her chances that the hearer will comply. Note that domain widening increases utility only in a situation in which no element in the enlarged domain is ruled out as an option. This allows us to derive from (13) the permission to take any card as an implicature. Since *any* can be used only in situations where domain widening increases utility, this explains why this implicature is hard to cancel. Since existential sentences can be seen as generalized disjunctive sentences, the free choice implicatures of disjuntive imperatives follow by the same reasoning. In this case, however, these implicatures can be canceled, like in Rescher & Robinson's example where the implicated material was in conflict with shared assumptions in the common ground.

2 Expected utility of imperatives

In this section I define the expected utility value of an imperative I in terms of how far I can help in increasing the probability of the occurrence of a desirable future world. Expected utility values will be calculated with respect to a state representing the speaker's beliefs and desires about the future.

2.1 States

A *state* σ is a pair (p, u) consisting of a probability function p on the set W of possible worlds and a utility function u.

The *probability* function p maps worlds to numbers in the interval $[0, 1]$, with the constraint that $\sum_{w \in W} p(w) = 1$. Probability distributions can be extended to subsets C of W as follows: $p(C) = \sum_{w \in C} p(w)$. In this context, a world represents a way in which things might turn out to be in the near future. The probability function p represents the belief of the agent with respect to the probability of the occurrence of a world w. The value $p(w)$ may depend on a number of factors, like physical possibility (relative to the laws of nature), temporal possibility (possible in the time), and, most important, active possibility (relative to the willingness of the other people to co-operate). If $p_\sigma(w) \neq 0$ we will say that w is possible in σ.

The *utility* function u is a mapping from W to the set $\{0, 1\}$ and expresses the desirability of a world w. Desirable worlds obtain value 1, undesirable worlds, value 0.

As an illustration of these notions, consider the following examples of a state (for simplicity we are considering only four worlds, where each world is indexed with the atomic propositions holding in it. For example, in w_q, only q holds, and in w, no atomic proposition holds):

(14) a.

	p	u
w_q	1/2	1
w_r	1/2	1
w_{qr}	0	0
w	0	0

b.

	p	u
w_q	0	1
w_r	3/4	0
w_{qr}	0	0
w	1/4	0

c.

	p	u
w_q	1/6	1
w_r	1/6	1
w_{qr}	0	0
w	2/3	0

In order to understand this notion it might be useful to ask ourself in which of these states one would rather be. Intuitively, (14a) is the best choice. Each world which is still possible there, is also desirable. State (14b) is the worst choice, none of the possible worlds is a desirable one. Finally, in (14c), which is probably the most realistic option, some of the possible worlds are desirable, some are not. The notion of the value of a state defined in the following paragraph is meant to capture these intuitions.

2.2 The value of a state

We can think of a state $\sigma = (p, u)$ as a degenerate decision problem in which the set of alternative actions has just one element. Following the standard notion of expected utility in Bayesian decision theory, I define the *value* of a state as follows:

(15) $V(\sigma) = \sum_{w \in W}(p_\sigma(w) \times u_\sigma(w))$

The value of a state σ expresses the probability in σ of the occurrence of a desirable world. A state with value 1 is one in which each possible world is also desirable, e.g. (14a) above. A state with value 0 is one in which none of the possible worlds are desirable, e.g (14b). More realistic states are those in which the value lies between 0 and 1, like (14c) above with value $(1/6 + 1/6) = 1/3$.

In order to increase the value of a state, an agent may do different things. She might change her desire or, better, she might act in order to change her probability function, for example, by using an imperative. Declaratives do not have the power to change the probability of a future world, imperatives do. The goal of a declarative is to update an information state. The goal of an imperative is to enlarge the chance of the occurrence of a desirable world.

In what follows I will characterize the expected utility of an imperative in a state σ in terms of how far it can help in increasing the value of σ. More precisely, the expected utility value of an imperative I will be defined in terms of the *utility value* and the *probability* of the proposition C_I expressing the *compliance conditions* of I.

2.3 Compliance conditions

Declaratives have truth conditions, interrogatives have answerhood conditions, imperatives have *compliance conditions*. Someone cannot be said to understand the meaning of an imperative I unless she recognizes what has to be true for the command (or request, advice, etc.) issued by an utterance of I to be complied with. I shall identify the compliance conditions $C_{!\phi}$ of imperative $!\phi$ with the proposition expressed by ϕ.[3] For example,

(16) I: 'Kill Bill!'

C_I: 'That the hearer kills Bill'

(17) I: 'Kill Bill or John!'

C_I: 'That the hearer kills Bill or John'

[3]But see (Mastop, 2005) or (Portner, 2004) who, among others, have argued that imperatives are better analyzed in terms of actions or properties rather than propositions.

2.4 Utility value of a proposition

Following (van Rooij, 2003), we define the *utility value* $UV(C, \sigma)$ of a proposition C in a state σ as the difference between the value of σ after updating with C and before updating with C, where updates are defined in terms of Bayesian conditionalizations.

(18) $UV(C, \sigma) = V(\sigma/C) - V(\sigma)$

where $\sigma/C = (p_C, u)$ and p_C is the old probability function p conditionalized on C, that is, for each world w:

(19) $p_C(w) = p(w \;\&\; C)/p(C)$

The utility value of a proposition C in a state σ expresses how much an update with C can enlarge the value of σ.[4]

As an illustration, let us calculate the utility value of the following three propositions in the state (14c) above.

(20) $q \vee r$, q, $\neg q$

In order to do this we need to update (14c) (rewritten as τ in (21)) with the propositions in (20) and calculate the value of the resulting states.

(21) τ

	p	u
w_q	1/6	1
w_r	1/6	1
w_{qr}	0	0
w	2/3	0

(22) a. $\tau/(q \vee r)$

	p	u
w_q	1/2	1
w_r	1/2	1
w_{qr}	0	0
w	0	0

b. τ/q

	p	u
w_q	1	1
w_r	0	1
w_{qr}	0	0
w	0	0

c. $\tau/\neg q$

	p	u
w_q	0	1
w_r	1/5	1
w_{qr}	0	0
w	4/5	0

States (22a) and (22b) have value 1. State (22c) has value $1/5$. Since $V(\tau) = 1/3$, we obtain for our three propositions the following utility values:

(23) a. $UV(q \vee r, \tau) = 1 - 1/3 = 2/3$

b. $UV(q, \tau) = 1 - 1/3 = 2/3$

c. $UV(\neg q, \tau) = 1/5 - 1/3 = -2/15$

We can now define the expected utility value of imperatives.

2.5 Expected utility of imperatives

The *expected utility value* of an imperative I is defined as the product of the utility value and the probability of its compliance conditions C_I.

(24) $EUV(I, \sigma) = UV(C_I, \sigma) \times p_\sigma(C_I)$

The expected utility of imperative I in σ depends not only on the utility value of C_I, $UV(C_I, \sigma)$, formalizing how much closer to your goal the imperative would lead you, if accepted, but also on the probability of its acceptance, $p_\sigma(C_I)$.

As an illustration consider again our state τ, with value $1/3$:

[4] This notion is different from the *value of sample information* of statistical decision theory, e.g. (Raiffa and Schlaifer, 1961).

		p	u
	w_q	1/6	1
τ	w_r	1/6	1
	w_{qr}	0	0
	w	2/3	0

Suppose one wants to increase $V(\tau)$ by using an imperative. The notions defined above can help us in making predictions on which imperative one should choose. We have three reasonable options:

(25) a. $!q$ 'Post this letter!'

b. $!r$ 'Burn this letter!'

c. $!(q \vee r)$ 'Post this letter or burn it!'

To see which is the best choice let us calculate their expected utility. In order to do so we need to determine the utility values and the probabilities of the propositions expressing their compliance conditions, namely q, r, and $q \vee r$.

As we have already seen, these three propositions obtain equivalent utility values since updating τ with any of them leads to a state of value 1.

(26) a. $UV(q, \tau) = UV(r, \tau) = 2/3$

b. $UV(q \vee r, \tau) = 2/3$

The probabilities, however, of the three propositions crucially differ, giving for the three imperatives the following expected utilities:

(27) a. $EUV(!q, \tau) = 2/3 \times 1/6 = 1/9$

b. $EUV(!r, \tau) = 2/3 \times 1/6 = 1/9$

c. $EUV(!(q \vee r), \tau) = 2/3 \times 1/3 = 2/9$

Among the options which have the potential to maximally increase the value of τ, $!(q \vee r)$ is the one with the highest probability of being accepted. Therefore, $!(q \vee r)$ is recommended as the best choice in this case.

3 Applications

In this section we discuss two applications of the previously defined notions. The first application concerns the potential of imperatives to license free choice *any*. The second concerns the free choice effects of *or* and *any* imperatives.

3.1 *Any* in imperatives

The utility value of a disjunction $UV(A \vee B)$ can never be higher than the utility values of both its disjuncts.

(28) For *no* state σ:

$$UV(A \vee B, \sigma) > UV(A, \sigma),\ UV(B, \sigma)$$

In declaratives, disjunctions cannot increase relevance. The use of *or*, in declaratives, usually signals either lack of information (it is unknown which of the disjuncts is true) or lack of relevance (none of the disjuncts would be strictly more relevant).

In imperatives, however, disjunctions can be used to increase relevance. The example discussed in the previous section, has shown that the expected utility of a disjunctive imperative $EUV(!(A \vee B))$ can be higher than the expected utility value of any of its disjuncts:

(29) There is a state σ:

$$EUV(!(A \vee B), \sigma) > EUV(!A, \sigma)\ \&$$
$$EUV(!(A \vee B), \sigma) > EUV(!B, \sigma)$$

Since existential sentences can be treated as generalized disjunctions:

(30) $\exists x\phi \equiv \phi(a) \vee \phi(b) \vee \phi(c) \vee ...$

we can then conclude that domain widening can increase the relevance of an existential imperative ($!\exists x\phi$), but not of an existential declarative ($\exists x\phi$). This explains why *any* is licensed in (31a), while it is out in (31b).

(31) a. Take any card!

b. # John took any card.

In (31a), domain widening can increase relevance because it can increase the probability that the hearer will comply. In (31b), it cannot. The utility of a declarative is not a function of its probability.

With imperatives, but not with declaratives, a weaker option can be more relevant than a stronger alternative.

3.2 Free choice implicatures

On this account, free choice effects are derived as implicatures arising from the following Gricean reasoning (again for ease of exposition we only consider the case of disjunction):

(32) The speaker used $!(A \vee B)$ rather than the shorter $!A$ or $!B$. Why? $!A$ and $!B$ must have had a lower expected utility. A disjunctive imperative $!(A \vee B)$ has a higher expected utility than $!A$ and $!B$ only in a situation in which both disjuncts are allowed. Then A and B must both be allowed.

To formalize (32), I first define the following semantics for deontic $\Diamond$, to be read as 'It is allowed', and $\Box$, to be read as 'it is obligatory':

(i) $\sigma \models \Diamond\phi$ iff $\exists w : u(w) = 1 \;\&\; w \in [\phi]$;

(ii) $\sigma \models \Box\phi$ iff $\forall w : u(w) = 1 \Rightarrow w \in [\phi]$.

ϕ is allowed in σ iff there is at least one desirable world in σ in which ϕ is true. ϕ is obligatory in σ iff in each desirable world in σ, ϕ is true.

Building on ideas from (Schulz, 2003), I then define the *implicatures* of an imperative I as the sentences not entailed by I holding in all σ/I where σ is an optimal states for I.

(33) I implicates ϕ, $I \approx\!\!\!\models \phi \;\Leftrightarrow$
$I \not\models \phi \;\&\; \forall\sigma \in opt(I) : \; \sigma/I \models \phi$

An optimal state for I is one in which I is the choice with highest expected utility among a set of alternatives.

(34) $opt(I) = \{\sigma \mid \forall I' \in alt(I) : EUV(I) > EUV(I')\}$

Now, it is easy to prove that a disjunctive imperative $!(\phi_1 \vee \phi_2)$ has a higher expected utility than any of its disjuncts $!\phi_i$ only in a state in which each ϕ_i is possible, $p([\phi_i]) \neq 0$, and allowed, $\exists w : u(w) = 1 \;\&\; w \in [\phi_i]$.

If we assume as set of alternatives for a disjunctive imperative $!(A \vee B)$, the set $\{!A, !B\}$, and for an existential imperative $!\exists_D x\phi$ the set $\{!(\exists_Z x\phi) \mid Z \subset D\}$, it then follows that choice-offering imperatives implicate that each alternative way of complying with them is allowed:

(35) a. $!(A \vee B) \approx\!\!\!\models \Diamond A \wedge \Diamond B$

b. $!\exists x\phi \approx\!\!\!\models \forall x \Diamond \phi$

On this account, all disjunctive and indefinite imperatives induce a free choice effect. Like all implicatures, this effect disappears in the scope of negation. As it is easy to see, reconstructing the optimal state for $!\neg(A \vee B)$ or $!\neg\exists x\phi$ does not yield any free choice inference. In the case of positive disjunctive or *a*-imperatives, free choice effects can be canceled depending on the circumstances of the utterance (examples (1), (3) and (10)). In the case of positive *any*-imperatives, free choice effects cannot be canceled. This fact can be explained if we assume that *any* is felicitous only in contexts in which domain widening is functional, i.e. it increases relevance. In a context in which not all elements in the enlarged domain are permitted options, domain widening would be unjustified and *any* would be infelicitous.

4 Conclusion

I have defined the expected utility of an imperative in terms of how far it can help in increasing the probability of the occurrence of a desirable world. This notion has been then applied to explain: (i) the potential of imperatives to license *any* in their scope; and (ii) the free choice effects of disjunctive and *any*-imperatives.

Any is licensed in imperatives, because enlarging the domain of an existential quantifier in an imperative can increase its expected utility. In this sense, imperatives meet Kadmon and Landman's requirement that domain widening should be for a reason.

Free choice effects have been derived as implicatures defined in terms of what must hold in a state in order for the used imperative to have maximal expected utility in that state.

References

Maria Aloni. 2003. On choice-offering imperatives. In Paul Dekker and Robert van Rooij, editors, *Proceedings of the 14th Amsterdam Colloquium*. ILLC-University of Amsterdam.

Lennart Åquist. 1965. Choice-offering and alternative-presenting disjunctive commands. *Analysis*, 25:182–184.

Jeroen Groenendijk and Martin Stokhof. 1984. *Studies on the Semantics of Questions and the Pragmatics of Answers*. Ph.D. thesis, University of Amsterdam.

Charles L. Hamblin. 1987. *Imperatives*. Basil Blackwell.

Martin Haspelmath. 1997. *Indefinite Pronouns*. Oxford University Press, Oxford.

Lawrence Horn. 2000. Pick a theory (not just *any* theory): Indiscriminatives and the free-choice indefinite. In L. Horn and Y. Kato, editors, *Studies in Negation and Polarity*. Oxford U. Press.

Nirit Kadmon and Fred Landman. 1993. Any. *Linguistics and Philosophy*, 16:353–422.

Rosja Mastop. 2005. *What can you do? Imperative mood in semantic theory*. Ph.D. thesis, University of Amsterdam.

Paul Portner. 2004. The semantics of imperatives within a theory of clause types. In Kazuha Watanabe and Robert B. Young, editors, *Proceedings of Semantics and Linguistic Theory 14*, Ithaca, NY. CLC Publications.

Howard Raiffa and Robert Schlaifer. 1961. *Applied statistical decision theory*. MIT Press, Cambridge MA.

Nicholas Rescher and John Robinson. 1964. Can one infer commands from commands. *Analysis*, 24:176–179.

Alf Ross. 1941. Imperatives and logic. *Theoria*, 7:53–71.

Katrin Schulz. 2003. You may read it now or later. A case study on the paradox of free choice permission. Master thesis. University of Amsterdam.

Robert van Rooij. 2003. Negative polarity items in questions: Strength as relevance. *Journal of Semantics*, 20:239–273.