An Improperly Replicated Rotational Grazing Experiment

One way to reduce herd size in grazing trials is to use a single herd to graze the replicates of each treatment in turn. For example, suppose an experiment is to be conducted to investigate the effects of three levels of pasture availability on the weight gain of cattle. Further, the 12 combinations of three levels of availability and four rotations are applied completely at random to 12 paddocks so that each treatment combination occurs on one paddock. Also, the three levels of availability are assigned completely at random to 15 animals so that each level of availability is assigned to five animals. The five animals are then grazed together in sequence on the four paddocks assigned to that level of availability; the sequence of 4 paddocks is determined by the order in which the rotations were assigned to them.

The sets for this experiment are paddocks, cattle and treatments and the tiers are $\mathcal{F}_{\text{paddocks}} = \{\text{Paddocks}\}$, $\mathcal{F}_{\text{cattle over time}} = \{\text{Animals}, \text{Rotations}\}$, and $\mathcal{F}_{\text{treatments}} = \{\text{Availability}, \text{Rotations}\}$. Note that Rotations occurs in the two tiers $\mathcal{F}_{\text{cattle over time}}$ and $\mathcal{F}_{\text{treatments}}$. It must be included in the first so that the set of factors in this tier uniquely indexes the observational units. It occurs in the other tier because Availability $\wedge$ Rotations is randomized to Paddocks. Further, it is not possible to have the Rotations in the Cattle-over-time tier randomized to Paddocks there is no viable unrandomized design. There are two randomizations that have to be taken into account in deriving the analysis: the randomization of levels of availability to animals and that of treatments to the paddocks. Note that treatments are randomized in both randomizations and so this experiment involves double multiple randomizations: see the randomization diagram in Figure 1, where the double randomizations are shown by two arrows which start from the same factor but go to different tiers. There is no arrow between the two Rotations in the different tiers because they are the same Rotations.

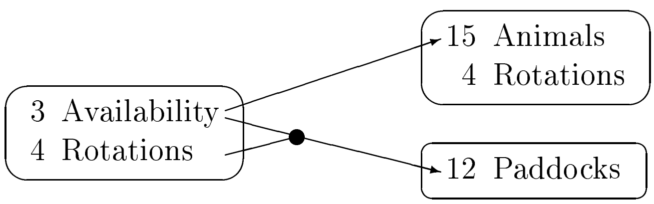

Figure 1: Double randomizations in the rotational grazing experiment

The structure formulae for the rotational grazing experiment are

$$
\begin{gathered}
15 \text{ Animals} * 4 \text{ Rotations} \\
12 \text{ Paddocks} // (3 \text{ Herds} + 4 \text{ Rotations}) \\
3 \text{ Availability} * 4 \text{ Rotations.}
\end{gathered}
$$

It is necessary to introduce two pseudofactors into the second formula to account for the association between cattle-over-time entities and paddocks that arises from the randomization. The first of these recognizes that certain groups of animals, or herds, were associated with particular paddocks in the randomization of Availablilty to both Animals and Paddocks. The second, Rotations, recognizes that different Rotations were associated with different paddocks when treatments were randomized to paddocks.

The Hasse diagrams corresponding to the first and third formulae are just standard ones for two crossed factors. That for the second formula is given Figure 2. The matrices of efficiences are straightforward with elements that are either 0 or 1. The analysis of variance that summarizes the relationships between the elements of the different structures is given in Table 1. It shows how the 11 degrees of freedom is split between the three terms and that there is no Residual degrees of freedom for testing Availability differenes and interactions — hence its being dubbed improperly replicated.

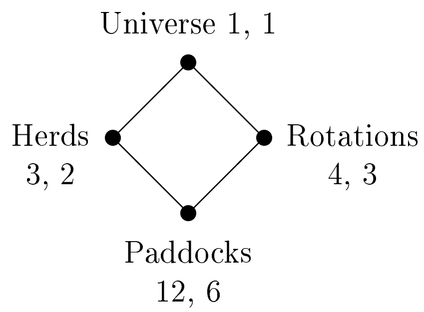

Figure 2: Hasse diagram for paddocks formula from the rotational grazing experiment

Source	DF		
Animals	14		
Paddocks		2	
Availability			2
Residual		12	
Rotations	3		
Paddocks		3	
Animals # Rotations	42		
Paddocks		6	
Availability # Rotations			6
Residual		36	
Total	59		

Table 1: Analysis variance table for the rotational grazing experiment