Superimposed Experiment using a Row-and-Column Design

Freeman (1959) describes a cherry experiment in which a large number of rootstocks had been tested using a randomized complete block design for 20 years. The trees from 10 rootstocks in three blocks were than to be used for investigating five virus treatments. The five virus treatments were assigned using the extended Youden square shown in table 1, the rootstocks corresponding to the columns and the blocks to the rows.

		Rootstocks									
		1	2	3	4	5	6	7	8	9	10
	I	A	B	A	C	D	C	B	E	E	D
Blocks	II	D	E	B	D	E	A	C	C	A	B
	III	E	A	C	E	B	D	D	B	C	A

Table 1: Virus Treatment for each Block-Rootstock combination

The sets for this experiment are trees, rootstocks and treatments and the tiers are $\mathcal{F}_{\text{trees}} = \{\text{Blocks}, \text{Trees}\}$, $\mathcal{F}_{\text{rootstocks}} = \{\text{Rootstocks}\}$ and $\mathcal{F}_{\text{treatments}} = \{\text{Viruses}\}$. There are two randomizations: rootstocks to trees in the initial experiment and virus treatments to trees, taking into account the rootstocks, in the revised experiment. In this experiment, the two randomizations are u-inclusive in that the unrandomized factors for the second randomization include factors from both tiers, $\mathcal{F}_{\text{trees}}$ and $\mathcal{F}_{\text{rootstocks}}$, of the first randomization.

In Figure 1 the dashed oval once again shows the new pseudotier created by the first randomization.

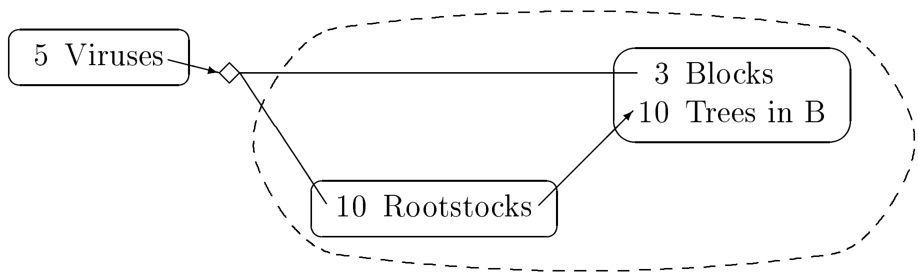

Figure 1: Unrandomized-inclusive randomizations in the superimposed experiment using a row-and-column design

Now the diamond after the arrow indicates that viruses are randomized to those combinations of Block and Rootstock that occur as a result of the first randomization. The group for the first randomization is $S_{10} \wr S_3$, while that for the second is $S_{10} \times S_3$, which is a subgroup of the former. In contrast to wheat variety experiment, both the group and the systematic plan for the second randomization are constrained by the result of the first. This double constraint is reflected in the number of plans which the designer must write out. If the result of the first randomization is shown in Table 2 and the constrained systematic plan in Table 1 is randomized by $S_{10} \times S_3$ to the plan in Table 3 then the designer will also have to write out Table 4 for the orchard worker to follow.

		Trees									
		1	2	3	4	5	6	7	8	9	10
	I	7	2	1	10	6	3	5	9	4	8
Blocks	II	5	6	3	4	9	1	8	7	2	10
	III	8	6	3	9	7	4	2	10	1	5

Table 2: Allocation of rootstocks after the first randomization

		Rootstocks									
		1	2	3	4	5	6	7	8	9	10
	I	C	D	B	E	A	D	B	E	A	C
Blocks	II	D	B	A	B	E	A	D	C	C	E
	III	A	E	E	C	D	B	C	A	B	D

Table 3: Allocation of viruses after the second randomization

		Trees									
		1	2	3	4	5	6	7	8	9	10
	I	7B	2D	1C	10C	6D	3B	5A	9A	4E	8E
Blocks	II	5E	6A	3A	4B	9C	1D	8C	7D	2B	10E
	III	8A	6B	3E	9B	7C	4C	2E	10D	1A	5D

Table 4: Plan for the orchard worker

The structure formulae for the superimposed experiment are

$$\begin{array}{c}\text{3 Blocks / 10 Trees}\\ \text{10 Rootstocks}\\ \text{5 Viruses.}\end{array}$$

The Hasse diagrams for this experiment are trivial as is the matrix of efficiencies for the structure on rootstocks in relation to that on trees. The matrix of efficiencies for the structure on treatments in relation to the joint decomposition of trees and rootstocks is given in Table 5. Note that for this example, we have that Viruses is orthogonal with respect to the structure on trees since

$$\begin{aligned}\lambda_{\text{Plots[Blocks]},\text{Viruses}} &= \lambda_{\text{Plots[Blocks]}\boxminus\text{Rootstocks},\text{Viruses}} \\ &\quad + \lambda_{\text{Plots[Blocks]}\square\text{Rootstocks},\text{Viruses}} \\ &= \frac{1}{6} + \frac{5}{6} \\ &= 1.\end{aligned}$$

$$\begin{array}{l c}
 & \begin{array}{cc}\text{Mean} & \text{Viruses}\end{array} \\
\begin{array}{l}\text{Mean}\\ \text{Blocks}\\ \text{Plots [Blocks]}\\ \quad\square\,\text{Rootstocks}\\ \quad\boxminus\,\text{Rootstocks}\end{array} &
\left[\begin{array}{cc}1 & 0\\ 0 & 0\\ & \\ 0 & \frac{1}{6}\\ 0 & \frac{5}{6}\end{array}\right]
\end{array}$$

Table 5: Matrix of efficiency factors $\Lambda_{\mathcal{P}\square\mathcal{Q},\mathcal{R}}$ for the superimposed experiment

The analysis of variance table derived from this structure is given in Table 6. It shows that Viruses is confounded with both Rootstocks and that part of Plots [Blocks] that is orthogonal to Rootstocks. A consequence of this is that four Rootstocks degrees of freedom are not able to be separated from Virus differences. However, there is five Rootstocks degrees of freedom that are free of Virus differences and this analysis provides the corresponding sums of squares. Further, we note that while Viruses is balanced with respect to Rootstocks, the reverse is not true. The aspect of this experiment that the analysis elucidates is that, in the second randomization, Rootstocks has acted as a 'block' factor in that it has been involved in restrictions that were placed on this randomization.

Source	DF			Efficiency factor
Blocks	2			
Plots [Blocks]	27			
Rootstocks		9		
Viruses			4	1/6
Residual			5	
Residual		18		
Viruses			4	5/6
Residual			14	
Total	29			

Table 6: Analysis variance table for the superimposed experiment