VII.D Determining the analysis of variance table – further examples

For each of the following studies determine the analysis of variance table (Source, df and E[MSq]) using the following seven steps:

A. Description of pertinent features of the study
B. The experimental structure
C. Terms derived from the structure formulae
D. Degrees of freedom
E. The analysis of variance table
F. Maximal expectation and variation models
G. The expected mean squares.

Example VII.8 Fertilizer effects on kale

An experiment is to be conducted to investigate the effect of two levels of potassium (K), three levels of nitrogen (N) and two levels of phosphorus (P) on the yield of three varieties of kale (couve). The two levels of potassium were none and 50 kg added, the three levels of nitrogen were none, 25 kg and 50 kg added and the two levels of phosphorus were none and 30 kg added. The 36 treatment combinations are to be applied using a randomized complete block design with 3 blocks of 36 plots.

1. Observational unit – a plot
2. Response variable – Yield
3. Unrandomized factors – Blocks/Plots
4. Randomized factors – N, P, K, Variety
5. Type of study – Complete $2^2 \times 3^2$ factorial RCBD

The experimental structure is:

Structure	Formula
unrandomized	*3* Blocks/*36* Plots
randomized	*3* N**2* P**2* K**3* Varieties

The terms derived from these structure formulae are:

Blocks.Plots = Blocks + Blocks.Plots

N*P*K*Varieties
= N + P + K + Variety
+ N.P + N.K + N.Variety + P.K + P.Variety + K.Variety
+ N.P.K + N.P.Variety + N.K.Variety + P.K.Variety
+ N.P.K.Variety

Note that factors in the randomized structure are completely crossed so that the degrees of freedom of a term from that structure can be obtained by computing the number of levels minus one for each factor in the term and forming their product.

Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Seems that randomized factors should be fixed and unrandomized factor should be random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Blocks} + \text{Blocks.Plots}$$

$$E[Y] = \text{N.P.K.Variety}$$

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{BP} \text{ and } 36\sigma^2_{B}$$

The analysis of variance table is:

Source	df	E[MSq]
Blocks	2	$\sigma^2_{BP} + 36\sigma^2_{B}$
Blocks.Plots	105	
N	2	$\sigma^2_{BP} + f_{N}(\psi)$
P	1	$\sigma^2_{BP} + f_{P}(\psi)$
K	1	$\sigma^2_{BP} + f_{K}(\psi)$
Variety	2	$\sigma^2_{BP} + f_{V}(\psi)$
N.P	2	$\sigma^2_{BP} + f_{NP}(\psi)$
N.K	2	$\sigma^2_{BP} + f_{NK}(\psi)$
N.Variety	4	$\sigma^2_{BP} + f_{NV}(\psi)$
P.K	1	$\sigma^2_{BP} + f_{PK}(\psi)$
P.Variety	2	$\sigma^2_{BP} + f_{PV}(\psi)$
K.Variety	2	$\sigma^2_{BP} + f_{KV}(\psi)$
N.P.K	2	$\sigma^2_{BP} + f_{NPK}(\psi)$
N.P.Variety	4	$\sigma^2_{BP} + f_{NPV}(\psi)$
N.K.Variety	4	$\sigma^2_{BP} + f_{NKV}(\psi)$
P.K.Variety	2	$\sigma^2_{BP} + f_{PKV}(\psi)$
N.P.K.Variety	4	$\sigma^2_{BP} + f_{NPKV}(\psi)$
Residual	70	σ^2_{BP}

Example VII.9 Mathematics teaching methods

An educational psychologist wants to determine the effect of three different methods of teaching mathematics to year 10 students. Five metropolitan schools with three mathematics classes in year 10 are selected and the methods of teaching randomized to the classes in each school. After being taught by one of the methods for a semester, the students sit a test and their average score is recorded.

1. the observational unit – a class
2. response variable – Test score
3. unrandomized factors – Schools, Classes
4. randomized factors – Methods
5. type of study – an RCBD

The experimental structure is:

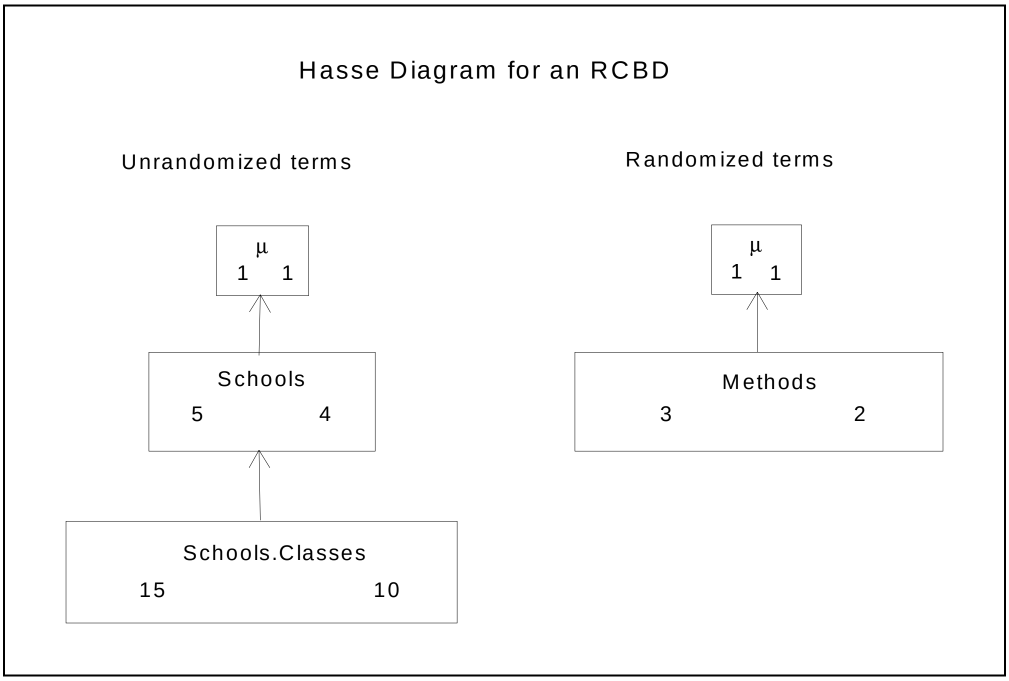
Structure	Formula
unrandomized	*5* Schools/*3* Classes
randomized	*3* Methods

The terms derived from these structure formulae are:

Schools/Classes = Schools + Schools.Classes

Methods = Methods

The Hasse diagrams, with degrees of freedom, for this study are:



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Seems that randomized factors should be fixed and unrandomized factors should be random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Schools} + \text{Schools.Classes}$$

$$\text{E}[Y] = \text{Methods}$$

What are the variation components, and their multipliers, for this study?

$$3\sigma^2_{\text{S}} \text{ and } \sigma^2_{\text{SC}}$$

The analysis of variance table is:

Source	df	E[MSq]
Schools	4	$\sigma^2_{\text{SC}} + 3\sigma^2_{\text{S}}$
Schools.Classes	10	
Methods	2	$\sigma^2_{\text{SC}} + f_{\text{M}}(\psi)$
Residual	8	σ^2_{SC}

Example VII.10 Lead concentration in hair

An investigation was performed to discover trace metal concentrations in humans in the five major cities in Australia. The concentration of lead in the hair of fourth grade school boys was determined. In each city, 10 primary schools were randomly selected and from each school ten students selected. Hair samples were taken from the selected boys and the concentration of lead in the hair determined.

1. the observational unit – hair of a boy
2. response variable – Lead concentration
3. unrandomized factors – Cities, Schools, Boys
4. randomized factors – not applicable
5. type of study – a multistage survey

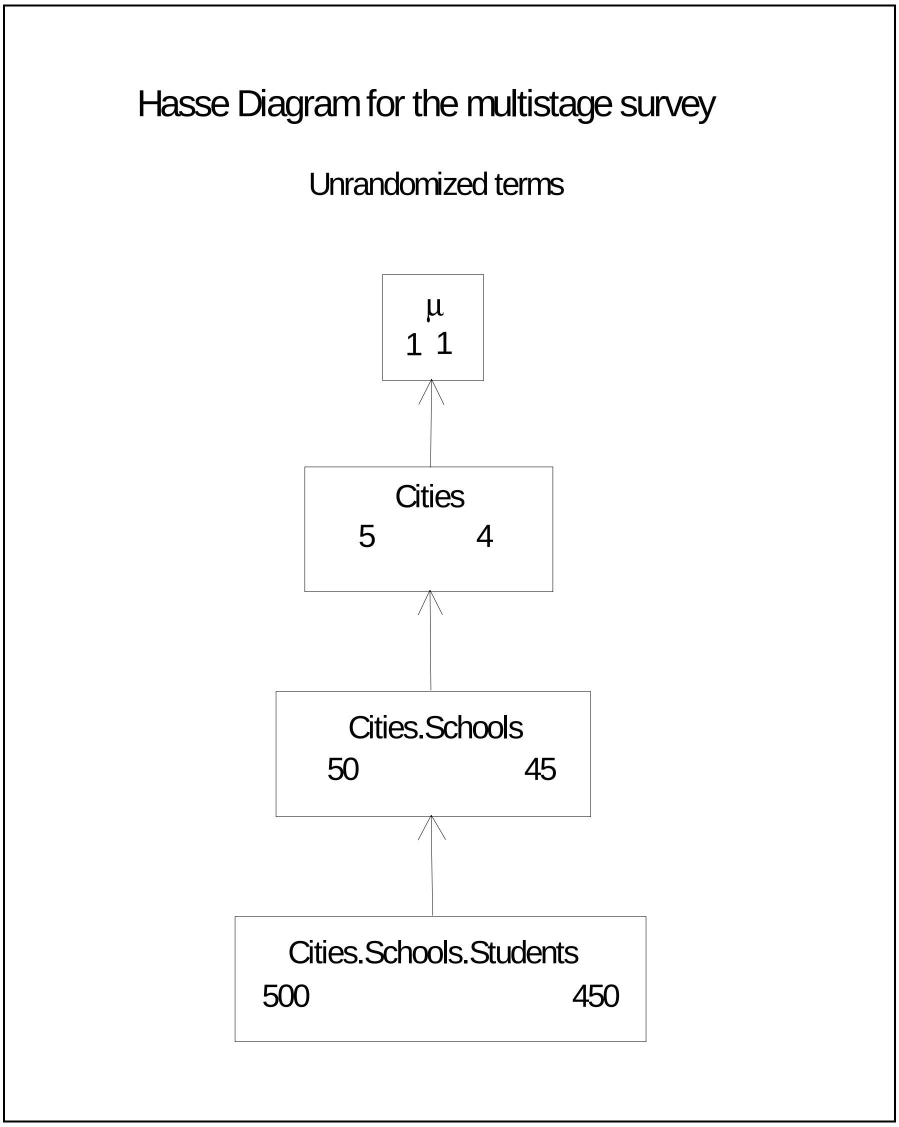
The experimental structure is:

Structure	Formula
unrandomized	*5* Cities/*10* Schools/*10* Boys
randomized	not applicable.

The terms derived from these structure formulae are:

$$\text{Cities/Schools/Boys} = \text{Cities} + \text{Cities.Schools} + \text{Cities.Schools.Boys}$$

The Hasse diagram, with degrees of freedom, for the unrandomized terms in this study is:



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Seems that Cities should be fixed and Schools and Boys should be random. Hence, the maximal variation and expectation models are:

Var[Y] = Cities.Schools + Cities.Schools.Boys and

E[Y] = Cities

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\mathrm{CSB}},\ \ 10\sigma^2_{\mathrm{CS}}$$

The analysis of variance table is

Source	df	E[MSq]
Cities	4	$\sigma^2_{\text{CSB}} + 10\sigma^2_{\text{CS}} + f_{\text{C}}(\psi)$
Cities.Schools	45	$\sigma^2_{\text{CSB}} + 10\sigma^2_{\text{CS}}$
Cities.Schools.Boys	450	σ^2_{CSB}

Example VII.11 Plant rehabilitation study

In a plant rehabilitation study, the increase in height of plants of a certain species during a 12 month period was to be determined at three sites differing in soil salinity. Each site was divided into five parcels of land containing 4 plots and four different management regimes applied to the plots, the regimes being randomized to the plots within a parcel. In each plot, six plants of the species were selected and marked and the total increase in height of all six plants measured.

1. the observational unit – a plot of plants
2. response variable – Plant height increase
3. unrandomized factors – Sites, Parcels, Plots
4. randomized factors – Regimes
5. type of study – a type of RCBD

The experimental structure is:

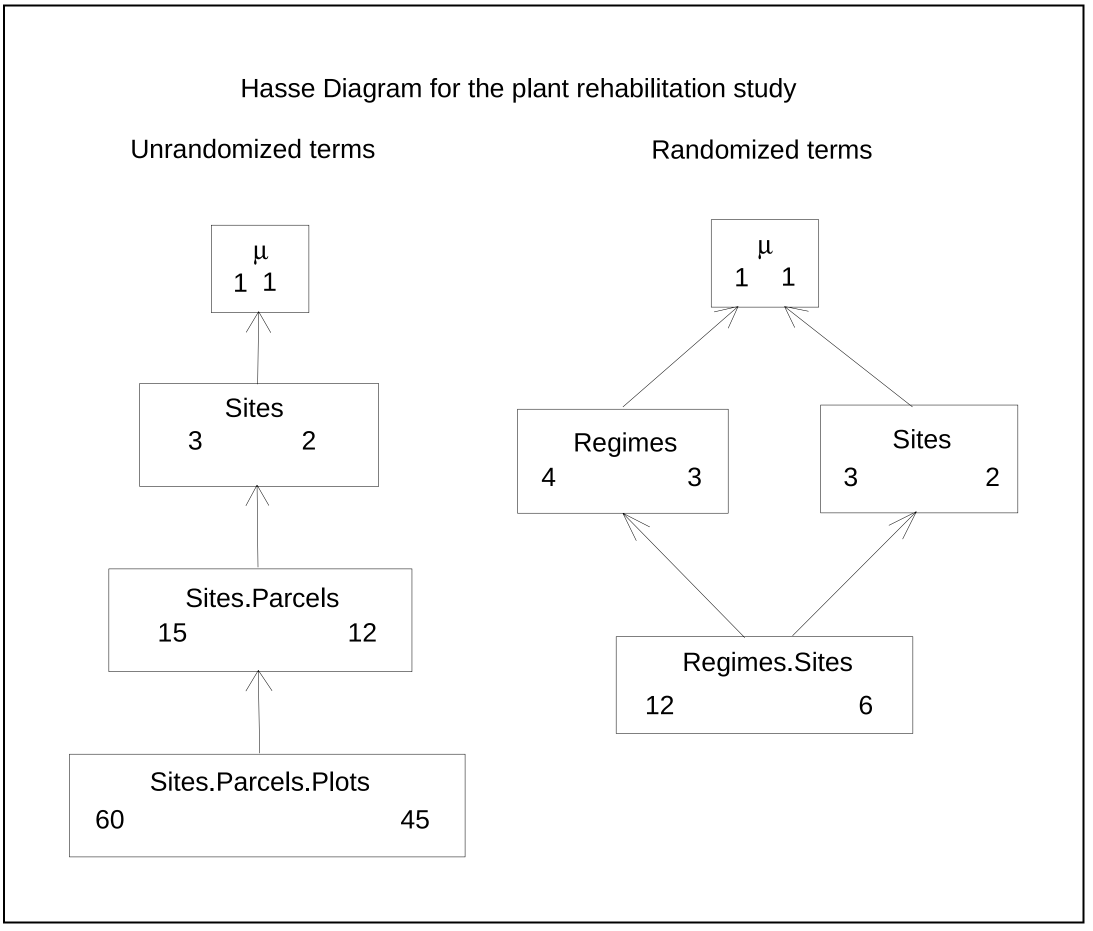
Structure	Formula
unrandomized	*3* Sites/*5* Parcels/*4* Plots
randomized	*4* Regimes*Sites

The terms derived from these structure formulae are:

Sites/Parcels/Plots = Sites + Sites.Parcels + Sites.Parcels.Plots

Regimes*Sites = Regimes + Sites + Regimes.Sites

The Hasse diagrams, with degrees of freedom, for this study are:



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Seems that Sites and Regimes should be fixed and Parcels and Plots should be random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Sites.Parcels} + \text{Sites.Parcels.Plots}$$

$$\text{E}[Y] = \text{Regimes} + \text{Sites} + \text{Regimes.Sites}$$

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{SPL}, \quad 4\sigma^2_{SP}$$

The analysis of variance table is

Source	df	E[MSq]		
Sites	2	σ^2_{SPL}	$+4\sigma^2_{\mathrm{SP}}$	$+f_{\mathrm{S}}(\psi)$
Sites.Parcels	12	σ^2_{SPL}	$+4\sigma^2_{SP}$	
Sites.Parcels.Plots	45			
Regimes	3	σ^2_{SPL}		$+f_{\mathrm{R}}(\psi)$
Regimes.Sites	6	σ^2_{SPL}		$+f_{\mathrm{RS}}(\psi)$
Residual	36	σ^2_{SPL}		

Example VII.12 Generalized randomized complete block design

A generalized randomized complete block design is the same as the ordinary randomized complete block design, except that each treatment occurs more than once in a block. For example, suppose four treatments are to be compared when applied to a new variety of wheat. I employed a generalized randomized complete block design with 12 plots in each of 4 blocks so that each treatment is replicated 3 times in each block. The yield of wheat from each plot was measured. A possible layout for this experiment is shown in the table given below.

Layout for a generalized randomized complete block experiment

		Plots											
		1	2	3	4	5	6	7	8	9	10	11	12
	I	C	D	B	D	B	C	A	B	A	D	A	C
	II	A	A	D	B	C	C	B	C	D	A	B	D
Blocks	III	D	C	C	B	B	C	A	D	A	B	A	D
	IV	B	B	A	D	C	D	B	D	C	C	A	A

In working out the analysis for this experiment include a term for Block $\times$ Treatment interaction and assume that the unrandomized factors are random and the randomized factors are fixed. Having done this derive the analysis for only Plots random and the rest of the factors fixed.

1. the observational unit – a plot
2. response variable – Yield
3. unrandomized factors – Blocks, Plots
4. randomized factors – Treatments
5. type of study – Generalized Randomized Complete Block Design

The experimental structure is:

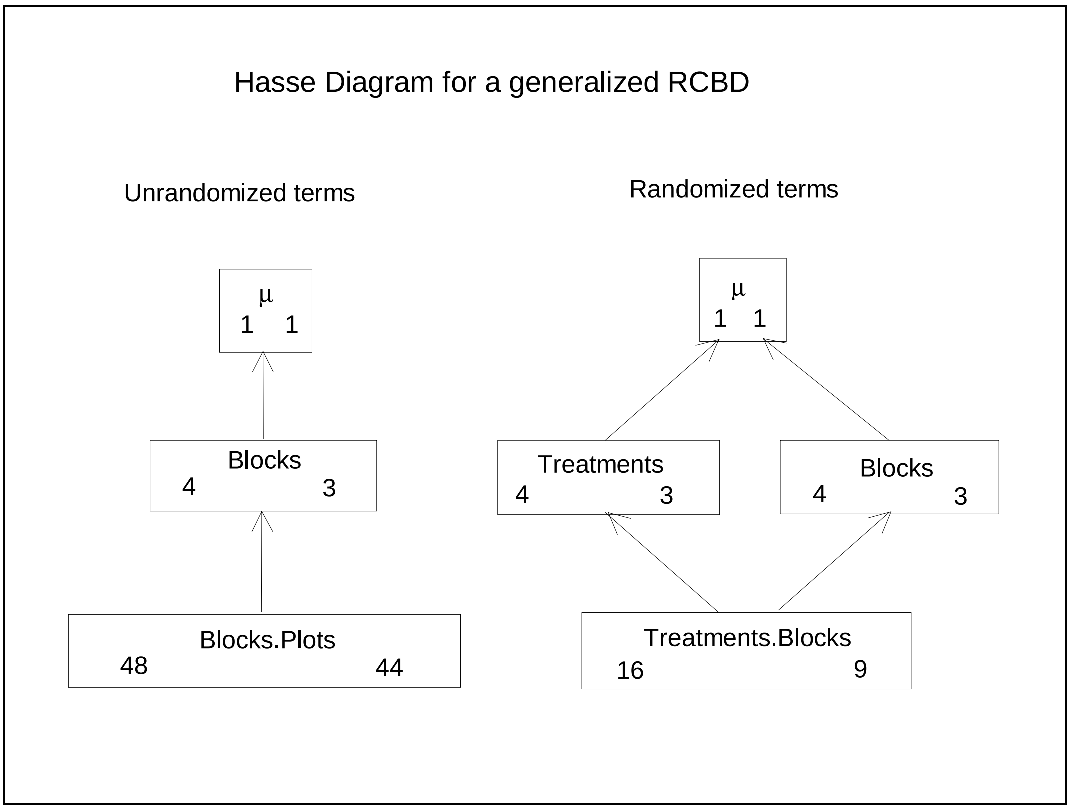
Structure	Formula
unrandomized	*4* Blocks/*12* Plots
randomized	*4* Treatments*Blocks

The terms derived from these structure formulae are:

Blocks/Plots = Blocks + Blocks.Plots

Treatments*Blocks = Treatments + Blocks + Treatments.Blocks

The Hasse diagrams, with degrees of freedom, for this study are:



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Take the randomized factors to be fixed and unrandomized factors to be random. Hence, the maximal variation and expectation models are:

Var[Y] = Blocks + Blocks.Plots + Treatments.Blocks

E[Y] = Treatments

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\text{BP}},\ 4\sigma^2_{\text{BT}},\ 12\sigma^2_{\text{B}}$$

The analysis of variance table is:

Source	df	E[MSq]
Blocks	3	$\sigma^2_{\text{BP}} + 4\sigma^2_{\text{BT}} + 12\sigma^2_{\text{B}}$
Blocks.Plots	44	
Treatments	3	$\sigma^2_{\text{BP}} + 4\sigma^2_{\text{BT}} + f_{\text{T}}(\psi)$
Treatments.Blocks	9	$\sigma^2_{\text{BP}} + 4\sigma^2_{\text{BT}}$
Residual	32	σ^2_{BP}
Total	47	

For only Plots random, the maximal variation and expectation models are:

$$\text{Var[Y]} = \text{Blocks.Plots}$$

$$\text{E[Y]} = \text{Blocks + Treatments + Treatments.Blocks}$$

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\text{BP}}$$

The analysis of variance table is:

Source	df	E[MSq]
Blocks	3	$\sigma^2_{\text{BP}} + f_{\text{B}}(\psi)$
Blocks.Plots	44	
Treatments	3	$\sigma^2_{\text{BP}} + f_{\text{T}}(\psi)$
Treatments.Blocks	9	$\sigma^2_{\text{BP}} + f_{\text{BT}}(\psi)$
Residual	32	σ^2_{BP}
Total	47	

Note the difference in the denominator of the test for Treatments between the two analyses. For the former analysis it is Treatments.Blocks and the latter analysis it is the Residual. Which analysis is correct depends on the nature of the Block-Treatment interaction.

Example VII.13 Controlled burning

Suppose an environmental scientist wants to investigate the effect on the biomass of burning areas of natural vegetation. There are available two areas separated by several kilometres for use in the investigation. It is only possible to either burn or not burn an entire area. The scientist randomly selects to burn one area and the other area is left unburnt as a control. She randomly samples 30 locations in each area and measures the biomass at each location.

1. the observational unit – a location
2. response variable – Biomass
3. unrandomized factors – Areas, Locations
4. randomized factors – Burning
5. type of study – a CRD with subsampling

The experimental structure is:

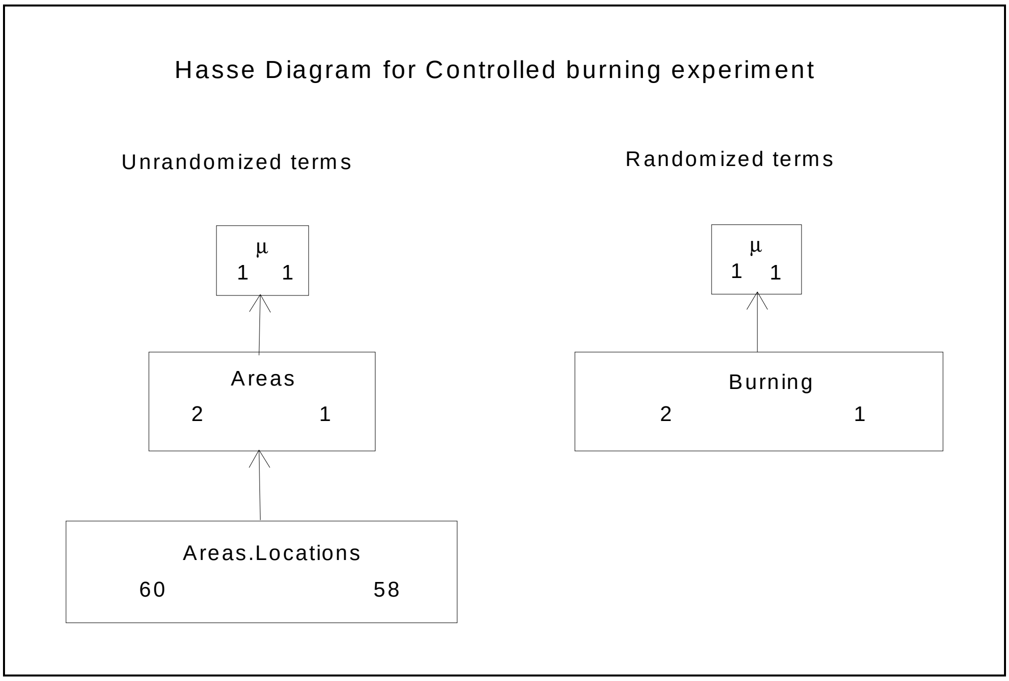
Structure	Formula
unrandomized	*2* Areas/*30* Locations
randomized	*2* Burning

The terms derived from these structure formulae are:

Areas/Locations = Areas + Areas.Locations

Burning = Burning

The Hasse diagrams, with degrees of freedom, for this study are:



Seems that randomized factor should be fixed and unrandomized factors should be random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Areas} + \text{Areas.Locations}$$

$$E[Y] = \text{Burning}$$

What are the variation components, and their multipliers, for this study?

$$30\sigma^2_{\text{A}},\ \sigma^2_{\text{AL}}$$

The analysis of variance table is:

Source	df	E[MSq]
Areas	1	
Burning	1	$\sigma^2_{\text{AL}} + 30\sigma^2_{\text{A}} + f_{\text{B}}(\psi)$
Areas.Locations	58	σ^2_{AL}

We see that we cannot test for Burning differences because there is no source with expected mean square $\sigma^2_{\text{AL}} + 30\sigma^2_{\text{A}}$. The problem is that Burning and Area differences are totally confounded and cannot be separated. This shows up in the analysis of variance table derived using our approach.

Example VII.14 Salt tolerance of lizards

To examine the salt tolerance of the lizard *Tiliqua rugosa*, eighteen lizards of this species were obtained. Each lizard was randomly selected to receive one of three salt treatments (injection with sodium, injection with potassium, no injection) so that 6 lizards received each treatment. Blood samples were then taken from each lizard on five occasions after injection and the concentration of Na in the sample determined.

1. the observational unit – a lizard on an occasion
2. response variable – Na concentration
3. unrandomized factors – Lizards, Occasions
4. randomized factors – Treatments
5. type of study – a repeated measures CRD

The experimental structure is:

Structure	Formula
unrandomized	*18* Lizards**5* Occasions
randomized	*6* Treatments*Occasions

The terms derived from these structure formulae are:

Lizards*Occasions = Lizards + Occasions + Lizards.Occasions

Treatments*Occasions = Treatments + Occasions + Treatments.Occasions

The degrees of freedom for this study can be worked out using the rule for completely crossed structures.

Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Seems that Treatments and Occasions should be fixed and Lizards should be random. Hence, the maximal variation and expectation models are:

Var[Y] = Lizards + Lizards.Occasions

E[Y] = Treatments.Occasions

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\mathrm{LO}},\ \ 5\sigma^2_{\mathrm{L}}$$

The analysis of variance table is:

Source	df	E[MSq]		
Occasions	4	σ^2_{LO}		$+f_{\mathrm{O}}(\psi)$
Lizards	17			
Treatments	2	σ^2_{LO}	$+5\sigma^2_{\mathrm{L}}$	$+f_{\mathrm{T}}(\psi)$
Residual	15	σ^2_{LO}	$+5\sigma^2_{\mathrm{L}}$	
Lizards.Occasions	68			
Treatments.Occasions	8	σ^2_{LO}		$+f_{\mathrm{TO}}(\psi)$
Residual	60	σ^2_{LO}		

Example VII.15 Eucalyptus growth

An experiment was planted in a forest in Queensland to study the effects of irrigation and fertilizer on 4 seedlots of a species of gum tree. There were two levels of irrigation (no and yes), two levels of fertilizer (no and yes) and four seedlots (Bulahdelah, Coffs Harbour, Pomona and Atherton). Because of the difficulties of irrigating and applying fertilizers to individual trees, these needed to be applied to groups of trees. So the experimental area was divided up into 8 stands of 20 trees, with four stands in one block and the other four in a second block. The four combinations of irrigation and fertilizer were randomized to the four stands in a block. Each stand of 20 trees consisted of 4 rows by 5 columns and the 4 seedlots were randomized to the four rows. The mean height of the five trees in a row was measured.

1. the observational unit – a row of trees
2. response variable – Mean height
3. unrandomized factors – Blocks, Stands, Rows
4. randomized factors – Irrigation, Fertilizer, Seedlots
5. type of study – a split-plot design with main plots in a RCBD
6. and subplots completely randomized

The experimental structure is:

Structure	Formula
unrandomized	*2* Blocks/*4* Stands/*4* Rows
randomized	*2* Irrigation**2* Fertilizer**4* Seedlots

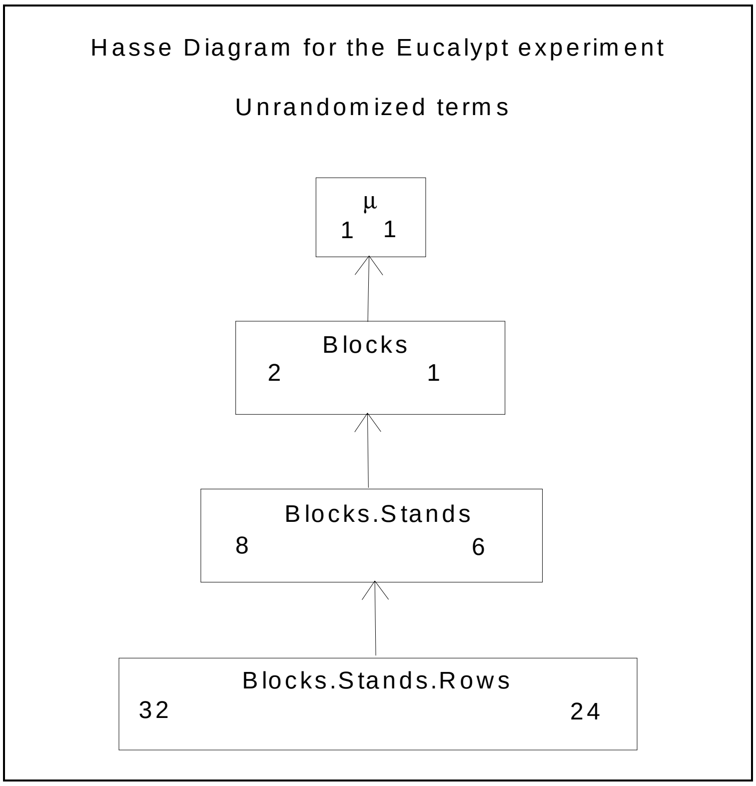
The terms derived from these structure formulae are:

Blocks/Stands/Rows = Blocks + Blocks.Stands + Blocks.Stands.Rows

Irrigation*Fertilizer*Seedlots = (Irrigation + Fertilizer + Irrigation.Fertilizer)*Seedlot
= Irrigation + Fertilizer + Irrigation.Fertilizer
+ Irrigation.Seedlots + Fertilizer.Seedlots
+ Irrigation.Fertilizer.Seedlots

The Hasse diagrams, with degrees of freedom, for this study are:

In this case we only do the Hasse diagram for the unrandomized factors because the degrees of freedom for the randomized factors can be obtained using the rule for all factors crossed.



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Take all the randomized factors and Blocks to be fixed; the remainder of the factors take as random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Blocks.Stands} + \text{Blocks.Stands.Rows}$$

$$\text{E}[Y] = \text{Blocks} + \text{Irrigation.Fertilizer.Seedlots}$$

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\text{BSR}},\quad 4\sigma^2_{\text{BS}}$$

The analysis of variance table is:

Source	df		E[MSq]
Blocks	1		$\sigma^2_{\text{BSR}} + 4\sigma^2_{\text{BS}} + f_{\text{B}}(\psi)$
Blocks.Stands	6		
Irrigation		1	$\sigma^2_{\text{BSR}} + 4\sigma^2_{\text{BS}} + f_{\text{I}}(\psi)$
Fertilizer		1	$\sigma^2_{\text{BSR}} + 4\sigma^2_{\text{BS}} + f_{\text{F}}(\psi)$
Irrigation.Fertilizer		1	$\sigma^2_{\text{BSR}} + 4\sigma^2_{\text{BS}} + f_{\text{IF}}(\psi)$
Residual		3	$\sigma^2_{\text{BSR}} + 4\sigma^2_{\text{BS}}$
Blocks.Stands.Rows	24		
Seedlots		3	$\sigma^2_{\text{BSR}} + f_{\text{S}}(\psi)$
Irrigation.Seedlots		3	$\sigma^2_{\text{BSR}} + f_{\text{IS}}(\psi)$
Fertilizer.Seedlots		3	$\sigma^2_{\text{BSR}} + f_{\text{FS}}(\psi)$
Irrigation.Fertilizer.Seedlots		3	$\sigma^2_{\text{BSR}} + f_{\text{IFS}}(\psi)$
Residual		12	σ^2_{BSR}

Example VII.16 Wheat samplers

Suppose a study is to be conducted to investigate whether four samplers differ in the amount by which they differ in the amount of error in their selection of wheat samples. Four areas and four intervals are to be employed in the study and the following Latin Square arrangement is to be used to assign the samplers to the interval-area combinations:

		Area			
		1	2	3	4
	I	A	B	D	C
	II	D	C	A	B
Intervals	III	B	D	C	A
	IV	C	A	B	D

(Samplers A, B, C, D)

1. the observational unit – a interval in a area
2. response variable – Kilometres per litre
3. unrandomized factors – Intervals, Areas
4. randomized factors – Samplers
5. type of study – a Latin Square

The experimental structure is:

Structure	Formula
unrandomized	*4* Intervals**4* Areas
randomized	*4* Samplers

The terms derived from these structure formulae are:

Intervals*Areas = Intervals + Areas + Intervals.Areas

Samplers = Samplers

The degrees of freedom for this study can be worked out using the rule for completely crossed structures.

Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Will take the random factors to be Areas and Samplers and the fixed factor to be Intervals. Then the maximal expectation and variation models are

E[Y] = Intervals and
var[Y] = Samplers + Area + Area.Interval

What are the variation components, and their multipliers, for this study?

$$4\sigma^2_{\mathrm{S}},\ 4\sigma^2_{\mathrm{A}} \text{ and } 1\sigma^2_{\mathrm{IA}}$$

The analysis of variance table is:

Source	df	E[MSq]
Intervals	3	$\sigma^2_{\text{IA}} + f_{\text{I}}(\psi)$
Areas	3	$\sigma^2_{\text{IA}} + 4\sigma^2_{\text{A}}$
Intervals.Areas	9	
Samplers	3	$\sigma^2_{\text{IA}} + 4\sigma^2_{\text{S}}$
Residual	6	σ^2_{IA}

Suppose that the experiment is to be repeated by replicating the Latin Square twice using the same areas but new intervals on a second occasion. What are the components of the study?

1. the observational unit – an interval in an area
2. response variable – Error
3. unrandomized factors – Occasions, Intervals, Areas
4. randomized factors – Samplers
5. type of study – a replicated Latin Square

The experimental structure is:

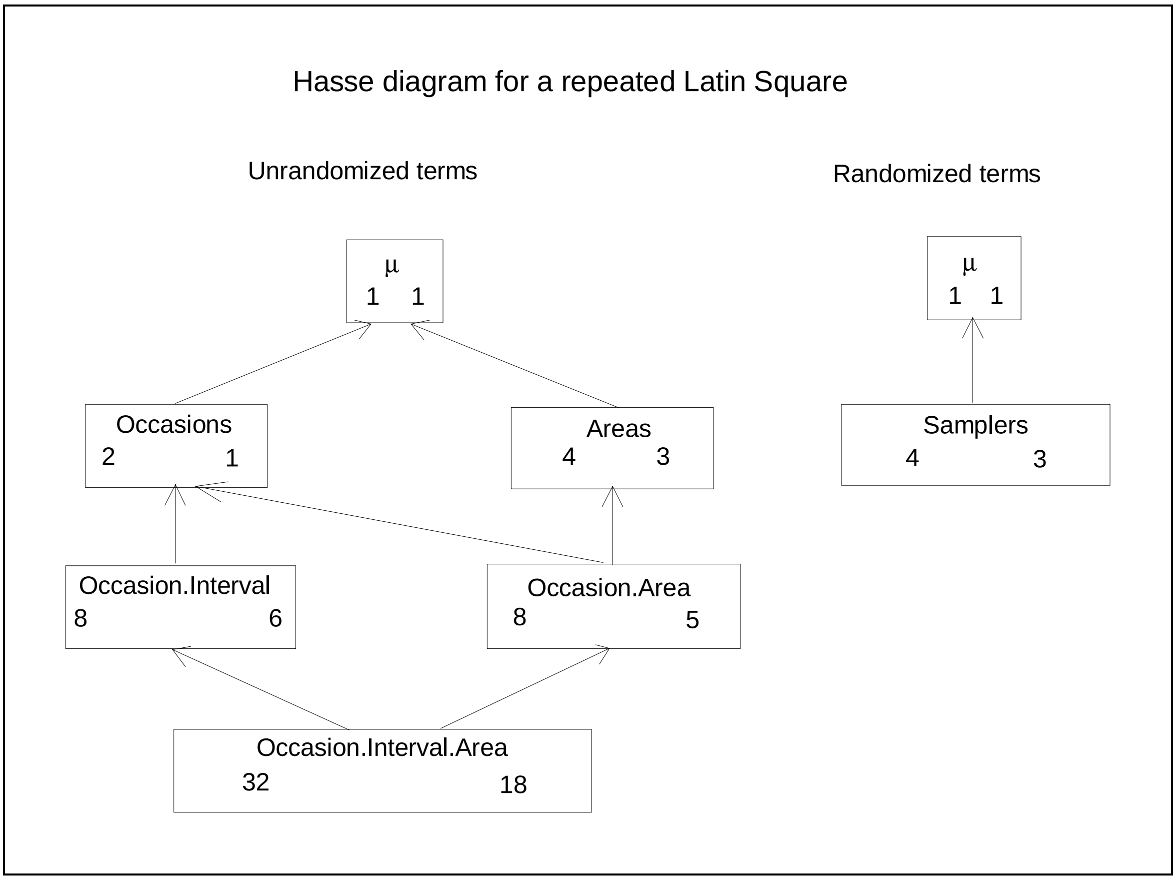
Structure	Formula
unrandomized	(*2* Occasions /*4* Intervals)**4* Areas
randomized	*4* Samplers

The terms derived from these structure formulae are:

(Occasions /Intervals)*Areas = Occasions + Occasions.Intervals + Areas + Occasions.Areas + Occasions.Intervals.Areas

Samplers = Samplers

The Hasse diagrams, with degrees of freedom, for this study are:



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Will take the random factors to be Areas and Samplers and the fixed factors to be Occasions and Intervals. Then the maximal expectation and variation models are

Var[Y] = Samplers + Areas + Occasions.Areas + Occasions.Intervals.Areas

E[Y] = Occasions + Occasions.Intervals

What are the variation components, and their multipliers, for this study?

$$8\sigma^2_{\mathrm{S}},\ 8\sigma^2_{\mathrm{A}},\ 4\sigma^2_{\mathrm{OA}} \text{ and } \sigma^2_{\mathrm{OIA}}$$

The analysis of variance table is:

Source	df	E[MSq]		
Occasions	1	σ^2_{OIA}	$+4\sigma^2_{\mathrm{OA}}$	$+f_{\mathrm{O}}(\psi)$
Occasions.Interval	6	σ^2_{OIA}		$+f_{\mathrm{OI}}(\psi)$
Areas	3	σ^2_{OIA}	$+4\sigma^2_{\mathrm{OA}}$	$+8\sigma^2_{\mathrm{A}}$
Occasions.Areas	3	σ^2_{OIA}	$+4\sigma^2_{\mathrm{OA}}$	
Occasions.Interval.Areas	18			
Samplers	3	σ^2_{OIA}		$+8\sigma^2_{\mathrm{S}}$
Residual	15	σ^2_{OIA}		

Example VII.17 Eelworm experiment

Cochran and Cox (1957, section 3.2) present the results of an experiment examining the effects of soil fumigants on the number of eelworms. There were four different fumigants each applied in both single and double dose rates as well as a control treatment in which no fumigant was applied. The experiment was laid out in 4 blocks each containing 12 plots; in each block, the 8 treatment combinations were each applied once and the control treatment four times and the 12 treatments randomly allocated to plots. The number of eelworm cysts in 400g samples of soil from each plot was determined.

1. the observational unit – a plot
2. response variable – Number of cysts
3. unrandomized factors – Blocks, Plots
4. randomized factors – Control, Fumigant, Dose
5. type of study – an RCBD

The experimental structure is:

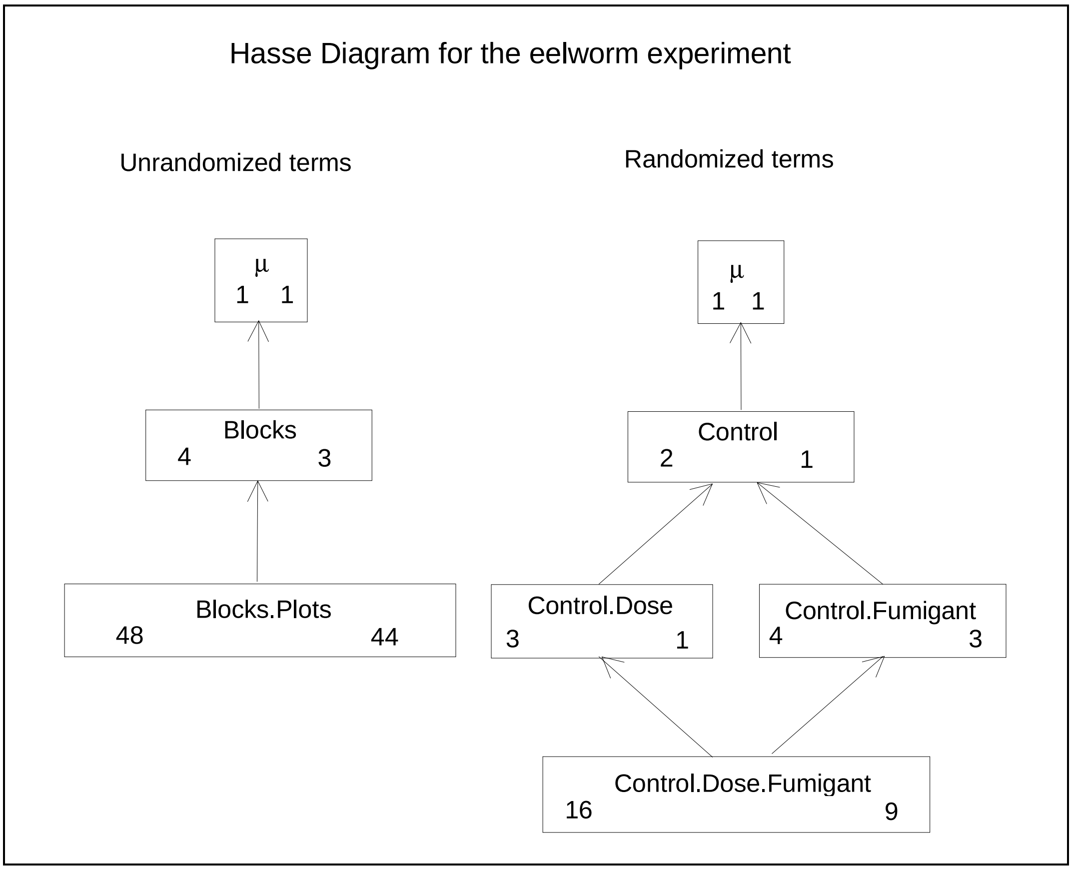
Structure	Formula
unrandomized	*4* Blocks/*12* Plots
randomized	*2* Control/(*4* Fumigant**2* Dose)

The terms derived from these structure formulae are:

Blocks/Plots = Blocks + Blocks.Plots

Control/(Fumigant*Dose)
= Control/(Fumigant + Dose + Fumigant.Dose)
= Control.Fumigant + Control.Dose + Control.Fumigant.Dose

The Hasse diagrams, with degrees of freedom, for this study are:



Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Take the randomized factors to be fixed and unrandomized factors to be random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Blocks} + \text{Blocks.Plots} + \text{Treatments.Blocks}$$

$$\text{E}[Y] = \text{Control.Fumigant.Dose}$$

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\text{BP}}\text{, } 12\sigma^2_{\text{B}}$$

The analysis of variance table is:

Source	df	E[MSq]
Blocks	3	$\sigma^2_{BP} + 12\sigma^2_{B}$
Blocks.Plots	44	
Control	1	$\sigma^2_{BP} + f_{C}(\psi)$
Control.Dose	1	$\sigma^2_{BP} + f_{CD}(\psi)$
Control.Fumigant	3	$\sigma^2_{BP} + f_{CF}(\psi)$
Control.Dose.Fumigant	3	$\sigma^2_{BP} + f_{CDF}(\psi)$
Residual	36	σ^2_{BP}
Total	47	

Example VII.18 A factorial experiment

An experiment is to be conducted on sugar cane to investigate 6 factor (A, B, C, D, E, F) each at two levels. This experiment is to involve 16 blocks each of eight plots. The 64 treatment combinations are divided into 8 sets of 8 so that the ABCD, ABEF and ACE interactions are associated with set differences. The 8 sets are randomized to the 8 sets so that each set occurs on two blocks and the 8 combinations in a set are randomized to the plots within a block. The sugar content of the cane is to be measured.

1. the observational unit – a plot
2. response variable – Sugar content
3. unrandomized factors – Blocks, Plots
4. randomized factors – A, B, C, D, E, F
5. type of study – two replicates of a confounded 2^k factorial

The experimental structure is:

Structure	Formula
unrandomized	*16* Blocks/*8* Runs
randomized	*2* A**2* B**2* C**2* D**2* E**2* F

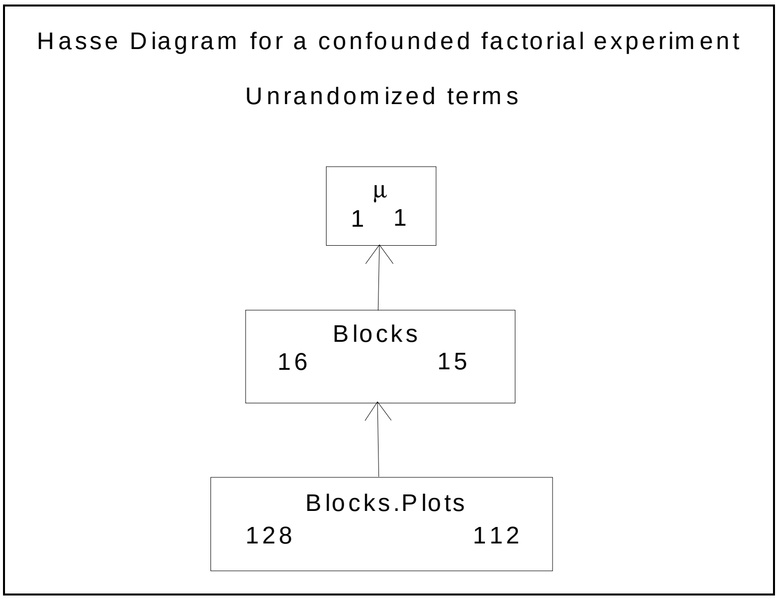
How many main effects, two factor interactions, three-factor interactions and interactions of more than 3 factors are there? What are the interactions confounded with blocks?

There are 6 main effects, 15 two-factor interactions, 20 three-factor interactions and 64 – 1 – 6 – 15 – 20 = 22 other interactions.

The interactions confounded with blocks are ABCD, ABEF and ACE and all the products of these. That is ABCD, ABEF and ACE and CDEF, BDE, BCF, ADF

What are the degrees of freedom for the unrandomized terms and for the randomized terms?

For the unrandomized terms, the Hasse diagram is as follows:



For the randomized terms, by the crossed structure formula rule, all degrees of freedom will be 1 as all factors have 2 levels.

Enter the sources and degrees of freedom for the study into the analysis of variance table below.

Seems that randomized factors should be fixed and unrandomized factors should be random. Hence, the maximal variation and expectation models are:

$$\text{Var}[Y] = \text{Blocks} + \text{Blocks.Plots}$$

$$\text{E}[Y] = \text{A.B.C.D.E.F}$$

What are the variation components, and their multipliers, for this study?

$$\sigma^2_{\text{BP}}\,,\ 8\sigma^2_{\text{B}}$$

The analysis of variance table is (just give the interaction confound with blocks and the numbers of main effects, two-factor, three-factor and other interactions):

Source	df	E[MSq]
Blocks	15	
ACE	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{ACE}}(\psi)$
ADF	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{ADF}}(\psi)$
BCF	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{BCF}}(\psi)$
BDE	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{BDE}}(\psi)$
ABCD	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{ABCD}}(\psi)$
ABEF	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{ABEF}}(\psi)$
CDEF	1	$\sigma_{BP}^2 + 8\sigma_B^2 + f_{\mathbf{CDEF}}(\psi)$
Residual	8	$\sigma_{BP}^2 + 8\sigma_B^2$
Blocks.Plots	112	
main effects	6	$\sigma_{BP}^2 + f_{\mathbf{i}}(\psi)$
2-factor interactions	15	$\sigma_{BP}^2 + f_{\mathbf{i.j}}(\psi)$
3-factor interactions	17	$\sigma_{BP}^2 + f_{\mathbf{i.j.k}}(\psi)$
other interactions	19	$\sigma_{BP}^2 + f_{\mathbf{i.j.k.l+}}(\psi)$
Residual	55	σ_{BP}^2