DESIGN AND MIXED-MODEL ANALYSIS OF EXPERIMENTS

XII. Repeated measurements experiments

(Payne, 1996; Mead, sec. 14.5)

XII.A	Introduction to repeated measurements experiments	XII-1
XII.B	Repeated measurements versus split-plot experiments	XII-2
	a) Determining the analysis of variance table	XII-5
	b) Analysis of the example	XII-7
	c) Computation in Genstat	XII-14
XII.C	Problems with ANOVA on individual measurements	
	timepoints	XII-15
XII.D	Separate analyses of each timepoint	
	Analyses of summary statistics	
	a) Ad hoc summary statistics	
	b) Summary statistics from fitted curves	
	c) Computation in Genstat	

XII.A Introduction to repeated measurements experiments

Definition XII.1: A **repeated measurements experiment** is one in which a sequence of measurements is made under constant treatment conditions. Generally, there will be u units (plots, animals and so) each of which is observed for s_k times.

Thus repeated measurement experiments are distinguished from split-plot experiments in which each unit is observed only once and from cross-over experiments in which each unit is observed more than once but the treatment conditions are not the same for all observations. Also, experiments involving the observation of different plots at different times are not repeated measurements experiments. That is, just because an experiment involves observations at different times, does not mean that it is necessarily a repeated measurements experiment.

Definition XII.2: A repeated measures experiment in which all units are observed for the same time intervals relative to the start of the experiment is called a **balanced repeated measurements experiment**.

Definition XII.3: The set of measurements made on a unit is called the unit's **profile**.

The design of repeated measurement experiments involves no special consideration. One has only to decide on the design to be applied to the units, most likely using one of the designs covered in the other chapters of this subject. For example, one might choose to run a factorial experiment laid out using an RCBD. This having been decided the experiment is laid out according to the chosen design and the repeated observations made on each unit.

There are now available several methods for the analysis of repeated measurements. These vary from an analysis of variance to complicated analyses allowing for non-linear fitting of trends and complex covariance structures between the observations from different times. We will be focussing on analysis of variance methods, although these have the limitation of often involving over-simplified models.

XII.B Repeated measurements versus split-plot experiments

In this section we consider the application of the analysis of variance to repeated measurements experiments and explore how this differs from that for split-plot experiments. In many textbooks you will see suggested that the analysis of variance for a repeated measurements experiment is a split-plot-in-time, with times being regarded as randomized to hypothetical subplots. For example, suppose that the experiment was laid out as an RCBD and then repeated measurements were made on each plot. The suggested analysis would be that for the standard split-plot experiment with the plots from the RCBD being treated as main plot and the times as being randomized to hypothetical subplots. However, the times are not randomized — they are the result of systematic observation of the plots. Further times are not nested within subplots, as is often the case with split-plot experiments, but time is crossed with plots in that each time is the same for all units. So this type of experiment is not of the split-plot kind.

First we consider experiments in which time is randomized. The first two exercises for chapter XI are examples of such experiments as they involved time factors that were randomized to subplots using a split-plot design.

Example XII.1 Celery experiment

Exercise XI.1 involved an experiment to investigate the effect on the yield of three methods of seedling propagation, two levels of nutrient and four harvest dates. The six combinations of propagation methods and nutrients were applied to main plots using a completely randomized design with three replicates of each treatment combination. The harvest dates were to be randomized to the four subplots within each main plot. The analysis of variance that was derived for this experiment was of the following form:

Source	df	E[MSq]	
MainPlots	17		
Propagation	2	$\sigma_{\rm MS}^2$ +4 $\sigma_{\rm M}^2$ + $f_{\rm P}$	$\psi)$
Nutrient	1	$\sigma_{\rm MS}^2$ +4 $\sigma_{\rm M}^2$ + $f_{\rm N}$	$\psi)$
Propagation.Nutrient	2	$\sigma_{\rm MS}^2$ +4 $\sigma_{\rm M}^2$ + $f_{\rm PN}$	(ψ)
Residual	12	σ_{MS}^2 +4 σ_{M}^2	
MainPlots.Subplots	54		
Harvests	3	$\sigma_{\rm MS}^2$ + $f_{\rm H}$	$\psi)$
Propagation.Harvests	6	$\sigma_{ ext{MS}}^2$ + $f_{ ext{PH}}$	(ψ)
Nutrient.Harvests	3	$\sigma_{ ext{MS}}^2$ + $f_{ ext{NH}}$	(ψ)
Propagation.Nutrient.Harvests	6	$\sigma_{\rm MS}^2$ + $f_{\rm PNH}$	$_{H}(\Psi)$
Residual	36	$\sigma_{ t MS}^2$	
Total	71		

Here the factor Harvests corresponds to times of harvest so that the experiment involves tracing the response variable, yield, over a period of time. However, the experiment is not a repeated measurements experiment because we did not repeatedly observe the same plot.

We next consider a repeated measurements experiment.

Example XII.2 Clones observed over several years

Consider a randomized complete block experiment in which several clones of some perennial crop are to be compared. The yield for each plot is measured in successive years without any change in the experimental layout. Generated data for such an experiment are given in the table below.

Layout and results for the repeated measurements experiment

					ots		_
		1		2			3
Blocks	Years	Clone	Yield	Clone	Yield	Clone	Yield
1	1	1	148.8	2	152.7	3	159.9
	2		142.4		142.3		150.6
	2		146.9		141.9		157.7
	4		155.4		142.9		152.2
2	1	3	160.5	1	160.5	2	156.2
	2		152.1		162.0		150.9
	3		136.7		148.1		135.0
	4		151.9		164.4		149.8
3	1	3	158.6	2	151.2	1	152.3
	2		157.5		145.1		156.6
	2		145.6		135.7		145.6
	4		163.1		154.6		165.3
4	1	1	152.5	3	159.0	2	151.4
			154.4		158.0		145.7
	2		162.7		166.5		151.6
	4		162.3		164.0		147.5
5	1	1	152.1	3	153.2	2	148.5
J	2	•	158.4	J	149.1	_	144.4
	3		168.3		157.4		153.6
	4		168.2		151.7		144.7

The split-plot-in-time analysis for the generated set of data is presented in table below. In working out the expected mean squares for this analysis, Years is taken to be a fixed factor as it is unlikely that Years will display the necessary homogeneity to be regarded as a variation factor. From this analysis we conclude that there is no interaction between Clones and Years and no overall differences between the Years but that there are overall differences between the Clones.

Split-plot-in-time analysis for the repeated measurements experiment

	16	140			
Source	df	MSq	E[MSq]	F	Prob
Blocks	4	75.38	σ_{BPY}^2 +4 σ_{BP}^2 +12 σ_{B}^2	1.35	0.332
Blocks.Plots	10				
Clones	2	490.52	$\sigma_{BPY}^2 + 4\sigma_{BP}^2 + f_{C}(\psi)$	8.77	0.010
Residual	8	55.96	σ_{BPY}^2 +4 σ_{BP}^2	1.50	0.192
Blocks.Plots.SubPlots	45				
Years	3	105.57	$\sigma_{BPY}^2 + f_{Y}(\psi)$	2.84	0.051
Clones.Years	6	48.52	$\sigma_{BPY}^2 + f_{CY}(\psi)$	1.30	0.282
Residual	36	37.22	σ_{BPY}^2		

Determining the analysis of variance table a)

We now derive the analysis for a repeated measurements experiment using the rules for determining the analysis of variance table.

Example XII.2 Clones observed over several years (continued)

What are the components of the study?

Observational unit — a plot in a year Response variable — Yield 1.

2.

3. Unrandomized factors – Blocks, Plots, Years

4.

Randomized factors – Clones
Type of study – Repeated measurements on an RCBD 5.

The experimental structure is:

Structure	Formula
unrandomized	(5 Blocks/3 Plots)*4 Years
randomized	3 Clones*Years

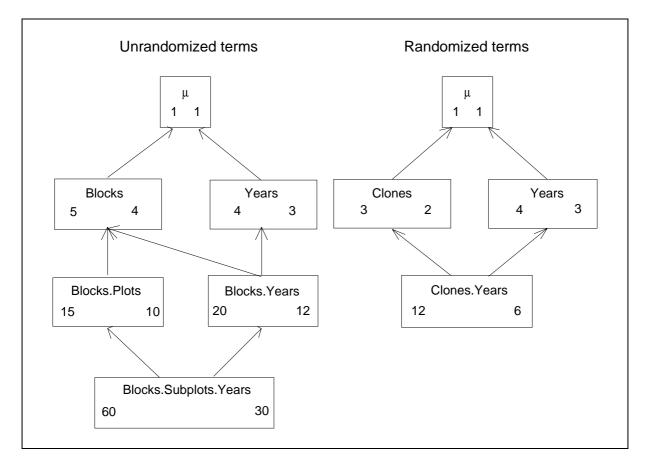
Note that we have included the factor Years in the randomized structure because we are interested its interaction with Clones.

The formulae expand to give:

(Blocks/Plots)*Years

- = (Blocks + Blocks.Plots)*Years
- = Blocks + Blocks.Plots + Years + Blocks.Years + Blocks.Plots.Years





Note, that in working out the degrees of freedom for the terms from the randomized structure, the rule for a set of crossed factors can be used. That is, for each factor in the term, calculate the number of levels minus one and multiply these together.

The models for this experiment, based on Blocks and Plots being random factors and Clones and Years being fixed factors are:

$$E[Y] = Clones. Years$$

and var[Y] = Blocks + Blocks.Plots + Blocks.Years + Blocks.Plots.Years.

The analysis of variance table for this experiment has the following form:

Skeleton analysis of variance table for the repeated measurements experiment

Source	df	E[MSq]
Blocks	4	σ_{BPY}^2 +4 σ_{BP}^2 +3 σ_{BY}^2 +12 σ_{B}^2
Blocks.Plots	10	
Clones	2	σ_{BPY}^2 +4 σ_{BP}^2 + $f_{C}(\psi)$
Residual	8	σ_{BPY}^2 +4 σ_{BP}^2
Years	3	$\sigma_{BPY}^2 + 3\sigma_{BY}^2 + f_{Y}(\psi)$
Blocks.Years	12	σ_{BPY}^2 +3 σ_{BY}^2
Blocks.Plots.Years	30	
Clones.Years	6	$\sigma_{BPY}^2 + f_{CY} \left(\psi \right)$
Residual	24	σ_{BPY}^2
Total	59	

The analysis of variance table includes a source for Blocks. Years, a term that was not included in the split-plot-in-time analysis. It should be included unless one is willing to argue that it is unlikely to occur. If this is an appreciable source of variability, one may get quite different results from an analysis that separates this term compared to one that does not. Further it is noted that a test for Years has Blocks. Years as the denominator, rather than Blocks. Plots. Years. However, this would not be the case if both Blocks and Years were designated as fixed factors.

b) Analysis of the example

The analysis in Genstat will start with plots of the profiles using the DREPMEASURES procedure. The analysis of variance will be conducted on the data and the usual diagnostic checking performed. Because time is a quantitative factor, we will investigate whether a linear trend can be used to describe the time trend for this data.

Example XII.2 Clones observed over several years (continued)

The Genstat output file for analyzing this experiment is:

Genstat 5 Release 4.1 (PC/Windows NT) 29 April 2000 11:30:52 Copyright 1998, Lawes Agricultural Trust (Rothamsted Experimental Station)

Genstat 5 Fourth Edition - (for Windows) Genstat 5 Procedure Library Release PL11

- 3 "Data taken from File: D:/ANALYSES/LM/REPEATMEASURE/RMECLONE.GSH"
- 4 DELETE [redefine=yes] Blocks, Plots, Years, Clones, Yields
- 5 FACTOR [modify=yes;nvalues=60;levels=5] Blocks 6 READ Blocks; frepresentation=ordinal

Identifier Values Missing Blocks 60

- 9 FACTOR [modify=yes;nvalues=60;levels=3] Plots
- 10 READ Plots; frepresentation=ordinal

Identifier Values Missing Levels Plots 60

- 13 FACTOR [modify=yes;nvalues=60;levels=4] Years
- 14 READ Years; frepresentation=ordinal

Identifier Values Missing 60 Years

- 17 FACTOR [modify=yes;nvalues=60;levels=3] Clones
 18 READ Clones; frepresentation=ordinal

Levels Identifier Values Missing Clones 60

- 21 VARIATE [nvalues=60] Yields 22 READ Yields

IdentifierMinimumMeanMaximumValuesYields135.0153.0168.360 60

29 PRINT Blocks, Plots, Years, Clones, Yields

Blocks	Plots	Years	Clones	Yields
1	1	1	1	148.8
1	2	1	2	152.7
1	3	1	3	159.9
1	1	2	1	142.4
1	2	2	2	142.3
1	3	2	3	150.6
1	1	3	1	146.9
1	2	3	2	141.9
1	3	3	3	157.7
1	1	4	1	155.4
1	2	4	2	142.9
1	3	4	3	152.2
2	1	1	3	160.5
2	2	1	1	160.5
2	3	1	2	156.2
2	1	2	3	152.1
2	2	2	1	162.0
2	3	2	2	150.9
2	1	3	3	136.7
2	2	3	1	148.1
2	3	3	2	135.0
2	1	4	3	151.9
2	2	4	1	164.4
2	3	4	2	149.8
3	1	1	3	158.6
3	2	1	2	151.2
3	3	1	1	152.3

```
157.5
                                                2
2
                                2
                                                                  2
                                                                               145.1
                                                                 1
                                                                              156.6
              3
                                3
                                                3
                                                                  3
                                                                              145.6
                                                3
              3
                                2
                                                                  2
                                                                              135.7
                                                3
4
                                3
                                                                   1
                                                                               145.6
                                                                              163.1
                               1
                                                                   3
                               2
                                                4
                                                                  2
                                                                              154.6
                               3
              3
                                                 4
                                                                   1
                                                                              165.3
                                                1
                                                                              152.5
              4
                               1
                                                                   1
                               2
                                                                              159.0
                                                1
                                                                   3
                               3
                                                1
                                                                   2
              4
                                                                              151.4
              4
                                1
                                                 2
                                                                   1
                                                                              154.4
                                                2
                                                                              158.0
              4
                               2
                                                                  3
                                                2
              4
                               3
                                                                  2
                                                                              145.7
                                                3
              4
                               1
                                                                   1
                                                                              162.7
                                                                              166.5
                               2.
                                                                   3
              4
                               3
                                                3
                                                                  2
                                                                              151.6
                               1
                                                4
              4
                                                                   1
                                                                              162.3
              4
                               2
                                                 4
                                                                   3
                                                                              164.0
                                                 4
                                                                  2
                                                                              147.5
              4
                               3
              5
                              1
                                                1
                                                                  1
                                                                              152.1
                                                1
1
              5
                                2
                                                                   3
                                                                              153.2
              5
                                3
                                                                   2
                                                                              148.5
                                                2
                                                                  1
                                                                              158.4
                                                2
2
              5
                                2
                                                                   3
                                                                              149.1
                                                                              144.4
              5
                                                                   2
                               3
                                                3
              5
                              1
                                                                 1
                                                                              168.3
                                                                 3
                                                3
              5
                              2
                                                                              157.4
                              3
                                               3
              5
                                                                   2
                                                                              153.6
                                                                              168.2
              5
                               1
                                                                   1
                                2
                                                 4
                                                                   3
                                                                              151.7
              5
                                                 4
                                                                   2
                                                                              144.7
      **** separate data for years and plot profiles
-31
-32
 33 SUBSET [CONDITION=Years==1] OLD=Blocks,Plots,Clones; \
                                               NEW=Block,Plot,Clone
 35 FOR i=1...4
       SUBSET [CONDITION=Years==i] OLD=Yields; NEW=Yield[i]
 36
 38 PRINT Block, Plot, Clone, Yield[1...4]; FIELD=9; DEC=1
                Plot Clone Yield[1] Yield[2] Yield[3] Yield[4]
    Block

      1
      1
      148.8
      142.4

      2
      2
      152.7
      142.3

      3
      3
      159.9
      150.6

      1
      3
      160.5
      152.1

      2
      1
      160.5
      162.0

      3
      2
      156.2
      150.9

      1
      3
      158.6
      157.5

      2
      2
      151.2
      145.1

      3
      1
      152.3
      156.6

      1
      1
      152.5
      154.4

      2
      3
      159.0
      158.0

      3
      2
      151.4
      145.7

      1
      1
      152.1
      158.4

      2
      3
      153.2
      149.1

      3
      2
      148.5
      144.4

                               1 148.8 142.4 146.9
          1
                   1
                                                                                155.4
          1
                                                                      141.9
                                                                                   142.9
                                                                    157.7
                                                                                  152.2
          1
                                                                    136.7
                                                                                 151.9
                                                                     148.1
          2
                                                                                  164.4
          2
                                                                      135.0
                                                                                   149.8
                                                                     145.6
          3
                                                                                  163.1
          3
                                                                     135.7
                                                                                  154.6
          3
                                                                      145.6
                                                                                   165.3
          4
                                                                      162.7
                                                                                   162.3
                                                                     166.5
                                                                                  164.0
                                                                     151.6
168.3
                                                                                 147.5
168.2
          4
          5
                                                                     157.4
                                                                                  151.7
          5
                       3
                                    2
                                           148.5
                                                        144.4
                                                                     153.6 144.7
      DREPMEASURES [GROUPS=Clone, Block] DATA=Yield
 39
 40
      DREPMEASURES [GROUPS=Clone] DATA=Yield
 41
      **** perform repeated measurements ANOVA
-42
-43
 44 DUPLICATE OLD=Years; NEW=Year
 45
      BLOCK (Blocks/Plots)*Years
 46 TREAT Clones*POL(Year;2)
 47 ANOVA [FPROB=Y; PSE=LSD] Yields
```

47						
**** Analysis	of variance *	***				
Variate: Yields						
Source of variat	tion d.f.	s.s.	m.s.	v.r.	F pr.	
Blocks stratum	4	302.268	75.567			
Years stratum Year Lin Quad Deviations	3 1 1 1	6.931 293.046	293.046			
Blocks.Plots str Clones Residual	ratum 2 8		490.783 56.171		0.010	
Blocks.Years st	ratum 12	1247.572	103.964	27.33		
Blocks.Plots.Year Clones.Year Clones.Lin Clones.Quad Deviations Residual	6	266.222 7.267 16.711	3.633	34.99	0.399	
Total	59	3678.226				
* MESSAGE: the i	following uni	ts have larg	e residuals			
Blocks 2 Year	rs 3	-9.66 s.	e. 4.56			
Blocks 1 Plot	ts 3 Years	3	2.98 s.	e. 1.23		
**** Tables of	means ****					
Variate: Yields						
Grand mean 152	.97					
Clones	1 2 .36 147.29	3 155.27				
Year 154	1 2 .49 151.30	3 150.22 1	4 55.87			
Clones Ye	1 153.24 152.00 158.24	145.68 1	3 54.32 163 43.56 147 52.78 156	.90		
*** Least significant differences of means (5% level) ***						
Table	Clones	Year	Clones Year			
rep. l.s.d. d.f. Except when comp Year d.f.	20 5.465 8 paring means	15 * * with the sam	5 * *	of		

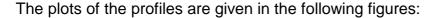
```
48 CALC pBP=1-FPROB(56.171/3.805; 8; 24)
           pBY=1-FPROB(103.964/3.805; 12; 24)
  49
     ۶
  50
     PRINT pBP, pBY
                     pBY
         pBP
 0.000000148 0.4532E-10
  51 APLOT METHOD=fit, normal
              Ι
             Ι
         2.5
             Ι
r
             Ι
              I
е
             Ι
s
i
              Ι
d
              Ι
u
         0.0
             I
             Ι
а
1
             I
              Ι
S
             Ι
              Ι
                                                      2
        -2.5 I
          132.0
                     138.0
                               144.0
                                          150.0
                                                     156.0
                                                               162.0
                                                                          168.0
                                           fitted values
                                            Normal plot
         3.0 I
r
              Ι
              I
е
s
             Т
                                   222*
2222222*
222*
*2*
                                                        2**2*
i
              I
d
             Ι
u
         0.0
             Ι
             Ι
а
1
             Ι
             I
S
             Ι
              Ι
        -3.0 I
            -2.4
                      -1.6
                                 -0.8
                                            0.0
                                                       0.8
                                                                 1.6
                                     expected Normal quantiles
 -53
      **** Tukey''s one-degree-of-freedom-for-non-additivity.
      **** It is the term designated covariate in the following analysis
 -54
 -55
      TREAT Clones*Year
  56
  57
      AKEEP [FIT=Fit]
      CALC ResSq=Fit*Fit
  58
  59
      ANOVA [PRINT=*] ResSq; RES=ResSq
  60
     COVAR ResSq
                                                 "A computational trick"
  61 ANOVA [PRINT=A; FPROB=Y] Yields
```

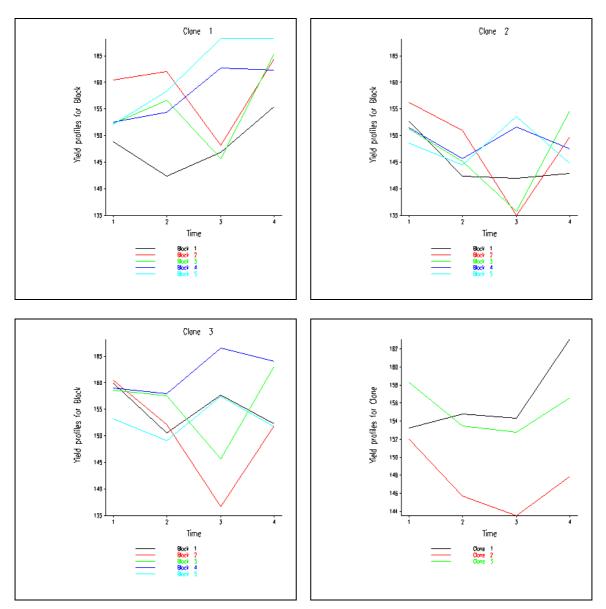
Stratum variance cannot be estimated Years stratum has zero residual sum of squares or degrees of freedom $% \left(1\right) =\left(1\right) \left(1\right) +\left(1\right) \left(1\right) \left(1\right) +\left(1\right) \left(1\right) \left(1\right) \left(1\right) \left(1\right) +\left(1\right) \left(1\right)$

******* Warning (Code AN 40). Statement 1 on Line 61

Command: ANOVA [PRINT=A; FPROB=Y] Yields

61							
**** Analysis of varia	ance (ad	djusted for	covariate)	****			
Variate: Yields Covariate: ResSq							
Source of variation	d.f.	s.s.	m.s.	v.r.	cov.ef.	F pr.	
Blocks stratum	4	302.268	75.567				
Years stratum Year	3	315.939	105.313		1.00		
Blocks.Plots stratum Clones Residual	2	981.567 449.371	490.783 56.171			0.010	
Blocks.Years stratum	12	1247.572	103.964	26.81			
Blocks.Plots.Years stra Clones.Year Covariate Residual Total	6 1 23	290.200 2.135 89.174 3678.226	48.367 2.135 3.877	12.47 0.55	1.00	<.001 0.466	
63 " -64 **** obtain fitted equations -65 " 66 VARI Yr 67 CALC Yr=Year 68 MODEL Yields 69 TERMS Clones/Yr 70 FIT Clones/Yr							
70				• • • • • •			
**** Regression Analys	sis ***	**					
Response variate: Yie. Fitted terms: Cons		Clones + Yr	Clones				
*** Summary of analysis	s ***						
d.f. Regression 5 Residual 54 Total 59	s.s 1259 2424 3678	5. 250 4. 44		.r. .59			
Percentage variance accounted for 28.0 Standard error of observations is estimated to be 6.70 * MESSAGE: The following units have large standardized residuals:							
*** Estimates of parame	eters *	* *					
Constant Clones 2 Clones 3 Yr.Clones 1 Yr.Clones 2 Yr.Clones 3		estimate 149.06 1.83 7.62 2.92 -1.44 -0.57	s.e. 3.67 5.19 5.19 1.34 1.34	40 0 1 2 -1	54) .62 .35 .47 .18 .08		





These plots indicate that there is considerable variability amongst the individual profiles for a particular clone. However, it also appears that the clones differ in their pattern over the years.

As far as diagnostic checking is concerned, there are no problems apparent. The residual-versus-fitted-values plot is displaying a homogenous pattern, the normal probability plot looks like a roughly linear pattern and Tukey's test for nonadditivity is nonsignificant.

The analysis of variance indicates that

- 1. The Deviations term for Clones. Times is not significant;
- 2. The Clones.Quad term is not significant;
- 3. The Clones.Lin term is significant.

That is, a linear trend describes the pattern over time, but that the slope of the trend differs between the clones. Regression was used to obtain the fitted equations and these are as follows:

```
For clone 1, Yield = 149.06 + 2.92 Year
For clone 2, Yield = 150.89 - 1.44 Year
For clone 3, Yield = 156.68 - 0.57 Year
```

We note that the Clones. Years term was not significant in the split-plot-in-time analysis, but is in this analysis. This is because the Blocks. Years term is highly significant and in the split-plot-in-time analysis it included in the Residual MSq of 37.22.

c) Computation in Genstat

The following commands were used in analyzing the example:

```
PRINT Blocks, Plots, Years, Clones, Yields
**** separate data for years and plot profiles
SUBSET [CONDITION=Years==1] OLD=Blocks,Plots,Clones; \
                            NEW=Block, Plot, Clone
FOR i=1...4
    SUBSET [CONDITION=Years==i] OLD=Yields; NEW=Yield[i]
ENDFOR
PRINT Block, Plot, Clone, Yield[1...4]; FIELD=9; DEC=1
DREPMEASURES [GROUPS=Clone, Block] DATA=Yield
DREPMEASURES [GROUPS=Clone] DATA=Yield
**** perform repeated measurements ANOVA
DUPLICATE OLD=Years; NEW=Year
BLOCK (Blocks/Plots) *Years
TREAT Clones*POL(Year;2)
ANOVA [FPROB=Y; PSE=LSD] Yields
CALC pBP=1-FPROB(56.171/3.805; 8; 24)
    pBY=1-FPROB(103.964/3.805; 12; 24)
PRINT pBP,pBY
APLOT METHOD=fit, normal
**** Tukey''s one-degree-of-freedom-for-non-additivity.
**** It is the term designated covariate in the following analysis
TREAT Clones*Year
AKEEP [FIT=Fit]
CALC ResSq=Fit*Fit
ANOVA [PRINT=*] ResSq; RES=ResSq
COVAR ResSq
                                         "A computational trick"
ANOVA [PRINT=A; FPROB=Y] Yields
**** obtain fitted equations
VARI Yr
CALC Yr=Year
MODEL Yields
TERMS Clones/Yr
FIT Clones/Yr
```

XII.C Problems with ANOVA on individual measurements for all timepoints

The major problem with using an ANOVA on the individual measurements for all timepoints is that it is very likely that the assumptions are not met. In practice it is assumed that variances at the different timepoints are equal and the correlations between pairs of timepoints are the same for all timepoints. That is the variance matrix for times should have the following form:

$$\sigma^{2} \begin{bmatrix} 1 & 2 & 3 & \dots & t \\ 1 & \rho & \rho & \dots & \rho \\ \rho & 1 & \rho & \dots & \rho \\ \rho & \rho & 1 & \dots & \rho \\ \vdots & \vdots & \vdots & & \vdots \\ \rho & \rho & \rho & \dots & 1 \end{bmatrix}$$

This pattern is known as **uniform covariance structure** and the matrix is said to show **compound symmetry**. Further the values of σ^2 and ρ in this matrix must be the same for each of the terms not involving times in the analysis. Tests are available for compound symmetry and equality of variance matrices.

There will be situations in which these assumptions are met and the single ANOVA on the individual measurements for all timepoints is appropriate. However, when the observations are made over a period of time during which there is a considerable change in the organisms being observed, it is very likely that these assumptions will not be met. If they are not met, alternative analyses must be used instead. Some of the possible alternative analyses are:

- a) separate analyses of each timepoint
- b) ad hoc summary statistics over timepoints
- c) summary statistics from fitted curves
- d) Greenhouse-Geisser and Huyhn-Feldt adjustments to the degrees of freedom of ANOVA on individual measurements for all timepoints
- e) multivariate ANOVA
- f) analysis assuming ante-dependence structure
- g) nonlinear-mixed models
- h) more general covariance models and nonparametric smoothing of time trends

Of these, only the first three will be discussed in this course.

XII.D Separate analyses of each timepoint

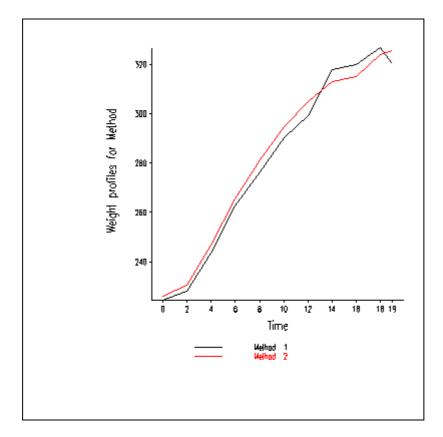
Often the data from repeated measurements experiments is analysed using separate analyses of the data from each timepoint. This is not invalid but separate tests must be interpreted correctly, and this means in a very limited sense. The major errors that occur in the use of this procedure are:

- The tests are incorrectly used to assess changes over time. These tests ignore the successive nature of the observations. They are based on between-unit comparisons. The test examines whether the group means over different units at a particular time are different using variability in the response variable between units within a group. On the other hand assessment of change requires within-unit comparisons. That is, a comparison of the means of the time differences computed for each unit using the variability of the within-unit time differences. The variability of within-unit time differences is likely to be quite different to the variability between units at a particular time. Further, a change in statistical significance is not equivalent to a statistically significant change.
- The tests may not detect differences because they are very inefficient. They ignore the within-unit correlations and treat each timepoint as independent.
- It is inappropriate to simply divide timepoints into those that are "significant" and those that are "not significant" for a process that is essentially continuous in time.

Example XII.3 Internal parasites in calves

An experiment to compare two methods for the control of intestinal parasites in calves involved 60 calves. At the start of the grazing season the calves were randomly assigned to the two methods so that 30 calves received each method. The weights of each calf were measured at weeks 0, 2, 4, 6, 8, 10,12, 14, 16, 18 and 19.

The plot of the mean profile for the two methods is shown in the following diagram.



There are only two treatments and so it is a simple matter to perform t-tests on the data for each timepoint (very easy to do in Excel). The results of the t-tests are shown in the following output:

	test	d.f.	prob
ttest[0]	-0.60	58.00	0.55
ttest[2]	-0.82	58.00	0.42
ttest[4]	-1.03	58.00	0.31
ttest[6]	-0.88	58.00	0.38
ttest[8]	-1.21	58.00	0.23
ttest[10]	-1.12	58.00	0.27
ttest[12]	-1.28	58.00	0.21
ttest[14]	1.10	58.00	0.28
ttest[16]	0.95	58.00	0.34
ttest[18]	0.53	58.00	0.60
ttest[19]	-0.85	58.00	0.40

These tests indicate that there are no differences between the methods over time. We will see how alternative analyses can give more useful information about the effects of treatments in this experiment.

XII.E Analyses of summary statistics

The summary statistic approach to the analysis of repeated measurements experiments is, in principle, very simple. A small number of statistics is computed for each unit from the observations over time for that unit.

The analysis of these summary statistics is generally straightforward. Each statistics is treated a single response and analyzed using conventional techniques, for example analysis of variance, regression or even a nonparametric test. If there is more than one-statistic from each subject then these will usually not be independent and multivariate analyses can be used, for example multivariate analysis of variance. So this method moves the problem from the realm of repeated measurements into that of more conventional methodology.

The advantage of this approach is that the unlikely-to-be-met assumptions about the behaviour through time are avoided. The choice of summary statistic depends very much on the context and there exist many possible alternatives. It is important that the chosen statistics have some meaning or interpretation in terms of the experiment. However, it is not critical that all information in a unit's profile is extracted, provided that features believed to be relevant are examined.

It is likely that in choosing the statistics there will be some trade-off between exploratory and confirmatory analysis. It is preferable that the statistics be chosen in the planning stage of the experiment. However, this may require knowledge in advance about the form of the profile and this may well not be the case. If there is insufficient knowledge available in the planning stage, one may be forced into a more exploratory analysis where an examination of the observed profiles is used to choose a suitable summary statistic.

The summary statistic approach is particularly valuable when

- individual profiles vary considerably;
- the repeated measurements are unbalanced because not all units are measured at the same set of timepoints, provided the differences between units are not too great.

We distinguish between two forms of summary statistics: a) ad hoc summary statistics and b) summary statistics from fitted curves.

a) Ad hoc summary statistics

Often a profile will not follow a simple functional form and many types of ad hoc statistics can be considered. Some commonly used ones are:

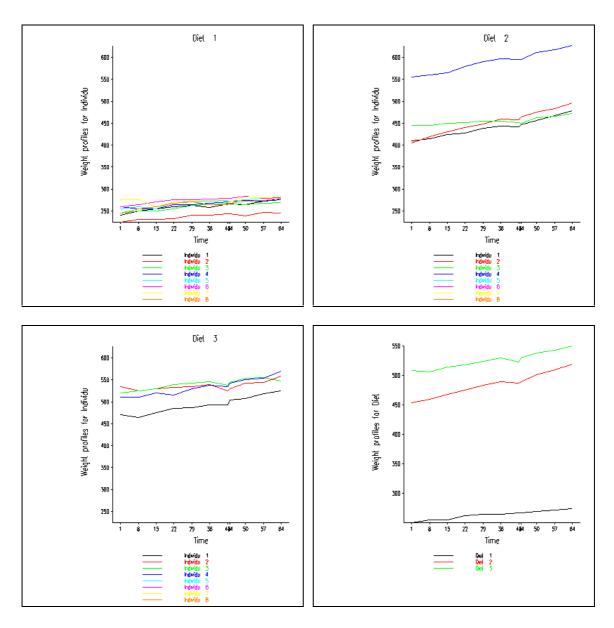
- particular end point(s)
- mean over the whole or part of the profile
- area under the profile
- maximum or minimum during the whole, or part, of the period of measurement
- change between two times
- time to a pre-defined threshold, to return to a baseline or to the maximum/minimum
- the number of events or changes
- proportion of successes

The great advantage of summary statistics is that they provide a way of handling categorical (particularly binary) repeated measurements without the need for sophisticated modelling. However, some summary statistics may be far from normally distributed whatever the distribution of the original measurements. Examples are extrema or times to particular events when there are only a few timepoints.

One danger with this approach is to create a very large number of statistics and to analyse them all. It is unlikely that all the statistics will be independent. Consequently, the analysis and interpretation of the results would not be independent and the statistical properties of the analyses would be compromised.

Example XII.4 Rat weight

An experiment to investigate the effects of 3 diets is described in Crowder and Hand (1996). It involved 16 rats, eight randomly assigned to diet 1 and four to each of diets 2 and 3. The body weights, in grams, were measured over 64 days. The profiles are plotted in the following diagrams.



The profiles show a roughly linear pattern with all rats having parallel patterns — rather well-behaved profiles.

In this case, the weight gain over the 64 days is likely to be of interest to the experiment and so we choose it as our summary statistic for analysis. Below is the Genstat output that contains the commands to produce the profiles and analyses and the output from these commands.

```
Genstat 5 Release 4.1 (PC/Windows NT) 30 April 2000 12:14:48
Copyright 1998, Lawes Agricultural Trust (Rothamsted Experimental Station)

Genstat 5 Fourth Edition - (for Windows)
Genstat 5 Procedure Library Release PL11

3 "Data taken from File: D:/ANALYSES/LM/REPEATMEASURE/RMERAT.GSH"
4 DELETE [redefine=yes] Days,Rats,Diets,Weights
5 FACTOR [modify=yes;nvalues=176;levels=!(1,8,15,22,29,36,43,44,50,57,64)\
6 ] Days
7 READ Days; frepresentation=ordinal
```

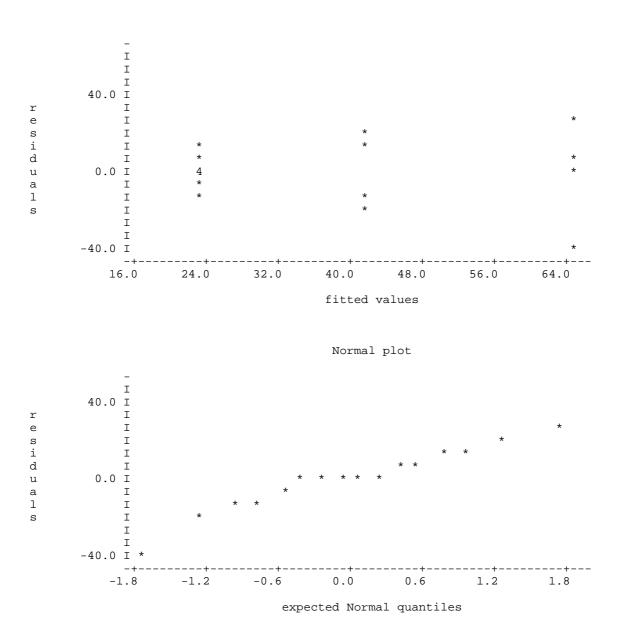
```
Identifier Values Missing
                                 Levels
        Days
                  176
                              0
                                      11
 14 FACTOR [modify=yes;nvalues=176;levels=16] Rats
 15 READ Rats; frepresentation=ordinal
   Identifier
                Values Missing
                                   Levels
        Rats
                176
 22 FACTOR [modify=yes;nvalues=176;levels=3] Diets
 23 READ Diets; frepresentation=ordinal
  Identifier
                Values
                        Missing
                                   Levels
       Diets
                 176
                                       3
 29 VARIATE [nvalues=176] Weights
 30 READ Weights
   Identifier
             Minimum
                          Mean
                                Maximum
                                           Values
                                                    Missing
                                  628.0
                          384.5
                                             176
     Weights
               225.0
 41
 42
-43
    **** separate data for Days and Rat profiles
-44
 45
    SUBSET [CONDITION=Days==1] OLD=Rats, Diets; \
 46
                              NEW=Rat,Diet
 47
    FOR i=1,8...43,44,50,57,64
 48
        SUBSET [CONDITION=Days==i] OLD=Weights; NEW=Weight[i]
    ENDFOR
 49
    PRINT Rat, Diet, Weight[1,8...43,44,50,57,64]; FIELD=9; DEC=0
            Diet Weight[1] Weight[8] Weight[15] Weight[22] Weight[29] Weight[36]
                                           260
             1 240
                          250 255
                                                    262
                                                                258
      1
      2
               1
                     225
                              230
                                       230
                                               232
                                                        240
                                                                240
      3
              1
                     245
                              250
                                       250
                                               255
                                                        262
                                                                265
              1
      4
                     260
                             255
                                      255
                                              265
                                                        265
                                                                268
                    255
                             260
      5
              1
                                     255
                                              270
                                                        270
                                                                273
              1
                                     270
      6
                     260
                              265
                                               275
                                                        275
                                                                277
      7
               1
                     275
                              275
                                       260
                                               270
                                                        273
                                                                274
      8
                    245
                             255
                                                        270
              1
                                     260
                                              268
                  410
                             415
              2
                                      425
      9
                                               428
                                                       438
                                                                443
                   405
445
     10
               2
                              420
                                      430
                                               440
                                                        448
                                                                460
              2
                             445
                                      450
                                              452
     11
                                                       455
                                                                455
     12
              2
                   555
                            560
                                     565
                                              580
                                                       590
                                                                597
     13
              3
                     470
                              465
                                      475
                                               485
                                                        487
                                                                493
                             525
               3
     14
                     535
                                      530
                                               533
                                                        535
                                                                540
                     520
                             525
                                              540
     15
               3
                                      530
                                                        543
                                                                546
                             510
              3
                    510
                                     520
                                               515
                                                       530
                                                                538
     16
Weight[43] Weight[44] Weight[50] Weight[57] Weight[64]
                                   278
    266
          266 265 272
    243
             244
                     238
                              247
                                       245
    267
             267
                     264
                              268
                                      269
                     274
                                       275
    270
             272
                              273
    274
             273
                     276
                              278
                                       280
    278
             278
                     284
                              279
                                       281
    276
             271
                     282
                              281
                                       284
                     273
                              274
    265
             267
                                       278
    442
             446
                     456
                              468
                                       478
    458
            464
                    475
                             484
                                      496
                     462
    451
             450
                              466
                                      472
    595
             595
                     612
                              618
                                      628
    493
             504
                     507
                              518
                                      525
    525
             530
                     543
                              544
                                      559
             544
                     553
                              555
                                       548
    538
                     550
                              553
                                      569
    535
             542
 51 FACTOR [LEV=30] Individuals
 52 CALC Individuals=NEWLEVELS(Rat; !v(1...8,(1...4)2))
```

```
* MESSAGE: There are missing values in the plot
****** Warning (Code HG 20). Statement 130 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
* MESSAGE: There are missing values in the plot
****** Warning (Code HG 20). Statement 130 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
* MESSAGE: There are missing values in the plot
******* Warning (Code HG 20). Statement 130 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
  54 DREPMEASURES [GROUPS=Diet] DATA=Weight
  55
 -56 **** analyse weight gain
 -57
  58 CALC WtGain=Weight[64]-Weight[1]
  59 BLOCK Rat
 60 TREAT Diet
61 ANOVA [FPROB=Y; PSE=LSD] WtGain
61.....
**** Analysis of variance ****
Variate: WtGain
Source of variation d.f. s.s. m.s. v.r. F pr.
Rat stratum
                        2
                             4681.1 2340.6 8.07 0.005
Diet
Residual
                              3772.6
                        13
                                         290.2
Total
                        15
                              8453.7
* MESSAGE: the following units have large residuals.
              -37.7 s.e. 15.4
Rat 11
**** Tables of means ****
Variate: WtGain
Grand mean 38.1
    Diet
             23.1
                    64.7
                             41.5
    rep.
               8
*** Least significant differences of means (5% level) ***
Table
                    Diet
rep.
                 unequal
                     13
d.f.
                   26.02 min.rep 22.54 max-min
l.s.d.
                   18.40X max.rep
```

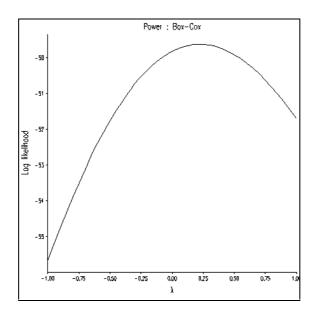
(No comparisons in categories where s.e.d. marked with an X)

53 DREPMEASURES [GROUPS=Diet, Individuals] DATA=Weight

62 APLOT METHOD=fit, normal



The assumptions underlying the analysis of weight gain appear not to be met in that the variance appears to be increasing with the weight gain. So we need to find modify the analysis to take this into account. The following diagram contains the plot produced by YTRANSFORM to identify a power transformation. The log transformation was chosen because it is close to the maximum and is interpretable. The output below shows the analysis of the natural logarithm of the weight gain.



- 63 YTRANSFORM [TERMS=Diet; LOWER=-1; UPPER=1] WtGain; SAVE=s
- 64 CALC TWtGain=LOG(WtGain) 65 ANOVA [FPROB=Y; PSE=LSD] TWtGain

65....

**** Analysis of variance ****

Variate: TWtGain

d.f. Source of variation s.s. m.s. v.r. F pr. Rat stratum Diet 2 2.9587 1.4794 6.55 0.011 Residual 2.9344 13 0.2257 Total 15 5.8932

* MESSAGE: the following units have large residuals.

Rat 7 -0.86 s.e. 0.43

**** Tables of means ****

Variate: TWtGain

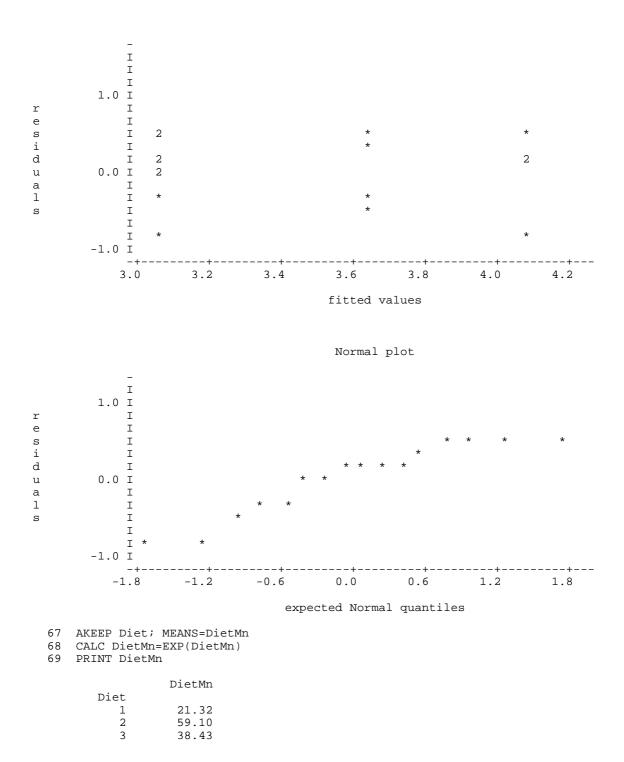
Grand mean 3.46

Diet 1 3.06 4.08 3.65 8 rep.

*** Least significant differences of means (5% level) ***

Table Diet rep. unequal d.f. 13 1.s.d. 0.726 min.rep 0.629 max-min 0.513X max.rep

(No comparisons in categories where s.e.d. marked with an X)



This analysis indicates a significant difference between the weight gains under the different diets (p = 0.011). To determine the differences between the diet we examine the table of log weight gain means and use the LSD; the relevant part of the output is given below.

```
***** Tables of means *****
Variate: TWtGain
Grand mean 3.46
            1 2 3
3.06 4.08 3.65
    Diet
    rep.
*** Least significant differences of means (5% level) ***
                     Diet
Table
rep.
                 unequal
                       13
d.f.
                    0.726 min.rep
l.s.d.
                    0.629 max-min
                    0.513X max.rep
(No comparisons in categories where s.e.d. marked with an X)
```

Note to compare diet 1 with the other diets, use the max-min LSD as these comparisons involve a mean of 8 with a mean of 4; for diet 2 versus 3, use the min-min LSD as these two means are both based on 4 observations. Using the max-min LSD we see that diet 1 results in less weight gain than diet 3 but is not significantly different to diet 2. Nor is there a significant difference between the mean weight gains of diets 2 and 3.

The back-transformed means have also been computed and are given at the end of the output — they are the geometric means of the original data. We can interpret the results from using the LSD as indicating which ratios of geometric means are significantly different. In this case, the only significant difference is between diets 1 and 2 and the ratio of diet 2 to diet 1 is 59.1/21.32 = 2.77.

b) Summary statistics from fitted curves

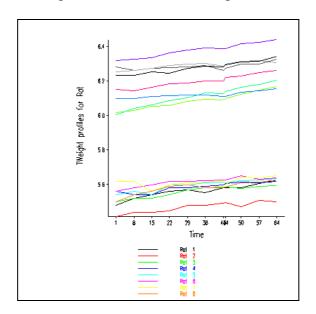
If the individual profiles are smooth, it may be possible to summarize an individual profile with a response function. In general, this response function can be any suitable function and the parameters of the fitted equation provide the summary statistics for analysis. We will examine the use of low-degree polynomials for this purpose.

Although the measurements at different timepoints for a subject are not independent, it is still suggested that ordinary, unweighted, least-squares estimates be used, possibly after transformation of the response. Provided that the variability of the repeated measurements does not change much over time (a transformation may be needed to ensure this), the ordinary, unweighted, least-squares estimates will be as efficient, or nearly as efficient, as more sophisticated methods that take into account the correlations between the timepoints. The ordinary, unweighted, least-squares estimates will always be valid, in the sense that they will be consistent, and they have the great advantage of simplicity. However, standard errors, R² and other inferential, regression procedures, in particular the use of differences in the residual mean squares to compare models, are **not** valid and can be very misleading. Here it is to be emphasized that we merely computed fitted parameters as summary statistics describing particular aspects of the behaviour of the data. The analysis that

we then perform on these statistics involves assumptions about a model for the distribution of the values of the fitted parameters over the sample of units, and not the distribution of the residuals from the fit of a curve to an individual's profile. One consequence of this is that it is not even required that the curves are particularly good fit to the data, although clearly this may be a desirable aid to interpretation.

Example XII.4 Rat weight (continued)

In the previous section we described an experiment involving 16 rats to investigate the effects of 3 diets. The weight gain over the period of observation was used as an ad hoc summary statistic to analyse this data. However, it was found that a log transformation was needed to stabilise the variance. A plot of the profiles is the natural logarithms of the weights is shown in the diagram below.



From this plot it is concluded that the trend remains linear, as would be expected if the range of the weight gains for each rat covers a narrow range (max/min < 3).

Given this approximately linear trend, another possible summary statistic is the slope of a fitted straight line. Genstat's VORTHPOL procedure can be used to compute the coefficients of orthogonal polynomials. The degree of the polynomials to be fitted is specified by the option MAXDEGREE and the coefficients are saved in variates specified by the CONTRAST parameter. For example CONTRAST=pol saves the mean for each unit in pol[0], the slope for each in pol[1], the quadratic coefficient for each in pol[2] and so on. The following Genstat output contains the commands to compute the mean and linear and quadratic coefficients and to analyse them. The output of these commands is also given.

```
Genstat 5 Release 4.1 (PC/Windows NT) 30 April 2000 13:11:19
Copyright 1998, Lawes Agricultural Trust (Rothamsted Experimental Station)

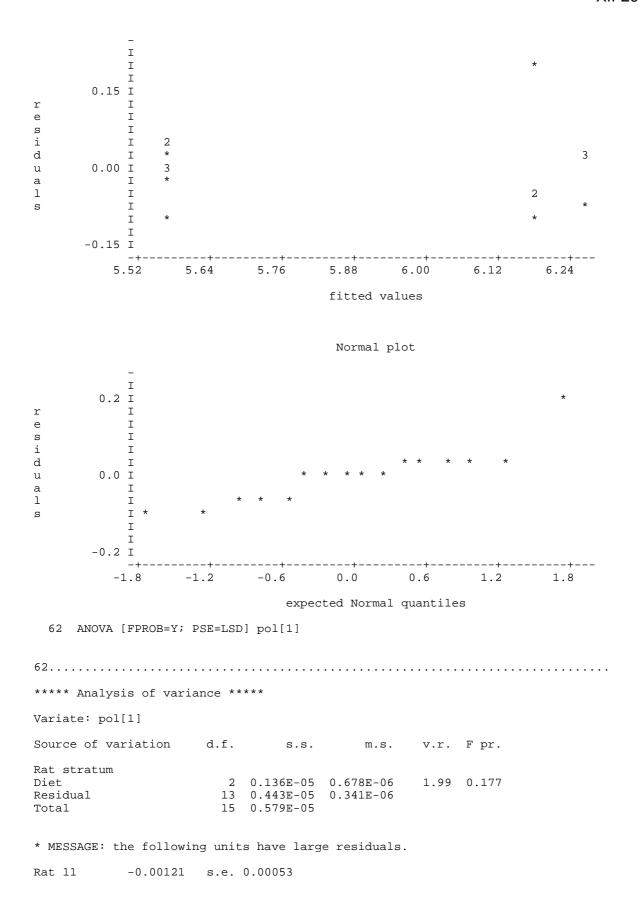
Genstat 5 Fourth Edition - (for Windows)
Genstat 5 Procedure Library Release PL11
```

B "Data taken from File: D:/ANALYSES/LM/REPEATMEASURE/RMERAT.GSH"

⁴ DELETE [redefine=yes] Days, Rats, Diets, Weights

```
5 FACTOR [modify=yes;nvalues=176;levels=!(1,8,15,22,29,36,43,44,50,57,64)\
     1 Davs
   7 READ Days; frepresentation=ordinal
   Identifier
                  Values
                           Missing
                                      Levels
         Days
                    176
 14 FACTOR [modify=yes;nvalues=176;levels=16] Rats
 15 READ Rats; frepresentation=ordinal
   Identifier
                Values
                         Missing
                                      Levels
                     176
         Rats
 22 FACTOR [modify=yes;nvalues=176;levels=3] Diets
 23 READ Diets; frepresentation=ordinal
    Identifier
                  Values
                           Missing
                                      Levels
        Diets
                    176
                                            3
 29 VARIATE [nvalues=176] Weights
 30 READ Weights
   Identifier
                 Minimum
                              Mean
                                     Maximum
                                                Values
                                                          Missina
                                                   176
                   225.0
                             384.5
      Weights
                                       628.0
 41
 42
 -43
     **** separate data for Days and Rat profiles
 -44
 45
     SUBSET [CONDITION=Days==1] OLD=Rats, Diets; \
 46
                                  NEW=Rat, Diet
 47
     FOR i=1,8...43,44,50,57,64
 48
         SUBSET [CONDITION=Days==i] OLD=Weights; NEW=Weight[i]
 49
     ENDFOR
 50 PRINT Rat, Diet, Weight[1,8...43,44,50,57,64]; FIELD=9; DEC=0
             Diet Weight[1] Weight[8] Weight[15] Weight[22] Weight[29] Weight[3
     Rat
6]
                        240
                                 250
                                           255
                                                    260
                                                             262
                                                                      258
        2
                        225
                                 230
                                           230
                                                    232
                                                             240
                                                                      240
                 1
        3
                 1
                        245
                                 250
                                           250
                                                    255
                                                             262
                                                                      265
                                 255
                                           255
                 1
                        260
                                                    265
                                                             265
                                                                      268
        5
                                           255
                                                    270
                                                             270
                 1
                        255
                                 260
                                                                      273
        6
                 1
                        260
                                 265
                                           270
                                                    275
                                                             275
                                                                      277
        7
                       275
                                 275
                                                    270
                                                             273
                                                                      274
                 1
                                          260
        8
                1
                       245
                                 255
                                          260
                                                    268
                                                             270
                                                                      265
        9
                 2
                       410
                                 415
                                          425
                                                    428
                                                             438
                                                                      443
       10
                 2
                       405
                                 420
                                          430
                                                    440
                                                             448
                                                                      460
                2
                       445
                                 445
                                          450
                                                    452
       11
                                                             455
                                                                      455
                 2
                                                    580
       12
                       555
                                 560
                                          565
                                                             590
                                                                      597
       13
                 3
                        470
                                 465
                                           475
                                                    485
                                                             487
                                                                      493
                                 525
      14
                 3
                       535
                                          530
                                                    533
                                                             535
                                                                      540
      15
                 3
                       520
                                 525
                                          530
                                                   540
                                                             543
                                                                      546
                 3
      16
                        510
                                 510
                                          520
                                                    515
                                                             530
                                                                      538
Weight[43] Weight[44] Weight[50] Weight[57] Weight[64]
               266
                        265
                                 272
                                           278
      266
      243
               244
                        238
                                 247
                                           245
      267
               267
                        264
                                 268
                                           269
      270
               272
                        274
                                 273
                                           275
      274
               273
                        276
                                 278
                                           280
      278
               278
                        284
                                 279
      276
               271
                        282
                                 281
                                           284
      265
               267
                        273
                                 274
                                           278
      442
              446
                       456
                                 468
                                           478
      458
               464
                       475
                                 484
                                           496
      451
               450
                        462
                                 466
                                           472
      595
               595
                       612
                                 618
                                          628
      493
              504
                       507
                                 518
                                          525
      525
              530
                        543
                                 544
                                          559
      538
               544
                        553
                                 555
                                          548
```

```
535 542 550 553 569
 51 "
 -52
     **** analyse means, linear and quadratic coefficients
 -53
 54 DUPLICATE OLD=Weight; NEW=TWeight 55 CALC #TWeight=LOG(#Weight)
 56 DREPMEASURES [GROUP=Rat] DATA=TWeight
******* Warning (Code HG 20). Statement 96 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
  57 VORTHPOL [MAXDEGREE=2] TWeight; CONTRAST=pol
  58 BLOCK Rat
 59 TREAT Diet
60 ANOVA [FPROB=Y; PSE=LSD] pol[0]
60.....
**** Analysis of variance ****
Variate: pol[0]
Source of variation d.f. s.s. m.s. v.r. F pr.
Rat stratum
Diet
                        2.
                           1.685232 0.842616 141.88 <.001
Residual
                        13
                            0.077208
                                     0.005939
                        15
                            1.762440
Total
* MESSAGE: the following units have large residuals.
Rat 12
             0.205 s.e. 0.069
**** Tables of means ****
Variate: pol[0]
Grand mean 5.897
            1
                  6.175
            5.573
    rep.
*** Least significant differences of means (5% level) ***
Table
                    Diet
rep.
                 unequal
d.f.
                     13
                  0.1177
l.s.d.
                         min.rep
                  0.1020 max-min
                  0.0832X max.rep
(No comparisons in categories where s.e.d. marked with an X)
  61 APLOT METHOD=fit, normal
```



```
**** Tables of means ****
Variate: pol[1]
Grand mean 0.00151
     Diet
                           2
           0.00138 0.00200 0.00126
     rep.
*** Least significant differences of means (5% level) ***
Table
                      Diet
rep.
                   unequal
d.f.
                         13
                  0.000892 min.rep
0.000773 max-min
l.s.d.
                   0.000631X max.rep
(No comparisons in categories where s.e.d. marked with an {\tt X})
  63 APLOT METHOD=fit, normal
             I
             Ι
             Ι
      0.0015 I
             Ι
е
s
             Ι
i
             Ι
d
             I
      0.0000 I
u
             I
а
1
             Ι
s
             Ι
     -0.0015 I
        0.00120 \quad 0.00135 \quad 0.00150 \quad 0.00165 \quad 0.00180 \quad 0.00195 \quad 0.00210
                                           fitted values
                                            Normal plot
      0.0015 I
r
             Ι
е
s
             Ι
i
             Ι
d
      0.0000 I
u
             Ι
а
             Ι
1
             I *
     -0.0015 I
            -1.8
                     -1.2
                                -0.6
                                      0.0 0.6 1.2 1.8
```

64 ANOVA [FPROB=Y; PSE=LSD] pol[2]

64.....

**** Analysis of variance ****

Variate: pol[2]

Source of variation d.f. s.s. m.s. v.r. F pr.

Rat stratum

Diet 2 0.117E-08 0.587E-09 2.31 0.138

Residual 13 0.330E-08 0.254E-09

Total 15 0.448E-08

* MESSAGE: the following units have large residuals.

Rat 7 0.0000401 s.e. 0.0000144

***** Tables of means *****

Variate: pol[2]

Grand mean -0.0000013

*** Least significant differences of means (5% level) ***

Table Diet rep. unequal d.f. 13

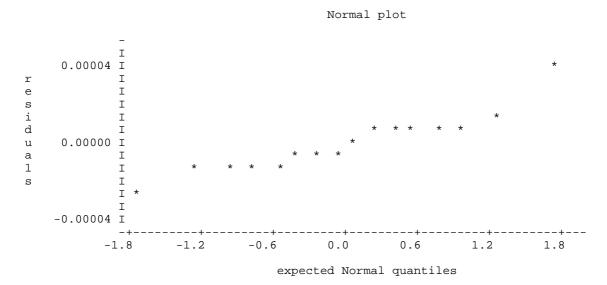
1.s.d. 0.00002435 min.rep 0.00002109 max-min 0.00001722X max.rep

(No comparisons in categories where s.e.d. marked with an X)

65 APLOT METHOD=fit, normal

Ι Ι 0.00003 I r е Ι I i Т d Ι 0.00000 I u а Ι Ι S Т -0.00003 I $-0.000012 \ -0.000008 \ -0.000004 \ \ 0.000000 \ \ 0.000004 \ \ 0.000008 \ \ 0.000012$

fitted values



The assumptions underlying these analyses appear to be met, although there is an outlier in the mean values that would require investigation to see if a reason could be established for it. The analyses indicate that there are significant differences between the diets in the overall rat means (p < 0.001) and that there are not significant differences between the diets in the slopes (p = 0.177) nor in the quadratic coefficients (p = 0.138). That is all the information about the differences between the diets appears to be contained in the overall rat means. The advantage of the overall rat means, compared to the weight gains, is that the overall rat means are based on all the observations for a rat. However, the final choice must be based on what is most important to the experimenter.

The diet means computed from the overall rat means are given in the following table. Again the exponential of these means is the geometric mean of the observations for that diet and the ratios of these could be reported.

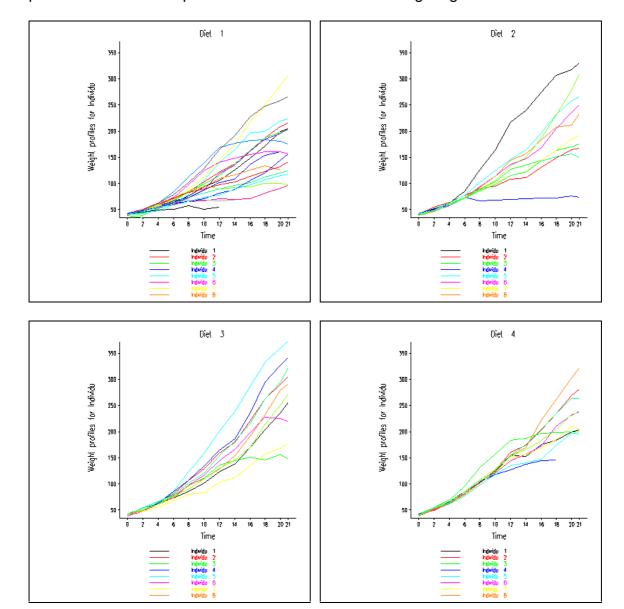
```
***** Tables of means *****
Variate: pol[0]
Grand mean 5.897
     Diet
             5.573
                     6.175
                               6.264
     rep.
*** Least significant differences of means (5% level) ***
                      Diet
Table
rep.
                   unequal
d.f.
                        13
                    0.1177
l.s.d.
                            min.rep
                    0.1020 max-min
                    0.0832X max.rep
(No comparisons in categories where s.e.d. marked with an X)
```

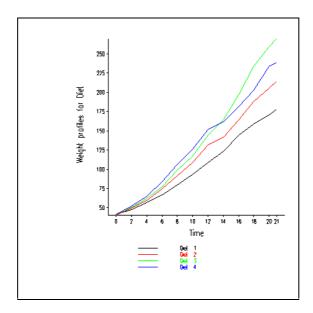
In this case diet 1 is significantly different to diets 2 and 3, the latter two diets not being significantly different. The back-transformed diet means and ratios are given in the following table.

Diet	Geometric mean	Ratio to diet 1
1	263.4	
2	480.7	1.82
3	525.2	1.99

Example XII.5 Chick body weights under 4 diets

Fifty chicks were involved in the study of the effect of four diets on the body weight of chicks. The chicks were observed on alternative days over a 3-week period. The plots of the individual profiles are shown in the following diagrams.





To analyse this data polynomials of order 6 are fitted. The results are shown in the following Genstat output.

```
Genstat 5 Release 4.1 (PC/Windows NT)
                                                       30 April 2000 21:36:45
Copyright 1998, Lawes Agricultural Trust (Rothamsted Experimental Station)
                 Genstat 5 Fourth Edition - (for Windows)
                 Genstat 5 Procedure Library Release PL11
     "Data taken from File: D:/ANALYSES/LM/REPEATMEASURE/RMECHICKDIET.GSH"
     DELETE [redefine=yes] Days, Chicks, Diets, Weights
     FACTOR [modify=yes;nvalues=600;levels=!(0,2,4,6,8,10,12,14,16,18,20,21)\
     ] Days
   7 READ Days; frepresentation=ordinal
    Identifier
                  Values
                           Missing
                                      Levels
          Days
                     600
                                 0
                                          12
  26 FACTOR [modify=yes;nvalues=600;levels=50] Chicks
     READ Chicks; frepresentation=ordinal
    Identifier
                  Values
                           Missing
                                      Levels
        Chicks
                     600
  51 FACTOR [modify=yes;nvalues=600;levels=4] Diets
  52 READ Diets; frepresentation=ordinal
    Identifier
                  Values
                           Missing
                                      Levels
         Diets
                     600
                                 0
                                           4
  69 VARIATE [nvalues=600] Weights
  70 READ Weights
    Identifier
                 Minimum
                              Mean
                                     Maximum
                                                 Values
                             121.8
                                                   600
       Weights
                    35.0
                                       373.0
  99
100
-101
      **** separate data for Days and Chick profiles
-102
     SUBSET [CONDITION=Days==0] OLD=Chicks, Diets; \
103
 104
                                  NEW=Chick, Diet
105
     FOR i=0,2...20,21
 106
          SUBSET [CONDITION=Days==i] OLD=Weights; NEW=Weight[i]
```

107 ENDFOR 108 PRINT Chick, Diet, Weight[0,2...20,21]; FIELD=9; DEC=0

Chick Diet Weight[0] Weight[2] Weight[4] Weight[6] Weight[10] 1	Chick	Diet	Weight[0]	Weight[2]	Weight[4]	Weight[6]	Weight[8]	Weight[10]
2 1 40 49 58 72 84 103 3 1 43 39 55 67 84 99 4 1 42 49 56 67 74 87 5 1 41 42 48 60 79 106 6 1 41 49 59 74 97 124 7 1 41 49 57 71 89 112 8 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 1 43 51 63 84 112 139 12 1 41 49 56 62 72 88 13 1 41 49 56 62 79 101 128 13 1 41 49 56 64 68 68 16 1 41 49 57 67 11 89 12 1 41 49 56 62 79 101 128 13 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 17 1 41 49 56 64 68 68 18 68 68 68 19 10 1 42 51 61 72 83 89 19 1 42 51 61 72 83 89 10 1 42 51 61 72 83 89 11 42 51 61 72 83 89 12 14 1 49 56 64 68 68 14 1 49 56 64 68 68 15 1 41 49 56 64 68 68 16 1 41 47 54 55 62 65 71 17 1 42 51 61 72 83 89 18 1 39 35 * * * * * * * * 19 1 43 48 55 62 65 71 20 1 41 47 75 45 88 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 42 42 52 58 74 66 68 24 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 48 59 74 87 106 30 2 42 42 48 57 74 93 114 27 2 39 48 59 72 85 98 31 3 42 53 62 73 85 102 24 2 42 42 52 58 74 66 68 31 39 48 59 74 87 106 30 2 42 48 57 74 93 114 27 28 2 39 46 58 73 87 100 28 2 39 48 59 72 85 98 31 3 41 49 63 85 107 129 33 3 3 39 50 63 77 96 111 34 49 61 74 98 109 39 34 42 53 62 73 85 102 31 3 42 53 62 73 85 102 32 3 41 49 61 74 98 109 39 34 44 42 55 66 88 80 83 38 3 41 49 61 74 98 109 39 3 42 55 69 96 131 157 44 44 42 55 66 79 101 120 41 44 42 55 66 79 101 120 41 44 44 55 66 86 103 118 45 44 44 45 55 66 79 101 120 46 44 40 50 62 82 82 101 120 47 4 44 52 51 66 85 103 124 48 4 39 50 62 80 104 125								
3 1 43 39 55 67 84 99 4 1 42 49 56 67 74 87 5 1 41 42 48 60 79 106 6 1 41 49 59 74 97 124 7 1 41 49 59 77 19 124 8 1 42 50 61 71 84 93 9 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 43 51 63 84 112 139 12 1 41 49 56 62 72 88 13 1 41 49 56 64 68 68 15 1 41 49								
4 1 42 49 56 67 74 87 5 1 41 42 48 60 79 106 6 1 41 49 59 74 97 124 7 1 41 49 57 71 89 112 8 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 43 51 63 84 112 139 12 1 41 49 56 62 72 88 13 1 41 49 56 62 79 101 128 14 1 41 49 56 62 79 101 128 15 1 41 49 56 64 68 68 16 1	3							
5 1 41 42 48 60 79 106 6 1 41 49 59 74 97 124 7 1 41 49 57 71 89 112 8 1 42 50 61 71 84 93 9 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 43 54 52 63 74 81 11 1 41 49 56 62 72 88 13 1 41 49 56 62 72 88 13 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 16 1 41 49	1							
6								
7 1 41 49 57 71 89 112 8 1 42 50 61 71 84 93 9 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 43 51 63 84 112 139 12 1 41 49 56 62 72 88 13 1 41 49 56 62 72 88 13 1 41 49 56 64 68 68 15 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 17 1 42 51 <td>5</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	5							
8 1 42 50 61 71 84 93 9 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 43 51 63 84 112 139 12 1 41 49 56 62 72 88 13 1 41 49 56 62 72 88 13 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 16 1 41 49 56 64 68 68 17 1 42 51 61 72 83 89 18 1 39 35 * * * * * * 19 1 43 48 55 62 65 73 21 20 1 <td>0</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	0							
9 1 42 51 59 68 85 96 10 1 41 44 52 63 74 81 11 1 43 51 63 84 112 139 12 1 41 48 53 60 65 67 14 1 41 49 56 62 72 88 13 1 41 49 62 79 101 128 15 1 41 45 49 55 64 68 68 16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 * * * * * * 19 1 43 48 55 62 65 71 20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 87 100 28 29 2 39 46 58 73 87 100 28 3 3 41 49 65 82 107 129 33 3 3 9 50 63 77 96 111 34 34 3 41 49 65 82 107 129 33 3 3 9 50 63 77 96 111 34 34 3 41 49 65 82 107 129 33 3 3 9 50 63 77 96 111 34 4 4 4 55 66 79 100 123 44 4 4 4 55 69 96 131 157 44 4 4 4 55 69 96 131 157 46 4 4 0 52 62 82 101 120 47 4 4 41 53 66 79 100 123 48 4 4 9 65 82 107 129 47 4 4 11 53 66 79 100 123 48 4 4 9 52 62 82 101 120 47 4 4 41 53 66 79 100 123 48 4 4 9 65 82 107 129 47 4 4 11 53 66 79 100 123 48 4 4 9 65 82 101 120 47 4 4 11 53 66 79 100 123 48 4 4 9 65 82 101 120 47 4 4 11 53 66 79 100 123 48 4 9 4 40 53 64 85 108 128								
10								
11 1 43 51 63 84 112 139 12 1 41 49 56 62 72 88 13 1 41 48 53 60 65 67 14 1 41 49 56 64 68 68 15 1 41 49 56 64 68 68 16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 *								
12 1 41 49 56 62 72 88 13 1 41 48 53 60 65 67 14 1 41 49 62 79 101 128 15 1 41 49 56 64 68 68 16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 * * * * * 19 1 43 48 55 62 65 71 20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42								
13 1 41 48 53 60 65 67 14 1 41 49 62 79 101 128 15 1 41 49 56 64 68 68 16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 *								
14 1 41 49 62 79 101 128 15 1 41 49 56 64 68 68 16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 *<								
15 1 41 49 56 64 68 68 16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 * * * * * * 19 1 43 48 55 62 65 71 20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2								
16 1 41 45 49 51 57 51 17 1 42 51 61 72 83 89 18 1 39 35 * * * * * 19 1 43 48 55 62 65 71 20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>								
17 1 42 51 61 72 83 89 18 1 39 35 * <			41					
18 1 39 35 * * * * * * 1 134 48 55 62 65 71 71 20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 42 52 58 74 66 68 68 25 2 40 49 62 78 102 124 24 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 33 41 49 65	16	1	41	45	49	51	57	51
19	17	1	42	51	61	72	83	89
19 1 43 48 55 62 65 71 20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 92 114 29 2 39 46 58 73 92 114 29 2 39 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49	18	1	39	35	*	*	*	*
20 1 41 47 54 58 65 73 21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 30 2 42 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49		1			55	62	65	71
21 2 40 50 62 86 125 163 22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 41 49 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>73</td>								73
22 2 41 55 64 77 90 95 23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49								
23 2 43 52 61 73 90 103 24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 49 <td></td> <td>2</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>		2						
24 2 42 52 58 74 66 68 25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 46 58 73 87 100 28 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
25 2 40 49 62 78 102 124 26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48<								
26 2 42 48 57 74 93 114 27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
27 2 39 46 58 73 87 100 28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 40 3 41 55 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
28 2 39 46 58 73 92 114 29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 49<								
29 2 39 48 59 74 87 106 30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 </td <td></td> <td>2</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>		2						
30 2 42 48 59 72 85 98 31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49<		2						
31 3 42 53 62 73 85 102 32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 49 63 84 103 124 42 4 42 49 63 84 103 126 43 4 42 5								
32 3 41 49 65 82 107 129 33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42								
33 3 39 50 63 77 96 111 34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 55 69 96 131 157 44 4 42		3						
34 3 41 49 63 85 107 134 35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40		3						
35 3 41 53 64 87 123 158 36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41								
36 3 39 48 61 76 98 116 37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128		3						
37 3 41 48 56 68 80 83 38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128		3						
38 3 41 49 61 74 98 109 39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128		3						
39 3 42 50 61 78 89 109 40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128		3						
40 3 41 55 66 79 101 120 41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128		3	41					
41 4 42 51 66 85 103 124 42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128	39		42	50	61	78	89	109
42 4 42 49 63 84 103 126 43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128	40	3	41	55	66	79	101	120
43 4 42 55 69 96 131 157 44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128	41	4	42	51	66	85	103	124
44 4 42 51 65 86 103 118 45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128	42	4	42	49	63	84	103	126
45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128	43	4	42	55	69	96	131	157
45 4 41 50 61 78 98 117 46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128	44	4	42	51	65	86	103	118
46 4 40 52 62 82 101 120 47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128								
47 4 41 53 66 79 100 123 48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128								
48 4 39 50 62 80 104 125 49 4 40 53 64 85 108 128								
49 4 40 53 64 85 108 128								
	50	-		J 1	5 /	0 1	100	

Weight[12] Weight[14] Weight[16] Weight[18] Weight[20] Weight[21]

```
* * * * * * 103 113 123 133
      98
                                                142
       *
                                     144
115
             88 106 120
89 98 107
                                                157
      82
      77
                                                117
     217
              240
                      275
                               307
                                       318
                                                331
                      131
                              148
                                       164
     108
            111 131
135 145
71 72
164 197
             111
                                                167
                             163
72
231
                                     170
76
259
     127
                                               175
      70
                                                 74
                                                265
     146
                     169
                             205
     136
             147
                                       236
                                                251
                      144
                              163
                                       185
     115
              123
                                                192
     145
              156
                      184
                               207
                                        212
                                                233
                      187
     134
             150
                                       279
                              230
                                                309
     115
             122
                      143
                              151
                                       157
                                                150
                      170
     123
              138
                               204
                                       235
                                                256
                              263
                                       291
     159
              179
                      221
                                                305
     137
              144
                      151
                                       156
                              146
                                                147
                      235
     164
              186
                              294
                                       327
                                                341
     201
              238
                      287
                               332
                                        361
                                                373
     145
                      198
                              227
             166
                                       225
                                                220
     103
             112
                      135
                              157
                                       169
                                                178
     128
              154
                      192
                               232
                                       280
                                                290
     130
                      170
                                                272
              146
                               214
                                       250
     154
             182
                      215
                              262
                                       295
                                                321
                      175
204
                              184
234
     155
                                       199
              153
                                                204
     160
              174
                                        269
                                                281
     184
             188
                      197
                              198
                                      199
                                               200
     127
             138
                     145
                              146
     135
              141
                      147
                               174
                                       197
                                                196
                              210
                      173
     144
             156
                                                238
                                       231
     148
             157
                      168
                              185
                                       210
                                                205
                      222
                               261
              170
                                       303
     154
                                                322
     152
              166
                      184
                               203
                                        233
                                                237
     155
              175
                      205
                               234
                                       264
                                                264
109 FACTOR [LEV=30] Individuals
110 CALC Individuals=NEWLEVELS(Chick; !v(1...20,(1...10)3))
111 DREPMEASURES [GROUPS=Diet, Individuals] DATA=Weight
****** WARNING message from procedure DREPMEAS :
There are missing values in the DATA pointer
Plots of means can be misleading
* MESSAGE: There are missing values in the plot
****** Warning (Code HG 20). Statement 130 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
* MESSAGE: There are missing values in the plot
****** Warning (Code HG 20). Statement 130 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
* MESSAGE: There are missing values in the plot
******* Warning (Code HG 20). Statement 130 in Procedure DREPMEAS
Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns
Key window full
* MESSAGE: There are missing values in the plot
```

****** Warning (Code HG 20). Statement 130 in Procedure DREPMEAS

Command: DGRAPH [WINDOW= window; KEYWINDOW= keywindow; TITLE= Title_] GroupMns

Key window full

```
112 DREPMEASURES [GROUPS=Diet] DATA=Weight
****** WARNING message from procedure DREPMEAS :
There are missing values in the DATA pointer
Plots of means can be misleading
-114 **** analyse polynomial coefficients to degree 6
-115
 116 VORTHPOL [MAXDEGREE=6] Weight; CONTRAST=pol
 117 BLOCK Chick
118 TREAT Diet
119 FOR k=0...6
      ANOVA [PRINT=aov; FPROB=Y; PSE=LSD] pol[k]
 120
 121
      APLOT METHOD=fit, normal
 122 ENDFOR
122.....
**** Analysis of variance ****
Variate: pol[0]
Source of variation d.f.(m.v.) s.s.
                                           m.s. v.r. F pr.
Chick stratum
                                 11015.2
                                           3671.7
                                                   5.76 0.002
Diet
                        3
Residual
                       41(5)
                                 26124.6
                                            637.2
Total
                       44(5)
                                 35824.9
                                                                      5
           Т
           Ι
           Ι
       60.0 I
r
           Ι
           I
е
           Ι
i
                 2
                                 2
           Ι
d
           Ι
                                 2
                                                         2
        0.0 I
u
                                 2
                                                         3
а
           Т
1
           Ι
                 3
                                 2
           Ι
s
           Ι
           I
      -60.0 I
                                    128.0 136.0 144.0 152.0
         104.0
                          120.0
                112.0
                                    fitted values
                                     Normal plot
                                                                      5
           Ι
       60.0 I
r
           Ι
е
           Ι
           Ι
S
i
           Ι
d
                                         **2**2**
           Ι
                                      2*2
        0.0 I
u
                                 2**2**
а
1
           Т
s
           Ι
           Ι
           Ι
      -60.0 I *
                           -0.8 0.0 0.8 1.6 2.4
          -2.4
                  -1.6
                               expected Normal quantiles
```

122								
**** A	nalysis of va	riance **	***					
Variate	: pol[1]							
Source	of variation	d.f.((m.v.)	s.s.	m.s.	v.r. F pr	· •	
Chick s Diet Residua Total		3 41 (44 (149.15 445.85 579.84	49.72 10.87	4.57 0.00)7	
								5
r e s i d u a 1 s	8.0 I I I I I I 0.0 I I I I I I I	*	+			* * * 3	: : : : :	5
	6.0	7.0	8.0	9.0	10.0	11.0	12.0	
				fitted	values			
				Norma	l plot			5
r e s i d u a 1 s	8.0 I I I I I 0.0 I I I I -8.0 I *	* * * +	*****	**2* *2**2***	*** *2**2**	* * ** *	*	
	-2.4	-1.6	-0.8	0.0	0.8	1.6	2.4	

***** Analysis of variance ***** Variate: pol[2] Source of variation
Source of variation d.f.(m.v.) s.s. m.s. v.r. F pr. Chick stratum Diet 3 0.35795 0.11932 2.54 0.069 Residual 41(5) 1.92227 0.04688 Total 44(5) 2.26415
Chick stratum Diet 3 0.35795 0.11932 2.54 0.069 Residual 41(5) 1.92227 0.04688 Total 44(5) 2.26415
Diet 3 0.35795 0.11932 2.54 0.069 Residual 41(5) 1.92227 0.04688 Total 44(5) 2.26415 5
0.6 I
0.00 0.05 0.10 0.15 0.20 0.25 0.30 fitted values Normal plot 0.6 I r
Normal plot $ \begin{matrix} & & & \\ & & \\ & & & \\ & &$
5 - I 0.6 I r I
0.6 I r I
e I * * * s I
s I i I ***** * * d I ***2**2*
u 0.0 I **2** a I ****2**2 1 I *** s I ** I * *
$-0.6\stackrel{\text{I}}{\text{I}}$ $-+$

122										
****	Analysis of	variance ****								
Variate	e: pol[3]									
Source	of variatio	n d.f.(m.v.)	s.s.	m.s.	v.r. F pr.					
Chick stratum Diet Residual Total		3 41(5) 44(5)	0.0003445 0.0169446 0.0172501	0.0001148 0.0004133	0.28 0.841					
r e s i d u a 1 s	0.06 I I I I I I I I I I I I I I I I I I I	2 2 4 2 2 2 * 2	++	3 4 * *	* 2 2 2 2 * 2	* 2 2 * 3				
	-0.0144	-0.0132 -0.012		-0.0096 ed values	-0.0084 -0.0)072				
		Normal plot								
r e s i d u a 1 s	0.06 I I I I I I I I I I I I I I I I I I I	* * * * * *		**** ***2**2**	* * *	*				
	I *									
	-+ -2.4		8 0.0		1.6	2.4				
	expected Normal quantiles									

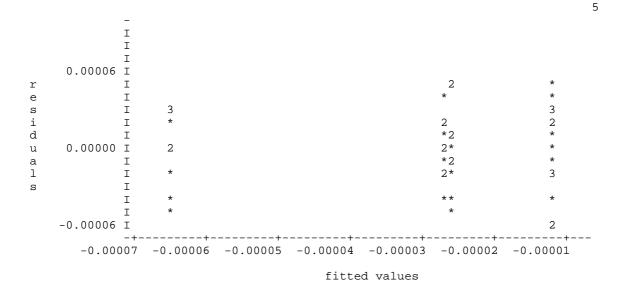
122										
****	Analysis	of vari	ance	****						
Varia	te: pol[4	1]								
Source	e of vari	lation	d.	f.(m.v.)	S.S.	m.s.	v.r.	F pr.		
Chick Diet Residu Total				3 41(5) 44(5)	0.323E-04 0.994E-04 0.130E-03	0.108E-04 0.242E-05	4.44	0.009		
										5
r e s i d u a l s	0.004		2 * 4 2 *		2 * 3 * * 3 2 * 2	* 3 2 3 *			* * 2 2 * *	
	-0.001	-0.0	010	-0.000	0.0000	0.0005	0.001	0.0	015	
					fitt	ed values				
					Nor	rmal plot				
	_	_			NOI	imai pioc				5
r e s	0.004	[[[** * *	* *	*	
i d u a l s	I **** I 2**2*									
		[[[*	*	* * *	**2* ***2**2	:2**				
		+	+- 1 <i>6</i>		+	0.8		+	+	-
	-2.	-	1.6	-0.8	3 0.0	. 0.8	1.	υ	2.4	

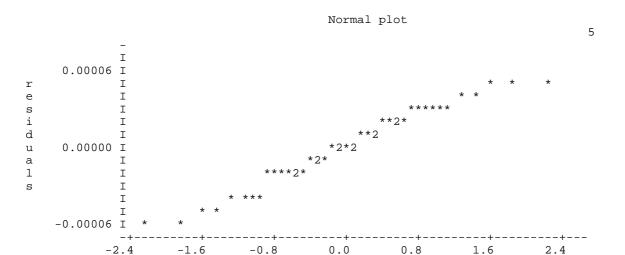
```
122.....
**** Analysis of variance ****
Variate: pol[5]
Source of variation d.f.(m.v.) s.s. m.s.
                                                 v.r. F pr.
Chick stratum
                            0.292E-06 0.974E-07 1.65 0.194
                      3
Diet
Residual
                      41(5) 0.243E-05 0.592E-07
Total
                      44(5) 0.270E-05
                                                                  5
           Ι
     0.0005 I
r
           Ι
           Ι
е
           Ι
s
i
           Ι
d
           Ι
                                                            3
     0.0000 I
u
                                            2 *
                                                            3
           I
а
                                            3 2
1
           Ι
               2
s
           Ι
           Ι
    -0.0005 I
     -0.000248 \ -0.000208 \ -0.000168 \ -0.000128 \ -0.000088 \ -0.000048 \ -0.000008
                                  fitted values
                                   Normal plot
                                                                  5
     0.0005 I
е
           Ι
           Ι
S
i
                                 2*
**2**
d
           Ι
     0.0000 I
u
а
           Ι
           Ι
1
           I
s
           Ι
           Ι
    -0.0005 I *
         -2.4
                 -1.6
                          -0.8
                                  0.0
                                           0.8
                                                   1.6
                                                            2.4
                             expected Normal quantiles
```

**** Analysis of variance ****

Variate: pol[6]

Source of variation	d.f.(m.v.)	s.s.	m.s.	v.r.	F pr.
Chick stratum					
Diet	3	0.189E-07	0.630E-08	6.69	<.001
Residual	41(5)	0.386E-07	0.942E-09		
Total	44(5)	0.551E-07			





While the assumptions for the analysis appear to be met, the fourth- and sixth-degree polynomial coefficients are significant. The analysis of polynomial coefficients does not appear to be appropriate for this example. Perhaps non-linear models would provide a more satisfactory basis for analysis of this example. In fitting nonlinear models one has to be particularly careful about the normality of the coefficients and very high correlations between the estimates of different parameters.

c) Computation in Genstat

Example XII.4 Rat weight (continued)

The commands used in the analysis of summary statistics for this example are as follows:

```
**** separate data for Days and Rat profiles
SUBSET [CONDITION=Days==1] OLD=Rats,Diets; \
                            NEW=Rat,Diet
FOR i=1,8...43,44,50,57,64
    SUBSET [CONDITION=Days==i] OLD=Weights; NEW=Weight[i]
ENDFOR
PRINT Rat, Diet, Weight[1,8...43,44,50,57,64]; FIELD=9; DEC=0
FACTOR [LEV=30] Individuals
CALC Individuals=NEWLEVELS(Rat; !v(1...8,(1...4)2))
DREPMEASURES [GROUPS=Diet, Individuals] DATA=Weight
DREPMEASURES [GROUPS=Diet] DATA=Weight
**** analyse weight gain
CALC WtGain=Weight[64]-Weight[1]
BLOCK Rat
TREAT Diet
ANOVA [FPROB=Y; PSE=LSD] WtGain
APLOT METHOD=fit, normal
YTRANSFORM [TERMS=Diet; LOWER=-1; UPPER=1] WtGain; SAVE=s
CALC TWtGain=LOG(WtGain)
ANOVA [FPROB=Y; PSE=LSD] TWtGain
APLOT METHOD=fit, normal
AKEEP Diet; MEANS=DietMn
CALC DietMn=EXP(DietMn)
PRINT DietMn
**** analyse means, linear and quadratic coefficients
DUPLICATE OLD=Weight; NEW=TWeight
CALC #TWeight=LOG(#Weight)
DREPMEASURES [GROUP=Rat] DATA=TWeight
VORTHPOL [MAXDEGREE=2] TWeight; CONTRAST=pol
BLOCK Rat
TREAT Diet
FOR k=0...2
 ANOVA [FPROB=Y; PSE=LSD] pol[k]
  APLOT METHOD=fit, normal
ENDFOR
```