Duplicated wheat measurements

Example 9 of **?** is an experiment that consists of a field phase and a laboratory phase. In the field phase 49 lines of wheat are investigated using a randomized complete-block design with four blocks. Here the laboratory phase is modified by supposing that the procedure described by **?** is repeated on a second occasion. That is, two samples will be obtained from each plot and one of them processed on the first occasion and the other on the second occasion. Figure 1 gives the randomization diagram for the modified experiment. Recall that a 7×7 balanced lattice square design with four replicates is used to assign the blocks, plots and lines to four intervals in each occasion. In each interval on one occasion there are seven runs at which samples are processed at seven consecutive times. Pseudofactors are introduced for lines and plots in order to define the design of the second phase. The sets of objects for this experiment are analyses, samples and lines.

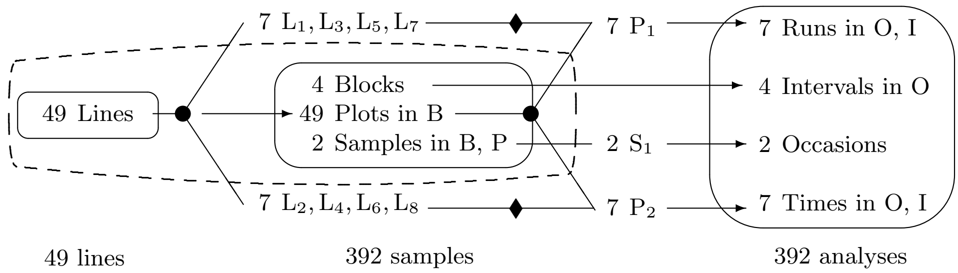

Figure 1: Randomized-inclusive randomizations in the wheat experiment

The covariance matrix under the randomizations is

$$\begin{aligned}\mathbf{C} &= \xi_0\mathbf{P}_0 + \xi_O\mathbf{P}_O + \xi_{OI}\mathbf{P}_{OI} + \xi_{OIR}\mathbf{P}_{OIR} + \xi_{OIT}\mathbf{P}_{OIT} + \xi_{OITR}\mathbf{P}_{OITR} \\ &\quad +2\eta_0\mathbf{Q}_0 + 2\eta_B\mathbf{Q}_B + 2\eta_{BP}\mathbf{Q}_{BP} + 2\eta_{BPS}\mathbf{Q}_{BPS}.\end{aligned}$$

Randomized-inclusive randomizations were used in this experiment, as the outcome of the randomization of lines to samples had to be known before the samples could be randomized to analyses. The plots pseudofactors P_1 and P_2 were used to ensure appropriate partial confounding of sources from the treatments tier with sources in the analyses tier. As in Example **??**, there are no idempotents for the Plot pseudofactors in $\mathbf{C}$ because the pseudofactors are irrelevant to the first randomization, that of treatments to samples, and are not one of the unrandomized factors, that gives rise to covariance, in the other randomization. However, as in Example **??**, $\mathbf{Q}_{BP}$ can be rewritten as the sum of three $\mathbf{Q}^*$-matrices each with coefficient η_{BP}. This results in the coefficient η_{BP} occuring with three different ξ-coefficients in the skeleton analysis of variance in Table 1, which is an extended version of the decomposition table given by **?** in their Example 4. To obtain structure balance, the projection space of the idempotent $\mathbf{Q}_{BPS}$ is decomposed as the sum of five subspaces involving the pseudofactor S_1 so that $\mathbf{Q}_{BPS}$ can be rewritten as the sum of five $\mathbf{Q}^*$-matrices each with coefficient η_{BPS}. As a consequence, the coefficient η_{BPS} occurs with five different ξ-coefficients in Table 1.

analyses tier	
Source	df
Mean	1
Occasions	1
$\mathit{Intervals}\,[O]$	6
$\mathit{Runs}\,[O \wedge I]$	48
$\mathit{Times}\,[O \wedge I]$	48
$R \# T\,[O \wedge I]$	288

analyses tier		plots tier	
1-2 4-5 Source	df	Source	df
Mean	1	Mean	1
Occasions	1	S_1	1
$\mathit{Intervals}\,[O]$	6	Blocks	3
		$S_1 \# B$	3
$\mathit{Runs}\,[O \wedge I]$	48	$P_1\,[B]$	24
		$S_1 \# P_1\,[B]$	24
$\mathit{Times}\,[O \wedge I]$	48	$P_2\,[B]$	24
		$S_1 \# P_2\,[B]$	24
$R \# T\,[O \wedge I]$	288	$\mathit{Plots}\,[B]_{\vdash}$	144
		$\mathit{Samples}\,[B \wedge P]_{\vdash}$	144

analyses tier		plots tier			lines tier	
1-2 4-5 7-9 Source	df	Source	df	eff	Source	df
Mean	1	Mean	1		Mean	1
Occasions	1	S_1	1			
$\mathit{Intervals}\,[O]$	6	Blocks	3			
		$S_1 \# B$	3			
$\mathit{Runs}\,[O \wedge I]$	48	$P_1\,[B]$	24	$\frac{1}{4}$	Lines_R	24
		$S_1 \# P_1\,[B]$	24			
$\mathit{Times}\,[O \wedge I]$	48	$P_2\,[B]$	24	$\frac{1}{4}$	Lines_T	24
		$S_1 \# P_2\,[B]$	24			
$R \# T\,[O \wedge I]$	288	$\mathit{Plots}\,[B]_{\vdash}$	144	$\frac{3}{4}$	Lines_R	24
				$\frac{3}{4}$	Lines_T	24
					Residual	96
4-9		$\mathit{Samples}\,[B \wedge P]_{\vdash}$	144			

Table 1: Skeleton analysis of variance for Example

analyses tier		plots tier		lines tier			
Source	df	Source	df	eff	Source	df	EMS
Mean	1	Mean	1		Mean	1	$\xi_0 + \eta_0$
Occasions	1	S_1	1				$\xi_O + \eta_{BPS}$
$Intervals\,[O]$	6	Blocks	3				$\xi_{OI} + \eta_B$
		$S_1 \# B$	3				$\xi_{OI} + \eta_{BPS}$
$Runs\,[O \wedge I]$	48	$P_1\,[B]$	24	$\frac{1}{4}$	$Lines_R$	24	$\xi_{OIR} + \eta_{BP} + \frac{1}{4}q(L_R)$
		$S_1 \# P_1\,[B]$	24				$\xi_{OIR} + \eta_{BPS}$
$Times\,[O \wedge I]$	48	$P_2\,[B]$	24	$\frac{1}{4}$	$Lines_T$	24	$\xi_{OIT} + \eta_{BP} + \frac{1}{4}q(L_T)$
		$S_1 \# P_2\,[B]$	24				$\xi_{OIT} + \eta_{BPS}$
$R \# T\,[O \wedge I]$	288	$Plots\,[B]_{\vdash}$	144	$\frac{3}{4}$	$Lines_R$	24	$\xi_{OIRT} + \eta_{BP} + \frac{3}{4}q(L_R)$
				$\frac{3}{4}$	$Lines_T$	24	$\xi_{OIRT} + \eta_{BP} + \frac{3}{4}q(L_T)$
					Residual	96	$\xi_{OIRT} + \eta_{BP}$
		$Samples\,[B \wedge P]_{\vdash}$	144				$\xi_{OIRT} + \eta_{BPS}$

analyses tier		plots tier		lines tier			EMS					
1-2 4-5 7-9 11-19 Source	df	Source	df	eff	Source	df	ϕ_{OIRT}	ϕ_{OIT}	ϕ_{OIR}	ϕ_{OI}	ϕ_O	ϕ_{BPS}
Mean	1	Mean	1		Mean	1	1	7	7	49	96	1
Occasions	1	S_1	1				1	7	7	49	96	1
$Intervals\,[O]$	6	Blocks	3				1	7	7	49		1
		$S_1 \# B$	3				1	7	7	49		1
$Runs\,[O \wedge I]$	48	$P_1\,[B]$	24	$\frac{1}{4}$	$Lines_R$	24	1		7			1
		$S_1 \# P_1\,[B]$	24				1		7			1
$Times\,[O \wedge I]$	48	$P_2\,[B]$	24	$\frac{1}{4}$	$Lines_T$	24	1	7				1
		$S_1 \# P_2\,[B]$	24				1	7				1
$R \# T\,[O \wedge I]$	288	$Plots\,[B]_{\vdash}$	144	$\frac{3}{4}$	$Lines_R$	24	1					1
				$\frac{3}{4}$	$Lines_T$	24	1					1
					Residual	96	1					1
4-19		$Samples\,[B \wedge P]_{\vdash}$	144				1					1