THE DESIGN AND MIXED-MODEL ANALYSIS OF EXPERIMENTS

PRACTICAL IX
SOLUTIONS

IX.1 Let **Y** be a n-vector of jointly-distributed random variables with

$$\psi = E[\mathbf{Y}] = \mathbf{X}_{AB}(\alpha\beta) \text{ and } \mathbf{V} = \sigma_U^2 \mathbf{I}_n,$$

where $(\alpha\beta)$ is the ab-vector of parameters specifying a different mean response for each A-B combinations,

$\mathbf{X}_{AB}$ is the $n{\times}ab$ matrix equal to $\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r$ that gives the combinations of A and B for each element of **Y**,

σ_U^2 is the variability arising from different units.

Prove that Then $\hat{\psi} = \mathbf{A.B}$ where **A.B** is the n-vector of means for the combinations of A and B, respectively.

The model $\psi = E[\mathbf{Y}] = \mathbf{X}_{AB}(\alpha\beta)$ is of full rank so that $\hat{\psi} = \mathbf{X}_{AB}(\mathbf{X}'_{AB}\mathbf{X}_{AB})^{-1}\mathbf{X}'_{AB}\mathbf{Y}$.

Now, for the observations arranged in standard order of A then B then the replicates,

$$\mathbf{X}_{AB}(\mathbf{X}'_{AB}\mathbf{X}_{AB})^{-1}\mathbf{X}'_{AB} = \mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r \left[(\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r)'(\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r) \right]^{-1} \mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}'_r$$

$$= \mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r [\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}'_r \mathbf{1}_r]^{-1} \mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}'_r$$

$$= \tfrac{1}{r}\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r (\mathbf{I}_a \otimes \mathbf{I}_b)\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}'_r$$

$$= \tfrac{1}{r}(\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r)(\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}'_r)$$

$$= \tfrac{1}{r}\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{1}_r\mathbf{1}'_r$$

$$= \tfrac{1}{r}\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{J}_r$$

and so $\hat{\psi}$ is clearly the n-vector of means for the combinations of A and B as claimed.

IX.2 Let $\mathbf{P}_G$, $\mathbf{P}_A$, $\mathbf{P}_B$ $\mathbf{R}_G$, $\mathbf{R}_A$ and $\mathbf{R}_B$ be as defined in lemma IX.2. Also, let $\mathbf{P}_{AB} = r^{-1}\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{J}_r$ and $\mathbf{R}_{AB} = \mathbf{I} - \mathbf{P}_{AB}$.

a) Prove that $\mathbf{R}_{AB}\mathbf{R}_B\mathbf{R}_A\mathbf{R}_G = \mathbf{R}_{AB}$, given that $\mathbf{P}_{AB}\mathbf{P}_A = \mathbf{P}_A\mathbf{P}_{AB} = \mathbf{P}_A$, $\mathbf{P}_{AB}\mathbf{P}_B = \mathbf{P}_B\mathbf{P}_{AB} = \mathbf{P}_B$ and $\mathbf{P}_{AB}\mathbf{P}_G = \mathbf{P}_G\mathbf{P}_{AB} = \mathbf{P}_G$

$$
\begin{aligned}
\mathbf{R}_{AB}\mathbf{R}_B\mathbf{R}_A\mathbf{R}_G &= (\mathbf{I} - \mathbf{P}_{AB})(\mathbf{I} - \mathbf{P}_B)\mathbf{R}_A\mathbf{R}_G \\
&= (\mathbf{I} - \mathbf{P}_{AB} - \mathbf{P}_B + \mathbf{P}_{AB}\mathbf{P}_B)\mathbf{R}_A\mathbf{R}_G \\
&= (\mathbf{I} - \mathbf{P}_{AB})\mathbf{R}_A\mathbf{R}_G \\
&= (\mathbf{I} - \mathbf{P}_{AB})\mathbf{R}_G \\
&= \mathbf{R}_{AB}
\end{aligned}
$$

b) Prove that $\mathbf{P}_{AB}\mathbf{R}_B\mathbf{R}_A\mathbf{R}_G = \mathbf{P}_{AB}\mathbf{R}_A\mathbf{R}_B\mathbf{R}_G = \mathbf{P}_{AB} - \mathbf{P}_A - \mathbf{P}_B + \mathbf{P}_G$.

First note that since by lemma IX.1 $\mathbf{P}_A\mathbf{P}_B = \mathbf{P}_B\mathbf{P}_A$, $\mathbf{R}_A\mathbf{R}_B = (\mathbf{I} - \mathbf{P}_A)(\mathbf{I} - \mathbf{P}_B) = \mathbf{I} - \mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_A\mathbf{P}_B = \mathbf{I} - \mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_B\mathbf{P}_A = \mathbf{R}_B\mathbf{R}_A$. This proves that $\mathbf{P}_{AB}\mathbf{R}_B\mathbf{R}_A\mathbf{R}_G = \mathbf{P}_{AB}\mathbf{R}_A\mathbf{R}_B\mathbf{R}_G$.

Next using lemmas IX.1 and IX.2

$$
\begin{aligned}
\mathbf{P}_{AB}\mathbf{R}_A\mathbf{R}_B\mathbf{R}_G &= \mathbf{P}_{AB}(\mathbf{I} - \mathbf{P}_A)(\mathbf{I} - \mathbf{P}_B)(\mathbf{I} - \mathbf{P}_G) \\
&= (\mathbf{P}_{AB} - \mathbf{P}_A)(\mathbf{I} - \mathbf{P}_B - \mathbf{P}_G + \mathbf{P}_G) \\
&= (\mathbf{P}_{AB} - \mathbf{P}_A)(\mathbf{I} - \mathbf{P}_B) \\
&= \mathbf{P}_{AB} - \mathbf{P}_A - \mathbf{P}_{AB}\mathbf{P}_B + \mathbf{P}_A\mathbf{P}_B \\
&= \mathbf{P}_{AB} - \mathbf{P}_A - \mathbf{P}_B + \mathbf{P}_G
\end{aligned}
$$

IX.3 Let $\mathbf{Y}$ be a n-vector of jointly-distributed random variables and $R(\alpha\beta \mid \alpha, \beta) = (\mathbf{A.B}_e)'(\mathbf{A.B}_e)$ where $\mathbf{A.B}_e = \mathbf{P}_{AB}\mathbf{R}_A\mathbf{R}_B\mathbf{R}_G\mathbf{Y} = (\mathbf{P}_{AB} - \mathbf{P}_A - \mathbf{P}_B + \mathbf{P}_G)\mathbf{Y}$. Prove that the degrees of freedom of $R(\alpha\beta \mid \alpha,\beta)$ is $(a-1)(b-1)$ where a is the number of levels of A and b is the number of levels of B.

First use the direct product expressions of the $\mathbf{P}s$ to show that $trace(\mathbf{P}_G) = 1$, $trace(\mathbf{P}_A) = a$, $trace(\mathbf{P}_B) = b$ and $trace(\mathbf{P}_{AB}) = ab$.

Now
$$
\begin{aligned}
trace(\mathbf{P}_{AB} - \mathbf{P}_A - \mathbf{P}_B + \mathbf{P}_G) &= trace(\mathbf{P}_{AB}) - trace(\mathbf{P}_A) - trace(\mathbf{P}_B) + trace(\mathbf{P}_G) \\
&= ab - a - b + 1 \\
&= (a-1)(b-1)
\end{aligned}
$$

IX.4 Suppose that an experiment is to be conducted to investigate the effect on the maize yield of two factors, Varieties (A, B and C) and Fertilizer treatments (Control, P, PM, PNK where P, M, N and K stand for Phosphorus, Manure, Nitrogen and Potash, respectively). It is planned to use a completely randomized design to assign the 12 Variety-Fertilizer combinations and it is estimated that the variance for this experiment will be about 4.

a) It is desired to be able to detect a difference of at least 5 in the overall differences between a pair of Fertilizers. How many replicates should be observed if the power is to be 0.90 and the level of significance 0.05?

In this case you are concerned with the Fertilizer main effect. The columns of ANOVAPower.xls are filled out as follows:

Column	Column heading	Cell contents
A	sample size (r)	3
B	alpha	0.05
C	DF numerator	3
D	DF denominator	=12*(A5-1)
E	central F	=FINV(B5,C5,D5)
F	no. values in a mean (m)	=3*A5
G	delta	5
H	standard deviation	2
I	lambda	=F5*G5*G5/2/H5/H5
J	power	=1-NCF(E5,C5,D5,I5)

The completed worksheet has the following values:

sample size (r)	alpha	DF numer-ator	DF denomin-ator	central F	no. values in a mean (m)	delta	standard deviation	lambda	power
3	0.05	3	24	3.0088	9	5	2.000000	28.125	0.9906

So 3 replicates needed to detect, with 90% power, a minimum change of 5 in the overall Fertilizer difference.

b) What power would be achieved with the number of replicates that you have computed in detecting a change, between two varieties, of at least 5 in the difference between a pair of Fertilizers? How many replicates would be required to detect this latter difference with power 0.90 and significance level 0.05?

In this case, you are concerned with the power of an interaction effect. You need to change DF numerator to 6 and the no. values in a mean (m) to be equal to the sample size (r). The completed worksheet has the following values:

sample size (r)	alpha	DF numer-ator	DF denomin-ator	central F	no. values in a mean (m)	delta	standard deviation	lambda	power
3	0.05	6	24	2.5082	3	5	2.000000	9.375	0.4887
7	0.05	6	72	2.2274	7	5	2.000000	21.875	0.9410

The number of replicates for the main effect was computed to be 3 and this achieves only a power of 0.4887 in detecting the interaction effect. To achieve a power of at least 0.90 requires 7 replicates.

c) Use Genstat to obtain a randomized layout for this experiment with the number of replicates you finally computed in b). Use a seed of 552312 in generating the design and set up the treatment factors with the letter codes above by using them as labels.

The number of replicates computed was 7 and so we generate a completely randomized design with two factors Variety and Fertilizers with 3 and 4 levels and a Dummy factor with 7 levels. The steps given in the lecture notes were followed and the following Genstat output generated.

```
Genstat 5  Release 4.1  (PC/Windows NT)              04 April 2000 10:49:39
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          _____

              Genstat 5 Fourth Edition - (for Windows)
              Genstat 5 Procedure Library Release PL11

          _____

  3  DESIGN
  4  %WSPREAD Plots,Variety,Fertiliz
  5  DELETE [redefine=yes] Variety,Fertiliz
  6  FACTOR [modify=yes;nvalues=84;levels=3;labels=!t('A','B','C')] Variety
  7  READ Variety; frepresentation=ordinal

   Identifier     Values    Missing     Levels
      Variety         84          0          3

 11  FACTOR [modify=yes;nvalues=84;levels=4;labels=!t('Control','P','PM','PNK')\
 12  ] Fertiliz
 13  READ Fertiliz; frepresentation=ordinal

   Identifier     Values    Missing     Levels
     Fertiliz         84          0          4

 17
```

```
  18  PDESIGN [BLOCK=Plots; TREAT=Variety,Fertilizer]
```

*** Treatment combinations on each unit of the design ***

```
Plots
     1    2 2
     2    3 2
     3    1 2
     4    1 3
     5    2 3
     6    1 1
     7    1 3
     8    3 3
     9    1 3
    10    3 4
    11    2 4
    12    2 4
    13    2 3
    14    1 3
    15    1 2
    16    3 2
    17    2 1
    18    2 1
    19    3 4
    20    1 4
    21    2 2
    22    1 4
    23    1 4
    24    3 4
    25    1 4
    26    3 2
    27    3 2
    28    1 4
    29    1 1
    30    3 2
    31    2 4
    32    1 4
    33    3 2
    34    1 1
    35    3 4
    36    3 3
    37    1 2
    38    2 1
    39    1 1
    40    2 3
    41    1 1
    42    2 1
    43    3 4
    44    3 1
    45    3 1
    46    3 3
    47    3 1
    48    2 4
    49    2 3
    50    2 4
    51    2 3
    52    2 3
    53    2 2
    54    3 2
    55    3 1
    56    2 4
    57    1 1
    58    1 2
    59    3 1
    60    3 3
    61    2 1
    62    1 3
    63    2 4
    64    3 4
    65    2 1
```

```
66   3 3
67   1 2
68   1 1
69   2 3
70   1 2
71   1 3
72   3 3
73   2 2
74   2 1
75   3 4
76   2 2
77   1 4
78   1 2
79   3 1
80   3 1
81   3 3
82   2 2
83   1 3
84   2 2
```

Treatment factors are listed in the order: Variety Fertiliz

```
Labels of Variety:
          1          A
          2          B
          3          C

Labels of Fertiliz:
          1     Control
          2          P
          3         PM
          4        PNK
```

IX.5 A completely randomized experiment was conducted to investigate the effect of vitamin B_{12} (0, 5 mg) and antibiotics (0, 40 mg) fed to swine. The response was the average daily gain in weight.

		Vitamin B_{12}	
		0	5
		1.30	1.26
	0	1.19	1.21
		1.08	1.19
Antibiotics			
		1.05	1.52
	40	1.00	1.56
		1.05	1.55

What are the components of this experiment?

1. Observational unit   – a swine
2. Response variable   – Weight gain
3. Unrandomized factors – Swines
4. Randomized factors   – Antibiotics, Vitamin
5. Type of study   – Factorial CRD

What is the experimental structure for this experiment?

Structure	Formula
unrandomized	*12* Swines
randomized	*2* Antibiotic**2* Vitamin

What are the expected mean squares for the lines in the analysis of variance table based on the same dichotomization of the factors into random/fixed factors as for unrandomized/randomized factors?

Source	df	E[MSq]
Swines	11	
Antibiotic	1	$\sigma_S^2 + f_A(\psi)$
Vitamin	1	$\sigma_S^2 + f_V(\psi)$
Antibiotic.Vitamin	1	$\sigma_S^2 + f_{AV}(\psi)$
Residual	8	σ_S^2
Total	11	

Obtain the usual analysis for a two-factor factorial experiment using Genstat, including diagnostic checking and the examination of treatment differences.

Also perform an analysis of the data that involves a nested factorial structure by identifying an appropriate grouping of the treatments using your previous analysis.

```
Genstat 5  Release 4.1  (PC/Windows NT)              04 April 2000 11:11:08
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             Genstat 5 Fourth Edition - (for Windows)
             Genstat 5 Procedure Library Release PL11
             _____

  3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FAC2SWIN.GSH"
  4  DELETE [redefine=yes] Swine,Antibiot,Vitamin,WtGain
  5  FACTOR [modify=yes;nvalues=12;levels=12] Swine
  6  READ Swine; frepresentation=ordinal

  Identifier    Values    Missing    Levels
      Swine         12          0        12

  8  FACTOR [modify=yes;nvalues=12;levels=!(0,5)] Antibiot
  9  READ Antibiot; frepresentation=ordinal

  Identifier    Values    Missing    Levels
    Antibiot        12          0         2

 11  FACTOR [modify=yes;nvalues=12;levels=!(0,40)] Vitamin
 12  READ Vitamin; frepresentation=ordinal
```

```
   Identifier    Values    Missing     Levels
      Vitamin        12          0          2

14  VARIATE [nvalues=12] WtGain
15  READ WtGain

   Identifier   Minimum      Mean   Maximum    Values   Missing
      WtGain     1.000     1.247     1.560        12         0

17
18  PRINT Swine,Antibiot,Vitamin,WtGain

      Swine    Antibiot     Vitamin       WtGain
         1       0.000        0.00        1.300
         2       0.000       40.00        1.260
         3       0.000        0.00        1.190
         4       0.000       40.00        1.210
         5       0.000        0.00        1.080
         6       0.000       40.00        1.190
         7       5.000        0.00        1.050
         8       5.000       40.00        1.520
         9       5.000        0.00        1.000
        10       5.000       40.00        1.560
        11       5.000        0.00        1.050
        12       5.000       40.00        1.550

19  BLOCK Swine
20  TREAT Antibiot*Vitamin
21  ANOVA [FPROB=Y; PSE=LSD] WtGain


21.................................................................................

***** Analysis of variance *****

Variate: WtGain

Source of variation     d.f.        s.s.         m.s.      v.r.   F pr.

Swine stratum
Antibiot                   1    0.020833    0.020833     5.68   0.044
Vitamin                    1    0.218700    0.218700    59.65   <.001
Antibiot.Vitamin           1    0.172800    0.172800    47.13   <.001
Residual                   8    0.029333    0.003667
Total                     11    0.441667


* MESSAGE: the following units have large residuals.

Swine 1          0.110   s.e. 0.049
Swine 5         -0.110   s.e. 0.049


***** Tables of means *****

Variate: WtGain

Grand mean  1.247

 Antibiot     0.00      5.00
             1.205     1.288


  Vitamin     0.00     40.00
             1.112     1.382


 Antibiot  Vitamin     0.00     40.00
    0.00               1.190     1.220
    5.00               1.033     1.543
```

```
*** Least significant differences of means (5% level) ***

Table            Antibiot    Vitamin    Antibiot
                                          Vitamin
rep.                    6          6            3
d.f.                    8          8            8
l.s.d.             0.0806     0.0806       0.1140



  22  APLOT METHOD=fit,normal



        -
        I
        I
        I
 0.12 I
r       I                    *
e       I
s       I
i       I                        *
d       I    2                                            *
u  0.00 I              *   *                              *
a       I                  *                              *
l       I   *
s       I
        I
        I             *
-0.12 I
        -+---------+---------+---------+---------+---------+---------+---------+---
        1.0       1.1       1.2       1.3       1.4       1.5       1.6

                              fitted values



                            Normal plot


        -
        I
 0.12 I
r       I                                                        *
e       I
s       I
i       I                                                 *
d       I                            *     *     *
u  0.00 I                   *    *    *
a       I              *    *
l       I        *
s       I
        I
        I   *
-0.12 I
        -+---------+---------+---------+---------+---------+---------+---------+---
       -1.8      -1.2      -0.6       0.0       0.6       1.2       1.8

                        expected Normal quantiles
```
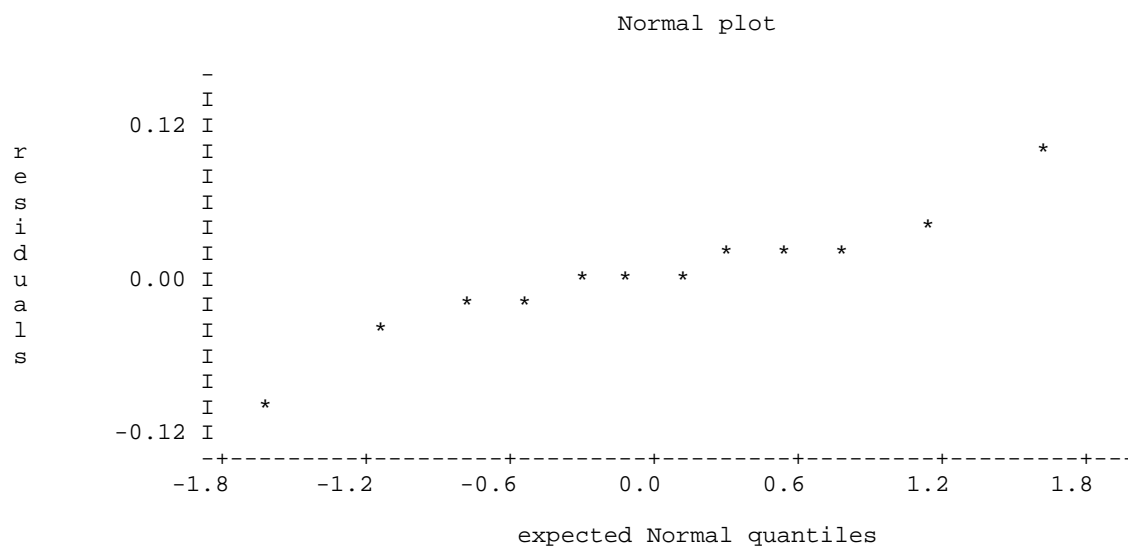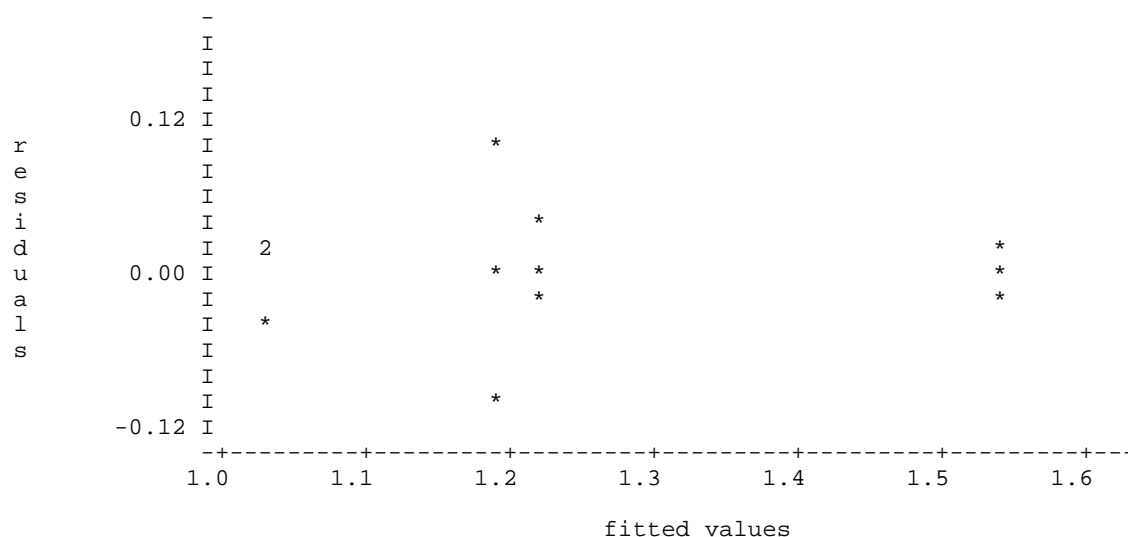
As this is a CRD we cannot perform Tukey's nonadditivity test on the Residual.

The residuals-versus-fitted-values plot displays an appropriate pattern except that one treatment (no Vitamin, no Antibiotic) appears to have a somewhat higher spread compared to the others. This is reflected in the Normal Probability plot where there are two extreme residuals. Otherwise the plot looks satisfactory. Without further evidence of the reason for these outliers, it is Advisable to leave them in the analysis.

The significance test for the example is a follows:

Step 1: Set up hypotheses

a) H_0: $\alpha_0 = \alpha_{40}$
   H_1: the population Antibiotic means are different

b) H_0: $\rho_0 = \rho_5$
   H_1: the population Vitamin means are different

c) H_0: there is no interaction between Antibiotic and Vitamin
$$\left((\alpha\rho)_{ij} - \overline{(\alpha\rho)}_{i.} - \overline{(\alpha\rho)}_{.j} + \overline{(\alpha\rho)}_{..} = 0 \quad \text{for all i,j}\right)$$

H_1: there is an interaction between Antibiotic and Vitamin
$$\left((\alpha\rho)_{ij} - \overline{(\alpha\rho)}_{i.} - \overline{(\alpha\rho)}_{.j} + \overline{(\alpha\rho)}_{..} \neq 0 \quad \text{for some i,j}\right)$$

Step 2: Calculate test statistics

The analysis of variance table for a two-factor CRD, with random factors being the unrandomized factors and fixed factors the randomized factors, is:

Source	df	SSq	MSq	E[MSq]	F	Prob
Swine	11	0.4417				
Antibiotic	1	0.0208	0.0208	$\sigma_S^2 + f_A(\psi)$	5.68	0.0443
Vitamin	1	0.2187	0.2187	$\sigma_S^2 + f_V(\psi)$	59.65	<0.001
Antibiotic.Vitamin	1	0.1728	0.1728	$\sigma_S^2 + f_{AV}(\psi)$	47.13	<0.001
Residual	8	0.0293	0.0037	σ_S^2		

Step 3: Decide between hypotheses

There is a significant interaction between Antibiotic and Vitamin level.

Because there is a significant interaction the means for each combination of Vitamin and Antibiotic are examined using a multiple comparison procedure.

		Vitamin B$_{12}$		
		0	5	LSD(5%)
	0	1.19	1.22	
Antibiotics				0.1140
	40	1.03	1.54	

Examination of the means reveals that Vitamin has no effect with 0 Antibiotic, but there is a significant extra weight gain when Vitamin is used with 40mg of Antibiotic.

However, it appears that there might be a response when both Antibiotic is at 40 and Vitamin B_{12} is at 5 and not a response with other combinations. To investigate this possibility we set up an analysis with a nested factorial structure.

```
  33  FACPRODUCT FACTORS=!p(Antibiot,Vitamin); PRODUCT=Treats
  34  FACTOR [LEV=2] Both
  35  CALC Both=NEWLEVELS(Treats; !v(2,2,2,1))
  36  BLOCK Swine
  37  TREAT Both/Treats
  38  ANOVA [FPROB=Y; PSE=LSD] WtGain
```

```
38........................................................................
```

***** Analysis of variance *****

Variate: WtGain

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Swine stratum					
Both	1	0.352044	0.352044	96.01	<.001
Both.Treats	2	0.060289	0.030144	8.22	0.011
Residual	8	0.029333	0.003667		
Total	11	0.441667			

* MESSAGE: the following units have large residuals.

Swine 1	0.110	s.e.	0.049
Swine 5	-0.110	s.e.	0.049

***** Tables of means *****

Variate: WtGain

Grand mean 1.247

Both	1	2
	1.543	1.148
rep.	3	9

Both	Treats	1	2	3	4
1					1.543
2		1.190	1.220	1.033	

*** Least significant differences of means (5% level) ***

Table	Both	Both Treats
rep.	unequal	3
d.f.	8	8
l.s.d.	0.0931	0.1140

The analysis indicates that our proposed model does not fit the data as there are difference between the three other treatment combinations.

IX.6 An experiment was conducted on the survival of *Salmonella typhimurium* under three different levels of sorbic acid and six levels of water activity (a_w). A randomized complete block design was used with three blocks of eighteen plates to which the eighteen treatment combinations are applied. The data analyzed are the log(density/ml) measured seven days after the imposition of treatments.

Sorbic acid	a_w	Block I	Block II	Block III
	0.98	3604.72	4315.64	4146.42
	0.94	772.78	812.41	518.01
	0.90	354.25	395.44	464.05
0	0.86	157.59	210.61	149.90
	0.82	127.74	74.44	91.84
	0.78	74.44	76.71	66.69
	0.98	2079.74	2416.32	1978.31
	0.94	678.58	487.85	671.83
	0.90	149.90	196.37	323.76
100	0.86	127.74	141.17	72.97
	0.82	72.97	83.93	65.37
	0.78	62.18	80.64	65.37
	0.98	1261.43	1012.32	1326.10
	0.94	561.16	482.99	620.17
	0.90	181.27	164.02	228.15
200	0.86	82.27	81.45	120.30
	0.82	70.81	71.52	79.04
	0.78	138.38	61.56	63.43

What are the components of this experiment?

1. Observational unit - a plate
2. Response variable - Density
3. Unrandomized factors - Blocks, Plates
4. Randomized factors - Activity, Sorbic
5. Type of study - Two-factor RCBD

What is the experimental structure for this experiment?

Structure	Formula
unrandomized	*3* Blocks/*18* Plates
randomized	*6* Activity**3* Sorbic

What are the expected mean squares for the lines in the analysis of variance table based on the same division of the factors into random/fixed factors as for unrandomized/randomized factors?

Source	df	E[MSq]
Blocks	2	$\sigma_{BP}^2 + 18\sigma_B^2$
Blocks.Plates	51	
Activity	5	$\sigma_{BP}^2 + f_A(\psi)$
Sorbic	2	$\sigma_{BP}^2 + f_S(\psi)$
Activity.Sorbic	10	$\sigma_{BP}^2 + f_{AS}(\psi)$
Residual	34	σ_{BP}^2
Total	53	

The data is available in the file *Fac2Salm.gsh* in the directory *G:\Disciplina\Genstat*. Analyze the data using Genstat, including diagnostic checking and the examination of treatment differences. If there is evidence of transformable non-additivity, use the YTRANSFORM procedure in Genstat to determine the best power transformation for this data. Reanalyze the data using the transformation that you identify.

```
Genstat 5  Release 4.1  (PC/Windows NT)              04 April 2000 14:51:05
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              Genstat 5 Fourth Edition - (for Windows)
              Genstat 5 Procedure Library Release PL11
              _____

  3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FAC2SALM.GSH"
  4  DELETE [redefine=yes] Block,Plates,Activity,Sorbic,Density,LnDensit
  5  FACTOR [modify=yes;nvalues=54;levels=3] Block
  6  READ Block; frepresentation=ordinal

   Identifier    Values   Missing    Levels
        Block        54         0         3

  9  FACTOR [modify=yes;nvalues=54;levels=18] Plates
 10  READ Plates; frepresentation=ordinal

   Identifier    Values   Missing    Levels
       Plates        54         0        18

 13  FACTOR [modify=yes;nvalues=54;levels=!(0.98,0.94,0.9,0.86,0.82,0.78)\
 14  ] Activity
 15  READ Activity; frepresentation=ordinal

   Identifier    Values   Missing    Levels
     Activity        54         0         6

 18  FACTOR [modify=yes;nvalues=54;levels=!(0,100,200)] Sorbic
 19  READ Sorbic; frepresentation=ordinal

   Identifier    Values   Missing    Levels
       Sorbic        54         0         3

 22  VARIATE [nvalues=54] Density
 23  READ Density

   Identifier   Minimum      Mean   Maximum    Values   Missing
      Density      61.6     606.9    4315.6        54         0    Skew
```

```
35   VARIATE [nvalues=54] LnDensit
36   READ LnDensit
```

Identifier	Minimum	Mean	Maximum	Values	Missing
LnDensit	4.120	5.519	8.370	54	0

```
41
42   PRINT Block,Plates,Sorbic,Activity,Density
```

Block	Plates	Sorbic	Activity	Density
1	1	0.0	0.9800	3604.7
2	1	0.0	0.9800	4315.6
3	1	0.0	0.9800	4146.4
1	2	0.0	0.9400	772.8
2	2	0.0	0.9400	812.4
3	2	0.0	0.9400	518.0
1	3	0.0	0.9000	354.2
2	3	0.0	0.9000	395.4
3	3	0.0	0.9000	464.1
1	4	0.0	0.8600	157.6
2	4	0.0	0.8600	210.6
3	4	0.0	0.8600	149.9
1	5	0.0	0.8200	127.7
2	5	0.0	0.8200	74.4
3	5	0.0	0.8200	91.8
1	6	0.0	0.7800	74.4
2	6	0.0	0.7800	76.7
3	6	0.0	0.7800	66.7
1	7	100.0	0.9800	2079.7
2	7	100.0	0.9800	2416.3
3	7	100.0	0.9800	1978.3
1	8	100.0	0.9400	678.6
2	8	100.0	0.9400	487.8
3	8	100.0	0.9400	671.8
1	9	100.0	0.9000	149.9
2	9	100.0	0.9000	196.4
3	9	100.0	0.9000	323.8
1	10	100.0	0.8600	127.7
2	10	100.0	0.8600	141.2
3	10	100.0	0.8600	73.0
1	11	100.0	0.8200	73.0
2	11	100.0	0.8200	83.9
3	11	100.0	0.8200	65.4
1	12	100.0	0.7800	62.2
2	12	100.0	0.7800	80.6
3	12	100.0	0.7800	65.4
1	13	200.0	0.9800	1261.4
2	13	200.0	0.9800	1012.3
3	13	200.0	0.9800	1326.1
1	14	200.0	0.9400	561.2
2	14	200.0	0.9400	483.0
3	14	200.0	0.9400	620.2
1	15	200.0	0.9000	181.3
2	15	200.0	0.9000	164.0
3	15	200.0	0.9000	228.1
1	16	200.0	0.8600	82.3
2	16	200.0	0.8600	81.5
3	16	200.0	0.8600	120.3
1	17	200.0	0.8200	70.8
2	17	200.0	0.8200	71.5
3	17	200.0	0.8200	79.0
1	18	200.0	0.7800	138.4
2	18	200.0	0.7800	61.6
3	18	200.0	0.7800	63.4

```
 43  BLOCK Block/Plates
 44  TREAT POL(Activity; 2)*POL(Sorbic; 2)
 45  ANOVA [FPROB=Y; PSE=LSD] Density


45.................................................................................

***** Analysis of variance *****

Variate: Density

Source of variation     d.f.       s.s.        m.s.      v.r.  F pr.

Block stratum              2     11587.       5793.      0.36

Block.Plates stratum
Activity                   5  38997864.    7799573.    487.17  <.001
  Lin                      1  24082914.   24082914.   1504.23  <.001
  Quad                     1  11539997.   11539997.    720.79  <.001
  Deviations               3   3374953.    1124984.     70.27  <.001
Sorbic                     2   2785828.    1392914.     87.00  <.001
  Lin                      1   2671748.    2671748.    166.88  <.001
  Quad                     1    114080.     114080.      7.13  0.012
Activity.Sorbic           10   9696336.     969634.     60.56  <.001
  Lin.Lin                  1   4623625.    4623625.    288.79  <.001
  Quad.Lin                 1   2879858.    2879858.    179.88  <.001
  Lin.Quad                 1    147971.     147971.      9.24  0.005
  Dev.Lin                  3   1883694.     627898.     39.22  <.001
  Quad.Quad                1     87311.      87311.      5.45  0.026
  Deviations               3     73876.      24625.      1.54  0.222
Residual                  34    544343.      16010.

Total                     53  52035958.


* MESSAGE: the following units have large residuals.

Block 1    Plates 1            -397.   s.e. 100.
Block 2    Plates 1             280.   s.e. 100.
Block 2    Plates 7             245.   s.e. 100.


***** Tables of means *****

Variate: Density

Grand mean  607.

 Activity    0.98     0.94     0.90     0.86     0.82     0.78
            2460.     623.     273.     127.      82.      77.

   Sorbic    0.00   100.00   200.00
             912.     542.     367.

 Activity  Sorbic     0.00   100.00   200.00
    0.98             4022.    2158.    1200.
    0.94              701.     613.     555.
    0.90              405.     223.     191.
    0.86              173.     114.      95.
    0.82               98.      74.      74.
    0.78               73.      69.      88.


*** Least significant differences of means (5% level) ***

Table              Activity    Sorbic   Activity
                                        Sorbic
rep.                      9        18          3
d.f.                    34        34         34
l.s.d.               121.2      85.7      210.0
```

```
  46   APLOT METHOD=fit,normal
             -
             I
             I
             I
     400.0 I
r            I
e            I                                *                      *
s            I
i            I                       *                               *
d            I 3 * * 2*2        *
u      0.0 I 96   2 *
a            I 2*3     *                    *
l            I           *
s            I         *      *           *
             I
             I
    -400.0 I                                                   *
             -+---------+---------+---------+---------+---------+---------+---
            0.0       800.0     1600.0    2400.0    3200.0    4000.0    4800.0

                                    fitted values

                                  Normal plot
             -
             I
     400.0 I
r            I
e            I                                                  *    *
s            I
i            I                                              * *
d            I                                  2***2* ***
u      0.0 I               2*2*22*222*22*2*
a            I           ** *2***
l            I         *
s            I     *   * *
             I
             I
    -400.0 I  *
             -+---------+---------+---------+---------+---------+---------+---
           -2.4      -1.6      -0.8       0.0       0.8       1.6       2.4

                             expected Normal quantiles
  47   "
 -48   **** Tukey''s one-degree-of-freedom-for-non-additivity.
 -49   **** It is the term designated covariate in the following analysis
 -50   "
  51   AKEEP [FIT=Fit]
  52   CALC ResSq=Fit*Fit
  53   TREAT Activity*Sorbic
  54   ANOVA [PRINT=*]  ResSq; RES=ResSq
  55   COVAR ResSq                         "A computational trick"
  56   ANOVA [PRINT=A; FPROB=Y] Density

56....................................................................

***** Analysis of variance (adjusted for covariate) *****

Variate: Density
Covariate: ResSq
```

Source of variation	d.f.	s.s.	m.s.	v.r.	cov.ef.	F pr.
Block stratum	2	11587.	5793.	0.59		
Block.Plates stratum						
Activity	5	38997864.	7799573.	792.65	1.00	<.001
Sorbic	2	2785828.	1392914.	141.56	1.00	<.001
Activity.Sorbic	10	9696336.	969634.	98.54	1.00	<.001
Covariate	1	219628.	219628.	22.32		<.001
Residual	33	324715.	9840.		1.63	

```
Total                    53  52035958.

   57  COVAR
```

We first perform the diagnostic checking. The residual-versus-fitted-values plot is displaying a funnel shape and the normal probability plot is not displaying a straight-line trend. The test for transformable non-additivity, summarized in the following table, is significant. So the assumptions are not met

Source	df	SSq	MSq	F	Prob
Block	2	11587.	5793.	0.59	
Block.Plates	51				
Activity	5	38997864.	7799573.	792.65	<.001
Sorbic	2	2785828.	1392914.	141.56	<.001
Activity.Sorbic	10	9696336.	969634.	98.54	<.001
Residual	34	544343.	16010.		
Nonadditivity	1	219628.	219628.	22.32	<.001
Deviations	33	324715.	9840		
Total	53	52035958.			

The following output is obtained using the YTRANSFORM procedure.

```
58  "Perform Box-Cox analysis"
  59   YTRANSFORM [TERMS=Activity*Sorbic; LOWER=-2.5; UPPER=2.5; GRAPHICS=line] \
  60                           Density; SAVE=s
```

```
                              Power : Box-Cox


           -
L          I
o   -200.0 I
g          I                        ....''''....
           I                  ....''''        '..
l          I            ....'''                  '..
i          I        ...'''                        ''.
k          I      ''                                '..
e   -400.0 I                                          ''.
l          I                                            '..
i          I                                               '..
h          I
o          I
o          I
d   -600.0 I
           -+---------+---------+---------+---------+---------+---------+---
          -3.0      -2.0      -1.0       0.0       1.0       2.0       3.0

                                   Lambda
```
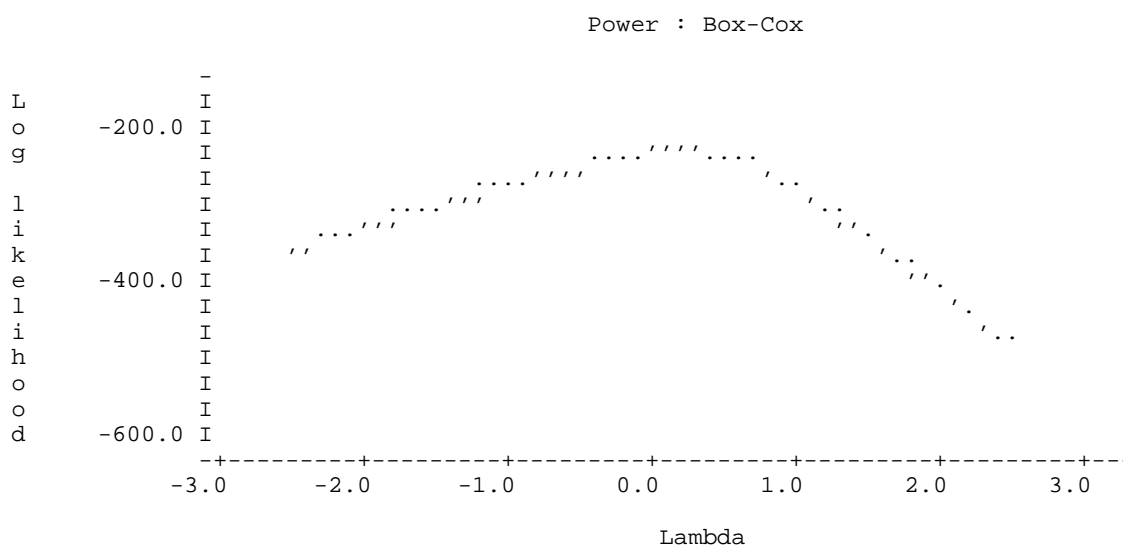
This plot indicates that the log-likelihood of λ is maximized at about 0.25. We apply this transformation to the data and reanalyze the transformed data. However, a log transformation ($\lambda = 0$) is also likely to be satisfactory.

```
  61  "Analyse data raised to the power 0.25"
  62  CALC TrDensity=Density**0.25
  63  BLOCK Block/Plates
  64  TREAT POL(Activity; 2)*POL(Sorbic; 2)
  65  ANOVA [FPROB=Y; PSE=LSD] TrDensity
```

65..

***** Analysis of variance *****

Variate: TrDensit

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Block stratum	2	0.00466	0.00233	0.05	
Block.Plates stratum					
Activity	5	104.81864	20.96373	441.51	<.001
Lin	1	89.25659	89.25659	1879.79	<.001
Quad	1	14.95629	14.95629	314.99	<.001
Deviations	3	0.60575	0.20192	4.25	0.012
Sorbic	2	3.65786	1.82893	38.52	<.001
Lin	1	3.49327	3.49327	73.57	<.001
Quad	1	0.16459	0.16459	3.47	0.071
Activity.Sorbic	10	4.53375	0.45337	9.55	<.001
Lin.Lin	1	2.78285	2.78285	58.61	<.001
Quad.Lin	1	0.33371	0.33371	7.03	0.012
Lin.Quad	1	0.00003	0.00003	0.00	0.982
Dev.Lin	3	1.35144	0.45048	9.49	<.001
Quad.Quad	1	0.01187	0.01187	0.25	0.620
Deviations	3	0.05385	0.01795	0.38	0.769
Residual	34	1.61439	0.04748		
Total	53	114.62930			

* MESSAGE: the following units have large residuals.

Block 1	Plates 18	0.407	s.e.	0.173
Block 3	Plates 9	0.406	s.e.	0.173

***** Tables of means *****

Variate: TrDensit

Grand mean 4.188

Activity	0.98	0.94	0.90	0.86	0.82	0.78
	6.883	4.980	4.006	3.323	2.998	2.940

Sorbic	0.00	100.00	200.00
	4.539	4.110	3.916

Activity	Sorbic	0.00	100.00	200.00
0.98		7.959	6.811	5.878
0.94		5.127	4.965	4.848
0.90		4.480	3.828	3.711
0.86		3.617	3.244	3.109
0.82		3.132	2.931	2.930
0.78		2.918	2.883	3.018

*** Least significant differences of means (5% level) ***

Table	Activity	Sorbic	Activity Sorbic
rep.	9	18	3
d.f.	34	34	34
l.s.d.	0.2088	0.1476	0.3616

```
   66   APLOT METHOD=fit,normal
                -
                I
                I
                I
        0.4 I               *          *
r               I
e               I
s               I            2*    **                  *                      *
i               I            2   *            *     *2*           *                      *
d               I            2                                *                          *
u       0.0 I            3                     *     *
a               I            5 2     ***                                  *
l               I            *     **         *   *                    *
s               I            2*                            *                      *
                I                                *
                I              *     *              *
       -0.4 I
                -+---------+---------+---------+---------+---------+---------+---------+---
               2.0       3.0       4.0       5.0       6.0       7.0       8.0

                                    fitted values

                                    Normal plot

                -
                I
        0.4 I                                                             *     *
r               I
e               I
s               I                                          ** *** * *
i               I                              *2*2****
d               I                        22
u       0.0 I                          22*
a               I                    *2*22*2
l               I              ****2
s               I          *** **
                I        *
                I  *    *    *
       -0.4 I
                -+---------+---------+---------+---------+---------+---------+---------+---
              -2.4      -1.6      -0.8       0.0       0.8       1.6       2.4

                             expected Normal quantiles
   67   "
  -68   **** Tukey''s one-degree-of-freedom-for-non-additivity.
  -69   **** It is the term designated covariate in the following analysis
  -70   "
   71   AKEEP [FIT=Fit]
   72   CALC ResSq=Fit*Fit
   73   TREAT Activity*Sorbic
   74   ANOVA [PRINT=*]  ResSq; RES=ResSq
   75   COVAR ResSq                          "A computational trick"
   76   ANOVA [PRINT=A; FPROB=Y] TrDensity

76.................................................................................
```

***** Analysis of variance (adjusted for covariate) *****

Variate: TrDensit
Covariate: ResSq

Source of variation	d.f.	s.s.	m.s.	v.r.	cov.ef.	F pr.
Block stratum	2	0.00466	0.00233	0.05		
Block.Plates stratum						
Activity	5	104.81864	20.96373	431.66	1.00	<.001
Sorbic	2	3.65786	1.82893	37.66	1.00	<.001
Activity.Sorbic	10	4.53375	0.45337	9.34	1.00	<.001
Covariate	1	0.01173	0.01173	0.24		0.626
Residual	33	1.60266	0.04857		0.98	

```
Total                53   114.62930

  77  COVAR
```

Source	df	SSq	MSq	F	Prob
Blocks	2	0.00466	0.00233	0.05	
Blocks.Plates	51				
Activity	5	104.81864	20.96373	441.51	<.001
Linear	1	89.25659	89.25659	1879.79	<.001
Quadratic	1	14.95629	14.95629	314.99	<.001
Deviations	3	0.60575	0.20192	4.25	0.012
Sorbic	2	3.65786	1.82893	38.52	<.001
Linear	1	3.49327	3.49327	73.57	<.001
Quadratic	1	0.16459	0.16459	3.47	0.071
Activity.Sorbic	10	4.53375	0.45337	9.55	<.001
$A_{Linear}.B_{Linear}$	1	2.78285	2.78285	58.61	<.001
$A_{Quadratic}.B_{Linear}$	1	0.33371	0.33371	7.03	0.012
$A_{Linear}.B_{quadratic}$	1	0.00003	0.00003	0.00	0.982
$Deviations.B_{Linear}$	1	1.35144	0.45048	9.49	<.001
$A_{Quadratic}.B_{Quadratic}$	1	0.01187	0.01187	0.25	0.620
Deviations	3	0.05385	0.01795	0.38	0.769
Residual	34	1.61439	0.04748		
Nonadditivity	1	0.01173	0.01173	0.24	0.626
Deviations	33	1.60266	0.04857		
Total	53	114.62930			

The diagnostic checking is now more satisfactory. The residual-versus-fitted-values and normal probability plots are acceptable, except for two outliers. The test for transformable non-additivity is now non-significant.

This analysis indicates that there are significant deviations from the $A_{Quadratic}.B_{Linear}$ interaction term and so a response function of degree two does not fit the data. A plot of the Activity.Sorbic means is given in the following Genstat output.

```
  78  "Examine trend in means"
  79  AGRAPH [GRAPHICS=line; METHOD=line] XFACTOR=Activity; GROUPS=Sorbic
        -
        I
        I
        I                                                      1
    7.5 I
        I
        I                                                      2
        I
        I                                                      3
        I
    5.0 I                                          :
        I                              1
        I -
        I                   1              :
        I           1       2
        I     :     :       3
    2.5 I
        -+---------+---------+---------+---------+---------+---------+---
      0.76      0.80      0.84      0.88      0.92      0.96      1.00
```
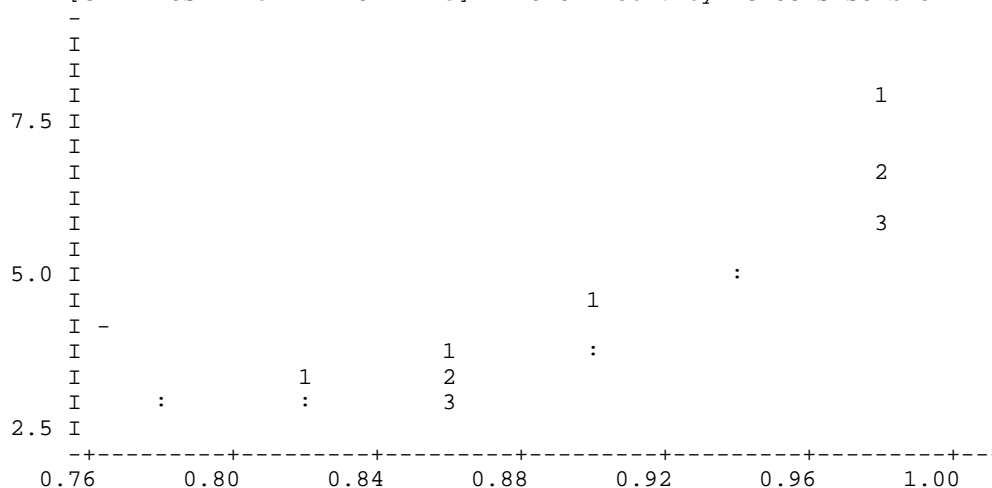
```
mean['0']  v. xlevel using symbol 1
mean['100']  v. xlevel using symbol 2
mean['200']  v. xlevel using symbol 3
sedbar  v. __ using symbol -
```

There appears to be a non-linear trend in the Activity means that differs between the levels of Sorbic acid. One could try to fit non-linear curves to these trends.

IX.7 The yields of an undesirable by-product of a process were measured from 12 runs in which 2 different catalysts and 2 different pressures were used; that is each combination was replicated 3 times. This experiment was repeated at two different laboratories. The data, given below, are the percentage of by-product produced and it is available in the file *Fac2ByPr.gsh* in the directory *G:\Disciplina\Genstat*.

	Catalyst	I		II	
	Laboratory	A	B	A	B
		53	27	40	45
	High	43	45	32	12
		45	57	29	69
Pressure					
		42	32	61	54
	Low	95	27	24	60
		60	98	11	26

The components of this experiment are:

1. Observational unit   – a run
2. Response variable   – % By-product
3. Unrandomized factors   – Laboratories, Tests
4. Randomized factors   – Catalyst, Pressure
5. Type of study   – Two-factor RCBD

In this experiment interactions between laboratories and treatments are likely to be of interest so that the experimental structure for this experiment would be:

Structure	Formula
unrandomized	*2* Labs/*12* Tests
randomized	*2* Catalyst**2* Pressure*Labs

What are the expected mean squares for the lines in the analysis of variance table based on the same dichotomization of the factors into random/fixed factors as for unrandomized/randomized factors?

Source	df	E[MSq]
Labs	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2 + 6\sigma_{CL}^2 + 6\sigma_{PL}^2 + 12\sigma_L^2$
Labs.Tests	22	
Pressure	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2 \qquad\qquad + 6\sigma_{PL}^2 + f_P(\psi)$
Catalyst	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2 + 6\sigma_{CL}^2 \qquad + f_C(\psi)$
Pressure.Catalyst	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2 \qquad\qquad\qquad + f_{PC}(\psi)$
Pressure.Labs	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2 \qquad + 6\sigma_{PL}^2$
Catalyst.Labs	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2 + 6\sigma_{CL}^2$
Pressure.Catalyst.Labs	1	$\sigma_{LT}^2 + 3\sigma_{PCL}^2$
Residual	16	σ_{LT}^2

Analyze the data using Genstat, including diagnostic checking and the examination of treatment differences.

NOTE: Genstat will omit BLOCK terms when they occur both in BLOCK and TREAT structures. Because of this you will need to declare two Laboratories factors, Lab and Labs say, so that the BLOCK and TREAT structures would be:

> BLOCK Labs/Runs
> TREAT Pressure*Catalyst*Lab

```
Genstat 5  Release 4.1  (PC/Windows NT)            14 April 2000 12:33:26
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            _____

            Genstat 5 Fourth Edition - (for Windows)
            Genstat 5 Procedure Library Release PL11
            _____

  3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FAC2BYPR.GSH"
  4  DELETE [redefine=yes] Labs,Runs,Lab,Pressure,Catalyst,Yield
  5  FACTOR [modify=yes;nvalues=24;levels=2] Labs
  6  READ Labs; frepresentation=ordinal

  Identifier    Values   Missing    Levels
        Labs        24         0         2

  8  FACTOR [modify=yes;nvalues=24;levels=12] Runs
  9  READ Runs; frepresentation=ordinal

  Identifier    Values   Missing    Levels
        Runs        24         0        12

 11  FACTOR [modify=yes;nvalues=24;levels=2] Lab
 12  READ Lab; frepresentation=ordinal
```

```
   Identifier    Values    Missing    Levels
        Lab        24         0          2

14  FACTOR [modify=yes;nvalues=24;levels=2] Pressure
15  READ Pressure; frepresentation=ordinal

   Identifier    Values    Missing    Levels
     Pressure     24         0          2

17  FACTOR [modify=yes;nvalues=24;levels=2] Catalyst
18  READ Catalyst; frepresentation=ordinal

   Identifier    Values    Missing    Levels
     Catalyst     24         0          2

20  VARIATE [nvalues=24] Yield
21  READ Yield

   Identifier   Minimum      Mean   Maximum    Values    Missing
        Yield     11.00     45.29     98.00        24          0


23
24  PRINT Labs,Runs,Pressure,Catalyst,Yield

        Labs         Runs      Pressure    Catalyst       Yield
           1            1             1           1       53.00
           2            1             1           1       27.00
           1            2             1           2       40.00
           2            2             1           2       45.00
           1            3             1           1       43.00
           2            3             1           1       45.00
           1            4             1           2       32.00
           2            4             1           2       12.00
           1            5             1           1       45.00
           2            5             1           1       57.00
           1            6             1           2       29.00
           2            6             1           2       69.00
           1            7             2           1       42.00
           2            7             2           1       32.00
           1            8             2           2       61.00
           2            8             2           2       54.00
           1            9             2           1       95.00
           2            9             2           1       27.00
           1           10             2           2       24.00
           2           10             2           2       60.00
           1           11             2           1       60.00
           2           11             2           1       98.00
           1           12             2           2       11.00
           2           12             2           2       26.00

25  BLOCK Labs/Runs
26  TREAT Pressure*Catalyst*Lab
27  ANOVA [FPROB=Y; PSE=LSD] Yield
```

```
27.........................................................................

***** Analysis of variance *****

Variate: Yield

Source of variation     d.f.       s.s.        m.s.     v.r.   F pr.

Labs stratum
Lab                       1        12.0        12.0

Labs.Runs stratum
Pressure                  1       360.4       360.4     0.65   0.430
Catalyst                  1      1080.0      1080.0     1.96   0.181
Pressure.Catalyst         1       234.4       234.4     0.43   0.523
Pressure.Lab              1         3.4         3.4     0.01   0.939
Catalyst.Lab              1       610.0       610.0     1.11   0.308
Pressure.Catalyst.Lab     1        92.0        92.0     0.17   0.688
Residual                 16      8812.7       550.8

Total                    23     11205.0


* MESSAGE: the following units have large residuals.

Labs 2    Runs 11          45.7    s.e. 19.2


***** Tables of means *****

Variate: Yield

Grand mean   45.3

  Pressure         1        2
                 41.4     49.2

  Catalyst         1        2
                 52.0     38.6

      Lab          1        2
                 44.6     46.0

  Pressure Catalyst       1        2
         1             45.0     37.8
         2             59.0     39.3

  Pressure      Lab       1        2
         1             40.3     42.5
         2             48.8     49.5

  Catalyst      Lab       1        2
         1             56.3     47.7
         2             32.8     44.3

          Catalyst       1                  2
  Pressure      Lab       1        2        1        2
         1             47.0     43.0     33.7     42.0
         2             65.7     52.3     32.0     46.7


*** Least significant differences of means (5% level) ***

Table             Pressure    Catalyst      Lab    Pressure
                                                   Catalyst
rep.                  12          12         12           6
d.f.                  16          16          *          16
l.s.d.              20.31       20.31         *       28.72
```
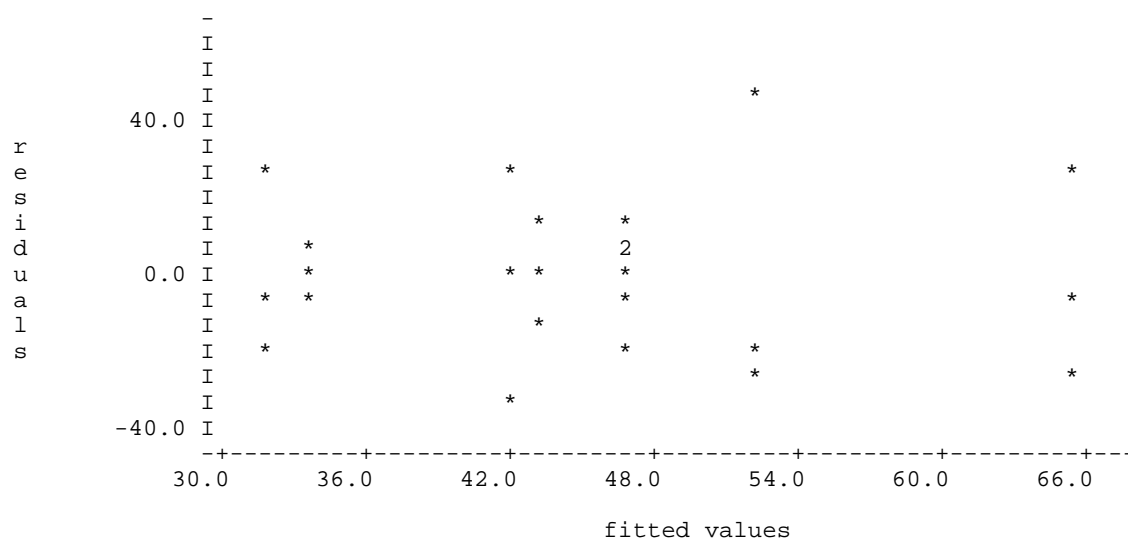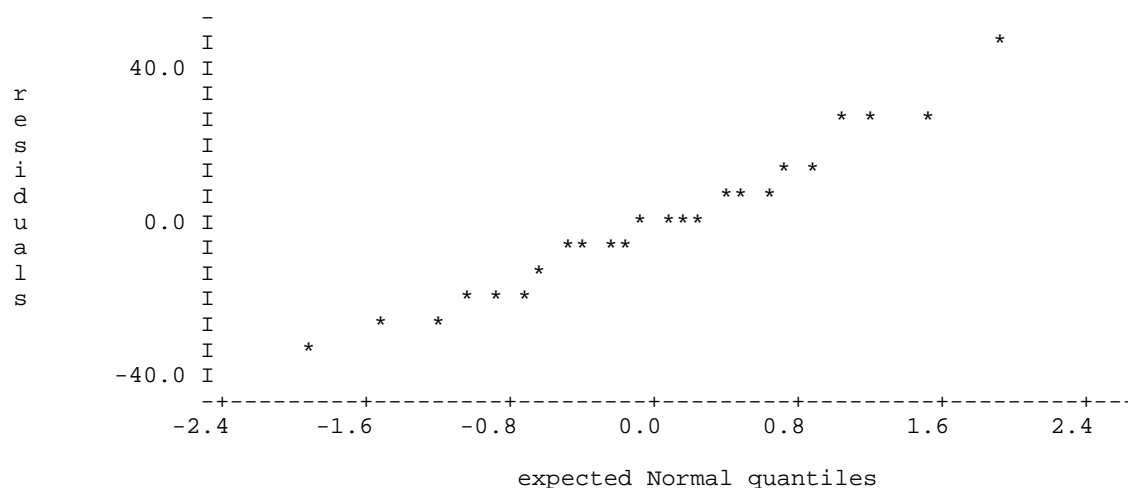
```
Table                Pressure   Catalyst   Pressure
                        Lab        Lab      Catalyst
                                              Lab
rep.                    6          6          3
d.f.                    *          *          *
l.s.d.                  *          *          *
Except when comparing means with the same level(s) of
 Lab                  28.72      28.72      40.62
 Pressure.Lab                               40.62
 Catalyst.Lab                               40.62
```

```
  28   APLOT METHOD=fit,normal
```

```
          -
          I
          I
          I                                    *
    40.0  I
 r        I
 e        I    *              *                                        *
 s        I
 i        I                       *     *
 d        I      *                      2
 u    0.0 I      *              * *     *
 a        I    *   *                    *                              *
 l        I                   *
 s        I    *                        *         *
          I                                       *                    *
          I                  *
   -40.0  I
          -+---------+---------+---------+---------+---------+---------+---
          30.0      36.0      42.0      48.0      54.0      60.0      66.0

                                    fitted values
```

```
                            Normal plot

          -
          I                                                      *
    40.0  I
 r        I
 e        I                                        *  *    *
 s        I
 i        I                                    *  *
 d        I                                 **  *
 u    0.0 I                            *  ***
 a        I                        ** **
 l        I                       *
 s        I                  *  *  *
          I          *     *
          I       *
   -40.0  I
          -+---------+---------+---------+---------+---------+---------+---
          -2.4      -1.6      -0.8       0.0       0.8       1.6       2.4

                          expected Normal quantiles
```

```
  29   "
 -30   **** Tukey''s one-degree-of-freedom-for-non-additivity.
 -31   **** It is the term designated covariate in the following analysis
 -32   "
  33   AKEEP [FIT=Fit]
  34   CALC ResSq=Fit*Fit
  35   ANOVA [PRINT=*]  ResSq; RES=ResSq
  36   COVAR ResSq                          "A computational trick"
  37   ANOVA [PRINT=A; FPROB=Y] Yield
```

```
******** Warning (Code AN 40). Statement 1 on Line 37
Command: ANOVA [PRINT=A; FPROB=Y] Yield
Stratum variance cannot be estimated
Labs stratum has zero residual sum of squares or degrees of freedom


37...................................................................................

***** Analysis of variance (adjusted for covariate) *****

Variate: Yield
Covariate: ResSq

Source of variation     d.f.       s.s.       m.s.     v.r. cov.ef.  F pr.

Labs stratum
Lab                       1        12.0       12.0            1.00

Labs.Runs stratum
Pressure                  1       360.4      360.4     0.65  1.00   0.430
Catalyst                  1      1080.0     1080.0     1.96  1.00   0.181
Pressure.Catalyst         1       234.4      234.4     0.43  1.00   0.523
Pressure.Lab              1         3.4        3.4     0.01  1.00   0.939
Catalyst.Lab              1       610.0      610.0     1.11  1.00   0.308
Pressure.Catalyst.Lab     1        92.0       92.0     0.17  1.00   0.688
Residual                 16      8812.7      550.8            1.00

Total                    23     11205.0

   38   COVAR
```

Note the warning message about not being to estimated Labs Residual variance — it can be ignored.

Step 1: Set up hypotheses

a) H_0: $\alpha_1 = \alpha_2$
    H_1: $\alpha_1 \neq \alpha_2$

b) H_0: $\beta_1 = \beta_2$
    H_1: $\beta_1 \neq \beta_2$

c) H_0: there is no interaction between A and B
    $$\left((\alpha\beta)_{ij} - \overline{(\alpha\beta)}_{i.} - \overline{(\alpha\beta)}_{.j} + \overline{(\alpha\beta)}_{..} = 0 \quad \text{for all i,j} \right)$$

    H_1: there is an interaction between A and B
    $$\left((\alpha\beta)_{ij} - \overline{(\alpha\beta)}_{i.} - \overline{(\alpha\beta)}_{.j} + \overline{(\alpha\beta)}_{..} \neq 0 \quad \text{for some i,j} \right)$$

d) H_0: $\sigma^2_{PCL} = 0$
    H_1: $\sigma^2_{PCL} \neq 0$

e) H_0: $\sigma^2_{PL} = 0$
    H_1: $\sigma^2_{PL} \neq 0$

f) H_0: $\sigma_{CL}^2 = 0$

H_1: $\sigma_{CL}^2 \neq 0$

g) H_0: $\sigma_L^2 = 0$

H_1: $\sigma_L^2 \neq 0$

Step 2: Calculate test statistics

Source	df	SSq	E[MSq]					MSq	F
Labs	1	12.0	σ_{LT}^2	$+3\sigma_{PCL}^2$	$+6\sigma_{CL}^2$	$+6\sigma_{PL}^2$	$+12\sigma_L^2$		
Labs.Tests	22								
Press	1	360.4	σ_{LT}^2	$+3\sigma_{PCL}^2$		$+6\sigma_{PL}^2$	$+f_P(\psi)$	0.65	0.430
Catal	1	1080.0	σ_{LT}^2	$+3\sigma_{PCL}^2$	$+6\sigma_{CL}^2$		$+f_C(\psi)$	1.96	0.181
Press.Catal	1	234.4	σ_{LT}^2	$+3\sigma_{PCL}^2$			$+f_{PC}(\psi)$	0.43	0.523
Press.Labs	1	3.4	σ_{LT}^2	$+3\sigma_{PCL}^2$		$+6\sigma_{PL}^2$		0.01	0.939
Catal.Labs	1	610.0	σ_{LT}^2	$+3\sigma_{PCL}^2$	$+6\sigma_{CL}^2$			1.11	0.308
Press.Catal.Labs	1	92.0	σ_{LT}^2	$+3\sigma_{PCL}^2$				0.17	0.688
Residual	16	550.8	σ_{LT}^2						

Step 3: Decide between hypotheses

For P.L.C interaction

The P.L.C interaction is not significant.

For P.L, P.C and L.C interactions

The two-factor interactions should be tested against the three factor interaction. However, as the three-factor interaction is not significant and has only one degree of freedom, the two-factor interactions should be tested against the three factor interaction. They are all are not significant.

For P, L and C

For similar reasons as the two-factor interactions, the main effects should be tested against the three factor interaction. The main effects are not significant.