THE DESIGN AND MIXED-MODEL ANALYSIS OF EXPERIMENTS

PRACTICAL X SOLUTIONS

X.1 A glasshouse experiment is to be run to investigate 6 factors each at two levels that affect the growth rate of plants. To save on resources the experimenter decides to use a quarter of the complete set of treatment combinations. Use the table given in subsection e) of section X.D, *Fractional factorial design at two levels*, to identify a suitable design.

a) What is the resolution of this design?

The design is a 2_{IV}^{6-2} *and so is of resolution IV.*

b) What are the implications of the design's resolution?

Being of resolution IV means that main effects are aliased with three factor interactions and tow-factor interactions are aliased with two-factor interactions.

c) What are the generators and defining relations for the design?

From the table the generators for the design are I = ABCE = BCDF.

Consequently the defining relations are: I = ABCE = BCDF = ADEF.

d) What is its aliasing pattern?

The aliasing pattern is obtained by multiplying all effects by the defining relations. It is given in the following table.

I + ABCE + ADEF + BCDF
A + BCE + DEF + ABCDF
B + ACE + CDF + ABDEF
C + ABE + BDF + ACDEF
D + AEF + BCF + ABCDE
E + ABC + ADF + BCDEF
F + ADE + BCD + ABCEF
AB + CE + ACDF + BDEF
AC + BE + ABDF + CDEF
AD + EF + ABCF + BCDE
AE + BC + DF + ABCDEF
AF + DE + ABCD + BCEF
BD + CF + ABEF + ACDE
BF + CD + ABDE + ACEF

ABD + ACF + BEF + CDE
ABF + ACD + BDE + CEF

e) What treatment combinations should the experimenter include in the experiment?

A	B	C	D	E	F
1	1	1	1	1	1
2	1	1	1	2	1
1	2	1	1	2	2
2	2	1	1	1	2
1	1	2	1	2	2
2	1	2	1	1	2
1	2	2	1	1	1
2	2	2	1	2	1
1	1	1	2	1	2
2	1	1	2	2	2
1	2	1	2	2	1
2	2	1	2	1	1
1	1	2	2	2	1
2	1	2	2	1	1
1	2	2	2	1	2
2	2	2	2	2	2

X.2 An experimenter wants to investigate 5 factors at 2 levels but considers that the number of plots in a block should be 8. Use Genstat to obtain a randomized layout for the experimenter using a seed of 965124.

After you have generated the design use the following command to check the analysis for the design:

ANOVA [FACTORIAL=5]

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 *** Treatment combinations on each unit of the design ***


 Blocks   1           2           3           4
  Plots
     1    1 2 1 1 1   1 1 1 1 2   2 1 2 1 2   1 2 1 1 2
     2    2 2 2 2 1   2 2 1 2 2   2 2 1 1 2   2 2 2 1 1
     3    2 1 1 1 2   1 1 1 2 1   2 1 2 2 1   1 2 1 2 1
     4    1 1 2 2 2   1 2 2 2 1   1 2 2 1 1   2 1 1 1 1
     5    2 1 1 2 1   2 1 2 2 2   1 2 2 2 2   1 1 2 2 1
     6    1 1 2 1 1   2 1 2 1 1   1 1 1 2 2   1 1 2 1 2
     7    2 2 2 1 2   1 2 2 1 2   1 1 1 1 1   2 1 1 2 2
     8    1 2 1 2 2   2 2 1 1 1   2 2 1 2 1   2 2 2 2 2

 Treatment factors are listed in the order: A B C D E
```

```
4  ANOVA [FACTORIAL=5]

4.................................................................................

***** Analysis of variance *****
Source of variation     d.f.

Blocks stratum
A.B.C                      1
A.D.E                      1
B.C.D.E                    1

Blocks.Plots stratum
A                          1
B                          1
C                          1
D                          1
E                          1
A.B                        1
A.C                        1
B.C                        1
A.D                        1
B.D                        1
C.D                        1
A.E                        1
B.E                        1
C.E                        1
D.E                        1
A.B.D                      1
A.C.D                      1
B.C.D                      1
A.B.E                      1
A.C.E                      1
B.C.E                      1
B.D.E                      1
C.D.E                      1
A.B.C.D                    1
A.B.C.E                    1
A.B.D.E                    1
A.C.D.E                    1
A.B.C.D.E                  1

Total                     31
```

X.3 A factorial experiment was carried out to investigate the weight gain of young chicks. There were eight cages of young chicks available and there were 3 factors each of two levels to investigate. The factors were two different amounts of zinc and two of copper added to two basic diets of either maize or wheat. The average weekly weight gains of the chicks in a cage are given in the following table:

Cage	Zinc	Copper	Base Diet	Weight Gain
1	1	1	M	23.20
2	1	1	W	20.50
3	1	2	M	24.30
4	1	2	W	16.40
5	2	1	M	24.60
6	2	1	W	20.70
7	2	2	M	22.90
8	2	2	W	14.10

What are the components of this experiment?

1. Observational unit - a cage
2. Response variable - Weight Gain
3. Unrandomized factors - Cages
4. Randomized factors - Zinc, Copper, Base Diet
5. Type of study - a 2^3 single-replicate, factorial CRD

What is the experimental structure for this experiment?

Structure	Formula
unrandomized	*8* Cages
randomized	*2* Zinc**2* Copper**2* BaseDiet

Analyze the data using Genstat, including diagnostic checking. What treatments will result in the most weight gain?

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   3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FAC3CHICK.GSH"
   4  DELETE [redefine=yes] Cage,Zinc,Copper,BaseDiet,WtGain
   5  FACTOR [modify=yes;nvalues=8;levels=8] Cage
   6  READ Cage; frepresentation=ordinal

    Identifier    Values   Missing    Levels
          Cage         8         0         8

   8  FACTOR [modify=yes;nvalues=8;levels=2] Zinc
   9  READ Zinc; frepresentation=ordinal

    Identifier    Values   Missing    Levels
          Zinc         8         0         2

  11  FACTOR [modify=yes;nvalues=8;levels=2] Copper
  12  READ Copper; frepresentation=ordinal

    Identifier    Values   Missing    Levels
        Copper         8         0         2

  14  FACTOR [modify=yes;nvalues=8;levels=2;labels=!t('M','W')] BaseDiet
  15  READ BaseDiet; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      BaseDiet         8         0         2

  17  VARIATE [nvalues=8] WtGain
  18  READ WtGain

    Identifier   Minimum      Mean   Maximum    Values   Missing
        WtGain     14.10     20.84     24.60         8         0

  20
```

```
  21  PRINT Cage,Zinc,Copper,BaseDiet,WtGain

        Cage        Zinc      Copper    BaseDiet      WtGain
           1           1           1           M       23.20
           2           1           1           W       20.50
           3           1           2           M       24.30
           4           1           2           W       16.40
           5           2           1           M       24.60
           6           2           1           W       20.70
           7           2           2           M       22.90
           8           2           2           W       14.10

  22  BLOCK Cage
  23  TREAT Zinc*Copper*BaseDiet
  24  "Produce normal plot of Yates effects"
  25  A2PLOT [PRINT=E; STRATUM=Cage; METHOD=normal; GRAPH=line] WtGain


25...............................................................................

***** Tables of effects *****

Variate: WtGain

***** Cage stratum *****

Zinc Y-effect               -0.52   s.e. *         rep. 4

Copper Y-effect             -2.83   s.e. *         rep. 4

BaseDiet Y-effect           -5.83   s.e. *         rep. 4

Zinc.Copper Y-effect        -1.33   s.e. *         rep. 2

Zinc.BaseDiet Y-effect      -0.53   s.e. *         rep. 2

Copper.BaseDiet Y-effect
                            -2.52   s.e. *         rep. 2

Zinc.Copper.BaseDiet Y-effect
                             0.08   s.e. *         rep. 1



             -
N            I
o            I                                                            *
r     1.0721 I
m            I                                                      *
a            I
l            I                                                      *
             I
Q            I                                               *
u    -0.2144 I
a            I                                    *
n            I
t            I                                 *
i            I
l            I     *
e    -1.5009 I
             -+---------+---------+---------+---------+---------+---------+-
        -6.4075   -5.3258   -4.2442   -3.1625   -2.0808   -0.9992    0.0825

                                            effect
```

This normal probability plot of Yates effects indicates that there is just one significant effect with a value of about –6. From the list of effects we see that this corresponds to the main effect for BaseDiet. (There is some evidence that *the Copper and Copper.BaseDiet effects are also significant but the evidence is not strong and so I have concluded that there is only one significant effect.)*

The analysis with just the significant effect included is given below. Because it involves only one factor, the analysis is the same as for a completely randomized design and so Tukey's-one-degree-of-freedom-for-nonadditivity is not appropriate.

```
  26  "Perform analysis including only significant effects
 -27     and do Residual analysis"
  28  BLOCK Cage
  29  TREAT BaseDiet
  30  ANOVA [PRINT=A,I,E,M; FPROB=Y] WtGain


30.................................................................................

***** Analysis of variance *****

Variate: WtGain

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Cage stratum
BaseDiet                   1     67.861     67.861   12.21  0.013
Residual                   6     33.338      5.556
Total                      7    101.199


***** Tables of effects *****

Variate: WtGain

***** Cage stratum *****

BaseDiet response          -5.82   s.e. 1.667     rep. 4


***** Tables of means *****

Variate: WtGain

Grand mean  20.84

 BaseDiet        M        W
            23.75    17.92


*** Standard errors of differences of means ***

Table             BaseDiet
rep.                     4
d.f.                     6
s.e.d.               1.667

  31  APLOT METHOD=fit,normal
```

```
            -
            I
            I
            I
        4.0 I
r           I
e           I         2
s           I
i           I
d           I                                                          2
u       0.0 I
a           I                                                          2
l           I         *
s           I
            I
            I
       -4.0 I         *
            -+---------+---------+---------+---------+---------+---------+---
           16.8      18.0      19.2      20.4      21.6      22.8      24.0

                                  fitted values
```

```
                                   Normal plot

            -
            I
        4.0 I
r           I
e           I                                               *           *
s           I
i           I
d           I                                 *     *
u       0.0 I
a           I                     *     *
l           I             *
s           I
            I
            I
       -4.0 I *
            -+---------+---------+---------+---------+---------+---------+---
           -1.5      -1.0      -0.5       0.0       0.5       1.0       1.5

                            expected Normal quantiles
```

The residual-versus-fitted-values and normal probability plots are not very informative as they are based on residuals with only 6 degrees of freedom. There is nothing unusual in the plots.

The table of means for the factor Base Diet is as follows:

BaseDiet	M	W
	23.75	17.92

Thus chicks fed with maize will have the greatest weight gain, irrrespective of the level of Zinc and Copper used. (Note there is no need for an LSD here as the F-test in the ANOVA tells us that the difference is significant.

X.4 An experiment was conducted at Rothamsted Experiment Station to investigate the effect of Dung, Nitrochalk (N), Superphosphate (P) and Muriate of Potash (K) on the yield of beans. The design involved 4 blocks of 8 plots with a complete set of treatments in each pair of the blocks. The treatments were assigned so that the four-factor interaction is confounded with blocks. The four sets of 8 treatment combinations that were to occur in a block were completely randomized to the 4 blocks. The experimental layout and yields are given in the following table.

Blocks	Plots	Dung	N	P	K	Yield
1	1	0	0.0	0.6	0	45
1	2	0	0.0	0.0	1	55
1	3	10	0.0	0.0	0	53
1	4	0	0.4	0.6	1	36
1	5	10	0.4	0.0	1	41
1	6	10	0.4	0.6	0	48
1	7	10	0.0	0.6	1	55
1	8	0	0.4	0.0	0	42
2	1	10	0.0	0.6	0	50
2	2	0	0.4	0.0	1	44
2	3	10	0.0	0.0	1	43
2	4	0	0.0	0.6	1	51
2	5	10	0.4	0.6	1	44
2	6	0	0.0	0.0	0	58
2	7	10	0.4	0.0	0	41
2	8	0	0.4	0.6	0	50
3	1	0	0.4	0.6	1	43
3	2	10	0.0	0.0	0	42
3	3	0	0.0	0.6	0	39
3	4	10	0.4	0.0	1	34
3	5	0	0.4	0.0	0	47
3	6	10	0.4	0.6	0	52
3	7	0	0.0	0.0	1	50
3	8	10	0.0	0.6	1	44
4	1	0	0.4	0.0	1	43
4	2	10	0.0	0.6	0	52
4	3	0	0.0	0.0	0	57
4	4	0	0.4	0.6	0	39
4	5	0	0.0	0.6	1	56
4	6	10	0.0	0.0	1	52
4	7	10	0.4	0.6	1	54
4	8	10	0.4	0.0	0	42

What are the components of this experiment?

1. Observational unit - a plot
2. Response variable - Yield
3. Unrandomized factors - Blocks, Plots
4. Randomized factors - Dung, N, P, K
5. Type of study - replicated, confounded 2^4 RCBD

What is the experimental structure for this experiment?

Structure	Formula
unrandomized	*4* Blocks/*8* Plots
randomized	*2* Dung**2* N**2* P**2* K

What are the expected mean squares for the lines in the analysis of variance table based on all unrandomized factors being random and all randomized factors being fixed?

Source	df	E[MSQ]
Blocks	3	
Dung.N.P.K	1	$\sigma^2_{BP} + 8\sigma^2_{P} + f_{DNPK}(\psi)$
Residual	2	$\sigma^2_{BP} + 8\sigma^2_{P}$
Blocks.Plots	28	
Dung	1	$\sigma^2_{BP} + f_{D}(\psi)$
N	1	$\sigma^2_{BP} + f_{N}(\psi)$
P	1	$\sigma^2_{BP} + f_{P}(\psi)$
K	1	$\sigma^2_{BP} + f_{K}(\psi)$
Dung.N	1	$\sigma^2_{BP} + f_{DN}(\psi)$
Dung.P	1	$\sigma^2_{BP} + f_{DP}(\psi)$
N.P	1	$\sigma^2_{BP} + f_{NP}(\psi)$
Dung.K	1	$\sigma^2_{BP} + f_{DK}(\psi)$
N.K	1	$\sigma^2_{BP} + f_{NK}(\psi)$
P.K	1	$\sigma^2_{BP} + f_{PK}(\psi)$
Dung.N.P	1	$\sigma^2_{BP} + f_{DNP}(\psi)$
Dung.N.K	1	$\sigma^2_{BP} + f_{DNK}(\psi)$
Dung.P.K	1	$\sigma^2_{BP} + f_{DPK}(\psi)$
N.P.K	1	$\sigma^2_{BP} + f_{NPK}(\psi)$
Residual	3	σ^2_{BP}

Analyze the data using Genstat, including diagnostic checking. What levels of Dung, N, P and K would you recommend be used to maximize the yield of beans? What yield would be achieved with these combinations of Dung, N, P and K?

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   3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FAC4BEAN.GSH"
   4  DELETE [redefine=yes] Blocks,Plots,Dung,N,P,K,Yield
   5  FACTOR [modify=yes;nvalues=32;levels=4] Blocks
   6  READ Blocks; frepresentation=ordinal

    Identifier    Values   Missing    Levels
        Blocks        32         0         4

   8  FACTOR [modify=yes;nvalues=32;levels=8] Plots
   9  READ Plots; frepresentation=ordinal

    Identifier    Values   Missing    Levels
         Plots        32         0         8

  11  FACTOR [modify=yes;nvalues=32;levels=!(0,10)] Dung
  12  READ Dung; frepresentation=ordinal

    Identifier    Values   Missing    Levels
          Dung        32         0         2

  14  FACTOR [modify=yes;nvalues=32;levels=!(0,0.4)] N
  15  READ N; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             N        32         0         2

  17  FACTOR [modify=yes;nvalues=32;levels=!(0,0.6)] P
  18  READ P; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             P        32         0         2

  20  FACTOR [modify=yes;nvalues=32;levels=!(0,1)] K
  21  READ K; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             K        32         0         2

  23  VARIATE [nvalues=32] Yield
  24  READ Yield

    Identifier   Minimum      Mean   Maximum    Values   Missing
         Yield     34.00     46.94     58.00        32         0

  27
  28  PRINT Blocks,Plots,Dung,N,P,K,Yield; FIELD=9; DEC=3(0),2(1,0)

   Blocks    Plots     Dung        N        P        K    Yield
        1        1        0      0.0      0.6        0       45
        1        2        0      0.0      0.0        1       55
        1        3       10      0.0      0.0        0       53
        1        4        0      0.4      0.6        1       36
        1        5       10      0.4      0.0        1       41
        1        6       10      0.4      0.6        0       48
        1        7       10      0.0      0.6        1       55
        1        8        0      0.4      0.0        0       42
        2        1       10      0.0      0.6        0       50
        2        2        0      0.4      0.0        1       44
        2        3       10      0.0      0.0        1       43
        2        4        0      0.0      0.6        1       51
        2        5       10      0.4      0.6        1       44
        2        6        0      0.0      0.0        0       58
```

```
       2        7       10      0.4      0.0        0       41
       2        8        0      0.4      0.6        0       50
       3        1        0      0.4      0.6        1       43
       3        2       10      0.0      0.0        0       42
       3        3        0      0.0      0.6        0       39
       3        4       10      0.4      0.0        1       34
       3        5        0      0.4      0.0        0       47
       3        6       10      0.4      0.6        0       52
       3        7        0      0.0      0.0        1       50
       3        8       10      0.0      0.6        1       44
       4        1        0      0.4      0.0        1       43
       4        2       10      0.0      0.6        0       52
       4        3        0      0.0      0.0        0       57
       4        4        0      0.4      0.6        0       39
       4        5        0      0.0      0.6        1       56
       4        6       10      0.0      0.0        1       52
       4        7       10      0.4      0.6        1       54
       4        8       10      0.4      0.0        0       42

  29  BLOCK Blocks/Plots
  30  TREAT Dung*N*P*K
  31  ANOVA [PRINT=A,I,E,M; FACTORIAL=4; TWOLEVEL=YATES; FPROB=Y; PSE=LSD] \
  32                                                              Yield


32................................................................................

***** Analysis of variance *****

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Blocks stratum
Dung.N.P.K                 1      78.12      78.12    3.24  0.214
Residual                   2      48.25      24.12    0.99

Blocks.Plots stratum
Dung                       1       2.00       2.00    0.08  0.778
N                          1     325.12     325.12   13.40  0.003
P                          1       6.12       6.12    0.25  0.623
K                          1       4.50       4.50    0.19  0.673
Dung.N                     1      32.00      32.00    1.32  0.270
Dung.P                     1     242.00     242.00    9.97  0.007
N.P                        1      78.12      78.12    3.22  0.094
Dung.K                     1       6.12       6.12    0.25  0.623
N.K                        1      32.00      32.00    1.32  0.270
P.K                        1      24.50      24.50    1.01  0.332
Dung.N.P                   1       2.00       2.00    0.08  0.778
Dung.N.K                   1      10.13      10.13    0.42  0.529
Dung.P.K                   1      15.13      15.13    0.62  0.443
N.P.K                      1      32.00      32.00    1.32  0.270
Residual                  14     339.75      24.27

Total                     31    1277.88


***** Tables of effects *****

Variate: Yield

***** Blocks stratum *****

Dung.N.P.K Y-effect         3.12   s.e. 1.737     rep. 2

***** Blocks.Plots stratum *****

Dung Y-effect              -0.50   s.e. 1.742     rep. 16

N Y-effect                 -6.37   s.e. 1.742     rep. 16

P Y-effect                  0.88   s.e. 1.742     rep. 16
```

K Y-effect	-0.75	s.e. 1.742	rep. 16
Dung.N Y-effect	2.00	s.e. 1.742	rep. 8
Dung.P Y-effect	5.50	s.e. 1.742	rep. 8
N.P Y-effect	3.12	s.e. 1.742	rep. 8
Dung.K Y-effect	-0.88	s.e. 1.742	rep. 8
N.K Y-effect	-2.00	s.e. 1.742	rep. 8
P.K Y-effect	1.75	s.e. 1.742	rep. 8
Dung.N.P Y-effect	0.50	s.e. 1.742	rep. 4
Dung.N.K Y-effect	1.12	s.e. 1.742	rep. 4
Dung.P.K Y-effect	-1.37	s.e. 1.742	rep. 4
N.P.K Y-effect	-2.00	s.e. 1.742	rep. 4

***** Tables of means *****

Variate: Yield

Grand mean 46.94

Dung	0.00	10.00
	47.19	46.69

N	0.00	0.40
	50.13	43.75

P	0.00	0.60
	46.50	47.38

K	0.00	1.00
	47.31	46.56

Dung	N	0.00	0.40
0.00		51.38	43.00
10.00		48.88	44.50

Dung	P	0.00	0.60
0.00		49.50	44.88
10.00		43.50	49.88

N	P	0.00	0.60
0.00		51.25	49.00
0.40		41.75	45.75

Dung	K	0.00	1.00
0.00		47.13	47.25
10.00		47.50	45.88

N	K	0.00	1.00
0.00		49.50	50.75
0.40		45.13	42.38

P	K	0.00	1.00
0.00		47.75	45.25
0.60		46.88	47.88

	N	0.00		0.40	
Dung	P	0.00	0.60	0.00	0.60
0.00		55.00	47.75	44.00	42.00
10.00		47.50	50.25	39.50	49.50

```
                 N     0.00              0.40
     Dung        K     0.00      1.00    0.00      1.00
     0.00             49.75     53.00   44.50     41.50
    10.00             49.25     48.50   45.75     43.25

                 P     0.00              0.60
     Dung        K     0.00      1.00    0.00      1.00
     0.00             51.00     48.00   43.25     46.50
    10.00             44.50     42.50   50.50     49.25

                 P     0.00              0.60
        N        K     0.00      1.00    0.00      1.00
     0.00             52.50     50.00   46.50     51.50
     0.40             43.00     40.50   47.25     44.25

                          P     0.00              0.60
     Dung        N        K     0.00      1.00    0.00      1.00
     0.00     0.00             57.50     52.50   42.00     53.50
              0.40             44.50     43.50   44.50     39.50
    10.00     0.00             47.50     47.50   51.00     49.50
              0.40             41.50     37.50   50.00     49.00

*** Least significant differences of means (5% level) ***

Table                  Dung         N           P           K
rep.                     16        16          16          16
d.f.                     14        14          14          14
l.s.d.                3.736     3.736       3.736       3.736

Table                  Dung      Dung           N        Dung
                          N         P           P           K
rep.                      8         8           8           8
d.f.                     14        14          14          14
l.s.d.                5.283     5.283       5.283       5.283

Table                     N         P        Dung        Dung
                          K         K           N           N
                                                P           K
rep.                      8         8           4           4
d.f.                     14        14          14          14
l.s.d.                5.283     5.283       7.471       7.471

Table                  Dung         N        Dung
                          P         P           N
                          K         K           P
                                                K
rep.                      4         4           2
l.s.d.                7.471     7.471      10.566
d.f.                     14        14          14

  33  APLOT METHOD=fit,normal
             -
             I
             I
             I
         8.0 I
r            I                   *
e            I     *
s            I                  *            2 * *  *
i            I
d            I        *        * *                         **     *
u        0.0 I            *   *                  *    *
a            I           *         *                *   *             *
l            I*
s            I                         2 *  2        *
             I             *
             I                       *
        -8.0 I
             -+---------+---------+---------+---------+---------+---------+---
           36.0      40.0      44.0      48.0      52.0      56.0      60.0

                                         fitted values
```

```
                              Normal plot

            -
            I
       8.0 I
r           I                                                          *
e           I                                                     *
s           I                                         ** ** * *
i           I
d           I                                  *** ***
u      0.0 I                                *2*
a           I                          ** ***
l           I                         *
s           I               * * ** **
            I           *
            I      *
      -8.0 I
            -+---------+---------+---------+---------+---------+---------+---
          -2.4      -1.6      -0.8       0.0       0.8       1.6       2.4

                          expected Normal quantiles

  34  "
 -35  **** Tukey''s one-degree-of-freedom-for-non-additivity.
 -36  **** It is the term designated covariate in the following analysis
 -37  "
  38  AKEEP [FIT=Fit]
  39  CALC ResSq=Fit*Fit
  40  ANOVA [PRINT=*]  ResSq; RES=ResSq
  41  COVAR ResSq                             "A computational trick"
  42  ANOVA [PRINT=A; FPROB=Y] Yield


42.............................................................................

***** Analysis of variance (adjusted for covariate) *****

Variate: Yield
Covariate: ResSq
```

Source of variation	d.f.	s.s.	m.s.	v.r.	cov.ef.	F pr.
Blocks stratum	3	126.37	42.13	1.69		
Blocks.Plots stratum						
Dung	1	2.00	2.00	0.08	1.00	0.782
N	1	325.12	325.12	13.02	1.00	0.003
P	1	6.12	6.12	0.25	1.00	0.629
K	1	4.50	4.50	0.18	1.00	0.678
Dung.N	1	32.00	32.00	1.28	1.00	0.278
Dung.P	1	242.00	242.00	9.69	1.00	0.008
N.P	1	78.12	78.12	3.13	1.00	0.100
Dung.K	1	6.12	6.12	0.25	1.00	0.629
N.K	1	32.00	32.00	1.28	1.00	0.278
P.K	1	24.50	24.50	0.98	1.00	0.340
Dung.N.P	1	2.00	2.00	0.08	1.00	0.782
Dung.N.K	1	10.13	10.13	0.41	1.00	0.535
Dung.P.K	1	15.13	15.13	0.61	1.00	0.450
N.P.K	1	32.00	32.00	1.28	1.00	0.278
Covariate	1	15.05	15.05	0.60		0.452
Residual	13	324.70	24.98		0.97	
Total	31	1277.88				

```
  43  COVAR
```

Source	df	SSq	MSq	F	Prob
Blocks	3				
Dung.N.P.K	1	78.12	78.12	3.24	0.214
Residual	2	48.25	24.12	0.99	
Blocks.Plots	28				
Dung	1	2.00	2.00	0.08	0.778
N	1	325.12	325.12	13.40	0.003
P	1	6.12	6.12	0.25	0.623
K	1	4.50	4.50	0.19	0.673
Dung.N	1	32.00	32.00	1.32	0.270
Dung.P	1	242.00	242.00	9.97	0.007
N.P	1	78.12	78.12	3.22	0.094
Dung.K	1	6.12	6.12	0.25	0.623
N.K	1	32.00	32.00	1.32	0.270
P.K	1	24.50	24.50	1.01	0.332
Dung.N.P	1	2.00	2.00	0.08	0.778
Dung.N.K	1	10.13	10.13	0.42	0.529
Dung.P.K	1	15.13	15.13	0.62	0.443
N.P.K	1	32.00	32.00	1.32	0.270
Residual	14	339.75	24.27		
Non-additivity	1	15.05	15.05	0.60	0.452
Deviations	13	324.70	24.98		
Total		1277.88			

The analysis indicates that the Dung.P interaction and the N main effect are significant.

The residuals-versus-fitted values plot is satisfactory as is the normal plot of the residuals. Tukey's test for non-additivity is not significant. there is no evidence that the assumptions are unmet.

The fitted equations for main effects are:

$$\text{grand mean} \pm \frac{\text{main effect}}{2}$$

These will lead to the single-factor tables of means given above. The table relevant here is N and it is given by:

		0	0.4
N	$46.94 \pm \frac{-6.37}{2}$	50.13	43.75

As the interaction of Dung and P is significant, we need to examine the means for the combinations of these two factors. The can be calculated using the following equation:

$$y_{\text{Dung, P}} = 46.94 - \frac{0.50}{2} x_{\text{Dung}} + \frac{0.88}{2} x_{\text{P}} + \frac{5.50}{2} x_{\text{Dung}} x_{\text{P}}$$

where x_{Dung} and x_{P} take the values ± 1 according as to whether the low or high level of the corresponding factor is involved.

From the output the table of means and LSD is:

		P	
		0	0.6
Dung	0	49.50	44.88
	10	43.50	49.88
LSD(0.05)		5.283	

```
44  AGRAPH [GRAPHICS=line; METHOD=lines] XFACTOR=P; GROUP=Dung
            -
            I                                                  2
            I          1
            I
       48.0 I-
            I-
            I-
            I-
            I
            I                                                  1
       44.0 I
            I          2
            I
            I
            I
            I
       40.0 I
            -+---------+---------+---------+---------+---------+---------+---
         -0.15      0.00      0.15      0.30      0.45      0.60      0.75

                      mean['0']  v. xlevel using symbol 1
                      mean['10']  v. xlevel using symbol 2
                      sedbar  v. __ using symbol -
```

It would appear that either use of neither Dung nor P or the use of both Dung and P is superior to using either Dung or P on their own. Note that while P and no Dung is not significantly different o any of the other treatments, it is close to being significant.

To maximize the bean yields no N should be used with neither Dung nor P or with both Dung and P. It does not matter what amount of K is used. So the cheapest combination to use would be no fertilizer at all.

The yield that would be achieved can be computed using the following equation for the response:

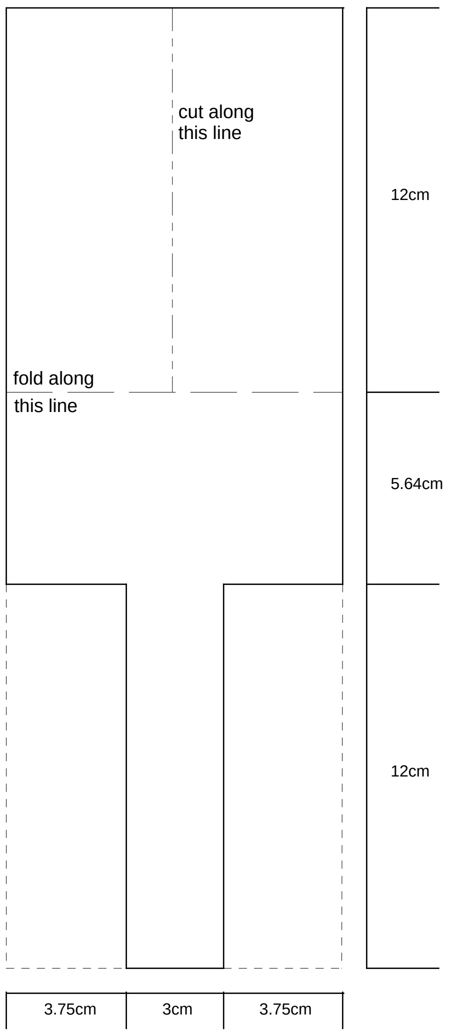
$$y = 46.94 - \frac{0.50}{2} x_{\text{Dung}} + \frac{0.88}{2} x_{\text{P}} + \frac{5.50}{2} x_{\text{Dung}} x_{\text{P}} - \frac{6.37}{2} x_{\text{N}}$$

The predicted yield with no fertilizer can be obtained by putting –1 for x_{Dung}, x_P and x_N into this equation. The predicted yield is

$$\begin{aligned} y &= 46.94 - \frac{0.50}{2}(-1) + \frac{0.88}{2}(-1) + \frac{5.50}{2}(-1)(-1) - \frac{6.37}{2}(-1) \\ &= 56.75 \end{aligned}$$

X.5 The Light Helicopter Corporation wishes to investigate ways in which the flight time of their helicopters can be increased. The standard design for the helicopters they produce in shown below.

The standard design



Improving the design

Engineers from their company have got together and had a brainstorming session to identify modifications to the design that might increase the flight time. They suggested that the following factors be investigated.

Factors		-	+
Paper type	(P)	light	heavy
Wing length	(W)	7.5cm	12cm
Body length	(L)	7.5cm	12cm
Body width	(B)	3cm	5cm
Paper clip	(C)	no	yes
Fold	(F)	no	yes
Taped body	(T)	no	yes
Taped wing	(M)	no	yes

Now there are 8 factors to be investigated. If all combinations of the factors were to be investigated, as in a complete factorial, how many helicopters would have to be produced?

It is decided that the full set cannot be run and that a fractional factorial must be employed. There are sufficient resources to make 16 helicopters at this stage. To study the 8 factors in 16 runs a 2_{IV}^{8-4} fractional factorial design is chosen. The design has generators **5** = **234**, **6** = **134**, **7** = **123** and **8** = **124**. The runs, given in standard order, are given in the following table:

	Factor							
Standard Order	**1** **P**	**2** **W**	**3** **L**	**4** **B**	**5** **C**	**6** **F**	**7** **T**	**8** **M**
1	-	-	-	-	-	-	-	-
2	+	-	-	-	-	+	+	+
3	-	+	-	-	+	-	+	+
4	+	+	-	-	+	+	-	-
5	-	-	+	-	+	+	+	-
6	+	-	+	-	+	-	-	+
7	-	+	+	-	-	+	-	+
8	+	+	+	-	-	-	+	-
9	-	-	-	+	+	+	-	+
10	+	-	-	+	+	-	+	-
11	-	+	-	+	-	+	+	-
12	+	+	-	+	-	-	-	+
13	-	-	+	+	-	-	+	+
14	+	-	+	+	-	+	-	-
15	-	+	+	+	+	-	-	-
16	+	+	+	+	+	+	+	+

The aliasing pattern (ignoring three- and more-factor interactions and substituting in factor names) for this experiment is as follows:

ℓ_1	➔ average	ℓ_0	➔ average
ℓ_2	➔ **1**	ℓ_P	➔ **P**
ℓ_3	➔ **2**	ℓ_W	➔ **W**
ℓ_4	➔ **12** + **37** + **48** + **56**	ℓ_{PW}	➔ **PW** + **LT** + **BM** + **CF**
ℓ_5	➔ **3**	ℓ_L	➔ **L**
ℓ_6	➔ **13** + **27** + **46** + **58**	ℓ_{PL}	➔ **PL** + **WT** + **BF** + **CM**
ℓ_7	➔ **23** + **17** + **45** + **68**	ℓ_{WL}	➔ **WL** + **PT** + **BC** + **FM**
ℓ_8	➔ **7**	ℓ_T	➔ **T**
ℓ_9	➔ **4**	ℓ_B	➔ **B**
ℓ_{10}	➔ **14** + **28** + **36** + **57**	ℓ_{PB}	➔ **PB** + **WM** + **LF** + **CT**
ℓ_{11}	➔ **24** + **18** + **35** + **68**	ℓ_{WB}	➔ **WB** + **PM** + **LC** + **FM**
ℓ_{12}	➔ **8**	ℓ_M	➔ **M**
ℓ_{13}	➔ **34** + **16** + **25** + **78**	ℓ_{LB}	➔ **LB** + **PF** + **WC** + **TM**
ℓ_{14}	➔ **6**	ℓ_F	➔ **F**
ℓ_{15}	➔ **5**	ℓ_C	➔ **C**
ℓ_{16}	➔ **15** + **26** + **38** + **47**	ℓ_{PF}	➔ **PC** + **WF** + **LM** + **BT**

Generators:

C = **WLB**, **F** = **PLB**, **T** = **PWL** and **M** = **PWB**.

Analysis of results

What is the experimental structure for this experiment?

Structure	Formula
unrandomized	Runs
randomized	*2* P**2* W**2* L**2* B**2* C**2* F**2* T**2* M

Use Genstat to analyse the results of the experiment and to perform appropriate diagnostic checking. What treatment combinations would give the longest flight time and what would you predict would be the flight time for these treatment combinations? The treatment combinations are available in the file *frf8heli.gsh* in the directory *G:\Disciplina\Genstat*.

The following Genstat output contains the analysis of the times recorded by the students (Time[1]), by the staff (Time[2]) and the mean of these two times (Time[3]). In all cases it would appear the normal probability plot of Yates *effects has one large negative effect and one large positive effect that are significant. The significant effects are Paper type* (**P**) *and wing length* (**W**)*. It would appear that there are no two-factor interactions in this experiment. The two significant terms have been fitted and diagnostic checking done on the residuals produced.*

```
Genstat 5  Release 4.1  (PC/Windows NT)              05 May 2000 11:52:01
Copyright 1998, Lawes Agricultural Trust (Rothamsted Experimental Station)
                 ______________________________________________

                 Genstat 5 Fourth Edition - (for Windows)
                 Genstat 5 Procedure Library Release PL11
                 ______________________________________________

   3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FRF8HELI00BRASIL.GSH"
   4  DELETE [redefine=yes] Tests,PaperTyp,Wing_Len,Body_Len,Body_Wid,Clip,Fold\
   5  ,BodyTape,WingTape,Time[1],Time[2]
   6  FACTOR [modify=yes;nvalues=16;levels=16] Tests
   7  READ Tests; frepresentation=ordinal

    Identifier    Values   Missing    Levels
         Tests        16         0        16

   9  FACTOR [modify=yes;nvalues=16;levels=2;labels=!t('light','heavy')] PaperTyp
  10  READ PaperTyp; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      PaperTyp        16         0         2

  12  FACTOR [modify=yes;nvalues=16;levels=!(7.5,12)] Wing_Len
  13  READ Wing_Len; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      Wing_Len        16         0         2

  15  FACTOR [modify=yes;nvalues=16;levels=!(7.5,12)] Body_Len
  16  READ Body_Len; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      Body_Len        16         0         2

  18  FACTOR [modify=yes;nvalues=16;levels=!(3,5)] Body_Wid
  19  READ Body_Wid; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      Body_Wid        16         0         2

  21  FACTOR [modify=yes;nvalues=16;levels=2;labels=!t('no','yes')] Clip
  22  READ Clip; frepresentation=ordinal

    Identifier    Values   Missing    Levels
          Clip        16         0         2

  24  FACTOR [modify=yes;nvalues=16;levels=2;labels=!t('no','yes')] Fold
  25  READ Fold; frepresentation=ordinal

    Identifier    Values   Missing    Levels
          Fold        16         0         2

  27  FACTOR [modify=yes;nvalues=16;levels=2;labels=!t('no','yes')] BodyTape
  28  READ BodyTape; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      BodyTape        16         0         2

  30  FACTOR [modify=yes;nvalues=16;levels=2;labels=!t('no','yes')] WingTape
  31  READ WingTape; frepresentation=ordinal

    Identifier    Values   Missing    Levels
      WingTape        16         0         2
```

```
  33  VARIATE [nvalues=16] Time[1]
  34  READ Time[1]

    Identifier   Minimum      Mean   Maximum    Values   Missing
       Time[1]     1.880     3.237     4.140        16         0

  37  VARIATE [nvalues=16] Time[2]
  38  READ Time[2]

    Identifier   Minimum      Mean   Maximum    Values   Missing
       Time[2]     1.800     3.296     4.530        16         0

  41
  42  PRINT Tests,PaperTyp,Wing_Len,Body_Len,Body_Wid,Clip,Fold,      \
  43                 BodyTape,WingTape,#Time; FIELD=6,4(9),5,5,9,9,5,5; DEC=1

 Tests PaperTyp Wing_Len Body_Len Body_Wid Clip Fold BodyTape WingTape Time[1] T
ime[2]
     1    light      7.5      7.5      3.0   no   no       no       no  3.3  2.9
     2    heavy      7.5      7.5      3.0   no  yes      yes      yes  3.0  3.2
     3    light     12.0      7.5      3.0  yes   no      yes      yes  4.1  4.2
     4    heavy     12.0      7.5      3.0  yes  yes       no       no  2.6  2.9
     5    light      7.5     12.0      3.0  yes  yes      yes       no  3.6  3.6
     6    heavy      7.5     12.0      3.0  yes   no       no      yes  2.8  2.8
     7    light     12.0     12.0      3.0   no  yes       no      yes  3.8  4.2
     8    heavy     12.0     12.0      3.0   no   no      yes       no  3.1  3.3
     9    light      7.5      7.5      5.0  yes  yes       no      yes  2.6  2.6
    10    heavy      7.5      7.5      5.0  yes   no      yes       no  1.9  1.8
    11    light     12.0      7.5      5.0   no  yes      yes       no  4.0  3.8
    12    heavy     12.0      7.5      5.0   no   no       no      yes  3.4  3.3
    13    light      7.5     12.0      5.0   no   no      yes      yes  4.1  4.1
    14    heavy      7.5     12.0      5.0   no  yes       no       no  2.5  2.6
    15    light     12.0     12.0      5.0  yes   no       no       no  3.9  4.5
    16    heavy     12.0     12.0      5.0  yes  yes      yes      yes  2.9  2.8

  44  CALC Time[3]=(Time[1]+Time[2])/2
  45  BLOCK Tests
  46  FOR t=1...3
  47    TREAT PaperTyp*Wing_Len*Body_Len*Body_Wid*Clip*Fold*BodyTape*WingTape
  48    A2PLOT [PRINT=inform,effect; FACTORIAL=2; STRATUM=Tests; \
  49            METHOD=normal; GRAPH=line] Time[t]
  50    "Perform analysis including only significant effects
 -51     and do Residual analysis"
  52    TREAT PaperTyp+Wing_Len
  53    ANOVA [FPROB=Y;  PSE=LSD] Time[t]
  54    APLOT METHOD=fit,normal
  55    "
 -56    **** Tukey''s one-degree-of-freedom-for-non-additivity.
 -57    **** It is the term designated covariate in the following analysis
 -58    "
  59    AKEEP [FIT=Fit]
  60    CALC ResSq=Fit*Fit
  61    ANOVA [PRINT=*]  ResSq; RES=ResSq
  62    COVAR ResSq                             "A computational trick"
  63    ANOVA [PRINT=A; FPROB=Y] Time[t]
  64    COVAR
  65  ENDFOR

65................................................................................

***** Information summary *****

Aliased model terms
Wing_Len.Clip
Body_Len.Clip
Body_Wid.Clip
PaperTyp.Fold
Wing_Len.Fold
Body_Len.Fold
Body_Wid.Fold
Clip.Fold
```

```
PaperTyp.BodyTape
Wing_Len.BodyTape
Body_Len.BodyTape
Body_Wid.BodyTape
Clip.BodyTape
Fold.BodyTape
PaperTyp.WingTape
Wing_Len.WingTape
Body_Len.WingTape
Body_Wid.WingTape
Clip.WingTape
Fold.WingTape
BodyTape.WingTape

***** Tables of effects *****

Variate: Time[1]

***** Tests stratum *****

PaperTyp Y-effect           -0.90   s.e. *         rep. 8
Wing_Len Y-effect            0.53   s.e. *         rep. 8
Body_Len Y-effect            0.23   s.e. *         rep. 8
Body_Wid Y-effect           -0.14   s.e. *         rep. 8
Clip Y-effect               -0.37   s.e. *         rep. 8
Fold Y-effect               -0.21   s.e. *         rep. 8
BodyTape Y-effect            0.23   s.e. *         rep. 8
WingTape Y-effect            0.22   s.e. *         rep. 8

PaperTyp.Wing_Len Y-effect
                            -0.03   s.e. *         rep. 4

PaperTyp.Body_Len Y-effect
                            -0.14   s.e. *         rep. 4

Wing_Len.Body_Len Y-effect
                            -0.32   s.e. *         rep. 4

PaperTyp.Body_Wid Y-effect
                            -0.07   s.e. *         rep. 4

Wing_Len.Body_Wid Y-effect
                             0.27   s.e. *         rep. 4

Body_Len.Body_Wid Y-effect
                             0.18   s.e. *         rep. 4

PaperTyp.Clip Y-effect      -0.09   s.e. *         rep. 4

             -
N            I
o            I                                                          *
r     1.3667 I                                                *
m            I
a            I                                               2
l            I                                              *
             I                                     *       *
Q            I                                   *
u    -0.2733 I                                *  *
a            I                                *
n            I                          *   *
t            I
i            I                        *
l            I   *
e    -1.9133 I
             -+---------+---------+---------+---------+---------+---------+-
        -0.9914   -0.7297   -0.4680   -0.2063    0.0555    0.3172    0.5789

                                            effect
```

65...

***** Analysis of variance *****

Variate: Time[1]

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Tests stratum					
PaperTyp	1	3.2490	3.2490	17.73	0.001
Wing_Len	1	1.1078	1.1078	6.05	0.029
Residual	13	2.3816	0.1832		
Total	15	6.7383			

***** Tables of means *****

Variate: Time[1]

Grand mean 3.24

PaperTyp	light	heavy
	3.69	2.79

Wing_Len	7.50	12.00
	2.97	3.50

*** Least significant differences of means (5% level) ***

Table	PaperTyp	Wing_Len
rep.	8	8
d.f.	13	13
l.s.d.	0.462	0.462

```
             -
             I
             I
             I
         1.0 I
r            I
e            I                                     *
s            I *
i            I                      *
d            I *                    *              *                    *
u        0.0 I *                                                        2
a            I                      *              *                    *
l            I                      *
s            I
             I *
             I                                     *
        -1.0 I
             -+---------+---------+---------+---------+---------+---------+---
           2.50      2.75      3.00      3.25      3.50      3.75      4.00
                                         fitted values

                                          Normal plot
             -
             I
         1.0 I
r            I
e            I                                                           *
s            I                                                   *
i            I                                              *
d            I                                  *  * *   *
u        0.0 I                          *  * *
a            I                 *   * *
l            I              *
s            I
             I         *
             I *
        -1.0 I
             -+---------+---------+---------+---------+---------+---------+---
           -1.8      -1.2      -0.6       0.0       0.6       1.2       1.8
                                   expected Normal quantiles
```

```
65..........................................................................

***** Analysis of variance (adjusted for covariate) *****

Variate: Time[1]
Covariate: ResSq

Source of variation     d.f.       s.s.       m.s.    v.r. cov.ef.  F pr.

Tests stratum
PaperTyp                   1     3.2490     3.2490   16.40    1.00  0.002
Wing_Len                   1     1.1078     1.1078    5.59    1.00  0.036
Covariate                  1     0.0046     0.0046    0.02          0.882
Residual                  12     2.3770     0.1981            0.92
Total                     15     6.7383


65..........................................................................

***** Information summary *****

Aliased model terms
Wing_Len.Clip
Body_Len.Clip
Body_Wid.Clip
PaperTyp.Fold
Wing_Len.Fold
Body_Len.Fold
Body_Wid.Fold
Clip.Fold
PaperTyp.BodyTape
Wing_Len.BodyTape
Body_Len.BodyTape
Body_Wid.BodyTape
Clip.BodyTape
Fold.BodyTape
PaperTyp.WingTape
Wing_Len.WingTape
Body_Len.WingTape
Body_Wid.WingTape
Clip.WingTape
Fold.WingTape
BodyTape.WingTape


***** Tables of effects *****

Variate: Time[2]

***** Tests stratum *****

PaperTyp Y-effect          -0.91   s.e. *         rep. 8

Wing_Len Y-effect           0.70   s.e. *         rep. 8

Body_Len Y-effect           0.41   s.e. *         rep. 8

Body_Wid Y-effect          -0.20   s.e. *         rep. 8

Clip Y-effect              -0.27   s.e. *         rep. 8

Fold Y-effect              -0.16   s.e. *         rep. 8

BodyTape Y-effect           0.13   s.e. *         rep. 8

WingTape Y-effect           0.21   s.e. *         rep. 8

PaperTyp.Wing_Len Y-effect
                           -0.20   s.e. *         rep. 4

PaperTyp.Body_Len Y-effect
```

```
                           -0.30   s.e. *         rep. 4

Wing_Len.Body_Len Y-effect
                           -0.24   s.e. *         rep. 4

PaperTyp.Body_Wid Y-effect
                           -0.21   s.e. *         rep. 4

Wing_Len.Body_Wid Y-effect
                            0.14   s.e. *         rep. 4

Body_Len.Body_Wid Y-effect
                            0.24   s.e. *         rep. 4

PaperTyp.Clip Y-effect     -0.25   s.e. *         rep. 4

             -
N            I
o            I                                                          *
r     1.3667 I                                                *
m            I
a            I                                         **
l            I                                       *
             I                            *         *
Q            I                           *
u    -0.2733 I                           2
a            I                          *
n            I                         2
t            I
i            I                        *
l            I   *
e    -1.9133 I
             -+---------+---------+---------+---------+---------+---------+-
        -0.9996   -0.7045   -0.4093   -0.1141    0.1810    0.4762    0.7714

                                            effect
```

65..

***** Analysis of variance *****

Variate: Time[2]

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Tests stratum					
PaperTyp	1	3.3033	3.3033	14.52	0.002
Wing_Len	1	1.9670	1.9670	8.65	0.011
Residual	13	2.9573	0.2275		
Total	15	8.2276			

***** Tables of means *****

Variate: Time[2]

Grand mean 3.30

PaperTyp	light	heavy
	3.75	2.84

Wing_Len	7.50	12.00
	2.94	3.65

*** Least significant differences of means (5% level) ***

Table	PaperTyp	Wing_Len
rep.	8	8
d.f.	13	13
l.s.d.	0.515	0.515

```
             -
             I
             I
             I
         1.0 I
r            I
e            I   *                             *
s            I                                                         *
i            I   *
d            I   *                      2      *                       2
u        0.0 I
a            I
l            I                          2                              *
s            I                             *
             I   *
             I                             *
        -1.0 I
             -+---------+---------+---------+---------+---------+---------+---
            2.4       2.7       3.0       3.3       3.6       3.9       4.2

                                    fitted values
```

Normal plot

```
             -
             I
         1.0 I
r            I
e            I                                                   *       *
s            I                                              *
i            I                                           *
d            I                          *  * *  *  * *
u        0.0 I
a            I
l            I                 *   * *
s            I              *
             I         *
             I *
        -1.0 I
             -+---------+---------+---------+---------+---------+---------+---
           -1.8      -1.2      -0.6       0.0       0.6       1.2       1.8

                              expected Normal quantiles
```

65..

***** Analysis of variance (adjusted for covariate) *****

Variate: Time[2]
Covariate: ResSq

Source of variation	d.f.	s.s.	m.s.	v.r.	cov.ef.	F pr.
Tests stratum						
PaperTyp	1	3.3033	3.3033	14.16	1.00	0.003
Wing_Len	1	1.9670	1.9670	8.43	1.00	0.013
Covariate	1	0.1580	0.1580	0.68		0.427
Residual	12	2.7993	0.2333		0.98	
Total	15	8.2276				

```
65...........................................................................

***** Information summary *****

Aliased model terms
Wing_Len.Clip
Body_Len.Clip
Body_Wid.Clip
PaperTyp.Fold
Wing_Len.Fold
Body_Len.Fold
Body_Wid.Fold
Clip.Fold
PaperTyp.BodyTape
Wing_Len.BodyTape
Body_Len.BodyTape
Body_Wid.BodyTape
Clip.BodyTape
Fold.BodyTape
PaperTyp.WingTape
Wing_Len.WingTape
Body_Len.WingTape
Body_Wid.WingTape
Clip.WingTape
Fold.WingTape
BodyTape.WingTape


***** Tables of effects *****

Variate: Time[3]

***** Tests stratum *****

PaperTyp Y-effect          -0.91   s.e. *         rep. 8

Wing_Len Y-effect           0.61   s.e. *         rep. 8

Body_Len Y-effect           0.32   s.e. *         rep. 8

Body_Wid Y-effect          -0.17   s.e. *         rep. 8

Clip Y-effect              -0.32   s.e. *         rep. 8

Fold Y-effect              -0.18   s.e. *         rep. 8

BodyTape Y-effect           0.18   s.e. *         rep. 8

WingTape Y-effect           0.22   s.e. *         rep. 8

PaperTyp.Wing_Len Y-effect
                           -0.12   s.e. *         rep. 4

PaperTyp.Body_Len Y-effect
                           -0.22   s.e. *         rep. 4

Wing_Len.Body_Len Y-effect
                           -0.28   s.e. *         rep. 4

PaperTyp.Body_Wid Y-effect
                           -0.14   s.e. *         rep. 4

Wing_Len.Body_Wid Y-effect
                            0.21   s.e. *         rep. 4

Body_Len.Body_Wid Y-effect
                            0.21   s.e. *         rep. 4

PaperTyp.Clip Y-effect     -0.17   s.e. *         rep. 4
```

```
             -
N            I
o            I                                                          *
r     1.3667 I                                               *
m            I
a            I                                           **
l            I                                           *
             I                                *         *
Q            I                               *
u    -0.2733 I                              2
a            I                             *
n            I                          * *
t            I
i            I                        *
l            I   *
e    -1.9133 I
             -+---------+---------+---------+---------+---------+---------+---------+-
        -0.9955   -0.7171   -0.4386   -0.1602    0.1182    0.3967    0.6751
                                                         effect
```

65...

***** Analysis of variance *****

Variate: Time[3]

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Tests stratum					
PaperTyp	1	3.2761	3.2761	17.12	0.001
Wing_Len	1	1.5068	1.5068	7.87	0.015
Residual	13	2.4880	0.1914		
Total	15	7.2709			

***** Tables of means *****

Variate: Time[3]

Grand mean 3.27

PaperTyp	light	heavy
	3.72	2.81

Wing_Len	7.50	12.00
	2.96	3.57

*** Least significant differences of means (5% level) ***

Table	PaperTyp	Wing_Len
rep.	8	8
d.f.	13	13
Pl.s.d.	0.473	0.473

```
             -
             I
             I
             I
         1.0 I
r            I
e            I                                    *
s            I*
i            I*
d            I                         2          *                        2
u        0.0 I*                                                            *
a            I                         *                                   *
l            I                         *          *
s            I
             I*
             I                                    *
        -1.0 I
             -+---------+---------+---------+---------+---------+---------+---------+---
           2.50      2.75      3.00      3.25      3.50      3.75      4.00
                                         fitted values
```

```
                                          Normal plot

             -
             I
         1.0 I
r            I
e            I                                                           *
s            I                                                   *
i            I                                              *
d            I                               *  *  * *   *
u        0.0 I                          *  *
a            I                     * *
l            I              *  *
s            I
             I         *
             I *
        -1.0 I
             -+---------+---------+---------+---------+---------+---------+---
           -1.8      -1.2      -0.6       0.0       0.6       1.2       1.8

                                   expected Normal quantiles
```

65...

***** Analysis of variance (adjusted for covariate) *****

Variate: Time[3]
Covariate: ResSq

Source of variation	d.f.	s.s.	m.s.	v.r.	cov.ef.	F pr.
Tests stratum						
PaperTyp	1	3.2761	3.2761	16.15	1.00	0.002
Wing_Len	1	1.5068	1.5068	7.43	1.00	0.018
Covariate	1	0.0541	0.0541	0.27		0.615
Residual	12	2.4340	0.2028		0.94	
Total	15	7.2709				

The residuals-versus-fitted-values plot appears to be satisfactory and so the homogeneity of variance assumption seems to be met. The normal probability plots display a roughly straight line pattern and so the normality assumptions appears to be met.

The tables of means from the mean of the two times to be used in summarizing the results of the experiment are as follows:

***** Tables of means *****

Variate: Time[3]

Grand mean 3.27

PaperTyp	light	heavy
	3.72	2.81

Wing_Len	7.50	12.00
	2.96	3.57

*** Least significant differences of means (5% level) ***

Table	PaperTyp	Wing_Len
rep.	8	8
d.f.	13	13
l.s.d.	0.473	0.473

To maximize flight time, use light paper with a wing length of 12 cm. The expected flight time with this combination is:

$$\begin{aligned} E[Y] &= 3.27 - \frac{0.91}{2}x_{\mathrm{P}} + \frac{0.61}{2}x_{\mathrm{W}} \\ &= 3.27 - \frac{0.91}{2}(-1) + \frac{0.61}{2}(1) \\ &= 3.27 + \frac{0.91 + 0.61}{2} \\ &= 4.03 \text{ sec} \end{aligned}$$

It would appear that the variability of the results was relatively low. An estimate of the variability is provided by the $\sqrt{\text{Residual MSq}}$ *from the second analysis. That is,* $s = \sqrt{0.1914} = 0.4375$*. So one can expect repeat runs with the same configuration to differ by as much as 0.44 of a second. This compares favourably with the previous values of s which were 0.40 and 0.31, respectively, for shorter flights.*

X.6 A half-replicate of a 2^6 fractional factorial design was employed in a study to investigate 6 factors related to the constituents that go into the manufacture of icing for cakes. The generator for the design was I = ABCDEF. What is the resolution of the design? How did you determine this resolution? What are the implications of its resolution?

The resolution of the design is VI because this is the length of the one word in the defining relation. This means that main effects will be confounded with five-factor interactions, two-factor interactions with four-factor interactions and three factor interactions with other three factor interactions.

Texture readings were taken for icing produced using each of the 32 treatment combinations in 32 experimental runs. The factors and data are contained in the file *frf6ice.gsh* in the directory *G:\Disciplina\Genstat*.

What is the experimental structure for this experiment?

Structure	Formula
unrandomized	*32* Runs
randomized	*2* A**2* B**2* C**2* D**2* E**2* F

Analyze this data using Genstat. Perform appropriate diagnostic checking. Which treatment combinations are likely to produce the highest texture value for the icing?

```
Genstat 5  Release 4.1  (PC/Windows NT)              12 April 2000 22:06:12
Copyright 1998, Lawes Agricultural Trust (Rothamsted Experimental Station)

                  ______________________________________________

                  Genstat 5 Fourth Edition - (for Windows)
                  Genstat 5 Procedure Library Release PL11
p                  ______________________________________________

   3  "Data taken from File: D:/ANALYSES/LM/MULTIFAC/FRF6ICE.GSH"
   4  DELETE [redefine=yes] Runs,A,B,C,D,E,F,Texture
   5  FACTOR [modify=yes;nvalues=32;levels=32] Runs
   6  READ Runs; frepresentation=ordinal

    Identifier    Values   Missing    Levels
          Runs        32         0        32

   9  FACTOR [modify=yes;nvalues=32;levels=2] A
  10  READ A; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             A        32         0         2

  12  FACTOR [modify=yes;nvalues=32;levels=2] B
  13  READ B; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             B        32         0         2

  15  FACTOR [modify=yes;nvalues=32;levels=2] C
  16  READ C; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             C        32         0         2

  18  FACTOR [modify=yes;nvalues=32;levels=2] D
  19  READ D; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             D        32         0         2

  21  FACTOR [modify=yes;nvalues=32;levels=2] E
  22  READ E; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             E        32         0         2

  24  FACTOR [modify=yes;nvalues=32;levels=2] F
  25  READ F; frepresentation=ordinal

    Identifier    Values   Missing    Levels
             F        32         0         2

  27  VARIATE [nvalues=32] Texture
  28  READ Texture

    Identifier   Minimum      Mean   Maximum    Values   Missing
       Texture     117.0     280.8     400.0        32         0

  31
  32  PRINT Runs,A,B,C,D,E,F,Texture; FIELD=5,6(4),9; DEC=1

 Runs   A   B   C   D   E   F  Texture
    1   1   1   1   1   1   1    233.0
    2   1   1   1   1   2   2    217.0
    3   1   1   1   2   1   2    267.0
    4   1   1   1   2   2   1    317.0
    5   1   1   2   1   1   2    250.0
    6   1   1   2   1   2   1    233.0
    7   1   1   2   2   1   1    333.0
    8   1   1   2   2   2   2    350.0
    9   1   2   1   1   1   2    250.0
```

```
  10   1   2   1   1   2   1    267.0
  11   1   2   1   2   1   1    400.0
  12   1   2   1   2   2   2    267.0
  13   1   2   2   1   1   1    400.0
  14   1   2   2   1   2   2    250.0
  15   1   2   2   2   1   2    283.0
  16   1   2   2   2   2   1    317.0
  17   2   1   1   1   1   2    200.0
  18   2   1   1   1   2   1    350.0
  19   2   1   1   2   1   1    283.0
  20   2   1   1   2   2   2    150.0
  21   2   1   2   1   1   1    400.0
  22   2   1   2   1   2   2    333.0
  23   2   1   2   2   1   2    317.0
  24   2   1   2   2   2   1    383.0
  25   2   2   1   1   1   1    267.0
  26   2   2   1   1   2   2    200.0
  27   2   2   1   2   1   2    150.0
  28   2   2   1   2   2   1    350.0
  29   2   2   2   1   1   2    117.0
  30   2   2   2   1   2   1    367.0
  31   2   2   2   2   1   1    267.0
  32   2   2   2   2   2   2    217.0

 33  BLOCK Runs
 34  TREAT A*B*C*D*E*F
 35  A2PLOT [PRINT=inform,effect; FACTORIAL=6; STRATUM=Runs; METHOD=normal; \
 36          GRAPH=line] Texture


36..............................................................................

***** Information summary *****

Aliased model terms
A.B.F
A.C.F
B.C.F
A.D.F
B.D.F
C.D.F
A.E.F
B.E.F
C.E.F
D.E.F
A.B.C.D
A.B.C.E
A.B.D.E
A.C.D.E
B.C.D.E
A.B.C.F
A.B.D.F
A.C.D.F
B.C.D.F
A.B.E.F
A.C.E.F
B.C.E.F
A.D.E.F
B.D.E.F
C.D.E.F
A.B.C.D.E
A.B.C.D.F
A.B.C.E.F
A.B.D.E.F
A.C.D.E.F
B.C.D.E.F
A.B.C.D.E.F
```

```
***** Tables of effects *****

Variate: Texture

***** Runs stratum *****

A Y-effect                 -17.69   s.e. *         rep. 16
B Y-effect                 -15.44   s.e. *         rep. 16
C Y-effect                  40.56   s.e. *         rep. 16
D Y-effect                  19.81   s.e. *         rep. 16
E Y-effect                   9.44   s.e. *         rep. 16
F Y-effect                 -84.31   s.e. *         rep. 16
A.B Y-effect               -44.69   s.e. *         rep. 8
A.C Y-effect                15.81   s.e. *         rep. 8
B.C Y-effect               -32.19   s.e. *         rep. 8
A.D Y-effect               -34.44   s.e. *         rep. 8
B.D Y-effect                -3.19   s.e. *         rep. 8
C.D Y-effect                -5.19   s.e. *         rep. 8
A.E Y-effect                34.19   s.e. *         rep. 8
B.E Y-effect                 3.19   s.e. *         rep. 8
C.E Y-effect                 0.94   s.e. *         rep. 8
D.E Y-effect                -3.06   s.e. *         rep. 8
A.F Y-effect               -38.56   s.e. *         rep. 8
B.F Y-effect               -28.31   s.e. *         rep. 8
C.F Y-effect                11.44   s.e. *         rep. 8
D.F Y-effect                 3.19   s.e. *         rep. 8
E.F Y-effect                 9.31   s.e. *         rep. 8
A.B.C Y-effect             -23.94   s.e. *         rep. 4
A.B.D Y-effect              26.06   s.e. *         rep. 4
A.C.D Y-effect              11.56   s.e. *         rep. 4
B.C.D Y-effect             -23.94   s.e. *         rep. 4
A.B.E Y-effect              36.44   s.e. *         rep. 4
A.C.E Y-effect               5.19   s.e. *         rep. 4
B.C.E Y-effect               7.44   s.e. *         rep. 4
A.D.E Y-effect             -19.81   s.e. *         rep. 4
B.D.E Y-effect               3.19   s.e. *         rep. 4
C.D.E Y-effect               9.44   s.e. *         rep. 4
```

```
             -
N            I
o            I                                                          *
r      1.614 I                                                        *
m            I                                                    *  *
a            I                                               * *
l            I                                             22
             I                                           ***
Q            I                                       * *3
u     -0.323 I                                  *   **
a            I                              2 **
n            I                          * *
t            I                        **
i            I                     *
l            I    *
e     -2.259 I
             -+---------+---------+---------+---------+---------+---------+-
        -92.744   -69.850   -46.956   -24.063    -1.169    21.725    44.619

                                            effect


   37  "Perform analysis including only significant effects
  -38     and do Residual analysis"
   39  BLOCK Runs
   40  TREAT F
   41  ANOVA [FPROB=Y;  PSE=LSD] Texture


41................................................................................

***** Analysis of variance *****

Variate: Texture

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Runs stratum
F                         1     56869.     56869.   14.46  <.001
Residual                 30    118003.      3933.
Total                    31    174871.


***** Tables of means *****

Variate: Texture

Grand mean  281.

        F       1        2
              323.     239.


*** Least significant differences of means (5% level) ***

Table                    F
rep.                    16
d.f.                    30
l.s.d.                45.3
```

```
42  APLOT METHOD=fit,normal
```

```
             -
             I
             I
             I
       150.0 I
r            I
e            I         2
s            I         *                                                    3
i            I         *                                                    2
d            I         2                                                    2
u        0.0 I         3                                                    3
a            I         2
l            I         2                                                    4
s            I
             I         2                                                    2
             I         *
      -150.0 I
             -+---------+---------+---------+---------+---------+---------+---
          224.0     240.0     256.0     272.0     288.0     304.0     320.0

                                         fitted values
```

```
                                          Normal plot

             -
             I
       150.0 I
r            I
e            I                                                   *    *
s            I                                          ** * *
i            I                                      ***
d            I                                 ** **
u        0.0 I                            **2**
a            I                          **
l            I                  * *****
s            I
             I         *   * * *
             I    *
      -150.0 I
             -+---------+---------+---------+---------+---------+---------+---
           -2.4      -1.6      -0.8       0.0       0.8       1.6       2.4

                                   expected Normal quantiles
```

The normal plot indicates that the only significant effect is F. The diagnostic checking with just this effect fitted reveals no problems.

The table of means for F is:

```
     F        1        2
            323.     239.
```

Level 1 of F will give the highest texture reading.