STATISTICAL MODELLING
PRACTICAL X
SOLUTIONS

X.1 A chemist is planning to run an experiment to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt of cloth to another, the chemist decides to use a randomized complete block design, with bolts of cloth used as blocks.

She wants to be at least 90% sure that she can detect a difference of 4 in the mean tensile strength and knows from previous experiments that a standard deviation of about 1.5 can be expected. Also, she is prepared to take a 5% risk of making a type I error. How many bolts of cloth should she observe?

She wants to use an RCBD with $\alpha = 0.05$, $m = 1$, $\Delta = 4$, $\sigma = 1.5$. The degrees of freedom of the numerator will be 3 and that of the denominator will be given by the expression `df.num*(r-1)`*. So the* `no.reps` *function is used as follows to compute the number of pure replicates required:*

```
> no.reps(multiple=1, df.num=3, df.denom=expression(df.num*(r-1)), delta=4,
  sigma=1.5, power=0.9, print=TRUE)
        rm df.num df.denom alpha delta sigma   lambda powr
1 20.33437      3  58.0031  0.05     4   1.5 72.29998    1
        rm df.num df.denom alpha delta sigma   lambda powr
1 31.66563      3  91.9969  0.05     4   1.5 112.5889    1
        rm df.num df.denom alpha delta sigma   lambda     powr
1 13.33126      3 36.99379  0.05     4   1.5 47.40005 0.999961
        rm df.num df.denom alpha delta sigma   lambda     powr
1 9.003106      3 24.00932  0.05     4   1.5 32.01104 0.996142
        rm df.num df.denom alpha delta sigma   lambda      powr
1 6.328157      3 15.98447  0.05     4   1.5 22.50011 0.9535301
        rm df.num df.denom alpha delta sigma   lambda      powr
1 4.674948      3 11.02484  0.05     4   1.5 16.62204 0.8258093
        rm df.num df.denom alpha delta sigma   lambda      powr
1 6.452931      3 16.35879  0.05     4   1.5 22.94375 0.9582721
        rm df.num df.denom alpha delta sigma   lambda     powr
1 5.721543      3 14.16463  0.05     4   1.5 20.34326 0.922711
        rm df.num df.denom alpha delta sigma   lambda      powr
1 5.321779      3 12.96534  0.05     4   1.5 18.92188 0.8934656
        rm df.num df.denom alpha delta sigma   lambda      powr
1 5.074712      3 12.22414  0.05     4   1.5 18.04342 0.8709092
        rm df.num df.denom alpha delta sigma   lambda      powr
1 5.422337      3 13.26701  0.05     4   1.5 19.27942 0.9016144
        rm df.num df.denom alpha delta sigma   lambda      powr
1 5.455671      3 13.36701  0.05     4   1.5 19.39794 0.9041924
        rm df.num df.denom alpha delta sigma   lambda      powr
1 5.389004      3 13.16701  0.05     4   1.5 19.16090 0.8989759
       rm df.num df.denom alpha delta sigma   lambda      powr
1 5.35567      3 13.06701  0.05     4   1.5 19.04238 0.8962756
        rm df.num df.denom alpha delta sigma   lambda      powr
1 5.389004      3 13.16701  0.05     4   1.5 19.16090 0.8989759
$nreps
[1] 6

$power
[1] 0.9386182
```

The number of replicates required is 6.

X.2 The effect of five different ingredients on the reaction time of a chemical process is to be studied. Each batch of material to be used in the process is only large enough to permit five runs to be made using it and at least five runs can be made in one day. In order to control for batch and day differences, it is decided to use a randomized complete block design in which each batch is used for a day.

The experimenter is intending to have 6 blocks, is willing to run a 5% chance of making a type I error and would like to have a 95% chance of detecting any difference of 6 minutes or more in the reaction time between ingredients. A variance of 3 minutes for the variation in runs on the same day is expected in the experiment. Will the experiment have the desired power?

The experiment will be an RCBD with $\alpha = 0.05$, $m = 1$, $r = rm = 6$, $\Delta = 6$, $\sigma =$ sqrt(3). The degrees of freedom of the numerator will be 4 and that of the denominator will be given by the expression `df.num*(r-1)`*. So the* `power.exp` *function is used as follows to compute the power:*

```
> rm <- 6
>  power.exp(rm=rm,  df.num=4,  df.denom=4*(rm-1),  delta=6,  sigma=sqrt(3),
print=TRUE)
  rm df.num df.denom alpha delta    sigma lambda    powr
1  6      4       20  0.05     6 1.732051     36 0.99533
[1] 0.99533
```

As the computed power is greater than 0.95, the experiment will have the desired power.

X.3 An experimenter wants to investigate the effects of four different rations on the apparent consumption of total carbohydrates (as a percentage) by calves. He has available four calves of around 280 kg. He plans to use a Latin square for two reasons. Firstly, so that each calf receives the four rations, one in each of four periods. Secondly, so that differences, such as climatic differences, between the periods are eliminated from treatment differences. The experimenter is willing to run a 5% chance of making a type I error and would like to have a 95% chance of detecting any difference of 7.5% or more in the apparent consumption between rations. A variance of 10% for the animal-period combinations is expected in the experiment. Will the Latin square have the desired power?

A Latin square is to be used with $\alpha = 0.05$, $m = 1$, rm = 4, $\Delta = 7.5$, $\sigma =$ sqrt(10). The degrees of freedom of the numerator will be `rm-1` *and that of the denominator will be given by the expression (*`rm-1)*(rm-2)`*. So the* `power.exp` *function is used as follows to compute the power that would be achieved:*

```
> rm <- 4
> power.exp(rm=rm, df.num=rm-1, df.denom=(rm-1)*(rm-2), delta=7.5,
+                                                         sigma=sqrt(10))
[1] 0.5155396
```

The power is well below the desired 95%.

X.4 In exercise X.3 you considered the power of an experiment to investigate the effects of four different rations on the apparent consumption of total carbohydrates (as a percentage) by calves. The design used a 4×4 Latin square. The values used in obtaining the power were that the experimenter is willing to run a 5% chance of making a type I error and would like to have a 95% chance of detecting any difference of 7.5% or more in the apparent consumption between rations. A variance of 10% for the animal-period combinations is expected in the experiment. For this design it was computed that the power would be just over 0.5 — not nearly enough!

Suppose the experimenter could obtain another 4 animals and so conduct the experiment with 8 animals over the 4 periods. Would this experiment have at least the level of power desired?

Now a repeated Latin square with the same period but different cows is to be used. Consequently, $\alpha = 0.05$, *m* $=$ *1, rm = 8,* $\Delta = 7.5$, $\sigma =$ *sqrt(10). The degrees of freedom of the numerator and denominator will be 3 and 15, respectively. So the* `power.exp` *function is used as follows to compute the power that would be achieved:*

```
> power.exp(rm=8, df.num=3, df.denom=15, delta=7.5, sigma=sqrt(10))
[1] 0.9501493
```

The power is just above the desired 95%.

X.5 Suppose that an experiment is to be conducted to investigate the effects of four temperatures and three pressures on the yield of a chemical process. It is planned to use a completely randomized design to assign the treatments and it is believed that the population standard deviation is about 0.1.

a) It is desired to be able to detect a difference of at least 0.25 in the overall differences between a pair of Temperatures. How many replicates should be observed if the power is to be 0.90 and the level of significance 0.05?

In this case you are concerned with the Temperature main effect. The required number of replicates is obtained using the R function `no.reps` *as follows:*

```
> no.reps(multiple=3, df.num=3, df.denom=expression(12*(r-1)), delta=0.25,
+                                         sigma=0.1, power=0.9)
$nreps
[1] 3

$power
[1] 0.9906208
```

So 3 replicates is required.

b) What power would be achieved in detecting a difference between a simple and the main Temperature effects of at least 0.25 with the number of replicates that you have computed in a)? How many replicates would be required to detect this latter difference with power 0.90 and significance level 0.05?

We require the power for a difference that is associated with the interaction when three replicates are used. Using `power.exp`*, as shown below, the power is 0.49.*

```
> rm <- 3
> power.exp(rm=rm, df.num=6, df.denom=12*(rm-1), delta=0.25, sigma=0.1)
[1] 0.4886767
```

Using `no.reps` *we find that 7 replicates are required to achieve a power of 0.9 in detecting change of 0.25 in the Temperature difference.*

```
> no.reps(multiple=1, df.num=6, df.denom=expression(12*(r-1)), delta=0.25,
sigma=0.1, power=0.9)
$nreps
[1] 7

$power
[1] 0.9409606
```

c) Use R to obtain a randomized layout for this experiment with the number of replicates you finally computed in b). Use a seed of 312 in generating the design.

The R expressions to generate 7 replicates of a 4 $\times$ 3 factorial in a completely randomized design is as follows:

```
> #
> # CRD
> #
> n <- 84
> fnames <- list(Temperature = 1:4, Pressure = 1:3)
> CRDFac2.Chem.ran <- fac.gen(generate = fnames, times = 7)
> CRDFac2.Chem.unit <- list(Run = n)
> CRDFac2.Chem.lay <- fac.layout(unrandomized = CRDFac2.Chem.unit,
+                                randomized = CRDFac2.Chem.ran,
+                                seed = 312)
> CRDFac2.Chem.lay
  Units Permutation Run Temperature Pressure
1      1          68   1           4        3
2      2          51   2           1        2
3      3          82   3           1        3
4      4          38   4           3        1
5      5          71   5           3        3
6      6          53   6           2        1
7      7          15   7           1        2
8      8          34   8           4        2
9      9          39   9           1        3
10    10          37  10           4        2
11    11          10  11           2        1
12    12          42  12           1        2
13    13          54  13           3        2
14    14           7  14           4        2
15    15          43  15           3        1
16    16           6  16           2        3
17    17          27  17           1        3
18    18          49  18           3        1
19    19           4  19           1        2
```

20	20	20	20	3	2
21	21	80	21	2	2
22	22	83	22	2	1
23	23	14	23	3	3
24	24	1	24	2	2
25	25	77	25	3	2
26	26	12	26	3	2
27	27	17	27	2	2
28	28	57	28	4	2
29	29	24	29	4	3
30	30	32	30	2	2
31	31	65	31	1	2
32	32	25	32	2	3
33	33	70	33	3	2
34	34	76	34	3	2
35	35	8	35	1	3
36	36	52	36	4	3
37	37	69	37	4	1
38	38	2	38	2	1
39	39	3	39	3	3
40	40	22	40	4	2
41	41	21	41	1	1
42	42	16	42	4	3
43	43	74	43	1	3
44	44	13	44	3	3
45	45	23	45	2	3
46	46	64	46	3	3
47	47	28	47	2	3
48	48	36	48	4	1
49	49	56	49	2	3
50	50	66	50	2	2
51	51	35	51	1	2
52	52	78	52	4	3
53	53	60	53	2	3
54	54	47	54	1	1
55	55	62	55	4	3
56	56	33	56	1	1
57	57	5	57	2	1
58	58	75	58	4	2
59	59	40	59	4	3
60	60	29	60	2	2
61	61	41	61	3	2
62	62	19	62	3	1
63	63	84	63	2	3
64	64	73	64	4	1
65	65	50	65	3	1
66	66	45	66	1	2
67	67	67	67	3	1
68	68	61	68	1	1
69	69	46	69	1	1
70	70	72	70	3	3
71	71	58	71	2	2
72	72	55	72	4	1
73	73	81	73	2	1
74	74	31	74	3	1
75	75	9	75	4	1
76	76	11	76	4	1
77	77	30	77	1	1
78	78	63	78	2	1
79	79	18	79	4	2
80	80	26	80	3	3
81	81	44	81	1	1
82	82	48	82	1	3
83	83	79	83	4	1
84	84	59	84	1	3