THE DESIGN AND MIXED-MODEL ANALYSIS OF EXPERIMENTS

PRACTICAL VII SOLUTIONS

VII.1 The following data are from a Latin Square experiment designed to investigate the moisture content of turnip greens. The experiment involved the measurement of the percent moisture content of five leaves of different sizes from each of five plants. The treatments were time of measurement in days since the beginning of the experiment.

		Plant				
		1	2	3	4	5
	A	5 6.67	2 5.40	3 7.32	1 4.92	4 4.88
	B	4 7.15	5 4.77	2 8.53	3 5.00	1 6.16
Leaf Size	C	1 8.29	4 5.40	5 8.50	2 7.29	3 7.83
(A = smallest,	D	3 8.95	1 7.54	4 9.99	5 7.85	2 5.83
E = largest)	E	2 9.62	3 6.93	1 9.68	4 7.08	5 8.51

What are the components of this experiment?

1. Observational unit — a leaf
2. Response variable — Moisture Content
3. Unrandomized factors — Plant, Leaf Size
4. Randomized factors — Time
5. Type of study — Latin Square

What is the experimental structure for this experiment?

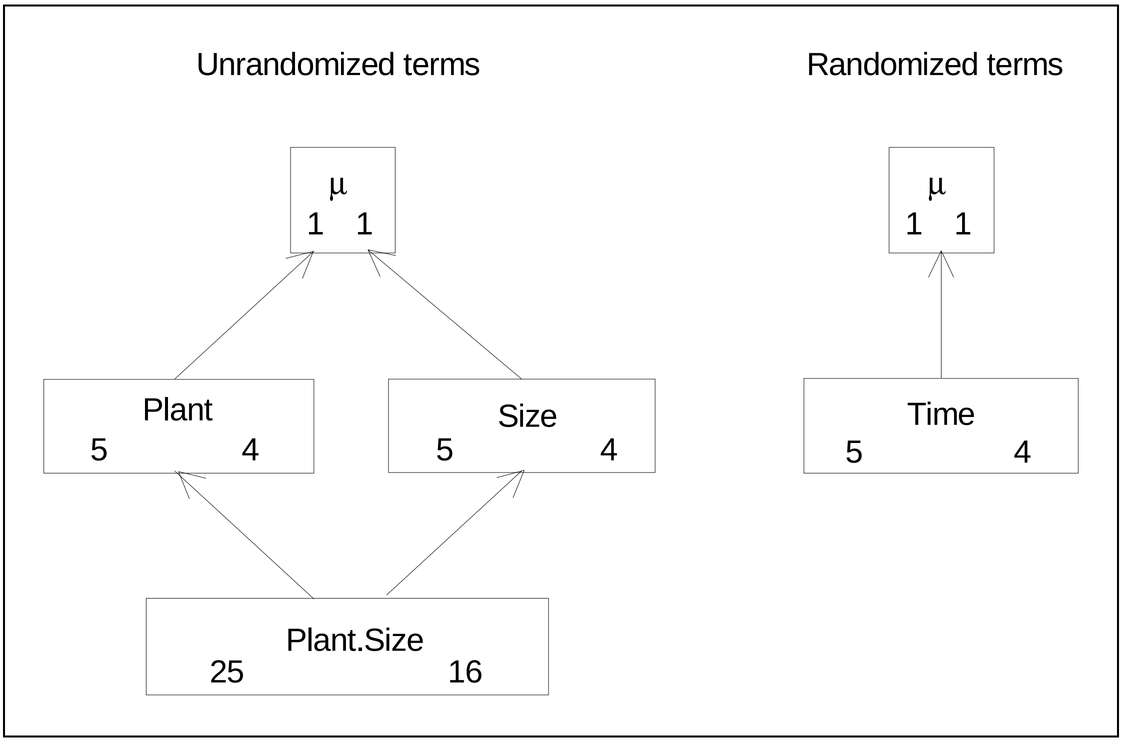
Structure	*Formula*
unrandomized	*5 Plant*5 Size*
randomized	*5 Time*

What are the Hasse diagrams of term marginalities for this study?

*The terms in the analysis are: Plant*Size = Plant + Size + Plant.Size and Time = Time.*

Consequently, the Hasse diagrams are as follows:

Hasse diagrams for the Turnip moisture example



Derive the maximal expectation and variation models for this study?

In this example, Plant is likely to be a random factor and Size and Time to be fixed factors. In particular, Size may well show a systematic trend and this would be modelled used expectation terms.

Hence, the maximal models are: E[Y] = Size + Time and var[Y] = Plant + Plant.Size.

What is the form of the analysis of variance table, including the expected mean squares?

Source	df	E[MSq]
Plant	4	$\sigma^2_{\text{PS}} + 5\sigma^2_{\text{P}}$
Size	4	$\sigma^2_{\text{PS}} + f_{\text{S}}(\psi)$
Plant.Size	16	
Time	4	$\sigma^2_{\text{PS}} + f_{\text{T}}(\psi)$
Residual	12	σ^2_{PS}
Total	24	

$f_{\text{S}}(\psi) = 5\sum\left(\beta_j - \overline{\beta}_{.}\right)^2 \big/ 4$, $f_{\text{T}}(\psi) = 5\sum\left(\tau_k - \overline{\tau}_{.}\right)^2 \big/ 4$

VII.2 A researcher has four different containers of soil. He wants to determine whether the moisture contents of these four soils differs. He randomly selects 10 samples from each container and determines the moisture content of each sample.

What are the components of this experiment?

1.	Observational unit	a sample
2.	Response variable	Moisture content
3.	Unrandomized factors	Soil, Sample
4.	Randomized factors	none
5.	Type of study	SRS

What is the experimental structure for this experiment?

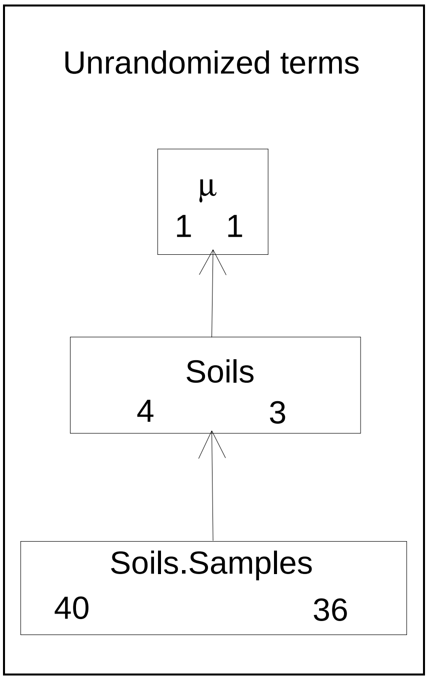
Structure	*Formula*
unrandomized	*4 Soils/10 Samples*
randomized	—

What are the Hasse diagrams of term marginalities for this study?

The terms in the analysis are: Soils/Samples = Soils + Soils.Samples.

Hence, the Hasse diagram is:

Hasse diagram for Soil survey



Derive the maximal expectation and variation models for this study?

In this survey Samples are likely to be random as these samples are meant to be representative of a larger population of samples and one envisages using a probability distribution function to model sample variability. I ti s not clear whether soils are fixed or random. Do we have four soils that difffer in an arbitrary way or are these 4 soils representative of a large group of similar soils? Assume they are fixed.

Based on this the maximal models would be: E[Y] = Soils and var[Y] = Soils.Samples.

What is the form of the analysis of variance table, including the expected mean squares?

Source	df	E[MSq]
Soils	3	$\sigma^2_{SS} + f_S(\psi)$
Soils.Samples	36	σ^2_{SS}
Total	39	

VII.3 Consider an experiment on cabbages to investigate the effects on yield of three sources of nitrogen. Each source was applied to 4 plots as was a control treatment for which no nitrogen was applied. So the layout for the experiment is given in the table below.

	Plots															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Treatment	n	0	n	s	0	c	0	s	c	n	c	s	s	n	0	c
	c = Nitro-chalk; s = Sulphate; n = Nitrate; 0 = Control															

In experiments that involve a control, such as this one, the control has special status in that it is the only treatment that has no nitrogen. It is useful to compare a model that allows for the difference between the control and the other treatments with a model that allows for all treatments to be different. This is done by including two factors for the treatments: one that we might call Nitrogen that has 1 or no for the control treatment and 2 or yes for the other treatments; the second factor is say Source that has four levels corresponding to the four treatments. In determining the analysis of variance table for this example, specify these two factors to be the randomized factors.

What are the components of this experiment?

6. Observational unit — a plot
7. Response variable — Yield
8. Unrandomized factors — Plot
9. Randomized factors — Nitrogen, Source
10. Type of study — CRD

What is the experimental structure for this experiment?

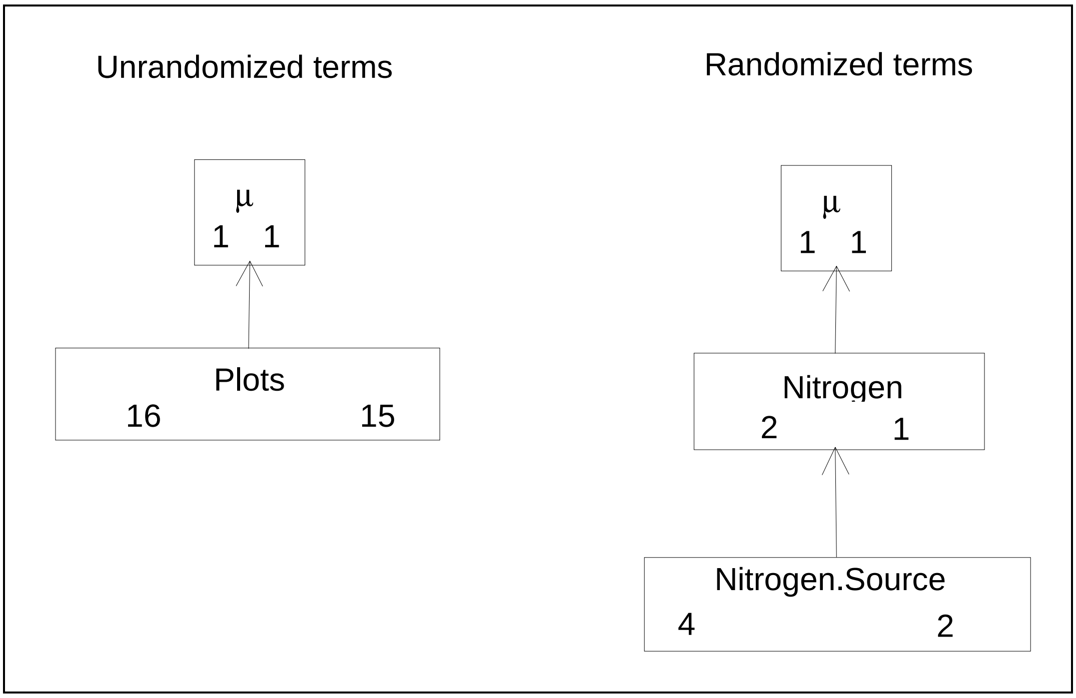
Structure	*Formula*
unrandomized	*16 Plots*
randomized	*2 Nitorgen/4 Source*

What are the Hasse diagrams of term marginalities for this study?

The terms in the analysis are: Plots = Plots and Nitrogen/Source = Nitrogen + Nitrogen.Source.

Hence, the Hasse diagrams are:

Hasse diagrams for Cabbage experiment



Derive the maximal expectation and variation models for this study?

In this experiment Plots must be random; Nitrogen and Source will be fixed.

Based on this the maximal models would be: E[Y] = Nitrogen + Nitrogen.Source and var[Y] = Plots.

What is the form of the analysis of variance table, including the expected mean squares?

Source	df	E[MSq]
Plots	15	
Nitrogen	1	$\sigma_{\mathrm{P}}^2 + f_{\mathrm{N}}(\psi)$
Nitrogen.Source	2	$\sigma_{\mathrm{P}}^2 + f_{\mathrm{N.S}}(\psi)$
Residual	12	σ_{P}^2
Total	19	