

<i>n, i, j, k</i>	Index variables for meta-lists
<i>num</i>	Numeric literals
<i>hex</i>	Bit vector literal, specified by C-style hex number
<i>bin</i>	Bit vector literal, specified by C-style binary number
<i>string</i>	String literals
<i>regexp</i>	Regular expressions, as a string literal
<i>x, y, z</i>	Variables
<i>ix</i>	Variables

l	$::=$ 	Source locations
$x^l, y^l, z^l, name$	$::=$ $x\ l$ $(ix)l$	Location-annotated names Remove infix status
ix^l	$::=$ $ix\ l$ $'x' l$	Location-annotated infix names Add infix status
α	$::=$ $'x$	Type variables
α^l	$::=$ $\alpha\ l$	Location-annotated type variables
N	$::=$ $''x$	numeric variables
N^l	$::=$ $N\ l$	Location-annotated numeric variables
id	$::=$ $x_1^l \dots x_n^l . x^l\ l$	Long identifiers
tnv	$::=$ α N	Union of type variables and Nexp type variables, without location
$tnvar^l$	$::=$ α^l N^l	Union of type variables and Nexp type variables, with location
$tnvs$	$::=$ $tnv_1 .. tnv_n$	Type variable lists
$tnvars^l$	$::=$ $tnvar_1^l .. tnvar_n^l$	Type variable lists
$Nexp_aux$	$::=$ N num $Nexp_1 * Nexp_2$ $Nexp_1 + Nexp_2$ $(Nexp)$	Numerical expressions for specifying vector lengths and indexes

$Nexp$	$::=$ $ \quad Nexp_aux \ l$	Location-annotated vector lengths
$Nexp_constraint$	$::=$ $ \quad Nexp$ $ \quad \geq Nexp$	Whether a vector is bounded or fixed size
typ_aux	$::=$ $ \quad -$ $ \quad \alpha^l$ $ \quad typ_1 \rightarrow typ_2$ $ \quad typ_1 * \dots * typ_n$ $ \quad Nexp$ $ \quad id \ typ_1 .. typ_n$ $ \quad (typ)$	Types Unspecified type Type variables Function types Tuple types As a typ to permit applications over Nexps, otherwise no Type applications
typ	$::=$ $ \quad typ_aux \ l$	Location-annotated types
lit_aux	$::=$ $ \quad \mathbf{true}$ $ \quad \mathbf{false}$ $ \quad num$ $ \quad hex$ $ \quad bin$ $ \quad string$ $ \quad ()$ $ \quad \mathbf{bitzero}$ $ \quad \mathbf{bitone}$	Literal constants hex and bin are constant bit vectors, entered as C-style L bitzero and bitone are constant bits, if commonly used w
lit	$::=$ $ \quad lit_aux \ l$	Location-annotated literal constants
$;\text{?}$	$::=$ $ $ $ \quad ;$	Optional semi-colons
pat_aux	$::=$ $ \quad -$ $ \quad (pat \ \mathbf{as} \ x^l)$ $ \quad (pat : typ)$ $ \quad id \ pat_1 .. pat_n$ $ \quad \langle fpat_1; \dots; fpat_n; ? \rangle$ $ \quad [pat_1; \dots; pat_n; ?]$ $ \quad [pat_1 .. pat_n]$ $ \quad (pat_1, \dots, pat_n)$ $ \quad [pat_1; \dots; pat_n; ?]$	Patterns Wildcards Named patterns Typed patterns Single variable and constructor patterns Record patterns Vector patterns Concatenated vector patterns Tuple patterns List patterns

		(<i>pat</i>)	
		$pat_1 :: pat_2$	Cons patterns
		$x^l + num$	constant addition patterns
		<i>lit</i>	Literal constant patterns
<i>pat</i>	::=		Location-annotated patterns
		<i>pat_aux l</i>	
<i>fpat</i>	::=		Field patterns
		<i>id = pat l</i>	
?	::=		Optional bars
<i>exp_aux</i>	::=		Expressions
		<i>id</i>	Identifiers
		<i>N</i>	Nexp var, has type num
		fun <i>psexp</i>	Curried functions
		function ? <i>pexp</i> ₁ ... <i>pexp</i> _{<i>n</i>} end	Functions with pattern matching
		<i>exp</i> ₁ <i>exp</i> ₂	Function applications
		<i>exp</i> ₁ <i>ix</i> ^{<i>l</i>} <i>exp</i> ₂	Infix applications
		⟨ <i>fexp</i> s ⟩	Records
		⟨ <i>exp with fexp</i> s ⟩	Functional update for records
		<i>exp.id</i>	Field projection for records
		[<i>exp</i> ₁ ; ..; <i>exp</i> _{<i>n</i>} ;?]	Vector instantiation
		<i>exp</i> .(<i>Nexp</i>)	Vector access
		<i>exp</i> .(<i>Nexp</i> ₁ .. <i>Nexp</i> ₂)	Subvector extraction
		match <i>exp with</i> ? <i>pexp</i> ₁ ... <i>pexp</i> _{<i>n</i>} <i>l end</i>	Pattern matching expressions
		(<i>exp</i> : <i>typ</i>)	Type-annotated expressions
		let <i>letbind in exp</i>	Let expressions
		(<i>exp</i> ₁ , ..., <i>exp</i> _{<i>n</i>})	Tuples
		[<i>exp</i> ₁ ; ..; <i>exp</i> _{<i>n</i>} ;?]	Lists
		(<i>exp</i>)	
		begin <i>exp end</i>	Alternate syntax for (<i>exp</i>)
		if <i>exp</i> ₁ then <i>exp</i> ₂ else <i>exp</i> ₃	Conditionals
		<i>exp</i> ₁ :: <i>exp</i> ₂	Cons expressions
		<i>lit</i>	Literal constants
		{ <i>exp</i> ₁ <i>exp</i> ₂ }	Set comprehensions
		{ <i>exp</i> ₁ forall <i>qbind</i> ₁ .. <i>qbind</i> _{<i>n</i>} <i>exp</i> ₂ }	Set comprehensions with explicit binding
		{ <i>exp</i> ₁ ; ..; <i>exp</i> _{<i>n</i>} ;?}	Sets
		<i>q qbind</i> ₁ ... <i>qbind</i> _{<i>n</i>} . <i>exp</i>	Logical quantifications
		[<i>exp</i> ₁ forall <i>qbind</i> ₁ .. <i>qbind</i> _{<i>n</i>} <i>exp</i> ₂]	List comprehensions (all binders must be qua
<i>exp</i>	::=		Location-annotated expressions
		<i>exp_aux l</i>	

q	$::=$ $ $ forall $ $ exists	Quantifiers
$qbind$	$::=$ $ $ x^l $ $ $(pat \textbf{IN} exp)$ $ $ $(pat \textbf{MEM} exp)$	Bindings for quantifiers Restricted quantifications over sets Restricted quantifications over lists
$fexp$	$::=$ $ $ $id = exp \ l$	Field-expressions
$fexps$	$::=$ $ $ $fexp_1; \dots; fexp_n; ? \ l$	Field-expression lists
$pexp$	$::=$ $ $ $pat \rightarrow exp \ l$	Pattern matches
$psexp$	$::=$ $ $ $pat_1 \dots pat_n \rightarrow exp \ l$	Multi-pattern matches
$tannot^?$	$::=$ $ $ $ $ $: typ$	Optional type annotations
$funcl_aux$	$::=$ $ $ $x^l \ pat_1 \dots pat_n \ tannot^? = exp$	Function clauses
$letbind_aux$	$::=$ $ $ $pat \ tannot^? = exp$ $ $ $funcl_aux$	Let bindings Value bindings Function bindings
$letbind$	$::=$ $ $ $letbind_aux \ l$	Location-annotated let bindings
$funcl$	$::=$ $ $ $funcl_aux \ l$	Location-annotated function clauses
$id^?$	$::=$ $ $ $ $ $x^l :$	Optional name for inductively defined relations
$rule_aux$	$::=$ $ $ $id^? \textbf{forall} \ x_1^l .. x_n^l. exp \implies x^l \ exp_1 .. exp_i$	Inductively defined relation clauses
$rule$	$::=$ $ $ $rule_aux \ l$	Location-annotated inductively defined relations

<i>typs</i>	$::=$ $ \quad typ_1 * \dots * typ_n$	Type lists
<i>ctor_def</i>	$::=$ $ \quad x^l \textbf{of} typs$ $ \quad x^l$	Datatype definition clauses S Constant constructors
<i>texp</i>	$::=$ $ \quad typ$ $ \quad \langle x_1^l : typ_1; \dots; x_n^l : typ_n; ? \rangle$ $ \quad ^? ctor_def_1 \dots ctor_def_n$	Type definition bodies Type abbreviations Record types Variant types
<i>name?</i>	$::=$ $ $ $ \quad [name = regexp]$	Optional name specification for variables of defined type
<i>td</i>	$::=$ $ \quad x^l tnvars^l name^? = texp$ $ \quad x^l tnvars^l name^?$	Type definitions Definitions of opaque types
<i>c</i>	$::=$ $ \quad id tnvar^l$	Typeclass constraints
<i>cs</i>	$::=$ $ $ $ \quad c_1, \dots, c_i \Rightarrow$	Typeclass constraint lists Must have > 0 constraints
<i>c_pre</i>	$::=$ $ $ $ \quad \textbf{forall} \, tnvar_1^l \dots tnvar_n^l . cs$	Type and instance scheme prefixes Must have > 0 type variables
<i>typschm</i>	$::=$ $ \quad c_pre \, typ$	Type schemes
<i>instschm</i>	$::=$ $ \quad c_pre(id \, typ)$	Instance schemes
<i>target</i>	$::=$ $ \quad \textbf{hol}$ $ \quad \textbf{isabelle}$ $ \quad \textbf{ocaml}$ $ \quad \textbf{coq}$ $ \quad \textbf{tex}$ $ \quad \textbf{html}$	Backend target names
τ	$::=$ $ \quad \{target_1; \dots; target_n\}$	Backend target name lists

$\tau^?$	$::=$ $ $ $ \quad \tau$	Optional targets
val_def	$::=$ $ \quad \mathbf{let} \tau^? \mathit{letbind}$ $ \quad \mathbf{let} \mathbf{rec} \tau^? \mathit{funcl}_1 \mathbf{and} \dots \mathbf{and} \mathit{funcl}_n$ $ \quad \mathbf{let} \mathbf{inline} \tau^? \mathit{letbind}$	Value definitions Non-recursive value definitions Recursive function definitions Function definitions to be inlined
val_spec	$::=$ $ \quad \mathbf{val} x^l : \mathit{typschm}$	Value type specifications
def_aux	$::=$ $ \quad \mathbf{type} \mathit{td}_1 \mathbf{and} \dots \mathbf{and} \mathit{td}_n$ $ \quad \mathit{val_def}$ $ \quad \mathbf{rename} \tau^? \mathit{id} = x^l$ $ \quad \mathbf{module} x^l = \mathbf{struct} \mathit{defs} \mathbf{end}$ $ \quad \mathbf{module} x^l = \mathit{id}$ $ \quad \mathbf{open} \mathit{id}$ $ \quad \mathbf{indreln} \tau^? \mathit{rule}_1 \mathbf{and} \dots \mathbf{and} \mathit{rule}_n$ $ \quad \mathit{val_spec}$ $ \quad \mathbf{class} (x^l \mathit{tnvar}^l) \mathbf{val} x_1^l : \mathit{typ}_1 l_1 \dots \mathbf{val} x_n^l : \mathit{typ}_n l_n \mathbf{end}$ $ \quad \mathbf{instance} \mathit{instschm} \mathit{val_def}_1 l_1 \dots \mathit{val_def}_n l_n \mathbf{end}$	Top-level definitions Type definitions Value definitions Rename constant or type Module definitions Module renamings Opening modules Inductively defined relations Top-level type constraints Typeclass definitions Typeclass instantiations
def	$::=$ $ \quad \mathit{def_aux} \mathit{l}$	Location-annotated definitions
$;;^?$	$::=$ $ $ $ \quad ;;$	Optional double-semi-colon
$defs$	$::=$ $ \quad \mathit{def}_1 ; ;_1^? \dots \mathit{def}_n ; ;_n^?$	Definition sequences
p	$::=$ $ \quad x_1 \dots x_n.x$ $ \quad \mathbf{_list}$ $ \quad \mathbf{_bool}$ $ \quad \mathbf{_num}$ $ \quad \mathbf{_set}$ $ \quad \mathbf{_string}$ $ \quad \mathbf{_unit}$ $ \quad \mathbf{_bit}$ $ \quad \mathbf{_vector}$	Unique paths
σ	$::=$ $ \quad \{ \mathit{tnv}_1 \mapsto t_1 \dots \mathit{tnv}_n \mapsto t_n \}$	Type variable substitutions

t, u	$::=$		Internal types
		α	
		$t_1 \rightarrow t_2$	
		$t_1 * \dots * t_n$	
		$p \ t_args$	
		ne	
		$\sigma(t)$	M Multiple substitutions
		$\sigma(tnv)$	M Single variable substitution
		curry (t_multi, t)	M Curried, multiple argument functions
ne	$::=$		internal numeric expressions
		N	
		num	
		$num * ne$	
		$ne_1 + ne_2$	
		$(-ne)$	
		normalize (ne)	M
		$ne_1 + \dots + ne_n$	M
		bitlength (bin)	M
		bitlength (hex)	M
		length $(pat_1 \dots pat_n)$	M
		length $(exp_1 \dots exp_n)$	M
t_args	$::=$		Lists of types
		$t_1 .. t_n$	
		$\sigma(t_args)$	M Multiple substitutions
t_multi	$::=$		Lists of types
		$(t_1 * .. * t_n)$	
		$\sigma(t_multi)$	M Multiple substitutions
nec	$::=$		Numeric expression constraints
		$ne \langle nec$	
		$ne = nec$	
		$ne \leq nec$	
		ne	
$names$	$::=$		Sets of names
		$\{x_1, .., x_n\}$	
\mathcal{C}	$::=$		Typeclass constraint lists
		$(p_1 \ tnv_1) .. (p_n \ tnv_n)$	
env_tag	$::=$		Tags for the (non-constructor) value descriptions
		method	Bound to a method
		val	Specified with val
		let	Defined with let or indreln

v_desc	$::=$ $ $ $ $	$\langle \mathbf{forall} \, tnvs.t_multi \rightarrow p, (x \mathbf{of} \, names) \rangle$ $\langle \mathbf{forall} \, tnvs.C \Rightarrow t, env_tag \rangle$	Value descriptions Constructors Values
f_desc	$::=$ $ $	$\langle \mathbf{forall} \, tnvs.p \rightarrow t, (x \mathbf{of} \, names) \rangle$	Fields
Σ^C	$::=$ $ $ $ $	$\{(p_1 \, t_1), \dots, (p_n \, t_n)\}$ $\Sigma_1^C \cup \dots \cup \Sigma_n^C$	Typeclass constraints M
Σ^N	$::=$ $ $ $ $	$\{nec_1, \dots, nec_n\}$ $\Sigma_1^N \cup \dots \cup \Sigma_n^N$	Nexp constraint lists M
E	$::=$ $ $ $ $ $ $	$\langle E^M, E^P, E^F, E^X \rangle$ $E_1 \uplus E_2$ ϵ	Environments M M
E^X	$::=$ $ $ $ $	$\{x_1 \mapsto v_desc_1, \dots, x_n \mapsto v_desc_n\}$ $E_1^X \uplus \dots \uplus E_n^X$	Value environments M
E^F	$::=$ $ $ $ $	$\{x_1 \mapsto f_desc_1, \dots, x_n \mapsto f_desc_n\}$ $E_1^F \uplus \dots \uplus E_n^F$	Field environments M
E^M	$::=$ $ $	$\{x_1 \mapsto E_1, \dots, x_n \mapsto E_n\}$	Module environments
E^P	$::=$ $ $ $ $	$\{x_1 \mapsto p_1, \dots, x_n \mapsto p_n\}$ $E_1^P \uplus \dots \uplus E_n^P$	Path environments M
E^L	$::=$ $ $ $ $	$\{x_1 \mapsto t_1, \dots, x_n \mapsto t_n\}$ $E_1^L \uplus \dots \uplus E_n^L$	Lexical bindings M
tc_abbrev	$::=$ $ $ $ $	$.t$	Type abbreviations
tc_def	$::=$ $ $	$tnvs \, tc_abbrev$	Type and class constructor definitions Type constructors
Δ	$::=$		Type constructor definitions

	$\begin{array}{ l} \{p_1 \mapsto tc_def_1, \dots, p_n \mapsto tc_def_n\} \\ \Delta_1 \uplus \Delta_2 \end{array}$	M	
δ	$\begin{array}{ l} ::= \\ \{p_1 \mapsto xs_1, \dots, p_n \mapsto xs_n\} \\ \delta_1 \uplus \delta_2 \end{array}$	M	Typeclass definitions
$inst$	$\begin{array}{ l} ::= \\ \mathcal{C} \Rightarrow (p \ t) \end{array}$		A typeclass instance, t must not contain nested type
I	$\begin{array}{ l} ::= \\ \{inst_1, \dots, inst_n\} \\ I_1 \cup I_2 \end{array}$	M	Global instances
D	$\begin{array}{ l} ::= \\ \langle \Delta, \delta, I \rangle \\ D_1 \uplus D_2 \\ \epsilon \end{array}$	M M	Global type definition store
xs	$\begin{array}{ l} ::= \\ x_1 \dots x_n \end{array}$		
$terminals$	$\begin{array}{ l} ::= \\ \geq \\ \rightarrow \\ \Rightarrow \\ \langle \\ \rangle \\ \cap \\ \cup \\ \uplus \\ \not\subseteq \\ \subset \\ \neq \\ \emptyset \\ \langle \\ \rangle \\ \vdash \\ , \\ \mapsto \\ \triangleright \\ \rightsquigarrow \\ \Rightarrow \\ - \\ \epsilon \end{array}$	$\begin{array}{ l} \geq \\ \rightarrow \\ \Rightarrow \\ \langle \\ \rangle \\ \cap \\ \cup \\ \uplus \\ \not\subseteq \\ \subset \\ \neq \\ \emptyset \\ \langle \\ \rangle \\ \vdash \\ , \\ \mapsto \\ \triangleright \\ \rightsquigarrow \\ \Rightarrow \\ - \\ \epsilon \end{array}$	
$formula$	$::=$		

	$judgement$	
	$formula_1 \dots formula_n$	
	$E^M(x) \triangleright E$	Module lookup
	$E^P(x) \triangleright p$	Path lookup
	$E^F(x) \triangleright f_desc$	Field lookup
	$E^X(x) \triangleright v_desc$	Value lookup
	$E^L(x) \triangleright t$	Lexical binding lookup
	$\Delta(p) \triangleright tc_def$	Type constructor lookup
	$\delta(p) \triangleright xs$	Type constructor lookup
	$\mathbf{dom}(E_1^M) \cap \mathbf{dom}(E_2^M) = \emptyset$	
	$\mathbf{dom}(E_1^X) \cap \mathbf{dom}(E_2^X) = \emptyset$	
	$\mathbf{dom}(E_1^F) \cap \mathbf{dom}(E_2^F) = \emptyset$	
	$\mathbf{dom}(E_1^P) \cap \mathbf{dom}(E_2^P) = \emptyset$	
	$\mathbf{disjoint\ doms}(E_1^L, \dots, E_n^L)$	Pairwise disjoint domains
	$\mathbf{disjoint\ doms}(E_1^X, \dots, E_n^X)$	Pairwise disjoint domains
	$\mathbf{compatible\ overlap}(x_1 \mapsto t_1, \dots, x_n \mapsto t_n)$	$(x_i = x_j) \implies (t_i = t_j)$
	$\mathbf{duplicates}(tnvs) = \emptyset$	
	$\mathbf{duplicates}(x_1, \dots, x_n) = \emptyset$	
	$x \notin \mathbf{dom}(E^L)$	
	$x \notin \mathbf{dom}(E^X)$	
	$x \notin \mathbf{dom}(E^F)$	
	$p \notin \mathbf{dom}(\delta)$	
	$p \notin \mathbf{dom}(\Delta)$	
	$\mathbf{FV}(t) \subset tnvs$	Free type variables
	$\mathbf{FV}(t_multi) \subset tnvs$	Free type variables
	$\mathbf{FV}(\mathcal{C}) \subset tnvs$	Free type variables
	$inst \mathbf{IN} I$	
	$(p\ t) \notin I$	
	$E_1^L = E_2^L$	
	$E_1^X = E_2^X$	
	$E_1^F = E_2^F$	
	$E_1 = E_2$	
	$\Delta_1 = \Delta_2$	
	$\delta_1 = \delta_2$	
	$I_1 = I_2$	
	$names_1 = names_2$	
	$t_1 = t_2$	
	$\sigma_1 = \sigma_2$	
	$p_1 = p_2$	
	$xs_1 = xs_2$	
	$tnvs_1 = tnvs_2$	
$convert_tnvars$	$::=$	
	$tnvars^l \rightsquigarrow tnvs$	
	$tnvar^l \rightsquigarrow tn timer$	

<i>look_m</i>	$::=$ $E_1(x_1^l \dots x_n^l) \triangleright E_2$	Name path lookup
<i>look_m_id</i>	$::=$ $E_1(id) \triangleright E_2$	Module identifier lookup
<i>look_tc</i>	$::=$ $E(id) \triangleright p$	Path identifier lookup
<i>check_t</i>	$::=$ $\Delta \vdash t \mathbf{ok}$ $\Delta, tnv \vdash t \mathbf{ok}$	Well-formed types Well-formed type/Nexps macro
<i>teq</i>	$::=$ $\Delta \vdash t_1 = t_2$	Type equality
<i>convert_typ</i>	$::=$ $\Delta, E \vdash typ \rightsquigarrow t$ $\vdash Nexp \rightsquigarrow ne$	Convert source types to internal Convert and normalize numbers
<i>convert_typs</i>	$::=$ $\Delta, E \vdash typs \rightsquigarrow t_multi$	
<i>check_lit</i>	$::=$ $\vdash lit : t$	Typing literal constants
<i>inst_field</i>	$::=$ $\Delta, E \vdash \mathbf{field} \ id : p \ t_args \rightarrow t \triangleright (x \mathbf{of} \ names)$	Field typing (also returns context)
<i>inst_ctor</i>	$::=$ $\Delta, E \vdash \mathbf{ctor} \ id : t_multi \rightarrow p \ t_args \triangleright (x \mathbf{of} \ names)$	Data constructor typing (also returns context)
<i>inst_val</i>	$::=$ $\Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^C$	Typing top-level bindings, context
<i>not_ctor</i>	$::=$ $E, E^\perp \vdash x \mathbf{not} \ \mathbf{ctor}$	v is not bound to a data constructor
<i>not_shadowed</i>	$::=$ $E^\perp \vdash id \mathbf{not} \ \mathbf{shadowed}$	id is not lexically shadowed
<i>check_pat</i>	$::=$ $\Delta, E, E_1^\perp \vdash pat : t \triangleright E_2^\perp$ $\Delta, E, E_1^\perp \vdash pat_aux : t \triangleright E_2^\perp$	Typing patterns, building type Typing patterns, building type
<i>id_field</i>	$::=$ $E \vdash id \mathbf{field}$	Check that the identifier is a field

<i>id_value</i>	$::=$ $ \quad E \vdash id \textbf{value}$	Check that the identifier is a
<i>check_exp</i>	$::=$ $ \quad \Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N$ $ \quad \Delta, E, E^L \vdash exp_aux : t \triangleright \Sigma^C, \Sigma^N$ $ \quad \Delta, E, E_1^L \vdash qbind_1 .. qbind_n \triangleright E_2^L, \Sigma^C$ $ \quad \Delta, E, E_1^L \vdash \textbf{list } qbind_1 .. qbind_n \triangleright E_2^L, \Sigma^C$ $ \quad \Delta, E, E^L \vdash func1 \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$ $ \quad \Delta, E, E_1^L \vdash letbind \triangleright E_2^L, \Sigma^C, \Sigma^N$	Typing expressions, collecting Typing expressions, collecting Build the environment for qua Build the environment for qua Build the environment for a f Build the environment for a l
<i>check_rule</i>	$::=$ $ \quad \Delta, E, E^L \vdash rule \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$	Build the environment for an
<i>check_texp_tc</i>	$::=$ $ \quad xs, \Delta_1, E \vdash \textbf{tc } td \triangleright \Delta_2, E^P$	Extract the type constructor i
<i>check_texp_tc</i>	$::=$ $ \quad xs, \Delta_1, E \vdash \textbf{tc } td_1 .. td_i \triangleright \Delta_2, E^P$	Extract the type constructor i
<i>check_texp</i>	$::=$ $ \quad \Delta, E \vdash tnvs p = texp \triangleright \langle E^F, E^X \rangle$	Check a type definition, with
<i>check_texp</i>	$::=$ $ \quad xs, \Delta, E \vdash td_1 .. td_n \triangleright \langle E^F, E^X \rangle$	
<i>convert_class</i>	$::=$ $ \quad \delta, E \vdash id \rightsquigarrow p$	Lookup a type class
<i>solve_class_constraint</i>	$::=$ $ \quad I \vdash (p \ t) \textbf{IN } \mathcal{C}$	Solve class constraint
<i>solve_class_constraints</i>	$::=$ $ \quad I \vdash \Sigma^C \triangleright \mathcal{C}$	Solve class constraints
<i>check_val_def</i>	$::=$ $ \quad \Delta, I, E \vdash val_def \triangleright E^X$	Check a value definition
<i>check_t_instance</i>	$::=$ $ \quad \Delta, (\alpha_1, .., \alpha_n) \vdash t \textbf{instance}$	Check that t be a typeclass in
<i>check_defs</i>	$::=$ $ \quad \overline{z}_j^j, D_1, E_1 \vdash def \triangleright D_2, E_2$ $ \quad \overline{z}_j^j, D_1, E_1 \vdash defs \triangleright D_2, E_2$	Check a definition Check definitions, given modu
<i>judgement</i>	$::=$ $ \quad convert_tnvars$	

		<i>look_m</i>
		<i>look_m_id</i>
		<i>look_tc</i>
		<i>check_t</i>
		<i>teq</i>
		<i>convert_typ</i>
		<i>convert_typs</i>
		<i>check_lit</i>
		<i>inst_field</i>
		<i>inst_ctor</i>
		<i>inst_val</i>
		<i>not_ctor</i>
		<i>not_shadowed</i>
		<i>check_pat</i>
		<i>id_field</i>
		<i>id_value</i>
		<i>check_exp</i>
		<i>check_rule</i>
		<i>check_texp_tc</i>
		<i>check_texprs_tc</i>
		<i>check_texp</i>
		<i>check_texprs</i>
		<i>convert_class</i>
		<i>solve_class_constraint</i>
		<i>solve_class_constraints</i>
		<i>check_val_def</i>
		<i>check_t_instance</i>
		<i>check_defs</i>
<i>user_syntax</i>	::=	
		<i>n</i>
		<i>num</i>
		<i>hex</i>
		<i>bin</i>
		<i>string</i>
		<i>regexp</i>
		<i>x</i>
		<i>ix</i>
		<i>l</i>
		x^l
		ix^l
		α
		α^l
		<i>N</i>
		N^l
		<i>id</i>

	<i>tnv</i>
	<i>tnvar^l</i>
	<i>tnvs</i>
	<i>tnvars^l</i>
	<i>Nexp_aux</i>
	<i>Nexp</i>
	<i>Nexp_constraint</i>
	<i>typ_aux</i>
	<i>typ</i>
	<i>lit_aux</i>
	<i>lit</i>
	<i>.?</i>
	<i>;</i>
	<i>pat_aux</i>
	<i>pat</i>
	<i>fpat</i>
	<i> ?</i>
	<i>exp_aux</i>
	<i>exp</i>
	<i>q</i>
	<i>qbind</i>
	<i>fexp</i>
	<i>fexps</i>
	<i>pexp</i>
	<i>psexp</i>
	<i>tannot?</i>
	<i>funcl_aux</i>
	<i>letbind_aux</i>
	<i>letbind</i>
	<i>funcl</i>
	<i>id?</i>
	<i>rule_aux</i>
	<i>rule</i>
	<i>typs</i>
	<i>ctor_def</i>
	<i>texp</i>
	<i>name?</i>
	<i>td</i>
	<i>c</i>
	<i>cs</i>
	<i>c_pre</i>
	<i>typschm</i>
	<i>instschm</i>
	<i>target</i>
	τ
	$\tau?$
	<i>val_def</i>

val_spec
 def_aux
 def
 $;;^?$
 $defs$
 p
 σ
 t
 ne
 t_args
 t_multi
 nec
 $names$
 \mathcal{C}
 env_tag
 v_desc
 f_desc
 $\Sigma^{\mathcal{C}}$
 $\Sigma^{\mathcal{N}}$
 E
 $E^{\mathbf{x}}$
 $E^{\mathbf{F}}$
 $E^{\mathbf{M}}$
 $E^{\mathbf{P}}$
 $E^{\mathbf{L}}$
 tc_abbrev
 tc_def
 Δ
 δ
 $inst$
 I
 D
 xs
 $terminals$
 $formula$

$$\boxed{tnvars^l \rightsquigarrow tnvs}$$

$$\frac{tnvar_1^l \rightsquigarrow tn timer_1 \quad .. \quad tnvar_n^l \rightsquigarrow tn timer_n}{tnvar_1^l .. tn timer_n^l \rightsquigarrow tn timer_1 .. tn timer_n} \quad \text{CONVERT_TNVARS_NONE}$$

$$\boxed{tnvar^l \rightsquigarrow tn timer}$$

$$\overline{\alpha \, l \rightsquigarrow \alpha} \quad \text{CONVERT_TNVAR_A}$$

$$\overline{N \, l \rightsquigarrow N} \quad \text{CONVERT_TNVAR_N}$$

$$\boxed{E_1(x_1^l .. x_n^l) \triangleright E_2}$$

Name path lookup

$$\overline{E() \triangleright E} \quad \text{LOOK_M_NONE}$$

$$\frac{\begin{array}{c} E^M(x) \triangleright E_1 \\ E_1(\overline{y_i^l}^i) \triangleright E_2 \end{array}}{\langle E^M, E^P, E^F, E^X \rangle(x \ l \ \overline{y_i^l}^i) \triangleright E_2} \text{ LOOK_M_SOME}$$

$E_1(id) \triangleright E_2$ Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i \ x \ l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i \ x \ l_1 \ l_2) \triangleright E_2} \text{ LOOK_M_ID_ALL}$$

$E(id) \triangleright p$ Path identifier lookup

$$\frac{\begin{array}{c} E(\overline{y_i^l}^i) \triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^P(x) \triangleright p \end{array}}{E(\overline{y_i^l}^i \ x \ l_1 \ l_2) \triangleright p} \text{ LOOK_TC_ALL}$$

$\Delta \vdash t \text{ ok}$ Well-formed types

$$\overline{\Delta \vdash \alpha \text{ ok}} \quad \text{CHECK_T_VAR}$$

$$\frac{\begin{array}{c} \Delta \vdash t_1 \text{ ok} \\ \Delta \vdash t_2 \text{ ok} \end{array}}{\Delta \vdash t_1 \rightarrow t_2 \text{ ok}} \quad \text{CHECK_T_FN}$$

$$\frac{\Delta \vdash t_1 \text{ ok} \quad \dots \quad \Delta \vdash t_n \text{ ok}}{\Delta \vdash t_1 * \dots * t_n \text{ ok}} \quad \text{CHECK_T_TUP}$$

$$\frac{\begin{array}{c} \Delta(p) \triangleright tnv_1 .. tnv_n \text{ tc_abbrev} \\ \Delta, tnv_1 \vdash t_1 \text{ ok} \quad \dots \quad \Delta, tnv_n \vdash t_n \text{ ok} \end{array}}{\Delta \vdash p \ t_1 .. t_n \text{ ok}} \quad \text{CHECK_T_APP}$$

$\Delta, tnv \vdash t \text{ ok}$ Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \text{ ok}}{\Delta, \alpha \vdash t \text{ ok}} \quad \text{CHECK_TLEN_T}$$

$$\overline{\Delta, N \vdash ne \text{ ok}} \quad \text{CHECK_TLEN_LEN}$$

$\Delta \vdash t_1 = t_2$ Type equality

$$\frac{\Delta \vdash t \text{ ok}}{\Delta \vdash t = t} \quad \text{TEQ_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ_SYM}$$

$$\frac{\begin{array}{c} \Delta \vdash t_1 = t_2 \\ \Delta \vdash t_2 = t_3 \end{array}}{\Delta \vdash t_1 = t_3} \quad \text{TEQ_TRANS}$$

$$\frac{\begin{array}{c} \Delta \vdash t_1 = t_3 \\ \Delta \vdash t_2 = t_4 \end{array}}{\Delta \vdash t_1 \rightarrow t_2 = t_3 \rightarrow t_4} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n}{\Delta \vdash t_1 * \dots * t_n = u_1 * \dots * u_n} \quad \text{TEQ_TUP}$$

$$\frac{\Delta(p) \triangleright \alpha_1 .. \alpha_n \quad \Delta \vdash t_1 = u_1 \quad .. \quad \Delta \vdash t_n = u_n}{\Delta \vdash p \ t_1 .. t_n = p \ u_1 .. u_n} \text{TEQ_APP}$$

$$\frac{\Delta(p) \triangleright \alpha_1 .. \alpha_n . u}{\Delta \vdash p \ t_1 .. t_n = \{\alpha_1 \mapsto t_1 .. \alpha_n \mapsto t_n\}(u)} \text{TEQ_EXPAND}$$

$$\frac{ne = \mathbf{normalize}(ne')}{\Delta \vdash ne = ne'} \text{TEQ_NEXP}$$

$\boxed{\Delta, E \vdash typ \rightsquigarrow t}$ Convert source types to internal types

$$\overline{\Delta, E \vdash \alpha \ l' \ l \rightsquigarrow \alpha} \text{CONVERT_TYP_VAR}$$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad \Delta, E \vdash typ_2 \rightsquigarrow t_2}{\Delta, E \vdash typ_1 \rightarrow typ_2 \ l \rightsquigarrow t_1 \rightarrow t_2} \text{CONVERT_TYP_FN}$$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad \quad \Delta, E \vdash typ_n \rightsquigarrow t_n}{\Delta, E \vdash typ_1 * * typ_n \ l \rightsquigarrow t_1 * * t_n} \text{CONVERT_TYP_TUP}$$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad .. \quad \Delta, E \vdash typ_n \rightsquigarrow t_n \quad E(id) \triangleright p \quad \Delta(p) \triangleright \alpha_1 .. \alpha_n \ tc_abbrev}{\Delta, E \vdash id \ typ_1 .. typ_n \ l \rightsquigarrow p \ t_1 .. t_n} \text{CONVERT_TYP_APP}$$

$$\frac{\vdash Nexp \rightsquigarrow ne}{\Delta, E \vdash Nexp \rightsquigarrow ne} \text{CONVERT_TYP_NEXP}$$

$$\frac{\Delta, E \vdash typ \rightsquigarrow t}{\Delta, E \vdash (typ) \ l \rightsquigarrow t} \text{CONVERT_TYP_PAREN}$$

$$\frac{\Delta, E \vdash typ \rightsquigarrow t_1 \quad \Delta \vdash t_1 = t_2}{\Delta, E \vdash typ \rightsquigarrow t_2} \text{CONVERT_TYP_EQ}$$

$\boxed{\vdash Nexp \rightsquigarrow ne}$ Convert and normalize numeric expressions

$$\overline{\vdash N \ l \rightsquigarrow N} \text{CONVERT_NEXP_VAR}$$

$$\overline{\vdash num \rightsquigarrow num} \text{CONVERT_NEXP_NUM}$$

$$\overline{\vdash num * N \rightsquigarrow num * N} \text{CONVERT_NEXP_MULT}$$

$$\frac{\vdash Nexp_1 \rightsquigarrow ne_1 \quad \vdash Nexp_2 \rightsquigarrow ne_2}{\vdash Nexp_1 + Nexp_2 \rightsquigarrow ne_1 + ne_2} \text{CONVERT_NEXP_ADD}$$

$\boxed{\Delta, E \vdash typs \rightsquigarrow t_multi}$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad .. \quad \Delta, E \vdash typ_n \rightsquigarrow t_n}{\Delta, E \vdash typ_1 * .. * typ_n \rightsquigarrow (t_1 * .. * t_n)} \text{CONVERT_TYP_ALL}$$

$\boxed{\vdash lit : t}$ Typing literal constants

$$\overline{\vdash \mathbf{true} \ l : _ \mathbf{bool}} \text{CHECK_LIT_TRUE}$$

$\frac{}{\vdash \text{false } l : \text{_bool}}$	CHECK_LIT_FALSE
$\frac{}{\vdash \text{num } l : \text{_num}}$	CHECK_LIT_NUM
$\frac{\text{num} = \text{bitlength}(\text{hex})}{\vdash \text{hex } l : \text{_vector num _bit}}$	CHECK_LIT_HEX
$\frac{\text{num} = \text{bitlength}(\text{bin})}{\vdash \text{bin } l : \text{_vector num _bit}}$	CHECK_LIT_BIN
$\frac{}{\vdash \text{string } l : \text{_string}}$	CHECK_LIT_STRING
$\frac{}{\vdash () l : \text{_unit}}$	CHECK_LIT_UNIT
$\frac{}{\vdash \text{bitzero } l : \text{_bit}}$	CHECK_LIT_BITZERO
$\frac{}{\vdash \text{bitone } l : \text{_bit}}$	CHECK_LIT_BITONE
$\boxed{\Delta, E \vdash \text{field } id : p \ t_args \rightarrow t \triangleright (x \text{ of names})}$	Field typing (also returns canonical field names)
$ \begin{array}{l} E(\overline{x_i^l}^i) \triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^F(y) \triangleright \langle \text{forall } tnv_1 .. tnv_n.p \rightarrow t, (z \text{ of names}) \rangle \\ \Delta \vdash t_1 \text{ ok} \quad \dots \quad \Delta \vdash t_n \text{ ok} \end{array} $	
$\Delta, E \vdash \text{field } \overline{x_i^l}^i \ y \ l_1 \ l_2 : p \ t_1 .. t_n \rightarrow \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}(t) \triangleright (z \text{ of names})$	INST_FIELD_ALL
$\boxed{\Delta, E \vdash \text{ctor } id : t_multi \rightarrow p \ t_args \triangleright (x \text{ of names})}$	Data constructor typing (also returns canonical constructors)
$ \begin{array}{l} E(\overline{x_i^l}^i) \triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^X(y) \triangleright \langle \text{forall } tnv_1 .. tnv_n.t_multi \rightarrow p, (z \text{ of names}) \rangle \\ \Delta \vdash t_1 \text{ ok} \quad \dots \quad \Delta \vdash t_n \text{ ok} \end{array} $	
$\Delta, E \vdash \text{ctor } \overline{x_i^l}^i \ y \ l_1 \ l_2 : \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}(t_multi) \rightarrow p \ t_1 .. t_n \triangleright (z \text{ of names})$	INST_CTOR_ALL
$\boxed{\Delta, E \vdash \text{val } id : t \triangleright \Sigma^C}$	Typing top-level bindings, collecting typeclass constraints
$ \begin{array}{l} E(\overline{x_i^l}^i) \triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^X(y) \triangleright \langle \text{forall } tnv_1 .. tnv_n.(p_1 \ tnv'_1) .. (p_i \ tnv'_i) \Rightarrow t, \text{env_tag} \rangle \\ \Delta \vdash t_1 \text{ ok} \quad \dots \quad \Delta \vdash t_n \text{ ok} \\ \sigma = \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\} \end{array} $	
$\Delta, E \vdash \text{val } \overline{x_i^l}^i \ y \ l_1 \ l_2 : \sigma(t) \triangleright \{(p_1 \ \sigma(tnv'_1)), \dots, (p_i \ \sigma(tnv'_i))\}$	INST_VAL_ALL
$\boxed{E, E^L \vdash x \text{ not ctor}}$	v is not bound to a data constructor
$\frac{E^L(x) \triangleright t}{E, E^L \vdash x \text{ not ctor}}$	NOT_CTOR_VAL
$\frac{x \notin \text{dom}(E^X)}{\langle E^M, E^P, E^F, E^X \rangle, E^L \vdash x \text{ not ctor}}$	NOT_CTOR_UNBOUND
$\frac{E^X(x) \triangleright \langle \text{forall } tnv_1 .. tnv_n.(p_1 \ tnv'_1) .. (p_i \ tnv'_i) \Rightarrow t, \text{env_tag} \rangle}{\langle E^M, E^P, E^F, E^X \rangle, E^L \vdash x \text{ not ctor}}$	NOT_CTOR_BOUND
$\boxed{E^L \vdash id \text{ not shadowed}}$	id is not lexically shadowed
$\frac{x \notin \text{dom}(E^L)}{E^L \vdash x \ l_1 \ l_2 \text{ not shadowed}}$	NOT_SHADOWED_SING

$\overline{E^L \vdash x_1^l \dots x_n^l . y^l . z^l l \text{ not shadowed}}$	NOT_SHADOWED_MULTI
$\boxed{\Delta, E, E_1^L \vdash pat : t \triangleright E_2^L}$	Typing patterns, building their binding environment
$\frac{\Delta, E, E_1^L \vdash pat_aux : t \triangleright E_2^L}{\Delta, E, E_1^L \vdash pat_aux l : t \triangleright E_2^L}$	CHECK_PAT_ALL
$\boxed{\Delta, E, E_1^L \vdash pat_aux : t \triangleright E_2^L}$	Typing patterns, building their binding environment
$\frac{\Delta \vdash t \text{ ok}}{\Delta, E, E^L \vdash _ : t \triangleright \{ \}}$	CHECK_PAT_AUX_WILD
$\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_2^L \quad x \notin \mathbf{dom}(E_2^L)}{\Delta, E, E_1^L \vdash (pat \text{ as } x l) : t \triangleright E_2^L \uplus \{x \mapsto t\}}$	CHECK_PAT_AUX_AS
$\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_2^L \quad \Delta, E \vdash typ \rightsquigarrow t}{\Delta, E, E_1^L \vdash (pat : typ) : t \triangleright E_2^L}$	CHECK_PAT_AUX_TYP
$\frac{\Delta, E \vdash \mathbf{ctor} id : (t_1 * \dots * t_n) \rightarrow p \ t_args \triangleright (x \text{ of } names) \quad E^L \vdash id \text{ not shadowed} \quad \Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L)}{\Delta, E, E^L \vdash id \ pat_1 \dots pat_n : p \ t_args \triangleright E_1^L \uplus \dots \uplus E_n^L}$	CHECK_PAT_AUX_IDENT_CONSTR
$\frac{\Delta \vdash t \text{ ok} \quad E, E^L \vdash x \text{ not ctor}}{\Delta, E, E^L \vdash x \ l_1 \ l_2 : t \triangleright \{x \mapsto t\}}$	CHECK_PAT_AUX_VAR
$\frac{\Delta, E \vdash \mathbf{field} id_i : p \ t_args \rightarrow t_i \triangleright (x_i \text{ of } names)^i \quad \Delta, E, E^L \vdash pat_i : t_i \triangleright E_i^L \quad \mathbf{disjoint doms}(\overline{E_i^L}^i) \quad \mathbf{duplicates}(\overline{x_i}^i) = \emptyset}{\Delta, E, E^L \vdash \langle id_i = pat_i \ l_i^i ; ? \rangle : p \ t_args \triangleright \uplus \overline{E_i^L}^i}$	CHECK_PAT_AUX_RECORD
$\frac{\Delta, E, E^L \vdash pat_1 : t \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t \triangleright E_n^L \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L) \quad \mathbf{length}(pat_1 \dots pat_n) = num}{\Delta, E, E^L \vdash [pat_1 ; \dots ; pat_n] : _ \mathbf{vector} \ num \ t \triangleright E_1^L \uplus \dots \uplus E_n^L}$	CHECK_PAT_AUX_VECTOR
$\frac{\Delta, E, E^L \vdash pat_1 : _ \mathbf{vector} \ ne_1 \ t \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : _ \mathbf{vector} \ ne_n \ t \triangleright E_n^L \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L) \quad ne' = ne_1 + \dots + ne_n}{\Delta, E, E^L \vdash [pat_1 \dots pat_n] : _ \mathbf{vector} \ ne' \ t \triangleright E_1^L \uplus \dots \uplus E_n^L}$	CHECK_PAT_AUX_VECTOR
$\frac{\Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L)}{\Delta, E, E^L \vdash (pat_1, \dots, pat_n) : t_1 * \dots * t_n \triangleright E_1^L \uplus \dots \uplus E_n^L}$	CHECK_PAT_AUX_TUP
$\frac{\Delta \vdash t \text{ ok} \quad \Delta, E, E^L \vdash pat_1 : t \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t \triangleright E_n^L \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L)}{\Delta, E, E^L \vdash [pat_1 ; \dots ; pat_n ; ?] : _ \mathbf{list} \ t \triangleright E_1^L \uplus \dots \uplus E_n^L}$	CHECK_PAT_AUX_LIST

$$\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_2^L}{\Delta, E, E_1^L \vdash (pat) : t \triangleright E_2^L} \text{ CHECK_PAT_AUX_PAREN}$$

$$\frac{\Delta, E, E_1^L \vdash pat_1 : t \triangleright E_2^L \quad \Delta, E, E_1^L \vdash pat_2 : _list t \triangleright E_3^L \quad \mathbf{disjoint\ doms}(E_2^L, E_3^L)}{\Delta, E, E_1^L \vdash pat_1 :: pat_2 : _list t \triangleright E_2^L \uplus E_3^L} \text{ CHECK_PAT_AUX_CONS}$$

$$\frac{\vdash lit : t}{\Delta, E, E^L \vdash lit : t \triangleright \{ \}} \text{ CHECK_PAT_AUX_LIT}$$

$$\frac{E, E^L \vdash x \mathbf{not\ ctor}}{\Delta, E, E^L \vdash x l + num : _num \triangleright \{ x \mapsto _num \}} \text{ CHECK_PAT_AUX_NUM_ADD}$$

$E \vdash id \mathbf{field}$ Check that the identifier is a permissible field identifier

$$\frac{E^F(x) \triangleright f_desc}{\langle E^M, E^P, E^F, E^X \rangle \vdash x l_1 l_2 \mathbf{field}} \text{ ID_FIELD_EMPTY}$$

$$\frac{E^M(x) \triangleright E \quad x \notin \mathbf{dom}(E^F) \quad E \vdash \overline{y_i^l}^i z^l l_2 \mathbf{field}}{\langle E^M, E^P, E^F, E^X \rangle \vdash x l_1. \overline{y_i^l}^i z^l l_2 \mathbf{field}} \text{ ID_FIELD_CONS}$$

$E \vdash id \mathbf{value}$ Check that the identifier is a permissible value identifier

$$\frac{E^X(x) \triangleright v_desc}{\langle E^M, E^P, E^F, E^X \rangle \vdash x l_1 l_2 \mathbf{value}} \text{ ID_VALUE_EMPTY}$$

$$\frac{E^M(x) \triangleright E \quad x \notin \mathbf{dom}(E^X) \quad E \vdash \overline{y_i^l}^i z^l l_2 \mathbf{value}}{\langle E^M, E^P, E^F, E^X \rangle \vdash x l_1. \overline{y_i^l}^i z^l l_2 \mathbf{value}} \text{ ID_VALUE_CONS}$$

$\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N$ Typing expressions, collecting typeclass and index constraints

$$\frac{\Delta, E, E^L \vdash exp_aux : t \triangleright \Sigma^C, \Sigma^N}{\Delta, E, E^L \vdash exp_aux l : t \triangleright \Sigma^C, \Sigma^N} \text{ CHECK_EXP_ALL}$$

$\Delta, E, E^L \vdash exp_aux : t \triangleright \Sigma^C, \Sigma^N$ Typing expressions, collecting typeclass and index constraints

$$\frac{E^L(x) \triangleright t}{\Delta, E, E^L \vdash x l_1 l_2 : t \triangleright \{ \}, \{ \}} \text{ CHECK_EXP_AUX_VAR}$$

$$\frac{}{\Delta, E, E^L \vdash N : num \triangleright \{ \}, \{ \}} \text{ CHECK_EXP_AUX_NVAR}$$

$E^L \vdash id \mathbf{not\ shadowed}$

$E \vdash id \mathbf{value}$

$$\frac{\Delta, E \vdash \mathbf{ctor} id : t_multi \rightarrow p t_args \triangleright (x \mathbf{of\ names})}{\Delta, E, E^L \vdash id : \mathbf{curry}(t_multi, p t_args) \triangleright \{ \}, \{ \}} \text{ CHECK_EXP_AUX_CTOR}$$

$E^L \vdash id \mathbf{not\ shadowed}$

$E \vdash id \mathbf{value}$

$$\frac{\Delta, E \vdash \mathbf{val} id : t \triangleright \Sigma^C}{\Delta, E, E^L \vdash id : t \triangleright \Sigma^C, \{ \}} \text{ CHECK_EXP_AUX_VAL}$$

$$\begin{array}{c}
\Delta, E, E^\perp \vdash pat_1 : t_1 \triangleright E_1^\perp \quad \dots \quad \Delta, E, E^\perp \vdash pat_n : t_n \triangleright E_n^\perp \\
\Delta, E, E^\perp \uplus E_1^\perp \uplus \dots \uplus E_n^\perp \vdash exp : u \triangleright \Sigma^C, \Sigma^N \\
\text{disjoint doms}(E_1^\perp, \dots, E_n^\perp) \\
\hline
\Delta, E, E^\perp \vdash \mathbf{fun} pat_1 \dots pat_n \rightarrow exp \, l : \mathbf{curry}((t_1 * \dots * t_n), u) \triangleright \Sigma^C, \Sigma^N \quad \text{CHECK_EXP_AUX_FN} \\
\\
\frac{\Delta, E, E^\perp \vdash pat_i : t \triangleright E_i^\perp}{\Delta, E, E^\perp \uplus E_i^\perp \vdash exp_i : u \triangleright \Sigma_i^C, \Sigma_i^N} \\
\hline
\Delta, E, E^\perp \vdash \mathbf{function} \, |^? pat_i \rightarrow exp_i \, \bar{l}_i^i \mathbf{end} : t \rightarrow u \triangleright \overline{\Sigma_i^C}^i, \overline{\Sigma_i^N}^i \quad \text{CHECK_EXP_AUX_FUNCTION} \\
\\
\frac{\Delta, E, E^\perp \vdash exp_1 : t_1 \rightarrow t_2 \triangleright \Sigma_1^C, \Sigma_1^N \quad \Delta, E, E^\perp \vdash exp_2 : t_1 \triangleright \Sigma_2^C, \Sigma_2^N}{\Delta, E, E^\perp \vdash exp_1 exp_2 : t_2 \triangleright \Sigma_1^C \cup \Sigma_2^C, \Sigma_1^N \cup \Sigma_2^N} \quad \text{CHECK_EXP_AUX_APP} \\
\\
\frac{\Delta, E, E^\perp \vdash (ix) : t_1 \rightarrow t_2 \rightarrow t_3 \triangleright \Sigma_1^C, \Sigma_1^N \quad \Delta, E, E^\perp \vdash exp_1 : t_1 \triangleright \Sigma_2^C, \Sigma_2^N \quad \Delta, E, E^\perp \vdash exp_2 : t_2 \triangleright \Sigma_3^C, \Sigma_3^N}{\Delta, E, E^\perp \vdash exp_1 ix \, l exp_2 : t_3 \triangleright \Sigma_1^C \cup \Sigma_2^C \cup \Sigma_3^C, \Sigma_1^N \cup \Sigma_2^N \cup \Sigma_3^N} \quad \text{CHECK_EXP_AUX_INFIX_APP1} \\
\\
\frac{\Delta, E, E^\perp \vdash x : t_1 \rightarrow t_2 \rightarrow t_3 \triangleright \Sigma_1^C, \Sigma_1^N \quad \Delta, E, E^\perp \vdash exp_1 : t_1 \triangleright \Sigma_2^C, \Sigma_2^N \quad \Delta, E, E^\perp \vdash exp_2 : t_2 \triangleright \Sigma_3^C, \Sigma_3^N}{\Delta, E, E^\perp \vdash exp_1 'x' l exp_2 : t_3 \triangleright \Sigma_1^C \cup \Sigma_2^C \cup \Sigma_3^C, \Sigma_1^N \cup \Sigma_2^N \cup \Sigma_3^N} \quad \text{CHECK_EXP_AUX_INFIX_APP2} \\
\\
\frac{\Delta, E \vdash \mathbf{field} id_i : p \, t_args \rightarrow t_i \triangleright (x_i \mathbf{of} names)^i \quad \Delta, E, E^\perp \vdash exp_i : t_i \triangleright \Sigma_i^C, \Sigma_i^N \quad \mathbf{duplicates}(\bar{x}_i^i) = \emptyset \quad names = \{\bar{x}_i^i\}}{\Delta, E, E^\perp \vdash \langle \bar{id}_i = exp_i \, \bar{l}_i^i ; ? \, l \rangle : p \, t_args \triangleright \overline{\Sigma_i^C}^i, \overline{\Sigma_i^N}^i} \quad \text{CHECK_EXP_AUX_RECORD} \\
\\
\frac{\Delta, E \vdash \mathbf{field} id_i : p \, t_args \rightarrow t_i \triangleright (x_i \mathbf{of} names)^i \quad \Delta, E, E^\perp \vdash exp_i : t_i \triangleright \Sigma_i^C, \Sigma_i^N \quad \mathbf{duplicates}(\bar{x}_i^i) = \emptyset \quad \Delta, E, E^\perp \vdash exp : p \, t_args \triangleright \Sigma^{C'}, \Sigma^{N'}}{\Delta, E, E^\perp \vdash \langle exp \mathbf{with} \bar{id}_i = exp_i \, \bar{l}_i^i ; ? \, l \rangle : p \, t_args \triangleright \Sigma^{C'} \cup \overline{\Sigma_i^C}^i, \Sigma^{N'} \cup \overline{\Sigma_i^N}^i} \quad \text{CHECK_EXP_AUX_RECUP} \\
\\
\frac{\Delta, E, E^\perp \vdash exp_1 : t \triangleright \Sigma_1^C, \Sigma_1^N \quad \dots \quad \Delta, E, E^\perp \vdash exp_n : t \triangleright \Sigma_n^C, \Sigma_n^N \quad \mathbf{length}(exp_1 \dots exp_n) = num}{\Delta, E, E^\perp \vdash [exp_1; \dots; exp_n] : \mathbf{vector} \, num \, t \triangleright \Sigma_1^C \cup \dots \cup \Sigma_n^C, \Sigma_1^N \cup \dots \cup \Sigma_n^N} \quad \text{CHECK_EXP_AUX_VECTOR} \\
\\
\frac{\Delta, E, E^\perp \vdash exp : \mathbf{vector} \, ne' \, t \triangleright \Sigma^C, \Sigma^N \quad \vdash Nexp \rightsquigarrow ne}{\Delta, E, E^\perp \vdash exp.(Nexp) : t \triangleright \Sigma^C, \Sigma^N \cup \{ne \langle ne' \rangle\}} \quad \text{CHECK_EXP_AUX_VECTORGET} \\
\\
\frac{\Delta, E, E^\perp \vdash exp : \mathbf{vector} \, ne' \, t \triangleright \Sigma^C, \Sigma^N \quad \vdash Nexp_1 \rightsquigarrow ne_1 \quad \vdash Nexp_2 \rightsquigarrow ne_2 \quad ne = ne_2 + (-ne_1)}{\Delta, E, E^\perp \vdash exp.(Nexp_1..Nexp_2) : \mathbf{vector} \, ne \, t \triangleright \Sigma^C, \Sigma^N \cup \{ne_1 \langle ne_2 \langle ne' \rangle\}} \quad \text{CHECK_EXP_AUX_VECTORSUB} \\
\\
\frac{E \vdash id \mathbf{field} \quad \Delta, E \vdash \mathbf{field} id : p \, t_args \rightarrow t \triangleright (x \mathbf{of} names) \quad \Delta, E, E^\perp \vdash exp : p \, t_args \triangleright \Sigma^C, \Sigma^N}{\Delta, E, E^\perp \vdash exp.id : t \triangleright \Sigma^C, \Sigma^N} \quad \text{CHECK_EXP_AUX_FIELD}
\end{array}$$

$$\begin{array}{c}
\frac{\overline{\Delta, E, E^L \vdash pat_i : t \triangleright E_i^L}^i}{\frac{\overline{\Delta, E, E^L \uplus E_i^L \vdash exp_i : u \triangleright \Sigma_i^C, \Sigma_i^N}^i}{\Delta, E, E^L \vdash exp : t \triangleright \Sigma^{C'}, \Sigma^{N'}}} \text{CHECK_EXP_AUX_CASE} \\
\hline
\Delta, E, E^L \vdash \mathbf{match} \ exp \ \mathbf{with} \ |^? \overline{pat_i \rightarrow exp_i} \overline{l_i}^i \ \mathbf{end} : u \triangleright \Sigma^{C'} \cup \overline{\Sigma_i^C}^i, \Sigma^{N'} \cup \overline{\Sigma_i^N}^i \\
\frac{\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N}{\Delta, E \vdash typ \rightsquigarrow t} \text{CHECK_EXP_AUX_TYPED} \\
\hline
\Delta, E, E_1^L \vdash letbind \triangleright E_2^L, \Sigma_1^C, \Sigma_1^N \\
\Delta, E, E_1^L \uplus E_2^L \vdash exp : t \triangleright \Sigma_2^C, \Sigma_2^N \\
\hline
\Delta, E, E^L \vdash \mathbf{let} \ letbind \ \mathbf{in} \ exp : t \triangleright \Sigma_1^C \cup \Sigma_2^C, \Sigma_1^N \cup \Sigma_2^N \text{CHECK_EXP_AUX_LET} \\
\hline
\Delta, E, E^L \vdash exp_1 : t_1 \triangleright \Sigma_1^C, \Sigma_1^N \quad \dots \quad \Delta, E, E^L \vdash exp_n : t_n \triangleright \Sigma_n^C, \Sigma_n^N \\
\hline
\Delta, E, E^L \vdash (exp_1, \dots, exp_n) : t_1 * \dots * t_n \triangleright \Sigma_1^C \cup \dots \cup \Sigma_n^C, \Sigma_1^N \cup \dots \cup \Sigma_n^N \text{CHECK_EXP_AUX_TUP} \\
\hline
\Delta \vdash t \ \mathbf{ok} \\
\Delta, E, E^L \vdash exp_1 : t \triangleright \Sigma_1^C, \Sigma_1^N \quad \dots \quad \Delta, E, E^L \vdash exp_n : t \triangleright \Sigma_n^C, \Sigma_n^N \\
\hline
\Delta, E, E^L \vdash [exp_1; \dots; exp_n; ?] : _list \ t \triangleright \Sigma_1^C \cup \dots \cup \Sigma_n^C, \Sigma_1^N \cup \dots \cup \Sigma_n^N \text{CHECK_EXP_AUX_LIST} \\
\hline
\frac{\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N}{\Delta, E, E^L \vdash (exp) : t \triangleright \Sigma^C, \Sigma^N} \text{CHECK_EXP_AUX_PAREN} \\
\hline
\frac{\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N}{\Delta, E, E^L \vdash \mathbf{begin} \ exp \ \mathbf{end} : t \triangleright \Sigma^C, \Sigma^N} \text{CHECK_EXP_AUX_BEGIN} \\
\hline
\Delta, E, E^L \vdash exp_1 : _bool \triangleright \Sigma_1^C, \Sigma_1^N \\
\Delta, E, E^L \vdash exp_2 : t \triangleright \Sigma_2^C, \Sigma_2^N \\
\Delta, E, E^L \vdash exp_3 : t \triangleright \Sigma_3^C, \Sigma_3^N \\
\hline
\Delta, E, E^L \vdash \mathbf{if} \ exp_1 \ \mathbf{then} \ exp_2 \ \mathbf{else} \ exp_3 : t \triangleright \Sigma_1^C \cup \Sigma_2^C \cup \Sigma_3^C, \Sigma_1^N \cup \Sigma_2^N \cup \Sigma_3^N \text{CHECK_EXP_AUX_IF} \\
\hline
\Delta, E, E^L \vdash exp_1 : t \triangleright \Sigma_1^C, \Sigma_1^N \\
\Delta, E, E^L \vdash exp_2 : _list \ t \triangleright \Sigma_2^C, \Sigma_2^N \\
\hline
\Delta, E, E^L \vdash exp_1 :: exp_2 : _list \ t \triangleright \Sigma_1^C \cup \Sigma_2^C, \Sigma_1^N \cup \Sigma_2^N \text{CHECK_EXP_AUX_CONS} \\
\hline
\frac{\vdash lit : t}{\Delta, E, E^L \vdash lit : t \triangleright \{\}, \{\}} \text{CHECK_EXP_AUX_LIT} \\
\hline
\overline{\Delta \vdash t_i \ \mathbf{ok}}^i \\
\Delta, E, E^L \uplus \{\overline{x_i \mapsto t_i}^i\} \vdash exp_1 : t \triangleright \Sigma_1^C, \Sigma_1^N \\
\Delta, E, E^L \uplus \{\overline{x_i \mapsto t_i}^i\} \vdash exp_2 : _bool \triangleright \Sigma_2^C, \Sigma_2^N \\
\mathbf{disjoint} \ \mathbf{doms} \ (E^L, \{\overline{x_i \mapsto t_i}^i\}) \\
E = \langle E^M, E^P, E^F, E^X \rangle \\
\overline{x_i \notin \mathbf{dom} \ (E^X)}^i \\
\hline
\Delta, E, E^L \vdash \{exp_1 | exp_2\} : _set \ t \triangleright \Sigma_1^C \cup \Sigma_2^C, \Sigma_1^N \cup \Sigma_2^N \text{CHECK_EXP_AUX_SET_COMP} \\
\hline
\Delta, E, E_1^L \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma_1^C \\
\Delta, E, E_1^L \uplus E_2^L \vdash exp_1 : t \triangleright \Sigma_2^C, \Sigma_2^N \\
\Delta, E, E_1^L \uplus E_2^L \vdash exp_2 : _bool \triangleright \Sigma_3^C, \Sigma_3^N \\
\hline
\Delta, E, E_1^L \vdash \{exp_1 | \mathbf{forall} \ \overline{qbind_i}^i \ | exp_2\} : _set \ t \triangleright \Sigma_1^C \cup \Sigma_2^C \cup \Sigma_3^C, \Sigma_1^N \cup \Sigma_2^N \cup \Sigma_3^N \text{CHECK_EXP_AUX_SET_COMP-} \\
\hline
\Delta \vdash t \ \mathbf{ok} \\
\Delta, E, E^L \vdash exp_1 : t \triangleright \Sigma_1^C, \Sigma_1^N \quad \dots \quad \Delta, E, E^L \vdash exp_n : t \triangleright \Sigma_n^C, \Sigma_n^N \\
\hline
\Delta, E, E^L \vdash \{exp_1; \dots; exp_n; ?\} : _set \ t \triangleright \Sigma_1^C \cup \dots \cup \Sigma_n^C, \Sigma_1^N \cup \dots \cup \Sigma_n^N \text{CHECK_EXP_AUX_SET}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta, E, E_1^L \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_1 \quad \Delta, E, E_1^L \uplus E_2^L \vdash exp : _ \mathbf{bool} \triangleright \Sigma^C_2, \Sigma^N_2}{\Delta, E, E_1^L \vdash q \overline{qbind_i}^i . exp : _ \mathbf{bool} \triangleright \Sigma^C_1 \cup \Sigma^C_2, \Sigma^N_2} \text{CHECK_EXP_AUX_QUANT} \\
\\
\frac{\Delta, E, E_1^L \vdash \mathbf{list} \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_1 \quad \Delta, E, E_1^L \uplus E_2^L \vdash exp_1 : t \triangleright \Sigma^C_2, \Sigma^N_2 \quad \Delta, E, E_1^L \uplus E_2^L \vdash exp_2 : _ \mathbf{bool} \triangleright \Sigma^C_3, \Sigma^N_3}{\Delta, E, E_1^L \vdash [exp_1 | \mathbf{forall} \overline{qbind_i}^i | exp_2] : _ \mathbf{list} t \triangleright \Sigma^C_1 \cup \Sigma^C_2 \cup \Sigma^C_3, \Sigma^N_2 \cup \Sigma^N_3} \text{CHECK_EXP_AUX_LIST_COMP} \\
\\
\boxed{\Delta, E, E_1^L \vdash qbind_1 .. qbind_n \triangleright E_2^L, \Sigma^C} \quad \text{Build the environment for quantifier bindings, collecting typeclass constraints} \\
\\
\frac{}{\Delta, E, E^L \vdash \triangleright \{\}, \{\}} \text{CHECK_LISTQUANT_BINDING_EMPTY} \\
\\
\frac{\Delta \vdash t \mathbf{ok} \quad \Delta, E, E_1^L \uplus \{x \mapsto t\} \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_1 \quad \mathbf{disjoint doms}(\{x \mapsto t\}, E_2^L)}{\Delta, E, E_1^L \vdash x \mathbf{l} \overline{qbind_i}^i \triangleright \{x \mapsto t\} \uplus E_2^L, \Sigma^C_1} \text{CHECK_LISTQUANT_BINDING_VAR} \\
\\
\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_3^L \quad \Delta, E, E_1^L \vdash exp : _ \mathbf{set} t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E_1^L \uplus E_3^L \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_2 \quad \mathbf{disjoint doms}(E_3^L, E_2^L)}{\Delta, E, E_1^L \vdash (pat \mathbf{IN} exp) \overline{qbind_i}^i \triangleright E_2^L \uplus E_3^L, \Sigma^C_1 \cup \Sigma^C_2} \text{CHECK_LISTQUANT_BINDING_RESTR} \\
\\
\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_3^L \quad \Delta, E, E_1^L \vdash exp : _ \mathbf{list} t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E_1^L \uplus E_3^L \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_2 \quad \mathbf{disjoint doms}(E_3^L, E_2^L)}{\Delta, E, E_1^L \vdash (pat \mathbf{MEM} exp) \overline{qbind_i}^i \triangleright E_2^L \uplus E_3^L, \Sigma^C_1 \cup \Sigma^C_2} \text{CHECK_LISTQUANT_BINDING_LIST_RESTR} \\
\\
\boxed{\Delta, E, E_1^L \vdash \mathbf{list} qbind_1 .. qbind_n \triangleright E_2^L, \Sigma^C} \quad \text{Build the environment for quantifier bindings, collecting typeclass constraints} \\
\\
\frac{}{\Delta, E, E^L \vdash \mathbf{list} \triangleright \{\}, \{\}} \text{CHECK_QUANT_BINDING_EMPTY} \\
\\
\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_3^L \quad \Delta, E, E_1^L \vdash exp : _ \mathbf{list} t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E_1^L \uplus E_3^L \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_2 \quad \mathbf{disjoint doms}(E_3^L, E_2^L)}{\Delta, E, E_1^L \vdash \mathbf{list} (pat \mathbf{MEM} exp) \overline{qbind_i}^i \triangleright E_2^L \uplus E_3^L, \Sigma^C_1 \cup \Sigma^C_2} \text{CHECK_QUANT_BINDING_RESTR} \\
\\
\boxed{\Delta, E, E^L \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N} \quad \text{Build the environment for a function definition clause, collecting typeclass constraints} \\
\\
\frac{\Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \quad \Delta, E, E^L \uplus E_1^L \uplus \dots \uplus E_n^L \vdash exp : u \triangleright \Sigma^C, \Sigma^N \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L) \quad \Delta, E \vdash typ \rightsquigarrow u}{\Delta, E, E^L \vdash x \mathbf{l}_1 pat_1 \dots pat_n : typ = exp \mathbf{l}_2 \triangleright \{x \mapsto \mathbf{curry}((t_1 * \dots * t_n), u)\}, \Sigma^C, \Sigma^N} \text{CHECK_FUNCL_ANNOT} \\
\\
\frac{\Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \quad \Delta, E, E^L \uplus E_1^L \uplus \dots \uplus E_n^L \vdash exp : u \triangleright \Sigma^C, \Sigma^N \quad \mathbf{disjoint doms}(E_1^L, \dots, E_n^L)}{\Delta, E, E^L \vdash x \mathbf{l}_1 pat_1 \dots pat_n = exp \mathbf{l}_2 \triangleright \{x \mapsto \mathbf{curry}((t_1 * \dots * t_n), u)\}, \Sigma^C, \Sigma^N} \text{CHECK_FUNCL_NOANNOT}
\end{array}$$

$\Delta, E, E_1^L \vdash \text{letbind} \triangleright E_2^L, \Sigma^C, \Sigma^N$	Build the environment for a let binding, collecting typeclass and index con
$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash \text{pat} : t \triangleright E_2^L \\ \Delta, E, E_1^L \vdash \text{exp} : t \triangleright \Sigma^C, \Sigma^N \\ \Delta, E \vdash \text{typ} \rightsquigarrow t \end{array}}{\Delta, E, E_1^L \vdash \text{pat} : \text{typ} = \text{exp } l \triangleright E_2^L, \Sigma^C, \Sigma^N}$	CHECK_LETBIND_VAL_ANNOT
$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash \text{pat} : t \triangleright E_2^L \\ \Delta, E, E_1^L \vdash \text{exp} : t \triangleright \Sigma^C, \Sigma^N \end{array}}{\Delta, E, E_1^L \vdash \text{pat} = \text{exp } l \triangleright E_2^L, \Sigma^C, \Sigma^N}$	CHECK_LETBIND_VAL_NOANNOT
$\frac{\Delta, E, E_1^L \vdash \text{funcl_aux } l \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N}{\Delta, E, E_1^L \vdash \text{funcl_aux } l \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N}$	CHECK_LETBIND_FN
$\Delta, E, E^L \vdash \text{rule} \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$	Build the environment for an inductive relation clause, collecting typecl
$\frac{\begin{array}{l} \overline{\Delta \vdash t_i \mathbf{ok}^i} \\ E_2^L = \{ \overline{y_i \mapsto t_i^i} \} \\ \Delta, E, E_1^L \uplus E_2^L \vdash \text{exp}' : _ \mathbf{bool} \triangleright \Sigma^{C'}, \Sigma^{N'} \\ \Delta, E, E_1^L \uplus E_2^L \vdash \text{exp}_1 : u_1 \triangleright \Sigma^C_1, \Sigma^N_1 \quad \dots \quad \Delta, E, E_1^L \uplus E_2^L \vdash \text{exp}_n : u_n \triangleright \Sigma^C_n, \Sigma^N_n \end{array}}{\Delta, E, E_1^L \vdash \text{id}^? \text{forall } \overline{y_i}^i \text{li}^i . \text{exp}' \implies x \text{ l exp}_1 \dots \text{exp}_n \text{ l}' \triangleright \{x \mapsto \mathbf{curry}((u_1 * \dots * u_n), _ \mathbf{bool})\}, \Sigma^{C'} \cup \Sigma^C_1 \cup \dots \cup \Sigma^C_n}$	
$xs, \Delta_1, E \vdash \mathbf{tc} \text{ td} \triangleright \Delta_2, E^P$	Extract the type constructor information
$\frac{\begin{array}{l} \text{tnvars}^l \rightsquigarrow \text{tnvs} \\ \Delta, E \vdash \text{typ} \rightsquigarrow t \\ \mathbf{duplicates}(\text{tnvs}) = \emptyset \\ \mathbf{FV}(t) \subset \text{tnvs} \\ \overline{y_i}^i x \notin \mathbf{dom}(\Delta) \end{array}}{\overline{y_i}^i, \Delta, E \vdash \mathbf{tc} \text{ x l tnvars}^l = \text{typ} \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs}.t \}, \{x \mapsto \overline{y_i}^i x\}}$	CHECK_TEXPS_TC_ABBREV
$\frac{\begin{array}{l} \text{tnvars}^l \rightsquigarrow \text{tnvs} \\ \mathbf{duplicates}(\text{tnvs}) = \emptyset \\ \overline{y_i}^i x \notin \mathbf{dom}(\Delta) \end{array}}{\overline{y_i}^i, \Delta, E_1 \vdash \mathbf{tc} \text{ x l tnvars}^l \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs} \}, \{x \mapsto \overline{y_i}^i x\}}$	CHECK_TEXPS_TC_ABSTRACT
$\frac{\begin{array}{l} \text{tnvars}^l \rightsquigarrow \text{tnvs} \\ \mathbf{duplicates}(\text{tnvs}) = \emptyset \\ \overline{y_i}^i x \notin \mathbf{dom}(\Delta) \end{array}}{\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} \text{ x l tnvars}^l = \langle x_1^l : \text{typ}_1; \dots; x_j^l : \text{typ}_j; ? \rangle \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs} \}, \{x \mapsto \overline{y_i}^i x\}}$	CHECK_TEXPS_TC_REC
$\frac{\begin{array}{l} \text{tnvars}^l \rightsquigarrow \text{tnvs} \\ \mathbf{duplicates}(\text{tnvs}) = \emptyset \\ \overline{y_i}^i x \notin \mathbf{dom}(\Delta) \end{array}}{\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} \text{ x l tnvars}^l = ^? \text{ctor_def}_1 \dots \text{ctor_def}_j \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs} \}, \{x \mapsto \overline{y_i}^i x\}}$	CHECK_TEXPS_TC_VAR
$xs, \Delta_1, E \vdash \mathbf{tc} \text{ td}_1 \dots \text{td}_i \triangleright \Delta_2, E^P$	Extract the type constructor information
$xs, \Delta, E \vdash \mathbf{tc} \triangleright \{\}, \{\}$	CHECK_TEXPS_TC_EMPTY
$\frac{\begin{array}{l} xs, \Delta_1, E \vdash \mathbf{tc} \text{ td} \triangleright \Delta_2, E_2^P \\ xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{\}, E_2^P, \{\}, \{\} \rangle \vdash \mathbf{tc} \overline{td_i}^i \triangleright \Delta_3, E_3^P \\ \mathbf{dom}(E_2^P) \cap \mathbf{dom}(E_3^P) = \emptyset \end{array}}{xs, \Delta_1, E \vdash \mathbf{tc} \text{ td } \overline{td_i}^i \triangleright \Delta_2 \uplus \Delta_3, E_2^P \uplus E_3^P}$	CHECK_TEXPS_TC_ABBREV

$\Delta, E \vdash tnvs\ p = texp \triangleright \langle E^F, E^X \rangle$ Check a type definition, with its path already resolved

$\overline{\Delta, E \vdash tnvs\ p = typ \triangleright \langle \{ \}, \{ \} \rangle}$ CHECK_TEXPS_EMPTY

$\overline{\Delta, E \vdash typ_i \rightsquigarrow t_i^i}$
 $names = \{ \overline{x_i^i} \}$
 $\mathbf{duplicates}(\overline{x_i^i}) = \emptyset$
 $\overline{\mathbf{FV}(t_i) \subset tnvs^i}$
 $\overline{E^F = \{ x_i \mapsto \langle \mathbf{forall}\ tnvs.p \rightarrow t_i, (x_i \mathbf{of}\ names) \rangle \}^i}$
 $\Delta, E \vdash tnvs\ p = \langle | \overline{x_i^l : typ_i^i} ; ? | \rangle \triangleright \langle E^F, \{ \} \rangle$ CHECK_TEXPS_REC

$\overline{\Delta, E \vdash typs_i \rightsquigarrow t_multi_i^i}$
 $names = \{ \overline{x_i^i} \}$
 $\mathbf{duplicates}(\overline{x_i^i}) = \emptyset$
 $\overline{\mathbf{FV}(t_multi_i) \subset tnvs^i}$
 $\overline{E^X = \{ x_i \mapsto \langle \mathbf{forall}\ tnvs.t_multi_i \rightarrow p, (x_i \mathbf{of}\ names) \rangle \}^i}$
 $\Delta, E \vdash tnvs\ p = | ? \overline{x_i^l \mathbf{of}\ typs_i} \triangleright \langle \{ \}, E^X \rangle$ CHECK_TEXPS_VAR

$xs, \Delta, E \vdash td_1 .. td_n \triangleright \langle E^F, E^X \rangle$

$\overline{\overline{y_i^i}, \Delta, E \vdash \triangleright \langle \{ \}, \{ \} \rangle}$ CHECK_TEXPS_EMPTY

$tnvars^l \rightsquigarrow tnvs$
 $\Delta, E_1 \vdash tnvs\ \overline{y_i^i} \cdot x = texp \triangleright \langle E_1^F, E_1^X \rangle$
 $\overline{y_i^i}, \Delta, E \vdash \overline{td_j^j} \triangleright \langle E_2^F, E_2^X \rangle$
 $\mathbf{dom}(E_1^X) \cap \mathbf{dom}(E_2^X) = \emptyset$
 $\mathbf{dom}(E_1^F) \cap \mathbf{dom}(E_2^F) = \emptyset$
 $\overline{\overline{y_i^i}, \Delta, E \vdash x\ l\ tnvars^l = texp\ \overline{td_j^j} \triangleright \langle E_1^F \uplus E_2^F, E_1^X \uplus E_2^X \rangle}$ CHECK_TEXPS_CONS_CONCRETE

$\overline{\overline{y_i^i}, \Delta, E \vdash \overline{td_j^j} \triangleright \langle E^F, E^X \rangle}$
 $\overline{\overline{y_i^i}, \Delta, E \vdash x\ l\ tnvars^l\ \overline{td_j^j} \triangleright \langle E^F, E^X \rangle}$ CHECK_TEXPS_CONS_ABSTRACT

$\delta, E \vdash id \rightsquigarrow p$ Lookup a type class

$E(id) \triangleright p$
 $\delta(p) \triangleright xs$
 $\delta, E \vdash id \rightsquigarrow p$ CONVERT_CLASS_ALL

$I \vdash (p\ t) \mathbf{INC}$ Solve class constraint

$\overline{I \vdash (p\ \alpha) \mathbf{IN}\ (p_1\ tnvs_1) .. (p_i\ tnvs_i)(p\ \alpha)(p'_1\ tnvs'_1) .. (p'_j\ tnvs'_j)}$ SOLVE_CLASS_CONSTRAINT_IMMEDIATE

$\overline{(p_1\ tnvs_1) .. (p_n\ tnvs_n) \Rightarrow (p\ t) \mathbf{IN}\ I}$
 $\overline{I \vdash (p_1\ \sigma(tnvs_1)) \mathbf{INC} \quad .. \quad I \vdash (p_n\ \sigma(tnvs_n)) \mathbf{INC}}$
 $I \vdash (p\ \sigma(t)) \mathbf{INC}$ SOLVE_CLASS_CONSTRAINT_CHAIN

$I \vdash \Sigma^C \triangleright C$ Solve class constraints

$\overline{I \vdash (p_1\ t_1) \mathbf{INC} \quad .. \quad I \vdash (p_n\ t_n) \mathbf{INC}}$
 $I \vdash \{(p_1\ t_1), .., (p_n\ t_n)\} \triangleright C$ SOLVE_CLASS_CONSTRAINTS_ALL

$\Delta, I, E \vdash \text{val_def} \triangleright E^x$ Check a value definition

$$\frac{\begin{array}{l} \Delta, E, \{ \} \vdash \text{letbind} \triangleright \{ \overline{x_i \mapsto t_i^i} \}, \Sigma^C, \Sigma^N \\ I \vdash \Sigma^C \triangleright C \\ \overline{\mathbf{FV}(t_i) \subset \text{tnvs}}^i \\ \mathbf{FV}(C) \subset \text{tnvs} \end{array}}{\Delta, I, E_1 \vdash \text{let } \tau^? \text{ letbind} \triangleright \{ \overline{x_i \mapsto \langle \text{forall } \text{tnvs}.C \Rightarrow t_i, \text{let} \rangle^i} \}} \text{CHECK_VAL_DEF_VAL}$$

$$\frac{\begin{array}{l} \Delta, E, E^L \vdash \text{funcl}_i \triangleright \{ x_i \mapsto t_i \}, \Sigma_i^C, \Sigma_i^N^i \\ I \vdash \Sigma^C \triangleright C \\ \overline{\mathbf{FV}(t_i) \subset \text{tnvs}}^i \\ \mathbf{FV}(C) \subset \text{tnvs} \\ \text{compatible overlap}(\overline{x_i \mapsto t_i^i}) \\ E^L = \{ \overline{x_i \mapsto t_i^i} \} \end{array}}{\Delta, I, E \vdash \text{let rec } \tau^? \text{ funcl}_i^i \triangleright \{ \overline{x_i \mapsto \langle \text{forall } \text{tnvs}.C \Rightarrow t_i, \text{let} \rangle^i} \}} \text{CHECK_VAL_DEF_RECFUN}$$

$\Delta, (\alpha_1, \dots, \alpha_n) \vdash t \text{ instance}$ Check that t be a typeclass instance

$$\frac{}{\Delta, (\alpha) \vdash \alpha \text{ instance}} \text{CHECK_T_INSTANCE_VAR}$$

$$\frac{}{\Delta, (\alpha_1, \dots, \alpha_n) \vdash \alpha_1 * \dots * \alpha_n \text{ instance}} \text{CHECK_T_INSTANCE_TUP}$$

$$\frac{}{\Delta, (\alpha_1, \alpha_2) \vdash \alpha_1 \rightarrow \alpha_n \text{ instance}} \text{CHECK_T_INSTANCE_FN}$$

$$\frac{\Delta(p) \triangleright \alpha'_1 \dots \alpha'_n}{\Delta, (\alpha_1, \dots, \alpha_n) \vdash p \alpha_1 \dots \alpha_n \text{ instance}} \text{CHECK_T_INSTANCE_TC}$$

$\overline{z_j^j}, D_1, E_1 \vdash \text{def} \triangleright D_2, E_2$ Check a definition

$$\frac{\begin{array}{l} \overline{z_j^j}, \Delta_1, E \vdash \text{tc } \overline{td_i^i} \triangleright \Delta_2, E^P \\ \overline{z_j^j}, \Delta_1 \uplus \Delta_2, E \uplus \langle \{ \}, E^P, \{ \}, \{ \} \rangle \vdash \overline{td_i^i} \triangleright \langle E^F, E^X \rangle \end{array}}{\overline{z_j^j}, \langle \Delta_1, \delta, I \rangle, E \vdash \text{type } \overline{td_i^i} l \triangleright \langle \Delta_2, \{ \}, \{ \} \rangle, \langle \{ \}, E^P, E^F, E^X \rangle} \text{CHECK_DEF_TYPE}$$

$$\frac{\Delta, I, E \vdash \text{val_def} \triangleright E^x}{\overline{z_j^j}, \langle \Delta, \delta, I \rangle, E \vdash \text{val_def } l \triangleright \epsilon, \langle \{ \}, \{ \}, \{ \}, E^X \rangle} \text{CHECK_DEF_VAL_DEF}$$

$$\frac{\begin{array}{l} \Delta, E_1, E^L \vdash \text{rule}_i \triangleright \{ x_i \mapsto t_i \}, \Sigma_i^C, \Sigma_i^N^i \\ I \vdash \overline{\Sigma_i^C}^i \triangleright C \\ \overline{\mathbf{FV}(t_i) \subset \text{tnvs}}^i \\ \mathbf{FV}(C) \subset \text{tnvs} \\ \text{compatible overlap}(\overline{x_i \mapsto t_i^i}) \\ E^L = \{ \overline{x_i \mapsto t_i^i} \} \\ E_2 = \langle \{ \}, \{ \}, \{ \}, \{ x_i \mapsto \langle \text{forall } \text{tnvs}.C \Rightarrow t_i, \text{let} \rangle^i \} \rangle \end{array}}{\overline{z_j^j}, \langle \Delta, \delta, I \rangle, E_1 \vdash \text{indreln } \tau^? \overline{\text{rule}_i^i} l \triangleright \epsilon, E_2} \text{CHECK_DEF_INDRELN}$$

$$\frac{\overline{z_j^j} x, D_1, E_1 \vdash \text{defs} \triangleright D_2, E_2}{\overline{z_j^j}, D_1, E_1 \vdash \text{module } x \text{ l}_1 = \text{struct } \text{defs} \text{ end } l_2 \triangleright D_2, \langle \{ x \mapsto E_2 \}, \{ \}, \{ \}, \{ \} \rangle} \text{CHECK_DEF_MODULE}$$

$$\frac{E_1(id) \triangleright E_2}{\overline{z_j^j}, D, E_1 \vdash \text{module } x \text{ l}_1 = id \text{ l}_2 \triangleright \epsilon, \langle \{ x \mapsto E_2 \}, \{ \}, \{ \}, \{ \} \rangle} \text{CHECK_DEF_MODULE_RENAME}$$

$$\begin{array}{c}
\Delta, E \vdash \text{typ} \rightsquigarrow t \\
\mathbf{FV}(t) \subset \overline{\alpha_i}^i \\
\mathbf{FV}(\overline{\alpha_k'}^k) \subset \overline{\alpha_i}^i \\
\overline{\delta, E \vdash id_k \rightsquigarrow p_k}^k \\
E' = \langle \{\}, \{\}, \{\}, \{x \mapsto \langle \mathbf{forall} \overline{\alpha_i}^i. (\overline{p_k \alpha_k'})^k \Rightarrow t, \mathbf{val} \rangle\} \rangle \\
\hline
\overline{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{val} x l_1 : \mathbf{forall} \overline{\alpha_i}^i. \overline{id_k \alpha_k' l_k''}^k \Rightarrow \text{typ } l_2 \triangleright \epsilon, E'} \quad \text{CHECK_DEF_SPEC} \\
\hline
\overline{\Delta, E_1 \vdash \text{typ}_i \rightsquigarrow t_i}^i \\
\overline{\mathbf{FV}(t_i) \subset \alpha}^i \\
p = \overline{z_j}^j. x \\
E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{y_i \mapsto \langle \mathbf{forall} \alpha. (p \alpha) \Rightarrow t_i, \mathbf{method} \rangle^i\} \rangle \\
\delta_2 = \{p \mapsto \overline{y_i}^i\} \\
p \notin \mathbf{dom}(\delta_1) \\
\hline
\overline{\overline{z_j}^j, \langle \Delta, \delta_1, I \rangle, E_1 \vdash \mathbf{class} (x l \alpha l'') \overline{\mathbf{val} y_i l_i : \text{typ}_i l_i}^i \mathbf{end} l' \triangleright \langle \{\}, \delta_2, \{\} \rangle, E_2} \quad \text{CHECK_DEF_CLASS} \\
\hline
\begin{array}{c}
E = \langle E^M, E^P, E^F, E^X \rangle \\
\Delta, E \vdash \text{typ}' \rightsquigarrow t' \\
\Delta, (\overline{\alpha_i}^i) \vdash t' \mathbf{instance} \\
tnvs = \overline{\alpha_i}^i \\
\mathbf{duplicates}(tnvs) = \emptyset \\
\overline{\delta, E \vdash id_k \rightsquigarrow p_k}^k \\
\mathbf{FV}(\overline{\alpha_k'}^k) \subset tnvs \\
E(id) \triangleright p \\
\delta(p) \triangleright \overline{z_j}^j \\
I_2 = \{ \Rightarrow (\overline{p_k \alpha_k'})^k \} \\
\overline{\Delta, I \cup I_2, E \vdash \text{val_def}_n \triangleright E_n^X}^n \\
\mathbf{disjoint doms}(\overline{E_n^X}^n) \\
\overline{E^X(x_k) \triangleright \langle \mathbf{forall} \alpha'' . (p \alpha'') \Rightarrow t_k, \mathbf{method} \rangle^k}^k \\
\overline{\{x_k \mapsto \langle \mathbf{forall} tnvs. \Rightarrow \{\alpha'' \mapsto t'\}(t_k), \mathbf{let} \rangle^k\} = \overline{E_n^X}^n}^k \\
\overline{x_k}^k = \overline{z_j}^j \\
I_3 = \{ (\overline{p_k \alpha_k'})^k \Rightarrow (\overline{p t'})^k \} \\
(p \{ \overline{\alpha_i \mapsto \alpha_i'''}^i \}(t')) \notin I \\
\hline
\overline{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{instance forall} \overline{\alpha_i}^i. \overline{id_k \alpha_k' l_k''}^k \Rightarrow (id \text{typ}') \overline{\text{val_def}_n l_n}^n \mathbf{end} l' \triangleright \langle \{\}, \{\}, I_3 \rangle, \epsilon} \quad \text{CHECK_DEF_} \\
\hline
\boxed{\overline{\overline{z_j}^j, D_1, E_1 \vdash \text{defs} \triangleright D_2, E_2}} \quad \text{Check definitions, given module path, definitions and environment}
\end{array}$$

$$\overline{\overline{z_j}^j, D, E \vdash \triangleright \epsilon, \epsilon} \quad \text{CHECK_DEFS_EMPTY}$$

$$\begin{array}{c}
\overline{\overline{z_j}^j, D_1, E_1 \vdash \text{def} \triangleright D_2, E_2} \\
\overline{\overline{z_j}^j, D_1 \uplus D_2, E_1 \uplus E_2 \vdash \overline{\text{def}_i ; ; \overline{?}^i}^i \triangleright D_3, E_3} \\
\hline
\overline{\overline{z_j}^j, D_1, E_1 \vdash \text{def} ; ; \overline{\text{def}_i ; ; \overline{?}^i}^i \triangleright D_2 \uplus D_3, E_2 \uplus E_3} \quad \text{CHECK_DEFS_RELEVANT_DEF} \\
\hline
\overline{E_1(id) \triangleright E_2} \\
\overline{\overline{z_j}^j, D_1, E_1 \uplus E_2 \vdash \overline{\text{def}_i ; ; \overline{?}^i}^i \triangleright D_3, E_3} \\
\hline
\overline{\overline{z_j}^j, D_1, E_1 \vdash \mathbf{open} id l ; ; \overline{\text{def}_i ; ; \overline{?}^i}^i \triangleright D_3, E_3} \quad \text{CHECK_DEFS_OPEN}
\end{array}$$

Definition rules: 145 good 0 bad
 Definition rule clauses: 437 good 0 bad