n, i, j, k Index variables for meta-lists

num Numeric literals

nat Internal literal numbers

hex Bit vector literal, specified by C-style hex number bin Bit vector literal, specified by C-style binary number

stringString literalsbacktick_stringString literals

regexp Regular expresions, as a string literal

x, y, z Variables ix Variables

```
l
                                                       Source locations
                        ::=
x^l, y^l, z^l, name
                                                       Location-annotated names
                              x l
                              (ix)l
                                                          Remove infix status
                              name_-t \to x^l
                                                  Μ
                                                          Extract x from a name_t
ix^l
                                                       Location-annotated infix names
                              ix l
                        ::=
                                                       Type variables
\alpha
                              \dot{x}
\alpha^l
                                                       Location-annotated type variables
                        ::=
                              \alpha l
N
                                                       numeric variables
                              ",x
N^l
                                                       Location-annotated numeric variables
                              N l
id
                                                       Long identifers
                              x_1^l \dots x_n^l . x^l l
tnv
                                                       Union of type variables and Nexp type variables, without lo
                              \alpha
                              N
tnvar^l
                        ::=
                                                       Union of type variables and Nexp type variables, with locati
                              \alpha^l
                                                       Type variable lists
tnvs
                              tnv_1 \dots tnv_n
tnvars^l
                                                       Type variable lists
                        ::=
                              tnvar_1^l ... tnvar_n^l
Nexp\_aux
                                                       Numerical expressions for specifying vector lengths and inde
                        ::=
                              N
                              num
                              Nexp_1 * Nexp_2
                              Nexp_1 + Nexp_2
```

(Nexp)

```
Nexp
                                                                  Location-annotated vector lengths
                           ::=
                                 Nexp\_aux\ l
Nexp\_constraint\_aux
                                                                  Whether a vector is bounded or fixed size
                           ::=
                                 Nexp = Nexp'
                                 Nexp \ge Nexp'
Nexp\_constraint
                                                                  Location-annotated Nexp range
                           ::=
                                 Nexp\_constraint\_aux\ l
                           ::=
                                                                  Types
typ\_aux
                                                                     Unspecified type
                                                                     Type variables
                                                                     Function types
                                 typ_1 \rightarrow typ_2
                                 typ_1*\ldots*typ_n
                                                                     Tuple types
                                                                     As a typ to permit applications over Nexps, o
                                 Nexp
                                 id\ typ_1 \dots typ_n
                                                                     Type applications
                                 backtick\_string\ typ_1\ ..\ typ_n
                                                                     Backend-Type applications
                                 (typ)
                                                                  Location-annotated types
typ
                                 typ_aux l
                                                                  Literal constants
lit\_aux
                           ::=
                                 true
                                 false
                                 num
                                 hex
                                                                     hex and bin are constant bit vectors, entered
                                 bin
                                 string
                                 ()
                                 bitzero
                                                                     bitzero and bitone are constant bits, if commo
                                 bitone
lit
                                                                     Location-annotated literal constants
                                 lit\_aux\ l
                                                                  Optional semi-colons
                           ::=
                                                                  Patterns
pat\_aux
                           ::=
                                                                     Wildcards
                                 (pat \mathbf{as} x^l)
                                                                     Named patterns
                                 (pat:typ)
                                                                     Typed patterns
                                 id\ pat_1 \dots pat_n
                                                                     Single variable and constructor patterns
                                 \langle |fpat_1; ...; fpat_n;^?| \rangle
                                                                     Record patterns
```

```
[|pat_1; ...; pat_n;^?|]
                                                                             Vector patterns
                      [|pat_1 ... pat_n|]
                                                                             Concatenated vector patterns
                     (pat_1, \ldots, pat_n)
[pat_1; \ldots; pat_n;^?]
                                                                             Tuple patterns
                                                                             List patterns
                      (pat)
                     pat_1 :: pat_2
                                                                             Cons patterns
                     x^l + num
                                                                             constant addition patterns
                      lit
                                                                             Literal constant patterns
                                                                          Location-annotated patterns
pat
               ::=
                     pat_{-}aux l
                                                                          Field patterns
fpat
                      id = pat l
                                                                          Optional bars
               ::=
                                                                          Expressions
exp\_aux
                                                                             Identifiers
                      id
                      backtick\_string
                                                                             identifier that should be literally used in out
                      N
                                                                             Nexp var, has type num
                                                                             Curried functions
                     fun psexp
                     function ||^{?}|pexp_{1}| \dots ||pexp_{n}| end
                                                                             Functions with pattern matching
                      exp_1 \ exp_2
                                                                             Function applications
                      exp_1 ix^{\bar{l}} exp_2
                                                                             Infix applications
                      \langle |fexps| \rangle
                                                                             Records
                      \langle |exp \mathbf{with} fexps| \rangle
                                                                             Functional update for records
                      exp.id
                                                                             Field projection for records
                      [|exp_1; ...; exp_n;^?|]
                                                                             Vector instantiation
                      exp.(Nexp)
                                                                             Vector access
                      exp.(Nexp_1..Nexp_2)
                                                                             Subvector extraction
                     match exp with |?| pexp_1| ... | pexp_n| l end
                                                                             Pattern matching expressions
                      (exp:typ)
                                                                             Type-annotated expressions
                     let letbind in exp
                                                                             Let expressions
                                                                             Tuples
                      (exp_1, \ldots, exp_n)
                      [exp_1; ...; exp_n; ?]
                                                                             Lists
                      (exp)
                     begin exp end
                                                                             Alternate syntax for (exp)
                     if exp_1 then exp_2 else exp_3
                                                                             Conditionals
                      exp_1 :: exp_2
                                                                             Cons expressions
                      lit
                                                                             Literal constants
                      \{exp_1|exp_2\}
                                                                             Set comprehensions
                      \{exp_1| \mathbf{forall} \ qbind_1 .. \ qbind_n| exp_2\}
                                                                             Set comprehensions with explicit binding
                      \{exp_1; ...; exp_n;^?\}
                                                                             Sets
                      q \ qbind_1 \dots qbind_n . exp
                                                                             Logical quantifications
```

```
[exp_1| \mathbf{forall} \ qbind_1 .. \ qbind_n| exp_2]
                                                                                     List comprehensions (all binders mu
                         do id pat_1 \leftarrow exp_1; .. pat_n \leftarrow exp_n; in exp end
                                                                                     Do notation for monads
                                                                                  Location-annotated expressions
exp
                  ::=
                         exp\_aux l
                                                                                  Quantifiers
q
                         forall
                         exists
                                                                                  Bindings for quantifiers
qbind
                  ::=
                         x^{l}
                         (pat \, \mathbf{IN} \, exp)
                                                                                     Restricted quantifications over sets
                         (pat \mathbf{MEM} \ exp)
                                                                                     Restricted quantifications over lists
                                                                                  Field-expressions
fexp
                         id = exp l
fexps
                                                                                  Field-expression lists
                         fexp_1; \dots; fexp_n; ^? l
                                                                                  Pattern matches
pexp
                         pat \rightarrow exp \ l
                                                                                  Multi-pattern matches
psexp
                         pat_1 \dots pat_n \to exp \ l
tannot?
                                                                                  Optional type annotations
                  ::=
                         : typ
                                                                                  Function clauses
funcl_aux
                         x^l pat_1 \dots pat_n tannot^? = exp
letbind\_aux
                                                                                  Let bindings
                         pat \ tannot? = exp
                                                                                     Value bindings
                         funcl_-aux
                                                                                     Function bindings
letbind
                                                                                  Location-annotated let bindings
                  ::=
                         letbind\_aux\ l
                                                                                  Location-annotated function clauses
funcl
                         funcl_aux l
                                                                                  Name or name with type for inductive
name_-t
```

```
Names with optional type
name\_ts
                             ::=
                                     name\_t_0 \dots name\_t_n
rule\_aux
                                                                                                               Inductively defined relat
                                     x^l: \mathbf{forall} \ name_-t_1 \dots name_-t_i.exp \Longrightarrow x_1^l \ exp_1 \dots exp_n
rule
                              ::=
                                                                                                               Location-annotated indu
                                     rule\_aux\ l
witness?
                                                                                                               Optional witness type na
                              ::=
                                     witness type x^l;
check?
                                                                                                               Option check name decla
                                     \mathbf{check}\,x^l;
functions?
                                                                                                               Optional names and typ
                                    x^{l}: typ
x^{l}: typ; functions^{?}
                                                                                                               Name for inductively de-
indreln\_name\_aux
                              ::=
                                     [x^l: typschm\ witness^?\ check^?\ functions^?]
indreln\_name
                             ::=
                                                                                                               Location-annotated nam
                                     indreln\_name\_aux\ l
                                                                                                               Type lists
typs
                                     typ_1 * ... * typ_n
                                                                                                               Datatype definition clau
ctor\_def
                                     x^l of typs
                                                                                                           S
                                                                                                                   Constant constructors
                                                                                                               Type definition bodies
texp
                                    \begin{array}{l} typ \\ \langle |x_1^l:typ_1;\ldots;x_n^l:typ_n;^?| \rangle \\ |^?\ ctor\_def_1|\ldots|ctor\_def_n \end{array}
                                                                                                                   Type abbreviations
                                                                                                                   Record types
                                                                                                                   Variant types
name?
                                                                                                               Optional name specificar
                                     [name = regexp]
td
                                                                                                               Type definitions
                                    x^{l} tnvars^{l} name^{?} = texp
x^{l} tnvars^{l} name^{?}
                                                                                                                   Definitions of opaque
```

```
::=
                                                                                        Typeclass constraints
c
                         id\ tnvar^l
                   ::=
                                                                                        Typeclass and length constraint
cs
                                                                                           Must have > 0 constraints
                         Nexp\_constraint_1, \, \ldots, Nexp\_constraint_i \Rightarrow
                                                                                          Must have > 0 constraints
                          c_1, ..., c_i; Nexp\_constraint_1, ..., Nexp\_constraint_n \Rightarrow
                                                                                          Must have > 0 of both form of
                   ::=
                                                                                        Type and instance scheme prefix
c\_pre
                         \mathbf{forall}\, tnvar_1^l \dots tnvar_n^l.cs
                                                                                          Must have > 0 type variables
                                                                                        Type schemes
typschm
                          c-pre typ
instschm
                                                                                        Instance schemes
                   ::=
                          c\_pre(id\ typ)
target
                                                                                       Backend target names
                   ::=
                         hol
                         isabelle
                         ocaml
                         coq
                          \mathbf{tex}
                         html
                          lem
open\_import
                                                                                        Open or import statements
                         open
                         import
                         open import
                         include
                         include import
                                                                                        Backend target name lists
                   ::=
\tau
                         \{target_1; ...; target_n\}
\{target_1; ...; target_n\}
                                                                                          all targets except the listed or
\tau?
                                                                                        Optional targets
                                                                                        Types of Lemmata
lemma\_typ
                         assert
                          lemma
                          theorem
```

$lemma_decl$::= le	$lemma_typ\ au^?\ x^l: exp$	Lemmata and Tests
dexp	fe	$egin{aligned} name_s &= string \ l \ \mathbf{format} &= string \ l \ \mathbf{arguments} &= exp_1 \dots exp_n \ l \ \mathbf{targuments} &= texp_1 \dots texp_n \ l \end{aligned}$	declaration field-expressions
$declare_arg$	$::= \ \mid \ s \ \mid \ \langle$	$string \ \langle dexp_1;; dexp_n;^? l \rangle$	arguments to a declaration
component	f	module function type field	components
$termination_setting$		automatic manual	termination settings
$exhaustivity_setting$		exhaustive inexhaustive	exhaustivity settings
$elim_opt$	$::=$ $\begin{vmatrix} & & & & & & & & & & & & & & & & & & &$	id	optional terms used as eliminators for patte
$fixity_decl$	le	$right_assocnat$ $left_assocnat$ $non_assocnat$	fixity declarations for infix identifiers
$target_rep_rhs$	$\begin{vmatrix} & & & & & & & & & & & & & & & & & & &$	$egin{aligned} & \mathbf{infix} \ fixity_decl \ backtick_string \ exp \ typ \ & \mathbf{special} \ string \ exp_1 \ \ exp_n \end{aligned}$	right hand side of a target representation de
$target_rep_lhs$::= to to	$target_rep component \ id \ x_1^l \dots x_n^l \ target_rep component \ id \ tnvars^l$	left hand side of a target representation dec

```
declare\_def
                          ::=
                                  \mathbf{declare} \, \tau^? compile\_messageid = string
                                  \mathbf{declare}\,\tau^{?}\,\mathbf{rename}\,\mathbf{module}\,=x^{l}
                                  declare \tau? rename component id = x^l
                                  \mathbf{declare} \, \tau^? ascii\_rep component \, id = backtick\_string
                                  \mathbf{declare}\ targettarget\_rep\_target\_rep\_lhs = target\_rep\_rhs
                                  \mathbf{declare}\, set\text{-}flagx_1^l = x_2^l
                                  \mathbf{declare} \ \tau^? termination\_argumentid = termination\_setting
                                  \mathbf{declare} \ \tau^? pattern\_matchex haustivity\_setting \ id \ tnvars^l = [\mathit{id}_1; \ \dots; \mathit{id}_n; ?] elim\_opt
val\_def
                                  let \tau? letbind
                                  let \operatorname{rec} \tau^? \operatorname{funcl}_1 and ... and \operatorname{funcl}_n let \operatorname{inline} \tau^? \operatorname{letbind}
                                  let lem\_transform\tau? letbind
ascii\_opt
                                  [backtick\_string]
instance\_decl
                                  instance
                                  default\_instance
class\_decl
                          ::=
                                  class
                                  class inline
val\_spec
                          ::=
                                  \mathbf{val} \ x^l \ ascii\_opt : typschm
def_{-}aux
                          ::=
                                  type td_1 and ... and td_n
                                  val\_def
                                  lemma\_decl
                                  declare\_def
                                  module x^l = struct defs end
                                  module x^l = id
                                  open\_import\ id_1\ ...\ id_n
                                  open\_import \ \tau^? \ backtick\_string_1 \dots backtick\_string_n
                                  indreln 	au^{?} indreln\_name_1 and ... and indreln\_name_i rule_1 and ... and rule_n
                                  class\_decl(x^l\ tnvar^l)\ \mathbf{val}\ \tau_1^?\ x_1^l\ ascii\_opt_1: typ_1\ l_1\dots \mathbf{val}\ \tau_n^?\ x_n^l\ ascii\_opt_n: typ_n\ l_n\ \mathbf{end}
                                  instance\_decl\ instschm\ val\_def_1\ l_1\ ...\ val\_def_n\ l_n\ \mathbf{end}
def
                          ::=
                                  def_{-}aux \ l
```

```
;;?
             ::=
                                                         Optional double-semi-colon
defs
                                                         Definition sequences
                    def_1; ;_1^? ... def_n; ;_n^?
                                                         Unique paths
p
                    x_1 \dots x_n \cdot x
                    \_list
                    \_bool
                    __num
                    \_set
                    \_string
                    _{-}unit
                    _{-}bit
                    \_vector
                                                         Type variable substitutions
                    \{tnv_1 \mapsto t_1 ... tnv_n \mapsto t_n\}
                                                         Internal types
t, u
                    \alpha
                    t_1 \rightarrow t_2
                    t_1 * \dots * t_n
                    p\ t\_args
                    ne
                    \sigma(t)
                                                    Μ
                                                            Multiple substitutions
                                                    Μ
                                                            Single variable substitution
                    \sigma(tnv)
                    \mathbf{curry}\left(t_{-}multi,t\right)
                                                    Μ
                                                            Curried, multiple argument functions
ne
                                                         internal numeric expressions
                    N
                    nat
                    ne_1 * ne_2
                    ne_1 + ne_2
                    (-ne)
                    normalize(ne)
                                                    Μ
                    ne_1 + \ldots + ne_n
                                                    Μ
                    bitlength(bin)
                                                    Μ
                    bitlength(hex)
                                                    Μ
                    length (pat_1 \dots pat_n)
                                                    Μ
                    length (exp_1 \dots exp_n)
                                                    Μ
t\_args
                                                         Lists of types
                    t_1 \dots t_n
                    \sigma(t\_args)
                                                    Μ
                                                            Multiple substitutions
```

```
Lists of types
t_{-}multi
                             (t_1*..*t_n)
                             \sigma(t\_multi)
                                                                                                  Μ
                                                                                                              Multiple substitutions
                                                                                                         Numeric expression constraints
nec
                              ne \langle nec
                              ne = nec
                              ne <= nec
                                                                                                          Sets of names
names
                             \{x_1,\ldots,x_n\}
\mathcal{C}
                                                                                                          Typeclass constraint lists
                             (p_1 tnv_1) \dots (p_n tnv_n)
                                                                                                          Tags for the (non-constructor) value description
env\_tag
                             method
                                                                                                              Bound to a method
                              val
                                                                                                              Specified with val
                              let
                                                                                                              Defined with let or indreln
v\_desc
                                                                                                          Value descriptions
                              \langle \mathbf{forall} \ tnvs.t\_multi \rightarrow p, (x \mathbf{of} \ names) \rangle
                                                                                                              Constructors
                              \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t, env\_tag \rangle
                                                                                                              Values
f\_desc
                              \langle  forall tnvs.p \rightarrow t, (x  of names) \rangle
                      Fields
xs
                      x_1 \dots x_n
\Sigma^{\mathcal{C}}
                                                                                                          Typeclass constraints
                            \{(p_1 t_1), \dots, (p_n t_n)\}
\Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n
                                                                                                  Μ
\Sigma^{\mathcal{N}}
                     ::=
                                                                                                         Nexp constraint lists
                              \{ nec_1, \dots, nec_n \} 
 \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n 
                                                                                                  Μ
E
                                                                                                          Environments
                      \begin{array}{ll} \mid & \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \\ \mid & E_{1} \uplus E_{2} \\ \mid & \epsilon \end{array} 
                                                                                                  Μ
                                                                                                  Μ
E^{\mathbf{X}}
                                                                                                          Value environments
                      Μ
```

```
E^{\mathrm{F}}
                                                                                                                      Field environments
                            ::=
                                       \begin{array}{l} \{x_1 \mapsto f\_desc_1, \, ..\,, x_n \mapsto f\_desc_n\} \\ E_1^{\scriptscriptstyle \mathrm{F}} \uplus \, ..\, \uplus E_n^{\scriptscriptstyle \mathrm{F}} \end{array} 
                                                                                                             Μ
E^{\mathrm{M}}
                                                                                                                      Module environments
                                      \{x_1 \mapsto E_1, \dots, x_n \mapsto E_n\}
E^{\mathrm{P}}
                                                                                                                      Path environments
                                       \{x_1 \mapsto p_1, \dots, x_n \mapsto p_n\} 
E_1^{\mathrm{P}} \uplus \dots \uplus E_n^{\mathrm{P}} 
                                                                                                             Μ
E^{\scriptscriptstyle 
m L}
                                                                                                                      Lexical bindings
                              \begin{vmatrix} \{x_1 \mapsto t_1, \dots, x_n \mapsto t_n\} \\ \{x_1^l \mapsto t_1, \dots, x_n^l \mapsto t_n\} \\ E_1^L \uplus \dots \uplus E_n^L \end{vmatrix} 
                                                                                                             Μ
tc\_abbrev
                                                                                                                      Type abbreviations
                                       .\,t
tc\_def
                                                                                                                      Type and class constructor definitions
                                       tnvs\ tc\_abbrev
                                                                                                                           Type constructors
\Delta
                                                                                                                      Type constructor definitions
                            ::=
                                       \{p_1 \mapsto tc\_def_1, ..., p_n \mapsto tc\_def_n\}
                                       \Delta_1 \uplus \Delta_2
                                                                                                             Μ
\delta
                                                                                                                      Typeclass definitions
                                      \{p_1 \mapsto xs_1, \dots, p_n \mapsto xs_n\}
\delta_1 \uplus \delta_2
                                                                                                             Μ
inst
                                                                                                                      A typeclass instance, t must not contain nested ty
                                      C \Rightarrow (p t)
Ι
                                                                                                                      Global instances
                                      \{inst_1, \dots, inst_n\}I_1 \cup I_2
                                                                                                             Μ
D
                                                                                                                      Global type definition store
                                       \langle \Delta, \delta, I \rangle
                                       D_1 \uplus D_2
                                                                                                             Μ
                                                                                                             Μ
terminals
```

>=

```
==>
                                                                                                                                                             <|
                                          |\rangle
                                                                                                                                                             |>
formula
                                         judgement
                                          formula_1 .. formula_n
                                          E^{\mathrm{M}}(x) \triangleright E
                                                                                                                                                             Module lookup
                                          E^{P}(x) \triangleright p
                                                                                                                                                             Path lookup
                                          E^{\mathrm{F}}(x) \triangleright f_{-}desc
                                                                                                                                                             Field lookup
                                          E^{\mathbf{X}}(x) \triangleright v_{-}desc
                                                                                                                                                             Value lookup
                                          E^{\mathrm{L}}(x) \triangleright t
                                                                                                                                                             Lexical binding lookup
                                          \Delta(p) \triangleright tc\_def
                                                                                                                                                             Type constructor lookup
                                          \delta(p) \triangleright xs
                                                                                                                                                             Type constructor lookup
                                         \mathbf{dom}\left(E_{1}^{\mathrm{M}}
ight)\cap\mathbf{dom}\left(E_{2}^{\mathrm{M}}
ight)=\emptyset \ \mathbf{dom}\left(E_{1}^{\mathrm{X}}
ight)\cap\mathbf{dom}\left(E_{2}^{\mathrm{X}}
ight)=\emptyset \ \mathbf{dom}\left(E_{1}^{\mathrm{F}}
ight)\cap\mathbf{dom}\left(E_{2}^{\mathrm{F}}
ight)=\emptyset
                                         \mathbf{dom}\left(E_{1}^{\mathrm{p}}\right)\,\cap\,\mathbf{dom}\left(E_{2}^{\mathrm{p}}\right)=\,\emptyset
                                         \begin{array}{l} \textbf{disjoint doms} \ (E_1^{\text{L}}, \ \ldots, E_n^{\text{L}}) \\ \textbf{disjoint doms} \ (E_1^{\text{X}}, \ \ldots, E_n^{\text{X}}) \end{array}
                                                                                                                                                             Pairwise disjoint domains
                                                                                                                                                             Pairwise disjoint domains
                                          compatible overlap (x_1 \mapsto t_1, ..., x_n \mapsto t_n)
                                                                                                                                                             (x_i = x_j) \Longrightarrow (t_i = t_j)
                                          \mathbf{duplicates}(tnvs) = \emptyset
                                          duplicates (x_1, ..., x_n) = \emptyset
                                          x \not\in \mathbf{dom}(E^{\mathrm{L}})
                                          x \not\in \mathbf{dom}(E^{X})
                                          x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                          p \not\in \mathbf{dom}(\delta)
                                          p \not\in \mathbf{dom}(\Delta)
                                          \mathbf{FV}(t) \subset tnvs
                                                                                                                                                             Free type variables
```

		$\mathbf{FV}(t_multi) \subset tnvs$ $\mathbf{FV}(\mathcal{C}) \subset tnvs$ $inst \mathbf{IN} I$ $(p t) \notin I$ $E_1^{\mathrm{L}} = E_2^{\mathrm{L}}$ $E_1^{\mathrm{X}} = E_2^{\mathrm{X}}$ $E_1^{\mathrm{F}} = E_2^{\mathrm{F}}$ $E_1 = E_2$ $\Delta_1 = \Delta_2$ $\delta_1 = \delta_2$ $I_1 = I_2$ $names_1 = names_2$ $t_1 = t_2$ $\sigma_1 = \sigma_2$ $p_1 = p_2$ $xs_1 = xs_2$ $tnvs_1 = tnvs_2$	Free type variables Free type variables
$convert_tnvars$::= 	$tnvars^l \leadsto tnvs \\ tnvar^l \leadsto tnv$	
$look_m$::=	$E_1(x_1^l \dots x_n^l) \triangleright E_2$	Name path lookup
$look_m_id$::=	$E_1(id) > E_2$	Module identifier lookup
$look_tc$::=	$E(id) \triangleright p$	Path identifier lookup
$check_t$::= 	$\begin{array}{l} \Delta \vdash t \mathbf{ok} \\ \Delta, tnv \vdash t \mathbf{ok} \end{array}$	Well-formed types Well-formed type/Nexps matching the application t
teq	::= 	$\Delta dash t_1 = t_2$	Type equality
$convert_typ$::= 	$\begin{array}{l} \Delta, E \vdash typ \leadsto t \\ \vdash Nexp \leadsto ne \end{array}$	Convert source types to internal types Convert and normalize numeric expressions
$convert_typs$::=	$\Delta, E \vdash typs \leadsto t_multi$	
$check_lit$::=	$\vdash \mathit{lit}:t$	Typing literal constants

 $typ\epsilon$

```
inst\_field
                                     ::=
                                                 \Delta, E \vdash \mathbf{field} \ id : p \ t\_args \rightarrow t \triangleright (x \ \mathbf{of} \ names)
                                                                                                                                                                   Field typing (also returns of
inst\_ctor
                                                 \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
                                                                                                                                                                   Data constructor typing (a
inst\_val
                                                 \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                                                                                                                   Typing top-level bindings,
not\_ctor
                                                E, E^{\text{L}} \vdash x \mathbf{not} \mathbf{ctor}
                                                                                                                                                                   v is not bound to a data co
not\_shadowed
                                                 E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                                                                                                                   id is not lexically shadowed
check\_pat
                                                \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_2^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash pat\_aux: t \vartriangleright E_2^{\text{\tiny L}} \end{array}
                                                                                                                                                                   Typing patterns, building t
                                                                                                                                                                   Typing patterns, building t
id_{-}field
                                                 E \vdash id \, \mathbf{field}
                                                                                                                                                                   Check that the identifier is
id\_value
                                      ::=
                                                 E \vdash id \mathbf{value}
                                                                                                                                                                   Check that the identifier is
check\_exp
                                                 \Delta, E, E^{\mathsf{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Typing expressions, collect
                                                \Delta, E, E^{\mathrm{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Typing expressions, collect
                                                \Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}
                                                                                                                                                                   Build the environment for
                                                \Delta, E, E_1^{\mathrm{L}} \vdash \mathbf{list} \ qbind_1 ... \ qbind_n \triangleright E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}
                                                                                                                                                                   Build the environment for
                                                \Delta, E, E^{\tilde{L}} \vdash funcl \rhd \{x \mapsto t\}, \Sigma^{C}, \Sigma^{\tilde{N}}
                                                                                                                                                                   Build the environment for
                                                \Delta, E, E_1^{\scriptscriptstyle L} \vdash letbind \triangleright E_2^{\scriptscriptstyle L}, \Sigma^{\circ}, \Sigma^{\circ}
                                                                                                                                                                   Build the environment for
check\_rule
                                                \Delta, E, E^{\text{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Build the environment for
check\_texp\_tc
                                                xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathrm{P}}
                                       Extract the type construct
check\_texps\_tc
                                                xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                                                                                   Extract the type construct
check\_texp
                                                \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                                                                                                                                                   Check a type definition, wi
```

 $xs, \Delta, E \vdash td_1 ... td_n \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle$

 $check_texps$

```
convert\_class
                                      ::=
                                             \delta, E \vdash id \leadsto p
                                       Lookup a type class
solve\_class\_constraint
                                      ::=
                                              I \vdash (p\ t)\ \mathbf{IN}\ \mathcal{C}
                                                                                              Solve class constraint
solve\_class\_constraints
                                              I \vdash \Sigma^{\mathcal{C}} \, \triangleright \, \mathcal{C}
                                                                                              Solve class constraints
check\_val\_def
                                      ::=
                                              \Delta, I, E \vdash val\_def \triangleright E^{\mathbf{X}}
                                       Check a value definition
check\_t\_instance
                                      ::=
                                              \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance
                                                                                              Check that t be a typeclass instance
                                       check\_defs
                                             \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2
                                                                                              Check a definition
                                             \overline{z_i}^j, D_1, E_1 \vdash defs \triangleright D_2, E_2
                                                                                              Check definitions, given module path, defi
judgement
                                      ::=
                                              convert\_tnvars
                                              look\_m
                                              look\_m\_id
                                              look\_tc
                                              check\_t
                                              teq
                                              convert\_typ
                                              convert\_typs
                                              check\_lit
                                              inst\_field
                                              inst\_ctor
                                              inst\_val
                                              not\_ctor
                                              not\_shadowed
                                              check\_pat
                                              id\_field
                                              id\_value
                                              check\_exp
                                              check\_rule
                                              check\_texp\_tc
```

check_texps_tc
check_texp
check_texps
convert_class

 $check_val_def$

 $solve_class_constraint$ $solve_class_constraints$

```
check\_defs
user\_syntax
                  ::=
                         n
                         num
                         nat
                         hex
                         bin
                         string
                         backtick\_string
                         regexp
                         \boldsymbol{x}
                         ix
                         l
                         x^l
                         ix^l
                         \alpha
                         \alpha^l
                         N
                         N^l
                         id
                         tnv
                         tnvar^l
                         tnvs
                         tnvars^l
                         Nexp\_aux
                         Nexp
                         Nexp\_constraint\_aux
                         Nexp\_constraint
                         typ\_aux
                         typ
                         lit\_aux
                         lit
                         pat\_aux
                         pat
                         fpat
                         exp\_aux
                         exp
                         qbind
                         fexp
                         fexps
```

pexp

 $check_t_instance$

```
psexp
tannot?
funcl\_aux
letbind\_aux
letbind
funcl
name\_t
name\_ts
rule\_aux
rule
witness?
check?
functions?
indreln\_name\_aux
indreln\_name
typs
ctor\_def
texp
name?
td
c
cs
c\_pre
typschm
instschm
target
open\_import
	au^{?}
lemma\_typ
lemma\_decl
dexp
declare\_arg
component
termination\_setting
exhaustivity\_setting
elim\_opt
fixity\_decl
target\_rep\_rhs
target\_rep\_lhs
declare\_def
val\_def
ascii\_opt
instance\_decl
class\_decl
val\_spec
```

```
def\_aux
defs
p
ne
t\_args
t\_multi
nec
names
\mathcal{C}
env\_tag
v\_desc
f\_desc
\Sigma^{\mathcal{C}}
\Sigma^{\mathcal{N}}
 E
E^{\mathbf{X}}
E^{\mathrm{F}}
E^{\scriptscriptstyle \mathrm{M}}
E^{\scriptscriptstyle \mathrm{P}}
E^{\scriptscriptstyle \rm L}
tc\_abbrev
tc\_def
\Delta
\delta
inst
I
 D
terminals
formula
```

 $tnvars^l \leadsto tnvs$

$$\frac{tnvar_1^l \leadsto tnv_1 \quad .. \quad tnvar_n^l \leadsto tnv_n}{tnvar_1^l \ .. \ tnvar_n^l \leadsto tnv_1 \ .. \ tnv_n} \quad \text{convert_tnvars_none}$$

 $tnvar^l \leadsto tnv$

$$\frac{}{\alpha\; l \leadsto \alpha} \quad \begin{array}{ll} \text{convert_tnvar_a} \\ \\ \hline N\; l \leadsto N \end{array}$$

 $E_1(x_1^l ... x_n^l) \triangleright E_2$ Name path lookup

$$\overline{E(\,) \triangleright E}$$
 LOOK_M_NONE

$$\frac{E^{\mathrm{M}}(x) \triangleright E_{1}}{E_{1}(\overline{y_{i}^{l}}^{i}) \triangleright E_{2}} \\ \frac{E_{1}(\overline{y_{i}^{l}}^{i}) \triangleright E_{2}}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle (x \, l \, \overline{y_{i}^{l}}^{i}) \triangleright E_{2}} \quad \text{LOOK_M_SOME}$$

 $E_1(id) \triangleright E_2$ Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i x l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i x l_1 l_2) \triangleright E_2} \quad \text{LOOK_M_ID_ALL}$$

 $E(id) \triangleright p$ Path identifier lookup

$$\frac{E(\overline{y_{i}^{l}}^{i}) \triangleright \langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle}{E^{\mathbf{P}}(x) \triangleright p}$$

$$E(\overline{y_{i}^{l}}^{i} x l_{1} l_{2}) \triangleright p$$
LOOK_TC_ALL

 $\Delta \vdash t \text{ ok}$ Well-formed types

$$\frac{\Delta \vdash \alpha \, \mathbf{ok}}{\Delta \vdash t_1 \, \mathbf{ok}} \quad \text{CHECK_T_VAR}$$

$$\frac{\Delta \vdash t_1 \, \mathbf{ok}}{\Delta \vdash t_1 \to t_2 \, \mathbf{ok}} \quad \text{CHECK_T_FN}$$

$$\frac{\Delta \vdash t_1 \, \mathbf{ok} \quad \dots \quad \Delta \vdash t_n \, \mathbf{ok}}{\Delta \vdash t_1 * \dots * t_n \, \mathbf{ok}} \quad \text{CHECK_T_TUP}$$

$$\frac{\Delta(p) \rhd tnv_1 \dots tnv_n \ tc_abbrev}{\Delta, tnv_1 \vdash t_1 \, \mathbf{ok} \quad \dots \quad \Delta, tnv_n \vdash t_n \, \mathbf{ok}}$$

$$\frac{\Delta \vdash p \ t_1 \dots t_n \, \mathbf{ok}}{\Delta \vdash p \ t_1 \dots t_n \, \mathbf{ok}} \quad \text{CHECK_T_APP}$$

 $\Delta, tnv \vdash t$ ok Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \, \mathbf{ok}}{\Delta, \alpha \vdash t \, \mathbf{ok}} \quad \text{CHECK_TLEN_T}$$

$$\frac{\Delta, N \vdash ne \, \mathbf{ok}}{\Delta, N \vdash ne \, \mathbf{ok}} \quad \text{CHECK_TLEN_LEN}$$

 $\Delta \vdash t_1 = t_2$ Type equality

$$\frac{\Delta \vdash t \text{ ok}}{\Delta \vdash t = t} \quad \text{TEQ_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ_SYM}$$

$$\frac{\Delta \vdash t_1 = t_2}{\Delta \vdash t_2 = t_3} \quad \text{TEQ_TRANS}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_1 = t_3} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_2 = t_4} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n}{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n} \quad \text{TEQ_TUP}$$

$$\frac{\Delta(p) \rhd \alpha_{1} ... \alpha_{n}}{\Delta \vdash t_{1} = u_{1} ... \Delta \vdash t_{n} = u_{n}} \underbrace{\Delta \vdash t_{1} = u_{1} ... \Delta \vdash t_{n} = u_{n}}_{\Delta \vdash p \ t_{1} ... t_{n} = p \ u_{1} ... u_{n}}$$
 TEQ_APP
$$\frac{\Delta(p) \rhd \alpha_{1} ... \alpha_{n} .u}{\Delta \vdash p \ t_{1} ... t_{n} = \{\alpha_{1} \mapsto t_{1} ... \alpha_{n} \mapsto t_{n}\}(u)}$$
 TEQ_EXPAND
$$\underbrace{\frac{ne = \mathbf{normalize} (ne')}{\Delta \vdash ne = ne'}}_{\Delta \vdash ne = ne'}$$
 TEQ_NEXP

 $\Delta, E \vdash typ \leadsto t$ Convert source types to internal types

 $\vdash Nexp \leadsto ne$ Convert and normalize numeric expressions

 $\Delta, E \vdash typs \leadsto t_multi$

$$\frac{\Delta, E \vdash typ_1 \leadsto t_1 \quad .. \quad \Delta, E \vdash typ_n \leadsto t_n}{\Delta, E \vdash typ_1 \ast .. \ast typ_n \leadsto (t_1 \ast .. \ast t_n)} \quad \text{CONVERT_TYPS_ALL}$$

 $\vdash lit: t$ Typing literal constants

```
CHECK_LIT_FALSE
                                                                            \vdash false l : \_bool
                                                                                                                                 CHECK_LIT_NUM
                                                                               \vdash num \ l : \_\_num
                                                                         nat = \mathbf{bitlength}(hex)
                                                                                                                                              CHECK_LIT_HEX
                                                                  \vdash hex \ l : \_\_\mathbf{vector} \ nat \ \_\_\mathbf{bit}
                                                                          nat = \mathbf{bitlength}(bin)
                                                                                                                                               CHECK_LIT_BIN
                                                                    \vdash bin \ l : \_	extbf{vector} \ nat \ \_	extbf{bit}
                                                                                                                                 CHECK_LIT_STRING
                                                                       \vdash string \ l : \_string
                                                                                                                             CHECK_LIT_UNIT
                                                                                  \vdash () l : \_\mathbf{unit}
                                                                                                                              CHECK_LIT_BITZERO
                                                                       \overline{\vdash \mathbf{bitzero}\ l : \_\mathbf{bit}}
                                                                                                                              CHECK_LIT_BITONE
                                                                         \vdash bitone l : __bit
    \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                                               Field typing (also returns canonical field names)
  \begin{split} E(\,\overline{x_{i}^{l}}^{i}\,) & \rhd \left\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \right\rangle \\ E^{\mathrm{F}}(y) & \rhd \left\langle \, \mathbf{forall} \, tnv_{1} \ldots tnv_{n}.p \, \rightarrow \, t, (z \, \mathbf{of} \, names) \right\rangle \\ \frac{\Delta \vdash t_{1} \, \mathbf{ok} \quad \ldots \quad \Delta \vdash t_{n} \, \mathbf{ok}}{\Delta, E \vdash \, \mathbf{field} \, \overline{x_{i}^{l}}^{i} \, y \, l_{1} \, l_{2} : p \, t_{1} \ldots t_{n} \, \rightarrow \, \{tnv_{1} \mapsto t_{1} \ldots tnv_{n} \mapsto t_{n}\}(t) \, \rhd \, (z \, \mathbf{of} \, names) \end{split} \quad \text{INST\_FIELD\_ALL}
    \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \to p\ t\_args \triangleright (x\ \mathbf{of}\ names) Data constructor typing (also returns canonical constru
                                    E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 ... \ tnv_n.t_-multi \to p, (z \mathbf{of} \ names) \rangle
\Delta \vdash t_1 \mathbf{ok} ... \Delta \vdash t_n \mathbf{ok}
                                                                                                                                                                                                                          INST_CTOR_ALL
\overline{\Delta, E \vdash \mathbf{ctor} \, \overline{x_i^l}^{\, l}} \, y \, l_1 \, l_2 : \{ tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n \} (t\_multi) \to p \, t_1 \dots t_n \rhd (z \, \mathbf{of} \, names)
     \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                      Typing top-level bindings, collecting typeclass constraints
                          E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle 
E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, \ env\_tag \rangle
                           \Delta \vdash t_1 \mathbf{ok} \quad .. \quad \Delta \vdash t_n \mathbf{ok}
                            \frac{\sigma = \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}}{\Delta, E \vdash \mathbf{val} \, \overline{x_i^l}^i \, y \, l_1 \, l_2 : \sigma(t) \triangleright \{(p_1 \, \sigma(tnv_1')), ..., (p_i \, \sigma(tnv_i'))\}} \quad \text{INST_VAL_ALL}
                          \sigma = \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}
      E, E^{\mathrm{L}} \vdash x \mathbf{not} \mathbf{ctor}
                                                              v is not bound to a data constructor
                                                                             \frac{E^{\text{L}}(x) \triangleright t}{E, E^{\text{L}} \vdash x \text{ not ctor}} \quad \text{NOT\_CTOR\_VAL}
                                                   \frac{x \not\in \mathbf{dom}\left(E^{\mathbf{X}}\right)}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}}\rangle, E^{\mathbf{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor}} \quad \text{NOT\_CTOR\_UNBOUND}
                    \frac{E^{\mathbf{X}}(x) \triangleright \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, env\_tag \rangle}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle, E^{\mathbf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}} \quad \text{NOT\_CTOR\_BOUND}
```

```
E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                          id is not lexically shadowed
                                                              \frac{x \not\in \mathbf{dom}\,(E^{\mathrm{L}})}{E^{\mathrm{L}} \vdash x \ l_1 \ l_2 \ \mathbf{not} \ \mathbf{shadowed}} \quad \text{NOT\_SHADOWED\_SING}
                                                                                                                                                     NOT_SHADOWED_MULTI
                                                 \overline{E^{\text{L}} \vdash x_1^l \dots x_n^l \cdot y^l \cdot z^l \ l \ \textbf{not shadowed}}
   \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}
                                                                             Typing patterns, building their binding environment
                                                                   \frac{\Delta, E, E_1^{\text{L}} \vdash pat\_aux : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash pat\_aux \ l : t \triangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_ALL}
\Delta, E, E_1^{\scriptscriptstyle L} \vdash pat\_aux : t \triangleright E_2^{\scriptscriptstyle L}
                                                                                        Typing patterns, building their binding environment
                                                                      \frac{\Delta \vdash t \text{ ok}}{\Delta, E, E^{\text{L}} \vdash \_: t \triangleright \{\}} \quad \text{CHECK\_PAT\_AUX\_WILD}
                                                                       \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}
                                            \frac{x \not\in \mathbf{dom}\,(E_2^{\mathrm{L}})}{\Delta, E, E_1^{\mathrm{L}} \vdash (pat\,\mathbf{as}\,x\,l): t \vartriangleright E_2^{\mathrm{L}} \uplus \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_AS}
                                                                     \Delta, E, E_1^{\mathrm{L}} \vdash pat : t \triangleright E_2^{\mathrm{L}}
                                                          \frac{\Delta, E, E_1 \vdash put : t \triangleright E_2}{\Delta, E \vdash typ \leadsto t}
\frac{\Delta, E, E_1^{\text{L}} \vdash (pat : typ) : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat : typ) : t \triangleright E_2^{\text{L}}}
CHECK_PAT_AUX_TYP
       \Delta, E \vdash \mathbf{ctor}\ id : (t_1 * ... * t_n) \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
        E^{\text{L}} \vdash id \text{ not shadowed}
       \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_1 : t_1 \, \triangleright \, E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_n : t_n \, \triangleright \, E_n^{\mathsf{L}}
       \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,..\,,E_n^{\scriptscriptstyle{\mathrm{L}}})
                                                                                                                                                                              CHECK_PAT_AUX_IDENT_CONSTR
                \Delta, E, E^{\text{\tiny L}} \vdash id \; pat_1 \ldots pat_n : p \; t\_args \; \triangleright \; E_1^{\text{\tiny L}} \uplus \; \ldots \; \uplus \; E_n^{\text{\tiny L}}
                                                                           \Delta \vdash t \mathbf{ok}
                                                        \frac{E, E^{\mathsf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}}{\Delta, E, E^{\mathsf{L}} \vdash x \ l_1 \ l_2 \ : t \, \rhd \, \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_VAR}
                            \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \mathbf{of}\ names)}^i
                            \overline{\Delta, E, E^{\mathrm{L}} \vdash pat_i : t_i \triangleright E_i^{\mathrm{L}}}
                            \mathbf{disjoint}\,\mathbf{doms}\,(\,\overline{E_i^{\scriptscriptstyle L}}^{\,i}\,)
                           \mathbf{duplicates}\,(\,\overline{x_i}^{\,i}\,)=\,\emptyset
                            \frac{\Delta \operatorname{Approximation}(|x_i|) - |v|}{\Delta, E, E^{\text{L}} \vdash \langle | \overline{id_i = pat_i l_i}^i; ?| \rangle : p \ t\_args \triangleright \uplus \overline{E_i^{\text{L}}}^i} \quad \text{CHECK\_PAT\_AUX\_RECORD}
                  \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_1 : t \, \rhd \, E_1^{\mathrm{L}} \quad \dots \quad \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_n : t \, \rhd \, E_n^{\mathrm{L}}
                   \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
        \frac{\mathbf{length}\,(pat_1\dots pat_n) = nat}{\Delta,E,E^\mathtt{L} \vdash [|pat_1;\dots;pat_n\,;^2|] : \_\mathbf{vector}\,nat\;t \,\triangleright\,E^\mathtt{L}_1 \uplus \dots \uplus E^\mathtt{L}_n} \quad \text{CHECK\_PAT\_AUX\_VECTOR}
\Delta, E, E^{\mathtt{L}} \vdash \mathit{pat}_1 : \_\mathtt{vector} \ \mathit{ne}_1 \ t \, \rhd \, E^{\mathtt{L}}_1 \quad \dots \quad \Delta, E, E^{\mathtt{L}} \vdash \mathit{pat}_n : \_\mathtt{vector} \ \mathit{ne}_n \ t \, \rhd \, E^{\mathtt{L}}_n
disjoint doms (E_1^L, \ldots, E_n^L)
ne' = ne_1 + \dots + ne_n
                                  \frac{\Delta}{\Delta, E, E^{\mathrm{L}} \vdash [|pat_{1} \dots pat_{n}|] : \_\mathbf{vector} \ ne' \ t \rhd E_{1}^{\mathrm{L}} \uplus \ \dots \ \uplus E_{n}^{\mathrm{L}}}
                                                                                                                                                                                                                                            CHECK_PAT_AUX_VECTOR
                    \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_1 : t_1 \, \rhd \, E_1^{\mathsf{L}} \quad .... \quad \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_n : t_n \, \rhd \, E_n^{\mathsf{L}}
                    \frac{\text{disjoint doms}\left(E_{1}^{\text{L}},\, \ldots,\, E_{n}^{\text{L}}\right)}{\Delta,\, E,\, E^{\text{L}} \vdash \left(pat_{1},\, \ldots,\, pat_{n}\right) :\, t_{1} \ast\, \ldots\, \ast\, t_{n}\, \triangleright\, E_{1}^{\text{L}} \uplus\, \ldots\, \uplus\, E_{n}^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_TUP}
```

```
\Delta \vdash t \mathbf{ok}
                          \Delta, E, E^{\mathsf{L}} \vdash pat_1 : t \triangleright E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash pat_n : t \triangleright E_n^{\mathsf{L}}
                         \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,..\,,E_n^{\scriptscriptstyle{\mathrm{L}}})
                              \Delta, E, E^{\text{L}} \vdash [pat_1; ...; pat_n;^?] : \_\textbf{list} \ t \triangleright E_1^{\text{L}} \uplus ... \uplus E_n^{\text{L}} CHECK_PAT_AUX_LIST
                                                               \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat) : t \vartriangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_PAREN}
                                         \begin{array}{c} \Delta, E, E_{1}^{\text{L}} \vdash pat_{1} : t \vartriangleright E_{2}^{\text{L}} \\ \Delta, E, E_{1}^{\text{L}} \vdash pat_{2} : \_\textbf{list} \ t \vartriangleright E_{3}^{\text{L}} \\ \textbf{disjoint doms} \left(E_{2}^{\text{L}}, E_{3}^{\text{L}}\right) \\ \hline \Delta, E, E_{1}^{\text{L}} \vdash pat_{1} :: pat_{2} : \_\textbf{list} \ t \vartriangleright E_{2}^{\text{L}} \uplus E_{3}^{\text{L}} \end{array} \quad \text{CHECK\_PAT\_AUX\_CONS} \end{array}
                                                                         \frac{\vdash lit:t}{\Delta,E,E^{\text{L}}\vdash lit:t\,\vartriangleright\,\big\{\,\big\}}\quad\text{CHECK\_PAT\_AUX\_LIT}
                                                                    E, E^{\text{\tiny L}} \vdash x \operatorname{\mathbf{not}} \operatorname{\mathbf{ctor}}
                          \frac{E, E^{\perp} \vdash x \text{ not ctor}}{\Delta, E, E^{\perp} \vdash x \ l + num : \_\text{num} \ \triangleright \{x \mapsto \_\text{num}\}}
  E \vdash id \mathbf{field}
                                                Check that the identifier is a permissible field identifier
                                                                  \frac{E^{\mathrm{F}}(x) \rhd f\_desc}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \vdash x \ l_1 \ l_2 \, \mathbf{field}} \quad \text{ID\_FIELD\_EMPTY}
                                                                                       E^{\mathrm{M}}(x) \triangleright E
                                                                                       x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                                           \frac{E \vdash \overline{y_{i}^{l}}.^{i} z^{l} \ l_{2} \, \mathbf{field}}{\langle E^{\text{\tiny{M}}}, E^{\text{\tiny{P}}}, E^{\text{\tiny{F}}}, E^{\text{\tiny{X}}} \rangle \vdash x \ l_{1}. \ \overline{y_{i}^{l}}.^{i} \ z^{l} \ l_{2} \, \mathbf{field}} \quad \text{\tiny{ID\_FIELD\_CONS}}
  E \vdash id value
                                                   Check that the identifier is a permissible value identifier
                                                                \frac{E^{\mathbf{X}}(x) \triangleright v\_desc}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{value}} \quad \text{id\_value\_empty}
                                                                                     E^{\mathrm{M}}(x) \triangleright E
                                                                                    x \notin \mathbf{dom}(E^{X})
                                                        \frac{E \vdash \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}}{\langle E^{\text{M}}, E^{\text{P}}, E^{\text{F}}, E^{\text{X}} \rangle \vdash x \ l_1. \ \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}} \quad \text{ID_VALUE\_CONS}
  \Delta, E, E^{\text{L}} \vdash exp: t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Typing expressions, collecting typeclass and index constraints
                                                             \frac{\Delta, E, E^{\mathsf{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\mathsf{L}} \vdash exp\_aux \ l : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_ALL}
\Delta, E, E^{\text{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Typing expressions, collecting typeclass and index constraints
                                                             \frac{E^{\mathrm{L}}(x) \triangleright t}{\Delta, E, E^{\mathrm{L}} \vdash x \ l_{1} \ l_{2} : t \triangleright \{\}, \{\}} \quad \text{CHECK\_EXP\_AUX\_VAR}
                                                       \overline{\Delta,E,E^{\text{\tiny L}} \vdash N: \_\mathbf{num} \ \triangleright \ \{\ \},\{\ \}} \quad \text{CHECK\_EXP\_AUX\_NVAR}
                            E^{\mathrm{L}} \vdash id \, \mathbf{not} \, \mathbf{shadowed}
                            E \vdash id \text{ value}
                           \Delta, E \vdash \mathbf{ctor} \ id : t\_multi \to p \ t\_args \rhd (x \ \mathbf{of} \ names)
CHECK\_EXP\_AUX\_CTOR
                               \Delta, E, E^{\text{L}} \vdash id : \mathbf{curry}(t\_multi, p \ t\_args) \triangleright \{\}, \{\}
```

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E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                        E \vdash id \text{ value}
                                                                        \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                    \frac{-}{\Delta, E, E^{\text{\tiny L}} \vdash id: t \, \triangleright \, \Sigma^{\mathcal{C}}, \{\,\,\}} \quad \text{CHECK\_EXP\_AUX\_VAL}
                           \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                           disjoint doms (E_1^L, ..., E_n^L)
                                                                                                                                                                                                                    CHECK_EXP_AUX_FN
       \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{fun} \ pat_1 \dots pat_n \rightarrow exp \ l : \mathbf{curry} \left( (t_1 * \dots * t_n), u \right) \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                           \overline{\Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{pat}_i : t \, \triangleright \, E_i^{\scriptscriptstyle L}}^{\, i}
                                           \frac{1}{\Delta, E, E^{\mathsf{L}} \uplus E_{i}^{\mathsf{L}} \vdash exp_{i} : u \rhd \Sigma^{\mathcal{C}}_{i}, \Sigma^{\mathcal{N}}_{i}}
     \Delta, E, E^{\text{L}} \vdash \mathbf{function} \mid^{?} \overline{pat_{i} \rightarrow exp_{i} l_{i}}^{i} \mathbf{end} : t \rightarrow u \triangleright \overline{\Sigma^{C}_{i}}^{i}, \overline{\Sigma^{N}_{i}}^{i}
                                                    \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \to t_2 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                    \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                \frac{1}{\Delta,E,E^{\text{L}} \vdash exp_1 \ exp_2 : t_2 \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2} \quad \text{Check_exp_aux_app}
                                      \begin{array}{l} \Delta, E, E^{\mathrm{L}} \vdash (ix) : t_1 \to t_2 \to t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                      \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
                                                                                                                                                                                                       CHECK_EXP_AUX_INFIX_APP1
\overline{\Delta, E, E^{\text{\tiny L}} \vdash \textit{exp}_{1} \textit{ ix } \textit{l} \textit{exp}_{2} : \textit{t}_{3} \vartriangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2} \cup \Sigma^{\mathcal{C}}_{3}, \Sigma^{\mathcal{N}}_{1} \cup \Sigma^{\mathcal{N}}_{2} \cup \Sigma^{\mathcal{N}}_{3}}
                                                                                                                                                    \Delta, E, E^{\mathrm{L}} \vdash x : t_1 \to t_2 \to t_3 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                                                                                                    \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                                                                                                                                    \Delta, E, E^{\mathsf{L}} \vdash exp_2 : t_2 \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
<<no parses (char 18): TD,E,E_l |- exp1 '***x' l exp2 : t3 gives S_c1 union S_c2 union S_c3,</pre>
                               \overline{\Delta}, E \vdash \mathbf{field} \ id_i : p \ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of} \ names)
                               \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                               duplicates (\overline{x_i}^i) = \emptyset
                               names = \{ \overline{x_i}^i \}
                                                                                                                                                                                   CHECK_EXP_AUX_RECORD
                      \Delta, E, E^{\text{\tiny L}} \vdash \langle | \overline{id_i} = \overline{\exp_i \ l_i}^i \ ; ^? \ l | \rangle : \overline{p \ t_{-}} \overline{args} \, \rhd \, \overline{\overline{\Sigma^{\mathcal{C}}_i}}^i . \overline{\overline{\Sigma^{\mathcal{N}_i}}^i}
                                          \overline{\Delta}, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)
                                          \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                          \mathbf{duplicates}(\overline{x_i}^i) = \emptyset
                                          \Delta, E, E^{\mathsf{L}} \vdash exp : p \ t_{-}args \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'}
\Delta, E, E^{\text{L}} \vdash \langle | exp \ \overline{\textbf{with}} \ \overline{id_i = exp_i \ l_i}^i \ ;^? \ l | \rangle : p \ t\_args \rhd \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_i}^i, \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}}_i}^i CHECK_EXP_AUX_RECUP
                     \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \rhd \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n}
                     length(exp_1 ... exp_n) = nat
\overline{\Delta}, E, E^{\text{L}} \vdash [|exp_1; ...; exp_n;^?|] : \_\text{vector } nat \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n]
                                    \Delta, E, E^{\mathrm{L}} \vdash exp : \_\mathbf{vector} \ ne' \ t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                   \vdash Nexp \leadsto ne
                                                                                                                                                                      CHECK_EXP_AUX_VECTORGET
                          \overline{\Delta, E, E^{\text{L}} \vdash exp.(Nexp) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne\langle ne'\}\}}
                                              \Delta, E, E^{L} \vdash exp : \_vector ne' t \triangleright \Sigma^{C}, \Sigma^{N}
                                              \vdash Nexp_1 \leadsto ne_1
                                              \vdash Nexp_2 \leadsto ne_2
                                              ne = ne_2 + (-ne_1)
                                                                                                                                                                                                                    CHECK_EXP_AUX_VECTORSUB
\Delta, E, E^{\text{L}} \vdash exp.(Nexp_1..Nexp_2) : \overline{ \_vector ne \ t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne_1 \langle ne_2 \langle ne' \} \} }
```

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E \vdash id \mathbf{field}
                                              \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                              \Delta, E, E^{\mathrm{L}} \vdash exp : p \ t\_args \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                                                                CHECK_EXP_AUX_FIELD
                                                                      \Delta, E, E^{\text{L}} \vdash exp.id : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                           \overline{\Delta, E, E^{\text{L}} \vdash pat_i : t \triangleright E_i^{\text{L}}}^i
                                                                           \Delta, E, E^{\text{L}} \vdash \mathbf{match} \ exp \ \mathbf{with} \ |^{?} \ \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ l \ \mathbf{end} : u \rhd \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_{i}}^{i}, \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}_{i}}}^{i}
                                                                         \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                         \Delta, E \vdash typ \leadsto t
                                                                                                                                                                                      CHECK_EXP_AUX_TYPED
                                                             \overline{\Delta, E, E^{\text{L}} \vdash (exp : tup) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                  \Delta, E, E_1^{\mathrm{L}} \vdash letbind \triangleright E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                 \Delta, E, E_1^{\mathrm{L}} \uplus E_2^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                                                                                                                                                                                                                  CHECK_EXP_AUX_LET
                           \Delta, E, E_1^{\text{L}} \vdash \text{let } \textit{letbind } \text{in } \textit{exp} : t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2
\frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t_n \rhd \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash (exp_1, \dots, exp_n) : t_1 * \dots * t_n \rhd \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_TUP}
 \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \vartriangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash [exp_1; \; ..; exp_n \; ;^?] : \_\mathbf{list} \; t \vartriangleright \Sigma^{\mathcal{C}}_1 \; \cup \; .. \; \cup \; \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \; \cup \; .. \; \cup \; \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_LIST}
                                                                        \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                      \Delta, E, E^{\perp} \vdash (exp) : t \triangleright \Sigma^{\mathcal{C}} \cdot \Sigma^{\mathcal{N}} CHECK_EXP_AUX_PAREN
                                                      \frac{\Delta, E, E^{\text{L}} \vdash exp: t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\text{L}} \vdash \mathbf{begin} \ exp \ \mathbf{end}: t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_BEGIN}
                                                                            \Delta, E, E^{\mathsf{L}} \vdash exp_1 : \_\mathbf{bool} \triangleright \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1}
                                                                            \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                                                            \Delta, E, E^{\mathsf{L}} \vdash exp_3 : t \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
\overline{\Delta, E, E^{\scriptscriptstyle L} \vdash \mathbf{if} \ exp_1 \, \mathbf{then} \ exp_2 \, \mathbf{else} \ exp_3 : t \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2 \, \cup \, \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \, \cup \, \Sigma^{\mathcal{N}}_2 \, \cup \, \Sigma^{\mathcal{N}}_3}
                                                             \Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{exp}_1 : t \, \rhd \, \Sigma^{\mathcal{C}}_{-1}, \Sigma^{\mathcal{N}}_{-1}
                                                              \Delta, E, E^{\mathsf{L}} \vdash exp_2 : \_\mathbf{list} \ t \, \triangleright \, \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                        \overline{\Delta, E, E^{\text{L}} \vdash exp_1 :: exp_2 : \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_CONS}
                                                                                 \frac{\vdash lit:t}{\Delta,E,E^{\text{L}}\vdash lit:t\,\vartriangleright\,\{\,\},\{\,\}}\quad\text{CHECK\_EXP\_AUX\_LIT}
                            \Delta \vdash t_i \mathbf{ok}^i
                             \Delta, E, E^{L} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_1 : t \rhd \Sigma^{C}_1, \Sigma^{N}_1
                             \Delta, E, E^{\mathsf{L}} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_2 : \_bool \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                             disjoint doms (E^{L}, \{\overline{x_i \mapsto t_i}^i\})
                             E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                            x_i \not\in \mathbf{dom}(E^{\mathbf{X}})^{i}
                \overline{\Delta, E, E^{\text{L}} \vdash \{exp_1 | exp_2\} : \_\mathbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_SET\_COMP}
                                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp_2 : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3 \end{array}
                                                                                                                                                                                                                                                                                  CHECK_EXP_AUX
\Delta, E, E_1^{\text{\tiny L}} \vdash \{ exp_1 | \mathbf{forall} \ \overline{qbind_i}^{\ i} \ | exp_2 \} : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2 \ \cup \ \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \ \cup \ \Sigma^{\mathcal{N}}_3
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\Delta \vdash t \mathbf{ok}
  \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \triangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash \{exp_1; ...; exp_n;^?\} : \_\mathbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_SET}
                                            \frac{\Delta, E, E_1^{\text{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}_1}{\Delta, E, E_1^{\text{L}} \vdash \overline{qbind_i}^i \cdot exp : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2} \xrightarrow{\text{CHECK\_EXP\_AUX\_QUANT}} \Delta, E, E_1^{\text{L}} \vdash q \ \overline{qbind_i}^i \cdot exp : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2}
\Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass cons
                                                                                                    \overline{\Delta,E,E^{\text{L}} \vdash \rhd\{\,\},\{\,\}} \quad \text{CHECK\_LISTQUANT\_BINDING\_EMPTY}
                                                          \Delta \vdash t \mathbf{ok}
                                                  \frac{\Delta, E, E_1^{\text{\tiny L}} \uplus \{x \mapsto t\} \vdash \overline{qbind_i}^i \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1}{\text{disjoint doms}\left(\{x \mapsto t\}, E_2^{\text{\tiny L}}\right)} \\ \frac{\Delta, E, E_1^{\text{\tiny L}} \vdash x \ l \ \overline{qbind_i}^i \rhd \{x \mapsto t\} \uplus E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1}{\Delta, E, E_1^{\text{\tiny L}} \vdash x \ l \ \overline{qbind_i}^i \rhd \{x \mapsto t\} \uplus E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1}
                                                                                                                                                                                                                                                                                        CHECK\_LISTQUANT\_BINDING\_VAR
                                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                                \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}_2}
                                                               \frac{\mathbf{disjoint}\,\mathbf{doms}\,(E_3^{\mathtt{L}},E_2^{\mathtt{L}}) }{} 
              \frac{\Delta B_{\mathbf{J}}^{\mathbf{DIR}} \mathbf{GOHB} \left(E_{3}, E_{2}\right)}{\Delta, E, E_{1}^{\mathbf{L}} \vdash \left(pat \, \mathbf{IN} \, exp\right) \overline{qbind_{i}}^{i} \triangleright E_{2}^{\mathbf{L}} \uplus E_{3}^{\mathbf{L}}, \Sigma^{\mathcal{C}}_{1} \, \cup \, \Sigma^{\mathcal{C}}_{2}} \quad \text{CHECK\_LISTQUANT\_BINDING\_RESTR}
                                                          \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \triangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp : \_\textbf{list} \ t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                          \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{\mathit{qbind}_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_2
\frac{\mathbf{disjoint \ doms} \ (E_{3}^{\mathrm{L}}, E_{2}^{\mathrm{L}})}{\Delta, E, E_{1}^{\mathrm{L}} \vdash (pat \ \mathbf{MEM} \ exp) \ \overline{qbind_{i}}^{i} \rhd E_{2}^{\mathrm{L}} \uplus E_{3}^{\mathrm{L}}, \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2}}
     \Delta, E, E_1^{\text{L}} \vdash \mathbf{list} \ qbind_1 ... \ qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass
                                                                                                                                                                                                                                              CHECK\_QUANT\_BINDING\_EMPTY
                                                                                                    \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{list} \triangleright \{\}, \{\}}
                                                                           \begin{array}{l} \Delta, E, E_1^{\rm L} \vdash pat : t \vartriangleright E_3^{\rm L} \\ \Delta, E, E_1^{\rm L} \vdash exp : \_{\bf list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                                         \begin{array}{l} \Delta, E, E_1^{\text{L}} \uplus E_3^{\text{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}_2 \\ \textbf{disjoint doms}\left(E_3^{\text{L}}, E_2^{\text{L}}\right) \end{array}
      \Delta, E, E_1^{\text{L}} \vdash \overline{\textbf{list}\left(pat\, \mathbf{MEM}\, exp\right)\, \overline{qbind_i}^{\,\,i}} \, \triangleright \, E_2^{\text{L}} \uplus E_3^{\text{L}}, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2
      \Delta, E, E^{\text{L}} \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a function definition clause, collecting typed
                                                                 \begin{array}{l} \Delta, E, E^{\text{\tiny L}} \vdash pat_1: t_1 \vartriangleright E_1^{\text{\tiny L}} \quad \dots \quad \Delta, E, E^{\text{\tiny L}} \vdash pat_n: t_n \vartriangleright E_n^{\text{\tiny L}} \\ \Delta, E, E^{\text{\tiny L}} \uplus E_1^{\text{\tiny L}} \uplus \quad \dots \quad \uplus E_n^{\text{\tiny L}} \vdash exp: u \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \textbf{disjoint doms} \ (E_1^{\text{\tiny L}}, \ \dots, E_n^{\text{\tiny L}}) \end{array}
                                                                  \Delta, E \vdash typ \leadsto u
\frac{-\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}\sum_{j=1
```

CHECK_FUNCL_ANNOT

```
\begin{array}{lll} \Delta,\,E,\,E^{\scriptscriptstyle L}\vdash\,pat_1:t_1\,\vartriangleright\,E_1^{\scriptscriptstyle L}&\ldots&\Delta,\,E,\,E^{\scriptscriptstyle L}\vdash\,pat_n:t_n\,\vartriangleright\,E_n^{\scriptscriptstyle L}\\ \Delta,\,E,\,E^{\scriptscriptstyle L}\uplus\,E_1^{\scriptscriptstyle L}\uplus\,\ldots\,\uplus\,E_n^{\scriptscriptstyle L}\vdash\,exp:u\,\vartriangleright\,\Sigma^{\scriptscriptstyle \mathcal{C}},\Sigma^{\scriptscriptstyle \mathcal{N}} \end{array}
                               disjoint doms (E_1^{\text{L}}, \ldots, E_n^{\text{L}})
\overline{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                                                                                                                                                                                             CHECK_FUNCL_NOANNOT
   \Delta, E, E_1^{\text{L}} \vdash letbind \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a let binding, collecting typeclass and index con
                                                           \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                           \Delta, E \vdash typ \leadsto t
                                      \frac{\Delta, E \vdash \textit{typ} \leadsto \textit{t}}{\Delta, E, E_1^{\text{L}} \vdash \textit{pat} : \textit{typ} = \textit{exp} \; \textit{l} \mathrel{\triangleright} E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_VAL\_ANNOT}
                                         \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \rhd E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash exp : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \\ \frac{\Delta, E, E_1^{\text{L}} \vdash exp : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\text{L}} \vdash pat = exp \ l \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_VAL\_NOANNOT}
                                                  \frac{\Delta, E, E_1^{\text{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\text{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_FN}
     \Delta, E, E^{\text{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for an inductive relation clause, collecting typed
                                                                                                  \overline{\Delta \vdash t_i \, \mathbf{ok}}^i
                                                                                                  E_2^{\mathrm{L}} = \{ \overline{name_- t_i \to x \mapsto t_i}^i \}
\Delta, E, E_{1}^{\mathsf{L}} \uplus E_{2}^{\mathsf{L}} \vdash exp' : \_\mathbf{bool} \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'}
\Delta, E, E_{1}^{\mathsf{L}} \uplus E_{2}^{\mathsf{L}} \vdash exp_{1} : u_{1} \rhd \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \quad ... \quad \Delta, E, E_{1}^{\mathsf{L}} \uplus E_{2}^{\mathsf{L}} \vdash exp_{n} : u_{n} \rhd \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n}
\Delta, E, E_{1}^{\mathsf{L}} \vdash x_{1}^{\mathsf{l}} : \mathbf{forall} \overline{name\_t_{i}}^{i} \cdot exp' \Longrightarrow x \ l \ exp_{1} ... \ exp_{n} \ l' \rhd \{x \mapsto \mathbf{curry} ((u_{1} * ... * u_{n}), \_\mathbf{bool})\}, \Sigma^{\mathcal{C}'} \cup \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{1})
      xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathsf{P}} Extract the type constructor information
                                                                        tnvars^l \leadsto tnvs
                                                                         \Delta, E \vdash typ \leadsto t
                                                                        \mathbf{duplicates}(tnvs) = \emptyset
                                                                        \mathbf{FV}(t) \subset tnvs
      \frac{\overline{y_i}.^i \, x \not\in \mathbf{dom} \, (\Delta)}{\overline{y_i}.^i, \Delta, E \vdash \mathbf{tc} \, x \, l \, tnvars^l \, = typ \, \triangleright \, \{\overline{y_i}.^i \, x \mapsto tnvs \, .t\}, \{x \mapsto \overline{y_i}.^i \, x\}} \quad \text{CHECK\_TEXP\_TC\_ABBREV}
                                                                    tnvars^l \leadsto tnvs
                                                                    \mathbf{duplicates}(tnvs) = \emptyset
             \frac{\overline{y_i.}^i \, x \not\in \mathbf{dom} \, (\Delta)}{\overline{y_i}^i, \Delta, E_1 \vdash \mathbf{tc} \, x \, l \, tnvars^l \, \triangleright \, \{\overline{y_i.}^i \, x \mapsto tnvs \, \}, \{x \mapsto \overline{y_i.}^i \, x\}} \quad \text{CHECK\_TEXP\_TC\_ABSTRACT}
                                                                                                   tnvars^l \leadsto tnvs
                                                                                                   \mathbf{duplicates}\left(tnvs\right) = \emptyset
\frac{\overline{y_i.}^i\,x\not\in\mathbf{dom}\,(\Delta)}{\overline{y_i}^i,\Delta_1,E\vdash\mathbf{tc}\,x\,l\,tnvars^l\,=\langle|x_1^l:typ_1;\ldots;x_j^l:typ_j\,;^?|\rangle\,\rhd\,\{\overline{y_i.}^i\,x\mapsto tnvs\,\},\{x\mapsto\overline{y_i.}^i\,x\}}
                                                                                                                                                                                                                                                                                   CHECK_TEXP_TC_REC
                                                                                                tnvars^l \leadsto tnvs
                                                                                                 \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                                                                \overline{y_i}. i \ x \not\in \mathbf{dom}(\Delta)
\overline{\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l = |? \ ctor\_def_1| \dots |ctor\_def_j| \triangleright \{\overline{y_i.}^i \ x \mapsto tnvs \}, \{x \mapsto \overline{y_i.}^i \ x\}
                                                                                                                                                                                                                                                                             CHECK_TEXP_TC_VAR
      xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                                    Extract the type constructor information
                                                                                                                                                 CHECK_TEXPS_TC_EMPTY
                                                                        \overline{xs, \Delta, E \vdash \mathbf{tc} \triangleright \{\}, \{\}}
```

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xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E_2^{\mathrm{P}}
                     xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{ \}, E_2^{\mathrm{P}}, \{ \}, \{ \} \rangle \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_3, E_3^{\mathrm{P}} \\ \mathbf{dom} (E_2^{\mathrm{P}}) \cap \mathbf{dom} (E_3^{\mathrm{P}}) = \emptyset

    — CHECK_TEXPS_TC_ABBREV

                                     xs, \Delta_1, E \vdash \mathbf{tc} \ td \ \overline{td_i}^i \rhd \Delta_2 \uplus \Delta_3, E_2^P \uplus E_2^P
     \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle Check a type definition, with its path already resolved
                                                                                                                                                           CHECK_TEXP_ABBREV
                                                           \overline{\Delta, E \vdash tnvs \ p = typ \, \triangleright \, \langle \{\,\}, \{\,\} \rangle}
                                      \overline{\Delta, E \vdash typ_i \leadsto t_i}^i
                                       names = \{ \overline{x_i}^i \}
                                     \frac{\text{duplicates}\left(\overline{x_{i}}^{i}\right)}{\text{FV}\left(t_{i}\right) \subset \textit{tnvs}^{i}} = \emptyset
                                      E^{F} = \{ \overline{x_i \mapsto \langle \mathbf{forall} \ tnvs. p \to t_i, (x_i \mathbf{of} \ names) \rangle^i} \}\Delta, E \vdash tnvs \ p = \langle |\overline{x_i^l : typ_i}^i; ?| \rangle \triangleright \langle E^F, \{ \} \rangle
                             \Delta, E \vdash typs_i \leadsto t\_multi_i{}^i
                              names = \{ \overline{x_i}^i \}
                             \mathbf{duplicates}\,(\,\overline{x_i}^{\,i}\,)=\,\emptyset
                             \overline{\mathbf{FV}\left(t\_multi_i\right) \subset tnvs}
                             E^{\mathbf{X}} = \{ \overline{x_i \mapsto \langle \mathbf{forall} \ tnvs.t_{-}multi_i \rightarrow p, (x_i \mathbf{of} \ names) \rangle}^i \}
                                                  \Delta, \overline{E \vdash tnvs \ p = |? \overline{x_i^l \ \textbf{of} \ typs_i}^i} \ \triangleright \ \langle \{\ \}, E^{\times} \rangle
    xs, \Delta, E \vdash td_1 ... td_n \rhd \langle E^{\scriptscriptstyle{\mathsf{F}}}, E^{\scriptscriptstyle{\mathsf{X}}} \rangle
                                                                                                                                              CHECK\_TEXPS\_EMPTY
                                                                        \overline{\overline{y_i}^i, \Delta, E \vdash \triangleright \langle \{\}, \{\} \rangle}
                                    tnvars^l \leadsto tnvs
                                    \Delta, E_1 \vdash tnvs \overline{y_i}_{\cdot}^i x = texp \triangleright \langle E_1^{\scriptscriptstyle F}, E_1^{\scriptscriptstyle X} \rangle
                                    \begin{array}{l} \overline{y_i}^{\,i}, \Delta, E \vdash \overline{td_j}^{\,j} \rhd \langle E_2^{\scriptscriptstyle \mathrm{F}}, E_2^{\scriptscriptstyle \mathrm{X}} \rangle \\ \mathbf{dom} \, (E_1^{\scriptscriptstyle \mathrm{X}}) \cap \mathbf{dom} \, (E_2^{\scriptscriptstyle \mathrm{X}}) = \emptyset \end{array}
                                   \mathbf{dom}\,(E_1^{\mathrm{F}})\,\cap\,\mathbf{dom}\,(E_2^{\mathrm{F}})=\emptyset
                                                                                                                                                                               CHECK_TEXPS_CONS_CONCRETE
        \overline{\overline{y_i}^i, \Delta, E \vdash x \ l \ tnvars^l \ = \ texp \ \overline{td_j}^j \ \triangleright \ \langle E_1^{\scriptscriptstyle \mathrm{F}} \uplus E_2^{\scriptscriptstyle \mathrm{F}}, E_1^{\scriptscriptstyle \mathrm{X}} \uplus E_2^{\scriptscriptstyle \mathrm{X}} \rangle}
                                  \frac{\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \rhd \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle}{\overline{y_i}^i, \Delta, E \vdash x \ l \ tnvars^l \ \overline{td_j}^j \rhd \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle} \quad \text{CHECK\_TEXPS\_CONS\_ABSTRACT}
    \delta, E \vdash id \leadsto p
                                                     Lookup a type class
                                                                                           E(id) \triangleright p
                                                                                     \frac{\delta(p) \triangleright xs}{\delta, E \vdash id \leadsto p} \quad \text{CONVERT\_CLASS\_ALL}
    I \vdash (p \ t) \mathbf{IN} \, \mathcal{C}
                                                      Solve class constraint
                                                                                                                                                                      SOLVE_CLASS_CONSTRAINT_IMMEDIATE
\overline{I \vdash (p \alpha) \mathbf{IN} (p_1 tnv_1) ... (p_i tnv_i) (p \alpha) (p'_1 tnv'_1) ... (p'_i tnv'_i)}
                  (p_1 tnv_1) ... (p_n tnv_n) \Rightarrow (p t) \mathbf{IN} I
                 I \vdash (p_1 \sigma(tnv_1)) \mathbf{IN} \mathcal{C} .. I \vdash (p_n \sigma(tnv_n)) \mathbf{IN} \mathcal{C}
                                                                                                                                                                      SOLVE_CLASS_CONSTRAINT_CHAIN
                                                              I \vdash (p \sigma(t)) \mathbf{IN} \mathcal{C}
```

```
I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C} Solve class constraints
                                \frac{I \vdash (p_1 \ t_1) \mathbf{IN} \, \mathcal{C} \quad .. \quad I \vdash (p_n \ t_n) \mathbf{IN} \, \mathcal{C}}{I \vdash \{(p_1 \ t_1), \dots, (p_n \ t_n)\} \triangleright \mathcal{C}} \quad \text{SOLVE\_CLASS\_CONSTRAINTS\_ALL}
     \Delta, I, E \vdash val\_def \triangleright E^{X} Check a value definition
                                              \Delta, E, \{\} \vdash letbind \rhd \{\overline{x_i \mapsto t_i}^i\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
I \vdash \Sigma^{\mathcal{C}} \rhd \mathcal{C}
                                              \overline{\mathbf{FV}\left(t_{i}\right)} \subset \mathit{tnvs}^{i}
                                              \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                                                                                              CHECK_VAL_DEF_VAL
                    \overline{\Delta, I, E_1 \vdash \mathbf{let} \, \tau^? \, letbind} \, \triangleright \, \{ \, \overline{x_i \mapsto \langle \, \mathbf{forall} \, tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle}^{\, i} \, \}
                                         \overline{\Delta, E, E^{\text{L}} \vdash funcl_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                         I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                         \overline{\mathbf{FV}\left(t_{i}
ight)} \subset tnvs^{i}
                                         \mathbf{FV}(\mathcal{C}) \subset tnvs
                                         \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
          \frac{E^{\mathrm{L}} = \{\overline{x_i \mapsto t_i}^i\}}{\Delta, I, E \vdash \mathbf{let} \operatorname{\mathbf{rec}} \tau^? \overline{funcl_i}^i \triangleright \{\overline{x_i \mapsto \langle \operatorname{\mathbf{forall}} tnvs.\mathcal{C} \Rightarrow t_i, \operatorname{\mathbf{let}} \rangle}^i\}} \quad \text{CHECK\_VAL\_DEF\_RECFUN}
      \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance Check that t be a typeclass instance
                                                                                                                                  CHECK_T_INSTANCE_VAR
                                                                      \overline{\Delta,(\alpha) \vdash \alpha \text{ instance}}
                                                                                                                                                                 CHECK_T_INSTANCE_TUP
                                          \overline{\Delta,(\alpha_1,\ldots,\alpha_n)} \vdash \alpha_1 * \ldots * \alpha_n  instance
                                                        \overline{\Delta, (\alpha_1, \alpha_2) \vdash \alpha_1 \rightarrow \alpha_n \text{ instance}} CHECK_T_INSTANCE_FN
                                                   \frac{\Delta(p) \rhd \alpha_1' \dots \alpha_n'}{\Delta, (\alpha_1, \dots, \alpha_n) \vdash p \alpha_1 \dots \alpha_n \text{ instance}} \quad \text{CHECK\_T\_INSTANCE\_TC}
     \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2 Check a definition
                                   \overline{z_i}^j, \Delta_1, E \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_2, E^{\mathrm{P}}

\frac{z_{j}^{j}, \Delta_{1}, E \vdash \iota \iota \iota \iota_{i} \lor \Delta_{2}, E}{\overline{z_{j}^{j}}, \Delta_{1} \uplus \Delta_{2}, E \uplus \langle \{\}, E^{P}, \{\}, \{\} \rangle \vdash \overline{td_{i}^{i}} \lor \langle E^{F}, E^{X} \rangle} \qquad \text{CHECK_DEF_TYPE}

                     \overline{\overline{z_i}^j, \langle \Delta_1, \delta, I \rangle, E \vdash \mathbf{type} \, \overline{td_i}^i \, l \, \triangleright \, \langle \Delta_2, \{ \}, \{ \} \rangle, \langle \{ \}, E^{\mathrm{p}}, E^{\mathrm{r}}, E^{\mathrm{x}} \rangle}
                                     \frac{\Delta, I, E \vdash val\_def \rhd E^{\mathbf{X}}}{\overline{z_{i}}^{j}, \langle \Delta, \delta, I \rangle, E \vdash val\_def \ l \rhd \epsilon, \langle \{ \}, \{ \}, \{ \}, E^{\mathbf{X}} \rangle} \quad \text{CHECK\_DEF\_VAL\_DEF}
                                                                                                                             \Delta, E_1, E^{\scriptscriptstyle L} \vdash rule_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\scriptscriptstyle C}_i, \Sigma^{\scriptscriptstyle N}_i^{i}
                                                                                                                             I \vdash \overline{\Sigma^{\mathcal{C}}_{i}}^{i} \triangleright \mathcal{C}
                                                                                                                             \overline{\mathbf{FV}(t_i)} \subset tnvs
                                                                                                                             \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                             \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
                                                                                                                             E^{L} = \{ \overline{x_i \mapsto t_i}^i \}
E_2 = \langle \{\}, \{\}, \{\}, \{\overline{x_i \mapsto \langle \mathbf{forall} \ tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle}^i \} \rangle <<no parses (char 59): </zj//j/>,<TD,TC,I>,E1 |- indreln targets_opt indreln_names*** </rule
```

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\frac{\overline{z_j}^j \ x, D_1, E_1 \vdash \mathit{defs} \rhd D_2, E_2}{\overline{z_j}^j \ , D_1, E_1 \vdash \mathbf{module} \ x \ l_1 = \mathbf{struct} \ \mathit{defs} \ \mathbf{end} \ l_2 \rhd D_2, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle}
                                                                                                                                                                                                                                                          CHECK_DEF_MODULE
                                                                                                                                                                                                              CHECK_DEF_MODULE_RENAME
        \overline{z_i}^j, D, E_1 \vdash \mathbf{module} \ x \ l_1 = id \ l_2 \triangleright \epsilon, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle
                                    \Delta, E \vdash typ \leadsto t
                                    \mathbf{FV}(t) \subset \overline{\alpha_i}^i
                                   \frac{\mathbf{FV}(\overline{\alpha'_k}^k) \subset \overline{\alpha_i}^i}{\delta, E \vdash id_k \leadsto p_k}^k
                 \frac{E' = \langle \{ \}, \{ \}, \{ \}, \{x \mapsto \langle \operatorname{\mathbf{forall}} \overline{\alpha_i}^i. \overline{(p_k \, \alpha_k')}^k \Rightarrow t, \operatorname{\mathbf{val}} \rangle \} \rangle}{\overline{z_j}^j \, , \langle \Delta, \delta, I \rangle, E \vdash \operatorname{\mathbf{val}} x \, l_1 \, : \operatorname{\mathbf{forall}} \overline{\alpha_i \, l_i''}^i \, . \overline{id_k \, \alpha_k' \, l_k'}^k \stackrel{k}{\Rightarrow} \, typ \, l_2 \, \rhd \, \epsilon, E'}
                                                                                                                                                                                                                                             CHECK_DEF_SPEC
                   \frac{\Delta, E_1 \vdash typ_i \leadsto t_i}{\mathbf{FV}(t_i) \subset \alpha}^i
                    p = \overline{z_i}^j x
                   E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{\overline{y_i \mapsto \langle \mathbf{forall} \, \alpha. (p \, \alpha) \Rightarrow t_i, \mathbf{method} \rangle}^i \} \rangle
                    \delta_2 = \{ p \mapsto \overline{y_i}^{\,i} \}
                   p \not\in \mathbf{dom}(\delta_1)
                                                                                                                                                                                                                                                        CHECK_DEF_CLASS
   \overline{z_j}^j, \langle \Delta, \delta_1, I \rangle, E_1 \vdash \mathbf{class}(x \ l \ \alpha \ l'') \ \overline{\mathbf{val} \ y_i \ l_i \ : typ_i \ l_i}^i \ \mathbf{end} \ l' \rhd \langle \{ \ \}, \delta_2, \{ \ \} \rangle, E_2
                                                                      E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                                                      \Delta, E \vdash typ' \leadsto t'
                                                                     \Delta, (\overline{\alpha_i}^i) \vdash t' instance
                                                                      tnvs = \overline{\alpha_i}^i
                                                                     \frac{\mathbf{duplicates}\left(tnvs\right)}{\delta, E \vdash id_k \leadsto p_k} = \emptyset
\mathbf{FV}\left(\overline{\alpha'_k}^k\right) \subset tnvs
                                                                      E(id) \triangleright p
                                                                      \delta(p) \triangleright \overline{z_j}^j

\frac{I_2 = \{ \Rightarrow (p_k \, \alpha'_k)^k \}}{\Delta, I \cup I_2, E \vdash val\_def_n \triangleright E_n^{\mathsf{X}}^n}

                                                                     disjoint doms (\overline{E_n^{\mathrm{X}}}^n)
                                                                     \overline{E^{\mathbf{X}}(x_k) \triangleright \langle \mathbf{forall} \, \alpha''. (p \, \alpha'') \Rightarrow t_k, \mathbf{method} \rangle}^{k}
                                                                    \{ \overline{x_k \mapsto \langle \operatorname{forall} tnvs.} \Rightarrow \{\alpha'' \mapsto t'\}(t_k), \operatorname{let} \rangle^k \} = \overline{E_n^{\mathrm{X}}}^n \\ \overline{x_k}^k = \overline{z_j}^j
                                                                     I_3 = \{ \overline{(p_k \, \alpha'_k) \Rightarrow (p \, t')}^k \, \}
                                                                     (p \{ \overline{\alpha_i \mapsto \alpha_i'''}^i \}(t')) \not\in I
\overline{z_{j}^{\;j}\;,\langle\Delta,\delta,I\rangle,E\vdash\mathbf{instance\,forall}}\,\overline{\alpha_{i}\;\overline{l_{i}^{'}}^{\;i}}\,.\overline{id_{k}\;\alpha_{k}^{'}\;\overline{l_{k}^{''}}^{\;k}}\;\Rightarrow\left(id\;typ'\right)\overline{val\_def_{n}\;l_{n}}^{\;n}\,\mathbf{end}\;l'\;\rhd\;\langle\{\;\},\{\;\},I_{3}\rangle,\epsilon^{*}
                                                                                                                                                                                                                                                                                                                          CHECK_DEF_
   \overline{z_j}^j, D_1, E_1 \vdash defs \triangleright D_2, E_2
                                                                                                    Check definitions, given module path, definitions and environment
                                                                                           \frac{1}{\overline{z_j}^j, D, E \vdash \triangleright \epsilon, \epsilon} CHECK_DEFS_EMPTY
                                        \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2
                             \frac{\overline{z_{j}}^{j}, D_{1} \uplus D_{2}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}}{\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash def;;_{0}^{?} \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{2} \uplus D_{3}, E_{2} \uplus E_{3}}
```

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CHECK_DEFS_RELEVANT_DEF

$$\frac{E_{1}(id) \triangleright E_{2}}{\overline{z_{j}}^{j}, D_{1}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}}{\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash \mathbf{open} \ id \ l;;_{i}^{?} \ \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}} \quad \text{CHECK_DEFS_OPEN}$$

Definition rules: 143 good 2 bad Definition rule clauses: 437 good 2 bad