n, i, j, k Index variables for meta-lists

num Numeric literals

nat Internal literal numbers

hex Bit vector literal, specified by C-style hex number bin Bit vector literal, specified by C-style binary number

string String literals

regexp Regular expresions, as a string literal

x, y, z Variables ix Variables

```
l
                                                       Source locations
                        ::=
x^l, y^l, z^l, name
                                                       Location-annotated names
                              x l
                              (ix)l
                                                          Remove infix status
                              name_-t \to x^l
                                                  Μ
                                                          Extract x from a name_t
ix^l
                                                       Location-annotated infix names
                              ix l
                              x'l
                                                          Add infix status
                                                       Type variables
\alpha
                              \dot{x}
\alpha^l
                                                       Location-annotated type variables
                              \alpha l
N
                                                       numeric variables
                              ",x
N^l
                                                       Location-annotated numeric variables
                        ::=
                              N l
id
                                                       Long identifers
                              x_1^l \dots x_n^l . x^l l
                                                       Union of type variables and Nexp type variables, without lo
tnv
                        ::=
                              \alpha
                              N
tnvar^l
                                                       Union of type variables and Nexp type variables, with locati
                        ::=
                              \alpha^l
tnvs
                                                       Type variable lists
                              tnv_1 \dots tnv_n
tnvars^l
                                                       Type variable lists
                              tnvar_1^l \dots tnvar_n^l
Nexp\_aux
                                                       Numerical expressions for specifying vector lengths and inde
                        ::=
                              N
                              num
                              Nexp_1 * Nexp_2
                              Nexp_1 + Nexp_2
```

(Nexp)

```
Nexp
                                                                 Location-annotated vector lengths
                            ::=
                                   Nexp\_aux\ l
Nexp\_constraint\_aux
                                                                 Whether a vector is bounded or fixed size
                            ::=
                                   Nexp = Nexp'
                                   Nexp \ge Nexp'
Nexp\_constraint
                                                                 Location-annotated Nexp range
                            ::=
                                   Nexp\_constraint\_aux\ l
                            ::=
                                                                 Types
typ\_aux
                                                                    Unspecified type
                                   \alpha^l
                                                                   Type variables
                                                                   Function types
                                   typ_1 \rightarrow typ_2
                                                                   Tuple types
                                   typ_1*....*typ_n
                                                                   As a typ to permit applications over Nexps, other
                                   Nexp
                                   id\ typ_1 \dots typ_n
                                                                   Type applications
                                   (typ)
typ
                                                                 Location-annotated types
                                   typ\_aux l
                                                                 Literal constants
lit\_aux
                            ::=
                                   true
                                   false
                                   num
                                   hex
                                                                   hex and bin are constant bit vectors, entered as C
                                   bin
                                   string
                                   bitzero
                                                                   bitzero and bitone are constant bits, if commonly
                                   bitone
lit
                                   lit_{-}aux\ l
                                                                   Location-annotated literal constants
                            ::=
                                                                 Optional semi-colons
                                                                 Patterns
pat_{-}aux
                            ::=
                                                                    Wildcards
                                   (pat \mathbf{as} \, x^l)
                                                                   Named patterns
                                   (pat:typ)
                                                                   Typed patterns
                                   id\ pat_1 \dots pat_n
                                                                   Single variable and constructor patterns
                                   \langle |fpat_1; \dots; fpat_n;^?| \rangle
[|pat_1; \dots; pat_n;^?|]
                                                                   Record patterns
                                                                    Vector patterns
```

```
Concatenated vector patterns
                     [|pat_1 ... pat_n|]
                     (pat_1, \ldots, pat_n)
                                                                                     Tuple patterns
                     [pat_1; ...; pat_n; ]
                                                                                     List patterns
                                                                                     Cons patterns
                     pat_1 :: pat_2
                     x^l + num
                                                                                     constant addition patterns
                     lit
                                                                                     Literal constant patterns
                                                                                  Location-annotated patterns
pat
               ::=
                     pat_{-}aux l
fpat
                                                                                  Field patterns
                     id = pat l
                                                                                  Optional bars
               ::=
                                                                                  Expressions
exp\_aux
                     id
                                                                                     Identifiers
                     N
                                                                                     Nexp var, has type num
                                                                                     Curried functions
                     fun psexp
                     function ||^{?}|pexp_{1}| \dots ||pexp_{n}| end
                                                                                     Functions with pattern matching
                                                                                     Function applications
                     exp_1 \ exp_2
                     exp_1 ix^l exp_2
                                                                                     Infix applications
                     \langle |fexps| \rangle
                                                                                     Records
                     \langle |exp \ \mathbf{with} \ fexps| \rangle
                                                                                     Functional update for records
                     exp.id
                                                                                     Field projection for records
                     [|exp_1; ...; exp_n;^?|]
                                                                                     Vector instantiation
                     exp.(Nexp)
                                                                                     Vector access
                     exp.(Nexp_1..Nexp_2)
                                                                                     Subvector extraction
                     match exp with |?| pexp_1| ... | pexp_n| l end
                                                                                     Pattern matching expressions
                     (exp:typ)
                                                                                     Type-annotated expressions
                     let letbind in exp
                                                                                     Let expressions
                     (exp_1, \ldots, exp_n)
                                                                                     Tuples
                     [exp_1; ...; exp_n; ?]
                                                                                     Lists
                     (exp)
                     begin exp end
                                                                                     Alternate syntax for (exp)
                     if exp_1 then exp_2 else exp_3
                                                                                     Conditionals
                                                                                     Cons expressions
                     exp_1 :: exp_2
                     lit
                                                                                     Literal constants
                     \{exp_1|exp_2\}
                                                                                     Set comprehensions
                     \{exp_1| \mathbf{forall} \ qbind_1 .. \ qbind_n| exp_2\}
                                                                                     Set comprehensions with explicit bin
                     \{exp_1; ...; exp_n; ?\}
                     q \ qbind_1 \dots qbind_n . exp
                                                                                     Logical quantifications
                     [exp_1| \mathbf{forall} \ qbind_1 \dots qbind_n| exp_2]
                                                                                     List comprehensions (all binders mus
                     do id pat_1 < -exp_1; .. pat_n < -exp_n; in exp end
                                                                                     Do notation for monads
```

```
Location-annotated expressions
                  ::=
exp
                        exp\_aux l
                                                            Quantifiers
q
                  ::=
                        forall
                        exists
qbind
                                                            Bindings for quantifiers
                        x^l
                        (pat \, \mathbf{IN} \, exp)
                                                              Restricted quantifications over sets
                        (pat \mathbf{MEM} \ exp)
                                                              Restricted quantifications over lists
                                                            Field-expressions
fexp
                        id = exp l
                                                            Field-expression lists
fexps
                        fexp_1; \dots; fexp_n; ^? l
                                                            Pattern matches
pexp
                        pat \rightarrow exp l
                                                            Multi-pattern matches
psexp
                  ::=
                        pat_1 \dots pat_n \to exp \ l
tannot?
                  ::=
                                                            Optional type annotations
                        : typ
                                                            Function clauses
funcl_aux
                        x^l pat_1 \dots pat_n tannot? = exp
                                                            Let bindings
letbind\_aux
                        pat\ tannot? = exp
                                                               Value bindings
                        funcl_aux
                                                              Function bindings
letbind
                                                            Location-annotated let bindings
                  ::=
                        letbind\_aux\ l
                                                            Location-annotated function clauses
funcl
                  ::=
                        funcl_aux l
name_-t
                                                            Name or name with type for inductively defined relation
                       (x^l:typ)
                                                            Names with optional types for inductively defined relat
name\_ts
                        name\_t_0 ... name\_t_n
```

```
Inductively defined relat
rule\_aux
                                    x^l: \mathbf{forall} \ name_-t_1 \dots name_-t_i.exp \Longrightarrow x_1^l \ exp_1 \dots exp_n
rule
                                                                                                            Location-annotated indu
                                    rule\_aux\ l
witness?
                                                                                                            Optional witness type na
                                    witness type x^l;
check?
                                                                                                            Option check name decla
                                    \mathbf{check}\,x^l;
functions?
                                                                                                            Optional names and typ
                                   x^{l}: typ
x^{l}: typ; functions^{?}
                                                                                                            Name for inductively de-
indreln\_name\_aux
                                    [x^l: typschm\ witness^?\ check^?\ functions^?]
indreln\_name
                                                                                                            Location-annotated name
                             ::=
                                    indreln\_name\_aux\ l
                                                                                                            Type lists
typs
                                    typ_1 * ... * typ_n
                                                                                                            Datatype definition clau
ctor\_def
                                    x^l of typs
                                                                                                       S
                                                                                                               Constant constructors
                                                                                                            Type definition bodies
texp
                                                                                                               Type abbreviations
                                     \begin{array}{c} \langle |x_1^l:typ_1;\ldots;x_n^l:typ_n\,;^?| \rangle \\ |^?\:ctor\_def_1|\ldots|ctor\_def_n \end{array} 
                                                                                                               Record types
                                                                                                               Variant types
name?
                                                                                                            Optional name specificar
                                    [name = regexp]
td
                                                                                                            Type definitions
                             ::=
                                    x^l tnvars^l name? = texp
                                    x^l tnvars^l name?
                                                                                                               Definitions of opaque
c
                                                                                                            Typeclass constraints
```

 $id\ tnvar^l$

```
Typeclass and length constraint l
cs
                   ::=
                                                                                              Must have > 0 constraints
                          Nexp\_constraint_1, ..., Nexp\_constraint_i \Rightarrow
                                                                                              Must have > 0 constraints
                          c_1, \dots, c_i; Nexp\_constraint_1, \dots, Nexp\_constraint_n \Rightarrow
                                                                                              Must have > 0 of both form of
                                                                                           Type and instance scheme prefixe
                   ::=
c\_pre
                          \mathbf{forall}\,tnvar_1^l \dots tnvar_n^l.cs
                                                                                              Must have > 0 type variables
typschm
                                                                                           Type schemes
                          c-pre typ
                                                                                           Instance schemes
instschm
                          c\_pre(id\ typ)
                                                                                           Backend target names
target
                          hol
                          isabelle
                          ocaml
                          coq
                          tex
                          html
                                                                                           Backend target name lists
                          \{target_1; ...; target_n\}
                                                                                           Optional targets
                   ::=
lemma\_typ
                                                                                           Types of Lemmata
                          assert
                          lemma
                          theorem
lemma\_decl
                                                                                           Lemmata and Tests
                          lemma\_typ\ \tau^?\ x^l:(exp)
                          lemma\_typ \ \tau^?(exp)
                                                                                           Value definitions
val\_def
                         let \tau^? letbind
let rec \tau^? funcl<sub>1</sub> and ... and funcl<sub>n</sub>
let inline \tau^? letbind
                                                                                              Non-recursive value definitions
                                                                                              Recursive function definitions
                                                                                              Function definitions to be inlin
                                                                                           Value type specifications
val\_spec
                         \mathbf{val}\,x^l:typschm
```

```
def\_aux
                                                                                                                         Top-level d
                      type td_1 and ... and td_n
                                                                                                                            Type det
                      val\_def
                                                                                                                            Value de
                      lemma\_decl
                                                                                                                            Lemmata
                      rename \tau? id = x^l
                                                                                                                            Rename
                      module x^l = struct defs end
                                                                                                                            Module (
                      module x^l = id
                                                                                                                            Module 1
                      \mathbf{open}\,id
                                                                                                                            Opening
                      indreln \tau^{?} indreln\_name_1 and ... and indreln\_name_i rule_1 and ... and rule_n
                                                                                                                            Inductive
                                                                                                                            Top-leve
                       class (x^l \, tnvar^l) valx_1^l : typ_1 \, l_1 \dots valx_n^l : typ_n \, l_n end
                                                                                                                            Typeclas
                      instance instschm\ val\_def_1\ l_1\ ...\ val\_def_n\ l_n end
                                                                                                                            Typeclas
def
                                                                                                                         Location-ar
                ::=
                      def\_aux\ l
;;?
                                                                                                                         Optional de
                ::=
                      ;;
defs
                                                                                                                         Definition s
                      def_1; ;_1^? ... def_n; ;_n^?
                                                                                                                         Unique pat
                ::=
p
                      x_1 \dots x_n x
                      _{-}list
                      __bool
                      __num
                      \_set
                      \_string
                      _{-}unit
                      _{-}bit
                       __vector
                ::=
                                                                                                                         Type varia
                      \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}
                                                                                                                         Internal typ
t, u
                      t_1 \rightarrow t_2
                      t_1 * \dots * t_n
                      p t\_args
                      ne
                      \sigma(t)
                                                                                                                    Μ
                                                                                                                            Multiple
                      \sigma(tnv)
                                                                                                                    Μ
                                                                                                                            Single va
```

Μ

Curried,

 $\mathbf{curry}(t_multi, t)$

```
internal numeric expressions
ne
                ::=
                       N
                       nat
                       ne_1 * ne_2
                       ne_1 + ne_2
                       (-ne)
                       normalize(ne)
                                                                              Μ
                       ne_1 + \dots + ne_n
                                                                              Μ
                       bitlength(bin)
                                                                              Μ
                       bitlength (hex)
                                                                              Μ
                       length (pat_1 \dots pat_n)
                                                                              Μ
                       length (exp_1 \dots exp_n)
                                                                              Μ
                                                                                   Lists of types
t\_args
                ::=
                       t_1 \dots t_n
                       \sigma(t\_args)
                                                                              Μ
                                                                                       Multiple substitutions
t_{-}multi
                                                                                   Lists of types
                       (t_1 * \ldots * t_n)
                       \sigma(t_{-}multi)
                                                                              Μ
                                                                                       Multiple substitutions
                                                                                   Numeric expression constraints
nec
                       ne\langle nec
                       ne = nec
                       ne \le nec
                       ne
names
                                                                                   Sets of names
                       \{x_1, ..., x_n\}
\mathcal{C}
                ::=
                                                                                   Typeclass constraint lists
                       (p_1 tnv_1) \dots (p_n tnv_n)
env\_tag
                                                                                   Tags for the (non-constructor) value description
                       method
                                                                                       Bound to a method
                                                                                       Specified with val
                       val
                       let
                                                                                       Defined with let or indreln
v\_desc
                ::=
                                                                                   Value descriptions
                        \langle \mathbf{forall} \ tnvs.t\_multi \rightarrow p, (x \mathbf{of} \ names) \rangle
                                                                                       Constructors
                        \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t, env\_tag \rangle
                                                                                       Values
f\_desc
                ::=
                        \langle \mathbf{forall} \ tnvs.p \rightarrow t, (x \ \mathbf{of} \ names) \rangle
                                                                                       Fields
                ::=
xs
```

 $x_1 \dots x_n$

```
\Sigma^{\mathcal{C}}
                                                                                                                                                 Typeclass constraints
                                               \{(p_1 t_1), \dots, (p_n t_n)\}
\Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n
                                                                                                                                      Μ
\Sigma^{\mathcal{N}}
                                                                                                                                                 Nexp constraint lists
                                                \{nec_1, \dots, nec_n\} 
\Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n 
                                                                                                                                      Μ
E
                                                                                                                                                 Environments
                                     \begin{vmatrix} \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \\ | E_1 \uplus E_2 \end{vmatrix} 
                                                                                                                                      Μ
                                                                                                                                      Μ
E^{\mathbf{X}}
                                                                                                                                                 Value environments
                                                \left\{ \begin{aligned} & \{x_1 \mapsto v\_desc_1, \, \dots, x_n \mapsto v\_desc_n \} \\ & E_1^{\mathsf{X}} \uplus \, \dots \, \uplus \, E_n^{\mathsf{X}} \end{aligned} \right. 
                                                                                                                                      Μ
E^{\mathrm{F}}
                                                                                                                                                 Field environments
                                               \{x_1 \mapsto f \_desc_1, \dots, x_n \mapsto f \_desc_n\} 
E_1^{\mathsf{F}} \uplus \dots \uplus E_n^{\mathsf{F}} 
                                                                                                                                      Μ
E^{\mathrm{M}}
                                                                                                                                                 Module environments
                                               \{x_1 \mapsto E_1, \dots, x_n \mapsto E_n\}
E^{\mathrm{P}}
                                                                                                                                                 Path environments
                                     | \{x_1 \mapsto p_1, \dots, x_n \mapsto p_n\} 
| E_1^P \uplus \dots \uplus E_n^P 
                                                                                                                                      Μ
E^{\mathrm{L}}
                                                                                                                                                 Lexical bindings
                                    \begin{vmatrix} \{x_1 \mapsto t_1, \dots, x_n \mapsto t_n\} \\ \{x_1^l \mapsto t_1, \dots, x_n^l \mapsto t_n\} \\ E_1^L \uplus \dots \uplus E_n^L \end{vmatrix} 
                                                                                                                                      Μ
tc\_abbrev
                                                                                                                                                 Type abbreviations
tc\_def
                                                                                                                                                 Type and class constructor definitions
                                                tnvs\ tc\_abbrev
                                                                                                                                                       Type constructors
\Delta
                                                                                                                                                 Type constructor definitions
                                                \begin{cases} p_1 \mapsto tc\_def_1, \, \dots, p_n \mapsto tc\_def_n \\ \Delta_1 \uplus \Delta_2 \end{cases} 
                                                                                                                                      Μ
                                                                                                                                                 Typeclass definitions
                                           \{p_1 \mapsto xs_1, \dots, p_n \mapsto xs_n\}\delta_1 \uplus \delta_2
```

Μ

```
A typeclass instance, t must not contain nested types
inst
                            \mathcal{C} \Rightarrow (p\ t)
                      Ι
                                                                                  Global instances
                             \{inst_1, ..., inst_n\}
                             I_1 \cup I_2
                                                                           Μ
D
                                                                                  Global type definition store
                             \langle \Delta, \delta, I \rangle
                                                                           Μ
                             D_1 \uplus D_2
                                                                           Μ
terminals
                                                                                     >=
                                                                                     ->
                                                                                     ==>
                                                                                      <|
                                                                                      |>
                             \forall
                             \not\in
formula
                             judgement
                             formula_1 .. formula_n
                             E^{\mathrm{M}}(x) \triangleright E
                                                                                     Module lookup
                             E^{\mathrm{P}}(x) \triangleright p
                                                                                     Path lookup
                             E^{\mathrm{F}}(x) \triangleright f_{-}desc
                                                                                     Field lookup
                             E^{\mathbf{X}}(x) \triangleright v_{-}desc
                                                                                      Value lookup
```

Lexical binding lookup

Type constructor lookup

Type constructor lookup

 $E^{\mathrm{L}}(x) \triangleright t$

 $\delta(p) \triangleright xs$

 $\Delta(p) \triangleright tc_def$

 $\operatorname{\mathbf{dom}}\left(E_{1}^{\scriptscriptstyle{\mathrm{M}}}\right)\,\cap\,\operatorname{\mathbf{dom}}\left(E_{2}^{\scriptscriptstyle{\mathrm{M}}}\right)=\,\emptyset$

```
\mathbf{dom}\left(E_{1}^{\mathrm{F}}\right)\,\cap\,\mathbf{dom}\left(E_{2}^{\mathrm{F}}\right)=\,\emptyset
                                              \operatorname{\mathbf{dom}}(E_1^{\operatorname{P}}) \cap \operatorname{\mathbf{dom}}(E_2^{\operatorname{P}}) = \emptyset
                                              disjoint doms (E_1^{\text{L}}, \dots, E_n^{\text{L}})
                                                                                                                                             Pairwise disjoint domains
                                              disjoint doms (E_1^{\mathrm{X}}, \ldots, E_n^{\mathrm{X}})
                                                                                                                                             Pairwise disjoint domains
                                               compatible overlap (x_1 \mapsto t_1, ..., x_n \mapsto t_n)
                                                                                                                                             (x_i = x_j) \Longrightarrow (t_i = t_j)
                                               \mathbf{duplicates}\left(\mathit{tnvs}\right) = \emptyset
                                               duplicates (x_1, ..., x_n) = \emptyset
                                               x \not\in \mathbf{dom}(E^{L})
                                               x \not\in \mathbf{dom}(E^{X})
                                               x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                              p \not\in \mathbf{dom}(\delta)
                                               p \not\in \mathbf{dom}(\Delta)
                                              \mathbf{FV}\left(t\right)\subset\mathit{tnvs}
                                                                                                                                             Free type variables
                                               \mathbf{FV}(t_{-}multi) \subset tnvs
                                                                                                                                             Free type variables
                                               \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                                             Free type variables
                                               inst \ \mathbf{IN} \ I
                                               (p t) \not\in I
                                              E_1^{\text{L}} = E_2^{\text{L}}

E_1^{\text{X}} = E_2^{\text{X}}
                                              E_1^{\mathrm{F}} = E_2^{\mathrm{F}}
                                               E_1 = E_2
                                               \Delta_1 = \Delta_2
                                              \delta_1 = \delta_2
                                               I_1 = I_2
                                              names_1 = names_2
                                               t_1 = t_2
                                              \sigma_1 = \sigma_2
                                              p_1 = p_2
                                               xs_1 = xs_2
                                               tnvs_1 = tnvs_2
convert\_tnvars
                                    ::=
                                              tnvars^l \leadsto tnvs
                                               tnvar^l \leadsto tnv
look\_m
                                    ::=
                                              E_1(x_1^l \dots x_n^l) \triangleright E_2
                                      Name path lookup
look\_m\_id
                                    ::=
                                               E_1(id) \triangleright E_2
                                                                                                                                             Module identifier lookup
look\_tc
                                    ::=
                                               E(id) \triangleright p
                                                                                                                                             Path identifier lookup
check\_t
                                               \Delta \vdash t \, \mathbf{ok}
                                                                                                                                             Well-formed types
```

 $\mathbf{dom}\left(E_{1}^{\mathbf{X}}\right) \cap \mathbf{dom}\left(E_{2}^{\mathbf{X}}\right) = \emptyset$

```
\Delta, tnv \vdash t ok
                                                                                                                                                                  Well-formed type/Nexps m
teq
                                     ::=
                                               \Delta \vdash t_1 = t_2
                                                                                                                                                                  Type equality
convert\_typ
                                     ::=
                                               \Delta, E \vdash typ \leadsto t
                                                                                                                                                                  Convert source types to int
                                               \vdash Nexp \leadsto ne
                                                                                                                                                                  Convert and normalize num
convert\_typs
                                               \Delta, E \vdash typs \leadsto t\_multi
check\_lit
                                               \vdash lit:t
                                                                                                                                                                  Typing literal constants
inst\_field
                                     ::=
                                               \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                                                                                  Field typing (also returns c
inst\_ctor
                                     ::=
                                                                                                                                                                  Data constructor typing (al
                                               \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
inst\_val
                                     ::=
                                               \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                                                                                                                  Typing top-level bindings, of
not\_ctor
                                               E, E^{\text{L}} \vdash x \text{ not ctor}
                                                                                                                                                                  v is not bound to a data co
not\_shadowed
                                               E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                                                                                                                  id is not lexically shadowed
check\_pat
                                     ::=
                                               \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_2^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash pat\_aux: t \vartriangleright E_2^{\text{\tiny L}} \end{array}
                                                                                                                                                                  Typing patterns, building t
                                                                                                                                                                  Typing patterns, building t
id_{-}field
                                     ::=
                                               E \vdash id \mathbf{field}
                                                                                                                                                                  Check that the identifier is
id\_value
                                    ::=
                                               E \vdash id value
                                                                                                                                                                  Check that the identifier is
check\_exp
                                     ::=
                                               \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                  Typing expressions, collecti
                                               \Delta, E, E^{\text{\tiny L}} \vdash \overrightarrow{exp\_aux} : t \, \triangleright \, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                  Typing expressions, collecti
                                               \Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}
                                                                                                                                                                  Build the environment for o
                                              \Delta, E, E_{1}^{L} \vdash \mathbf{list} \ qbind_{1} ... \ qbind_{n} \triangleright E_{2}^{L}, \Sigma^{\mathcal{C}}
\Delta, E, E^{L} \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
\Delta, E, E_{1}^{L} \vdash letbind \triangleright E_{2}^{L}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                  Build the environment for o
```

Build the environment for a Build the environment for a

```
check\_rule
                                             ::=
                                                      \Delta, E, E^{\mathrm{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                              Build the environment for an induc
check\_texp\_tc
                                             ::=
                                                      xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                                             Extract the type constructor inform
check\_texps\_tc
                                             ::=
                                                      xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{P}
                                                                                                                            Extract the type constructor inform
check\_texp
                                             ::=
                                                      \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                                                                                                             Check a type definition, with its pa
                                              check\_texps
                                             ::=
                                                      xs, \Delta, E \vdash td_1 ... td_n \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                              convert\_class
                                                      \delta, E \vdash id \leadsto p
                                                                                                                            Lookup a type class
solve\_class\_constraint
                                                      I \vdash (p\ t)\ \mathbf{IN}\ \mathcal{C}
                                                                                                                            Solve class constraint
solve\_class\_constraints
                                                      I \vdash \Sigma^{\mathcal{C}} \mathrel{\triangleright} \mathcal{C}
                                                                                                                            Solve class constraints
check_val_def
                                                      \Delta, I, E \vdash val\_def \triangleright E^{X}
                                                                                                                             Check a value definition
check\_t\_instance
                                             ::=
                                                      \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance
                                                                                                                             Check that t be a typeclass instance
check\_defs
                                             ::=
                                                       \overline{z_j}^j , D_1, E_1 \vdash def \rhd D_2, E_2 
\overline{z_j}^j , D_1, E_1 \vdash defs \rhd D_2, E_2 
                                                                                                                             Check a definition
                                                                                                                             Check definitions, given module pa
judgement
                                             ::=
                                                       convert\_tnvars
                                                      look\_m
                                                      look\_m\_id
                                                      look\_tc
                                                      check\_t
```

teq

convert_typ
convert_typs
check_lit
inst_field
inst_ctor
inst_val

```
not\_ctor
      not\_shadowed
      check\_pat
      id\_field
      id\_value
      check\_exp
      check\_rule
      check\_texp\_tc
      check\_texps\_tc
      check\_texp
      check\_texps
      convert\_class
      solve\_class\_constraint
      solve\_class\_constraints
      check\_val\_def
      check\_t\_instance
      check\_defs
::=
      n
      num
      nat
      hex
      bin
      string
      regexp
      \boldsymbol{x}
      ix
      l
      x^{l}
      ix^l
      \alpha
      \alpha^l
      N
      N^l
      id
      tnv
      tnvar^l
      tnvs
      tnvars^l
      Nexp\_aux
```

Nexp

 $\begin{array}{c} typ_aux \\ typ \end{array}$

 $Nexp_constraint_aux$ $Nexp_constraint$

 $user_syntax$

```
lit\_aux
lit
pat\_aux
pat
fpat|?
exp\_aux
exp
q
qbind
fexp
fexps
pexp
psexp
tannot?
funcl_aux
letbind\_aux
letbind
funcl
name\_t
name\_ts
rule\_aux
rule
witness?
check?
functions?
indreln\_name\_aux
indreln\_name
typs
ctor\_def
texp
name?
td
c
cs
c-pre
typschm
instschm
target
\tau?
lemma\_typ
lemma\_decl
val\_def
val\_spec
```

```
def\_aux
defs
p
ne
t\_args
t\_multi
nec
names
\mathcal{C}
env\_tag
v\_desc
f\_desc
\Sigma^{\mathcal{C}}
\Sigma^{\mathcal{N}}
 E
E^{\mathbf{X}}
E^{\mathrm{F}}
E^{\scriptscriptstyle \mathrm{M}}
E^{\scriptscriptstyle \mathrm{P}}
E^{\scriptscriptstyle \rm L}
tc\_abbrev
tc\_def
\Delta
\delta
inst
I
 D
terminals
formula
```

 $tnvars^l \leadsto tnvs$

$$\frac{tnvar_1^l \leadsto tnv_1 \quad .. \quad tnvar_n^l \leadsto tnv_n}{tnvar_1^l \ .. \ tnvar_n^l \leadsto tnv_1 \ .. \ tnv_n} \quad \text{convert_tnvars_none}$$

 $tnvar^l \leadsto tnv$

$$\frac{}{\alpha\; l \leadsto \alpha} \quad \begin{array}{ll} \text{convert_tnvar_a} \\ \\ \hline N\; l \leadsto N \end{array}$$

 $E_1(x_1^l ... x_n^l) \triangleright E_2$ Name path lookup

$$\overline{E(\,) \triangleright E}$$
 LOOK_M_NONE

$$\frac{E^{\mathrm{M}}(x) \triangleright E_{1}}{E_{1}(\overline{y_{i}^{l}}^{i}) \triangleright E_{2}} \\ \frac{E_{1}(\overline{y_{i}^{l}}^{i}) \triangleright E_{2}}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle (x \, l \, \overline{y_{i}^{l}}^{i}) \triangleright E_{2}} \quad \text{LOOK_M_SOME}$$

 $E_1(id) \triangleright E_2$ Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i x l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i x l_1 l_2) \triangleright E_2} \quad \text{LOOK_M_ID_ALL}$$

 $E(id) \triangleright p$ Path identifier lookup

$$\frac{E(\overline{y_{i}^{l}}^{i}) \triangleright \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{E^{\mathrm{P}}(x) \triangleright p}$$

$$E(\overline{y_{i}^{l}}^{i} x l_{1} l_{2}) \triangleright p$$
LOOK_TC_ALL

 $\Delta \vdash t \text{ ok}$ Well-formed types

$$\begin{array}{c|c} \overline{\Delta \vdash \alpha \, \mathbf{ok}} & \text{CHECK_T_VAR} \\ \hline \Delta \vdash t_1 \, \mathbf{ok} \\ \underline{\Delta \vdash t_2 \, \mathbf{ok}} \\ \hline \Delta \vdash t_1 \to t_2 \, \mathbf{ok} \end{array} \text{ CHECK_T_FN} \\ \hline \\ \frac{\Delta \vdash t_1 \, \mathbf{ok} \quad \quad \Delta \vdash t_n \, \mathbf{ok}}{\Delta \vdash t_1 * * t_n \, \mathbf{ok}} \quad \text{CHECK_T_TUP} \\ \hline \\ \Delta(p) \rhd tnv_1 ...tnv_n \ tc_abbrev \\ \underline{\Delta, tnv_1 \vdash t_1 \, \mathbf{ok}} \quad ... \quad \Delta, tnv_n \vdash t_n \, \mathbf{ok}} \quad \text{CHECK_T_APP} \\ \hline \\ \Delta \vdash p \ t_1 ... t_n \, \mathbf{ok} \end{array}$$

 $\Delta, tnv \vdash t$ ok Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \, \mathbf{ok}}{\Delta, \alpha \vdash t \, \mathbf{ok}} \quad \text{CHECK_TLEN_T}$$

$$\frac{\Delta, N \vdash ne \, \mathbf{ok}}{\Delta, N \vdash ne \, \mathbf{ok}} \quad \text{CHECK_TLEN_LEN}$$

 $\Delta \vdash t_1 = t_2$ Type equality

$$\frac{\Delta \vdash t \text{ ok}}{\Delta \vdash t = t} \quad \text{TEQ_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ_SYM}$$

$$\frac{\Delta \vdash t_1 = t_2}{\Delta \vdash t_2 = t_3} \quad \text{TEQ_TRANS}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_1 = t_3} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_2 = t_4} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n}{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n} \quad \text{TEQ_TUP}$$

$$\frac{\Delta(p) \rhd \alpha_{1} ... \alpha_{n}}{\Delta \vdash t_{1} = u_{1} ... \Delta \vdash t_{n} = u_{n}} \underbrace{\Delta \vdash t_{1} = u_{1} ... \Delta \vdash t_{n} = u_{n}}_{\Delta \vdash p \ t_{1} ... t_{n} = p \ u_{1} ... u_{n}}$$
 TEQ_APP
$$\frac{\Delta(p) \rhd \alpha_{1} ... \alpha_{n} .u}{\Delta \vdash p \ t_{1} ... t_{n} = \{\alpha_{1} \mapsto t_{1} ... \alpha_{n} \mapsto t_{n}\}(u)}$$
 TEQ_EXPAND
$$\underbrace{\frac{ne = \mathbf{normalize} (ne')}{\Delta \vdash ne = ne'}}_{\Delta \vdash ne = ne'}$$
 TEQ_NEXP

 $\Delta, E \vdash typ \leadsto t$

Convert source types to internal types

 $\vdash Nexp \leadsto ne$ Convert and normalize numeric expressions

 $\Delta, E \vdash typs \leadsto t_multi$

$$\frac{\Delta, E \vdash typ_1 \leadsto t_1 \quad .. \quad \Delta, E \vdash typ_n \leadsto t_n}{\Delta, E \vdash typ_1 \ast .. \ast typ_n \leadsto (t_1 \ast .. \ast t_n)} \quad \text{CONVERT_TYPS_ALL}$$

 $\vdash lit: t$ Typing literal constants

```
CHECK_LIT_FALSE
                                                                            \vdash false l : \_bool
                                                                                                                                 CHECK\_LIT\_NUM
                                                                               \vdash num \ l : \_\_num
                                                                         nat = \mathbf{bitlength}(hex)
                                                                                                                                              CHECK_LIT_HEX
                                                                  \vdash hex \ l : \_\_\mathbf{vector} \ nat \ \_\_\mathbf{bit}
                                                                          nat = \mathbf{bitlength}(bin)
                                                                                                                                               CHECK_LIT_BIN
                                                                    \vdash bin \ l : \_	extbf{vector} \ nat \ \_	extbf{bit}
                                                                                                                                 CHECK_LIT_STRING
                                                                       \vdash string \ l : \_string
                                                                                                                             CHECK_LIT_UNIT
                                                                                  \vdash () l : \_\mathbf{unit}
                                                                                                                              CHECK_LIT_BITZERO
                                                                       \overline{\vdash \mathbf{bitzero}\ l : \_\mathbf{bit}}
                                                                                                                              CHECK_LIT_BITONE
                                                                          \vdash bitone l : __bit
    \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                                               Field typing (also returns canonical field names)
  \begin{split} E(\,\overline{x_{i}^{l}}^{i}\,) & \rhd \left\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \right\rangle \\ E^{\mathrm{F}}(y) & \rhd \left\langle \, \mathbf{forall} \, tnv_{1} \ldots tnv_{n}.p \, \rightarrow \, t, (z \, \mathbf{of} \, names) \right\rangle \\ \frac{\Delta \vdash t_{1} \, \mathbf{ok} \quad \ldots \quad \Delta \vdash t_{n} \, \mathbf{ok}}{\Delta, E \vdash \, \mathbf{field} \, \overline{x_{i}^{l}}^{i} \, y \, l_{1} \, l_{2} : p \, t_{1} \ldots t_{n} \, \rightarrow \, \{tnv_{1} \mapsto t_{1} \ldots tnv_{n} \mapsto t_{n}\}(t) \, \rhd \, (z \, \mathbf{of} \, names) \end{split} \quad \text{INST\_FIELD\_ALL}
    \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \to p\ t\_args \triangleright (x\ \mathbf{of}\ names) Data constructor typing (also returns canonical constru
                                    E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 ... \ tnv_n.t_-multi \to p, (z \mathbf{of} \ names) \rangle
\Delta \vdash t_1 \mathbf{ok} ... \Delta \vdash t_n \mathbf{ok}
                                                                                                                                                                                                                          INST_CTOR_ALL
\overline{\Delta, E \vdash \mathbf{ctor} \, \overline{x_i^l}^{\, l}} \, y \, l_1 \, l_2 : \{ tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n \} (t\_multi) \to p \, t_1 \dots t_n \rhd (z \, \mathbf{of} \, names)
     \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                      Typing top-level bindings, collecting typeclass constraints
                          E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle 
E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, \ env\_tag \rangle
                           \Delta \vdash t_1 \mathbf{ok} \quad .. \quad \Delta \vdash t_n \mathbf{ok}
                            \frac{\sigma = \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}}{\Delta, E \vdash \mathbf{val} \, \overline{x_i^l}^i \, y \, l_1 \, l_2 : \sigma(t) \triangleright \{(p_1 \, \sigma(tnv_1')), ..., (p_i \, \sigma(tnv_i'))\}} \quad \text{INST_VAL_ALL}
                          \sigma = \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}
      E, E^{\mathrm{L}} \vdash x \mathbf{not} \mathbf{ctor}
                                                              v is not bound to a data constructor
                                                                             \frac{E^{\text{L}}(x) \triangleright t}{E, E^{\text{L}} \vdash x \text{ not ctor}} \quad \text{NOT\_CTOR\_VAL}
                                                   \frac{x \not\in \mathbf{dom}\left(E^{\mathbf{X}}\right)}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}}\rangle, E^{\mathbf{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor}} \quad \text{NOT\_CTOR\_UNBOUND}
                    \frac{E^{\mathbf{X}}(x) \triangleright \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, env\_tag \rangle}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle, E^{\mathbf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}} \quad \text{NOT\_CTOR\_BOUND}
```

```
E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                           id is not lexically shadowed
                                                               \frac{x \not\in \mathbf{dom}\,(E^{\mathrm{L}})}{E^{\mathrm{L}} \vdash x \ l_1 \ l_2 \ \mathbf{not} \ \mathbf{shadowed}} \quad \text{NOT\_SHADOWED\_SING}
                                                                                                                                                       NOT_SHADOWED_MULTI
                                                 \overline{E^{\text{L}} \vdash x_1^l \dots x_n^l . y^l . z^l \, l \, \text{not shadowed}}
   \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}
                                                                              Typing patterns, building their binding environment
                                                                   \frac{\Delta, E, E_1^{\text{L}} \vdash pat\_aux : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash pat\_aux \ l : t \triangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_ALL}
\Delta, E, E_1^{\scriptscriptstyle L} \vdash pat\_aux : t \triangleright E_2^{\scriptscriptstyle L}
                                                                                         Typing patterns, building their binding environment
                                                                       \frac{\Delta \vdash t \text{ ok}}{\Delta, E, E^{\text{L}} \vdash \_: t \triangleright \{\}} \quad \text{CHECK\_PAT\_AUX\_WILD}
                                                                        \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}
                                             \frac{x \not\in \mathbf{dom}\,(E_2^{\mathrm{L}})}{\Delta, E, E_1^{\mathrm{L}} \vdash (pat\,\mathbf{as}\,x\,l): t \vartriangleright E_2^{\mathrm{L}} \uplus \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_AS}
                                                                      \Delta, E, E_1^{\mathrm{L}} \vdash pat : t \triangleright E_2^{\mathrm{L}}
                                                           \frac{\Delta, E, E_1 \vdash put : t \triangleright E_2}{\Delta, E \vdash typ \leadsto t}
\frac{\Delta, E, E_1^{\text{L}} \vdash (pat : typ) : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat : typ) : t \triangleright E_2^{\text{L}}}
CHECK_PAT_AUX_TYP
       \Delta, E \vdash \mathbf{ctor}\ id : (t_1 * ... * t_n) \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
        E^{\text{L}} \vdash id \text{ not shadowed}
       \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_1 : t_1 \, \triangleright \, E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_n : t_n \, \triangleright \, E_n^{\mathsf{L}}
       \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,..\,,E_n^{\scriptscriptstyle{\mathrm{L}}})
                                                                                                                                                                                CHECK_PAT_AUX_IDENT_CONSTR
                \Delta, E, E^{\text{\tiny L}} \vdash id \; pat_1 \ldots pat_n : p \; t\_args \; \triangleright \; E^{\text{\tiny L}}_1 \; \uplus \; \ldots \; \uplus \; E^{\text{\tiny L}}_n
                                                                           \Delta \vdash t \mathbf{ok}
                                                         \frac{E, E^{\mathsf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}}{\Delta, E, E^{\mathsf{L}} \vdash x \ l_1 \ l_2 \ : t \, \rhd \, \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_VAR}
                            \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \mathbf{of}\ names)}^i
                            \overline{\Delta, E, E^{\mathrm{L}} \vdash pat_i : t_i \triangleright E_i^{\mathrm{L}}}
                            \mathbf{disjoint}\,\mathbf{doms}\,(\,\overline{E_i^{\scriptscriptstyle \mathrm{L}}}^{\,i}\,)
                           \mathbf{duplicates}\,(\,\overline{x_i}^{\,i}\,)=\,\emptyset
                            \frac{\Delta \operatorname{Approximation}(|x_i|) - |v|}{\Delta, E, E^{\text{L}} \vdash \langle | \overline{id_i = pat_i l_i}^i; ?| \rangle : p \ t\_args \triangleright \uplus \overline{E_i^{\text{L}}}^i} \quad \text{CHECK\_PAT\_AUX\_RECORD}
                  \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_1 : t \, \rhd \, E_1^{\mathrm{L}} \quad \dots \quad \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_n : t \, \rhd \, E_n^{\mathrm{L}}
                   \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
        \frac{\mathbf{length}\,(pat_1\dots pat_n) = nat}{\Delta,E,E^\mathtt{L} \vdash [|pat_1;\dots;pat_n\,;^2|] : \_\mathbf{vector}\,nat\;t \,\triangleright\,E^\mathtt{L}_1 \uplus \dots \uplus E^\mathtt{L}_n} \quad \text{CHECK\_PAT\_AUX\_VECTOR}
\Delta, E, E^{\mathtt{L}} \vdash \mathit{pat}_1 : \_\mathtt{vector} \ \mathit{ne}_1 \ t \, \rhd \, E^{\mathtt{L}}_1 \quad \dots \quad \Delta, E, E^{\mathtt{L}} \vdash \mathit{pat}_n : \_\mathtt{vector} \ \mathit{ne}_n \ t \, \rhd \, E^{\mathtt{L}}_n
disjoint doms (E_1^L, \ldots, E_n^L)
ne' = ne_1 + \dots + ne_n
                                  \frac{\Delta}{\Delta, E, E^{\mathrm{L}} \vdash [|pat_{1} \dots pat_{n}|] : \_\mathbf{vector} \ ne' \ t \rhd E_{1}^{\mathrm{L}} \uplus \ \dots \ \uplus E_{n}^{\mathrm{L}}}
                                                                                                                                                                                                                                              CHECK_PAT_AUX_VECTOR
                     \Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{pat}_1 : t_1 \, \rhd \, E_1^{\scriptscriptstyle L} \quad .... \quad \Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{pat}_n : t_n \, \rhd \, E_n^{\scriptscriptstyle L}
                     \frac{\text{disjoint doms}\left(E_{1}^{\text{L}},\, \ldots,\, E_{n}^{\text{L}}\right)}{\Delta,\, E,\, E^{\text{L}} \vdash \left(pat_{1},\, \ldots,\, pat_{n}\right) :\, t_{1} \ast\, \ldots\, \ast\, t_{n}\, \triangleright\, E_{1}^{\text{L}} \uplus\, \ldots\, \uplus\, E_{n}^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_TUP}
```

```
\Delta \vdash t \mathbf{ok}
                          \Delta, E, E^{\mathsf{L}} \vdash pat_1 : t \triangleright E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash pat_n : t \triangleright E_n^{\mathsf{L}}
                         disjoint doms (E_1^{\scriptscriptstyle \rm L},\,..\,,E_n^{\scriptscriptstyle \rm L})
                             \Delta, E, E^{\text{L}} \vdash [pat_1; ...; pat_n;^?] : \_\textbf{list} \ t \triangleright E_1^{\text{L}} \uplus ... \uplus E_n^{\text{L}} CHECK_PAT_AUX_LIST
                                                               \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat) : t \vartriangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_PAREN}
                                         \begin{array}{c} \Delta, E, E_{1}^{\text{L}} \vdash pat_{1} : t \vartriangleright E_{2}^{\text{L}} \\ \Delta, E, E_{1}^{\text{L}} \vdash pat_{2} : \_\textbf{list} \ t \vartriangleright E_{3}^{\text{L}} \\ \textbf{disjoint doms} \left(E_{2}^{\text{L}}, E_{3}^{\text{L}}\right) \\ \hline \Delta, E, E_{1}^{\text{L}} \vdash pat_{1} :: pat_{2} : \_\textbf{list} \ t \vartriangleright E_{2}^{\text{L}} \uplus E_{3}^{\text{L}} \end{array} \quad \text{CHECK\_PAT\_AUX\_CONS} \end{array}
                                                                        \frac{\vdash lit:t}{\Delta,E,E^{\text{L}}\vdash lit:t\,\vartriangleright\,\big\{\,\big\}}\quad\text{CHECK\_PAT\_AUX\_LIT}
                                                                   E, E^{\text{\tiny L}} \vdash x \operatorname{\mathbf{not}} \operatorname{\mathbf{ctor}}
                         \frac{L, E + x \text{ not ctor}}{\Delta, E, E^{\text{\tiny L}} \vdash x \ l + num : \_\text{num} \ \triangleright \{x \mapsto \_\text{num} \ \}}
  E \vdash id \mathbf{field}
                                                Check that the identifier is a permissible field identifier
                                                                  \frac{E^{\mathrm{F}}(x) \rhd f\_desc}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \vdash x \ l_1 \ l_2 \, \mathbf{field}} \quad \text{ID\_FIELD\_EMPTY}
                                                                                       E^{\mathrm{M}}(x) \triangleright E
                                                                                       x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                                           \frac{E \vdash \overline{y_{i}^{l}}.^{i} z^{l} \ l_{2} \, \mathbf{field}}{\langle E^{\text{\tiny{M}}}, E^{\text{\tiny{P}}}, E^{\text{\tiny{F}}}, E^{\text{\tiny{X}}} \rangle \vdash x \ l_{1}. \ \overline{y_{i}^{l}}.^{i} \ z^{l} \ l_{2} \, \mathbf{field}} \quad \text{\tiny{ID\_FIELD\_CONS}}
  E \vdash id value
                                                   Check that the identifier is a permissible value identifier
                                                               \frac{E^{\mathbf{X}}(x) \triangleright v\_desc}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{value}} \quad \text{ID\_VALUE\_EMPTY}
                                                                                    E^{\mathrm{M}}(x) \triangleright E
                                                                                   x \notin \mathbf{dom}(E^{X})
                                                        \frac{E \vdash \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}}{\langle E^{\text{M}}, E^{\text{P}}, E^{\text{F}}, E^{\text{X}} \rangle \vdash x \ l_1. \ \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}} \quad \text{ID_VALUE\_CONS}
  \Delta, E, E^{\text{L}} \vdash exp: t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Typing expressions, collecting typeclass and index constraints
                                                             \frac{\Delta, E, E^{\mathsf{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\mathsf{L}} \vdash exp\_aux \ l : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_ALL}
\Delta, E, E^{\text{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Typing expressions, collecting typeclass and index constraints
                                                            \frac{E^{\mathrm{L}}(x) \triangleright t}{\Delta, E, E^{\mathrm{L}} \vdash x \ l_{1} \ l_{2} : t \triangleright \{\}, \{\}} \quad \text{CHECK\_EXP\_AUX\_VAR}
                                                      \overline{\Delta,E,E^{\text{\tiny L}} \vdash N: \_\mathbf{num} \ \triangleright \ \{\ \},\{\ \}} \quad \text{CHECK\_EXP\_AUX\_NVAR}
                            E^{\mathrm{L}} \vdash id \, \mathbf{not} \, \mathbf{shadowed}
                            E \vdash id \text{ value}
                           \Delta, E \vdash \mathbf{ctor} \ id : t\_multi \to p \ t\_args \rhd (x \ \mathbf{of} \ names)
CHECK\_EXP\_AUX\_CTOR
                              \Delta, E, E^{\text{L}} \vdash id : \mathbf{curry}(t\_multi, p \ t\_args) \triangleright \{\}, \{\}
```

```
E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                           E \vdash id \text{ value}
                                                                           \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                       \overline{\Delta,E,E^{\text{\tiny L}} \vdash id:t \vartriangleright \Sigma^{\mathcal{C}},\{\,\,\}} \quad \text{CHECK\_EXP\_AUX\_VAL}
                            \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                            disjoint doms (E_1^L, ..., E_n^L)
                                                                                                                                                                                                                            CHECK_EXP_AUX_FN
        \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{fun} \ pat_1 \dots pat_n \rightarrow exp \ l : \mathbf{curry} \left( (t_1 * \dots * t_n), u \right) \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                             \overline{\Delta,E,E^{\scriptscriptstyle L} \vdash \mathit{pat}_i : t \, \triangleright \, E_i^{\scriptscriptstyle L}}^i
                                             \frac{1}{\Delta, E, E^{L} \uplus E_{i}^{L} \vdash exp_{i} : u \rhd \Sigma^{C}_{i}, \Sigma^{N}_{i}}
     \Delta, E, E^{\text{L}} \vdash \overline{\mathbf{function}} \mid^{?} \overline{pat_{i} \rightarrow exp_{i} l_{i}}^{i} \mathbf{end} : t \rightarrow u \triangleright \overline{\Sigma^{C}_{i}}^{i}, \overline{\Sigma^{N_{i}}}^{i}
                                                      \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t_1 \to t_2 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                      \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                 \frac{1}{\Delta,E,E^{\text{L}} \vdash exp_1 \ exp_2 : t_2 \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2} \quad \text{Check_exp_aux_approx}
                                        \begin{array}{l} \Delta, E, E^{\mathrm{L}} \vdash (ix) : t_1 \to t_2 \to t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                        \Delta, E, E^{\mathsf{L}} \vdash exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
                                                                                                                                                                                                                CHECK_EXP_AUX_INFIX_APP1
\overline{\Delta, E, E^{\text{\tiny L}} \vdash exp_1 \ ix \ l \ exp_2 : t_3 \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2 \cup \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2 \cup \Sigma^{\mathcal{N}}_3}
                                           \Delta, E, E^{\mathsf{L}} \vdash x : t_1 \to t_2 \to t_3 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                            \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}
                                           \Delta, E, E^{\mathsf{L}} \vdash exp_2 : t_2 \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
                                                                                                                                                                                                               CHECK_EXP_AUX_INFIX_APP2
\overline{\Delta, E, E^{\text{\tiny L}} \vdash exp_1 \text{'}x\text{'}l \ exp_2 : t_3 \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2 \cup \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2 \cup \Sigma^{\mathcal{N}}_3}
                                \overline{\Delta}, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)^i
                               \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                \mathbf{duplicates} \left( \overline{x_i}^i \right) = \emptyset
                                names = \{ \overline{x_i}^i \}
                                                                                                                                                                                         CHECK_EXP_AUX_RECORD
                       \Delta, E, E^{\text{\tiny L}} \vdash \langle | \overline{id_i = exp_i \ l_i}^{\ i}; ? \ l | \rangle : p \ t\_args \rhd \overline{\Sigma^{\mathcal{C}_i}}^{\ i}. \overline{\Sigma^{\mathcal{N}_i}}^{\ i}
                                            \Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \mathbf{of}\ names)
                                            \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                           \mathbf{duplicates}(\overline{x_i}^i) = \emptyset
                                            \Delta, E, E^{\mathsf{L}} \vdash exp : p \ t\_args \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'}
\Delta, E, E^{\text{L}} \vdash \langle | \overline{exp \ \textbf{with}} \ \overline{id_i = exp_i \ l_i}^i \ ; ^? \ l | \rangle : p \ t\_args \rhd \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_i}^i , \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}}_i}^i  CHECK_EXP_AUX_RECUP
                      \Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{exp}_1 : t \, \rhd \, \Sigma^{\mathcal{C}}_{\phantom{\mathcal{C}}1}, \Sigma^{\mathcal{N}}_{\phantom{\mathcal{N}}1} \quad \dots \quad \Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{exp}_n : t \, \rhd \, \Sigma^{\mathcal{C}}_{\phantom{\mathcal{C}}n}, \Sigma^{\mathcal{N}}_{\phantom{\mathcal{N}}n}
                      length(exp_1 ... exp_n) = nat
\overline{\Delta, E, E^{\text{\tiny L}} \vdash [|exp_1; \dots; exp_n;^?|] : \_\textbf{vector} \ nat \ t \, \triangleright \, \overline{\Sigma^{\mathcal{C}}}_1 \, \cup \, \dots \, \cup \, \underline{\Sigma^{\mathcal{C}}}_n, \underline{\Sigma^{\mathcal{N}}}_1 \, \cup \, \dots \, \cup \, \underline{\Sigma^{\mathcal{N}}}_n}
                                                                                                                                                                                                                                                         CHECK_EXP_AUX_VECTOR
                                     \Delta, E, E^{\perp} \vdash exp : \_vector ne' t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                     \vdash Nexp \leadsto ne
                                                                                                                                                                             CHECK_EXP_AUX_VECTORGET
                           \overline{\Delta, E, E^{\text{L}} \vdash exp.(Nexp) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne\langle ne'\}\}}
                                               \Delta, E, E^{\perp} \vdash exp : \_vector ne' t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                               \vdash Nexp_1 \leadsto ne_1
                                               \vdash Nexp_2 \leadsto ne_2
                                               ne = ne_2 + (-ne_1)
                                                                                                                                                                                                                            CHECK_EXP_AUX_VECTORSUB
\Delta, E, E^{\text{L}} \vdash exp.(Nexp_1..Nexp_2) : \_vector ne t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne_1 \langle ne_2 \langle ne' \} \}
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E \vdash id \mathbf{field}
                                              \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                              \Delta, E, E^{\mathrm{L}} \vdash exp : p \ t\_args \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                                                               CHECK_EXP_AUX_FIELD
                                                                      \Delta, E, E^{\text{L}} \vdash exp.id : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                           \overline{\Delta, E, E^{\text{L}} \vdash pat_i : t \triangleright E_i^{\text{L}}}^i
                                                                          \Delta, E, E^{\text{L}} \vdash \mathbf{match} \ exp \ \mathbf{with} \ |^{?} \ \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ l \ \mathbf{end} : u \rhd \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_{i}}^{i}, \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}_{i}}}^{i}
                                                                         \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                         \Delta, E \vdash typ \leadsto t
                                                                                                                                                                                     CHECK_EXP_AUX_TYPED
                                                             \overline{\Delta, E, E^{\text{L}} \vdash (exp : tup) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                 \Delta, E, E_1^{\mathrm{L}} \vdash letbind \triangleright E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                 \Delta, E, E_1^{\mathrm{L}} \uplus E_2^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                                                                                                                                                                                                                 CHECK_EXP_AUX_LET
                           \frac{1}{\Delta, E, E_1^{\text{L}} \vdash \mathbf{let} \ let bind \ \mathbf{in} \ exp: t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2}
\frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t_n \rhd \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash (exp_1, \dots, exp_n) : t_1 * \dots * t_n \rhd \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_TUP}
 \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \vartriangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash [exp_1; \; ..; exp_n \; ;^?] : \_\mathbf{list} \; t \vartriangleright \Sigma^{\mathcal{C}}_1 \; \cup \; .. \; \cup \; \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \; \cup \; .. \; \cup \; \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_LIST}
                                                                        \Delta, E, E^{\mathrm{L}} \vdash exp : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                      \Delta, E, E^{\perp} \vdash (exp) : t \triangleright \Sigma^{\mathcal{C}} \cdot \Sigma^{\mathcal{N}} CHECK_EXP_AUX_PAREN
                                                      \frac{\Delta, E, E^{\text{L}} \vdash exp: t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\text{L}} \vdash \mathbf{begin} \ exp \ \mathbf{end}: t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_BEGIN}
                                                                           \Delta, E, E^{\mathsf{L}} \vdash exp_1 : \_\mathbf{bool} \triangleright \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1}
                                                                           \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                                                           \Delta, E, E^{\mathsf{L}} \vdash exp_3 : t \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
\overline{\Delta, E, E^{\scriptscriptstyle L} \vdash \mathbf{if} \ exp_1 \, \mathbf{then} \ exp_2 \, \mathbf{else} \ exp_3 : t \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2 \, \cup \, \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \, \cup \, \Sigma^{\mathcal{N}}_2 \, \cup \, \Sigma^{\mathcal{N}}_3}
                                                             \Delta, E, E^{\scriptscriptstyle L} \vdash \mathit{exp}_1 : t \, \rhd \, \Sigma^{\mathcal{C}}_{-1}, \Sigma^{\mathcal{N}}_{-1}
                                                              \Delta, E, E^{\mathsf{L}} \vdash exp_2 : \_\mathbf{list} \ t \, \triangleright \, \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                        \overline{\Delta, E, E^{\text{L}} \vdash exp_1 :: exp_2 : \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_CONS}
                                                                                 \frac{\vdash lit:t}{\Delta,E,E^{\text{L}}\vdash lit:t\,\rhd\,\{\,\},\{\,\}}\quad\text{CHECK\_EXP\_AUX\_LIT}
                            \Delta \vdash t_i \mathbf{ok}^i
                            \Delta, E, E^{L} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_1 : t \rhd \Sigma^{C}_1, \Sigma^{N}_1
                            \Delta, E, E^{\mathsf{L}} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_2 : \_bool \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                            disjoint doms (E^{L}, \{\overline{x_i \mapsto t_i}^i\})
                            E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                            x_i \not\in \mathbf{dom}(E^{\mathbf{X}})^{i}
                \overline{\Delta, E, E^{\text{L}} \vdash \{exp_1 | exp_2\} : \_\mathbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_SET\_COMP}
                                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp_2 : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3 \end{array}
                                                                                                                                                                                                                                                                                 CHECK_EXP_AUX
\Delta, E, E_1^{\text{\tiny L}} \vdash \{ exp_1 | \mathbf{forall} \ \overline{qbind_i}^{\ i} \ | exp_2 \} : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2 \ \cup \ \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \ \cup \ \Sigma^{\mathcal{N}}_3
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\Delta \vdash t \mathbf{ok}
  \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \triangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash \{exp_1; ...; exp_n;^?\} : \_\mathbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_SET}
                                               \frac{\Delta, E, E_1^{\text{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}_1}{\Delta, E, E_1^{\text{L}} \vdash \overline{qbind_i}^i \cdot exp : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2} \xrightarrow{\text{CHECK\_EXP\_AUX\_QUANT}} \Delta, E, E_1^{\text{L}} \vdash q \ \overline{qbind_i}^i \cdot exp : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2}
\Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass cons
                                                                                                           \overline{\Delta,E,E^{\text{L}} \vdash \rhd\{\,\},\{\,\}} \quad \text{CHECK\_LISTQUANT\_BINDING\_EMPTY}
                                                             \Delta \vdash t \mathbf{ok}
                                                     \frac{\Delta, E, E_1^{\text{\tiny L}} \uplus \{x \mapsto t\} \vdash \overline{qbind_i}^i \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1}{\text{disjoint doms}\left(\{x \mapsto t\}, E_2^{\text{\tiny L}}\right)} \\ \frac{\Delta, E, E_1^{\text{\tiny L}} \vdash x \ l \ \overline{qbind_i}^i \rhd \{x \mapsto t\} \uplus E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1}{\Delta, E, E_1^{\text{\tiny L}} \vdash x \ l \ \overline{qbind_i}^i \rhd \{x \mapsto t\} \uplus E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1}
                                                                                                                                                                                                                                                                                                          CHECK\_LISTQUANT\_BINDING\_VAR
                                                                  \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                                    \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}_2}
                                                                   \frac{\mathbf{disjoint}\,\mathbf{doms}\,(E_3^{\mathtt{L}},E_2^{\mathtt{L}}) }{} 
               \frac{\Delta B_{\mathbf{J}}^{\mathbf{DIR}} \mathbf{GOHB} \left(E_{3}, E_{2}\right)}{\Delta, E, E_{1}^{\mathbf{L}} \vdash \left(pat \, \mathbf{IN} \, exp\right) \overline{qbind_{i}}^{i} \triangleright E_{2}^{\mathbf{L}} \uplus E_{3}^{\mathbf{L}}, \Sigma^{\mathcal{C}}_{1} \, \cup \, \Sigma^{\mathcal{C}}_{2}} \quad \text{CHECK\_LISTQUANT\_BINDING\_RESTR}
                                                             \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \triangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp : \_\textbf{list} \ t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                             \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{\mathit{qbind}_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_2
\frac{\mathbf{disjoint \ doms} \ (E_{3}^{\mathrm{L}}, E_{2}^{\mathrm{L}})}{\Delta, E, E_{1}^{\mathrm{L}} \vdash (pat \ \mathbf{MEM} \ exp) \ \overline{qbind_{i}}^{i} \rhd E_{2}^{\mathrm{L}} \uplus E_{3}^{\mathrm{L}}, \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2}}
     \Delta, E, E_1^{\text{L}} \vdash \mathbf{list} \ qbind_1 ... \ qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass
                                                                                                                                                                                                                                                            CHECK\_QUANT\_BINDING\_EMPTY
                                                                                                          \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{list} \triangleright \{\}, \{\}}
                                                                               \begin{array}{l} \Delta, E, E_1^{\rm L} \vdash pat : t \vartriangleright E_3^{\rm L} \\ \Delta, E, E_1^{\rm L} \vdash exp : \_{\bf list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                                             \begin{array}{l} \Delta, E, E_1^{\text{L}} \uplus E_3^{\text{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}_2 \\ \textbf{disjoint doms}\left(E_3^{\text{L}}, E_2^{\text{L}}\right) \end{array}
      \Delta, E, E_1^{\text{L}} \vdash \overline{\textbf{list}\left(pat\, \mathbf{MEM}\, exp\right)\, \overline{qbind_i}^{\,\,i}} \, \triangleright \, E_2^{\text{L}} \uplus E_3^{\text{L}}, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2
      \Delta, E, E^{\text{L}} \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a function definition clause, collecting typed
                                                                     \begin{array}{l} \Delta, E, E^{\text{\tiny L}} \vdash pat_1: t_1 \vartriangleright E_1^{\text{\tiny L}} \quad \dots \quad \Delta, E, E^{\text{\tiny L}} \vdash pat_n: t_n \vartriangleright E_n^{\text{\tiny L}} \\ \Delta, E, E^{\text{\tiny L}} \uplus E_1^{\text{\tiny L}} \uplus \quad \dots \quad \uplus E_n^{\text{\tiny L}} \vdash exp: u \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \textbf{disjoint doms} \ (E_1^{\text{\tiny L}}, \ \dots, E_n^{\text{\tiny L}}) \end{array}
                                                                       \Delta, E \vdash typ \leadsto u
\frac{-\sum_{l} \sum_{l} \sum
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CHECK_FUNCL_ANNOT

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\begin{array}{lll} \Delta,\,E,\,E^{\scriptscriptstyle L}\vdash\,pat_1:t_1\,\vartriangleright\,E_1^{\scriptscriptstyle L}&\ldots&\Delta,\,E,\,E^{\scriptscriptstyle L}\vdash\,pat_n:t_n\,\vartriangleright\,E_n^{\scriptscriptstyle L}\\ \Delta,\,E,\,E^{\scriptscriptstyle L}\uplus\,E_1^{\scriptscriptstyle L}\uplus\,\ldots\,\uplus\,E_n^{\scriptscriptstyle L}\vdash\,exp:u\,\vartriangleright\,\Sigma^{\scriptscriptstyle \mathcal{C}},\Sigma^{\scriptscriptstyle \mathcal{N}} \end{array}
                               disjoint doms (E_1^{\text{L}}, \ldots, E_n^{\text{L}})
\overline{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                                                                                                                                                                                                  CHECK_FUNCL_NOANNOT
   \Delta, E, E_1^{\text{L}} \vdash letbind \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a let binding, collecting typeclass and index con
                                                           \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                           \Delta, E \vdash typ \leadsto t
                                      \frac{\Delta, E \vdash \textit{typ} \leadsto \textit{t}}{\Delta, E, E_1^{\text{L}} \vdash \textit{pat} : \textit{typ} = \textit{exp} \; \textit{l} \mathrel{\triangleright} E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_VAL\_ANNOT}
                                         \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \rhd E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash exp : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \\ \frac{\Delta, E, E_1^{\text{L}} \vdash exp : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\text{L}} \vdash pat = exp \ l \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_VAL\_NOANNOT}
                                                  \frac{\Delta, E, E_1^{\text{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\text{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_FN}
     \Delta, E, E^{\text{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for an inductive relation clause, collecting typed
                                                                                                  \overline{\Delta \vdash t_i \, \mathbf{ok}}^i
                                                                                                  E_2^{\mathrm{L}} = \{ \overline{name_- t_i \to x \mapsto t_i}^i \}
\Delta, E, E_{1}^{\mathsf{L}} \uplus E_{2}^{\mathsf{L}} \vdash exp' : \_\mathbf{bool} \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'}
\Delta, E, E_{1}^{\mathsf{L}} \uplus E_{2}^{\mathsf{L}} \vdash exp_{1} : u_{1} \rhd \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \quad ... \quad \Delta, E, E_{1}^{\mathsf{L}} \uplus E_{2}^{\mathsf{L}} \vdash exp_{n} : u_{n} \rhd \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n}
\Delta, E, E_{1}^{\mathsf{L}} \vdash x_{1}^{\mathsf{l}} : \mathbf{forall} \overline{name\_t_{i}}^{i} \cdot exp' \Longrightarrow x \ l \ exp_{1} ... \ exp_{n} \ l' \rhd \{x \mapsto \mathbf{curry} ((u_{1} * ... * u_{n}), \_\mathbf{bool})\}, \Sigma^{\mathcal{C}'} \cup \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{1})
      xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathsf{P}} Extract the type constructor information
                                                                        tnvars^l \leadsto tnvs
                                                                         \Delta, E \vdash typ \leadsto t
                                                                        \mathbf{duplicates}(tnvs) = \emptyset
                                                                        \mathbf{FV}(t) \subset tnvs
      \frac{\overline{y_i}.^i \, x \not\in \mathbf{dom} \, (\Delta)}{\overline{y_i}.^i, \Delta, E \vdash \mathbf{tc} \, x \, l \, tnvars^l \, = typ \, \triangleright \, \{\overline{y_i}.^i \, x \mapsto tnvs \, .t\}, \{x \mapsto \overline{y_i}.^i \, x\}} \quad \text{CHECK\_TEXP\_TC\_ABBREV}
                                                                     tnvars^l \leadsto tnvs
                                                                     \mathbf{duplicates}(tnvs) = \emptyset
             \frac{\overline{y_i.}^i \, x \not\in \mathbf{dom} \, (\Delta)}{\overline{y_i}^i, \Delta, E_1 \vdash \mathbf{tc} \, x \, l \, tnvars^l \, \triangleright \, \{\overline{y_i.}^i \, x \mapsto tnvs \, \}, \{x \mapsto \overline{y_i.}^i \, x\}} \quad \text{CHECK\_TEXP\_TC\_ABSTRACT}
                                                                                                   tnvars^l \leadsto tnvs
                                                                                                   \mathbf{duplicates}\left(tnvs\right) = \emptyset
\frac{\overline{y_i.}^i\,x\not\in\mathbf{dom}\,(\Delta)}{\overline{y_i}^i,\Delta_1,E\vdash\mathbf{tc}\,x\,l\,tnvars^l\,=\langle|x_1^l:typ_1;\ldots;x_j^l:typ_j\,;^?|\rangle\,\rhd\,\{\overline{y_i.}^i\,x\mapsto tnvs\,\},\{x\mapsto\overline{y_i.}^i\,x\}}
                                                                                                                                                                                                                                                                                    CHECK_TEXP_TC_REC
                                                                                                 tnvars^l \leadsto tnvs
                                                                                                 \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                                                                 \overline{y_i}. i \ x \not\in \mathbf{dom}(\Delta)
\overline{\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l = |? \ ctor\_def_1| \dots |ctor\_def_j| \triangleright \{\overline{y_i.}^i \ x \mapsto tnvs \}, \{x \mapsto \overline{y_i.}^i \ x\}
                                                                                                                                                                                                                                                                             CHECK_TEXP_TC_VAR
      xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                                    Extract the type constructor information
                                                                                                                                                 CHECK_TEXPS_TC_EMPTY
                                                                        \overline{xs, \Delta, E \vdash \mathbf{tc} \triangleright \{\}, \{\}}
```

```
xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E_2^{P}
                     xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{ \}, E_2^{\mathrm{P}}, \{ \}, \{ \} \rangle \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_3, E_3^{\mathrm{P}} \\ \mathbf{dom} (E_2^{\mathrm{P}}) \cap \mathbf{dom} (E_3^{\mathrm{P}}) = \emptyset

    — CHECK_TEXPS_TC_ABBREV

                                     xs, \Delta_1, E \vdash \mathbf{tc} \ td \ \overline{td_i}^i \rhd \Delta_2 \uplus \Delta_3, E_2^P \uplus E_3^P
    \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle Check a type definition, with its path already resolved
                                                                                                                                                          CHECK_TEXP_ABBREV
                                                           \overline{\Delta, E \vdash tnvs \ p = typ \, \triangleright \, \langle \{\,\}, \{\,\} \rangle}
                                      \overline{\Delta, E \vdash typ_i \leadsto t_i}^i
                                      names = \{ \overline{x_i}^i \}
                                     \frac{\mathbf{duplicates}\left(\overline{x_{i}}^{i}\right)}{\mathbf{FV}\left(t_{i}\right) \subset \mathit{tnvs}^{i}} = \emptyset
                                     E^{F} = \{ \overline{x_i \mapsto \langle \mathbf{forall} \ tnvs. p \to t_i, (x_i \mathbf{of} \ names) \rangle^i} \}\Delta, E \vdash tnvs \ p = \langle |\overline{x_i^l : typ_i}^i; ?| \rangle \triangleright \langle E^F, \{ \} \rangle
                            \Delta, E \vdash typs_i \leadsto t\_multi_i{}^i
                             names = \{ \overline{x_i}^i \}
                            \mathbf{duplicates}\,(\,\overline{x_i}^{\,i}\,)=\,\emptyset
                            \overline{\mathbf{FV}\left(t\_multi_i\right) \subset tnvs}
                             E^{X} = \{ \overline{x_i \mapsto \langle \mathbf{forall} \ tnvs. t\_multi_i \to p, (x_i \mathbf{of} \ names) \rangle}^i \}
                                                  \Delta, \overline{E \vdash tnvs \ p = |? \overline{x_i^l \ \textbf{of} \ typs_i}^i} \ \triangleright \ \langle \{\ \}, E^{\times} \rangle
    xs, \Delta, E \vdash td_1 ... td_n \rhd \langle E^{\scriptscriptstyle{\mathsf{F}}}, E^{\scriptscriptstyle{\mathsf{X}}} \rangle
                                                                                                                                             CHECK\_TEXPS\_EMPTY
                                                                        \overline{y_i}^i, \Delta, E \vdash \triangleright \langle \{\}, \{\} \rangle
                                    tnvars^l \leadsto tnvs
                                    \Delta, E_1 \vdash tnvs \overline{y_i}_{\cdot}^i x = texp \triangleright \langle E_1^{\scriptscriptstyle F}, E_1^{\scriptscriptstyle X} \rangle
                                    \begin{array}{l} \overline{y_i}^{\,i}, \Delta, E \vdash \overline{td_j}^{\,j} \rhd \langle E_2^{\scriptscriptstyle \mathrm{F}}, E_2^{\scriptscriptstyle \mathrm{X}} \rangle \\ \mathbf{dom} \, (E_1^{\scriptscriptstyle \mathrm{X}}) \cap \mathbf{dom} \, (E_2^{\scriptscriptstyle \mathrm{X}}) = \emptyset \end{array}
                                   \mathbf{dom}\,(E_1^{\mathrm{F}})\,\cap\,\mathbf{dom}\,(E_2^{\mathrm{F}})=\emptyset
                                                                                                                                                                             CHECK_TEXPS_CONS_CONCRETE
        \overline{\overline{y_i}^i, \Delta, E \vdash x \ l \ tnvars^l \ = \ texp \ \overline{td_j}^j \ \triangleright \ \langle E_1^{\scriptscriptstyle \mathrm{F}} \uplus E_2^{\scriptscriptstyle \mathrm{F}}, E_1^{\scriptscriptstyle \mathrm{X}} \uplus E_2^{\scriptscriptstyle \mathrm{X}} \rangle}
                                  \frac{\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \rhd \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle}{\overline{y_i}^i, \Delta, E \vdash x \ l \ tnvars^l \ \overline{td_j}^j \rhd \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle} \quad \text{CHECK\_TEXPS\_CONS\_ABSTRACT}
    \delta, E \vdash id \leadsto p
                                                    Lookup a type class
                                                                                     \frac{\delta(p) \triangleright xs}{\delta, E \vdash id \leadsto p} \quad \text{CONVERT\_CLASS\_ALL}
    I \vdash (p \ t) \mathbf{IN} \, \mathcal{C}
                                                     Solve class constraint
                                                                                                                                                                     SOLVE_CLASS_CONSTRAINT_IMMEDIATE
\overline{I \vdash (p \alpha) \mathbf{IN} (p_1 tnv_1) ... (p_i tnv_i) (p \alpha) (p'_1 tnv'_1) ... (p'_i tnv'_i)}
                  (p_1 tnv_1) ... (p_n tnv_n) \Rightarrow (p t) \mathbf{IN} I
                 I \vdash (p_1 \sigma(tnv_1)) \mathbf{IN} \mathcal{C} .. I \vdash (p_n \sigma(tnv_n)) \mathbf{IN} \mathcal{C}
                                                                                                                                                                    SOLVE_CLASS_CONSTRAINT_CHAIN
                                                              I \vdash (p \sigma(t)) \mathbf{IN} \mathcal{C}
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I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C} Solve class constraints
                                \frac{I \vdash (p_1 \ t_1) \mathbf{IN} \, \mathcal{C} \quad .. \quad I \vdash (p_n \ t_n) \mathbf{IN} \, \mathcal{C}}{I \vdash \{(p_1 \ t_1), \dots, (p_n \ t_n)\} \triangleright \mathcal{C}} \quad \text{SOLVE\_CLASS\_CONSTRAINTS\_ALL}
     \Delta, I, E \vdash val\_def \triangleright E^{X} Check a value definition
                                              \Delta, E, \{\} \vdash letbind \rhd \{\overline{x_i \mapsto t_i}^i\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
I \vdash \Sigma^{\mathcal{C}} \rhd \mathcal{C}
                                              \overline{\mathbf{FV}\left(t_{i}\right)} \subset \mathit{tnvs}^{i}
                                              \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                                                                                             CHECK_VAL_DEF_VAL
                    \overline{\Delta, I, E_1 \vdash \mathbf{let} \, \tau^? \, letbind} \, \triangleright \, \{ \, \overline{x_i \mapsto \langle \, \mathbf{forall} \, tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle}^{\, i} \, \}
                                         \overline{\Delta, E, E^{\text{L}} \vdash funcl_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                         I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                         \overline{\mathbf{FV}\left(t_{i}
ight)} \subset tnvs^{i}
                                         \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                         \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
          \frac{E^{\mathrm{L}} = \{\overline{x_i \mapsto t_i}^i\}}{\Delta, I, E \vdash \mathbf{let} \operatorname{\mathbf{rec}} \tau^? \overline{funcl_i}^i \triangleright \{\overline{x_i \mapsto \langle \operatorname{\mathbf{forall}} tnvs.\mathcal{C} \Rightarrow t_i, \operatorname{\mathbf{let}} \rangle}^i\}} \quad \text{CHECK\_VAL\_DEF\_RECFUN}
      \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance Check that t be a typeclass instance
                                                                                                                                  CHECK_T_INSTANCE_VAR
                                                                      \overline{\Delta,(\alpha) \vdash \alpha \text{ instance}}
                                                                                                                                                                 CHECK_T_INSTANCE_TUP
                                          \overline{\Delta,(\alpha_1,\ldots,\alpha_n)} \vdash \alpha_1 * \ldots * \alpha_n  instance
                                                        \overline{\Delta, (\alpha_1, \alpha_2) \vdash \alpha_1 \rightarrow \alpha_n \text{ instance}} CHECK_T_INSTANCE_FN
                                                   \frac{\Delta(p) \rhd \alpha_1' \dots \alpha_n'}{\Delta, (\alpha_1, \dots, \alpha_n) \vdash p \alpha_1 \dots \alpha_n \text{ instance}} \quad \text{CHECK\_T\_INSTANCE\_TC}
     \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2 Check a definition
                                   \overline{z_i}^j, \Delta_1, E \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_2, E^{\mathrm{P}}

\frac{z_{j}^{j}, \Delta_{1}, E \vdash \iota \iota \iota \iota_{i} \lor \Delta_{2}, E}{\overline{z_{j}^{j}}, \Delta_{1} \uplus \Delta_{2}, E \uplus \langle \{\}, E^{P}, \{\}, \{\} \rangle \vdash \overline{td_{i}^{i}} \lor \langle E^{F}, E^{X} \rangle} \qquad \text{CHECK_DEF_TYPE}

                     \overline{\overline{z_i}^j, \langle \Delta_1, \delta, I \rangle, E \vdash \mathbf{type} \, \overline{td_i}^i \, l \, \triangleright \, \langle \Delta_2, \{ \}, \{ \} \rangle, \langle \{ \}, E^{\mathrm{p}}, E^{\mathrm{r}}, E^{\mathrm{x}} \rangle}
                                     \frac{\Delta, I, E \vdash val\_def \rhd E^{\mathbf{X}}}{\overline{z_{i}}^{j}, \langle \Delta, \delta, I \rangle, E \vdash val\_def \ l \rhd \epsilon, \langle \{ \}, \{ \}, \{ \}, E^{\mathbf{X}} \rangle} \quad \text{CHECK\_DEF\_VAL\_DEF}
                                                                                                                            \Delta, E_1, E^{\scriptscriptstyle L} \vdash rule_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\scriptscriptstyle C}_i, \Sigma^{\scriptscriptstyle N}_i^{i}
                                                                                                                            I \vdash \overline{\Sigma^{\mathcal{C}}_{i}}^{i} \triangleright \mathcal{C}
                                                                                                                            \overline{\mathbf{FV}(t_i)} \subset tnvs
                                                                                                                             \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                            compatible overlap (\overline{x_i \mapsto t_i}^i)
                                                                                                                            E^{L} = \{ \overline{x_i \mapsto t_i}^i \}
E_2 = \langle \{\}, \{\}, \{\}, \{\overline{x_i \mapsto \langle \mathbf{forall} \ tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle}^i \} \rangle <<no parses (char 59): </zj//j/>,<TD,TC,I>,E1 |- indreln targets_opt indreln_names*** </rule
```

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\frac{\overline{z_j}^j \ x, D_1, E_1 \vdash \mathit{defs} \rhd D_2, E_2}{\overline{z_j}^j \ , D_1, E_1 \vdash \mathbf{module} \ x \ l_1 = \mathbf{struct} \ \mathit{defs} \ \mathbf{end} \ l_2 \rhd D_2, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle}
                                                                                                                                                                                                                                                 CHECK_DEF_MODULE
                                                                                                                                                                                                      CHECK_DEF_MODULE_RENAME
        \overline{z_i}^j, D, E_1 \vdash \mathbf{module} \ x \ l_1 = id \ l_2 \triangleright \epsilon, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle
                                   \Delta, E \vdash typ \leadsto t
                                   \mathbf{FV}(t) \subset \overline{\alpha_i}^i
                                  \frac{\mathbf{FV}(\overline{\alpha'_k}^k) \subset \overline{\alpha_i}^i}{\delta, E \vdash id_k \leadsto p_k}^k
                 \frac{E' = \langle \{ \}, \{ \}, \{ \}, \{x \mapsto \langle \operatorname{\mathbf{forall}} \overline{\alpha_i}^i. \overline{(p_k \, \alpha_k')}^k \Rightarrow t, \operatorname{\mathbf{val}} \rangle \} \rangle}{\overline{z_j}^j \,, \langle \Delta, \delta, I \rangle, E \vdash \operatorname{\mathbf{val}} x \, l_1 : \operatorname{\mathbf{forall}} \overline{\alpha_i \, l_i''}^i . \overline{id_k \, \alpha_k' \, l_k'}^k \stackrel{k}{\Rightarrow} typ \, l_2 \, \rhd \, \epsilon, E'}
                                                                                                                                                                                                                                    CHECK_DEF_SPEC
                  \frac{\Delta, E_1 \vdash typ_i \leadsto t_i}{\mathbf{FV}(t_i) \subset \alpha}^i
                   p = \overline{z_i}^j x
                   E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{\overline{y_i \mapsto \langle \mathbf{forall} \alpha. (p \alpha) \Rightarrow t_i, \mathbf{method} \rangle}^i \} \rangle
                   \delta_2 = \{ p \mapsto \overline{y_i}^{\,i} \}
                   p \not\in \mathbf{dom}(\delta_1)
                                                                                                                                                                                                                                              CHECK_DEF_CLASS
    \overline{z_j}^j, \langle \Delta, \overline{\delta_1, I} \rangle, \overline{E_1 \vdash \mathbf{class}\,(x\,\,l\,\alpha\,\,l'')}\,\overline{\mathbf{val}\,y_i\,\,l_i: typ_i\,l_i}^i\,\mathbf{end}\,\,l' \, \triangleright \, \langle \{\,\}, \overline{\delta_2}, \{\,\} \rangle, E_2
                                                                   E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                                                   \Delta, E \vdash typ' \leadsto t'
                                                                   \Delta, (\overline{\alpha_i}^i) \vdash t' instance
                                                                   tnvs = \overline{\alpha_i}^{\ i}
                                                                  \frac{\mathbf{duplicates} (tnvs)}{\delta, E \vdash id_k \leadsto p_k} = \emptyset
\mathbf{FV} (\overline{\alpha'_k}^k) \subset tnvs
                                                                   E(id) \triangleright p
                                                                   \delta(p) \triangleright \overline{z_j}^j

\frac{I_2 = \{ \Rightarrow (p_k \, \alpha'_k)^k \}}{\Delta, I \cup I_2, E \vdash val\_def_n \triangleright E_n^{\mathsf{X}}^n}

                                                                   disjoint doms (\overline{E_n^{\mathrm{X}}}^n)
                                                                  \overline{E^{\mathbf{x}}(x_k)} \triangleright \langle \mathbf{forall} \, \alpha''. (p \, \alpha'') \Rightarrow t_k, \mathbf{method} \rangle^k
                                                                  \{ \overline{x_k \mapsto \langle \operatorname{forall} tnvs.} \Rightarrow \{\alpha'' \mapsto t'\}(t_k), \operatorname{let} \rangle^k \} = \overline{E_n^{\mathrm{X}}}^n \\ \overline{x_k}^k = \overline{z_j}^j
                                                                  I_3 = \{ \overline{(p_k \, \alpha'_k) \Rightarrow (p \, t')}^k \, \}
                                                                  (p \{ \overline{\alpha_i \mapsto \alpha_i'''}^i \}(t')) \not \in I
\frac{1}{\overline{z_{j}}^{j},\langle\Delta,\delta,I\rangle,E\vdash\mathbf{instance\,forall}}\frac{1}{\alpha_{i}\;l_{i}^{\prime}}^{i}\cdot\overline{id_{k}\;\alpha_{k}^{\prime}\;l_{k}^{\prime\prime}}^{k}}\Rightarrow\left(id\;typ^{\prime}\right)\overline{val\_def_{n}\;l_{n}}^{n}\,\mathbf{end}\;l^{\prime}\,\vartriangleright\,\langle\{\;\},\{\;\},I_{3}\rangle,\epsilon}
                                                                                                                                                                                                                                                                                                              CHECK_DEF_
   \overline{z_j}^j, D_1, E_1 \vdash defs \triangleright D_2, E_2
                                                                                                Check definitions, given module path, definitions and environment
                                                                                        \frac{1}{\overline{z_j}^j, D, E \vdash \triangleright \epsilon, \epsilon} CHECK_DEFS_EMPTY
                                       \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2
                                     \overline{z_j}^j, D_1 \uplus D_2, E_1 \uplus E_2 \vdash \overline{def_i;_i^2}^i \rhd D_3, E_3
```

 $\overline{z_i}^j, D_1, E_1 \vdash def; ? \overline{def_i; ?}^i \triangleright D_2 \uplus D_3, E_2 \uplus E_3$

CHECK_DEFS_RELEVANT_DEF

$$\frac{E_{1}(id) \triangleright E_{2}}{\overline{z_{j}}^{j}, D_{1}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}}{\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash \mathbf{open} \ id \ l;;_{i}^{?} \ \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}} \quad \text{CHECK_DEFS_OPEN}$$

Definition rules: 144 good 1 bad Definition rule clauses: 438 good 1 bad