n, i, j, k Index variables for meta-lists

num Numeric literals

nat Internal literal numbers

hex Bit vector literal, specified by C-style hex number bin Bit vector literal, specified by C-style binary number

stringString literalsbacktick_stringString literals

regexp Regular expresions, as a string literal

x, y, z Variables ix Variables

```
l
                                                       Source locations
                        ::=
x^l, y^l, z^l, name
                                                       Location-annotated names
                              x l
                              (ix)l
                                                          Remove infix status
                              name_-t \to x^l
                                                  Μ
                                                          Extract x from a name_t
ix^l
                                                       Location-annotated infix names
                              ix l
                        ::=
                                                       Type variables
\alpha
                              \dot{x}
\alpha^l
                                                       Location-annotated type variables
                        ::=
                              \alpha l
N
                                                       numeric variables
                              ",x
N^l
                                                       Location-annotated numeric variables
                              N l
id
                                                       Long identifers
                              x_1^l \dots x_n^l . x^l l
tnv
                                                       Union of type variables and Nexp type variables, without lo
                              \alpha
                              N
tnvar^l
                        ::=
                                                       Union of type variables and Nexp type variables, with locati
                              \alpha^l
                                                       Type variable lists
tnvs
                              tnv_1 \dots tnv_n
tnvars^l
                                                       Type variable lists
                        ::=
                              tnvar_1^l ... tnvar_n^l
Nexp_aux
                                                       Numerical expressions for specifying vector lengths and inde
                        ::=
                              N
                              num
                              Nexp_1 * Nexp_2
                              Nexp_1 + Nexp_2
```

(Nexp)

```
Nexp
                                                                 Location-annotated vector lengths
                           ::=
                                 Nexp\_aux\ l
Nexp\_constraint\_aux
                                                                 Whether a vector is bounded or fixed size
                           ::=
                                 Nexp = Nexp'
                                 Nexp \ge Nexp'
Nexp\_constraint
                                                                 Location-annotated Nexp range
                           ::=
                                 Nexp\_constraint\_aux\ l
                           ::=
                                                                 Types
typ\_aux
                                                                    Unspecified type
                                                                    Type variables
                                                                    Function types
                                 typ_1 \rightarrow typ_2
                                 typ_1*....*typ_n
                                                                    Tuple types
                                                                    As a typ to permit applications over Nexps, o
                                 Nexp
                                 id\ typ_1 \dots typ_n
                                                                    Type applications
                                 backtick\_string\ typ_1\ ..\ typ_n
                                                                    Backend-Type applications
                                 (typ)
                                                                 Location-annotated types
typ
                                 typ_aux l
                                                                 Literal constants
lit\_aux
                           ::=
                                 true
                                 false
                                 string
                                                                   hex and bin are constant bit vectors, entered
                                 hex
                                 bin
                                 string
                                 string
                                 ()
                                 bitzero
                                                                   bitzero and bitone are constant bits, if commo
                                 bitone
lit
                           ::=
                                 lit\_aux\ l
                                                                    Location-annotated literal constants
;?
                           ::=
                                                                 Optional semi-colons
pat\_aux
                                                                 Patterns
                                                                    Wildcards
                                 (pat \mathbf{as} x^l)
                                                                   Named patterns
                                 (pat:typ)
                                                                    Typed patterns
                                                                    Single variable and constructor patterns
                                 id pat_1 ... pat_n
```

```
\langle |fpat_1; \dots; fpat_n; ?| \rangle
                                                                             Record patterns
                      [|pat_1; ...; pat_n;^?|]
                                                                             Vector patterns
                      [|pat_1 ... pat_n|]
                                                                             Concatenated vector patterns
                      (pat_1, \ldots, pat_n)
                                                                             Tuple patterns
                      [pat_1; ...; pat_n; ?]
                                                                             List patterns
                      (pat)
                      pat_1 :: pat_2
                                                                             Cons patterns
                      x^l + num
                                                                             constant addition patterns
                      lit
                                                                             Literal constant patterns
pat
                                                                          Location-annotated patterns
                      pat\_aux l
                                                                          Field patterns
fpat
                      id = pat l
               ::=
                                                                          Optional bars
                                                                          Expressions
exp_aux
                      id
                                                                             Identifiers
                      backtick_string
                                                                             identifier that should be literally used in out
                      N
                                                                             Nexp var, has type num
                      \mathbf{fun}\;psexp
                                                                             Curried functions
                     function ||^{?} pexp_{1}| \dots || pexp_{n}| end
                                                                             Functions with pattern matching
                      exp_1 \ exp_2
                                                                             Function applications
                      exp_1 ix^l exp_2
                                                                             Infix applications
                      \langle |fexps| \rangle
                                                                             Records
                      \langle |exp \mathbf{with} fexps| \rangle
                                                                             Functional update for records
                      exp.id
                                                                             Field projection for records
                      [|exp_1; ...; exp_n;^?|]
                                                                             Vector instantiation
                      exp.(Nexp)
                                                                             Vector access
                      exp.(Nexp_1..Nexp_2)
                                                                             Subvector extraction
                      match exp with |?| pexp_1| ... | pexp_n| l end
                                                                             Pattern matching expressions
                      (exp:typ)
                                                                             Type-annotated expressions
                      \mathbf{let}\ letbind\ \mathbf{in}\ exp
                                                                             Let expressions
                                                                             Tuples
                      (exp_1, \ldots, exp_n)
                      [exp_1; ...; exp_n; ?]
                                                                             Lists
                      (exp)
                      begin exp end
                                                                             Alternate syntax for (exp)
                      if exp_1 then exp_2 else exp_3
                                                                             Conditionals
                                                                             Cons expressions
                      exp_1 :: exp_2
                      lit
                                                                             Literal constants
                      \{exp_1|exp_2\}
                                                                             Set comprehensions
                      \{exp_1 | \mathbf{forall} \ qbind_1 .. \ qbind_n | exp_2\}
                                                                             Set comprehensions with explicit binding
                      \{exp_1; ...; exp_n;^?\}
                                                                             Sets
```

```
q \ qbind_1 \dots qbind_n.exp
                                                                                       Logical quantifications
                         [exp_1| forall qbind_1 ... qbind_n|exp_2]
                                                                                       List comprehensions (all binders mu
                         do id \ pat_1 \leftarrow exp_1; \ ... \ pat_n \leftarrow exp_n; \ \mathbf{in} \ exp \ \mathbf{end}
                                                                                       Do notation for monads
                                                                                    Location-annotated expressions
                   ::=
exp
                          exp\_aux l
                                                                                    Quantifiers
q
                   ::=
                         forall
                         exists
qbind
                                                                                    Bindings for quantifiers
                         x^l
                         (pat \, \mathbf{IN} \, exp)
                                                                                       Restricted quantifications over sets
                          (pat \mathbf{MEM} \ exp)
                                                                                       Restricted quantifications over lists
                                                                                    Field-expressions
fexp
                   ::=
                         id = exp l
                                                                                    Field-expression lists
fexps
                   ::=
                         fexp_1; \dots; fexp_n; ? l
                                                                                    Pattern matches
                   ::=
pexp
                         pat \rightarrow exp l
                                                                                    Multi-pattern matches
psexp
                         pat_1 \dots pat_n \to exp \ l
tannot?
                   ::=
                                                                                    Optional type annotations
                         : typ
funcl_aux
                                                                                    Function clauses
                         x^l pat_1 \dots pat_n tannot? = exp
letbind\_aux
                                                                                    Let bindings
                   ::=
                         pat\ tannot? = exp
                                                                                       Value bindings
                         funcl_{-}aux
                                                                                       Function bindings
letbind
                                                                                    Location-annotated let bindings
                          letbind\_aux\ l
funcl
                                                                                    Location-annotated function clauses
                         funcl_aux l
                                                                                    Name or name with type for inductive
name_{-}t
                         x^l
```

```
(x^l:typ)
                                                                                                            Names with optional type
name\_ts
                                    name\_t_0 ... name\_t_n
                                                                                                            Inductively defined relat
rule\_aux
                                   x^l: \mathbf{forall} \ name_-t_1 \dots name_-t_i.exp \Longrightarrow x_1^l \ exp_1 \dots exp_n
                                                                                                            Location-annotated indu
rule
                                    rule\_aux\ l
witness?
                                                                                                            Optional witness type na
                                    witness type x^l;
check?
                                                                                                            Option check name decla
                                    \mathbf{check}\,x^l;
functions?
                                                                                                            Optional names and typ
                                   x^{l}: typ
x^{l}: typ; functions^{?}
indreln\_name\_aux
                                                                                                            Name for inductively de-
                                    [x^l: typschm\ witness?\ check?\ functions?]
indreln\_name
                                                                                                            Location-annotated name
                            ::=
                                    indreln\_name\_aux\;l
                                                                                                            Type lists
typs
                                    typ_1 * ... * typ_n
ctor\_def
                                                                                                            Datatype definition clau
                                    x^l of typs
                                                                                                       S
                                                                                                               Constant constructors
texp
                                                                                                            Type definition bodies
                                                                                                               Type abbreviations
                                     \begin{array}{c} \langle |x_1^l:typ_1;\ldots;x_n^l:typ_n\,;^?|\rangle \\ |?\;ctor\_def_1|\ldots|ctor\_def_n \end{array} 
                                                                                                               Record types
                                                                                                               Variant types
name?
                                                                                                            Optional name specifica
                             ::=
                                    [name = regexp]
                                                                                                            Type definitions
td
                            ::=
```

```
\begin{array}{l} x^{l} \, tnvars^{l} \, name^{?} = texp \\ x^{l} \, tnvars^{l} \, name^{?} \end{array}
                                                                                                      Definitions of opaque types
                      ::=
                                                                                                   Typeclass constraints
c
                             id\ tnvar^l
cs
                                                                                                   Typeclass and length constraint
                                                                                                      Must have > 0 constraints
                             Nexp\_constraint_1, \dots, Nexp\_constraint_i \Rightarrow
                                                                                                      Must have > 0 constraints
                             c_1, \dots, c_i; Nexp\_constraint_1, \dots, Nexp\_constraint_n \Rightarrow
                                                                                                      Must have > 0 of both form of
                                                                                                   Type and instance scheme prefix
c\_pre
                      ::=
                             \mathbf{forall}\,tnvar_1^l\dots tnvar_n^l.cs
                                                                                                      Must have > 0 type variables
typschm
                                                                                                   Type schemes
                             c\_pre\ typ
instschm
                                                                                                   Instance schemes
                     ::=
                             c\_pre(id\ typ)
                                                                                                   Backend target names
target
                      ::=
                             hol
                             isabelle
                             ocaml
                             coq
                             tex
                             html
                             lem
open\_import
                                                                                                   Open or import statements
                             open
                             import
                             open import
                             include
                             include import
                                                                                                   Backend target name lists
                              \begin{aligned} & \{target_1; \dots; target_n\} \\ & \{target_1; \dots; target_n\} \end{aligned} 
                                                                                                      all targets except the listed or
                             non\_exec
                                                                                                      all non-executable targets, use
\tau?
                                                                                                   Optional targets
```

$lemma_typ$::= 	assert lemma theorem	Types of Lemmata
$lemma_decl$::=	$lemma_typ \ au^? \ x^l : exp$	Lemmata and Tests
dexp	::= 	$name_s = string \ l$ $\mathbf{format} = string \ l$ $\mathbf{arguments} = exp_1 \dots exp_n \ l$ $\mathbf{targuments} = texp_1 \dots texp_n \ l$	declaration field-expressions
$declare_arg$::= 	$string \ \langle dexp_1;; dexp_n;^2 l \rangle$	arguments to a declaration
component	::= 	module function type field	components
$termination_setting$::= 	automatic manual	termination settings
$exhaustivity_setting$::= 	exhaustive inexhaustive	exhaustivity settings
$elim_opt$::=	id	optional terms used as eliminators for patter
$fixity_decl$::= 	$right_assocnat$ $left_assocnat$ $non_assocnat$	fixity declarations for infix identifiers
$target_rep_rhs$::=	$infix fixity_decl \ backtick_string$ exp typ $special \ string \ exp_1 \dots \ exp_n$	right hand side of a target representation de

```
target\_rep\_lhs
                            ::=
                                    target\_rep component id x_1^l \dots x_n^l
                                    target\_rep component id tnvars^l
declare\_def
                            ::=
                                    \mathbf{declare} \, \tau^? compile\_messageid = string
                                    \operatorname{declare} \tau^{?}\operatorname{rename\,module} = x^{l}
                                    \mathbf{declare}\,\tau^{?}\,\mathbf{rename}\,component\,id=x^{l}
                                    \mathbf{declare} \, \tau^? ascii\_rep component \, id = backtick\_string
                                    \mathbf{declare}\ targettarget\_rep\_target\_rep\_lhs = target\_rep\_rhs
                                    \begin{aligned} &\mathbf{declare} \ set\_flagx_1^l = x_2^{\bar{l}} \\ &\mathbf{declare} \ \tau^? termination\_argumentid = termination\_setting \end{aligned}
                                    declare \tau? pattern_match exhaustivity_setting id tnvars^l = [id_1; ...; id_n; ?] elim\_opt
val\_def
                            ::=
                                    let \tau? letbind
                                    let \operatorname{\mathbf{rec}} \tau^? \operatorname{\mathit{funcl}}_1 and ... and \operatorname{\mathit{funcl}}_n
                                    let inline \tau? letbind
                                    let lem\_transform\tau? letbind
ascii\_opt
                            ::=
                                    [backtick\_string]
instance\_decl
                            ::=
                                    instance
                                    default\_instance
class\_decl
                            ::=
                                    class
                                    class inline
val\_spec
                            ::=
                                    \mathbf{val} \ x^l \ ascii\_opt : typschm
def_{-}aux
                            ::=
                                    type td_1 and ... and td_n
                                    val\_def
                                    lemma\_decl
                                    declare\_def
                                    module x^{\hat{l}} = struct defs end
                                    \mathbf{module}\, x^l = id
                                    open\_import\ id_1\ ...\ id_n
                                    open\_import \ \tau^? \ backtick\_string_1 \dots backtick\_string_n
                                    \mathbf{indreln} \ \tau^? \ indreln\_name_1 \ \mathbf{and} \ ... \ \mathbf{and} \ indreln\_name_i \ rule_1 \ \mathbf{and} \ ... \ \mathbf{and} \ rule_n
```

```
Top-
                     class\_decl(x^l \ tnvar^l) val \tau_1^? \ x_1^l \ ascii\_opt_1 : typ_1 \ l_1 \dots val \tau_n^? \ x_n^l \ ascii\_opt_n : typ_n \ l_n end instance\_decl instschm val\_def_1 \ l_1 \dots val\_def_n \ l_n end
                                                                                                                                                           Type
                                                                                                                                                           Type
def
                                                                                                                                                        Locatio
             ::=
                      def\_aux l
              ;;?
             ::=
                                                                                                                                                        Option
                     ;;
defs
                                                                                                                                                        Definit
             ::=
                     def_1; ;_1^? ... def_n; ;_n^?
              Unique
p
             ::=
                     x_1 \dots x_n x
                     _{-}list
                     \_bool
                     \_num
                     \_set
                     _string
                     _{-}unit
                     _{--}bit
                      \_vector
                                                                                                                                                        Type v
\sigma
             ::=
                     \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}
t, u
                                                                                                                                                        Interna
                     \alpha
                      t_1 \rightarrow t_2
                      t_1 * .... * t_n
                     p t\_args
                     ne
                     \sigma(t)
                                                                                                                                                 Μ
                                                                                                                                                           Mult
                     \sigma(tnv)
                                                                                                                                                 Μ
                                                                                                                                                           Sing
                     \mathbf{curry}(t\_multi, t)
                                                                                                                                                 Μ
                                                                                                                                                           Curr
             ::=
                                                                                                                                                       interna
ne
                     N
                     nat
                     ne_1 * ne_2
                     ne_1 + ne_2
                      (-ne)
                     normalize(ne)
                                                                                                                                                 Μ
```

Μ

Μ

 $ne_1 + \dots + ne_n$

bitlength(bin)

```
bitlength(hex)
                                                                                              Μ
                             \mathbf{length}(\mathit{pat}_1 \dots \mathit{pat}_n)
                                                                                              Μ
                             length (exp_1 \dots exp_n)
                                                                                              Μ
                                                                                                     Lists of types
t\_args
                             t_1 \dots t_n
                             \sigma(t\_args)
                                                                                              Μ
                                                                                                          Multiple substitutions
t\_multi
                                                                                                     Lists of types
                            (t_1 * \ldots * t_n)
                            \sigma(t_{-}multi)
                                                                                              Μ
                                                                                                         Multiple substitutions
                                                                                                     Numeric expression constraints
nec
                             ne\langle nec
                             ne = nec
                             ne \le nec
names
                                                                                                     Sets of names
                             \{x_1,\ldots,x_n\}
                     \mathcal{C}
                                                                                                      Typeclass constraint lists
                             (p_1 tnv_1) \dots (p_n tnv_n)
                                                                                                      Tags for the (non-constructor) value description
env\_tag
                            method
                                                                                                          Bound to a method
                             val
                                                                                                          Specified with val
                             let
                                                                                                         Defined with let or indreln
v\_desc
                    ::=
                                                                                                      Value descriptions
                             \langle \mathbf{forall} \ tnvs.t\_multi \rightarrow p, (x \mathbf{of} \ names) \rangle
                                                                                                          Constructors
                             \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t, env\_taq \rangle
                                                                                                          Values
f\_desc
                     \langle \mathbf{forall} \ tnvs.p \rightarrow t, (x \mathbf{of} \ names) \rangle
                                                                                                         Fields
xs
                     x_1 \dots x_n
\Sigma^{\mathcal{C}}
                                                                                                      Typeclass constraints
                            \{(p_1 t_1), \ldots, (p_n t_n)\}
\Sigma^{\mathcal{C}}_1 \cup \ldots \cup \Sigma^{\mathcal{C}}_n
                                                                                              Μ
\Sigma^{\mathcal{N}}
                                                                                                     Nexp constraint lists
                             \{nec_1, \dots, nec_n\} 
\Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n 
                                                                                              Μ
```

Μ

```
D
                                                                                                                                                  Global type definition store
                                                    \langle \Delta, \delta, I \rangle
                                                    D_1 \uplus D_2
                                                                                                                                      Μ
                                                                                                                                      Μ
terminals
                                                                                                                                                        >=
                                                                                                                                                        ->
                                                                                                                                                         <-
                                                                                                                                                         ==>
                                                     \langle |
                                                                                                                                                         <|
                                                                                                                                                         |>
                                                    \bigcup
                                                    \forall
                                                    \not\in
                                                    \subset
formula
                                      ::=
                                                    judgement
                                                    formula_1 .. formula_n
                                                    E^{\mathrm{M}}(x) \triangleright E
                                                                                                                                                         Module lookup
                                                    E^{P}(x) \triangleright p
                                                                                                                                                        Path lookup
                                                    E^{\mathrm{F}}(x) \triangleright f_{-}desc
                                                                                                                                                        Field lookup
                                                    E^{\mathbf{X}}(x) \triangleright v_{-}desc
                                                                                                                                                         Value lookup
                                                    E^{\mathrm{L}}(x) \triangleright t
                                                                                                                                                        Lexical binding lookup
                                                    \Delta(p) \triangleright tc\_def
                                                                                                                                                        Type constructor lookup
                                                    \delta(p) \triangleright xs
                                                                                                                                                        Type constructor lookup
                                                    \mathbf{dom}\left(E_{1}^{\mathrm{M}}
ight)\cap\mathbf{dom}\left(E_{2}^{\mathrm{M}}
ight)=\emptyset \ \mathbf{dom}\left(E_{1}^{\mathrm{M}}
ight)\cap\mathbf{dom}\left(E_{2}^{\mathrm{M}}
ight)=\emptyset \ \mathbf{dom}\left(E_{1}^{\mathrm{F}}
ight)\cap\mathbf{dom}\left(E_{2}^{\mathrm{F}}
ight)=\emptyset 
                                                    \mathbf{dom}\left(E_{1}^{\mathrm{p}}\right)\,\cap\,\mathbf{dom}\left(E_{2}^{\mathrm{p}}\right)=\,\emptyset
                                                    \begin{array}{l} \textbf{disjoint doms} \ (E_1^{\scriptscriptstyle \rm L}, \ \dots, E_n^{\scriptscriptstyle \rm L}) \\ \textbf{disjoint doms} \ (E_1^{\scriptscriptstyle \rm X}, \ \dots, E_n^{\scriptscriptstyle \rm X}) \end{array}
                                                                                                                                                        Pairwise disjoint domains
```

Pairwise disjoint domains

```
compatible overlap (x_1 \mapsto t_1, ..., x_n \mapsto t_n) (x_i = x_j) \Longrightarrow (t_i = t_j)
                                          \mathbf{duplicates}\left(\mathit{tnvs}\right) = \emptyset
                                          duplicates (x_1, ..., x_n) = \emptyset
                                          x \not\in \mathbf{dom}(E^{\mathrm{L}})
                                          x \not\in \mathbf{dom}(E^{\mathbf{X}})
                                          x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                          p \not\in \mathbf{dom}(\delta)
                                          p \not\in \mathbf{dom}(\Delta)
                                          \mathbf{FV}(t) \subset tnvs
                                                                                                                               Free type variables
                                          \mathbf{FV}(t_{-}multi) \subset tnvs
                                                                                                                               Free type variables
                                          \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                               Free type variables
                                          inst IN I
                                          (p t) \not\in I
                                          E_1^{\text{L}} = E_2^{\text{L}}
E_1^{\text{X}} = E_2^{\text{X}}
                                          E_1^{^{\mathrm{F}}} = E_2^{^{\mathrm{F}}}

E_1 = E_2
                                          \Delta_1 = \Delta_2
                                          \delta_1 = \delta_2
                                          I_1 = I_2
                                          names_1 = names_2
                                          t_1 = t_2
                                          \sigma_1 = \sigma_2
                                          p_1 = p_2
                                          xs_1 = xs_2
                                          tnvs_1 = tnvs_2
convert\_tnvars
                                          tnvars^l \leadsto tnvs
                                          tnvar^l \leadsto tnv
look\_m
                                 ::=
                                          E_1(x_1^l \dots x_n^l) \triangleright E_2
                                                                                                                               Name path lookup
                                  look\_m\_id
                                ::=
                                          E_1(id) \triangleright E_2
                                                                                                                               Module identifier lookup
look\_tc
                                 ::=
                                          E(id) \triangleright p
                                                                                                                              Path identifier lookup
check\_t
                                ::=
                                          \Delta \vdash t \mathbf{ok}
                                                                                                                               Well-formed types
                                          \Delta, tnv \vdash t ok
                                                                                                                               Well-formed type/Nexps matching
teq
                                         \Delta \vdash t_1 = t_2
                                                                                                                               Type equality
```

```
convert\_typ
                                     ::=
                                               \Delta, E \vdash typ \leadsto t
                                                                                                                                                                   Convert source types to int
                                               \vdash Nexp \leadsto ne
                                                                                                                                                                   Convert and normalize num
convert\_typs
                                     ::=
                                                \Delta, E \vdash typs \leadsto t\_multi
check\_lit
                                     ::=
                                               \vdash lit : t
                                                                                                                                                                   Typing literal constants
inst\_field
                                     ::=
                                                \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                                                                                   Field typing (also returns c
inst\_ctor
                                     ::=
                                                \Delta, E \vdash \mathbf{ctor} id : t\_multi \rightarrow p \ t\_args \triangleright (x \ \mathbf{of} \ names)
                                                                                                                                                                   Data constructor typing (al
inst\_val
                                                \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                                                                                                                   Typing top-level bindings, of
not\_ctor
                                               E, E^{\scriptscriptstyle 	ext{L}} \vdash x \operatorname{\mathbf{not}} \operatorname{\mathbf{ctor}}
                                                                                                                                                                   v is not bound to a data co
not\_shadowed
                                     ::=
                                               E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                                                                                                                   id is not lexically shadowed
check\_pat
                                     ::=
                                               \Delta, E, E_1^{\scriptscriptstyle 
m L} \vdash pat : t \triangleright E_2^{\scriptscriptstyle 
m L}
                                                                                                                                                                   Typing patterns, building t
                                               \Delta, E, E_1^{\mathrm{L}} \vdash pat\_aux : t \triangleright E_2^{\mathrm{L}}
                                                                                                                                                                   Typing patterns, building t
id_{-}field
                                     ::=
                                                E \vdash id \mathbf{field}
                                                                                                                                                                   Check that the identifier is
id\_value
                                     ::=
                                                E \vdash id value
                                                                                                                                                                   Check that the identifier is
check\_exp
                                               \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Typing expressions, collecti
                                               \Delta, E, E^{\mathrm{L}} \vdash exp_{-}aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Typing expressions, collecti
                                               \Delta, E, E_1^{\text{\tiny L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}
                                                                                                                                                                   Build the environment for o
                                               \Delta, E, E_{1}^{\text{L}} \vdash \mathbf{list} \ qbind_{1} ... \ qbind_{n} \triangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}
\Delta, E, E^{\text{L}} \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
\Delta, E, E_{1}^{\text{L}} \vdash letbind \triangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Build the environment for o
                                                                                                                                                                   Build the environment for a
                                                                                                                                                                   Build the environment for a
check\_rule
                                     ::=
                                               \Delta, E, E^{\mathrm{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                   Build the environment for a
```

 $check_texp_tc$

::=

		$xs, \Delta_1, E \vdash \mathbf{tc} \ td \rhd \Delta_2, E^{\mathrm{P}}$	Extract the type constructor informat
$check_texps_tc$::=	$xs, \Delta_1, E \vdash \mathbf{tc} \ td_1 \dots td_i \rhd \Delta_2, E^{\mathrm{P}}$	Extract the type constructor informat
$check_texp$::=	$\Delta, E \vdash \mathit{tnvs} \ p = \mathit{texp} \rhd \langle E^{\scriptscriptstyle{\mathrm{F}}}, E^{\scriptscriptstyle{\mathrm{X}}} \rangle$	Check a type definition, with its path
$check_texps$::=	$xs, \Delta, E \vdash td_1 td_n \rhd \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle$	
$convert_class$::=	$\delta, E \vdash id \leadsto p$	Lookup a type class
$solve_class_constraint$::=	$I \vdash (p\ t)\ \mathbf{IN}\ \mathcal{C}$	Solve class constraint
$solve_class_constraints$::=	$I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}$	Solve class constraints
$check_val_def$::=	$\Delta, I, E \vdash val_def \rhd E^{\mathbf{X}}$	Check a value definition
$check_t_instance$::=	$\Delta, (\alpha_1,, \alpha_n) \vdash t $ instance	Check that t be a typeclass instance
$check_defs$::= 	$ \overline{z_j}^j, D_1, E_1 \vdash def \rhd D_2, E_2 \overline{z_j}^j, D_1, E_1 \vdash defs \rhd D_2, E_2 $	Check a definition Check definitions, given module path,
judgement	::= 	convert_tnvars look_m look_m_id look_tc check_t teq convert_typ convert_typ check_lit	

 $inst_field$ $inst_ctor$ $inst_val$ not_ctor $not_shadowed$ $check_pat$ id_field

```
id\_value
      check\_exp
      check\_rule
      check\_texp\_tc
      check\_texps\_tc
      check\_texp
      check\_texps
      convert\_class
      solve\_class\_constraint
      solve\_class\_constraints
      check\_val\_def
      check\_t\_instance
      check\_defs
::=
      n
      num
      nat
      hex
      bin
      string
      backtick\_string
      regexp
      \boldsymbol{x}
      ix
      l
      x^{l}
      ix^l
      \alpha
      \alpha^l
      N
      N^l
      id
      tnv
      tnvar^l
      tnvs
      tnvars^l
      Nexp\_aux
      Nexp
      Nexp\_constraint\_aux
      Nexp\_constraint
      typ\_aux
      typ
      lit\_aux
```

 $\lim_{;?}$

 $user_syntax$

```
pat\_aux
pat
fpat|?
exp\_aux
exp
qbind
fexp
fexps
pexp
psexp
tannot?
funcl_{-}aux
letbind\_aux
letbind
funcl
name\_t
name\_ts
rule\_aux
rule
witness?
check?
functions?
indreln\_name\_aux
indreln\_name
typs
ctor\_def
texp
name?
td
c
cs
c\_pre
typschm
instschm
target
open\_import
lemma\_typ
lemma\_decl
dexp
declare\_arg
component
termination\_setting
```

```
exhaustivity\_setting
elim\_opt
fixity\_decl
target\_rep\_rhs
target\_rep\_lhs
declare\_def
val\_def
ascii\_opt
instance\_decl
class\_decl
val\_spec
def\_aux
def
;;?
de\!f\!s
p
ne
t\_args
t-multi
nec
names
\mathcal{C}
env\_tag
v\_desc
f\_desc
xs
\Sigma^{\mathcal{C}}
\Sigma^{\mathcal{N}}
E
E^{\mathbf{X}}
E^{\scriptscriptstyle \mathrm{F}}
E^{\mathrm{M}}
E^{\mathrm{P}}
E^{\scriptscriptstyle 
m L}
tc\_abbrev
tc\_def
\Delta
\delta
inst
Ι
D
terminals
formula
```

 $tnvars^l \leadsto tnvs$

$$\frac{tnvar_1^l \leadsto tnv_1 \quad .. \quad tnvar_n^l \leadsto tnv_n}{tnvar_1^l \ .. \ tnvar_n^l \leadsto tnv_1 \ .. \ tnv_n} \quad \text{Convert_tnvars_none}$$

 $tnvar^l \leadsto tnv$

$$\frac{\alpha \ l \leadsto \alpha}{N \ l \leadsto N} \quad \begin{array}{ll} \text{Convert_tnvar_a} \\ \hline \\ \hline N \ l \leadsto N \end{array}$$

 $E_1(x_1^l \dots x_n^l) \triangleright E_2$ Name path lookup

$$\frac{E(\) \rhd E \qquad \text{LOOK_M_NONE}}{E^{\text{M}}(x) \rhd E_{1}}$$

$$\frac{E_{1}(\ \overline{y_{i}^{l}}^{i}\) \rhd E_{2}}{\langle E^{\text{M}}, E^{\text{P}}, E^{\text{F}}, E^{\text{X}} \rangle (x \ l \ \overline{y_{i}^{l}}^{i}\) \rhd E_{2}} \quad \text{LOOK_M_SOME}$$

 $E_1(id) \triangleright E_2$ Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i x l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i x l_1 l_2) \triangleright E_2} \quad \text{LOOK_M_ID_ALL}$$

 $E(id) \triangleright p$ Path identifier lookup

$$\frac{E(\overline{y_i^l}^i) \triangleright \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{E^{\mathrm{P}}(x) \triangleright p} \qquad \text{LOOK_TC_ALL}$$

$$E(\overline{y_i^l}^i x \ l_1 \ l_2) \triangleright p$$

 $\Delta \vdash t \text{ ok}$ Well-formed types

$$\begin{array}{cccc} \overline{\Delta \vdash \alpha \, \mathbf{ok}} & \text{CHECK_T_VAR} \\ & \Delta \vdash t_1 \, \mathbf{ok} \\ & \underline{\Delta \vdash t_2 \, \mathbf{ok}} \\ & \overline{\Delta \vdash t_1 \to t_2 \, \mathbf{ok}} & \text{CHECK_T_FN} \\ \\ & \underline{\Delta \vdash t_1 \, \mathbf{ok} \quad \quad \Delta \vdash t_n \, \mathbf{ok}} \\ & \underline{\Delta \vdash t_1 \, \mathbf{ok} \quad \quad x \, t_n \, \mathbf{ok}} & \text{CHECK_T_TUP} \\ \\ & \underline{\Delta(p) \rhd tnv_1 .. tnv_n \, tc_abbrev} \\ & \underline{\Delta, tnv_1 \vdash t_1 \, \mathbf{ok} \quad ... \quad \Delta, tnv_n \vdash t_n \, \mathbf{ok}} \\ & \underline{\Delta \vdash p \, t_1 ... t_n \, \mathbf{ok}} & \text{CHECK_T_APP} \\ \hline \end{array}$$

 $\Delta, tnv \vdash t$ ok Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \text{ ok}}{\Delta, \alpha \vdash t \text{ ok}} \quad \text{CHECK_TLEN_T}$$

$$\frac{\Delta, N \vdash ne \text{ ok}}{\Delta, N \vdash ne \text{ ok}} \quad \text{CHECK_TLEN_LEN}$$

 $\Delta \vdash t_1 = t_2$ Type equality

$$\frac{\Delta \vdash t \text{ ok}}{\Delta \vdash t = t} \quad \text{TEQ_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ_SYM}$$

$$\begin{array}{c} \Delta \vdash t_1 = t_2 \\ \Delta \vdash t_2 = t_3 \\ \hline \Delta \vdash t_1 = t_3 \\ \hline \Delta \vdash t_1 = t_3 \\ \hline \Delta \vdash t_2 = t_4 \\ \hline \Delta \vdash t_1 \to t_2 = t_3 \to t_4 \\ \hline \Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n \\ \hline \Delta \vdash t_1 * \dots * t_n = u_1 * \dots * u_n \\ \hline \Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n \\ \hline \Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n \\ \hline \Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n \\ \hline \Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n \\ \hline \Delta \vdash p \ t_1 \dots t_n = p \ u_1 \dots u_n \\ \hline \hline \Delta \vdash p \ t_1 \dots t_n = \{\alpha_1 \mapsto t_1 \dots \alpha_n \mapsto t_n\}(u) \\ \hline \hline \qquad \begin{array}{c} D \vdash p \ t_1 \dots t_n = \{\alpha_1 \mapsto t_1 \dots \alpha_n \mapsto t_n\}(u) \\ \hline \hline \Delta \vdash p \ t_1 \dots t_n = ne' \end{array} \end{array} \quad \text{TEQ_EXPAND}$$

 $\Delta, E \vdash typ \leadsto t$

Convert source types to internal types

 $\vdash Nexp \leadsto ne$

Convert and normalize numeric expressions

```
\frac{\vdash Nexp_2 \leadsto ne_2}{\vdash Nexp_1 + Nexp_2 \ l \leadsto ne_1 + ne_2} \quad \text{CONVERT\_NEXP\_ADD}
     \Delta, E \vdash typs \leadsto t\_multi
                                     \frac{\Delta, E \vdash typ_1 \leadsto t_1 \quad .. \quad \Delta, E \vdash typ_n \leadsto t_n}{\Delta, E \vdash typ_1 * .. * typ_n \leadsto (t_1 * .. * t_n)} \quad \text{CONVERT\_TYPS\_ALL}
   \vdash lit:t
                            Typing literal constants
                                                                   \frac{}{\vdash \mathbf{true} \, l : \_\mathbf{bool}} \quad \text{CHECK\_LIT\_TRUE}
                                                                  \frac{}{\vdash \mathbf{false} \ l : \_\mathbf{bool}} \quad \text{CHECK\_LIT\_FALSE}
                                                                                                                                                         CHECK_LIT_NUM
                           <<no parses (char 10): |- num 1 :*** __num >>
                                                               nat = \mathbf{bitlength}(hex)
                                                                                                                        CHECK_LIT_HEX
                                                          \vdash hex \ l : \_vector \ nat \_bit
                                                                nat = \mathbf{bitlength}(bin)
                                                                                                                            CHECK_LIT_BIN
                                                           \overline{\vdash bin \ l : \_\mathbf{vector} \ nat \_\mathbf{bit}}
                                                                                                                   CHECK_LIT_STRING
                                                           <<multiple parses>>
                                                                       \frac{}{\vdash ()\: l: \_\mathtt{unit}} \quad \texttt{CHECK\_LIT\_UNIT}
                                                                                                             CHECK_LIT_BITZERO
                                                              \vdash bitzero l : __bit
                                                                                                              CHECK_LIT_BITONE
                                                                \vdash bitone l : __bit
   \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                              Field typing (also returns canonical field names)
                                  E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle 
E^{\mathrm{F}}(y) \rhd \langle \mathbf{forall} \ tnv_1 \dots tnv_n.p \to t, (z \ \mathbf{of} \ names) \rangle
                                  \Delta \vdash t_1 \mathbf{ok} \quad .. \quad \Delta \vdash t_n \mathbf{ok}
  \Delta, E \vdash \mathbf{field} \ \overline{x_i^l}^i \ y \ l_1 \ l_2 : p \ t_1 \dots t_n \to \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}(t) \rhd (z \ \mathbf{of} \ names)
   \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
                                                                                                                           Data constructor typing (also returns canonical constru
                               \begin{array}{l} E(\,\overline{x_i^l}^{\,i}\,) \,\rhd\, \langle E^{\scriptscriptstyle{\mathrm{M}}}, E^{\scriptscriptstyle{\mathrm{P}}}, E^{\scriptscriptstyle{\mathrm{F}}}, E^{\scriptscriptstyle{\mathrm{X}}}\rangle \\ E^{\scriptscriptstyle{\mathrm{X}}}(y) \,\rhd\, \langle\, \mathbf{forall}\, tnv_1 \mathinner{\ldotp\ldotp} tnv_n.t\_multi \to p, (z\,\mathbf{of}\,\, names)\rangle \end{array}
\frac{\Delta \vdash t_1 \text{ ok} \quad .. \quad \Delta \vdash t_n \text{ ok}}{\Delta, E \vdash \text{ ctor } \overline{x_i^l}^i \quad y \ l_1 \ l_2 : \{tnv_1 \mapsto t_1 .. \ tnv_n \mapsto t_n\}(t\_multi) \to p \ t_1 .. \ t_n \rhd (z \text{ of } names)}
                                                                                                                                                                                  — INST_CTOR_ALL
    \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}} Typing top-level bindings, collecting typeclass constraints
                       E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle

E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 \dots tnv_n . (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, \ env\_tag \rangle
                       \Delta \vdash t_1 \mathbf{ok} .. \Delta \vdash t_n \mathbf{ok}
                      \frac{\sigma = \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}}{\Delta, E \vdash \mathbf{val} \, \overline{x_i^l}^i \, y \, l_1 \, l_2 : \sigma(t) \triangleright \{(p_1 \, \sigma(tnv_1')), \dots, (p_i \, \sigma(tnv_i'))\}} INST_VAL_ALL
    E, E^{\perp} \vdash x \text{ not ctor} \mid v \text{ is not bound to a data constructor}
```

 $\vdash Nexp_1 \leadsto ne_1$

```
\frac{E^{\text{L}}(x) \triangleright t}{E, E^{\text{L}} \vdash x \text{ not ctor}} \quad \text{NOT\_CTOR\_VAL}
                                                   \frac{x \not\in \mathbf{dom}\left(E^{\mathbf{X}}\right)}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}}\rangle, E^{\mathbf{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor}} \quad \text{NOT\_CTOR\_UNBOUND}
                  \frac{E^{\mathbf{X}}(x) \triangleright \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, env\_tag \rangle}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle, E^{\mathbf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}} \quad \text{NOT\_CTOR\_BOUND}
 E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                      id is not lexically shadowed
                                                           \frac{x \not\in \mathbf{dom}(E^{L})}{E^{L} \vdash x l_{1} l_{2} \mathbf{not shadowed}} \quad \text{NOT\_SHADOWED\_SING}
                                               \overline{E^{\text{L}} \vdash x_1^l \dots x_n^l. y^l. z^l \ l \ \textbf{not shadowed}} \quad \text{NOT\_SHADOWED\_MULTI}
 \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}
                                                                           Typing patterns, building their binding environment
                                                                \frac{\Delta, E, E_1^{\text{L}} \vdash pat\_aux : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash pat\_aux \ l : t \triangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_ALL}
\Delta, E, E_1^{\text{L}} \vdash pat\_aux : t \triangleright E_2^{\text{L}} Typing patterns, building their binding environment
                                                                   \frac{\Delta \vdash t \, \mathbf{ok}}{\Delta, E, E^{\text{\tiny L}} \vdash \_ : t \, \triangleright \, \{\,\}} \quad \text{CHECK\_PAT\_AUX\_WILD}
                                          \begin{split} &\Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_2^{\text{\tiny L}} \\ &x \not\in \mathbf{dom}\left(E_2^{\text{\tiny L}}\right) \\ &\overline{\Delta, E, E_1^{\text{\tiny L}} \vdash \left(pat \, \mathbf{as} \, x \, l\right): t \vartriangleright E_2^{\text{\tiny L}} \uplus \left\{x \mapsto t\right\}} \end{split} \quad \text{CHECK\_PAT\_AUX\_AS}
                                                       \begin{split} & \Delta, E, E_1^{\text{L}} \vdash pat: t \vartriangleright E_2^{\text{L}} \\ & \Delta, E \vdash typ \leadsto t \\ & \overline{\Delta, E, E_1^{\text{L}} \vdash (pat: typ): t \vartriangleright E_2^{\text{L}}} \end{split} \quad \text{CHECK\_PAT\_AUX\_TYP} \end{split}
      \Delta, E \vdash \mathbf{ctor}\ id : (t_1 * ... * t_n) \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
      E^{\text{L}} \vdash id \text{ not shadowed}
     \frac{E, E^{\mathsf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}}{\Delta, E, E^{\mathsf{L}} \vdash x \ l_1 \ l_2 \ : t \, \triangleright \, \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_VAR}
                          \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \mathbf{of}\ names)}^i
                          \Delta, E, E^{\mathrm{L}} \vdash pat_i : t_i \triangleright E_i^{\mathrm{L}}^i
                         egin{aligned} 	extbf{disjoint doms} \left( \, \overline{E_i^{	ext{L}}}^i \, 
ight) \ 	extbf{duplicates} \left( \, \overline{x_i}^i \, 
ight) = \, \emptyset \end{aligned}
                          \frac{\Delta apricaces (|x_i||_j - |v|)}{\Delta, E, E^{\text{L}} \vdash \langle ||\overline{id_i} = pat_i|\overline{l_i}^i|^i; ?|\rangle : p \ t\_args \rhd \uplus \overline{E_i^{\text{L}}}^i} \quad \text{CHECK\_PAT\_AUX\_RECORD}
                 \Delta, E, E^{\mathsf{L}} \vdash pat_1 : t \triangleright E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash pat_n : t \triangleright E_n^{\mathsf{L}}
                 disjoint doms (E_1^L, \ldots, E_n^L)
                 \mathbf{length}\left(pat_1 \dots pat_n\right) = nat
        \overline{\Delta, E, E^{\text{L}} \vdash [|pat_1; \dots; pat_n;^?|] : \_\text{vector } nat \ t \vartriangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_VECTOR}
```

```
\Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_1: \_\textbf{vector} \; \textit{ne}_1 \; t \, \rhd \, E^{\text{\tiny L}}_1 \quad \dots \quad \Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_n: \_\textbf{vector} \; \textit{ne}_n \; t \, \rhd \, E^{\text{\tiny L}}_n
\mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
\frac{ne' = ne_1 + \dots + ne_n}{\Delta, E, E^{\text{L}} \vdash [|pat_1 \dots pat_n|] : \_\textbf{vector } ne' \ t \rhd E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}}
                                                                                                                                                                                                                                                                             CHECK_PAT_AUX_VECTOR
                       \frac{\Delta, E, E^{\text{L}} \vdash pat_1 : t_1 \triangleright E_1^{\text{L}} \quad \dots \quad \Delta, E, E^{\text{L}} \vdash pat_n : t_n \triangleright E_n^{\text{L}}}{\text{disjoint doms}\left(E_1^{\text{L}}, \dots, E_n^{\text{L}}\right)}
\frac{\Delta, E, E^{\text{L}} \vdash \left(pat_1, \dots, pat_n\right) : t_1 * \dots * t_n \triangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}}{\Delta, E, E^{\text{L}} \vdash \left(pat_1, \dots, pat_n\right) : t_1 * \dots * t_n \triangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}}
CHECK_PAT_AUX_TUP
                            \Delta \vdash t \mathbf{ok}
                            \Delta, E, E^{\mathsf{L}} \vdash pat_1 : t \triangleright E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash pat_n : t \triangleright E_n^{\mathsf{L}}
                           disjoint doms (E_1^{\text{L}}, ..., E_n^{\text{L}})
                                 \overline{\Delta, E, E^{\text{L}} \vdash [pat_1; ...; pat_n;^?] : \_\textbf{list} \ t \triangleright E_1^{\text{L}} \uplus ... \uplus E_n^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_LIST}
                                                                      \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat) : t \vartriangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_PAREN}
                                                                  \begin{array}{l} \Delta,E,E_1^{\rm L} \vdash pat_1: t \vartriangleright E_2^{\rm L} \\ \Delta,E,E_1^{\rm L} \vdash pat_2: \_{\bf list} \ t \vartriangleright E_3^{\rm L} \end{array}
                                              \frac{\mathbf{disjoint}\,\mathbf{doms}\,(E_2^{\scriptscriptstyle L},E_3^{\scriptscriptstyle L})}{\Delta,E,E_1^{\scriptscriptstyle L}\vdash\mathit{pat}_1::\mathit{pat}_2:\_\mathbf{list}\;t\,\vartriangleright\,E_2^{\scriptscriptstyle L}\uplus E_3^{\scriptscriptstyle L}}\quad\text{CHECK\_PAT\_AUX\_CONS}
                                                                                \frac{\vdash lit:t}{\Delta,E,E^{\text{L}}\vdash lit:t\,\vartriangleright\,\big\{\,\big\}}\quad\text{CHECK\_PAT\_AUX\_LIT}
                                                                          E, E^{\mathrm{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}
                            \frac{E, E^{\perp} \vdash x \text{ not ctor}}{\Delta, E, E^{\perp} \vdash x \ l + num : \_num \ \triangleright \{x \mapsto \_num \}}
   E \vdash id \, \mathbf{field}
                                                     Check that the identifier is a permissible field identifier
                                                                         \frac{E^{\mathrm{F}}(x) \rhd f\_desc}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{field}} \quad \text{ID\_FIELD\_EMPTY}
                                                                                                 E^{\mathrm{M}}(x) \triangleright E
                                                                                                 x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                                                  \frac{E \vdash \overline{y_i^l.}^i \ z^l \ l_2 \, \mathbf{field}}{\langle E^{\text{\tiny M}}, E^{\text{\tiny P}}, E^{\text{\tiny F}}, E^{\text{\tiny X}} \rangle \vdash x \ l_1. \, \overline{y_i^l.}^i \ z^l \ l_2 \, \mathbf{field}} \quad \text{id\_field\_cons}
   E \vdash id value
                                                         Check that the identifier is a permissible value identifier
                                                                       \frac{E^{\mathbf{X}}(x) \rhd v\_desc}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{value}} \quad \text{ID\_VALUE\_EMPTY}
                                                                                              E^{\mathrm{M}}(x) \triangleright E
                                                                                              x \not\in \mathbf{dom}(E^{X})
                                                               \frac{E \vdash \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \vdash x \ l_1. \ \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}} \quad \text{id_value\_cons}
   \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                        Typing expressions, collecting typeclass and index constraints
```

 $\Delta, E, E^{\perp} \vdash exp_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$ Typing expressions, collecting typeclass and index constraints

 $\frac{\Delta, E, E^{\mathsf{L}} \vdash exp_aux : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta. E. E^{\mathsf{L}} \vdash exp_aux \ l : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK_EXP_ALL}$

```
\frac{E^{\mathsf{L}}(x) \triangleright t}{\Delta, E, E^{\mathsf{L}} \vdash x \, l_1 \, l_2 : t \triangleright \{\}, \{\}} \quad \text{CHECK\_EXP\_AUX\_VAR}
                                                                                                                                                          CHECK_EXP_AUX_NVAR
                                                         \overline{\Delta, E, E^{\text{L}} \vdash N : \_\mathbf{num} \triangleright \{\}, \{\}}
                              E^{\mathsf{L}} \vdash id \text{ not shadowed}
                              E \vdash id \mathbf{value}
                              \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \to p\ t\_args \rhd (x\ \mathbf{of}\ names)
                                                                                                                                                                                       CHECK_EXP_AUX_CTOR
                                 \Delta, E, E^{\text{L}} \vdash id : \mathbf{curry} (t\_multi, p \ t\_args) \triangleright \{\}, \{\}
                                                                       E^{\mathsf{L}} \vdash id \text{ not shadowed}
                                                                       E \vdash id value
                                                                       \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                   \frac{-}{\Delta, E, E^{\text{\tiny L}} \vdash id: t \vartriangleright \Sigma^{\mathcal{C}}, \{\,\}} \quad \text{CHECK\_EXP\_AUX\_VAL}
                          \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                           disjoint doms (E_1^L, ..., E_n^L)
                                                                                                                                                                                                                  CHECK_EXP_AUX_FN
       \overline{\Delta, E, E^{\text{\tiny L}} \vdash \mathbf{fun} \ pat_1 \dots pat_n \rightarrow exp \ l : \mathbf{curry} \left( (t_1 * \dots * t_n), u \right) \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                          \frac{\Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_i : t \vartriangleright \overline{E_i^{\text{\tiny L}}}^i}{\Delta, E, E^{\text{\tiny L}} \uplus E_i^{\text{\tiny L}} \vdash \textit{exp}_i : u \vartriangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}_i}^i}
     \Delta, E, E^{\text{L}} \vdash \overline{\mathbf{function}} \mid^{?} \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ \mathbf{end} : t \rightarrow u \triangleright \overline{\Sigma^{\mathcal{C}}_{i}}^{i}, \overline{\Sigma^{\mathcal{N}_{i}}}^{i}
                                                    \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t_1 \to t_2 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                    \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_1 \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                               \frac{1}{\Delta, E, E^{\text{L}} \vdash exp_1 \ exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_APP}
                                      \Delta, E, E^{\text{L}} \vdash (ix) : t_1 \to t_2 \to t_3 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                      \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \triangleright \Sigma^{\bar{\mathcal{C}}}_{2}, \Sigma^{\bar{\mathcal{N}}}_{2}
                                      \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
\overline{\Delta, E, E^{\text{L}} \vdash exp_1 \ ix \ l \ exp_2 : t_3 \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2 \cup \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2 \cup \Sigma^{\mathcal{N}}_3} \quad \text{CHECK\_EXP\_AUX\_INFIX\_APP1}
                                                                                                                                                   \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash x &: t_1 \to t_2 \to t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\mathrm{L}} \vdash exp_1 &: t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                                                                                                                                   \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
<<no parses (char 18): TD,E,E_l |- exp1 '***x' l exp2 : t3 gives S_c1 union S_c2 union S_c3,</pre>
                              \Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i\ \mathbf{of}\ names)
                              \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                              \mathbf{duplicates} (\overline{x_i}^i) = \emptyset
                              names = \{ \overline{x_i}^i \}
                                                                                                                                                                                    CHECK_EXP_AUX_RECORD
                      \Delta, E, E^{\text{\tiny L}} \vdash \langle | \overline{id_i = exp_i} \overline{l_i}^i; ? \ l | \rangle : p \ t\_args \rhd \overline{\Sigma^{\mathcal{C}}_i}^i, \overline{\Sigma^{\mathcal{N}_i}^i}
                                         \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)}^i
                                         \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}^i
                                         \operatorname{duplicates}(\overline{x_i}^i) = \emptyset
                                          \Delta, E, E^{L} \vdash exp : p \ t\_args \triangleright \Sigma^{C'}, \Sigma^{N'}
\Delta, E, E^{\text{L}} \vdash \langle | exp \ \overline{\text{with}} \ \overline{id_i = exp_i \ l_i}^i ; ? \ l | \rangle : p \ t\_args \rhd \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_i}^i , \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}_i}}^i  CHECK_EXP_AUX_RECUP
                     \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \triangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n
                    length (exp_1 \dots exp_n) = nat
\overline{\Delta, E, E^{\text{L}} \vdash [|exp_1; \dots; exp_n;^?|] : \_\text{vector } nat \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_VECTOR}
```

```
\Delta, E, E^{\mathsf{L}} \vdash exp : \_\mathbf{vector} \ ne' \ t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                        \vdash Nexp \leadsto ne
                                                                                                                                                                                             CHECK_EXP_AUX_VECTORGET
                             \overline{\Delta, E, E^{\perp} \vdash exp.(Nexp) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne\langle ne' \}\}}
                                                    \Delta, E, E^{L} \vdash exp : \_vector ne' t \triangleright \Sigma^{C}, \Sigma^{N}
                                                    \vdash Nexp_1 \leadsto ne_1
                                                    \vdash Nexp_2 \leadsto ne_2
                                                    ne = ne_2 + (-ne_1)
                                                                                                                                                                                                                                                 CHECK_EXP_AUX_VECTORSUB
\overline{\Delta, E, E^{\text{\tiny L}} \vdash exp.(Nexp_1..Nexp_2) : \_\textbf{vector} \ ne \ t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \ \cup \ \{ne_1 \langle ne_2 \langle ne' \} \}
                                            E \vdash id \mathbf{field}
                                           \Delta, E \vdash field id : p \ t\_args \rightarrow t \triangleright (x \ \textbf{of} \ names)
                                           \Delta, E, E^{\mathsf{L}} \vdash exp : p \ t\_args \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                                                      CHECK_EXP_AUX_FIELD
                                                                  \Delta, E, E^{\text{L}} \vdash exp.id : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                      \begin{split} & \frac{\Delta, E, E^{\text{L}} \vdash \textit{pat}_i : t \rhd \overline{E_i^{\text{L}}}^i}{\Delta, E, E^{\text{L}} \uplus E_i^{\text{L}} \vdash \textit{exp}_i : u \rhd \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}^i \\ & \Delta, E, E^{\text{L}} \vdash \textit{exp} : t \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'} \end{split}
\Delta, E, E^{\text{L}} \vdash \mathbf{match} \ \underline{exp} \ \mathbf{with} \ |^{?} \ \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ l \ \mathbf{end} : u \vartriangleright \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_{i}}^{i}, \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}_{i}}}^{i} CHECK_EXP_AUX_CASE
                                                                     \Delta, E, E^{L} \vdash exp : t \triangleright \Sigma^{C}, \Sigma^{N}
                                                                     \Delta, E \vdash typ \leadsto t
                                                          \frac{1}{\Delta, E, E^{\text{L}} \vdash (exp: typ): t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_TYPED}
                                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash letbind \, \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}{}_1, \Sigma^{\mathcal{N}}{}_1 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp : t \, \rhd \, \Sigma^{\mathcal{C}}{}_2, \Sigma^{\mathcal{N}}{}_2 \end{array}
                         \frac{1}{\Delta, E, E_1^{\tt L} \vdash \mathbf{let} \ letbind \ \mathbf{in} \ exp: t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2}{} \quad \text{CHECK\_EXP\_AUX\_LET}
\frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t_n \rhd \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash (exp_1, \dots, exp_n) : t_1 * \dots * t_n \rhd \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_TUP}
         \Delta \vdash t \mathbf{ok}
 \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \rhd \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash [exp_1; \ ..; exp_n \ ;^?] : \_\mathbf{list} \ t \rhd \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_LIST}
                                                                    \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                  \frac{1}{\Delta, E, E^{\text{L}} \vdash (exp) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_PAREN}
                                                                      \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                    \frac{\cdot \quad - \ , -}{\Delta, E, E^{\text{\tiny L}} \vdash \mathbf{begin} \ exp \ \mathbf{end} : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{Check_exp_aux_begin}
                                                                        \Delta, E, E^{\mathsf{L}} \vdash exp_1 : \_\mathbf{bool} \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                        \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}
                                                                        \Delta, E, E^{\mathrm{L}} \vdash exp_3 : t \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
\overline{\Delta, E, E^{\text{L}} \vdash \textbf{if } \textit{exp}_{1} \textbf{ then } \textit{exp}_{2} \textbf{ else } \textit{exp}_{3} : t \vartriangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2} \cup \Sigma^{\mathcal{C}}_{3}, \Sigma^{\mathcal{N}}_{1} \cup \Sigma^{\mathcal{N}}_{2} \cup \Sigma^{\mathcal{N}}_{3}} \quad \text{CHECK\_EXP\_AUX\_IF}
                                                          \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                           \Delta, E, E^{\mathsf{L}} \vdash exp_2 : \_\mathbf{list} \ t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                       \frac{1}{\Delta,E,E^{\text{\tiny L}} \vdash exp_1 :: exp_2 : \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2} \quad \text{Check_exp_aux_cons}
                                                                             \frac{ \vdash lit:t}{\Delta,E,E^{\text{\tiny L}} \vdash lit:t \, \rhd \, \{\,\},\{\,\}} \quad \text{CHECK\_EXP\_AUX\_LIT}
```

```
\Delta \vdash t_i \mathbf{ok}^i
                                 \Delta, E, E^{L} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \}
                                 \Delta, E, E^{L} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_2 : \_bool \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                 disjoint doms (E^{\mathrm{L}}, \{\overline{x_i \mapsto t_i}^i\})
                                 E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                 \overline{x_i \not\in \mathbf{dom}(E^{\mathrm{X}})}^{i}
                   \overline{\Delta, E, E^{\text{L}} \vdash \{exp_1 | exp_2\} : \_\mathbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_SET\_COMP}
                                                                         \begin{array}{l} \Delta, E, E_{1}^{\text{\tiny L}} \vdash \overline{qbind_{i}}^{i} \rhd E_{2}^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_{1} \\ \Delta, E, E_{1}^{\text{\tiny L}} \uplus E_{2}^{\text{\tiny L}} \vdash exp_{1} : t \rhd \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2} \\ \Delta, E, E_{1}^{\text{\tiny L}} \uplus E_{2}^{\text{\tiny L}} \vdash exp_{2} : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_{3}, \Sigma^{\mathcal{N}}_{3} \end{array}
\Delta, E, E_1^{\text{\tiny L}} \vdash \{\overline{exp_1|\operatorname{\mathbf{forall}} \overline{qbind_i}^i|exp_2}\} : \_\operatorname{\mathbf{set}} t \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2 \, \cup \, \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \, \cup \, \Sigma^{\mathcal{N}}_3}
 \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \,\triangleright\, \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \,\triangleright\, \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash \{exp_1; \, ..; exp_n; ?\} : \_\mathbf{set} \, t \,\triangleright\, \Sigma^{\mathcal{C}}_1 \,\cup\, .. \,\cup\, \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \,\cup\, .. \,\cup\, \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_SET}
                                 \frac{\Delta, E, E_{1}^{\text{L}} \vdash \overline{qbind_{i}}^{i} \triangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}_{1}}{\Delta, E, E_{1}^{\text{L}} \vdash \overline{qbind_{i}}^{i} \cdot exp : \_\textbf{bool} \quad \triangleright \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2}} \quad \text{CHECK\_EXP\_AUX\_QUANT}}
\frac{\Delta, E, E_{1}^{\text{L}} \vdash q \ \overline{qbind_{i}}^{i} \cdot exp : \_\textbf{bool} \quad \triangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2}}}{\Delta, E, E_{1}^{\text{L}} \vdash q \ \overline{qbind_{i}}^{i} \cdot exp : \_\textbf{bool} \quad \triangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2}}}
                                                                        \begin{array}{l} \Delta, E, E_1^{\scriptscriptstyle L} \vdash \ \mathbf{list} \ \overline{qbind_i}^{\ i} \rhd E_2^{\scriptscriptstyle L}, \Sigma^{\mathcal{C}}_1 \\ \Delta, E, E_1^{\scriptscriptstyle L} \uplus E_2^{\scriptscriptstyle L} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \\ \Delta, E, E_1^{\scriptscriptstyle L} \uplus E_2^{\scriptscriptstyle L} \vdash exp_2 : \_\mathbf{bool} \ \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3 \end{array}
\overline{\Delta, E, E_1^{\tt L} \vdash [exp_1 | \mathbf{forall} \ \overline{qbind_i}^i | exp_2] : \_\mathbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2 \ \cup \ \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \ \cup \ \Sigma^{\mathcal{N}}_3}
   \Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass cons
                                                                                                                                                                CHECK_LISTQUANT_BINDING_EMPTY
                                                                              \overline{\Delta, E, E^{\mathrm{L}} \vdash \triangleright \{\}, \{\}}
                                            \Delta \vdash t \mathbf{ok}
                                      CHECK_LISTQUANT_BINDING_VAR
                                                \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_3^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : \_{\textbf{set}} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                \Delta, E, E_1^{\text{L}} \uplus E_3^{\text{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{L}}, \Sigma^{\mathcal{C}}_2
\mathbf{disjoint} \ \mathbf{doms} \ (E_3^{\text{L}}, E_2^{\text{L}})
          \frac{\Box}{\Delta, E, E_1^{\text{\tiny L}} \vdash (pat \, \mathbf{IN} \, exp) \, \overline{qbind_i}^{\, i} \, \triangleright \, E_2^{\text{\tiny L}} \uplus E_3^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2}
                                                                                                                                                                                                                                     CHECK_LISTQUANT_BINDING_RESTR
                                            \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp: \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
\frac{\Delta, E, E_{1}^{\text{L}} \uplus E_{3}^{\text{L}} \vdash \overline{qbind_{i}}^{i} \triangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}_{2}}{\text{disjoint doms}\left(E_{3}^{\text{L}}, E_{2}^{\text{L}}\right)} \\ \frac{\text{disjoint doms}\left(E_{3}^{\text{L}}, E_{2}^{\text{L}}\right)}{\Delta, E, E_{1}^{\text{L}} \vdash \left(pat \, \mathbf{MEM} \, exp\right) \, \overline{qbind_{i}}^{i} \triangleright E_{2}^{\text{L}} \uplus E_{3}^{\text{L}}, \Sigma^{\mathcal{C}}_{1} \, \cup \, \Sigma^{\mathcal{C}}_{2}} \\ \text{CHECK\_LISTQUANT\_BINDING\_LIST\_RESTR}
   \Delta, E, E_1^{\text{L}} \vdash \mathbf{list} \ qbind_1 ... \ qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}
                                                                                                                                                                         Build the environment for quantifier bindings, collecting typeclass
```

 $\overline{\Delta, E, E^{\text{L}} \vdash \mathbf{list} \triangleright \{\}, \{\}}$

CHECK_QUANT_BINDING_EMPTY

```
\begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp: \_\_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                 \Delta, E, E_1^{\scriptscriptstyle L} \uplus E_3^{\scriptscriptstyle L} \vdash \ \overline{qbind_i}^{\ i} \rhd E_2^{\scriptscriptstyle L}, \Sigma^{\mathcal{C}}_2
                                                 \mathbf{disjoint}\,\mathbf{doms}\,(E_3^{\scriptscriptstyle{\mathrm{L}}},E_2^{\scriptscriptstyle{\mathrm{L}}})
    \frac{1}{\Delta, E, E_1^{\text{L}} \vdash \textbf{list} \left(pat \, \textbf{MEM} \, exp\right) \overline{qbind_i}^i \triangleright E_2^{\text{L}} \uplus E_3^{\text{L}}, \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2} \quad \text{CHECK\_QUANT\_BINDING\_RESTR}
    \Delta, E, E^{\mathrm{L}} \vdash funcl \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                      Build the environment for a function definition clause, collecting typec
                                           \begin{array}{l} \Delta,\,E,\,E^{\scriptscriptstyle L}\vdash\,pat_1:t_1\,\vartriangleright\,E_1^{\scriptscriptstyle L}\quad\dots\quad\Delta,E,\,E^{\scriptscriptstyle L}\vdash\,pat_n:t_n\,\vartriangleright\,E_n^{\scriptscriptstyle L}\\ \Delta,\,E,\,E^{\scriptscriptstyle L}\uplus\,E_1^{\scriptscriptstyle L}\uplus\,\dots\,\uplus\,E_n^{\scriptscriptstyle L}\vdash\,exp:u\,\vartriangleright\,\Sigma^{\mathcal{C}},\Sigma^{\mathcal{N}} \end{array}
                                           disjoint doms (E_1^{\text{L}}, \dots, E_n^{\text{L}})
                                            \Delta, E \vdash typ \leadsto u
\frac{\Delta, E \vdash typ \leadsto u}{\Delta, E, E^{\perp} \vdash x \ l_1 \ pat_1 \dots pat_n : typ = exp \ l_2 \rhd \{x \mapsto \mathbf{curry} ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}  CHECK_FUNCL_ANNOT
                                  \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1}: t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n}: t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp: u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                  \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
 \frac{\Delta}{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
    \Delta, E, E_1^{\text{L}} \vdash letbind \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a let binding, collecting typeclass and index con
                                                                 \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                                  \Delta, E \vdash typ \leadsto t
                                          \frac{\Delta, E + igp \lor \lor \iota}{\Delta, E, E_1^{\mathsf{L}} \vdash pat : typ = exp \ l \rhd E_2^{\mathsf{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                                                                                                                                               CHECK_LETBIND_VAL_ANNOT
                                              \begin{array}{c} \Delta, E, E_{1}^{\text{\tiny L}} \vdash pat : t \vartriangleright E_{2}^{\text{\tiny L}} \\ \Delta, E, E_{1}^{\text{\tiny L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \hline \Delta, E, E_{1}^{\text{\tiny L}} \vdash pat = exp \ l \vartriangleright E_{2}^{\text{\tiny L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                                                                                                                                                CHECK_LETBIND_VAL_NOANNOT
                                                       \frac{\Delta, E, E_1^{\text{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\text{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_FN}
     \Delta, E, E^{\mathrm{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                      Build the environment for an inductive relation clause, collecting typed
                                                                                                             \overline{\Delta \vdash t_i \, \mathbf{ok}}^{\,i}
                                                                                                             E_2^{\rm L} = \{ \overline{name_-t_i \rightarrow x \mapsto t_i}^i \}
\Delta, E, E_{1}^{L} \uplus E_{2}^{L} \vdash exp' : \_bool \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'}
\Delta, E, E_{1}^{L} \uplus E_{2}^{L} \vdash exp_{1} : u_{1} \rhd \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \quad ... \quad \Delta, E, E_{1}^{L} \uplus E_{2}^{L} \vdash exp_{n} : u_{n} \rhd \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n}
\Delta, E, E_{1}^{L} \vdash x_{1}^{l} : \mathbf{forall} \ \overline{name\_t_{i}}^{i} \cdot exp' \Longrightarrow x \ l \ exp_{1} ... \ exp_{n} \ l' \rhd \{x \mapsto \mathbf{curry} ((u_{1} * ... * u_{n}), \_bool)\}, \Sigma^{\mathcal{C}'} \cup \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{n})
       xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                         Extract the type constructor information
                                                                                tnvars^l \leadsto tnvs
                                                                                 \Delta, E \vdash typ \leadsto t
                                                                                \mathbf{duplicates}(tnvs) = \emptyset
                                                                                \mathbf{FV}(t) \subset tnvs
                                                                                \overline{y_i}. i \ x \not\in \mathbf{dom}(\Delta)
       \frac{\overline{y_i}^i, \Delta, E \vdash \mathbf{tc} \, x \, l \, tnvars^l = typ \, \triangleright \, \{\overline{y_i}^i \, x \mapsto tnvs \, .t\}, \{x \mapsto \overline{y_i}^i \, x\}}{\{\overline{y_i}^i, \Delta, E \vdash \mathbf{tc} \, x \, l \, tnvars^l = typ \, \triangleright \, \{\overline{y_i}^i \, x \mapsto tnvs \, .t\}, \{x \mapsto \overline{y_i}^i \, x\}}
                                                                            tnvars^l \leadsto tnvs
                                                                            \mathbf{duplicates}(tnvs) = \emptyset
                                                                            \overline{y_i} x \notin \mathbf{dom}(\Delta)
               \frac{g_{i}. \ x \neq \mathbf{uoii} \ (\Delta)}{\overline{y_{i}}^{i}, \Delta, E_{1} \vdash \mathbf{tc} \ x \ l \ tnvars^{l} \ \rhd \{\overline{y_{i}.}^{i} \ x \mapsto tnvs \}, \{x \mapsto \overline{y_{i}.}^{i} \ x\}} \quad \text{CHECK\_TEXP\_TC\_ABSTRACT}
```

```
tnvars^l \leadsto tnvs
                                                                                      duplicates(tnvs) = \emptyset
                                                                                      \overline{y_i} x \notin \mathbf{dom}(\Delta)
\overline{y_i^{\ i}, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l \ = \langle |x_1^l : typ_1; \dots; x_j^l : typ_j \ ;^?| \rangle} \ \triangleright \ \{ \overline{y_i.}^i \ x \mapsto tnvs \ \}, \{ x \mapsto \overline{y_i.}^i \ x \}
                                                                                   tnvars^l \leadsto tnvs
                                                                                   \mathbf{duplicates}(tnvs) = \emptyset
                                                                                   \overline{y_i}^i x \not\in \mathbf{dom}(\Delta)
                                                                                                                                                                                                                                        CHECK_TEXP_TC_VAR
\overline{\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l \ = |^? \ ctor\_def_1| \dots | ctor\_def_j \ \rhd \ \{\overline{y_i.}^i \ x \mapsto tnvs \ \}, \{x \mapsto \overline{y_i.}^i \ x\}
     xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                  Extract the type constructor information
                                                                                                                                {\tt CHECK\_TEXPS\_TC\_EMPTY}
                                                              \overline{xs, \Delta, E \vdash \mathbf{tc} \triangleright \{\}, \{\}}
                      xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E_2^{\mathrm{P}}
                     xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{\}, E_2^{\mathtt{P}}, \{\}, \{\} \rangle \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_3, E_3^{\mathtt{P}} \\ \mathbf{dom} (E_2^{\mathtt{P}}) \cap \mathbf{dom} (E_3^{\mathtt{P}}) = \emptyset
                                   xs, \Delta_1, E \vdash \mathbf{tc} \ td \ \overline{td_i}^i \rhd \Delta_2 \uplus \Delta_3, E_2^P \uplus E_3^P
     \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                                                                                    Check a type definition, with its path already resolved
                                                                                                                                                CHECK\_TEXP\_ABBREV
                                                        \overline{\Delta, E \vdash tnvs \ p = typ \, \triangleright \, \langle \{\,\}, \{\,\} \rangle}
                                     \overline{\Delta, E \vdash typ_i \leadsto t_i}^i
                                     names = \{ \overline{x_i}^i \}
                                    \frac{\text{duplicates}\left(\overline{x_{i}}^{i}\right)}{\text{FV}\left(t_{i}\right) \subset tnvs}^{i} = \emptyset
                                    E^{F} = \{ \overline{x_{i} \mapsto \langle \text{ forall } tnvs.p \to t_{i}, (x_{i} \text{ of } names) \rangle}^{i} \}  CHECK_TEXP_REC
                                               \Delta, E \vdash tnvs \ p = \langle | \overline{x_i^l : typ_i}^i ; ? | \rangle \rhd \langle E^F, \{ \} \rangle
                             \overline{\Delta, E \vdash typs_i \leadsto t\_multi_i}^i
                             names = \{ \overline{x_i}^i \}
                            \mathbf{duplicates}\left(\,\overline{x_i}^{\,i}\,\right) = \emptyset
                            \overline{\mathbf{FV}(t_{-}multi_{i})} \subset tnvs
                            E^{\mathbf{X}} = \{ \, \overline{x_i \mapsto \langle \, \mathbf{forall} \, tnvs.t\_multi_i \rightarrow p, (x_i \, \mathbf{of} \, names) \rangle}^{\, \imath} \, \}
                                                                                                                                                                                          CHECK_TEXP_VAR
                                                \Delta, E \vdash \overline{tnvs} \, \overline{p = |? \, \overline{x_i^l \, \mathbf{of} \, typs_i}^{\, \, i}} \, \, \triangleright \, \langle \{ \, \}, E^{\scriptscriptstyle \mathrm{X}} \rangle
     xs, \Delta, E \vdash td_1 ... td_n \triangleright \langle E^{\scriptscriptstyle{\text{F}}}, E^{\scriptscriptstyle{\text{X}}} \rangle
                                                                                                                                  CHECK_TEXPS_EMPTY
                                                                    \overline{y_i}^i, \Delta, E \vdash \triangleright \langle \{\}, \{\} \rangle
                                   tnvars^l \leadsto tnvs
                                   \Delta, E_1 \vdash tnvs \overline{y_i}.^i x = texp \triangleright \langle E_1^F, E_1^X \rangle
                                   \overline{y_i}^{\;i}, \Delta, E \vdash \; \overline{td_j}^{\;j} \, \rhd \, \langle E_2^{\scriptscriptstyle \mathrm{F}}, E_2^{\scriptscriptstyle \mathrm{X}} \rangle
                                   \mathbf{dom}\left(E_{1}^{\mathrm{X}}\right)\cap\mathbf{dom}\left(E_{2}^{\mathrm{X}}\right)=\emptyset
                                   \mathbf{dom}\,(E_1^{\mathrm{F}})\,\cap\,\mathbf{dom}\,(E_2^{\mathrm{F}})=\,\emptyset
         \overline{y_i}^{\;i}, \Delta, E \vdash x \; l \; tnvars^l \; = \; texp \; \overline{td_j}^{\;j} \; \rhd \; \langle E_1^{\scriptscriptstyle \mathrm{F}} \uplus E_2^{\scriptscriptstyle \mathrm{F}}, E_1^{\scriptscriptstyle \mathrm{X}} \uplus E_2^{\scriptscriptstyle \mathrm{X}} \rangle
                                                                                                                                                                CHECK_TEXPS_CONS_CONCRETE
                                 \frac{\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \rhd \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{\overline{y_i}^i, \Delta, E \vdash x \ l \ tnvars^l \ \overline{td_j}^j \rhd \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle} \quad \text{CHECK\_TEXPS\_CONS\_ABSTRACT}
```

```
\delta, E \vdash id \leadsto p Lookup a type class
                                                                                   E(id) \triangleright p
                                                                             \frac{\delta(p) \stackrel{\frown}{\triangleright} xs}{\delta, E \vdash id \leadsto p} \quad \text{Convert\_class\_all}
   I \vdash (p \ t) \mathbf{IN} \mathcal{C}
                                                 Solve class constraint
\overline{I \vdash (p \alpha) \mathbf{IN} (p_1 tnv_1) ... (p_i tnv_i) (p \alpha) (p'_1 tnv'_1) ... (p'_j tnv'_j)}
                                                                                                                                                       SOLVE\_CLASS\_CONSTRAINT\_IMMEDIATE
                (p_1 \ tnv_1) \dots (p_n \ tnv_n) \Rightarrow (p \ t) \mathbf{IN} I
               \frac{I \vdash (p_1 \, \sigma(tnv_1)) \, \mathbf{IN} \, \mathcal{C} \quad \dots \quad I \vdash (p_n \, \sigma(tnv_n)) \, \mathbf{IN} \, \mathcal{C}}{I \vdash (p \, \sigma(t)) \, \mathbf{IN} \, \mathcal{C}}
                                                                                                                                                   SOLVE_CLASS_CONSTRAINT_CHAIN
   I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C} Solve class constraints
                             \frac{I \vdash (p_1 \ t_1) \ \mathbf{IN} \ \mathcal{C} \quad . \quad I \vdash (p_n \ t_n) \ \mathbf{IN} \ \mathcal{C}}{I \vdash \{(p_1 \ t_1), \ ..., (p_n \ t_n)\} \rhd \mathcal{C}} \quad \text{SOLVE\_CLASS\_CONSTRAINTS\_ALL}
    \Delta, I, E \vdash val\_def \triangleright E^{X} Check a value definition
                                          \Delta, E, \{\,\} \vdash letbind \, \triangleright \, \{\, \overline{x_i \mapsto t_i}^{\,\,i} \,\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                          \overline{\mathbf{FV}\left(t_{i}\right)} \subset tnvs
                                          \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                                                                                 CHECK_VAL_DEF_VAL
                 \overline{\Delta, I, E_1 \vdash \mathbf{let} \, \tau^? \, letbind} \, \triangleright \, \{ \, \overline{x_i \mapsto \langle \, \mathbf{forall} \, tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle}^{\, i} \, \}
                                    \frac{\Delta, E, E^{\mathsf{L}} \vdash funcl_i \, \triangleright \, \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}{I \vdash \Sigma^{\mathcal{C}} \, \triangleright \, \mathcal{C}}
                                     \overline{\mathbf{FV}(t_i) \subset tnvs}^i
                                     \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                     \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
                                     E^{\mathrm{L}} = \{ \overline{x_i \mapsto t_i}^i \}
       \frac{E^{-} = \{ x_{i} \mapsto t_{i}^{-} \}}{\Delta, I, E \vdash \mathbf{let} \operatorname{\mathbf{rec}} \tau^{?} \overline{funcl_{i}}^{i} \rhd \{ \overline{x_{i} \mapsto \langle \operatorname{\mathbf{forall}} tnvs. \mathcal{C} \Rightarrow t_{i}, \operatorname{\mathbf{let}} \rangle}^{i} \}} \quad \text{CHECK\_VAL\_DEF\_RECFUN}
    \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance | Check that t be a typeclass instance
                                                                                                                         CHECK_T_INSTANCE_VAR
                                                                \overline{\Delta}. (\alpha) \vdash \alpha instance
                                                                                                                                                       CHECK\_T\_INSTANCE\_TUP
                                     \overline{\Delta,(\alpha_1,\ldots,\alpha_n)\vdash\alpha_1*\ldots*\alpha_n} instance
                                                  \overline{\Delta,(\alpha_1,\alpha_2) \vdash \alpha_1 \rightarrow \alpha_n \, \mathbf{instance}} CHECK_T_INSTANCE_FN
                                              \frac{\Delta(p) \, \triangleright \, \alpha_1' \dots \alpha_n'}{\Delta, (\alpha_1, \dots, \alpha_n) \vdash p \, \alpha_1 \dots \alpha_n \, \mathbf{instance}} \quad \text{CHECK\_T\_INSTANCE\_TC}
   \overline{z_i}^j, D_1, E_1 \vdash def \triangleright D_2, E_2 Check a definition
                               \overline{z_i}^j, \Delta_1, E \vdash \mathbf{tc} \, \overline{td_i}^i \triangleright \Delta_2, E^{\mathrm{P}}
                 \frac{\overline{z_{j}}^{j}, \Delta_{1} \uplus \Delta_{2}, E \uplus \langle \{\}, E^{P}, \{\}, \{\} \rangle \vdash \overline{td_{i}}^{i} \triangleright \langle E^{F}, E^{X} \rangle}{\overline{z_{j}}^{j}, \langle \Delta_{1}, \delta, I \rangle, E \vdash \mathbf{type} \overline{td_{i}}^{i} l \triangleright \langle \Delta_{2}, \{\}, \{\} \rangle, \langle \{\}, E^{P}, E^{F}, E^{X} \rangle}  CHECK_DEF_TYPE
```

```
\frac{\Delta, I, E \vdash val\_def \triangleright E^{\mathbf{x}}}{\overline{z_i}^j, \langle \Delta, \delta, I \rangle, E \vdash val\_def \ l \triangleright \epsilon, \langle \{ \}, \{ \}, \{ \}, E^{\mathbf{x}} \rangle}
                                                                                                                                                                                                                   CHECK_DEF_VAL_DEF
                                                                                                                                                           \overline{\Delta}, E_1, E^{\scriptscriptstyle L} \vdash rule_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\scriptscriptstyle C}_i, \Sigma^{\scriptscriptstyle N}_i^{i}
                                                                                                                                                          \frac{I \vdash \overline{\Sigma^{C}}_{i}^{i} \triangleright C}{\mathbf{FV}(t_{i}) \subset tnvs}^{i}
                                                                                                                                                          \mathbf{FV}(\mathcal{C}) \subset tnvs
                                                                                                                                                          \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
                                                                                                                                                          E^{L} = \{ \overline{x_i \mapsto t_i}^i \}
E_2 = \langle \{\}, \{\}, \{\}, \{\overline{x_i} \mapsto \langle \mathbf{forall} \ tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle^i \} \rangle <<no parses (char 59): </zj//j/>,<TD,TC,I>,E1 |- indreln targets_opt indreln_names*** </rule
\frac{\overline{z_j}^j \ x, D_1, E_1 \vdash \mathit{defs} \rhd D_2, E_2}{\overline{z_j}^j \ , D_1, E_1 \vdash \mathbf{module} \ x \ l_1 = \mathbf{struct} \ \mathit{defs} \ \mathbf{end} \ l_2 \rhd D_2, \langle \{x \mapsto E_2\}, \{\,\}, \{\,\}, \{\,\} \rangle}
         \frac{E_1(id) \triangleright E_2}{\overline{z_j}^j, D, E_1 \vdash \mathbf{module} \ x \ l_1 = id \ l_2 \triangleright \epsilon, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle}
                                       \Delta, E \vdash typ \leadsto t

\mathbf{FV}(t) \subset \overline{\alpha_i}^i \\
\mathbf{FV}(\overline{\alpha'_k}^k) \subset \overline{\alpha_i}^i \\
\overline{\delta, E \vdash id_k \leadsto p_k}^k

                   \frac{E' = \langle \{ \}, \{ \}, \{ \}, \{x \mapsto \langle \operatorname{\mathbf{forall}} \overline{\alpha_i}^i. \overline{(p_k \alpha_k')}^k \Rightarrow t, \operatorname{\mathbf{val}} \rangle \} \rangle}{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \operatorname{\mathbf{val}} x \ l_1 : \operatorname{\mathbf{forall}} \overline{\alpha_i \ l_i''}^i . \overline{id_k \alpha_k' \ l_k'}^k \Rightarrow typ \ l_2 \rhd \epsilon, E'}
                     \frac{\Delta, E_1 \vdash typ_i \leadsto t_i}{\mathbf{FV}(t_i) \subset \alpha^i}^i
                      E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{\overline{y_i \mapsto \langle \mathbf{forall} \, \alpha. (p \, \alpha) \Rightarrow t_i, \mathbf{method} \rangle}^i \} \rangle
                     \delta_2 = \{ p \mapsto \overline{y_i}^{\ i} \}
                      p \not\in \mathbf{dom}(\delta_1)
                                                                                                                                                                                                                                                                            CHECK_DEF_CLASS
    \overline{z_i^{\ j}, \langle \Delta, \delta_1, I \rangle, E_1 \vdash \mathbf{class}(x \ l \ \alpha \ l'')} \overline{\mathbf{val} \ y_i \ l_i : typ_i \ l_i}^i \ \mathbf{end} \ l' \rhd \langle \{ \}, \delta_2, \{ \} \rangle, E_2
```

$$E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle$$

$$\Delta, E \vdash typ' \leadsto t'$$

$$\Delta, (\overline{\alpha_i}^i) \vdash t' \text{ instance}$$

$$tnvs = \overline{\alpha_i}^i$$

$$\mathbf{duplicates}(tnvs) = \emptyset$$

$$\overline{\delta, E \vdash id_k \leadsto p_k}^k$$

$$\mathbf{FV}(\overline{\alpha_k'}^k) \subset tnvs$$

$$E(id) \rhd p$$

$$\delta(p) \rhd \overline{z_j}^j$$

$$I_2 = \{ \overline{\Rightarrow} (p_k \alpha_k')^k \}$$

$$\overline{\Delta, I \cup I_2, E \vdash val_def_n \rhd E_n^{\mathrm{X}}}^n$$

$$\mathbf{disjoint} \, \mathbf{doms}(\overline{E_n^{\mathrm{X}}}^n)$$

$$\overline{E^{\mathrm{X}}(x_k)} \rhd \langle \mathbf{forall} \, \alpha''.(p \, \alpha'') \Rightarrow t_k, \mathbf{method} \rangle^k$$

$$\{ \overline{x_k} \mapsto \langle \mathbf{forall} \, tnvs. \Rightarrow \{\alpha'' \mapsto t'\}(t_k), \mathbf{let} \rangle^k \} = \overline{E_n^{\mathrm{X}}}^n$$

$$\overline{x_k}^k = \overline{z_j}^j$$

$$I_3 = \{ (p_k \alpha_k') \Rightarrow (p \, t')^k \}$$

$$(p \{ \overline{\alpha_i} \mapsto \alpha_i'''^i \}(t')) \not\in I$$

 $\frac{\overline{z_{j}^{j}}, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{instance forall}}{\overline{z_{i}^{j}}^{i}, \overline{id_{k} \alpha_{k}^{\prime} l_{k}^{\prime\prime}}^{i}} \xrightarrow{k} (id \ typ^{\prime}) \overline{val} \underline{-def_{n} \ l_{n}}^{n} \ \mathbf{end} \ l^{\prime} \rhd \langle \{ \}, \{ \}, I_{3} \rangle, \epsilon}$

 $\overline{z_j}^j$, D_1 , $E_1 \vdash defs \triangleright D_2$, E_2 Check definitions, given module path, definitions and environment

CHECK_DEF_

$$\overline{z_{j}}^{j}, D, E \vdash \triangleright \epsilon, \epsilon$$
 CHECK_DEFS_EMPTY
$$\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash def \triangleright D_{2}, E_{2}$$

$$\overline{z_{j}}^{j}, D_{1} \uplus D_{2}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash def;;_{i}^{?} \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{2} \uplus D_{3}, E_{2} \uplus E_{3}$$

$$E_{1}(id) \triangleright E_{2}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash \mathbf{open} \ id \ l;;_{i}^{?} \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}$$
CHECK_DEFS_OPEN

Definition rules: 141 good 4 bad Definition rule clauses: 435 good 4 bad