

<i>n, i, j, k</i>	Index variables for meta-lists
<i>num</i>	Numeric literals
<i>hex</i>	Bit vector literal, specified by C-style hex number
<i>bin</i>	Bit vector literal, specified by C-style binary number
<i>string</i>	String literals
<i>regexp</i>	Regular expressions, as a string literal
<i>x, y, z</i>	Variables
<i>ix</i>	Variables

l	$::=$ 	Source locations
$x^l, y^l, z^l, name$	$::=$ $x\ l$ $(ix)l$	Location-annotated names Remove infix status
ix^l	$::=$ $ix\ l$ $'x' l$	Location-annotated infix names Add infix status
α	$::=$ $'x$	Type variables
α^l	$::=$ $\alpha\ l$	Location-annotated type variables
N	$::=$ $''x$	numeric variables
N^l	$::=$ $N\ l$	Location-annotated numeric variables
id	$::=$ $x_1^l \dots x_n^l . x^l\ l$	Long identifiers
tnv	$::=$ α N	Union of type variables and Nexp type variables, without location
$tnvar^l$	$::=$ α^l N^l	Union of type variables and Nexp type variables, with location
$tnvs$	$::=$ $tnv_1 .. tnv_n$	Type variable lists
$tnvars^l$	$::=$ $tnvar_1^l .. tnvar_n^l$	Type variable lists
$Nexp_aux$	$::=$ N num $Nexp_1 * Nexp_2$ $Nexp_1 + Nexp_2$ $(Nexp)$	Numerical expressions for specifying vector lengths and indexes

$Nexp$	$::=$ $ \quad Nexp_aux \ l$	Location-annotated vector lengths
$Nexp_constraint_aux$	$::=$ $ \quad Nexp = Nexp'$ $ \quad Nexp \geq Nexp'$	Whether a vector is bounded or fixed size
$Nexp_constraint$	$::=$ $ \quad Nexp_constraint_aux \ l$	Location-annotated Nexp range
typ_aux	$::=$ $ \quad -$ $ \quad \alpha^l$ $ \quad typ_1 \rightarrow typ_2$ $ \quad typ_1 * \dots * typ_n$ $ \quad Nexp$ $ \quad id \ typ_1 .. typ_n$ $ \quad (typ)$	Types Unspecified type Type variables Function types Tuple types As a typ to permit applications over Nexps, other Type applications
typ	$::=$ $ \quad typ_aux \ l$	Location-annotated types
lit_aux	$::=$ $ \quad \mathbf{true}$ $ \quad \mathbf{false}$ $ \quad num$ $ \quad hex$ $ \quad bin$ $ \quad string$ $ \quad ()$ $ \quad \mathbf{bitzero}$ $ \quad \mathbf{bitone}$	Literal constants hex and bin are constant bit vectors, entered as C bitzero and bitone are constant bits, if commonly
lit	$::=$ $ \quad lit_aux \ l$	Location-annotated literal constants
$;^?$	$::=$ $ $ $ \quad ;$	Optional semi-colons
pat_aux	$::=$ $ \quad -$ $ \quad (pat \ \mathbf{as} \ x^l)$ $ \quad (pat : typ)$ $ \quad id \ pat_1 .. pat_n$ $ \quad \langle fpat_1; \dots; fpat_n; ? \rangle$ $ \quad [pat_1; ..; pat_n; ?]$	Patterns Wildcards Named patterns Typed patterns Single variable and constructor patterns Record patterns Vector patterns

		$[pat_1 .. pat_n]$	Concatenated vector patterns
		(pat_1, \dots, pat_n)	Tuple patterns
		$[pat_1; ..; pat_n; ?]$	List patterns
		(pat)	
		$pat_1 :: pat_2$	Cons patterns
		$x^l + num$	constant addition patterns
		lit	Literal constant patterns
pat	$::=$		Location-annotated patterns
		$pat_aux\ l$	
$fpat$	$::=$		Field patterns
		$id = pat\ l$	
$ ^?$	$::=$		Optional bars
exp_aux	$::=$		Expressions
		id	Identifiers
		N	Nexp var, has type num
		fun $psexp$	Curried functions
		function $ ^? pexp_1 \dots pexp_n$ end	Functions with pattern matching
		$exp_1\ exp_2$	Function applications
		$exp_1\ ix^l\ exp_2$	Infix applications
		$\langle fexps \rangle$	Records
		$\langle exp\ \mathbf{with}\ fexps \rangle$	Functional update for records
		$exp.id$	Field projection for records
		$[exp_1; ..; exp_n; ?]$	Vector instantiation
		$exp.(Nexp)$	Vector access
		$exp.(Nexp_1..Nexp_2)$	Subvector extraction
		match exp with $ ^? pexp_1 \dots pexp_n\ l$ end	Pattern matching expressions
		$(exp : typ)$	Type-annotated expressions
		let $letbind$ in exp	Let expressions
		(exp_1, \dots, exp_n)	Tuples
		$[exp_1; ..; exp_n; ?]$	Lists
		(exp)	
		begin exp end	Alternate syntax for (exp)
		if exp_1 then exp_2 else exp_3	Conditionals
		$exp_1 :: exp_2$	Cons expressions
		lit	Literal constants
		$\{exp_1 exp_2\}$	Set comprehensions
		$\{exp_1 \mathbf{forall}\ qbind_1 .. qbind_n exp_2\}$	Set comprehensions with explicit binders
		$\{exp_1; ..; exp_n; ?\}$	Sets
		$q\ qbind_1 \dots qbind_n.exp$	Logical quantifications
		$[exp_1 \mathbf{forall}\ qbind_1 .. qbind_n exp_2]$	List comprehensions (all binders must be quantified)
		do $id\ pat_1 < - exp_1; .. pat_n < - exp_n; \mathbf{in}\ exp$ end	Do notation for monads

exp	$::=$ $exp_aux\ l$	Location-annotated expressions
q	$::=$ forall exists	Quantifiers
$qbind$	$::=$ x^l $(pat\ \mathbf{IN}\ exp)$ $(pat\ \mathbf{MEM}\ exp)$	Bindings for quantifiers Restricted quantifications over sets Restricted quantifications over lists
$fexp$	$::=$ $id = exp\ l$	Field-expressions
$fexps$	$::=$ $fexp_1; \dots; fexp_n; ?\ l$	Field-expression lists
$pexp$	$::=$ $pat \rightarrow exp\ l$	Pattern matches
$psexp$	$::=$ $pat_1 \dots pat_n \rightarrow exp\ l$	Multi-pattern matches
$tannot^?$	$::=$ $: typ$	Optional type annotations
$funcl_aux$	$::=$ $x^l\ pat_1 \dots pat_n\ tannot^? = exp$	Function clauses
$letbind_aux$	$::=$ $pat\ tannot^? = exp$ $funcl_aux$	Let bindings Value bindings Function bindings
$letbind$	$::=$ $letbind_aux\ l$	Location-annotated let bindings
$funcl$	$::=$ $funcl_aux\ l$	Location-annotated function clauses
$id^?$	$::=$ $x^l :$	Optional name for inductively defined relations
$rule_aux$	$::=$ $id^? \mathbf{forall}\ x_1^l .. x_n^l. exp \implies x^l\ exp_1 .. exp_i$	Inductively defined relation clauses

<i>rule</i>	$::=$ $ \quad rule_aux \ l$	Location-annotated inductively defined
<i>typs</i>	$::=$ $ \quad typ_1 * \dots * typ_n$	Type lists
<i>ctor_def</i>	$::=$ $ \quad x^l \mathbf{of} \ typs$ $ \quad x^l$	Datatype definition clauses
<i>texp</i>	$::=$ $ \quad typ$ $ \quad \langle x_1^l : typ_1; \dots; x_n^l : typ_n; ? \rangle$ $ \quad ^? \ ctor_def_1 \dots \ ctor_def_n$	S Type definition bodies Type abbreviations Record types Variant types
<i>name[?]</i>	$::=$ $ $ $ \quad [name = regexp]$	Optional name specification for variants
<i>td</i>	$::=$ $ \quad x^l \ tnvars^l \ name^? = texp$ $ \quad x^l \ tnvars^l \ name^?$	Type definitions Definitions of opaque types
<i>c</i>	$::=$ $ \quad id \ tnvar^l$	Typeclass constraints
<i>cs</i>	$::=$ $ $ $ \quad c_1, \dots, c_i \Rightarrow$ $ \quad Nexp_constraint_1, \dots, Nexp_constraint_i \Rightarrow$ $ \quad c_1, \dots, c_i; Nexp_constraint_1, \dots, Nexp_constraint_n \Rightarrow$	Typeclass and length constraint lists Must have > 0 constraints Must have > 0 constraints Must have > 0 of both form of constraints
<i>c_pre</i>	$::=$ $ $ $ \quad \mathbf{forall} \ tnvar_1^l \dots tnvar_n^l. cs$	Type and instance scheme prefixes Must have > 0 type variables
<i>typschm</i>	$::=$ $ \quad c_pre \ typ$	Type schemes
<i>instschm</i>	$::=$ $ \quad c_pre(id \ typ)$	Instance schemes
<i>target</i>	$::=$ $ \quad \mathbf{hol}$ $ \quad \mathbf{isabelle}$ $ \quad \mathbf{ocaml}$ $ \quad \mathbf{coq}$	Backend target names

$defs$	$::=$ $def_1 ; ;_1^? .. def_n ; ;_n^?$	Definition sequences
p	$::=$ $x_1 .. x_n . x$ __list __bool __num __set __string __unit __bit __vector	Unique paths
σ	$::=$ $\{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}$	Type variable substitutions
t, u	$::=$ α $t_1 \rightarrow t_2$ $t_1 * * t_n$ $p \ t_args$ ne $\sigma(t)$ $\sigma(tnv)$ curry (t_multi, t)	Internal types M Multiple substitutions M Single variable substitution M Curried, multiple argument functions
ne	$::=$ N num $num * ne$ $ne_1 + ne_2$ $(-ne)$ normalize (ne) $ne_1 + ... + ne_n$ bitlength (bin) bitlength (hex) length ($pat_1 ... pat_n$) length ($exp_1 ... exp_n$)	internal numeric expressions M M M M M
t_args	$::=$ $t_1 .. t_n$ $\sigma(t_args)$	Lists of types M Multiple substitutions
t_multi	$::=$ $(t_1 * .. * t_n)$ $\sigma(t_multi)$	Lists of types M Multiple substitutions

nec	$::=$ $ \quad ne \langle nec$ $ \quad ne = nec$ $ \quad ne \leq nec$ $ \quad ne$	Numeric expression constraints
$names$	$::=$ $ \quad \{x_1, \dots, x_n\}$	Sets of names
\mathcal{C}	$::=$ $ \quad (p_1 \text{ } tnv_1) \dots (p_n \text{ } tnv_n)$	Typeclass constraint lists
env_tag	$::=$ $ \quad \mathbf{method}$ $ \quad \mathbf{val}$ $ \quad \mathbf{let}$	Tags for the (non-constructor) value descriptions Bound to a method Specified with val Defined with let or indreln
v_desc	$::=$ $ \quad \langle \mathbf{forall} \text{ } tnv.s.t_multi \rightarrow p, (x \text{ of } names) \rangle$ $ \quad \langle \mathbf{forall} \text{ } tnv.s.\mathcal{C} \Rightarrow t, env_tag \rangle$	Value descriptions Constructors Values
f_desc	$::=$ $ \quad \langle \mathbf{forall} \text{ } tnv.s.p \rightarrow t, (x \text{ of } names) \rangle$	Fields
$\Sigma^{\mathcal{C}}$	$::=$ $ \quad \{(p_1 \text{ } t_1), \dots, (p_n \text{ } t_n)\}$ $ \quad \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n$	Typeclass constraints M
$\Sigma^{\mathcal{N}}$	$::=$ $ \quad \{nec_1, \dots, nec_n\}$ $ \quad \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n$	Nexpe constraint lists M
E	$::=$ $ \quad \langle E^M, E^P, E^F, E^X \rangle$ $ \quad E_1 \uplus E_2$ $ \quad \epsilon$	Environments M M
E^X	$::=$ $ \quad \{x_1 \mapsto v_desc_1, \dots, x_n \mapsto v_desc_n\}$ $ \quad E_1^X \uplus \dots \uplus E_n^X$	Value environments M
E^F	$::=$ $ \quad \{x_1 \mapsto f_desc_1, \dots, x_n \mapsto f_desc_n\}$ $ \quad E_1^F \uplus \dots \uplus E_n^F$	Field environments M
E^M	$::=$ $ \quad \{x_1 \mapsto E_1, \dots, x_n \mapsto E_n\}$	Module environments

E^P	$::=$ $\{x_1 \mapsto p_1, \dots, x_n \mapsto p_n\}$ $E_1^P \uplus \dots \uplus E_n^P$	Path environments M
E^L	$::=$ $\{x_1 \mapsto t_1, \dots, x_n \mapsto t_n\}$ $E_1^L \uplus \dots \uplus E_n^L$	Lexical bindings M
tc_abbrev	$::=$ $.t$ 	Type abbreviations
tc_def	$::=$ $tnvs\ tc_abbrev$	Type and class constructor definitions Type constructors
Δ	$::=$ $\{p_1 \mapsto tc_def_1, \dots, p_n \mapsto tc_def_n\}$ $\Delta_1 \uplus \Delta_2$	Type constructor definitions M
δ	$::=$ $\{p_1 \mapsto xs_1, \dots, p_n \mapsto xs_n\}$ $\delta_1 \uplus \delta_2$	Typeclass definitions M
$inst$	$::=$ $C \Rightarrow (p\ t)$	A typeclass instance, t must not contain nested typ
I	$::=$ $\{inst_1, \dots, inst_n\}$ $I_1 \cup I_2$	Global instances M
D	$::=$ $\langle \Delta, \delta, I \rangle$ $D_1 \uplus D_2$ ϵ	Global type definition store M M
xs	$::=$ $x_1 \dots x_n$	
$terminals$	$::=$ \geq \rightarrow \Rightarrow $\langle $ $ \rangle$ \subset \cup \uplus	\geq \rightarrow \Rightarrow $\langle $ $ \rangle$ \subset \cup \uplus

	\notin	
	\subset	
	\neq	
	\emptyset	
	\langle	
	\rangle	
	\vdash	
	$,$	
	\mapsto	
	\triangleright	
	\rightsquigarrow	
	\Rightarrow	
	$-$	
	ϵ	
<i>formula</i>	$::=$	
	<i>judgement</i>	
	$formula_1 \dots formula_n$	
	$E^M(x) \triangleright E$	Module lookup
	$E^P(x) \triangleright p$	Path lookup
	$E^F(x) \triangleright f_desc$	Field lookup
	$E^X(x) \triangleright v_desc$	Value lookup
	$E^L(x) \triangleright t$	Lexical binding lookup
	$\Delta(p) \triangleright tc_def$	Type constructor lookup
	$\delta(p) \triangleright xs$	Type constructor lookup
	$\mathbf{dom}(E_1^M) \cap \mathbf{dom}(E_2^M) = \emptyset$	
	$\mathbf{dom}(E_1^X) \cap \mathbf{dom}(E_2^X) = \emptyset$	
	$\mathbf{dom}(E_1^F) \cap \mathbf{dom}(E_2^F) = \emptyset$	
	$\mathbf{dom}(E_1^P) \cap \mathbf{dom}(E_2^P) = \emptyset$	
	$\mathbf{disjoint\ doms}(E_1^L, \dots, E_n^L)$	Pairwise disjoint domains
	$\mathbf{disjoint\ doms}(E_1^X, \dots, E_n^X)$	Pairwise disjoint domains
	$\mathbf{compatible\ overlap}(x_1 \mapsto t_1, \dots, x_n \mapsto t_n)$	$(x_i = x_j) \implies (t_i = t_j)$
	$\mathbf{duplicates}(tnvs) = \emptyset$	
	$\mathbf{duplicates}(x_1, \dots, x_n) = \emptyset$	
	$x \notin \mathbf{dom}(E^L)$	
	$x \notin \mathbf{dom}(E^X)$	
	$x \notin \mathbf{dom}(E^F)$	
	$p \notin \mathbf{dom}(\delta)$	
	$p \notin \mathbf{dom}(\Delta)$	
	$\mathbf{FV}(t) \subset tnvs$	Free type variables
	$\mathbf{FV}(t_multi) \subset tnvs$	Free type variables
	$\mathbf{FV}(\mathcal{C}) \subset tnvs$	Free type variables
	$inst \mathbf{IN} I$	
	$(p\ t) \notin I$	
	$E_1^L = E_2^L$	
	$E_1^X = E_2^X$	

	$ \begin{array}{ l} E_1^F = E_2^F \\ E_1 = E_2 \\ \Delta_1 = \Delta_2 \\ \delta_1 = \delta_2 \\ I_1 = I_2 \\ names_1 = names_2 \\ t_1 = t_2 \\ \sigma_1 = \sigma_2 \\ p_1 = p_2 \\ xs_1 = xs_2 \\ tnvs_1 = tnvs_2 \end{array} $	
<i>convert_tnvars</i>	$ \begin{array}{ l} ::= \\ tnvars^l \rightsquigarrow tnvs \\ tnvar^l \rightsquigarrow tnv \end{array} $	
<i>look_m</i>	$ \begin{array}{ l} ::= \\ E_1(x_1^l \dots x_n^l) \triangleright E_2 \end{array} $	Name path lookup
<i>look_m_id</i>	$ \begin{array}{ l} ::= \\ E_1(id) \triangleright E_2 \end{array} $	Module identifier lookup
<i>look_tc</i>	$ \begin{array}{ l} ::= \\ E(id) \triangleright p \end{array} $	Path identifier lookup
<i>check_t</i>	$ \begin{array}{ l} ::= \\ \Delta \vdash t \mathbf{ok} \\ \Delta, tnv \vdash t \mathbf{ok} \end{array} $	Well-formed types Well-formed type/Nexps m
<i>teq</i>	$ \begin{array}{ l} ::= \\ \Delta \vdash t_1 = t_2 \end{array} $	Type equality
<i>convert_typ</i>	$ \begin{array}{ l} ::= \\ \Delta, E \vdash typ \rightsquigarrow t \\ \vdash Nexp \rightsquigarrow ne \end{array} $	Convert source types to im Convert and normalize num
<i>convert_typs</i>	$ \begin{array}{ l} ::= \\ \Delta, E \vdash typs \rightsquigarrow t_multi \end{array} $	
<i>check_lit</i>	$ \begin{array}{ l} ::= \\ \vdash lit : t \end{array} $	Typing literal constants
<i>inst_field</i>	$ \begin{array}{ l} ::= \\ \Delta, E \vdash \mathbf{field} \ id : p \ t_args \rightarrow t \triangleright (x \mathbf{of} \ names) \end{array} $	Field typing (also returns c
<i>inst_ctor</i>	$ \begin{array}{ l} ::= \\ \Delta, E \vdash \mathbf{ctor} \ id : t_multi \rightarrow p \ t_args \triangleright (x \mathbf{of} \ names) \end{array} $	Data constructor typing (a

<i>inst_val</i>	$::=$ $\mid \Delta, E \vdash \mathbf{val} \, id : t \triangleright \Sigma^C$	Typing top-level bindings, collecting
<i>not_ctor</i>	$::=$ $\mid E, E^L \vdash x \mathbf{not} \, \mathbf{ctor}$	v is not bound to a data constructor
<i>not_shadowed</i>	$::=$ $\mid E^L \vdash id \mathbf{not} \, \mathbf{shadowed}$	id is not lexically shadowed
<i>check_pat</i>	$::=$ $\mid \Delta, E, E_1^L \vdash pat : t \triangleright E_2^L$ $\mid \Delta, E, E_1^L \vdash pat_aux : t \triangleright E_2^L$	Typing patterns, building their Typing patterns, building their
<i>id_field</i>	$::=$ $\mid E \vdash id \mathbf{field}$	Check that the identifier is a p
<i>id_value</i>	$::=$ $\mid E \vdash id \mathbf{value}$	Check that the identifier is a p
<i>check_exp</i>	$::=$ $\mid \Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N$ $\mid \Delta, E, E^L \vdash exp_aux : t \triangleright \Sigma^C, \Sigma^N$ $\mid \Delta, E, E_1^L \vdash qbind_1 \dots qbind_n \triangleright E_2^L, \Sigma^C$ $\mid \Delta, E, E_1^L \vdash \mathbf{list} \, qbind_1 \dots qbind_n \triangleright E_2^L, \Sigma^C$ $\mid \Delta, E, E^L \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$ $\mid \Delta, E, E_1^L \vdash letbind \triangleright E_2^L, \Sigma^C, \Sigma^N$	Typing expressions, collecting Typing expressions, collecting Build the environment for quan Build the environment for quan Build the environment for a fu Build the environment for a let
<i>check_rule</i>	$::=$ $\mid \Delta, E, E^L \vdash rule \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$	Build the environment for an i
<i>check_texp_tc</i>	$::=$ $\mid xs, \Delta_1, E \vdash \mathbf{tc} \, td \triangleright \Delta_2, E^P$	Extract the type constructor in
<i>check_texps_tc</i>	$::=$ $\mid xs, \Delta_1, E \vdash \mathbf{tc} \, td_1 \dots td_i \triangleright \Delta_2, E^P$	Extract the type constructor in
<i>check_texp</i>	$::=$ $\mid \Delta, E \vdash tnvs \, p = texp \triangleright \langle E^F, E^X \rangle$	Check a type definition, with i
<i>check_texps</i>	$::=$ $\mid xs, \Delta, E \vdash td_1 \dots td_n \triangleright \langle E^F, E^X \rangle$	
<i>convert_class</i>	$::=$ $\mid \delta, E \vdash id \rightsquigarrow p$	Lookup a type class
<i>solve_class_constraint</i>	$::=$ $\mid I \vdash (p \, t) \mathbf{IN} \, C$	Solve class constraint

<i>solve_class_constraints</i>	$::=$ $ \quad I \vdash \Sigma^C \triangleright \mathcal{C}$	Solve class constraints
<i>check_val_def</i>	$::=$ $ \quad \Delta, I, E \vdash \text{val_def} \triangleright E^x$	Check a value definition
<i>check_t_instance</i>	$::=$ $ \quad \Delta, (\alpha_1, \dots, \alpha_n) \vdash t \textbf{instance}$	Check that t be a typeclass instance
<i>check_defs</i>	$::=$ $ \quad \overline{z}_j^j, D_1, E_1 \vdash \text{def} \triangleright D_2, E_2$ $ \quad \overline{z}_j^j, D_1, E_1 \vdash \text{defs} \triangleright D_2, E_2$	Check a definition Check definitions, given module path, definitions
<i>judgement</i>	$::=$ $ \quad \text{convert_tnvars}$ $ \quad \text{look_m}$ $ \quad \text{look_m_id}$ $ \quad \text{look_tc}$ $ \quad \text{check_t}$ $ \quad \text{teq}$ $ \quad \text{convert_typ}$ $ \quad \text{convert_typs}$ $ \quad \text{check_lit}$ $ \quad \text{inst_field}$ $ \quad \text{inst_ctor}$ $ \quad \text{inst_val}$ $ \quad \text{not_ctor}$ $ \quad \text{not_shadowed}$ $ \quad \text{check_pat}$ $ \quad \text{id_field}$ $ \quad \text{id_value}$ $ \quad \text{check_exp}$ $ \quad \text{check_rule}$ $ \quad \text{check_terp_tc}$ $ \quad \text{check_terps_tc}$ $ \quad \text{check_terp}$ $ \quad \text{check_terps}$ $ \quad \text{convert_class}$ $ \quad \text{solve_class_constraint}$ $ \quad \text{solve_class_constraints}$ $ \quad \text{check_val_def}$ $ \quad \text{check_t_instance}$ $ \quad \text{check_defs}$	
<i>user_syntax</i>	$::=$ $ \quad n$ $ \quad \text{num}$	

	<i>hex</i>
	<i>bin</i>
	<i>string</i>
	<i>regexp</i>
	<i>x</i>
	<i>ix</i>
	<i>l</i>
	x^l
	ix^l
	α
	α^l
	<i>N</i>
	N^l
	<i>id</i>
	<i>tnv</i>
	$tnvar^l$
	<i>tnvs</i>
	$tnvars^l$
	<i>Nexp_aux</i>
	<i>Nexp</i>
	<i>Nexp_constraint_aux</i>
	<i>Nexp_constraint</i>
	<i>typ_aux</i>
	<i>typ</i>
	<i>lit_aux</i>
	<i>lit</i>
	$;$
	$;$
	<i>pat_aux</i>
	<i>pat</i>
	<i>fpat</i>
	$ $
	<i>exp_aux</i>
	<i>exp</i>
	<i>q</i>
	<i>qbind</i>
	<i>fexp</i>
	<i>fexps</i>
	<i>pexp</i>
	<i>psexp</i>
	<i>tannot</i>
	<i>funcl_aux</i>
	<i>letbind_aux</i>
	<i>letbind</i>
	<i>funcl</i>
	<i>id</i>
	<i>rule_aux</i>

	<i>rule</i>
	<i>typs</i>
	<i>ctor_def</i>
	<i>terp</i>
	<i>name?</i>
	<i>td</i>
	<i>c</i>
	<i>cs</i>
	<i>c_pre</i>
	<i>typschm</i>
	<i>instschm</i>
	<i>target</i>
	τ
	$\tau?$
	<i>lemma_typ</i>
	<i>lemma_decl</i>
	<i>val_def</i>
	<i>val_spec</i>
	<i>def_aux</i>
	<i>def</i>
	$::?$
	<i>defs</i>
	<i>p</i>
	σ
	<i>t</i>
	<i>ne</i>
	<i>t_args</i>
	<i>t_multi</i>
	<i>nec</i>
	<i>names</i>
	\mathcal{C}
	<i>env_tag</i>
	<i>v_desc</i>
	<i>f_desc</i>
	$\Sigma^{\mathcal{C}}$
	$\Sigma^{\mathcal{N}}$
	<i>E</i>
	$E^{\mathbf{X}}$
	$E^{\mathbf{F}}$
	$E^{\mathbf{M}}$
	$E^{\mathbf{P}}$
	$E^{\mathbf{L}}$
	<i>tc_abbrev</i>
	<i>tc_def</i>
	Δ
	δ

$|$ *inst*
 $|$ *I*
 $|$ *D*
 $|$ *xs*
 $|$ *terminals*
 $|$ *formula*

$tnvars^l \rightsquigarrow tnvs$

$$\frac{tnvar_1^l \rightsquigarrow tn v_1 \quad \dots \quad tnvar_n^l \rightsquigarrow tn v_n}{tnvar_1^l \dots tnvar_n^l \rightsquigarrow tn v_1 \dots tn v_n} \text{ CONVERT_TNVARS_NONE}$$

$tnvar^l \rightsquigarrow tn v$

$$\frac{}{\alpha \ l \rightsquigarrow \alpha} \text{ CONVERT_TNVAR_A}$$

$$\frac{}{N \ l \rightsquigarrow N} \text{ CONVERT_TNVAR_N}$$

$E_1(x_1^l \dots x_n^l) \triangleright E_2$

Name path lookup

$$\frac{}{E() \triangleright E} \text{ LOOK_M_NONE}$$

$$\frac{E^M(x) \triangleright E_1 \quad E_1(\overline{y_i^l}^i) \triangleright E_2}{\langle E^M, E^P, E^F, E^X \rangle (x \ l \ \overline{y_i^l}^i) \triangleright E_2} \text{ LOOK_M_SOME}$$

$E_1(id) \triangleright E_2$

Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i \ x \ l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i \ x \ l_1 \ l_2) \triangleright E_2} \text{ LOOK_M_ID_ALL}$$

$E(id) \triangleright p$

Path identifier lookup

$$\frac{E(\overline{y_i^l}^i) \triangleright \langle E^M, E^P, E^F, E^X \rangle \quad E^P(x) \triangleright p}{E(\overline{y_i^l}^i \ x \ l_1 \ l_2) \triangleright p} \text{ LOOK_TC_ALL}$$

$\Delta \vdash t \text{ ok}$

Well-formed types

$$\frac{}{\Delta \vdash \alpha \text{ ok}} \text{ CHECK_T_VAR}$$

$$\Delta \vdash t_1 \text{ ok}$$

$$\Delta \vdash t_2 \text{ ok}$$

$$\frac{}{\Delta \vdash t_1 \rightarrow t_2 \text{ ok}} \text{ CHECK_T_FN}$$

$$\frac{\Delta \vdash t_1 \text{ ok} \quad \dots \quad \Delta \vdash t_n \text{ ok}}{\Delta \vdash t_1 * \dots * t_n \text{ ok}} \text{ CHECK_T_TUP}$$

$$\Delta(p) \triangleright tn v_1 \dots tn v_n \text{ tc_abbrev}$$

$$\frac{\Delta, tn v_1 \vdash t_1 \text{ ok} \quad \dots \quad \Delta, tn v_n \vdash t_n \text{ ok}}{\Delta \vdash p \ t_1 \dots t_n \text{ ok}} \text{ CHECK_T_APP}$$

$\Delta, tn v \vdash t \text{ ok}$

Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \mathbf{ok}}{\Delta, \alpha \vdash t \mathbf{ok}} \quad \text{CHECK_TLEN_T}$$

$$\frac{}{\Delta, N \vdash ne \mathbf{ok}} \quad \text{CHECK_TLEN_LEN}$$

$\boxed{\Delta \vdash t_1 = t_2}$ Type equality

$$\frac{\Delta \vdash t \mathbf{ok}}{\Delta \vdash t = t} \quad \text{TEQ_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ_SYM}$$

$$\frac{\Delta \vdash t_1 = t_2 \quad \Delta \vdash t_2 = t_3}{\Delta \vdash t_1 = t_3} \quad \text{TEQ_TRANS}$$

$$\frac{\Delta \vdash t_1 = t_3 \quad \Delta \vdash t_2 = t_4}{\Delta \vdash t_1 \rightarrow t_2 = t_3 \rightarrow t_4} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n}{\Delta \vdash t_1 * \dots * t_n = u_1 * \dots * u_n} \quad \text{TEQ_TUP}$$

$$\frac{\Delta(p) \triangleright \alpha_1 .. \alpha_n \quad \Delta \vdash t_1 = u_1 \quad .. \quad \Delta \vdash t_n = u_n}{\Delta \vdash p \, t_1 .. t_n = p \, u_1 .. u_n} \quad \text{TEQ_APP}$$

$$\frac{\Delta(p) \triangleright \alpha_1 .. \alpha_n . u}{\Delta \vdash p \, t_1 .. t_n = \{\alpha_1 \mapsto t_1 .. \alpha_n \mapsto t_n\}(u)} \quad \text{TEQ_EXPAND}$$

$$\frac{ne = \mathbf{normalize}(ne')}{\Delta \vdash ne = ne'} \quad \text{TEQ_NEXP}$$

$\boxed{\Delta, E \vdash typ \rightsquigarrow t}$ Convert source types to internal types

$$\frac{}{\Delta, E \vdash \alpha \, l' \, l \rightsquigarrow \alpha} \quad \text{CONVERT_TYP_VAR}$$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad \Delta, E \vdash typ_2 \rightsquigarrow t_2}{\Delta, E \vdash typ_1 \rightarrow typ_2 \, l \rightsquigarrow t_1 \rightarrow t_2} \quad \text{CONVERT_TYP_FN}$$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad \dots \quad \Delta, E \vdash typ_n \rightsquigarrow t_n}{\Delta, E \vdash typ_1 * \dots * typ_n \, l \rightsquigarrow t_1 * \dots * t_n} \quad \text{CONVERT_TYP_TUP}$$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad .. \quad \Delta, E \vdash typ_n \rightsquigarrow t_n \quad E(id) \triangleright p \quad \Delta(p) \triangleright \alpha_1 .. \alpha_n \, tc_abbrev}{\Delta, E \vdash id \, typ_1 .. typ_n \, l \rightsquigarrow p \, t_1 .. t_n} \quad \text{CONVERT_TYP_APP}$$

$$\frac{\vdash Nexp \rightsquigarrow ne}{\Delta, E \vdash Nexp \rightsquigarrow ne} \quad \text{CONVERT_TYP_NEXP}$$

$$\frac{\Delta, E \vdash typ \rightsquigarrow t}{\Delta, E \vdash (typ) \, l \rightsquigarrow t} \quad \text{CONVERT_TYP_PAREN}$$

$$\frac{\Delta, E \vdash typ \rightsquigarrow t_1 \quad \Delta \vdash t_1 = t_2}{\Delta, E \vdash typ \rightsquigarrow t_2} \quad \text{CONVERT_TYP_EQ}$$

$\boxed{\vdash Nexp \rightsquigarrow ne}$ Convert and normalize numeric expressions

$$\frac{}{\vdash N l \rightsquigarrow N} \quad \text{CONVERT_NEXP_VAR}$$

$$\frac{}{\vdash num \rightsquigarrow num} \quad \text{CONVERT_NEXP_NUM}$$

$$\frac{}{\vdash num * N \rightsquigarrow num * N} \quad \text{CONVERT_NEXP_MULT}$$

$$\frac{\vdash Nexp_1 \rightsquigarrow ne_1 \quad \vdash Nexp_2 \rightsquigarrow ne_2}{\vdash Nexp_1 + Nexp_2 \rightsquigarrow ne_1 + ne_2} \quad \text{CONVERT_NEXP_ADD}$$

$\boxed{\Delta, E \vdash typs \rightsquigarrow t_multi}$

$$\frac{\Delta, E \vdash typ_1 \rightsquigarrow t_1 \quad \dots \quad \Delta, E \vdash typ_n \rightsquigarrow t_n}{\Delta, E \vdash typ_1 * \dots * typ_n \rightsquigarrow (t_1 * \dots * t_n)} \quad \text{CONVERT_TYP_ALL}$$

$\boxed{\vdash lit : t}$ Typing literal constants

$$\frac{}{\vdash \mathbf{true} l : _ \mathbf{bool}} \quad \text{CHECK_LIT_TRUE}$$

$$\frac{}{\vdash \mathbf{false} l : _ \mathbf{bool}} \quad \text{CHECK_LIT_FALSE}$$

$$\frac{}{\vdash num l : _ \mathbf{num}} \quad \text{CHECK_LIT_NUM}$$

$$\frac{num = \mathbf{bitlength}(hex)}{\vdash hex l : _ \mathbf{vector} num _ \mathbf{bit}} \quad \text{CHECK_LIT_HEX}$$

$$\frac{num = \mathbf{bitlength}(bin)}{\vdash bin l : _ \mathbf{vector} num _ \mathbf{bit}} \quad \text{CHECK_LIT_BIN}$$

$$\frac{}{\vdash string l : _ \mathbf{string}} \quad \text{CHECK_LIT_STRING}$$

$$\frac{}{\vdash () l : _ \mathbf{unit}} \quad \text{CHECK_LIT_UNIT}$$

$$\frac{}{\vdash \mathbf{bitzero} l : _ \mathbf{bit}} \quad \text{CHECK_LIT_BITZERO}$$

$$\frac{}{\vdash \mathbf{bitone} l : _ \mathbf{bit}} \quad \text{CHECK_LIT_BITONE}$$

$\boxed{\Delta, E \vdash \mathbf{field} id : p \ t_args \rightarrow t \triangleright (x \mathbf{of} names)}$ Field typing (also returns canonical field names)

$$\begin{aligned} E(\overline{x_i^l}^i) &\triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^F(y) &\triangleright \langle \mathbf{forall} \ tnv_1 \dots tnv_n. p \rightarrow t, (z \mathbf{of} names) \rangle \\ \Delta \vdash t_1 \mathbf{ok} \quad \dots \quad \Delta \vdash t_n \mathbf{ok} \end{aligned}$$

$$\frac{}{\Delta, E \vdash \mathbf{field} \overline{x_i^l}^i \ y \ l_1 \ l_2 : p \ t_1 \dots t_n \rightarrow \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}(t) \triangleright (z \mathbf{of} names)} \quad \text{INST_FIELD_ALL}$$

$\boxed{\Delta, E \vdash \mathbf{ctor} id : t_multi \rightarrow p \ t_args \triangleright (x \mathbf{of} names)}$ Data constructor typing (also returns canonical constructors)

$$\begin{aligned} E(\overline{x_i^l}^i) &\triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^X(y) &\triangleright \langle \mathbf{forall} \ tnv_1 \dots tnv_n. t_multi \rightarrow p, (z \mathbf{of} names) \rangle \\ \Delta \vdash t_1 \mathbf{ok} \quad \dots \quad \Delta \vdash t_n \mathbf{ok} \end{aligned}$$

$$\frac{}{\Delta, E \vdash \mathbf{ctor} \overline{x_i^l}^i \ y \ l_1 \ l_2 : \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}(t_multi) \rightarrow p \ t_1 \dots t_n \triangleright (z \mathbf{of} names)} \quad \text{INST_CTOR_ALL}$$

$\Delta, E \vdash \mathbf{val} \, id : t \triangleright \Sigma^C$ Typing top-level bindings, collecting typeclass constraints

$$\frac{\begin{array}{l} E(\overline{x_i^l}^i) \triangleright \langle E^M, E^P, E^F, E^X \rangle \\ E^X(y) \triangleright \langle \mathbf{forall} \, tnv_1 \dots tnv_n. (p_1 \, tnv'_1) \dots (p_i \, tnv'_i) \Rightarrow t, env_tag \rangle \\ \Delta \vdash t_1 \mathbf{ok} \quad \dots \quad \Delta \vdash t_n \mathbf{ok} \\ \sigma = \{ tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n \} \end{array}}{\Delta, E \vdash \mathbf{val} \, \overline{x_i^l}^i \, y \, l_1 \, l_2 : \sigma(t) \triangleright \{(p_1 \, \sigma(tnv'_1)), \dots, (p_i \, \sigma(tnv'_i))\}} \quad \text{INST_VAL_ALL}$$

$E, E^L \vdash x \mathbf{not} \mathbf{ctor}$ v is not bound to a data constructor

$$\frac{E^L(x) \triangleright t}{E, E^L \vdash x \mathbf{not} \mathbf{ctor}} \quad \text{NOT_CTOR_VAL}$$

$$\frac{x \notin \mathbf{dom}(E^X)}{\langle E^M, E^P, E^F, E^X \rangle, E^L \vdash x \mathbf{not} \mathbf{ctor}} \quad \text{NOT_CTOR_UNBOUND}$$

$$\frac{E^X(x) \triangleright \langle \mathbf{forall} \, tnv_1 \dots tnv_n. (p_1 \, tnv'_1) \dots (p_i \, tnv'_i) \Rightarrow t, env_tag \rangle}{\langle E^M, E^P, E^F, E^X \rangle, E^L \vdash x \mathbf{not} \mathbf{ctor}} \quad \text{NOT_CTOR_BOUND}$$

$E^L \vdash id \mathbf{not} \mathbf{shadowed}$ id is not lexically shadowed

$$\frac{x \notin \mathbf{dom}(E^L)}{E^L \vdash x \, l_1 \, l_2 \mathbf{not} \mathbf{shadowed}} \quad \text{NOT_SHADOWED_SING}$$

$$\frac{}{E^L \vdash x_1^l \dots x_n^l. y^l. z^l \, l \mathbf{not} \mathbf{shadowed}} \quad \text{NOT_SHADOWED_MULTI}$$

$\Delta, E, E_1^L \vdash pat : t \triangleright E_2^L$ Typing patterns, building their binding environment

$$\frac{\Delta, E, E_1^L \vdash pat_aux : t \triangleright E_2^L}{\Delta, E, E_1^L \vdash pat_aux \, l : t \triangleright E_2^L} \quad \text{CHECK_PAT_ALL}$$

$\Delta, E, E_1^L \vdash pat_aux : t \triangleright E_2^L$ Typing patterns, building their binding environment

$$\frac{\Delta \vdash t \mathbf{ok}}{\Delta, E, E^L \vdash _ : t \triangleright \{ \}} \quad \text{CHECK_PAT_AUX_WILD}$$

$$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash pat : t \triangleright E_2^L \\ x \notin \mathbf{dom}(E_2^L) \end{array}}{\Delta, E, E_1^L \vdash (pat \mathbf{as} \, x \, l) : t \triangleright E_2^L \uplus \{x \mapsto t\}} \quad \text{CHECK_PAT_AUX_AS}$$

$$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash pat : t \triangleright E_2^L \\ \Delta, E \vdash typ \rightsquigarrow t \end{array}}{\Delta, E, E_1^L \vdash (pat : typ) : t \triangleright E_2^L} \quad \text{CHECK_PAT_AUX_TYP}$$

$\Delta, E \vdash \mathbf{ctor} \, id : (t_1 * \dots * t_n) \rightarrow p \, t_args \triangleright (x \mathbf{of} \, names)$

$E^L \vdash id \mathbf{not} \mathbf{shadowed}$

$\Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L$

$\mathbf{disjoint} \, \mathbf{doms} \, (E_1^L, \dots, E_n^L)$

$$\frac{}{\Delta, E, E^L \vdash id \, pat_1 \dots pat_n : p \, t_args \triangleright E_1^L \uplus \dots \uplus E_n^L} \quad \text{CHECK_PAT_AUX_IDENT_CONSTR}$$

$$\frac{\begin{array}{l} \Delta \vdash t \mathbf{ok} \\ E, E^L \vdash x \mathbf{not} \mathbf{ctor} \end{array}}{\Delta, E, E^L \vdash x \, l_1 \, l_2 : t \triangleright \{x \mapsto t\}} \quad \text{CHECK_PAT_AUX_VAR}$$

$$\begin{array}{c}
\frac{\Delta, E \vdash \mathbf{field} \, id_i : p \, t_args \rightarrow t_i \triangleright (x_i \mathbf{of} \, names)^i}{\Delta, E, E^L \vdash pat_i : t_i \triangleright E_i^L} \\
\mathbf{disjoint \, doms}(\overline{E_i^L}^i) \\
\mathbf{duplicates}(\overline{x_i}^i) = \emptyset \\
\hline
\Delta, E, E^L \vdash \langle | \overline{id_i} = pat_i \, \overline{l_i}^i ; ? | \rangle : p \, t_args \triangleright \uplus \overline{E_i^L}^i \quad \text{CHECK_PAT_AUX_RECORD} \\
\\
\Delta, E, E^L \vdash pat_1 : t \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t \triangleright E_n^L \\
\mathbf{disjoint \, doms}(E_1^L, \dots, E_n^L) \\
\mathbf{length}(pat_1 \dots pat_n) = num \\
\hline
\Delta, E, E^L \vdash [pat_1; \dots; pat_n] : \mathbf{--vector} \, num \, t \triangleright E_1^L \uplus \dots \uplus E_n^L \quad \text{CHECK_PAT_AUX_VECTOR} \\
\\
\Delta, E, E^L \vdash pat_1 : \mathbf{--vector} \, ne_1 \, t \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : \mathbf{--vector} \, ne_n \, t \triangleright E_n^L \\
\mathbf{disjoint \, doms}(E_1^L, \dots, E_n^L) \\
ne' = ne_1 + \dots + ne_n \\
\hline
\Delta, E, E^L \vdash [pat_1 \dots pat_n] : \mathbf{--vector} \, ne' \, t \triangleright E_1^L \uplus \dots \uplus E_n^L \quad \text{CHECK_PAT_AUX_VECTOR} \\
\\
\Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \\
\mathbf{disjoint \, doms}(E_1^L, \dots, E_n^L) \\
\hline
\Delta, E, E^L \vdash (pat_1, \dots, pat_n) : t_1 * \dots * t_n \triangleright E_1^L \uplus \dots \uplus E_n^L \quad \text{CHECK_PAT_AUX_TUP} \\
\\
\Delta \vdash t \mathbf{ok} \\
\Delta, E, E^L \vdash pat_1 : t \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t \triangleright E_n^L \\
\mathbf{disjoint \, doms}(E_1^L, \dots, E_n^L) \\
\hline
\Delta, E, E^L \vdash [pat_1; \dots; pat_n; ?] : \mathbf{--list} \, t \triangleright E_1^L \uplus \dots \uplus E_n^L \quad \text{CHECK_PAT_AUX_LIST} \\
\\
\frac{\Delta, E, E_1^L \vdash pat : t \triangleright E_2^L}{\Delta, E, E_1^L \vdash (pat) : t \triangleright E_2^L} \quad \text{CHECK_PAT_AUX_PAREN} \\
\\
\frac{\Delta, E, E_1^L \vdash pat_1 : t \triangleright E_2^L \quad \Delta, E, E_1^L \vdash pat_2 : \mathbf{--list} \, t \triangleright E_3^L}{\mathbf{disjoint \, doms}(E_2^L, E_3^L)} \quad \text{CHECK_PAT_AUX_CONS} \\
\hline
\Delta, E, E_1^L \vdash pat_1 :: pat_2 : \mathbf{--list} \, t \triangleright E_2^L \uplus E_3^L \\
\\
\frac{\vdash lit : t}{\Delta, E, E^L \vdash lit : t \triangleright \{ \}} \quad \text{CHECK_PAT_AUX_LIT} \\
\\
\frac{E, E^L \vdash x \mathbf{not \, ctor}}{\Delta, E, E^L \vdash x \, l + num : \mathbf{--num} \triangleright \{ x \mapsto \mathbf{--num} \}} \quad \text{CHECK_PAT_AUX_NUM_ADD}
\end{array}$$

$E \vdash id \mathbf{field}$ Check that the identifier is a permissible field identifier

$$\begin{array}{c}
\frac{E^F(x) \triangleright f_desc}{\langle E^M, E^P, E^F, E^X \rangle \vdash x \, l_1 \, l_2 \mathbf{field}} \quad \text{ID_FIELD_EMPTY} \\
\\
\frac{E^M(x) \triangleright E \quad x \notin \mathbf{dom}(E^F) \quad E \vdash \overline{y_i^L}^i \, z^l \, l_2 \mathbf{field}}{\langle E^M, E^P, E^F, E^X \rangle \vdash x \, l_1 . \overline{y_i^L}^i \, z^l \, l_2 \mathbf{field}} \quad \text{ID_FIELD_CONS}
\end{array}$$

$E \vdash id \mathbf{value}$ Check that the identifier is a permissible value identifier

$$\frac{E^X(x) \triangleright v_desc}{\langle E^M, E^P, E^F, E^X \rangle \vdash x \, l_1 \, l_2 \mathbf{value}} \quad \text{ID_VALUE_EMPTY}$$

$$\begin{array}{c}
\frac{
\begin{array}{l}
E^M(x) \triangleright E \\
x \notin \mathbf{dom}(E^X) \\
E \vdash \overline{y_i^l}^i z^l l_2 \mathbf{value}
\end{array}
}{
\langle E^M, E^P, E^F, E^X \rangle \vdash x l_1. \overline{y_i^l}^i z^l l_2 \mathbf{value}
} \text{ID_VALUE_CONS}
\\
\boxed{\Delta, E, E^L \vdash \mathit{exp} : t \triangleright \Sigma^C, \Sigma^N} \quad \text{Typing expressions, collecting typeclass and index constraints}
\\
\frac{
\Delta, E, E^L \vdash \mathit{exp_aux} : t \triangleright \Sigma^C, \Sigma^N
}{
\Delta, E, E^L \vdash \mathit{exp_aux} l : t \triangleright \Sigma^C, \Sigma^N
} \text{CHECK_EXP_ALL}
\\
\boxed{\Delta, E, E^L \vdash \mathit{exp_aux} : t \triangleright \Sigma^C, \Sigma^N} \quad \text{Typing expressions, collecting typeclass and index constraints}
\\
\frac{
E^L(x) \triangleright t
}{
\Delta, E, E^L \vdash x l_1 l_2 : t \triangleright \{\}, \{\}
} \text{CHECK_EXP_AUX_VAR}
\\
\frac{
}{
\Delta, E, E^L \vdash N : \mathit{num} \triangleright \{\}, \{\}
} \text{CHECK_EXP_AUX_NVAR}
\\
\begin{array}{l}
E^L \vdash \mathit{id} \mathbf{not shadowed} \\
E \vdash \mathit{id} \mathbf{value} \\
\Delta, E \vdash \mathbf{ctor} \mathit{id} : t_multi \rightarrow p t_args \triangleright (x \mathbf{of} names)
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathit{id} : \mathbf{curry} (t_multi, p t_args) \triangleright \{\}, \{\}
} \text{CHECK_EXP_AUX_CTOR}
\\
\begin{array}{l}
E^L \vdash \mathit{id} \mathbf{not shadowed} \\
E \vdash \mathit{id} \mathbf{value} \\
\Delta, E \vdash \mathbf{val} \mathit{id} : t \triangleright \Sigma^C
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathit{id} : t \triangleright \Sigma^C, \{\}
} \text{CHECK_EXP_AUX_VAL}
\\
\begin{array}{l}
\Delta, E, E^L \vdash \mathit{pat}_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash \mathit{pat}_n : t_n \triangleright E_n^L \\
\Delta, E, E^L \uplus E_1^L \uplus \dots \uplus E_n^L \vdash \mathit{exp} : u \triangleright \Sigma^C, \Sigma^N \\
\mathbf{disjoint doms}(E_1^L, \dots, E_n^L)
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathbf{fun} \mathit{pat}_1 \dots \mathit{pat}_n \rightarrow \mathit{exp} l : \mathbf{curry} ((t_1 * \dots * t_n), u) \triangleright \Sigma^C, \Sigma^N
} \text{CHECK_EXP_AUX_FN}
\\
\frac{
\Delta, E, E^L \vdash \mathit{pat}_i : t \triangleright E_i^L
}{
\Delta, E, E^L \uplus E_i^L \vdash \mathit{exp}_i : u \triangleright \Sigma^C_i, \Sigma^N_i
}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathbf{function} [^? \overline{\mathit{pat}_i \rightarrow \mathit{exp}_i l_i}^i \mathbf{end} : t \rightarrow u \triangleright \overline{\Sigma^C_i}^i, \overline{\Sigma^N_i}^i
} \text{CHECK_EXP_AUX_FUNCTION}
\\
\begin{array}{l}
\Delta, E, E^L \vdash \mathit{exp}_1 : t_1 \rightarrow t_2 \triangleright \Sigma^C_1, \Sigma^N_1 \\
\Delta, E, E^L \vdash \mathit{exp}_2 : t_1 \triangleright \Sigma^C_2, \Sigma^N_2
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathit{exp}_1 \mathit{exp}_2 : t_2 \triangleright \Sigma^C_1 \cup \Sigma^C_2, \Sigma^N_1 \cup \Sigma^N_2
} \text{CHECK_EXP_AUX_APP}
\\
\begin{array}{l}
\Delta, E, E^L \vdash (ix) : t_1 \rightarrow t_2 \rightarrow t_3 \triangleright \Sigma^C_1, \Sigma^N_1 \\
\Delta, E, E^L \vdash \mathit{exp}_1 : t_1 \triangleright \Sigma^C_2, \Sigma^N_2 \\
\Delta, E, E^L \vdash \mathit{exp}_2 : t_2 \triangleright \Sigma^C_3, \Sigma^N_3
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathit{exp}_1 ix l \mathit{exp}_2 : t_3 \triangleright \Sigma^C_1 \cup \Sigma^C_2 \cup \Sigma^C_3, \Sigma^N_1 \cup \Sigma^N_2 \cup \Sigma^N_3
} \text{CHECK_EXP_AUX_INFIX_APP1}
\\
\begin{array}{l}
\Delta, E, E^L \vdash x : t_1 \rightarrow t_2 \rightarrow t_3 \triangleright \Sigma^C_1, \Sigma^N_1 \\
\Delta, E, E^L \vdash \mathit{exp}_1 : t_1 \triangleright \Sigma^C_2, \Sigma^N_2 \\
\Delta, E, E^L \vdash \mathit{exp}_2 : t_2 \triangleright \Sigma^C_3, \Sigma^N_3
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \mathit{exp}_1 'x' l \mathit{exp}_2 : t_3 \triangleright \Sigma^C_1 \cup \Sigma^C_2 \cup \Sigma^C_3, \Sigma^N_1 \cup \Sigma^N_2 \cup \Sigma^N_3
} \text{CHECK_EXP_AUX_INFIX_APP2}
\\
\begin{array}{l}
\Delta, E \vdash \mathbf{field} \mathit{id}_i : p t_args \rightarrow t_i \triangleright (x_i \mathbf{of} names)^i \\
\Delta, E, E^L \vdash \mathit{exp}_i : t_i \triangleright \Sigma^C_i, \Sigma^N_i
\end{array}
\\
\begin{array}{l}
\mathbf{duplicates}(\overline{x_i}^i) = \emptyset \\
names = \{\overline{x_i}^i\}
\end{array}
\\
\frac{
}{
\Delta, E, E^L \vdash \langle \overline{\mathit{id}_i = \mathit{exp}_i l_i}^i, ? l \rangle : p t_args \triangleright \overline{\Sigma^C_i}^i, \overline{\Sigma^N_i}^i
} \text{CHECK_EXP_AUX_RECORD}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta, E \vdash \mathbf{field} \, id_i : p \, t_args \rightarrow t_i \triangleright (x_i \mathbf{of} \, names)^i}{\Delta, E, E^L \vdash exp_i : t_i \triangleright \Sigma^C_i, \Sigma^N_i} \\
\frac{\mathbf{duplicates}(\overline{x_i}^i) = \emptyset}{\Delta, E, E^L \vdash exp : p \, t_args \triangleright \Sigma^{C'}, \Sigma^{N'}} \\
\hline
\Delta, E, E^L \vdash \langle |exp \mathbf{with} \overline{id_i = exp_i \, l_i^i; ? \, l}| \rangle : p \, t_args \triangleright \Sigma^{C'} \cup \overline{\Sigma^C_i}^i, \Sigma^{N'} \cup \overline{\Sigma^N_i}^i \quad \text{CHECK_EXP_AUX_RECUP} \\
\Delta, E, E^L \vdash exp_1 : t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \dots \quad \Delta, E, E^L \vdash exp_n : t \triangleright \Sigma^C_n, \Sigma^N_n \\
\mathbf{length}(exp_1 \dots exp_n) = num \\
\hline
\Delta, E, E^L \vdash [|exp_1; \dots; exp_n|] : \mathbf{vector} \, num \, t \triangleright \Sigma^C_1 \cup \dots \cup \Sigma^C_n, \Sigma^N_1 \cup \dots \cup \Sigma^N_n \quad \text{CHECK_EXP_AUX_VECTOR} \\
\Delta, E, E^L \vdash exp : \mathbf{vector} \, ne' \, t \triangleright \Sigma^C, \Sigma^N \\
\vdash Nexp \rightsquigarrow ne \\
\hline
\Delta, E, E^L \vdash exp.(Nexp) : t \triangleright \Sigma^C, \Sigma^N \cup \{ne\langle ne' \rangle\} \quad \text{CHECK_EXP_AUX_VECTORGET} \\
\Delta, E, E^L \vdash exp : \mathbf{vector} \, ne' \, t \triangleright \Sigma^C, \Sigma^N \\
\vdash Nexp_1 \rightsquigarrow ne_1 \\
\vdash Nexp_2 \rightsquigarrow ne_2 \\
ne = ne_2 + (-ne_1) \\
\hline
\Delta, E, E^L \vdash exp.(Nexp_1..Nexp_2) : \mathbf{vector} \, ne \, t \triangleright \Sigma^C, \Sigma^N \cup \{ne_1\langle ne_2\langle ne' \rangle\} \quad \text{CHECK_EXP_AUX_VECTORSUB} \\
E \vdash id \, \mathbf{field} \\
\Delta, E \vdash \mathbf{field} \, id : p \, t_args \rightarrow t \triangleright (x \mathbf{of} \, names) \\
\Delta, E, E^L \vdash exp : p \, t_args \triangleright \Sigma^C, \Sigma^N \\
\hline
\Delta, E, E^L \vdash exp.id : t \triangleright \Sigma^C, \Sigma^N \quad \text{CHECK_EXP_AUX_FIELD} \\
\Delta, E, E^L \vdash pat_i : t \triangleright E_i^L{}^i \\
\Delta, E, E^L \uplus E_i^L \vdash exp_i : u \triangleright \Sigma^C_i, \Sigma^N_i{}^i \\
\Delta, E, E^L \vdash exp : t \triangleright \Sigma^{C'}, \Sigma^{N'} \\
\hline
\Delta, E, E^L \vdash \mathbf{match} \, exp \, \mathbf{with} \, |? \, pat_i \rightarrow exp_i \, \overline{l_i}^i \, \mathbf{end} : u \triangleright \Sigma^{C'} \cup \overline{\Sigma^C_i}^i, \Sigma^{N'} \cup \overline{\Sigma^N_i}^i \quad \text{CHECK_EXP_AUX_CASE} \\
\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N \\
\Delta, E \vdash typ \rightsquigarrow t \\
\hline
\Delta, E, E^L \vdash (exp : typ) : t \triangleright \Sigma^C, \Sigma^N \quad \text{CHECK_EXP_AUX_TYPED} \\
\Delta, E, E_1^L \vdash letbind \triangleright E_2^L, \Sigma^C_1, \Sigma^N_1 \\
\Delta, E, E_1^L \uplus E_2^L \vdash exp : t \triangleright \Sigma^C_2, \Sigma^N_2 \\
\hline
\Delta, E, E^L \vdash \mathbf{let} \, letbind \, \mathbf{in} \, exp : t \triangleright \Sigma^C_1 \cup \Sigma^C_2, \Sigma^N_1 \cup \Sigma^N_2 \quad \text{CHECK_EXP_AUX_LET} \\
\Delta, E, E^L \vdash exp_1 : t_1 \triangleright \Sigma^C_1, \Sigma^N_1 \quad \dots \quad \Delta, E, E^L \vdash exp_n : t_n \triangleright \Sigma^C_n, \Sigma^N_n \\
\hline
\Delta, E, E^L \vdash (exp_1, \dots, exp_n) : t_1 * \dots * t_n \triangleright \Sigma^C_1 \cup \dots \cup \Sigma^C_n, \Sigma^N_1 \cup \dots \cup \Sigma^N_n \quad \text{CHECK_EXP_AUX_TUP} \\
\Delta \vdash t \, \mathbf{ok} \\
\Delta, E, E^L \vdash exp_1 : t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \dots \quad \Delta, E, E^L \vdash exp_n : t \triangleright \Sigma^C_n, \Sigma^N_n \\
\hline
\Delta, E, E^L \vdash [exp_1; \dots; exp_n; ?] : \mathbf{list} \, t \triangleright \Sigma^C_1 \cup \dots \cup \Sigma^C_n, \Sigma^N_1 \cup \dots \cup \Sigma^N_n \quad \text{CHECK_EXP_AUX_LIST} \\
\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N \\
\hline
\Delta, E, E^L \vdash (exp) : t \triangleright \Sigma^C, \Sigma^N \quad \text{CHECK_EXP_AUX_PAREN} \\
\Delta, E, E^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N \\
\hline
\Delta, E, E^L \vdash \mathbf{begin} \, exp \, \mathbf{end} : t \triangleright \Sigma^C, \Sigma^N \quad \text{CHECK_EXP_AUX_BEGIN} \\
\Delta, E, E^L \vdash exp_1 : \mathbf{bool} \triangleright \Sigma^C_1, \Sigma^N_1 \\
\Delta, E, E^L \vdash exp_2 : t \triangleright \Sigma^C_2, \Sigma^N_2 \\
\Delta, E, E^L \vdash exp_3 : t \triangleright \Sigma^C_3, \Sigma^N_3 \\
\hline
\Delta, E, E^L \vdash \mathbf{if} \, exp_1 \, \mathbf{then} \, exp_2 \, \mathbf{else} \, exp_3 : t \triangleright \Sigma^C_1 \cup \Sigma^C_2 \cup \Sigma^C_3, \Sigma^N_1 \cup \Sigma^N_2 \cup \Sigma^N_3 \quad \text{CHECK_EXP_AUX_IF}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta, E, E^L \vdash \text{exp}_1 : t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E^L \vdash \text{exp}_2 : _ \text{list } t \triangleright \Sigma^C_2, \Sigma^N_2}{\Delta, E, E^L \vdash \text{exp}_1 :: \text{exp}_2 : _ \text{list } t \triangleright \Sigma^C_1 \cup \Sigma^C_2, \Sigma^N_1 \cup \Sigma^N_2} \text{CHECK_EXP_AUX_CONS} \\
\\
\frac{\vdash \text{lit} : t}{\Delta, E, E^L \vdash \text{lit} : t \triangleright \{\}, \{\}} \text{CHECK_EXP_AUX_LIT} \\
\\
\frac{\overline{\Delta \vdash t_i \text{ok}}^i \quad \Delta, E, E^L \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash \text{exp}_1 : t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E^L \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash \text{exp}_2 : _ \text{bool} \triangleright \Sigma^C_2, \Sigma^N_2 \quad \text{disjoint doms}(E^L, \{ \overline{x_i \mapsto t_i}^i \}) \quad E = \langle E^M, E^P, E^F, E^X \rangle}{\overline{x_i \notin \text{dom}(E^X)}^i} \text{CHECK_EXP_AUX_SET_COMP} \\
\\
\frac{\Delta, E, E^L_1 \vdash \overline{qbind_i}^i \triangleright E^L_2, \Sigma^C_1 \quad \Delta, E, E^L_1 \uplus E^L_2 \vdash \text{exp}_1 : t \triangleright \Sigma^C_2, \Sigma^N_2 \quad \Delta, E, E^L_1 \uplus E^L_2 \vdash \text{exp}_2 : _ \text{bool} \triangleright \Sigma^C_3, \Sigma^N_3}{\Delta, E, E^L_1 \vdash \{ \text{exp}_1 | \text{forall } \overline{qbind_i}^i | \text{exp}_2 \} : _ \text{set } t \triangleright \Sigma^C_1 \cup \Sigma^C_2 \cup \Sigma^C_3, \Sigma^N_2 \cup \Sigma^N_3} \text{CHECK_EXP_AUX_SET_COMP} \\
\\
\frac{\Delta \vdash t \text{ok} \quad \Delta, E, E^L \vdash \text{exp}_1 : t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \dots \quad \Delta, E, E^L \vdash \text{exp}_n : t \triangleright \Sigma^C_n, \Sigma^N_n}{\Delta, E, E^L \vdash \{ \text{exp}_1; \dots; \text{exp}_n; ? \} : _ \text{set } t \triangleright \Sigma^C_1 \cup \dots \cup \Sigma^C_n, \Sigma^N_1 \cup \dots \cup \Sigma^N_n} \text{CHECK_EXP_AUX_SET} \\
\\
\frac{\Delta, E, E^L_1 \vdash \overline{qbind_i}^i \triangleright E^L_2, \Sigma^C_1 \quad \Delta, E, E^L_1 \uplus E^L_2 \vdash \text{exp} : _ \text{bool} \triangleright \Sigma^C_2, \Sigma^N_2}{\Delta, E, E^L_1 \vdash q \overline{qbind_i}^i . \text{exp} : _ \text{bool} \triangleright \Sigma^C_1 \cup \Sigma^C_2, \Sigma^N_2} \text{CHECK_EXP_AUX_QUANT} \\
\\
\frac{\Delta, E, E^L_1 \vdash \text{list } \overline{qbind_i}^i \triangleright E^L_2, \Sigma^C_1 \quad \Delta, E, E^L_1 \uplus E^L_2 \vdash \text{exp}_1 : t \triangleright \Sigma^C_2, \Sigma^N_2 \quad \Delta, E, E^L_1 \uplus E^L_2 \vdash \text{exp}_2 : _ \text{bool} \triangleright \Sigma^C_3, \Sigma^N_3}{\Delta, E, E^L_1 \vdash [\text{exp}_1 | \text{forall } \overline{qbind_i}^i | \text{exp}_2] : _ \text{list } t \triangleright \Sigma^C_1 \cup \Sigma^C_2 \cup \Sigma^C_3, \Sigma^N_2 \cup \Sigma^N_3} \text{CHECK_EXP_AUX_LIST_COMP} \\
\\
\boxed{\Delta, E, E^L_1 \vdash qbind_1 \dots qbind_n \triangleright E^L_2, \Sigma^C} \quad \text{Build the environment for quantifier bindings, collecting typeclass constraints} \\
\\
\frac{}{\overline{\Delta, E, E^L \vdash \triangleright \{\}, \{\}}} \text{CHECK_LISTQUANT_BINDING_EMPTY} \\
\\
\frac{\Delta \vdash t \text{ok} \quad \Delta, E, E^L_1 \uplus \{ x \mapsto t \} \vdash \overline{qbind_i}^i \triangleright E^L_2, \Sigma^C_1 \quad \text{disjoint doms}(\{ x \mapsto t \}, E^L_2)}{\Delta, E, E^L_1 \vdash x \text{ l } \overline{qbind_i}^i \triangleright \{ x \mapsto t \} \uplus E^L_2, \Sigma^C_1} \text{CHECK_LISTQUANT_BINDING_VAR} \\
\\
\frac{\Delta, E, E^L_1 \vdash \text{pat} : t \triangleright E^L_3 \quad \Delta, E, E^L_1 \vdash \text{exp} : _ \text{set } t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E^L_1 \uplus E^L_3 \vdash \overline{qbind_i}^i \triangleright E^L_2, \Sigma^C_2 \quad \text{disjoint doms}(E^L_3, E^L_2)}{\Delta, E, E^L_1 \vdash (\text{pat IN exp}) \overline{qbind_i}^i \triangleright E^L_2 \uplus E^L_3, \Sigma^C_1 \cup \Sigma^C_2} \text{CHECK_LISTQUANT_BINDING_RESTR} \\
\\
\frac{\Delta, E, E^L_1 \vdash \text{pat} : t \triangleright E^L_3 \quad \Delta, E, E^L_1 \vdash \text{exp} : _ \text{list } t \triangleright \Sigma^C_1, \Sigma^N_1 \quad \Delta, E, E^L_1 \uplus E^L_3 \vdash \overline{qbind_i}^i \triangleright E^L_2, \Sigma^C_2 \quad \text{disjoint doms}(E^L_3, E^L_2)}{\Delta, E, E^L_1 \vdash (\text{pat MEM exp}) \overline{qbind_i}^i \triangleright E^L_2 \uplus E^L_3, \Sigma^C_1 \cup \Sigma^C_2} \text{CHECK_LISTQUANT_BINDING_LIST_RESTR}
\end{array}$$

$\Delta, E, E_1^L \vdash \mathbf{list} \ qbind_1 .. qbind_n \triangleright E_2^L, \Sigma^C$	Build the environment for quantifier bindings, collecting typeclass
$\frac{\Delta, E, E^L \vdash \mathbf{list} \triangleright \{\}, \{\}}{\text{CHECK_QUANT_BINDING_EMPTY}}$	
$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash pat : t \triangleright E_3^L \\ \Delta, E, E_1^L \vdash exp : \mathbf{list} \ t \triangleright \Sigma^C_1, \Sigma^N_1 \\ \Delta, E, E_1^L \uplus E_3^L \vdash \overline{qbind_i}^i \triangleright E_2^L, \Sigma^C_2 \\ \mathbf{disjoint\ doms} (E_3^L, E_2^L) \end{array}}{\Delta, E, E_1^L \vdash \mathbf{list} (pat \mathbf{MEM} exp) \overline{qbind_i}^i \triangleright E_2^L \uplus E_3^L, \Sigma^C_1 \cup \Sigma^C_2}$	CHECK_QUANT_BINDING_RESTR
$\Delta, E, E^L \vdash funcl \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$	Build the environment for a function definition clause, collecting typeclass
$\frac{\begin{array}{l} \Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \\ \Delta, E, E^L \uplus E_1^L \uplus \dots \uplus E_n^L \vdash exp : u \triangleright \Sigma^C, \Sigma^N \\ \mathbf{disjoint\ doms} (E_1^L, \dots, E_n^L) \\ \Delta, E \vdash typ \rightsquigarrow u \end{array}}{\Delta, E, E^L \vdash x \ l_1 \ pat_1 \dots pat_n : typ = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} ((t_1 * \dots * t_n), u)\}, \Sigma^C, \Sigma^N}$	CHECK_FUNCL_ANNOT
$\frac{\begin{array}{l} \Delta, E, E^L \vdash pat_1 : t_1 \triangleright E_1^L \quad \dots \quad \Delta, E, E^L \vdash pat_n : t_n \triangleright E_n^L \\ \Delta, E, E^L \uplus E_1^L \uplus \dots \uplus E_n^L \vdash exp : u \triangleright \Sigma^C, \Sigma^N \\ \mathbf{disjoint\ doms} (E_1^L, \dots, E_n^L) \end{array}}{\Delta, E, E^L \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} ((t_1 * \dots * t_n), u)\}, \Sigma^C, \Sigma^N}$	CHECK_FUNCL_NOANNOT
$\Delta, E, E_1^L \vdash letbind \triangleright E_2^L, \Sigma^C, \Sigma^N$	Build the environment for a let binding, collecting typeclass and index con
$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash pat : t \triangleright E_2^L \\ \Delta, E, E_1^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N \\ \Delta, E \vdash typ \rightsquigarrow t \end{array}}{\Delta, E, E_1^L \vdash pat : typ = exp \ l \triangleright E_2^L, \Sigma^C, \Sigma^N}$	CHECK_LETBIND_VAL_ANNOT
$\frac{\begin{array}{l} \Delta, E, E_1^L \vdash pat : t \triangleright E_2^L \\ \Delta, E, E_1^L \vdash exp : t \triangleright \Sigma^C, \Sigma^N \end{array}}{\Delta, E, E_1^L \vdash pat = exp \ l \triangleright E_2^L, \Sigma^C, \Sigma^N}$	CHECK_LETBIND_VAL_NOANNOT
$\frac{\Delta, E, E_1^L \vdash funcl_aux \ l \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N}{\Delta, E, E_1^L \vdash funcl_aux \ l \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N}$	CHECK_LETBIND_FN
$\Delta, E, E^L \vdash rule \triangleright \{x \mapsto t\}, \Sigma^C, \Sigma^N$	Build the environment for an inductive relation clause, collecting typeclass
$\frac{\begin{array}{l} \overline{\Delta \vdash t_i \mathbf{ok}}^i \\ E_2^L = \{ \overline{y_i \mapsto t_i}^i \} \\ \Delta, E, E_1^L \uplus E_2^L \vdash exp' : \mathbf{bool} \triangleright \Sigma^{C'}, \Sigma^{N'} \\ \Delta, E, E_1^L \uplus E_2^L \vdash exp_1 : u_1 \triangleright \Sigma^C_1, \Sigma^N_1 \quad \dots \quad \Delta, E, E_1^L \uplus E_2^L \vdash exp_n : u_n \triangleright \Sigma^C_n, \Sigma^N_n \end{array}}{\Delta, E, E_1^L \vdash id^? \mathbf{forall} \ \overline{y_i \ l_i}^i . exp' \implies x \ l \ exp_1 .. exp_n \ l' \triangleright \{x \mapsto \mathbf{curry} ((u_1 * \dots * u_n), \mathbf{bool})\}, \Sigma^{C'} \cup \Sigma^C_1 \cup \dots \cup \Sigma^C_n}$	
$xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^P$	Extract the type constructor information
$\frac{\begin{array}{l} tnvars^l \rightsquigarrow tnvs \\ \Delta, E \vdash typ \rightsquigarrow t \\ \mathbf{duplicates} (tnvs) = \emptyset \\ \mathbf{FV} (t) \subset tnvs \\ \overline{y_i}^i . x \notin \mathbf{dom} (\Delta) \end{array}}{\overline{y_i}^i . \Delta, E \vdash \mathbf{tc} \ x \ l \ tnvars^l = typ \triangleright \{ \overline{y_i}^i . x \mapsto tnvs . t \}, \{x \mapsto \overline{y_i}^i . x\}}$	CHECK_TEXP_TC_ABBREV

$$\begin{array}{c}
\begin{array}{c}
\text{tnvars}^l \rightsquigarrow \text{tnvs} \\
\mathbf{duplicates}(\text{tnvs}) = \emptyset \\
\overline{y_i}^i x \notin \mathbf{dom}(\Delta)
\end{array} \\
\hline
\overline{y_i}^i, \Delta, E_1 \vdash \mathbf{tc} x l \text{tnvars}^l \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs} \}, \{ x \mapsto \overline{y_i}^i x \} \quad \text{CHECK_TEXP_TC_ABSTRACT}
\end{array}$$

$$\begin{array}{c}
\begin{array}{c}
\text{tnvars}^l \rightsquigarrow \text{tnvs} \\
\mathbf{duplicates}(\text{tnvs}) = \emptyset \\
\overline{y_i}^i x \notin \mathbf{dom}(\Delta)
\end{array} \\
\hline
\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} x l \text{tnvars}^l = \langle |x_1^l : \text{typ}_1; \dots; x_j^l : \text{typ}_j; ?| \rangle \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs} \}, \{ x \mapsto \overline{y_i}^i x \} \quad \text{CHECK_TEXP_TC_REC}
\end{array}$$

$$\begin{array}{c}
\begin{array}{c}
\text{tnvars}^l \rightsquigarrow \text{tnvs} \\
\mathbf{duplicates}(\text{tnvs}) = \emptyset \\
\overline{y_i}^i x \notin \mathbf{dom}(\Delta)
\end{array} \\
\hline
\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} x l \text{tnvars}^l = |? \text{ctor_def}_1| \dots | \text{ctor_def}_j \triangleright \{ \overline{y_i}^i x \mapsto \text{tnvs} \}, \{ x \mapsto \overline{y_i}^i x \} \quad \text{CHECK_TEXP_TC_VAR}
\end{array}$$

$$\boxed{xs, \Delta_1, E \vdash \mathbf{tc} td_1 .. td_i \triangleright \Delta_2, E^P} \quad \text{Extract the type constructor information}$$

$$\begin{array}{c}
\overline{xs, \Delta, E \vdash \mathbf{tc} \triangleright \{ \}, \{ \}} \quad \text{CHECK_TEXPS_TC_EMPTY} \\
\hline
\begin{array}{c}
xs, \Delta_1, E \vdash \mathbf{tc} td \triangleright \Delta_2, E_2^P \\
xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{ \}, E_2^P, \{ \}, \{ \} \rangle \vdash \mathbf{tc} \overline{td_i}^i \triangleright \Delta_3, E_3^P \\
\mathbf{dom}(E_2^P) \cap \mathbf{dom}(E_3^P) = \emptyset
\end{array} \\
\hline
xs, \Delta_1, E \vdash \mathbf{tc} td \overline{td_i}^i \triangleright \Delta_2 \uplus \Delta_3, E_2^P \uplus E_3^P \quad \text{CHECK_TEXPS_TC_ABBREV}
\end{array}$$

$$\boxed{\Delta, E \vdash \text{tnvs } p = \text{texp} \triangleright \langle E^F, E^X \rangle} \quad \text{Check a type definition, with its path already resolved}$$

$$\begin{array}{c}
\overline{\Delta, E \vdash \text{tnvs } p = \text{typ} \triangleright \langle \{ \}, \{ \} \rangle} \quad \text{CHECK_TEXP_ABBREV} \\
\hline
\begin{array}{c}
\overline{\Delta, E \vdash \text{typ}_i \rightsquigarrow t_i}^i \\
\text{names} = \{ \overline{x_i}^i \} \\
\mathbf{duplicates}(\overline{x_i}^i) = \emptyset \\
\mathbf{FV}(t_i) \subset \text{tnvs}^i \\
E^F = \{ x_i \mapsto \langle \mathbf{forall} \text{tnvs}. p \rightarrow t_i, (x_i \mathbf{of} \text{names}) \rangle \}^i
\end{array} \\
\hline
\Delta, E \vdash \text{tnvs } p = \langle |x_i^l : \text{typ}_i^l; ?| \rangle \triangleright \langle E^F, \{ \} \rangle \quad \text{CHECK_TEXP_REC}
\end{array}$$

$$\begin{array}{c}
\overline{\Delta, E \vdash \text{typs}_i \rightsquigarrow t_multi_i}^i \\
\text{names} = \{ \overline{x_i}^i \} \\
\mathbf{duplicates}(\overline{x_i}^i) = \emptyset \\
\mathbf{FV}(t_multi_i) \subset \text{tnvs}^i \\
E^X = \{ x_i \mapsto \langle \mathbf{forall} \text{tnvs}. t_multi_i \rightarrow p, (x_i \mathbf{of} \text{names}) \rangle \}^i
\end{array}$$

$$\overline{\Delta, E \vdash \text{tnvs } p = |? \overline{x_i^l \mathbf{of} \text{typs}_i}^i \triangleright \langle \{ \}, E^X \rangle} \quad \text{CHECK_TEXP_VAR}$$

$$\boxed{xs, \Delta, E \vdash td_1 .. td_n \triangleright \langle E^F, E^X \rangle}$$

$$\begin{array}{c}
\overline{\overline{y_i}^i, \Delta, E \vdash \triangleright \langle \{ \}, \{ \} \rangle} \quad \text{CHECK_TEXPS_EMPTY} \\
\hline
\begin{array}{c}
\text{tnvars}^l \rightsquigarrow \text{tnvs} \\
\Delta, E_1 \vdash \text{tnvs } \overline{y_i}^i x = \text{texp} \triangleright \langle E_1^F, E_1^X \rangle \\
\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \triangleright \langle E_2^F, E_2^X \rangle \\
\mathbf{dom}(E_1^X) \cap \mathbf{dom}(E_2^X) = \emptyset \\
\mathbf{dom}(E_1^F) \cap \mathbf{dom}(E_2^F) = \emptyset
\end{array} \\
\hline
\overline{\overline{y_i}^i, \Delta, E \vdash x l \text{tnvars}^l = \text{texp } \overline{td_j}^j \triangleright \langle E_1^F \uplus E_2^F, E_1^X \uplus E_2^X \rangle} \quad \text{CHECK_TEXPS_CONS_CONCRETE}
\end{array}$$

$\frac{\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \triangleright \langle E^F, E^X \rangle}{\overline{y_i}^i, \Delta, E \vdash x \text{ ltnvars}^l \overline{td_j}^j \triangleright \langle E^F, E^X \rangle}$	CHECK_TEXPS_CONS_ABSTRACT
$\boxed{\delta, E \vdash id \rightsquigarrow p}$	Lookup a type class
$\frac{E(id) \triangleright p \quad \delta(p) \triangleright xs}{\delta, E \vdash id \rightsquigarrow p}$	CONVERT_CLASS_ALL
$\boxed{I \vdash (p \ t) \text{ IN } \mathcal{C}}$	Solve class constraint
$\overline{I \vdash (p \ \alpha) \text{ IN } (p_1 \text{ tnv}_1) .. (p_i \text{ tnv}_i)(p \ \alpha)(p'_1 \text{ tnv}'_1) .. (p'_j \text{ tnv}'_j)}$	SOLVE_CLASS_CONSTRAINT_IMMEDIATE
$\frac{(p_1 \text{ tnv}_1) .. (p_n \text{ tnv}_n) \Rightarrow (p \ t) \text{ IN } I \quad I \vdash (p_1 \ \sigma(\text{tnv}_1)) \text{ IN } \mathcal{C} \quad .. \quad I \vdash (p_n \ \sigma(\text{tnv}_n)) \text{ IN } \mathcal{C}}{I \vdash (p \ \sigma(t)) \text{ IN } \mathcal{C}}$	SOLVE_CLASS_CONSTRAINT_CHAIN
$\boxed{I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}}$	Solve class constraints
$\frac{I \vdash (p_1 \ t_1) \text{ IN } \mathcal{C} \quad .. \quad I \vdash (p_n \ t_n) \text{ IN } \mathcal{C}}{I \vdash \{(p_1 \ t_1), .., (p_n \ t_n)\} \triangleright \mathcal{C}}$	SOLVE_CLASS_CONSTRAINTS_ALL
$\boxed{\Delta, I, E \vdash \text{val_def} \triangleright E^X}$	Check a value definition
$\frac{\Delta, E, \{\} \vdash \text{letbind} \triangleright \{\overline{x_i \mapsto t_i}^i\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \quad I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C} \quad \overline{\mathbf{FV}(t_i) \subset \text{tnvs}}^i \quad \mathbf{FV}(\mathcal{C}) \subset \text{tnvs}}{\Delta, I, E \vdash \text{let } \tau^? \text{ letbind} \triangleright \{\overline{x_i \mapsto \langle \text{forall tnv} \cdot \mathcal{C} \Rightarrow t_i, \text{let} \rangle}^i\}}$	CHECK_VAL_DEF_VAL
$\frac{\Delta, E, E^L \vdash \text{funcl}_i \triangleright \{\overline{x_i \mapsto t_i}\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i^i \quad I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C} \quad \overline{\mathbf{FV}(t_i) \subset \text{tnvs}}^i \quad \mathbf{FV}(\mathcal{C}) \subset \text{tnvs} \quad \text{compatible overlap } (\overline{x_i \mapsto t_i}^i) \quad E^L = \{\overline{x_i \mapsto t_i}^i\}}{\Delta, I, E \vdash \text{let rec } \tau^? \text{ funcl}_i^i \triangleright \{\overline{x_i \mapsto \langle \text{forall tnv} \cdot \mathcal{C} \Rightarrow t_i, \text{let} \rangle}^i\}}$	CHECK_VAL_DEF_REC_FUN
$\boxed{\Delta, (\alpha_1, .., \alpha_n) \vdash t \text{ instance}}$	Check that t be a typeclass instance
$\overline{\Delta, (\alpha) \vdash \alpha \text{ instance}}$	CHECK_T_INSTANCE_VAR
$\overline{\Delta, (\alpha_1, .., \alpha_n) \vdash \alpha_1 * .. * \alpha_n \text{ instance}}$	CHECK_T_INSTANCE_TUP
$\overline{\Delta, (\alpha_1, \alpha_2) \vdash \alpha_1 \rightarrow \alpha_n \text{ instance}}$	CHECK_T_INSTANCE_FN
$\frac{\Delta(p) \triangleright \alpha'_1 .. \alpha'_n}{\Delta, (\alpha_1, .., \alpha_n) \vdash p \ \alpha_1 .. \alpha_n \text{ instance}}$	CHECK_T_INSTANCE_TC
$\boxed{\overline{z_j}^j, D_1, E_1 \vdash \text{def} \triangleright D_2, E_2}$	Check a definition

$$\begin{array}{c}
\frac{\overline{z_j}^j, \Delta_1, E \vdash \mathbf{tc} \overline{td_i}^i \triangleright \Delta_2, E^P}{\overline{z_j}^j, \Delta_1 \uplus \Delta_2, E \uplus \langle \{\}, E^P, \{\}, \{\} \rangle \vdash \overline{td_i}^i \triangleright \langle E^F, E^X \rangle} \text{CHECK_DEF_TYPE} \\
\overline{z_j}^j, \langle \Delta_1, \delta, I \rangle, E \vdash \mathbf{type} \overline{td_i}^i l \triangleright \langle \Delta_2, \{\}, \{\}, \langle \{\}, E^P, E^F, E^X \rangle \\
\\
\frac{\Delta, I, E \vdash \mathbf{val_def} \triangleright E^X}{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{val_def} l \triangleright \epsilon, \langle \{\}, \{\}, \{\}, E^X \rangle} \text{CHECK_DEF_VAL_DEF} \\
\\
\frac{\begin{array}{l} \Delta, E_1, E^L \vdash \mathbf{rule}_i \triangleright \{x_i \mapsto t_i\}, \Sigma^C_i, \Sigma^N_i^i \\ I \vdash \overline{\Sigma^C_i}^i \triangleright \mathcal{C} \\ \mathbf{FV}(t_i) \subset \mathbf{tnvs}^i \\ \mathbf{FV}(\mathcal{C}) \subset \mathbf{tnvs} \\ \mathbf{compatible\ overlap}(\overline{x_i \mapsto t_i}^i) \\ E^L = \{x_i \mapsto t_i^i\} \\ E_2 = \langle \{\}, \{\}, \{\}, \{x_i \mapsto \langle \mathbf{forall} \mathbf{tnvs} \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle^i\} \rangle \end{array}}{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E_1 \vdash \mathbf{indreln} \tau^? \overline{\mathbf{rule}_i}^i l \triangleright \epsilon, E_2} \text{CHECK_DEF_INDRELN} \\
\\
\frac{\overline{z_j}^j x, D_1, E_1 \vdash \mathbf{defs} \triangleright D_2, E_2}{\overline{z_j}^j, D_1, E_1 \vdash \mathbf{module} x l_1 = \mathbf{struct} \mathbf{defs} \mathbf{end} l_2 \triangleright D_2, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle} \text{CHECK_DEF_MODULE} \\
\\
\frac{E_1(\mathbf{id}) \triangleright E_2}{\overline{z_j}^j, D, E_1 \vdash \mathbf{module} x l_1 = \mathbf{id} l_2 \triangleright \epsilon, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle} \text{CHECK_DEF_MODULE_RENAME} \\
\\
\frac{\begin{array}{l} \Delta, E \vdash \mathbf{typ} \rightsquigarrow t \\ \mathbf{FV}(t) \subset \overline{\alpha_i}^i \\ \mathbf{FV}(\overline{\alpha'_k}^k) \subset \overline{\alpha_i}^i \\ \overline{\delta, E \vdash \mathbf{id}_k \rightsquigarrow p_k}^k \\ E' = \langle \{\}, \{\}, \{\}, \{x \mapsto \langle \mathbf{forall} \overline{\alpha_i}^i. (\overline{p_k \alpha'_k})^k \Rightarrow t, \mathbf{val} \rangle^i \} \rangle \end{array}}{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{val} x l_1 : \mathbf{forall} \overline{\alpha_i}^i \overline{l''_i}^i. \overline{\mathbf{id}_k \alpha'_k l'_k}^k \Rightarrow \mathbf{typ} l_2 \triangleright \epsilon, E'} \text{CHECK_DEF_SPEC} \\
\\
\frac{\begin{array}{l} \Delta, E_1 \vdash \mathbf{typ}_i \rightsquigarrow t_i^i \\ \mathbf{FV}(t_i) \subset \overline{\alpha}^i \\ p = \overline{z_j}^j x \\ E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{y_i \mapsto \langle \mathbf{forall} \alpha. (p \alpha) \Rightarrow t_i, \mathbf{method} \rangle^i\} \rangle \\ \delta_2 = \{p \mapsto \overline{y_i}^i\} \\ p \notin \mathbf{dom}(\delta_1) \end{array}}{\overline{z_j}^j, \langle \Delta, \delta_1, I \rangle, E_1 \vdash \mathbf{class} (x l \alpha l'') \mathbf{val} y_i l_i : \mathbf{typ}_i l_i^i \mathbf{end} l' \triangleright \langle \{\}, \delta_2, \{\}, E_2} \text{CHECK_DEF_CLASS}
\end{array}$$

$$\begin{array}{c}
E = \langle E^M, E^P, E^F, E^X \rangle \\
\Delta, E \vdash \text{typ}' \rightsquigarrow t' \\
\Delta, (\overline{\alpha_i}^i) \vdash t' \text{ \textbf{instance}} \\
tnvs = \overline{\alpha_i}^i \\
\text{duplicates}(tnvs) = \emptyset \\
\overline{\delta, E \vdash id_k \rightsquigarrow p_k}^k \\
\mathbf{FV}(\overline{\alpha'_k}^k) \subset tnvs \\
E(id) \triangleright p \\
\delta(p) \triangleright \overline{z_j}^j \\
I_2 = \{ \Rightarrow (p_k \alpha'_k)^k \} \\
\overline{\Delta, I \cup I_2, E \vdash val_def_n \triangleright E_n^X}^n \\
\text{disjoint doms}(\overline{E_n^X}^n) \\
\overline{E^X(x_k) \triangleright \langle \text{forall } \alpha''.(p \alpha'') \Rightarrow t_k, \text{method} \rangle}^k \\
\{ x_k \mapsto \langle \text{forall } tnvs. \Rightarrow \{ \alpha'' \mapsto t' \}(t_k), \text{let} \rangle^k \} = \overline{E_n^X}^n \\
\overline{x_k}^k = \overline{z_j}^j \\
I_3 = \{ (p_k \alpha'_k) \Rightarrow (p t')^k \} \\
(p \{ \alpha_i \mapsto \alpha_i''' \}(t')) \notin I
\end{array}$$

$$\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \text{instance forall } \overline{\alpha_i l'_i}^i . id_k \alpha'_k l''_k \Rightarrow (id \text{ typ}') \overline{val_def_n l_n}^n \text{ \textbf{end} } l' \triangleright \langle \{ \}, \{ \}, I_3 \rangle, \epsilon$$

CHECK_DEF_

$\overline{z_j}^j, D_1, E_1 \vdash \text{defs} \triangleright D_2, E_2$

Check definitions, given module path, definitions and environment

$$\begin{array}{c}
\overline{\overline{z_j}^j, D, E \vdash \triangleright \epsilon, \epsilon} \quad \text{CHECK_DEFS_EMPTY} \\
\\
\overline{\overline{z_j}^j, D_1, E_1 \vdash \text{def} \triangleright D_2, E_2} \\
\overline{\overline{z_j}^j, D_1 \uplus D_2, E_1 \uplus E_2 \vdash \overline{def_i ; ; ?^i_i} \triangleright D_3, E_3} \\
\hline
\overline{\overline{z_j}^j, D_1, E_1 \vdash \text{def} ; ; ?^i_i \overline{def_i ; ; ?^i_i} \triangleright D_2 \uplus D_3, E_2 \uplus E_3} \quad \text{CHECK_DEFS_RELEVANT_DEF} \\
\\
\overline{E_1(id) \triangleright E_2} \\
\overline{\overline{z_j}^j, D_1, E_1 \uplus E_2 \vdash \overline{def_i ; ; ?^i_i} \triangleright D_3, E_3} \\
\hline
\overline{\overline{z_j}^j, D_1, E_1 \vdash \text{open } id \text{ l} ; ; ?^i_i \overline{def_i ; ; ?^i_i} \triangleright D_3, E_3} \quad \text{CHECK_DEFS_OPEN}
\end{array}$$

Definition rules: 145 good 0 bad
 Definition rule clauses: 437 good 0 bad