n, i, j, k Index variables for meta-lists

num Numeric literals

nat Internal literal numbers

hex Bit vector literal, specified by C-style hex number bin Bit vector literal, specified by C-style binary number

string String literals

regexp Regular expresions, as a string literal

x, y, z Variables ix Variables

l	::= 	Source locations
$x^l,\ y^l,\ z^l,\ name$		Location-annotated names  Remove infix status
$ix^l$	$::= \\   ix l$	Location-annotated infix names
$\alpha$	::= $  $ ' $x$	Type variables
$lpha^l$	$::= \\ \mid  \alpha \ l$	Location-annotated type variables
N	::= $ $ $ $ $ $ $ $	numeric variables
$N^l$	$::= \\ \mid  N \; l$	Location-annotated numeric variables
id	$::= \\ \mid  x_1^l \dots x_n^l . x^l \ l$	Long identifers
tnv		Union of type variables and Nexp type variables, without locar
$tnvar^l$		Union of type variables and Nexp type variables, with location
tnvs	$::= \\   tnv_1 \dots tnv_n$	Type variable lists
$tnvars^l$	$ ::= \\   tnvar_1^l tnvar_n^l $	Type variable lists
$Nexp\_aux$		Numerical expressions for specifying vector lengths and indexe

Location-annotated vector lengths

Nexp

::=

```
Nexp_aux l
                                                                 Whether a vector is bounded or fixed size
Nexp\_constraint\_aux
                             ::=
                                   Nexp = Nexp'
                                   Nexp \ge Nexp'
Nexp\_constraint
                             ::=
                                                                 Location-annotated Nexp range
                                   Nexp\_constraint\_aux\ l
                                                                 Types
typ\_aux
                             ::=
                                                                    Unspecified type
                                   \alpha^l
                                                                    Type variables
                                                                    Function types
                                   typ_1 \rightarrow typ_2
                                   typ_1*\,\ldots\,*\,typ_n
                                                                    Tuple types
                                   Nexp
                                                                    As a typ to permit applications over Nexps, other
                                                                    Type applications
                                   id\ typ_1 \dots typ_n
                                   (typ)
                                                                 Location-annotated types
typ
                             ::=
                                   typ\_aux l
lit\_aux
                                                                 Literal constants
                             ::=
                                   true
                                   false
                                   num
                                   hex
                                                                    hex and bin are constant bit vectors, entered as C
                                   bin
                                   string
                                   ()
                                   bitzero
                                                                    bitzero and bitone are constant bits, if commonly
                                   bitone
lit
                             ::=
                                   lit\_aux l
                                                                    Location-annotated literal constants
;?
                                                                 Optional semi-colons
                             ::=
                                                                 Patterns
pat_{-}aux
                             ::=
                                                                    Wildcards
                                   (pat \mathbf{as} x^l)
                                                                    Named patterns
                                   (pat:typ)
                                                                    Typed patterns
                                   id\ pat_1 \dots pat_n
                                                                    Single variable and constructor patterns
                                   \langle |fpat_1; \dots; fpat_n; ?| \rangle

[|pat_1; \dots; pat_n; ?|]
                                                                    Record patterns
                                                                    Vector patterns
                                   [|pat_1 ... pat_n|]
                                                                    Concatenated vector patterns
```

```
(pat_1, \ldots, pat_n)
[pat_1; \ldots; pat_n;^?]
                                                                                     Tuple patterns
                                                                                     List patterns
                     (pat)
                     pat_1 :: pat_2
                                                                                     Cons patterns
                     x^l + num
                                                                                     constant addition patterns
                                                                                     Literal constant patterns
                                                                                  Location-annotated patterns
pat
               ::=
                     pat_aux l
                                                                                  Field patterns
fpat
                     id = pat l
                                                                                  Optional bars
                                                                                  Expressions
exp\_aux
                     id
                                                                                     Identifiers
                     N
                                                                                     Nexp var, has type num
                                                                                     Curried functions
                     fun psexp
                     function |?| pexp_1| ... | pexp_n| end
                                                                                     Functions with pattern matching
                     exp_1 exp_2
                                                                                     Function applications
                      exp_1 ix^l exp_2
                                                                                     Infix applications
                     \langle |fexps| \rangle
                                                                                     Records
                      \langle |exp \mathbf{with} fexps| \rangle
                                                                                     Functional update for records
                      exp.id
                                                                                     Field projection for records
                     [|exp_1; ...; exp_n;^?|]
                                                                                     Vector instantiation
                      exp.(Nexp)
                                                                                     Vector access
                      exp.(Nexp_1..Nexp_2)
                                                                                     Subvector extraction
                     match exp with |||pexp_1|| ... ||pexp_n|| l end
                                                                                     Pattern matching expressions
                     (exp:typ)
                                                                                     Type-annotated expressions
                     let letbind in exp
                                                                                     Let expressions
                                                                                     Tuples
                     (exp_1, \ldots, exp_n)
                     [exp_1; ...; exp_n; ?]
                                                                                     Lists
                     (exp)
                     begin exp end
                                                                                     Alternate syntax for (exp)
                     if exp_1 then exp_2 else exp_3
                                                                                     Conditionals
                      exp_1 :: exp_2
                                                                                     Cons expressions
                     lit
                                                                                     Literal constants
                      \{exp_1|exp_2\}
                                                                                     Set comprehensions
                      \{exp_1| \mathbf{forall} \ qbind_1 .. \ qbind_n| exp_2\}
                                                                                     Set comprehensions with explicit bin
                      \{exp_1; ...; exp_n;^?\}
                                                                                     Sets
                     \textit{q qbind}_1 \dots \textit{qbind}_n.\textit{exp}
                                                                                     Logical quantifications
                     [exp_1| forall qbind_1 .. qbind_n|exp_2]
                                                                                     List comprehensions (all binders mus
                     do id pat_1 < -exp_1; .. pat_n < -exp_n; in exp end
                                                                                     Do notation for monads
```

```
Location-annotated expressions
                  ::=
exp
                        exp\_aux l
                                                                         Quantifiers
                  ::=
q
                        forall
                        exists
                                                                         Bindings for quantifiers
qbind
                        x^l
                        (pat IN exp)
                                                                           Restricted quantifications over sets
                        (pat \mathbf{MEM} \ exp)
                                                                           Restricted quantifications over lists
                                                                         Field-expressions
fexp
                        id = exp l
                                                                         Field-expression lists
fexps
                        fexp_1; \dots; fexp_n; ^? l
                                                                         Pattern matches
pexp
                        pat \rightarrow exp l
                                                                         Multi-pattern matches
psexp
                  ::=
                        pat_1 \dots pat_n \to exp \ l
tannot?
                  ::=
                                                                         Optional type annotations
                        : typ
                                                                         Function clauses
funcl_aux
                        x^l pat_1 \dots pat_n tannot^? = exp
letbind\_aux
                                                                         Let bindings
                        pat\ tannot? = exp
                                                                           Value bindings
                        funcl_aux
                                                                           Function bindings
letbind
                                                                         Location-annotated let bindings
                  ::=
                        letbind\_aux\ l
funcl
                                                                         Location-annotated function clauses
                  ::=
                        funcl_aux l
id?
                  ::=
                                                                         Optional name for inductively defined relatio
                                                                         Inductively defined relation clauses
rule\_aux
                       id^? forall x_1^l ... x_n^l .exp \Longrightarrow x^l \ exp_1 ... \ exp_i
```

```
Location-annotated inductively de-
rule
                 ::=
                        rule\_aux\ l
                                                                                             Type lists
typs
                 ::=
                        typ_1 * ... * typ_n
ctor\_def
                                                                                             Datatype definition clauses
                        x^l of typs
                                                                                        S
                                                                                                Constant constructors
                                                                                             Type definition bodies
texp
                                                                                                Type abbreviations
                         \begin{array}{l} \langle |x_1^l:typ_1;\ldots;x_n^l:typ_n;^?|\rangle \\ |^?\:ctor\_def_1|\ldots|ctor\_def_n \end{array} 
                                                                                                Record types
                                                                                                Variant types
name?
                                                                                             Optional name specification for va-
                        [name = regexp]
td
                                                                                             Type definitions
                        x^{l} tnvars^{l} name^{?} = texp
x^{l} tnvars^{l} name^{?}
                                                                                                Definitions of opaque types
c
                                                                                             Typeclass constraints
                        id\ tnvar^l
cs
                 ::=
                                                                                             Typeclass and length constraint lis
                                                                                                Must have > 0 constraints
                        Nexp\_constraint_1, \dots, Nexp\_constraint_i \Rightarrow
                                                                                                Must have > 0 constraints
                        c_1, \dots, c_i; Nexp\_constraint_1, \dots, Nexp\_constraint_n \Rightarrow
                                                                                                Must have > 0 of both form of o
                                                                                             Type and instance scheme prefixes
c\_pre
                 ::=
                        \mathbf{forall}\, tnvar_1^l \dots tnvar_n^l.cs
                                                                                                Must have > 0 type variables
                                                                                             Type schemes
typschm
                        c-pre typ
instschm
                                                                                             Instance schemes
                        c\_pre(id\ typ)
                                                                                             Backend target names
target
                        hol
                        isabelle
                        ocaml
                        coq
```

		tex html	
au	::=	$\{target_1;; target_n\}$ $\{target_1;; target_n\}$	Backend target name lists all targets except the listed ones
$ au^?$	::=   	au	Optional targets
$lemma\_typ$	::=     	assert lemma theorem	Types of Lemmata
$lemma\_decl$	::=   	$lemma\_typ \  au^? \ x^l : (exp)$ $lemma\_typ \  au^? (exp)$	Lemmata and Tests
dexp	::=	$name\_s = string \ l$ $format = string \ l$ $arguments = exp_1 \dots exp_n \ l$ $targuments = texp_1 \dots texp_n \ l$	declaration field-expressions
$declare\_arg$	::=   	$string \ \langle  dexp_1;; dexp_n;^? l  \rangle$	agruments to a declaration
component	::=	module function type constant field	components
$termination\_setting$	::=   	automatic manual	
$exhaustivity\_setting$	::=   	exhaustive inexhaustive	
$elim\_opt$	::=		

```
exp
declare\_def
                    ::=
                                                                                                                                     decla
                           declare \tau? rename component id = x^l
                           \mathbf{declare} \, \tau^? ascii\_rep component \, id = x^l
                           declare set_-flag = string
                           \mathbf{declare} \, \tau^? termination\_argumentid = termination\_setting
                           declare \tau? pattern_match exhaustivity_setting x^l threat l = [exp_1 ... exp_n] elim_opt
val\_def
                                                                                                                                     Valu
                    ::=
                           let \tau? letbind
                                                                                                                                        N
                           let rec \tau? funcl<sub>1</sub> and ... and funcl<sub>n</sub>
                                                                                                                                        Re
                           let inline \tau? letbind
                                                                                                                                        F\iota
                                                                                                                                     Valu
val\_spec
                    ::=
                           \mathbf{val} \ x^l : typschm
def_{-}aux
                    ::=
                                                                                                                                     Top-
                           type td_1 and ... and td_n
                                                                                                                                        Ţ
                                                                                                                                        Va
                           val\_def
                           lemma\_decl
                                                                                                                                        Le
                           declare\_def
                                                                                                                                        a
                           module x^l = struct defs end
                                                                                                                                        Μ
                           module x^l = id
                                                                                                                                        Μ
                           open id
                                                                                                                                        O.
                           indreln \tau? rule_1 and ... and rule_n
                                                                                                                                        In
                                                                                                                                        To
                           class (x^l tnvar^l) val x_1^l : typ_1 l_1 \dots val x_n^l : typ_n l_n end
                                                                                                                                        T
                           instance instschm\ val\_def_1\ l_1\ ...\ val\_def_n\ l_n end
                                                                                                                                        T
def
                                                                                                                                     Loca
                    ::=
                           def\_aux\ l
;;?
                                                                                                                                     Opti
                    ::=
                           ;;
defs
                                                                                                                                     Defin
                    ::=
                           def_1; ;_1^? ... def_n; ;_n^?
                                                                                                                                     Uniq
p
                    ::=
                           x_1 \dots x_n \cdot x
                           _{-}list
                           _{--}bool
                           __num
                           \_set
```

 $\_string$ 

```
_{--}unit
                     _{-}bit
                     __vector
                                                           Type variable substitutions
\sigma
                     \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}
                                                           Internal types
              ::=
t, u
                     t_1 \rightarrow t_2
                     t_1 * \dots * t_n
                     p t\_args
                     ne
                     \sigma(t)
                                                     Μ
                                                              Multiple substitutions
                                                     Μ
                                                              Single variable substitution
                     \sigma(tnv)
                     \mathbf{curry}\left(t_{-}multi,t\right)
                                                     Μ
                                                              Curried, multiple argument functions
                                                           internal numeric expressions
ne
              ::=
                     N
                     nat
                     ne_1 * ne_2
                     ne_1 + ne_2
                     (-ne)
                     \mathbf{normalize}\,(ne)
                                                     Μ
                                                     Μ
                     ne_1 + \dots + ne_n
                                                     Μ
                     bitlength(bin)
                     bitlength(hex)
                                                     Μ
                     length (pat_1 \dots pat_n)
                                                     Μ
                     length (exp_1 \dots exp_n)
                                                     Μ
t\_args
              ::=
                                                           Lists of types
                     t_1 \dots t_n
                                                     Μ
                     \sigma(t\_args)
                                                              Multiple substitutions
                                                           Lists of types
t\_multi
                     (t_1 * .. * t_n)
\sigma(t_-multi)
                                                     Μ
                                                              Multiple substitutions
              ::=
                                                           Numeric expression constraints
nec
                     ne\langle nec
                     ne = nec
                     ne <= nec
                     ne
                                                           Sets of names
names
                     \{x_1,\ldots,x_n\}
```

```
\mathcal{C}
                           ::=
                                                                                                                                             Typeclass constraint lists
                             (p_1 tnv_1) \dots (p_n tnv_n)
env\_tag
                                                                                                                                             Tags for the (non-constructor) value description
                            ::=
                                       method
                                                                                                                                                  Bound to a method
                                        val
                                                                                                                                                  Specified with val
                                        let
                                                                                                                                                  Defined with let or indreln
v\_desc
                            ::=
                                                                                                                                             Value descriptions
                                        \langle \mathbf{forall} \ tnvs.t\_multi \rightarrow p, (x \mathbf{of} \ names) \rangle
                                                                                                                                                  Constructors
                                        \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t, env\_tag \rangle
                                                                                                                                                   Values
f\_desc
                             \langle  forall tnvs.p \rightarrow t, (x  of names) \rangle
                                                                                                                                                  Fields
xs
                              x_1 \dots x_n
\Sigma^{\mathcal{C}}
                                                                                                                                             Typeclass constraints
                                        \{(p_1 t_1), \ldots, (p_n t_n)\}
\Sigma^{\mathcal{C}}_1 \cup \ldots \cup \Sigma^{\mathcal{C}}_n
                                                                                                                                   Μ
\Sigma^{\mathcal{N}}
                                                                                                                                            Nexp constraint lists

\{nec_1, \dots, nec_n\} \\
\Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n

                                                                                                                                   Μ
E
                                                                                                                                            Environments
                                      \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
E_1 \uplus E_2
                                                                                                                                   Μ
                                                                                                                                   Μ
E^{\mathbf{X}}
                            ::=
                                                                                                                                             Value environments
                                        \begin{cases} x_1 \mapsto v\_desc_1, \, ..., x_n \mapsto v\_desc_n \rbrace \\ E_1^{\scriptscriptstyle \mathrm{X}} \uplus \, ... \uplus E_n^{\scriptscriptstyle \mathrm{X}} \end{cases} 
                                                                                                                                   Μ
E^{\mathrm{F}}
                                                                                                                                            Field environments
                            ::=
                                        \begin{array}{l} \{x_1 \mapsto f \_desc_1, \, ..\,, x_n \mapsto f \_desc_n\} \\ E_1^{\scriptscriptstyle \mathrm{F}} \uplus \, ..\, \uplus E_n^{\scriptscriptstyle \mathrm{F}} \end{array} 
                                                                                                                                   Μ
E^{\mathrm{M}}
                                                                                                                                             Module environments
                                       \{x_1 \mapsto E_1, \dots, x_n \mapsto E_n\}
E^{\mathrm{P}}
                                                                                                                                            Path environments
                                       \{x_1 \mapsto p_1, \dots, x_n \mapsto p_n\} 
E_1^{\mathsf{P}} \uplus \dots \uplus E_n^{\mathsf{P}} 
                                                                                                                                   Μ
```

Lexical bindings

 $E^{\scriptscriptstyle 
m L}$ 

::=

```
formula
                                       judgement
                                        formula_1 .. formula_n
                                        E^{\mathrm{M}}(x) \triangleright E
                                                                                                                                                     Module lookup
                                        E^{P}(x) \triangleright p
                                                                                                                                                     Path lookup
                                        E^{\mathrm{F}}(x) \triangleright f_{-}desc
                                                                                                                                                     Field lookup
                                        E^{\mathbf{X}}(x) \triangleright v_{-}desc
                                                                                                                                                     Value lookup
                                        E^{\mathrm{L}}(x) \triangleright t
                                                                                                                                                     Lexical binding lookup
                                        \Delta(p) \triangleright tc\_def
                                                                                                                                                     Type constructor lookup
                                        \delta(p) \triangleright xs
                                                                                                                                                     Type constructor lookup
                                       \mathbf{dom}\left(E_{1}^{\mathrm{M}}\right)\cap\mathbf{dom}\left(E_{2}^{\mathrm{M}}\right)=\emptyset
\mathbf{dom}\left(E_{1}^{\mathrm{X}}\right)\cap\mathbf{dom}\left(E_{2}^{\mathrm{X}}\right)=\emptyset
                                       \mathbf{dom}\,(E_1^{\mathrm{F}}) \,\cap\, \mathbf{dom}\,(E_2^{\mathrm{F}}) = \emptyset
                                        \mathbf{dom}\left(E_{1}^{\mathrm{P}}\right)\cap\mathbf{dom}\left(E_{2}^{\mathrm{P}}\right)=\emptyset
                                       disjoint doms (E_1^{\scriptscriptstyle 
m L},\,\ldots,\,E_n^{\scriptscriptstyle 
m L})
                                                                                                                                                     Pairwise disjoint domains
                                       \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{X}}},\,\ldots,E_n^{\scriptscriptstyle{\mathrm{X}}})
                                                                                                                                                     Pairwise disjoint domains
                                        compatible overlap (x_1 \mapsto t_1, ..., x_n \mapsto t_n)
                                                                                                                                                     (x_i = x_j) \Longrightarrow (t_i = t_j)
                                        \mathbf{duplicates}(tnvs) = \emptyset
                                        duplicates (x_1, ..., x_n) = \emptyset
                                        x\not\in\mathbf{dom}\,(E^{\scriptscriptstyle{\mathrm{L}}})
                                        x \not\in \mathbf{dom}(E^{X})
                                        x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                        p \not\in \mathbf{dom}(\delta)
                                        p \not\in \mathbf{dom}(\Delta)
                                        \mathbf{FV}(t) \subset tnvs
                                                                                                                                                     Free type variables
                                        \mathbf{FV}(t_{-}multi) \subset tnvs
                                                                                                                                                     Free type variables
                                        \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                                                     Free type variables
                                        inst \ \mathbf{IN} \ I
                                        (p t) \not\in I
                                        E_1^{\rm L} = E_2^{\rm L}
                                       E_1^{^{\mathrm{I}}} = E_2^{^{\mathrm{X}}}
E_1^{^{\mathrm{F}}} = E_2^{^{\mathrm{F}}}
                                       E_1 = E_2
                                       \Delta_1 = \Delta_2
                                        \delta_1 = \delta_2
                                       I_1 = I_2
                                        names_1 = names_2
                                        t_1 = t_2
                                        \sigma_1 = \sigma_2
```

```
xs_1 = xs_2
                                     tnvs_1 = tnvs_2
convert\_tnvars
                                     tnvars^l \leadsto tnvs
                                     tnvar^l \leadsto tnv
look\_m
                                     E_1(x_1^l \dots x_n^l) \triangleright E_2
                                                                                                                            Name path lookup
look\_m\_id
                             ::=
                                     E_1(id) \triangleright E_2
                                                                                                                            Module identifier lookup
look\_tc
                             ::=
                                     E(id) \triangleright p
                                                                                                                            Path identifier lookup
check\_t
                             ::=
                                     \Delta \vdash t \mathbf{ok}
                                                                                                                            Well-formed types
                                     \Delta, tnv \vdash t ok
                                                                                                                            Well-formed type/Nexps n
teq
                                     \Delta \vdash t_1 = t_2
                                                                                                                            Type equality
convert\_typ
                                     \Delta, E \vdash \mathit{typ} \leadsto \mathit{t}
                                                                                                                            Convert source types to in
                                     \vdash Nexp \leadsto ne
                                                                                                                            Convert and normalize nur
convert\_typs
                                     \Delta, E \vdash typs \leadsto t\_multi
check\_lit
                                     \vdash lit:t
                              Typing literal constants
inst\_field
                             ::=
                                     \Delta, E \vdash field id : p \ t\_args \rightarrow t \triangleright (x \ \textbf{of} \ names)
                                                                                                                            Field typing (also returns
inst\_ctor
                             ::=
                                     \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
                                                                                                                            Data constructor typing (a
inst\_val
                             ::=
                                     \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
                                                                                                                            Typing top-level bindings,
not\_ctor
                                     E, E^{\text{L}} \vdash x \mathbf{not} \mathbf{ctor}
                                                                                                                            v is not bound to a data c
```

 $not\_shadowed$ 

		$E^{ ext{ iny L}} dash id \ \mathbf{not} \ \mathbf{shadowed}$	id is not lexically shadowed
$check\_pat$	::=   	$\begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \vartriangleright E_2^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash pat\_aux : t \vartriangleright E_2^{\text{\tiny L}} \end{array}$	Typing patterns, building the Typing patterns, building the
$id\_field$	::=	$E \vdash id  \mathbf{field}$	Check that the identifier is a p
$id\_value$	::=	$E \vdash id$ value	Check that the identifier is a p
$check\_exp$	::=	$\Delta, E, E^{L} \vdash exp : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$ $\Delta, E, E^{L} \vdash exp\_aux : t \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$ $\Delta, E, E^{L}_{1} \vdash qbind_{1} qbind_{n} \rhd E^{L}_{2}, \Sigma^{\mathcal{C}}$ $\Delta, E, E^{L}_{1} \vdash \mathbf{list} qbind_{1} qbind_{n} \rhd E^{L}_{2}, \Sigma^{\mathcal{C}}$ $\Delta, E, E^{L} \vdash funcl \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$ $\Delta, E, E^{L}_{1} \vdash letbind \rhd E^{L}_{2}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$	Typing expressions, collecting Typing expressions, collecting Build the environment for qua Build the environment for a fu Build the environment for a fu
$check\_rule$	::=	$\Delta, E, E^{\text{L}} \vdash rule \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$	Build the environment for an
$check\_texp\_tc$	::=	$xs, \Delta_1, E \vdash \mathbf{tc} \ td \rhd \Delta_2, E^{\mathrm{P}}$	Extract the type constructor i
$check\_texps\_tc$	::=	$xs, \Delta_1, E \vdash \mathbf{tc} \ td_1 td_i \triangleright \Delta_2, E^{\scriptscriptstyle \mathrm{P}}$	Extract the type constructor i
$check\_texp$	::=	$\Delta, E \vdash tnvs \ p = texp  \rhd  \langle E^{\scriptscriptstyle \mathrm{F}}, E^{\scriptscriptstyle \mathrm{X}} \rangle$	Check a type definition, with
$check\_texps$	::=	$\mathit{xs}, \Delta, E \vdash \mathit{td}_1 \mathinner{\ldotp\ldotp} \mathit{td}_n  \rhd  \langle E^{\scriptscriptstyle{F}}, E^{\scriptscriptstyle{X}} \rangle$	
$convert\_class$	::=	$\delta, E \vdash id \leadsto p$	Lookup a type class
$solve\_class\_constraint$	::=	$I \vdash (p\ t)  \mathbf{IN}  \mathcal{C}$	Solve class constraint
$solve\_class\_constraints$	::=	$I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}$	Solve class constraints
$check\_val\_def$	::=	$\Delta, I, E \vdash val\_def  \rhd  E^{\mathbf{X}}$	Check a value definition

```
check\_t\_instance
                          ::=
                           \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance
                                                                            Check that t be a type
class instance
check\_defs
                          ::=
                                 \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2
                                                                            Check a definition
                                 \overline{z_j}^j, D_1, E_1 \vdash defs \triangleright D_2, E_2
                                                                            Check definitions, given module path, definitions
judgement
                          ::=
                                 convert\_tnvars
                                 look\_m
                                 look\_m\_id
                                 look\_tc
                                 check\_t
                                 teq
                                 convert\_typ
                                 convert\_typs
                                 check\_lit
                                 inst\_field
                                 inst\_ctor
                                 inst\_val
                                 not\_ctor
                                 not\_shadowed
                                 check\_pat
                                 id\_field
                                 id\_value
                                 check\_exp
                                 check\_rule
                                 check\_texp\_tc
                                 check\_texps\_tc
                                 check\_texp
                                 check\_texps
                                 convert\_class
                                 solve\_class\_constraint
                                 solve\_class\_constraints
                                 check\_val\_def
                                 check\_t\_instance
                                 check\_defs
user\_syntax
                          ::=
                                 n
                                 num
                                 nat
                                 hex
                                 bin
                                 string
```

 $\begin{array}{c} regexp \\ x \end{array}$ 

```
ix
l
x^{l}
ix^l
\alpha
\alpha^l
N
N^l
id
tnv
tnvar^l
tnvs
tnvars^l
Nexp\_aux
Nexp
Nexp\_constraint\_aux
Nexp\_constraint
typ\_aux
typ
lit\_aux
lit
;?
pat\_aux
pat
fpat|?
exp\_aux
exp
q
qbind
fexp
fexps
pexp
psexp
tannot?
funcl\_aux
letbind\_aux
letbind
funcl
id?
rule\_aux
rule
typs
ctor\_def
texp
name?
```

```
td
c
cs
c\_pre
typschm
instschm
target
\tau \\ \tau^?
lemma\_typ
lemma\_decl
dexp
declare\_arg
component
termination\_setting
exhaustivity\_setting
elim\_opt
declare\_def
val\_def
val\_spec
def\_aux
def ;;?
defs
p
\sigma
ne
t\_args
t\_multi
nec
names
\mathcal{C}
env\_tag
v\_desc
f\_desc
xs
\Sigma^{\mathcal{C}}
\Sigma^{\mathcal{N}}
E
E^{\mathbf{X}}
E^{\mathrm{F}}
E^{\mathrm{M}}
E^{\mathrm{P}}
E^{\mathrm{L}}
```

 $tc\_abbrev$ 

$$\mid tc\_def$$
 $\mid \Delta$ 
 $\mid \delta$ 
 $\mid inst$ 
 $\mid I$ 
 $\mid D$ 
 $\mid terminals$ 
 $\mid formula$ 

 $tnvars^l \leadsto tnvs$ 

$$\frac{tnvar_1^l \rightsquigarrow tnv_1 \quad .. \quad tnvar_n^l \rightsquigarrow tnv_n}{tnvar_1^l \dots tnvar_n^l \rightsquigarrow tnv_n} \quad \text{Convert_tnvars_none}$$

 $tnvar^l \leadsto tnv$ 

$$\frac{\overline{\alpha \ l \leadsto \alpha}}{N \ l \leadsto N} \quad \begin{array}{ll} \text{Convert\_tnvar\_A} \\ \end{array}$$

 $E_1(x_1^l \dots x_n^l) \triangleright E_2$  Name path lookup

$$\frac{E(\ ) \rhd E \qquad \text{LOOK\_M\_NONE}}{E^{\text{M}}(x) \rhd E_{1}}$$
 
$$\frac{E_{1}(\ \overline{y_{i}^{l}}^{i}\ ) \rhd E_{2}}{\langle E^{\text{M}}, E^{\text{P}}, E^{\text{F}}, E^{\text{X}} \rangle (x \ l \ \overline{y_{i}^{l}}^{i}\ ) \rhd E_{2}} \quad \text{LOOK\_M\_SOME}$$

 $E_1(id) \triangleright E_2$  Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i x l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i x l_1 l_2) \triangleright E_2} \quad \text{LOOK\_M\_ID\_ALL}$$

E(id) > p Path identifier lookup

$$\frac{E(\overline{y_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{E^{\mathrm{P}}(x) \rhd p} \qquad \text{LOOK\_TC\_ALL}$$

$$E(\overline{y_i^l}^i x \ l_1 \ l_2) \rhd p$$

 $\Delta \vdash t \text{ ok}$  Well-formed types

$$\begin{array}{cccc} \overline{\Delta \vdash \alpha \, \mathbf{ok}} & & & \\ \overline{\Delta \vdash \alpha \, \mathbf{ok}} & & \\ \Delta \vdash t_1 \, \mathbf{ok} & & \\ \underline{\Delta \vdash t_2 \, \mathbf{ok}} & & & \\ \overline{\Delta \vdash t_1 \to t_2 \, \mathbf{ok}} & & & \\ \hline \Delta \vdash t_1 \, \mathbf{ok} & \dots & \Delta \vdash t_n \, \mathbf{ok} \\ \hline \Delta \vdash t_1 * \dots * t_n \, \mathbf{ok} & & & \\ \hline \Delta \vdash t_1 * \dots * t_n \, \mathbf{ok} & & & \\ \hline \Delta(p) \rhd tnv_1 \dots tnv_n \ tc\_abbrev & \\ \underline{\Delta, tnv_1 \vdash t_1 \, \mathbf{ok}} & \dots & \Delta, tnv_n \vdash t_n \, \mathbf{ok} \\ \hline \Delta \vdash p \ t_1 \dots t_n \, \mathbf{ok} & & & \\ \hline \end{array}$$
 CHECK\_T\_APP

 $\Delta, tnv \vdash t$  ok Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \, \mathbf{ok}}{\Delta, \alpha \vdash t \, \mathbf{ok}} \quad \text{CHECK\_TLEN\_T}$$
 
$$\frac{\Delta, N \vdash ne \, \mathbf{ok}}{\Delta, N \vdash ne \, \mathbf{ok}} \quad \text{CHECK\_TLEN\_LEN}$$

 $\Delta \vdash t_1 = t_2$  Type equality

$$\frac{\Delta \vdash t \text{ ok}}{\Delta \vdash t = t} \quad \text{TEQ\_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ\_SYM}$$

$$\frac{\Delta \vdash t_1 = t_2}{\Delta \vdash t_2 = t_3} \quad \text{TEQ\_TRANS}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_1 = t_3} \quad \text{TEQ\_ARROW}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_1 \to t_2 = t_3 \to t_4} \quad \text{TEQ\_ARROW}$$

$$\frac{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n}{\Delta \vdash t_1 * \dots * t_n = u_1 * \dots * u_n} \quad \text{TEQ\_TUP}$$

$$\frac{\Delta(p) \rhd \alpha_1 \dots \alpha_n}{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n} \quad \text{TEQ\_APP}$$

$$\frac{\Delta(p) \rhd \alpha_1 \dots \alpha_n}{\Delta \vdash p \ t_1 \dots t_n = p \ u_1 \dots u_n} \quad \text{TEQ\_APP}$$

$$\frac{\Delta(p) \rhd \alpha_1 \dots \alpha_n \dots u}{\Delta \vdash p \ t_1 \dots t_n = \{\alpha_1 \mapsto t_1 \dots \alpha_n \mapsto t_n\}(u)} \quad \text{TEQ\_EXPAND}$$

$$\frac{ne = \text{normalize} (ne')}{\Delta \vdash ne = ne'} \quad \text{TEQ\_NEXP}$$

 $\Delta, E \vdash typ \leadsto t$  Convert source types to internal types

$$\begin{array}{c} \Delta, E \vdash typ \leadsto t_1 \\ \Delta \vdash t_1 = t_2 \\ \hline \Delta, E \vdash typ \leadsto t_2 \end{array} \quad \text{CONVERT\_TYP\_EQ}$$

 $\vdash Nexp \leadsto ne$  Convert and normalize numeric expressions

 $\Delta, E \vdash typs \leadsto t\_multi$ 

$$\frac{\Delta, E \vdash typ_1 \leadsto t_1 \quad .. \quad \Delta, E \vdash typ_n \leadsto t_n}{\Delta, E \vdash typ_1 * .. * typ_n \leadsto (t_1 * .. * t_n)} \quad \text{CONVERT\_TYPS\_ALL}$$

 $\vdash lit : t$  Typing literal constants

 $\overline{\vdash \mathbf{bitzero}\ l : \_\mathbf{bit}}$  CHECK\_LIT\_BITZERO

 $\Delta, E \vdash$  **field**  $id : p \ t\_args \rightarrow t \triangleright (x \ \textbf{of} \ names)$  Field typing (also returns canonical field names)

$$E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \\ E^{\mathrm{F}}(y) \rhd \langle \text{ for all } tnv_1 \dots tnv_n.p \to t, (z \text{ of } names) \rangle \\ \Delta \vdash t_1 \text{ ok} \quad \dots \quad \Delta \vdash t_n \text{ ok} \\ \overline{\Delta, E \vdash \text{ field } \overline{x_i^l}^i} \quad y \; l_1 \; l_2 : p \; t_1 \dots t_n \to \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}(t) \rhd (z \text{ of } names)$$
 INST\_FIELD\_ALL

 $\Delta, E \vdash \mathbf{ctor}\ id : t\_multi \to p\ t\_args \rhd (x\ \mathbf{of}\ names)$  Data constructor typing (also returns canonical constructor)

```
\begin{array}{l} E(\,\overline{x_i^l}^{\,i}\,) \,\rhd\, \langle E^{\scriptscriptstyle{\mathrm{M}}}, E^{\scriptscriptstyle{\mathrm{P}}}, E^{\scriptscriptstyle{\mathrm{F}}}, E^{\scriptscriptstyle{\mathrm{X}}} \rangle \\ E^{\scriptscriptstyle{\mathrm{X}}}(y) \,\rhd\, \langle\, \mathbf{forall}\, tnv_1 \mathinner{\ldotp\ldotp} tnv_n.t\_multi \,\to\, p, (z\,\mathbf{of}\,\, names) \rangle \end{array}
                                          \Delta \vdash t_1 \mathbf{ok} \quad ... \quad \Delta \vdash t_n \mathbf{ok}
                                                                                                                                                                                                                                          — INST_CTOR_ALL
\overline{\Delta, E \vdash \mathbf{ctor} \, \overline{x_i^l}^i} \, y \, l_1 \, l_2 : \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}(t\_multi) \to p \, t_1 \dots t_n \rhd (z \, \mathbf{of} \, names)
     \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}} Typing top-level bindings, collecting typeclass constraints
                               E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle 
E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, \ env\_tag \rangle
                                \Delta \vdash t_1 \mathbf{ok} \quad \dots \quad \Delta \vdash t_n \mathbf{ok}
                              \sigma = \{ \underbrace{t_n v_1 \mapsto t_1 \dots t_n \mapsto t_n} \}
                                \frac{\sigma = \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}}{\Delta, E \vdash \mathbf{val} \, \overline{x_i^l}^{\ i} \, y \, l_1 \, l_2 : \sigma(t) \triangleright \{(p_1 \, \sigma(tnv_1')), ..., (p_i \, \sigma(tnv_1'))\}}  INST_VAL_ALL
      E, E^{\mathsf{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor} \, | \, v \, \text{is not bound to a data constructor}
                                                                                          \frac{E^{\text{L}}(x) \vartriangleright t}{E.\,E^{\text{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor}} \quad \text{Not\_ctor\_val}
                                                           \frac{x \not\in \mathbf{dom}\left(E^{\mathbf{X}}\right)}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}}\rangle, E^{\mathbf{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor}} \quad \text{NOT\_CTOR\_UNBOUND}
                       \frac{E^{\mathbf{X}}(x) \triangleright \langle \mathbf{forall} \ tnv_{1} ... \ tnv_{n}.(p_{1} \ tnv_{1}') ... (p_{i} \ tnv_{i}') \Rightarrow t, env\_tag \rangle}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle, E^{\mathbf{L}} \vdash x \ \mathbf{not} \ \mathbf{ctor}}  NOT_CTOR_BOUND
      E^{\text{L}} \vdash id \text{ not shadowed}
                                                                                id is not lexically shadowed
                                                                    \frac{x \not\in \mathbf{dom}(E^{L})}{E^{L} \vdash x \ l_{1} \ l_{2} \ \mathbf{not \ shadowed}} \quad \text{NOT\_SHADOWED\_SING}
                                                      \overline{E^{\text{L}} \vdash x_1^l \dots x_n^l. y^l. z^l \ l \ \textbf{not shadowed}} \quad \text{NOT\_SHADOWED\_MULTI}
      \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}} Typing patterns, building their binding environment
                                                                         \frac{\Delta, E, E_1^{\text{L}} \vdash pat\_aux : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash pat\_aux \ l : t \triangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_ALL}
   \Delta, E, E_1^{\mathrm{L}} \vdash pat\_aux : t \triangleright E_2^{\mathrm{L}}
                                                                                               Typing patterns, building their binding environment
                                                                             \frac{\Delta \vdash t \text{ ok}}{\Delta, E, E^{\text{L}} \vdash \_: t \vartriangleright \{\,\}} \quad \text{CHECK\_PAT\_AUX\_WILD}
                                                 \begin{array}{c} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \vartriangleright E_2^{\text{\tiny L}} \\ x \not\in \mathbf{dom}\,(E_2^{\text{\tiny L}}) \\ \overline{\Delta, E, E_1^{\text{\tiny L}} \vdash (pat \ \mathbf{as} \ x \ l) : t \vartriangleright E_2^{\text{\tiny L}} \uplus \{x \mapsto t\}} \end{array} \quad \text{CHECK\_PAT\_AUX\_AS}
                                                                \begin{split} & \Delta, E, E_1^{\text{L}} \vdash pat: t \vartriangleright E_2^{\text{L}} \\ & \frac{\Delta, E \vdash typ \leadsto t}{\Delta, E, E_1^{\text{L}} \vdash (pat: typ): t \vartriangleright E_2^{\text{L}}} & \text{CHECK\_PAT\_AUX\_TYP} \end{split}
          \Delta, E \vdash \mathbf{ctor}\ id : (t_1 * ... * t_n) \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
           E^{\text{L}} \vdash id \text{ not shadowed}
          \Delta, E, E^{\scriptscriptstyle L} \vdash pat_1: t_1 \, \rhd \, E_1^{\scriptscriptstyle L} \quad \dots \quad \Delta, E, E^{\scriptscriptstyle L} \vdash pat_n: t_n \, \rhd \, E_n^{\scriptscriptstyle L}
          disjoint doms (E_1^{\text{L}}, \dots, E_n^{\text{L}})
                    \frac{\text{cjoint doms} \ (E_1^{\text{\tiny L}}, \dots, E_n^{\text{\tiny L}})}{\Delta, E, E^{\text{\tiny L}} \vdash id \ pat_1 \dots pat_n : p \ t\_args \rhd E_1^{\text{\tiny L}} \uplus \dots \uplus E_n^{\text{\tiny L}}} \quad \text{CHECK\_PAT\_AUX\_IDENT\_CONSTR}
```

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\Delta \vdash t \mathbf{ok}
                                                           \frac{E, E^{\text{L}} \vdash x \text{ not ctor}}{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ l_2 \ : t \rhd \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_VAR}
                             \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)}^i
                             \Delta, E, E^{\mathrm{L}} \vdash pat_i : t_i \triangleright E_i^{\mathrm{L}}^i
                            \begin{array}{l} \operatorname{disjoint \, doms} \left( \, \overline{E_i^{\scriptscriptstyle \mathrm{L}}}^{\, i} \, \right) \\ \operatorname{duplicates} \left( \, \overline{x_i}^{\, i} \, \right) = \, \emptyset \end{array}
                             \frac{1}{\Delta,E,E^{\text{\tiny L}} \vdash \langle | \ \overline{id_i = pat_i \ l_i}^i \ ;^?| \rangle : p \ t\_args \ \rhd \ \uplus \ \overline{E_i^{\text{\tiny L}}}^i} \quad \text{CHECK\_PAT\_AUX\_RECORD}
                   \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_1 : t \, \rhd \, E_1^{\mathrm{L}} \quad \dots \quad \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_n : t \, \rhd \, E_n^{\mathrm{L}}
                   disjoint doms (E_1^L, \ldots, E_n^L)
                   \mathbf{length}\left(pat_1 \dots pat_n\right) = nat
          \overline{\Delta, E, E^{\text{L}} \vdash [|pat_1; \dots; pat_n;^?|]} : \_\text{vector } nat \ t \vartriangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}} \quad \text{CHECK\_PAT\_AUX\_VECTOR}
\Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_1: \_\textbf{vector} \; \textit{ne}_1 \; t \, \rhd \, E^{\text{\tiny L}}_1 \quad \dots \quad \Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_n: \_\textbf{vector} \; \textit{ne}_n \; t \, \rhd \, E^{\text{\tiny L}}_n
\mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
ne' = ne_1 + \dots + ne_n
                                                                                                                                                                                                                                                        CHECK_PAT_AUX_VECTOR
                                  \Delta, E, E^{\text{L}} \vdash [|pat_1 \dots pat_n|] : \_\text{vector } ne' \ t \triangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}
                     \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_1 : t_1 \mathrel{\vartriangleright} E_1^{\mathrm{L}} \quad .... \quad \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_n : t_n \mathrel{\vartriangleright} E_n^{\mathrm{L}}
                     \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\ldots,E_n^{\scriptscriptstyle{\mathrm{L}}})
                      \frac{\Delta, E, E^{\text{L}} \vdash (pat_1, \dots, pat_n)}{\Delta, E, E^{\text{L}} \vdash (pat_1, \dots, pat_n) : t_1 * \dots * t_n \triangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_TUP}
                          \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_1 : t \, \rhd \, E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_n : t \, \rhd \, E_n^{\mathsf{L}}
                          disjoint doms (E_1^{\scriptscriptstyle \rm L},\,..\,,E_n^{\scriptscriptstyle \rm L})
                              \Delta, \overline{E, E^{\text{\tiny L}} \vdash [pat_1; \ ..; pat_n \ ;^?] : \_\textbf{list} \ t \vartriangleright E_1^{\text{\tiny L}} \uplus \ .. \uplus E_n^{\text{\tiny L}}} \quad \text{CHECK\_PAT\_AUX\_LIST}
                                                                \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat) : t \triangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_PAREN}
                                                           \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat_1 : t \vartriangleright E_2^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash pat_2 : \_\_\textbf{list} \ t \vartriangleright E_3^{\text{L}} \end{array}
                                          \frac{\mathbf{disjoint\,doms}\,(E_2^{\mathrm{L}},E_3^{\mathrm{L}})}{\Delta,E,E_1^{\mathrm{L}}\vdash\mathit{pat}_1::\mathit{pat}_2:\_\mathbf{list}\;t\,\vartriangleright\,E_2^{\mathrm{L}}\uplus E_3^{\mathrm{L}}}
                                                                                                                                                                       CHECK_PAT_AUX_CONS
                                                                          \frac{\vdash lit:t}{\Delta,E,E^{\text{L}}\vdash lit:t\,\vartriangleright\,\{\,\}} \quad \text{CHECK\_PAT\_AUX\_LIT}
                                                                     E, E^{\text{\tiny L}} \vdash x \text{ not ctor}
                                                                                                                                                                                CHECK\_PAT\_AUX\_NUM\_ADD
                          \overline{\Delta, E, E^{\text{L}} \vdash x \ l + num : \_\text{num} \ \triangleright \{x \mapsto \_\text{num} \ \}}
  E \vdash id \mathbf{field}
                                                 Check that the identifier is a permissible field identifier
                                                                   \frac{E^{\mathrm{F}}(x) \triangleright f\_desc}{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{field}} \quad \text{ID\_FIELD\_EMPTY}
                                                                                         E^{\mathrm{M}}(x) \triangleright E
                                                                                         x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                                            \frac{E \vdash \overline{y_{i}^{l}}.^{i} z^{l} \ l_{2} \, \mathbf{field}}{\langle E^{\text{\tiny{M}}}, E^{\text{\tiny{P}}}, E^{\text{\tiny{F}}}, E^{\text{\tiny{X}}} \rangle \vdash x \ l_{1}. \, \overline{y_{i}^{l}}.^{i} z^{l} \ l_{2} \, \mathbf{field}} \quad \text{id_{FIELD\_CONS}}
  E \vdash id value
                                                    Check that the identifier is a permissible value identifier
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\frac{E^{\mathbf{X}}(x) \vartriangleright v\_desc}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{value}} \quad \text{ID\_VALUE\_EMPTY}
                                                                                                 E^{\mathrm{M}}(x) \triangleright E
                                                                                                x \not\in \mathbf{dom}(E^{X})
                                                                  \frac{E \vdash \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}}{\langle E^{\text{\tiny M}}, E^{\text{\tiny P}}, E^{\text{\tiny F}}, E^{\text{\tiny X}} \rangle \vdash x \ l_1. \ \overline{y_i^l}.^i \ z^l \ l_2 \, \mathbf{value}} \quad \text{id\_value\_cons}
       \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                          Typing expressions, collecting typeclass and index constraints
                                                                       \frac{\Delta, E, E^{\mathsf{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\mathsf{L}} \vdash exp\_aux \ l : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_ALL}
   \Delta, E, E^{\mathsf{L}} \vdash exp\_aux : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                Typing expressions, collecting typeclass and index constraints
                                                                      \frac{E^{\text{L}}(x) \triangleright t}{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ l_2 : t \triangleright \{\}, \{\}} \quad \text{CHECK\_EXP\_AUX\_VAR}
                                                                \overline{\Delta, E, E^{\perp} \vdash N : \_\mathbf{num} \triangleright \{\}, \{\}}
                                                                                                                                                                         CHECK_EXP_AUX_NVAR
                                  E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                  E \vdash id \text{ value}
                                  \Delta, E \vdash \mathbf{ctor} id : t\_multi \to p \ t\_args \rhd (x \ \mathbf{of} \ names)
                                                                                                                                                                                                               CHECK_EXP_AUX_CTOR
                                     \Delta, E, E^{\text{\tiny L}} \vdash id : \mathbf{curry} (t\_multi, p \ t\_args) \rhd \{ \}, \{ \}
                                                                                 E^{\mathsf{L}} \vdash id \text{ not shadowed}
                                                                                 E \vdash id value
                                                                            \frac{\Delta, E \vdash \mathbf{val} \, id : t \triangleright \Sigma^{\mathcal{C}}}{\Delta, E, E^{\mathsf{L}} \vdash id : t \triangleright \Sigma^{\mathcal{C}}, \{\}} \quad \text{CHECK\_EXP\_AUX\_VAL}
                              \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                              \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
                                                                                                                                                                                                                                              CHECK_EXP_AUX_FN
        \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{fun} \ pat_{1} \dots pat_{n} \rightarrow exp \ l : \mathbf{curry} \left( (t_{1} * \dots * t_{n}), u \right) \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                               \frac{\overline{\Delta}, E, E^{\text{\tiny L}} \vdash \textit{pat}_i : t \vartriangleright E_i^{\text{\tiny L}}{}^i}{\overline{\Delta}, E, E^{\text{\tiny L}} \uplus E_i^{\text{\tiny L}} \vdash \textit{exp}_i : u \vartriangleright \Sigma^{\mathcal{C}}{}_i, \Sigma^{\mathcal{N}}{}_i{}^i}
      \Delta, E, E^{\text{\tiny L}} \vdash \mathbf{function} \mid^{?} \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ \mathbf{end} : t \rightarrow u \ \triangleright \ \overline{\Sigma^{\mathcal{C}_{i}}}^{i} . \overline{\Sigma^{\mathcal{N}_{i}}}^{i}
                                                          \begin{array}{l} \Delta, E, E^{\text{\tiny L}} \vdash exp_1 : t_1 \rightarrow t_2 \rhd \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \\ \Delta, E, E^{\text{\tiny L}} \vdash exp_2 : t_1 \rhd \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2} \end{array}
                                    \frac{1}{\Delta,E,E^{\text{L}} \vdash exp_1 \; exp_2 : t_2 \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \, \cup \, \Sigma^{\mathcal{N}}_2} \quad \text{Check_exp_aux_app}
                                          \begin{array}{l} \Delta, E, E^{\mathrm{L}} \vdash (\mathit{ix}) \, : t_1 \to t_2 \to t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\mathrm{L}} \vdash \mathit{exp}_1 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                           \Delta, E, E^{\mathsf{L}} \vdash exp_2 : t_2 \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
\frac{1}{\Delta,E,E^{\text{L}} \vdash exp_1 \ ix \ l \ exp_2 : t_3 \rhd \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2 \ \cup \ \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2 \ \cup \ \Sigma^{\mathcal{N}}_3} \quad \text{Check_exp_aux_infix_app1}
                                                                                                                                                                      \begin{array}{lll} \Delta, E, E^{\text{\tiny L}} \vdash & x & : t_1 \rightarrow t_2 \rightarrow t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\text{\tiny L}} \vdash & exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                                                                                                                                                      \Delta, E, E^{\mathsf{L}} \vdash exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
 <<no parses (char 18): TD,E,E_l |- exp1 '***x' l exp2 : t3 gives S_c1 union S_c2 union S_c3,</pre>
```

```
\overline{\Delta}, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)^i
                                  \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                 \mathbf{duplicates} (\overline{x_i}^i) = \emptyset
                                  names = \{ \overline{x_i}^i \}
                                                                                                                                                                                                CHECK_EXP_AUX_RECORD
                        \Delta, E, E^{\scriptscriptstyle L} \vdash \langle | \overline{id_i = \overline{exp_i \; l_i}^i} \; ;^? \; l | \rangle : p \; t\_args \; \rhd \; \overline{\Sigma^{\mathcal{C}}_i}^i \; , \overline{\Sigma^{\mathcal{N}_i}}^i
                                             \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)}^i
                                              \overline{\Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                             \mathbf{duplicates} \, (\, \overline{x_i}^{\, i} \, ) = \, \emptyset
                                             \Delta, E, E^{L} \vdash exp : p \ t\_args \triangleright \Sigma^{C'}, \Sigma^{N'}
\Delta, E, E^{\text{L}} \vdash \langle | exp \ \overline{\textbf{with}} \ \overline{id_i = exp_i \ l_i}^i ; ^? \ l | \rangle : p \ t\_args \rhd \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_i}^i , \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}}_i}^i  CHECK_EXP_AUX_RECUP
                       \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \triangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n
                       length(exp_1 ... exp_n) = nat
\frac{\Delta, E, E^{\text{L}} \vdash [|exp_1; \dots; exp_n;^?|] : \_\textbf{vector} \ nat \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_VECTOR}}{}
                                      \Delta, E, E^{L} \vdash exp : \_vector ne' t \triangleright \Sigma^{C}, \Sigma^{N}
                                     \vdash Nexp \leadsto ne
                                                                                                                                                                                    CHECK_EXP_AUX_VECTORGET
                           \overline{\Delta, E, E^{\mathsf{L}} \vdash exp.(Nexp) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne\langle ne'\}\}}
                                                 \Delta, E, E^{L} \vdash exp : \_vector ne' t \triangleright \Sigma^{C}, \Sigma^{N}
                                                 \vdash Nexp_1 \leadsto ne_1
                                                 \vdash Nexp_2 \leadsto ne_2
                                                 ne = ne_2 + (-ne_1)
                                                                                                                                                                                                                                     CHECK_EXP_AUX_VECTORSUB
\overline{\Delta, E, E^{\text{\tiny{L}}} \vdash exp.(Nexp_1..Nexp_2) : \_\textbf{vector} \ ne \ t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne_1 \langle ne_2 \langle ne' \}
                                         E \vdash id \mathbf{field}
                                         \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                          \Delta, E, E^{\text{L}} \vdash exp : p \ t\_args \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                                             CHECK_EXP_AUX_FIELD
                                                                \Delta, E, E^{\text{L}} \vdash exp.id : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                    \overline{\Delta,E,E^{\scriptscriptstyle L} \vdash \mathit{pat}_i : t \, \rhd \, E_i^{\scriptscriptstyle L}}^{\,\,i}
                                                                   \frac{1}{\Delta, E, E^{L} \uplus E_{i}^{L} \vdash exp_{i} : u \rhd \Sigma^{C}_{i}, \Sigma^{N}_{i}}^{i}}{\Delta, E, E^{L} \vdash exp : t \rhd \Sigma^{C'}, \Sigma^{N'}}^{i}
\Delta, E, E^{\text{\tiny L}} \vdash \mathbf{match} \ \overline{exp} \ \overline{\mathbf{with}} \ |^{?} \ \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ l \ \mathbf{end} : u \vartriangleright \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}}_{i}}^{i}, \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}_{i}}}^{i} CHECK_EXP_AUX_CASE
                                                                  \Delta, E, E^{L} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                  \Delta, E \vdash typ \leadsto t
                                                        \frac{1}{\Delta, E, E^{\text{L}} \vdash (exp: typ): t \triangleright \Sigma^{\mathcal{C}}. \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_TYPED}
                                                           \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash letbind \, \rhd \, E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}{}_1, \Sigma^{\mathcal{N}}{}_1 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus \, E_2^{\text{\tiny L}} \vdash exp : t \, \rhd \, \Sigma^{\mathcal{C}}{}_2, \Sigma^{\mathcal{N}}{}_2 \end{array}
                         \frac{1}{\Delta, E, E^{\text{L}} \vdash \textbf{let} \ letbind \ \textbf{in} \ exp: t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_LET}
\frac{\Delta, E, E^{\text{\tiny L}} \vdash exp_1 : t_1 \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\text{\tiny L}} \vdash exp_n : t_n \vartriangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\text{\tiny L}} \vdash (exp_1, \dots, exp_n) : t_1 \ast \dots \ast t_n \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n}
                                                                                                                                                                                                                                                        CHECK_EXP_AUX_TUP
         \Delta \vdash t \mathbf{ok}
 \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \vartriangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash [exp_1; \; ..; exp_n; ?] : \_\mathbf{list} \; t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_LIST}
                                                                 \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                               \frac{1}{\Delta, E, E^{\text{L}} \vdash (exp) : t \triangleright \Sigma^{\mathcal{C}}.\Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_PAREN}
```

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\frac{\Delta, E, E^{\mathsf{L}} \vdash \mathit{exp} : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E^{\mathsf{L}} \vdash \mathbf{begin} \, \mathit{exp} \, \mathbf{end} : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                                                                                                                                                                               CHECK_EXP_AUX_BEGIN
                                                                                         \begin{array}{l} \Delta, E, E^{\mathrm{L}} \vdash exp_{1} : \_\mathbf{bool} \rhd \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \\ \Delta, E, E^{\mathrm{L}} \vdash exp_{2} : t \rhd \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2} \end{array}
                                                                                         \Delta, E, E^{\mathrm{L}} \vdash exp_3 : t \triangleright \Sigma^{\mathcal{C}_3}, \Sigma^{\mathcal{N}_3}
\overline{\Delta, E, E^{\scriptscriptstyle L} \vdash \mathbf{if} \ exp_1 \, \mathbf{then} \ exp_2 \, \mathbf{else} \ exp_3 : t \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2 \, \cup \, \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \, \cup \, \Sigma^{\mathcal{N}}_2 \, \cup \, \Sigma^{\mathcal{N}}_3}
                                                                                                                                                                                                                                                                                                                                                CHECK_EXP_AUX_IF
                                                                         \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                        \Delta, E, E^{\text{\tiny L}} \vdash exp_2 : \_\textbf{list} \ t \ \triangleright \ \Sigma^{\mathcal{C}_2}, \Sigma^{\mathcal{N}_2}
                             \frac{1}{\Delta, E, E^{\scriptscriptstyle L} \vdash exp_1 :: exp_2 : \_\mathbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2}
                                                                                                \frac{\vdash lit:t}{\Delta,E,E^{\text{\tiny L}}\vdash lit:t\,\vartriangleright\,\{\,\},\{\,\}}\quad\text{CHECK\_EXP\_AUX\_LIT}
                                 \Delta \vdash t_i \mathbf{ok}^i
                                 \Delta, E, E^{\mathrm{L}} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                  \Delta, E, E^{L} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_2 : \_bool \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                 \mathbf{disjoint}\,\mathbf{doms}\,(E^{\scriptscriptstyle{L}},\{\,\overline{x_i\mapsto t_i}^{\,i}\,\})
                                  E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                  x_i \not\in \mathbf{dom}(E^{\mathrm{X}})
                   \overline{\Delta, E, E^{\text{L}} \vdash \{exp_1 | exp_2\}} : \_\mathbf{set} \ t \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2}
                                                                                                                                                                                                                                                              CHECK_EXP_AUX_SET_COMP
\begin{array}{c} \Delta, E, E_{1}^{\text{L}} \vdash \overline{qbind_{i}}^{i} \triangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}_{1} \\ \Delta, E, E_{1}^{\text{L}} \uplus E_{2}^{\text{L}} \vdash exp_{1} : t \triangleright \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2} \\ \Delta, E, E_{1}^{\text{L}} \uplus E_{2}^{\text{L}} \vdash exp_{2} : \_\textbf{bool} \ \triangleright \Sigma^{\mathcal{C}}_{3}, \Sigma^{\mathcal{N}}_{3} \\ \hline \Delta, E, E_{1}^{\text{L}} \vdash \{exp_{1} | \textbf{forall} \ \overline{qbind_{i}}^{i} | exp_{2}\} : \_\textbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2} \cup \Sigma^{\mathcal{C}}_{3}, \Sigma^{\mathcal{N}}_{2} \cup \Sigma^{\mathcal{N}}_{3} \end{array}
             \Delta \vdash t \mathbf{ok}
 \frac{\Delta, E, E^{\text{L}} \vdash exp_1 : t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\text{L}} \vdash exp_n : t \vartriangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\text{L}} \vdash \{exp_1; ...; exp_n; ?\} : \_\mathbf{set} \; t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_SET}
                                  \frac{\Delta, E, E_{1}^{\text{L}} \vdash \overline{qbind_{i}}^{i} \triangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}_{1}}{\Delta, E, E_{1}^{\text{L}} \uplus E_{2}^{\text{L}} \vdash exp: \_bool \triangleright \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2}} \xrightarrow{\text{CHECK\_EXP\_AUX\_QUANT}} \Delta, E, E_{1}^{\text{L}} \vdash q \overline{qbind_{i}}^{i} \cdot exp: \_bool \triangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2}}
                                                                        \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash \textbf{list} \ \overline{qbind_i}^{\ i} \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_1 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \\ \Delta, E, E_1^{\text{\tiny L}} \uplus E_2^{\text{\tiny L}} \vdash exp_2 : \_\textbf{bool} \ \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3 \end{array}
                                                                                                                                                                                                                                                                                                                              CHECK_EXP_AUX_LIST_COMP_
\overline{\Delta, E, E_1^{\text{L}} \vdash [exp_1| \mathbf{forall} \overline{qbind_i}^i | exp_2] : \_\mathbf{list} \ t \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2 \cup \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \cup \Sigma^{\mathcal{N}}_3}
    \Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass cons
                                                                                                                                                                      CHECK\_LISTQUANT\_BINDING\_EMPTY
                                                                               \overline{\Delta, E, E^{\mathrm{L}} \vdash \triangleright \{\}, \{\}}
                                            \Delta \vdash t \mathbf{ok}
                                            \Delta, E, E_1^{\mathrm{L}} \uplus \{x \mapsto t\} \vdash \overline{qbind_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_1
                                            \mathbf{disjoint} \mathbf{doms} (\{x \mapsto \overline{t}\}, E_2^{\mathrm{L}})
                                       \frac{\text{disjoint doins}\left( \left\{ x \mapsto t \right\}, E_{2}^{\text{L}} \right)}{\Delta, E, E_{1}^{\text{L}} \vdash x \ l \ \overline{qbind_{i}}^{\ i} \ \rhd \left\{ x \mapsto t \right\} \uplus E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}_{1}}
```

 $CHECK\_LISTQUANT\_BINDING\_VAR$ 

```
\begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_3^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_{\ 1}, \Sigma^{\mathcal{N}}_{\ 1} \end{array}
                                       \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_2
        \frac{\mathbf{disjoint} \, \mathbf{doms} \, (E_{3}^{\mathtt{L}}, E_{2}^{\mathtt{L}})}{\Delta, E, E_{1}^{\mathtt{L}} \vdash (\mathit{pat} \, \mathbf{IN} \, \mathit{exp}) \, \overline{\mathit{qbind}_{i}}^{i} \rhd E_{2}^{\mathtt{L}} \uplus E_{3}^{\mathtt{L}}, \Sigma^{\mathcal{C}}_{1} \, \cup \, \Sigma^{\mathcal{C}}_{2}}
                                                                                                                                                                                            CHECK\_LISTQUANT\_BINDING\_RESTR
                                    \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp : \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                   \Delta, E, E_1^{\mathsf{L}} \uplus E_3^{\mathsf{L}} \vdash \overline{\mathit{qbind}_i}^i \rhd E_2^{\mathsf{L}}, \Sigma^{\mathcal{C}}_2
                                   disjoint doms (E_3^{\scriptscriptstyle 
m L},E_2^{\scriptscriptstyle 
m L})
\frac{\Delta E_3, E_2, E_3}{\Delta, E, E_1^{\text{L}} \vdash (pat \, \mathbf{MEM} \, exp) \, \overline{qbind_i}^i \, \triangleright \, E_2^{\text{L}} \uplus E_3^{\text{L}}, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2} \quad \text{Check_listquant_binding_list_restr}
   \Delta, E, E_1^{\text{L}} \vdash \text{list } qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass
                                                                                                                                                   CHECK\_QUANT\_BINDING\_EMPTY
                                                             \overline{\Delta, E, E^{\text{L}} \vdash \text{list} \triangleright \{\}, \{\}}
                                             \begin{array}{l} \Delta, E, E_{1}^{\text{\tiny L}} \vdash pat : t \triangleright E_{3}^{\text{\tiny L}} \\ \Delta, E, E_{1}^{\text{\tiny L}} \vdash exp : \_\textbf{list} \ t \triangleright \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \end{array}
                                              \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_2
                                             disjoint doms (E_3^{\scriptscriptstyle 
m L},E_2^{\scriptscriptstyle 
m L})
   \frac{\Box}{\Delta, E, E_1^{\text{L}} \vdash \ \textbf{list} \ (pat \ \textbf{MEM} \ exp) \ \overline{qbind_i}^{\ i} \ \rhd \ E_2^{\text{L}} \uplus E_3^{\text{L}}, \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2} \\ } \quad \text{CHECK\_QUANT\_BINDING\_RESTR}
   \Delta, E, E^{\text{L}} \vdash funcl \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                              Build the environment for a function definition clause, collecting typec
                                        \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                        disjoint doms (E_1^{\text{L}}, \dots, E_n^{\text{L}})
                                        \Delta, E \vdash typ \leadsto u
\frac{-}{\Delta, E, E^{\mathsf{L}} \vdash x \ l_1 \ pat_1 \dots pat_n : typ = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{} CHECK_FUNCL_ANNOT
                                \begin{array}{l} \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_1 : t_1 \rhd E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash \mathit{pat}_n : t_n \rhd E_n^{\mathsf{L}} \\ \Delta, E, E^{\mathsf{L}} \uplus E_1^{\mathsf{L}} \uplus \ \dots \ \uplus E_n^{\mathsf{L}} \vdash \mathit{exp} : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                disjoint doms (E_1^{\text{L}}, \dots, E_n^{\text{L}})
\overline{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_FUNCL\_NOANNOT}
   \Delta, E, E_1^{\text{L}} \vdash letbind \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a let binding, collecting typeclass and index con
                                                             \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                             \Delta, E \vdash typ \leadsto t
                                       \frac{\Delta, E + egp + v}{\Delta, E, E_1^{\mathsf{L}} \vdash pat : typ = exp \ l \rhd E_2^{\mathsf{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                                                                                                                                  CHECK_LETBIND_VAL_ANNOT
                                          \begin{array}{c} \Delta, E, E_{1}^{\text{L}} \vdash pat : t \vartriangleright E_{2}^{\text{L}} \\ \Delta, E, E_{1}^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \hline \Delta, E, E_{1}^{\text{L}} \vdash pat = exp \ l \vartriangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                                                                                                                                    CHECK_LETBIND_VAL_NOANNOT
                                                    \frac{\Delta, E, E_1^{\text{L}} \vdash funcl\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\text{L}} \vdash funcl\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_FN}
     \Delta, E, E^{\mathrm{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                             Build the environment for an inductive relation clause, collecting typed
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\Delta \vdash t_i \mathbf{ok}^i
                                                                                 E_2^{\rm L} = \{ \overline{y_i \mapsto t_i}^i \}
\begin{array}{c} \Delta, E, E_{1}^{\text{L}} \uplus E_{2}^{\text{L}} \vdash exp' : \_\textbf{bool} \quad \triangleright \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'} \\ \Delta, E, E_{1}^{\text{L}} \uplus E_{2}^{\text{L}} \vdash exp_{1} : u_{1} \triangleright \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \quad .. \quad \Delta, E, E_{1}^{\text{L}} \uplus E_{2}^{\text{L}} \vdash exp_{n} : u_{n} \triangleright \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n} \\ \hline \Delta, E, E_{1}^{\text{L}} \vdash id^{?} \, \textbf{forall} \, \overline{y_{i} \, l_{i}}^{i} \cdot exp' \Longrightarrow x \, l \, exp_{1} \dots exp_{n} \, l' \triangleright \{x \mapsto \textbf{curry} \, ((u_{1} * \dots * u_{n}), \_\textbf{bool} \, )\}, \Sigma^{\mathcal{C}'} \cup \Sigma^{\mathcal{C}}_{1} \cup \dots \cup \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n} \\ \hline \end{array}
      xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                        Extract the type constructor information
                                                                 tnvars^l \leadsto tnvs
                                                                 \Delta, E \vdash typ \leadsto t
                                                                 \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                                 \mathbf{FV}(t) \subset tnvs
                                                                 \overline{y_i}. i \ x \not\in \mathbf{dom}(\Delta)
      \overline{\overline{y_i}^{\;i}, \Delta, E \vdash \mathbf{tc} \; x \; l \; tnvars^l = typ \; \triangleright \; \{\overline{y_i.}^i \; x \mapsto tnvs \; .t\}, \{x \mapsto \overline{y_i.}^i \; x\}}
                                                                                                                                                                                              CHECK_TEXP_TC_ABBREV
                                                             tnvars^l \leadsto tnvs
                                                             \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                             \overline{y_i} x \notin \mathbf{dom}(\Delta)
            \overline{\overline{y_i}^i, \Delta, E_1 \vdash \mathbf{tc} \, x \, l \, tnvars^l \, \triangleright \{\overline{y_i}^i \, x \mapsto tnvs \}, \{x \mapsto \overline{y_i}^i \, x\}}
                                                                                                                                                                                   CHECK\_TEXP\_TC\_ABSTRACT
                                                                                         tnvars^l \leadsto tnvs
                                                                                         \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                                                         \overline{y_i} x \notin \mathbf{dom}(\Delta)
\overline{\overline{y_i}^{\;i}, \Delta_1, E \vdash \mathbf{tc} \; x \; l \; tnvars^l \; = \langle |x_1^l: typ_1; \, \dots; x_j^l: typ_j \; ;^?| \rangle \; \triangleright \; \{\overline{y_i.}^{\;i} \; x \mapsto tnvs \; \}, \{x \mapsto \overline{y_i.}^{\;i} \; x \}
                                                                                                                                                                                                                                                     CHECK_TEXP_TC_REC
                                                                                      tnvars^l \leadsto tnvs
                                                                                       \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                                                       \overline{y_i} x \notin \mathbf{dom}(\Delta)
\overline{\overline{y_i}^i, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l = |? \ ctor\_def_1| \dots | ctor\_def_j \rhd \{\overline{y_i}.^i \ x \mapsto tnvs \}, \{x \mapsto \overline{y_i}.^i \ x\}} \quad \text{CHECK\_TEXP\_TC\_VAR}
      xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                      Extract the type constructor information
                                                                \overline{xs, \Delta, E \vdash \mathbf{tc} \triangleright \{\}, \{\}} \quad \text{CHECK\_TEXPS\_TC\_EMPTY}
                       xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E_2^{\mathrm{P}}
                      xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{\}, E_2^{\mathtt{P}}, \{\}, \{\} \rangle \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_3, E_3^{\mathtt{P}} \\ \mathbf{dom} \, (E_2^{\mathtt{P}}) \cap \mathbf{dom} \, (E_3^{\mathtt{P}}) = \emptyset
                                     \overline{xs}, \Delta_1, E \vdash \mathbf{tc} \ td \ \overline{td_i}^i \rhd \Delta_2 \uplus \Delta_3, E_2^P \uplus E_3^P
      \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle Check a type definition, with its path already resolved
                                                                                                                                                  CHECK_TEXP_ABBREV
                                                          \overline{\Delta, E \vdash tnvs \ p = typ \, \triangleright \, \langle \{ \, \}, \{ \, \} \rangle}
                                      \overline{\Delta, E \vdash typ_i \leadsto t_i}^i
                                       names = \{ \overline{x_i}^i \}
                                      \mathbf{duplicates}\left(\,\overline{x_{i}}^{\,i}\,\right) = \,\emptyset
                                      \overline{\mathbf{F}\mathbf{V}\left(t_{i}\right)\ \subset\ tnvs}
                                      E^{\text{F}} = \{ \overline{x_i \mapsto \langle \text{ forall } tnvs.p \to t_i, (x_i \text{ of } names) \rangle^i} \}  CHECK_TEXP_REC
                                                \Delta, E \vdash tnvs \ p = \langle | \overline{x_i^l : typ_i}^i ; ? | \rangle \triangleright \langle E^F, \{ \} \rangle
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\overline{\Delta, E \vdash typs_i \leadsto t\_multi_i}^i
                              names = \{ \overline{x_i}^i \}
                              \mathbf{duplicates}\left(\,\overline{x_i}^{\,i}\,\right) = \emptyset
                             \overline{\mathbf{FV}(t_{-}multi_{i})} \subset tnvs^{i}
                              E^{X} = \{ \overline{x_i \mapsto \langle \mathbf{forall} \ tnvs.t_{-}multi_i \rightarrow p, (x_i \mathbf{of} \ names) \rangle}^i \}
                                                                                                                                                                                                               CHECK_TEXP_VAR
                                                   \Delta, E \vdash tnvs \ p = |? \overline{x_i^l \text{ of } typs_i}^i \rhd \langle \{ \}, E^{\mathbf{X}} \rangle
   xs, \Delta, E \vdash td_1 ... td_n \triangleright \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle
                                                                                                                                              CHECK_TEXPS_EMPTY
                                                                          \overline{y_i}^i, \Delta, E \vdash \triangleright \langle \{\}, \{\} \rangle
                                     tnvars^l \leadsto tnvs
                                     \Delta, E_1 \vdash tnvs \overline{y_i}.^i x = texp \triangleright \langle E_1^F, E_1^X \rangle
                                    \begin{array}{l} \overline{y_i}^{\,i}, \Delta, E \vdash \overline{td_j}^{\,j} \, \rhd \, \langle E_2^{\scriptscriptstyle \mathrm{F}}, E_2^{\scriptscriptstyle \mathrm{X}} \rangle \\ \mathbf{dom}\,(E_1^{\scriptscriptstyle \mathrm{X}}) \, \cap \, \mathbf{dom}\,(E_2^{\scriptscriptstyle \mathrm{X}}) = \emptyset \\ \mathbf{dom}\,(E_1^{\scriptscriptstyle \mathrm{F}}) \, \cap \, \mathbf{dom}\,(E_2^{\scriptscriptstyle \mathrm{F}}) = \emptyset \end{array}
                                                                                                                                                                                  CHECK_TEXPS_CONS_CONCRETE
       \overline{y_i}^{\;i}, \Delta, \overline{E \vdash x \; l \; tnvars^l \; = \; texp \; \overline{td_j}^{\;j} \; \rhd \; \langle E_1^{\scriptscriptstyle \mathrm{F}} \uplus E_2^{\scriptscriptstyle \mathrm{F}}, E_1^{\scriptscriptstyle \mathrm{X}} \uplus E_2^{\scriptscriptstyle \mathrm{X}} \rangle}
                                   \frac{\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \, \triangleright \, \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle}{\overline{y_i}^i, \Delta, E \vdash x \, l \, tnvars^l \, \, \overline{td_j}^j \, \triangleright \, \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle} \quad \text{CHECK\_TEXPS\_CONS\_ABSTRACT}
   \delta, E \vdash id \leadsto p Lookup a type class
                                                                                              E(id) \triangleright p
                                                                                        \frac{\delta(p) \triangleright xs}{\delta, E \vdash id \leadsto p} \quad \text{CONVERT\_CLASS\_ALL}
   I \vdash (p \ t) \mathbf{IN} \, \mathcal{C}
                                                       Solve class constraint
                                                                                                                                                                           SOLVE_CLASS_CONSTRAINT_IMMEDIATE
\overline{I \vdash (p \alpha) \mathbf{IN} (p_1 tnv_1) ... (p_i tnv_i) (p \alpha) (p'_1 tnv'_1) ... (p'_i tnv'_j)}
                  (p_1 tnv_1) ... (p_n tnv_n) \Rightarrow (p t) \mathbf{IN} I
                 \frac{I \vdash (p_1 \, \sigma(tnv_1)) \, \mathbf{IN} \, \mathcal{C} \quad \dots \quad I \vdash (p_n \, \sigma(tnv_n)) \, \mathbf{IN} \, \mathcal{C}}{I \vdash (p \, \sigma(t)) \, \mathbf{IN} \, \mathcal{C}}
                                                                                                                                                                      SOLVE_CLASS_CONSTRAINT_CHAIN
   I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                               Solve class constraints
                                 \frac{I \vdash (p_1 \ t_1) \ \mathbf{IN} \ \mathcal{C} \quad .. \quad I \vdash (p_n \ t_n) \ \mathbf{IN} \ \mathcal{C}}{I \vdash \{(p_1 \ t_1), \dots, (p_n \ t_n)\} \triangleright \mathcal{C}} \quad \text{SOLVE\_CLASS\_CONSTRAINTS\_ALL}
    \Delta, I, E \vdash val\_def \triangleright E^{x} Check a value definition
                                               \Delta, E, \{\} \vdash letbind \triangleright \{\overline{x_i \mapsto t_i}^i\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}

I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                               \overline{\mathbf{FV}\left(t_{i}\right)} \subset \mathit{tnvs}^{i}
                                                \mathbf{FV}(\mathcal{C}) \subset tnvs
                                                                                                                                                                                                            CHECK_VAL_DEF_VAL
                   \overline{\Delta, I, E_1 \vdash \mathbf{let} \, \tau^? \, letbind} \, \triangleright \, \{ \, \overline{x_i \mapsto \langle \, \mathbf{forall} \, tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle}^{\, i} \, \}
```

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\overline{\Delta, E, E^{\text{L}} \vdash funcl_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}^{i}}
                                                 I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                                 \overline{\mathbf{FV}(t_i) \subset tnvs}^i
                                                 \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                 \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
                                                 E^{L} = \{ \overline{x_i \mapsto t_i}^i \}
            \frac{\mathcal{L} - \{ x_i + r \in r \}}{\Delta, I, E \vdash \mathbf{let} \operatorname{rec} \tau^? \overline{funcl_i}^i \triangleright \{ \overline{x_i \mapsto \langle \operatorname{\mathbf{forall}} tnvs. \mathcal{C} \Rightarrow t_i, \operatorname{\mathbf{let}} \rangle}^i \}}
                                                                                                                                                                                                                                CHECK_VAL_DEF_RECFUN
       \Delta, (\alpha_1, ..., \alpha_n) \vdash t  instance
                                                                                                            Check that t be a typeclass instance
                                                                                                                                                           CHECK_T_INSTANCE_VAR
                                                                                   \overline{\Delta}. (\alpha) \vdash \alpha instance
                                                                                                                                                                                             CHECK_T_INSTANCE_TUP
                                                 \overline{\Delta,(\alpha_1,\ldots,\alpha_n)} \vdash \alpha_1 * \ldots * \alpha_n  instance
                                                                                                                                                                             CHECK_T_INSTANCE_FN
                                                                 \overline{\Delta,(\alpha_1,\alpha_2)\vdash\alpha_1\to\alpha_n\ \mathbf{instance}}
                                                           \frac{\Delta(p) \rhd \alpha_1' \dots \alpha_n'}{\Delta, (\alpha_1, \dots, \alpha_n) \vdash p \alpha_1 \dots \alpha_n \, \mathbf{instance}} \quad \text{CHECK\_T\_INSTANCE\_TC}
      \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2 Check a definition
                                         \overline{z_i}^j, \Delta_1, E \vdash \mathbf{tc} \, \overline{td_i}^i \triangleright \Delta_2, E^{\mathrm{P}}
                         \frac{\overline{z_{j}}^{j}, \Delta_{1} \uplus \Delta_{2}, E \uplus \langle \{\}, E^{\mathrm{P}}, \{\}, \{\} \rangle \vdash \overline{td_{i}}^{i} \rhd \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{\overline{z_{j}}^{j}, \langle \Delta_{1}, \delta, I \rangle, E \vdash \mathbf{type} \, \overline{td_{i}}^{i} \, l \rhd \langle \Delta_{2}, \{\}, \{\} \rangle, \langle \{\}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle} \quad \text{CHECK_DEF_TYPE}
                                            \frac{\Delta, I, E \vdash val\_def \triangleright E^{\mathbf{X}}}{\overline{z_i}^j, \langle \Delta, \delta, I \rangle, E \vdash val\_def \ l \triangleright \epsilon, \langle \{ \}, \{ \}, \{ \}, E^{\mathbf{X}} \rangle} \quad \text{CHECK\_DEF\_VAL\_DEF}
                                    \overline{\Delta, E_1, E^{\text{L}} \vdash rule_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}_i}}^i
                                    I \vdash \overline{\Sigma^{\mathcal{C}}_{i}}^{i} \triangleright \mathcal{C}
                                    \overline{\mathbf{FV}(t_i) \subset tnvs}^i
                                    \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                    compatible overlap (\overline{x_i \mapsto t_i}^i)
                                    E^{L} = \{ \overline{x_i \mapsto t_i}^i \}
                                  E_{2} = \langle \{ \}, \{ \}, \{ \}, \{ \overline{x_{i}} \mapsto \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t_{i}, \mathbf{let} \rangle^{i} \} \rangle
\overline{z_{i}}^{j}, \langle \Delta, \delta, I \rangle, E_{1} \vdash \mathbf{indreln} \tau^{?} \overline{rule_{i}}^{i} \ l \rhd \epsilon, E_{2}
CHECK_DEF_INDRELN
\frac{\overline{z_j}^j \ x, D_1, E_1 \vdash \mathit{defs} \, \triangleright \, D_2, E_2}{\overline{z_j}^j \ , D_1, E_1 \vdash \mathbf{module} \, x \, \mathit{l}_1 = \mathbf{struct} \, \mathit{defs} \, \mathbf{end} \, \mathit{l}_2 \, \triangleright \, D_2, \langle \{x \mapsto E_2\}, \{\,\}, \{\,\}, \{\,\}\rangle}
         \frac{E_1(id) \triangleright E_2}{\overline{z_j}^j, D, E_1 \vdash \mathbf{module} \ x \ l_1 = id \ l_2 \triangleright \epsilon, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle}
                                                                                                                                                                                                                  CHECK_DEF_MODULE_RENAME
                                     \Delta, E \vdash typ \leadsto t
                                   \mathbf{FV}(t) \subset \overline{\alpha_i}^i
\mathbf{FV}(\overline{\alpha'_k}^k) \subset \overline{\alpha_i}^i
\overline{\delta, E \vdash id_k \leadsto p_k}^k
                  \frac{E' = \langle \{ \}, \{ \}, \{ \}, \{x \mapsto \langle \operatorname{\mathbf{forall}} \overline{\alpha_i}^i. \overline{(p_k \alpha_k')}^k \Rightarrow t, \operatorname{\mathbf{val}} \rangle \} \rangle}{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \operatorname{\mathbf{val}} x \ l_1 : \operatorname{\mathbf{forall}} \overline{\alpha_i \ l_i''}^i. \overline{id_k \alpha_k' \ l_k'}^k \Rightarrow \mathit{typ} \ l_2 \rhd \epsilon, E'} \quad \text{CHECK_DEF_SPEC}
```

$$\begin{array}{c} \overline{\Delta,E_1 \vdash typ_i \leadsto t_i}^i \\ \hline \mathbf{FV}(t_i) \subset \alpha^i \\ p = \overline{z_j}^{-j} x \\ E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{\overline{y_i \mapsto \langle \mathbf{forall} \alpha.(p \, \alpha) \Rightarrow t_i, \mathbf{method} \rangle^i} \} \rangle \\ \delta_2 = \{p \mapsto \overline{y_i}^i\} \\ p \not \in \mathbf{dom}(\delta_1) \\ \hline \overline{z_j}^j, \langle \Delta, \delta_1, I \rangle, E_1 \vdash \mathbf{class}(x \, l \, \alpha \, l'') \, \overline{\mathbf{val} \, y_i \, l_i : typ_i \, l_i^i} \, \mathbf{end} \, l' \, \triangleright \, \langle \{\}, \delta_2, \{\} \rangle, E_2 \\ \hline E = \langle E^{\mathsf{M}}, E^{\mathsf{P}}, E^{\mathsf{F}}, E^{\mathsf{N}} \rangle \\ \Delta, E \vdash typ' \leadsto t' \\ \Delta, (\overline{\alpha_i}^i) \vdash t' \, \mathbf{instance} \\ twus = \overline{\alpha_i}^i \\ \mathbf{duplicates}(tnvs) = \emptyset \\ \overline{\delta, E \vdash id_k \leadsto p_k}^k \\ \hline \mathbf{FV}(\overline{\alpha_k'}^k) \subset tnvs \\ E(id) \triangleright p \\ \delta(p) \triangleright \overline{z_j}^j \\ \underline{I_2} = \{ \overline{\Rightarrow (p_k \, \alpha_k')^k} \} \\ \overline{\Delta, I \cup I_2, E \vdash val. def_n \triangleright E_n^{\mathsf{N}}^n} \\ \mathbf{disjoint} \, \mathbf{doms}(\overline{E_n^{\mathsf{N}}}^n) \\ \hline E^{\mathsf{X}}(x_k) \triangleright \langle \mathbf{forall} \, tnvs. \Rightarrow \{\alpha'' \mapsto t'\}(t_k), \mathbf{let} \rangle^k \} = \overline{E_n}^n \\ \overline{x_k}^k = \overline{z_j}^j \\ I_3 = \{ \overline{(p_k \, \alpha_k')^k} \ni p \, t' \rangle^k \} \\ (p \, \{\overline{\alpha_i \mapsto \alpha_i''^i}\}(t')) \not \in I \\ \hline \overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{instance} \, \mathbf{forall} \, \overline{\alpha_i'^i}^i, id_k \, \alpha_k' \, \overline{y_i''}^k \Rightarrow (id \, typ') \, \overline{val\_def_n \, l_n}^n \, \mathbf{end} \, t' \triangleright \langle \{\}, \{\}, I_3 \rangle, \epsilon \\ \end{array} \right.$$

 $\overline{z_j}^j, D_1, E_1 \vdash defs \triangleright D_2, E_2$ Check definitions, given module path, definitions and environment

Definition rules: 144 good 1 bad Definition rule clauses: 438 good 1 bad