n, i, j, k Index variables for meta-lists

num Numeric literals

bin Bit vector literal, specified by C-style hex numberbin Bit vector literal, specified by C-style binary number

string String literals

regexp Regular expresions, as a string literal

x, y, z Variables ix Variables

(Nexp)

```
Nexp
                                                        Location-annotated vector lengths
                       ::=
                        Nexp\_aux\ l
Nexp\_constraint
                                                        Whether a vector is bounded or fixed size
                       ::=
                             Nexp
                             \geq Nexp
                                                        Types
typ_aux
                                                           Unspecified type
                             \alpha^l
                                                           Type variables
                                                           Function types
                             typ_1 \rightarrow typ_2
                             typ_1 * \dots * typ_n
                                                           Tuple types
                             Nexp
                                                           As a typ to permit applications over Nexps, otherwise no
                                                           Type applications
                             id\ typ_1 ... typ_n
                             (typ)
                       ::=
                                                        Location-annotated types
typ
                             typ_aux l
lit\_aux
                                                        Literal constants
                             true
                             false
                             num
                                                           hex and bin are constant bit vectors, entered as C-style l
                             hex
                             bin
                             string
                             ()
                             bitzero
                                                           bitzero and bitone are constant bits, if commonly used w
                             bitone
lit
                       ::=
                             lit\_aux\ l
                                                           Location-annotated literal constants
                                                        Optional semi-colons
                                                        Patterns
pat_{-}aux
                       ::=
                                                           Wildcards
                             (pat \mathbf{as} x^l)
                                                           Named patterns
                             (pat:typ)
                                                           Typed patterns
                             id\ pat_1 \dots pat_n
                                                           Single variable and constructor patterns
                             \langle |fpat_1; ...; fpat_n; ?| \rangle
                                                           Record patterns
                             [|pat_1; ...; pat_n; ?|]
                                                           Vector patterns
                             [|pat_1 ... pat_n|]
                                                           Concatenated vector patterns
                             (pat_1, \ldots, pat_n)
                                                           Tuple patterns
                             [pat_1; ..; pat_n; ?]
                                                           List patterns
```

```
(pat)
                                                                           Cons patterns
                     pat_1 :: pat_2
                     x^l + num
                                                                           constant addition patterns
                                                                           Literal constant patterns
                                                                        Location-annotated patterns
pat
                     pat\_aux l
fpat
                                                                        Field patterns
                     id = pat l
|?
               ::=
                                                                        Optional bars
                                                                        Expressions
exp\_aux
                                                                           Identifiers
                     id
                     N
                                                                           Nexp var, has type num
                                                                           Curried functions
                     fun psexp
                     function |?| pexp_1| \dots | pexp_n| end
                                                                           Functions with pattern matching
                     exp_1 \ exp_2
                                                                           Function applications
                     exp_1 ix^l exp_2
                                                                           Infix applications
                     \langle |fexps| \rangle
                                                                           Records
                     \langle |exp \mathbf{with} fexps| \rangle
                                                                           Functional update for records
                                                                           Field projection for records
                     exp.id
                     [|exp_1; ...; exp_n;^?|]
                                                                           Vector instantiation
                     exp.(Nexp)
                                                                           Vector access
                     exp.(Nexp_1..Nexp_2)
                                                                           Subvector extraction
                     match exp with ||^{?} pexp_1| \dots || pexp_n| l end
                                                                           Pattern matching expressions
                     (exp:typ)
                                                                           Type-annotated expressions
                     let letbind in exp
                                                                           Let expressions
                     (exp_1, \ldots, exp_n)
                                                                           Tuples
                     [exp_1; ...; exp_n; ?]
                                                                           Lists
                     (exp)
                     begin exp end
                                                                           Alternate syntax for (exp)
                     if exp_1 then exp_2 else exp_3
                                                                           Conditionals
                                                                           Cons expressions
                     exp_1 :: exp_2
                                                                           Literal constants
                     lit
                     \{exp_1|exp_2\}
                                                                           Set comprehensions
                     \{exp_1| \mathbf{forall} \ qbind_1 .. \ qbind_n| exp_2\}
                                                                           Set comprehensions with explicit binding
                     \{exp_1; ...; exp_n;^?\}
                                                                           Sets
                     q \ qbind_1 \dots qbind_n . exp
                                                                           Logical quantifications
                     [exp_1| \mathbf{forall} \ qbind_1 .. \ qbind_n| exp_2]
                                                                           List comprehensions (all binders must be qua
                                                                        Location-annotated expressions
exp
```

 $exp_aux l$

```
Quantifiers
                  ::=
q
                         forall
                         exists
qbind
                                                                           Bindings for quantifiers
                         (pat IN exp)
                                                                              Restricted quantifications over sets
                         (pat \mathbf{MEM} \ exp)
                                                                              Restricted quantifications over lists
                                                                           Field-expressions
fexp
                         id = exp l
                                                                           Field-expression lists
fexps
                         fexp_1; \dots; fexp_n; ^? l
                                                                           Pattern matches
pexp
                         pat \rightarrow exp l
                                                                           Multi-pattern matches
psexp
                         pat_1 \dots pat_n \to exp \ l
tannot?
                  ::=
                                                                           Optional type annotations
                         : typ
funcl_aux
                                                                           Function clauses
                         x^l pat_1 \dots pat_n tannot? = exp
                                                                           Let bindings
letbind\_aux
                  ::=
                         pat \ tannot? = exp
                                                                              Value bindings
                         funcl_aux
                                                                              Function bindings
letbind
                  ::=
                                                                           Location-annotated let bindings
                         letbind\_aux\ l
funcl
                                                                           Location-annotated function clauses
                  ::=
                         funcl_aux l
id?
                  ::=
                                                                           Optional name for inductively defined relatio
                         x^l:
                                                                           Inductively defined relation clauses
rule\_aux
                         id^{?} forall x_{1}^{l} \dots x_{n}^{l} \cdot exp \Longrightarrow x^{l} \cdot exp_{1} \dots exp_{i}
rule
                  ::=
                                                                           Location-annotated inductively defined relati
```

 $rule_aux\ l$

```
Type lists
typs
                       typ_1*...*typ_n
ctor\_def
                                                              Datatype definition clauses
                                                         S
                                                                 Constant constructors
                                                              Type definition bodies
texp
                                                                 Type abbreviations
                        \begin{array}{c} \langle |x_1^l:typ_1;\ldots;x_n^l:typ_n;^?| \rangle \\ |?\;ctor\_def_1|\ldots|ctor\_def_n \end{array} 
                                                                 Record types
                                                                 Variant types
name?
                                                              Optional name specification for variables of defined type
                       [name = regexp]
td
                                                              Type definitions
                       x^l tnvars^l name? = texp
                       x^l tnvars^l name?
                                                                 Definitions of opaque types
c
                                                              Typeclass constraints
                       id\ tnvar^l
                                                              Typeclass constraint lists
                       c_1, \ldots, c_i \Rightarrow
                                                                 Must have > 0 constraints
                                                              Type and instance scheme prefixes
c\_pre
                ::=
                       forall tnvar_1^l ... tnvar_n^l.cs
                                                                 Must have > 0 type variables
typschm
                                                              Type schemes
                       c\_pre\ typ
                                                              Instance schemes
instschm
                       c\_pre(id\ typ)
target
                                                              Backend target names
                ::=
                       hol
                       isabelle
                       ocaml
                       coq
                       \mathbf{tex}
                       html
                                                              Backend target name lists
\tau
                       \{target_1; ..; target_n\}
```

```
\tau?
                                                                                                        Optional targets
                                                                                                        Types of Lemmata
lemma\_typ
                             assert
                             lemma
                             theorem
lemma\_decl
                                                                                                        Lemmata and Tests
                             lemma\_typ\ \tau^?\ x^l:(exp)
                             lemma\_typ \tau^{?}(exp)
val\_def
                                                                                                        Value definitions
                            \begin{array}{l} \mathbf{let}\,\tau^{?}\,letbind\\ \mathbf{let}\,\mathbf{rec}\,\tau^{?}\,funcl_{1}\,\mathbf{and}\,...\,\mathbf{and}\,funcl_{n}\\ \mathbf{let}\,\mathbf{inline}\,\tau^{?}\,letbind \end{array}
                                                                                                           Non-recursive value definition
                                                                                                           Recursive function definitions
                                                                                                           Function definitions to be inl
                                                                                                        Value type specifications
val\_spec
                      ::=
                             \mathbf{val} \ x^l : typschm
def_{-}aux
                                                                                                        Top-level definitions
                             type td_1 and ... and td_n
                                                                                                           Type definitions
                             val\_def
                                                                                                           Value definitions
                             lemma\_decl
                                                                                                           Lemmata
                             \mathbf{rename}\,\tau^?\,id=x^l
                                                                                                           Rename constant or type
                             \mathbf{module} \ x^l = \mathbf{struct} \ \mathit{defs} \ \mathbf{end}
                                                                                                           Module definitions
                             module x^l = id
                                                                                                           Module renamings
                             open id
                                                                                                           Opening modules
                             indreln \tau? rule_1 and ... and rule_n
                                                                                                           Inductively defined relations
                             val\_spec
                                                                                                           Top-level type constraints
                             class (x^l tnvar^l) val x_1^l : typ_1 l_1 ... val x_n^l : typ_n l_n end
                                                                                                           Typeclass definitions
                             instance instschm\ val\_def_1\ l_1\ ...\ val\_def_n\ l_n end
                                                                                                           Typeclass instantiations
def
                                                                                                        Location-annotated definitions
                             def_aux l
;;?
                                                                                                        Optional double-semi-colon
                      ::=
                             ;;
defs
                                                                                                        Definition sequences
                             def_1; ? ... def_n; ? ...
                                                                                                        Unique paths
p
                             x_1 \dots x_n \cdot x
```

 $_$ list

```
\_bool
                     __num
                    \_set
                    _string
                     _{-}unit
                     \_bit
                     \_vector
              ::=
                                                        Type variable substitutions
\sigma
                    \{tnv_1 \mapsto t_1 .. tnv_n \mapsto t_n\}
                                                        Internal types
t, u
                    t_1 \rightarrow t_2
                    t_1 * .... * t_n
                    p\ t\_args
                    ne
                                                   Μ
                                                           Multiple substitutions
                    \sigma(t)
                    \sigma(tnv)
                                                   Μ
                                                           Single variable substitution
                    \mathbf{curry}\left(t_{-}multi,t\right)
                                                   Μ
                                                           Curried, multiple argument functions
                                                        internal numeric expressions
ne
                    N
                    num
                    num*ne
                    ne_1 + ne_2
                    (-ne)
                                                   Μ
                    normalize(ne)
                    ne_1 + \dots + ne_n
                                                   Μ
                    bitlength (bin)
                                                   Μ
                    bitlength(hex)
                                                   Μ
                    length (pat_1 \dots pat_n)
                                                   Μ
                    length (exp_1 \dots exp_n)
                                                   Μ
                                                        Lists of types
t\_args
                    t_1 \dots t_n
                    \sigma(t\_args)
                                                   Μ
                                                           Multiple substitutions
t_{-}multi
                                                        Lists of types
                    (t_1 * ... * t_n)
                    \sigma(t_{-}multi)
                                                   Μ
                                                           Multiple substitutions
                                                        Numeric expression constraints
nec
                    ne\langle nec
                    ne = nec
                    ne \le nec
                    ne
```

```
Sets of names
names
                            ::=
                                        \{x_1, ..., x_n\}
                             \mathcal{C}
                                                                                                                                              Typeclass constraint lists
                                        (p_1 tnv_1) \dots (p_n tnv_n)
                                                                                                                                              Tags for the (non-constructor) value description
env\_tag
                                                                                                                                                   Bound to a method
                                        method
                                                                                                                                                   Specified with val
                                        val
                                        let
                                                                                                                                                   Defined with let or indreln
v\_desc
                                                                                                                                              Value descriptions
                            ::=
                                        \langle \mathbf{forall} \ tnvs.t\_multi \rightarrow p, (x \mathbf{of} \ names) \rangle
                                                                                                                                                   Constructors
                                        \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t, env\_tag \rangle
                                                                                                                                                    Values
f\_desc
                                        \langle \mathbf{forall} \ tnvs.p \rightarrow t, (x \mathbf{of} \ names) \rangle
                                                                                                                                                   Fields
                              \Sigma^{\mathcal{C}}
                                                                                                                                             Typeclass constraints
                            ::=
                                        \{(p_1 t_1), \ldots, (p_n t_n)\}
\Sigma^{\mathcal{C}}_1 \cup \ldots \cup \Sigma^{\mathcal{C}}_n
                                                                                                                                    Μ
\Sigma^{\mathcal{N}}
                                                                                                                                             Nexp constraint lists

\{nec_1, \dots, nec_n\} \\
\Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n

                                                                                                                                    Μ
E
                                                                                                                                             Environments
                                      \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
E_1 \uplus E_2
                                                                                                                                    Μ
                                                                                                                                    Μ
E^{\mathbf{X}}
                            ::=
                                                                                                                                              Value environments
                                        \begin{cases} x_1 \mapsto v\_desc_1, \, ..., x_n \mapsto v\_desc_n \rbrace \\ E_1^{\scriptscriptstyle \mathrm{X}} \uplus \, ... \uplus E_n^{\scriptscriptstyle \mathrm{X}} \end{cases} 
                                                                                                                                    Μ
E^{\mathrm{F}}
                                                                                                                                             Field environments
                            ::=
                                        \begin{array}{l} \{x_1 \mapsto f \_desc_1, \, ..\,, x_n \mapsto f \_desc_n\} \\ E_1^{\scriptscriptstyle \mathrm{F}} \uplus \, ..\, \uplus E_n^{\scriptscriptstyle \mathrm{F}} \end{array} 
                                                                                                                                    Μ
E^{\mathrm{M}}
                                                                                                                                              Module environments
                                       \{x_1 \mapsto E_1, \dots, x_n \mapsto E_n\}
E^{\mathrm{P}}
                                                                                                                                             Path environments
                                       \{x_1 \mapsto p_1, \dots, x_n \mapsto p_n\} 
E_1^{\mathsf{P}} \uplus \dots \uplus E_n^{\mathsf{P}} 
                                                                                                                                    Μ
E^{\scriptscriptstyle 
m L}
```

Lexical bindings

::=

$$\left| \begin{array}{c} \{x_1 \mapsto t_1, \dots, x_n \mapsto t_n\} \\ \mid E_1^1 \uplus \dots \uplus E_n^k \end{array} \right| \\ \mid E_1^1 \uplus \dots \uplus E_n^k \end{array} \right|$$
 Type abbreviations
$$\left| \begin{array}{c} tc.def \\ \mid tnvs \ tc.abbrev \end{array} \right|$$
 Type and class constructor definitions
$$\left| \begin{array}{c} \{p_1 \mapsto tc.def_1, \dots, p_n \mapsto tc.def_n\} \\ \mid \Delta_1 \uplus \Delta_2 \end{array} \right|$$
 Type constructors definitions
$$\left| \begin{array}{c} \{p_1 \mapsto tc.def_1, \dots, p_n \mapsto tc.def_n\} \\ \mid \Delta_1 \uplus \Delta_2 \end{array} \right|$$
 M
$$\left| \begin{array}{c} \delta \\ \vdots \\ \left| \begin{array}{c} \{p_1 \mapsto xs_1, \dots, p_n \mapsto xs_n\} \\ \mid \delta_1 \uplus \delta_2 \end{array} \right|$$
 M
$$\left| \begin{array}{c} trype \ constructor \ definitions \end{array} \right|$$
 Type constructor definitions
$$\left| \begin{array}{c} \left(p_1 \mapsto xs_1, \dots, p_n \mapsto xs_n \right) \\ \mid \delta_1 \uplus \delta_2 \end{array} \right|$$
 M
$$\left| \begin{array}{c} trype \ constructor \ definitions \end{array} \right|$$
 Type constructor definitions
$$\left| \begin{array}{c} trype \ constructor \ definitions \end{array} \right|$$
 Type constructor definitions
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 Type constructor definitions
$$\left| \begin{array}{c} trype \ constructor \ definitions \end{array} \right|$$
 Type constructor definitions
$$\left| \begin{array}{c} trype \ cons$$

 $\begin{picture}(60,0)(0,0) \put(0,0){\line(1,0){10}} \pu$

```
formula
                                          judgement
                                          formula_1 .. formula_n
                                           E^{\mathrm{M}}(x) \triangleright E
                                                                                                                                                                Module lookup
                                           E^{\mathrm{P}}(x) \triangleright p
                                                                                                                                                                Path lookup
                                           E^{\mathrm{F}}(x) \triangleright f_{-}desc
                                                                                                                                                                Field lookup
                                           E^{X}(x) \triangleright v_{-}desc
                                                                                                                                                                Value lookup
                                           E^{\mathrm{L}}(x) \triangleright t
                                                                                                                                                                Lexical binding lookup
                                           \Delta(p) \triangleright tc_-def
                                                                                                                                                                Type constructor lookup
                                           \delta(p) \triangleright xs
                                                                                                                                                                Type constructor lookup
                                           \operatorname{\mathbf{dom}}(E_1^{\mathrm{M}}) \cap \operatorname{\mathbf{dom}}(E_2^{\mathrm{M}}) = \emptyset
                                          \operatorname{\mathbf{dom}}(E_1^{\mathrm{x}}) \cap \operatorname{\mathbf{dom}}(E_2^{\mathrm{x}}) = \emptyset
                                          \mathbf{dom}\left(E_{1}^{\scriptscriptstyle{\mathrm{F}}}\right)\,\cap\,\mathbf{dom}\left(E_{2}^{\scriptscriptstyle{\mathrm{F}}}\right)=\,\emptyset
                                          \mathbf{dom}\left(E_{1}^{\mathrm{P}}\right)\,\cap\,\mathbf{dom}\left(E_{2}^{\mathrm{P}}\right)=\,\emptyset
                                          \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\ldots,E_n^{\scriptscriptstyle{\mathrm{L}}})
                                                                                                                                                                Pairwise disjoint domains
                                          \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathbf{X}}},\,\ldots,E_n^{\scriptscriptstyle{\mathbf{X}}})
                                                                                                                                                                Pairwise disjoint domains
                                           compatible overlap (x_1 \mapsto t_1, ..., x_n \mapsto t_n)
                                                                                                                                                                (x_i = x_j) \Longrightarrow (t_i = t_j)
                                           \mathbf{duplicates}(tnvs) = \emptyset
                                           \mathbf{duplicates}\left(x_{1}, \ldots, x_{n}\right) = \emptyset
                                           x \not\in \mathbf{dom}(E^{\mathrm{L}})
                                           x \not\in \mathbf{dom}(E^{\mathbf{X}})
                                           x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                           p \not\in \mathbf{dom}(\delta)
                                           p \not\in \mathbf{dom}(\Delta)
                                           \mathbf{FV}(t) \subset tnvs
                                                                                                                                                                Free type variables
                                           \mathbf{FV}(t_{-}multi) \subset tnvs
                                                                                                                                                                Free type variables
                                           \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                                                                                                                                                                Free type variables
                                           inst IN I
                                           (p t) \not\in I
                                          E_{1}^{\mathrm{L}} = E_{2}^{\mathrm{L}}
E_{1}^{\mathrm{X}} = E_{2}^{\mathrm{X}}
E_{1}^{\mathrm{F}} = E_{2}^{\mathrm{F}}
                                          E_1 = E_2
\Delta_1 = \Delta_2
\delta_1 = \delta_2
                                           I_1 = I_2
```

```
names_1 = names_2
                                    \sigma_1 = \sigma_2
                                    p_1 = p_2
                                    xs_1 = xs_2
                                     tnvs_1 = tnvs_2
convert\_tnvars
                             ::=
                                    tnvars^l \leadsto tnvs
                                     tnvar^l \leadsto tnv
look\_m
                                    E_1(x_1^l \dots x_n^l) \triangleright E_2
                                                                                                                           Name path lookup
look\_m\_id
                            ::=
                                    E_1(id) \triangleright E_2
                                                                                                                           Module identifier lookup
look\_tc
                             ::=
                                     E(id) \triangleright p
                                                                                                                           Path identifier lookup
check\_t
                             ::=
                                     \Delta \vdash t \mathbf{ok}
                                                                                                                           Well-formed types
                                     \Delta, tnv \vdash t ok
                                                                                                                           Well-formed type/Nexps n
                             ::=
teq
                                    \Delta \vdash t_1 = t_2
                                                                                                                           Type equality
convert\_typ
                             ::=
                                    \Delta, E \vdash typ \leadsto t
                                                                                                                           Convert source types to in
                                    \vdash Nexp \leadsto ne
                                                                                                                           Convert and normalize nur
convert\_typs
                                     \Delta, E \vdash typs \leadsto t\_multi
check\_lit
                             ::=
                                    \vdash lit:t
                                                                                                                           Typing literal constants
inst\_field
                             ::=
                                     \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                                           Field typing (also returns
inst\_ctor
                             ::=
                                                                                                                           Data constructor typing (a
                                     \Delta, E \vdash \mathbf{ctor} id : t\_multi \rightarrow p \ t\_args \triangleright (x \ \mathbf{of} \ names)
inst\_val
                             ::=
                                     \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}}
```

 not_ctor

::=

Typing top-level bindings,

		$E, E^{ t L} \vdash x \operatorname{f not}\operatorname{f ctor}$	v is not bound to a data cons
$not_shadowed$::=	$E^{ ext{L}} \vdash id \mathbf{not} \mathbf{shadowed}$	id is not lexically shadowed
$check_pat$::= 	$\begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \rhd E_2^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash pat_aux : t \rhd E_2^{\text{\tiny L}} \end{array}$	Typing patterns, building the Typing patterns, building the
id_field	::=	$E \vdash id \mathbf{field}$	Check that the identifier is a p
id_value	::=	$E \vdash id$ value	Check that the identifier is a p
$check_exp$::=	$\begin{array}{l} \Delta, E, E^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \Delta, E, E^{\text{L}} \vdash exp_aux : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \Delta, E, E_{1}^{\text{L}} \vdash qbind_{1} qbind_{n} \vartriangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}} \\ \Delta, E, E_{1}^{\text{L}} \vdash \textbf{list } qbind_{1} qbind_{n} \vartriangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}} \\ \Delta, E, E^{\text{L}} \vdash funcl \vartriangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \Delta, E, E_{1}^{\text{L}} \vdash letbind \vartriangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}$	Typing expressions, collecting Typing expressions, collecting Build the environment for quanching Build the environment for a function of the supplies that the supplies the supplies that the suppl
$check_rule$::=	$\Delta, E, E^{\mathrm{L}} \vdash rule \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}$	Build the environment for an
$check_texp_tc$::=	$xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathrm{P}}$	Extract the type constructor i
$check_texps_tc$::=	$xs, \Delta_1, E \vdash \mathbf{tc} \ td_1 \dots td_i \rhd \Delta_2, E^{\mathrm{P}}$	Extract the type constructor i
$check_texp$::=	$\Delta, E \vdash \mathit{tnvs}\ p = \mathit{texp}\ \triangleright\ \langle E^{\scriptscriptstyle{\mathrm{F}}}, E^{\scriptscriptstyle{\mathrm{X}}} \rangle$	Check a type definition, with
$check_texps$::=	$xs, \Delta, E \vdash td_1 td_n \triangleright \langle E^{\scriptscriptstyle F}, E^{\scriptscriptstyle X} \rangle$	
$convert_class$::=	$\delta, E \vdash id \leadsto p$	Lookup a type class
$solve_class_constraint$::=	$I \vdash (p\ t) \mathbf{IN} \mathcal{C}$	Solve class constraint
$solve_class_constraints$::=	$I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}$	Solve class constraints

```
check\_val\_def
                          ::=
                                 \Delta, I, E \vdash val\_def \triangleright E^{X}
                                                                             Check a value definition
                           check\_t\_instance
                          ::=
                                 \Delta, (\alpha_1, ..., \alpha_n) \vdash t  instance
                                                                             Check that t be a typeclass instance
check\_defs
                          ::=
                                 \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2
                                                                             Check a definition
                                 \overline{z_i}^j, D_1, E_1 \vdash defs \triangleright D_2, E_2
                                                                             Check definitions, given module path, definitions
judgement
                          ::=
                                 convert\_tnvars
                                 look\_m
                                 look\_m\_id
                                 look\_tc
                                 check\_t
                                 teq
                                  convert\_typ
                                  convert\_typs
                                  check\_lit
                                 inst\_field
                                 inst\_ctor
                                 inst\_val
                                 not\_ctor
                                 not\_shadowed
                                  check\_pat
                                 id\_field
                                 id\_value
                                  check\_exp
                                 check\_rule
                                  check\_texp\_tc
                                  check_texps_tc
                                  check\_texp
                                  check\_texps
                                 convert\_class
                                  solve\_class\_constraint
                                  solve\_class\_constraints
                                  check\_val\_def
                                  check\_t\_instance
                                  check\_defs
user\_syntax
                          ::=
                                 n
                                 num
                                 hex
                                 bin
```

string

```
regexp
\boldsymbol{x}
ix
l
x^{l}
ix^l
\alpha
\alpha^l
N
N^l
id
tnv
tnvar^l
tnvs
tnvars^l
Nexp\_aux
Nexp
Nexp\_constraint
typ\_aux
typ
lit\_aux
lit
pat\_aux
pat
fpat
exp\_aux
exp
q
qbind
fexp
fexps
pexp
psexp
tannot?
funcl_aux
letbind\_aux
letbind
funcl
id?
rule\_aux
rule
typs
ctor\_def
texp
```

```
name?
td
c
cs
c_pre
typschm
instschm
target
\tau \\ \tau^?
lemma\_typ
lemma\_decl
val\_def
val\_spec
def_{-}aux
def;;?
defs
p
\sigma
ne
t\_args
t\_multi
nec
names
env\_tag
v\_desc
f\_desc
\Sigma^{\mathcal{C}}
\Sigma^{\mathcal{N}}
E
E^{\mathbf{X}}
E^{\mathrm{F}}
E^{\mathrm{M}}
E^{\scriptscriptstyle \mathrm{P}}
E^{\scriptscriptstyle 
m L}
tc\_abbrev
tc\_def
\Delta
\delta
inst
Ι
D
```

xs

terminals formula

 $tnvars^l \leadsto tnvs$

$$\frac{tnvar_1^l \leadsto tnv_1 \quad .. \quad tnvar_n^l \leadsto tnv_n}{tnvar_1^l \dots tnvar_n^l \leadsto tnv_1 \dots tnv_n} \quad \text{CONVERT_TNVARS_NONE}$$

 $tnvar^l \leadsto tnv$

$$\frac{\alpha \ l \leadsto \alpha}{N \ l \leadsto N} \quad \begin{array}{ll} \text{Convert_tnvar_A} \\ \hline \\ \hline \\ \hline \\ N \ l \leadsto N \end{array}$$

 $E_1(x_1^l ... x_n^l) \triangleright E_2$ Name path lookup

$$\frac{E(\) \rhd E \qquad \text{LOOK_M_NONE}}{E^{\text{M}}(x) \rhd E_{1}}$$

$$\frac{E^{\text{M}}(x) \rhd E_{2}}{\langle E^{\text{M}}, E^{\text{P}}, E^{\text{F}}, E^{\text{X}} \rangle (x \ l \ \overline{y_{i}^{l}}^{i}) \rhd E_{2}} \qquad \text{LOOK_M_SOME}$$

 $E_1(id) \triangleright E_2$ Module identifier lookup

$$\frac{E_1(\overline{y_i^l}^i x l_1) \triangleright E_2}{E_1(\overline{y_i^l}^i x l_1 l_2) \triangleright E_2} \quad \text{LOOK_M_ID_ALL}$$

 $E(id) \triangleright p$ Path identifier lookup

$$\frac{E(\overline{y_{i}^{l}}^{i}) \triangleright \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{E^{\mathrm{P}}(x) \triangleright p}$$

$$E(\overline{y_{i}^{l}}^{i} x \ l_{1} \ l_{2}) \triangleright p$$
LOOK_TC_ALL

 $\Delta \vdash t \text{ ok}$ Well-formed types

$$\begin{array}{cccc} \overline{\Delta \vdash \alpha \, \mathbf{ok}} & \text{CHECK_T_VAR} \\ & \Delta \vdash t_1 \, \mathbf{ok} \\ & \underline{\Delta \vdash t_2 \, \mathbf{ok}} \\ & \overline{\Delta \vdash t_1 \to t_2 \, \mathbf{ok}} & \text{CHECK_T_FN} \\ \\ & \underline{\Delta \vdash t_1 \, \mathbf{ok} \quad \quad \Delta \vdash t_n \, \mathbf{ok}} \\ & \underline{\Delta \vdash t_1 \, \mathbf{ok} \quad \quad \Delta \vdash t_n \, \mathbf{ok}} & \text{CHECK_T_TUP} \\ \\ & \underline{\Delta(p) \rhd tnv_1 .. tnv_n \, tc_abbrev} \\ & \underline{\Delta, tnv_1 \vdash t_1 \, \mathbf{ok} \quad ... \quad \Delta, tnv_n \vdash t_n \, \mathbf{ok}} & \underline{\Delta \vdash p \, t_1 ... t_n \, \mathbf{ok}} & \text{CHECK_T_APP} \\ \hline \end{array}$$

 $\Delta, tnv \vdash t$ ok Well-formed type/Nexps matching the application type variable

$$\frac{\Delta \vdash t \, \mathbf{ok}}{\Delta, \alpha \vdash t \, \mathbf{ok}} \quad \text{CHECK_TLEN_T}$$

$$\Delta$$
. $N \vdash ne \, \mathbf{ok}$ CHECK_TLEN_LEN

 $\Delta \vdash t_1 = t_2$ Type equality

$$\frac{\Delta \vdash t \text{ ok}}{\Delta \vdash t = t} \quad \text{TEQ_REFL}$$

$$\frac{\Delta \vdash t_2 = t_1}{\Delta \vdash t_1 = t_2} \quad \text{TEQ_SYM}$$

$$\frac{\Delta \vdash t_1 = t_2}{\Delta \vdash t_2 = t_3} \quad \text{TEQ_TRANS}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_1 = t_3} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = t_3}{\Delta \vdash t_1 \to t_2 = t_3 \to t_4} \quad \text{TEQ_ARROW}$$

$$\frac{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n}{\Delta \vdash t_1 * \dots * t_n = u_1 * \dots * u_n} \quad \text{TEQ_TUP}$$

$$\frac{\Delta(p) \rhd \alpha_1 \dots \alpha_n}{\Delta \vdash t_1 = u_1 \quad \dots \quad \Delta \vdash t_n = u_n} \quad \text{TEQ_APP}$$

$$\frac{\Delta(p) \rhd \alpha_1 \dots \alpha_n \dots u}{\Delta \vdash p \ t_1 \dots t_n = p \ u_1 \dots u_n} \quad \text{TEQ_APP}$$

$$\frac{\Delta(p) \rhd \alpha_1 \dots \alpha_n \dots u}{\Delta \vdash p \ t_1 \dots t_n = \{\alpha_1 \mapsto t_1 \dots \alpha_n \mapsto t_n\}(u)} \quad \text{TEQ_EXPAND}$$

$$\frac{ne = \text{normalize}(ne')}{\Delta \vdash ne = ne'} \quad \text{TEQ_NEXP}$$

 $\Delta, E \vdash typ \leadsto t$ Convert source types to internal types

 $\vdash Nexp \leadsto ne$ Convert and normalize numeric expressions

```
\overline{\vdash N \; l \leadsto N} \quad \text{Convert_nexp_var}
                                                                 \frac{}{\vdash num \leadsto num} \quad \text{CONVERT\_NEXP\_NUM}
                                                      \frac{}{\vdash num * N \iff num * N} \quad \text{CONVERT\_NEXP\_MULT}
                                                                  \vdash Nexp_1 \leadsto ne_1
                                                 \frac{\vdash Nexp_2 \leadsto ne_2}{\vdash Nexp_1 + Nexp_2 \leadsto ne_1 + ne_2} \quad \text{CONVERT\_NEXP\_ADD}
    \Delta, E \vdash typs \leadsto t\_multi
                                     \frac{\Delta, E \vdash typ_1 \leadsto t_1 \quad .. \quad \Delta, E \vdash typ_n \leadsto t_n}{\Delta, E \vdash typ_1 * .. * typ_n \leadsto (t_1 * .. * t_n)} \quad \text{CONVERT\_TYPS\_ALL}
   \vdash lit:t
                            Typing literal constants
                                                                                                              CHECK_LIT_TRUE
                                                                   \vdash true l : __bool
                                                                                                               CHECK_LIT_FALSE
                                                                   \overline{\vdash \mathbf{false}\, l : \_\mathbf{bool}}
                                                                                                             CHECK_LIT_NUM
                                                                    \vdash num \ l : \_\_\mathbf{num}
                                                        \frac{\textit{num} = \mathbf{bitlength}\,(\textit{hex})}{\vdash \textit{hex}\; l: \_\mathbf{vector}\; \textit{num}\; \_\mathbf{bit}}
                                                                                                                             CHECK_LIT_HEX
                                                               num = \mathbf{bitlength}(bin)
                                                                                                                              CHECK_LIT_BIN
                                                          \vdash bin \ l : \_\mathbf{vector} \ num \ \_\mathbf{bit}
                                                                                                                 CHECK_LIT_STRING
                                                              \overline{\vdash string \ l : \_string}
                                                                                                             CHECK_LIT_UNIT
                                                                        \vdash () l : __unit
                                                                                                              CHECK_LIT_BITZERO
                                                              \overline{\vdash \mathbf{bitzero}\ l: \_\mathbf{bit}}
                                                                                                              CHECK_LIT_BITONE
                                                                \vdash bitone l: \_\_bit
   \Delta, E \vdash \mathbf{field}\ id : p\ t\_args \rightarrow t \triangleright (x\ \mathbf{of}\ names)
                                                                                                               Field typing (also returns canonical field names)
                                  \begin{array}{l} E(\left.\overline{x_{i}^{l}}^{i}\right.) \,\rhd\, \left\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}}\right\rangle \\ E^{\mathrm{F}}(y) \,\rhd\, \left\langle\, \mathbf{forall}\, tnv_{1} \ldots tnv_{n}.p \,\rightarrow\, t, (z\,\mathbf{of}\,\, names)\right\rangle \end{array}
                                   \Delta \vdash t_1 \mathbf{ok} \quad .. \quad \Delta \vdash t_n \mathbf{ok}
  \overline{\Delta, E \vdash \mathbf{field} \ \overline{x_i^l}^i \ y \ l_1 \ l_2 : p \ t_1 \dots t_n \rightarrow \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}(t) \triangleright (z \ \mathbf{of} \ names)}
   \Delta, E \vdash \mathbf{ctor}\ id : t\_multi \to p\ t\_args \triangleright (x\ \mathbf{of}\ names) Data constructor typing (also returns canonical constru
                               E(\overline{x_i^l}^i) \rhd \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
E^{\mathrm{X}}(y) \rhd \langle \mathbf{forall} \ tnv_1 \dots tnv_n. t\_multi \to p, (z \ \mathbf{of} \ names) \rangle
\Delta \vdash t_1 \ \mathbf{ok} \quad \dots \quad \Delta \vdash t_n \ \mathbf{ok}
\frac{\Delta \vdash t_1 \text{ ok} \quad .. \quad \Delta \vdash t_n \text{ ok}}{\Delta, E \vdash \text{ ctor } \overline{x_i^l}^i \text{ } y \text{ } l_1 \text{ } l_2 : \{tnv_1 \mapsto t_1 .. \text{ } tnv_n \mapsto t_n\}(t\_multi) \rightarrow p \text{ } t_1 .. \text{ } t_n \triangleright (z \text{ of } names)}
                                                                                                                                                                           ——— INST_CTOR_ALL
     \Delta, E \vdash \mathbf{val} \ id : t \triangleright \Sigma^{\mathcal{C}} Typing top-level bindings, collecting typeclass constraints
```

```
\begin{array}{l} E(\,\overline{x_i^l}^{\,i}\,\,) \,\rhd\, \langle E^{\scriptscriptstyle{\mathrm{M}}}, E^{\scriptscriptstyle{\mathrm{P}}}, E^{\scriptscriptstyle{\mathrm{F}}}, E^{\scriptscriptstyle{\mathrm{X}}} \rangle \\ E^{\scriptscriptstyle{\mathrm{X}}}(y) \,\rhd\, \langle\, \mathbf{forall}\, tnv_1 \mathinner{\ldotp\ldotp\ldotp} tnv_n \ldotp. (p_1\, tnv_1') \mathinner{\ldotp\ldotp\ldotp} (p_i\, tnv_i') \Rightarrow t, \, env\, \lrcorner tag \rangle \end{array}
                              \Delta \vdash t_1 \mathbf{ok} \quad .. \quad \Delta \vdash t_n \mathbf{ok}
                             \sigma = \{tnv_1 \mapsto t_1 \dots tnv_n \mapsto t_n\}
                               \Delta, E \vdash \mathbf{val} \overline{x_i^l}^i \ y \ l_1 \ l_2 : \sigma(t) \triangleright \{(p_1 \ \sigma(tnv_1')), \dots, (p_i \ \sigma(tnv_i'))\}
  E, E^{\mathrm{L}} \vdash x \mathbf{not} \mathbf{ctor}
                                                                          v is not bound to a data constructor
                                                                                             \frac{E^{\mathrm{L}}(x) \triangleright t}{E, E^{\mathrm{L}} \vdash x \, \mathbf{not} \, \mathbf{ctor}} \quad \text{NOT\_CTOR\_VAL}
                                                             \frac{x \not\in \mathbf{dom}(E^{\mathbf{X}})}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle, E^{\mathbf{L}} \vdash x \mathbf{\ not\ ctor}} \quad \text{NOT\_CTOR\_UNBOUND}
                     \frac{E^{\mathbf{X}}(x) \triangleright \langle \mathbf{forall} \ tnv_1 \dots tnv_n. (p_1 \ tnv_1') \dots (p_i \ tnv_i') \Rightarrow t, env\_tag \rangle}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle, E^{\mathbf{L}} \vdash x \mathbf{not} \mathbf{ctor}} \quad \text{NOT\_CTOR\_BOUND}
  E^{L} \vdash id \text{ not shadowed} id is not lexically shadowed
                                                                      \frac{x \notin \mathbf{dom}(E^{L})}{E^{L} \vdash x l_{1} l_{2} \mathbf{not shadowed}} \quad \text{NOT\_SHADOWED\_SING}
                                                       \overline{E^{\text{L}} \vdash x_1^l \dots x_n^l. y^l. z^l \ l \ \textbf{not shadowed}} \quad \text{NOT\_SHADOWED\_MULTI}
  \Delta, E, E_1^{\text{L}} \vdash pat : t \triangleright E_2^{\text{L}}
                                                                                        Typing patterns, building their binding environment
                                                                            \frac{\Delta, E, E_1^{\text{L}} \vdash pat\_aux : t \triangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash pat\_aux \ l : t \triangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_ALL}
\Delta, E, E_1^{\text{L}} \vdash \textit{pat\_aux} : \underline{t} \triangleright E_2^{\text{L}} Typing patterns, building their binding environment
                                                                                \frac{\Delta \vdash t \text{ ok}}{\Delta, E, E^{\text{L}} \vdash \_: t \vartriangleright \{\}} \quad \text{CHECK\_PAT\_AUX\_WILD}
                                                  \begin{array}{c} \Delta, E, E_{1}^{\text{\tiny L}} \vdash pat : t \vartriangleright E_{2}^{\text{\tiny L}} \\ x \not\in \mathbf{dom}\left(E_{2}^{\text{\tiny L}}\right) \\ \overline{\Delta, E, E_{1}^{\text{\tiny L}} \vdash (pat \ \mathbf{as} \ x \ l) : t \vartriangleright E_{2}^{\text{\tiny L}} \uplus \left\{x \mapsto t\right\}} \end{array} \quad \text{CHECK\_PAT\_AUX\_AS}
                                                                 \begin{split} & \Delta, E, E_1^{\text{L}} \vdash pat: t \vartriangleright E_2^{\text{L}} \\ & \frac{\Delta, E \vdash typ \leadsto t}{\Delta, E, E_1^{\text{L}} \vdash (pat: typ): t \vartriangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_TYP} \end{split}
       \Delta, E \vdash \mathbf{ctor}\ id : (t_1 * ... * t_n) \rightarrow p\ t\_args \triangleright (x\ \mathbf{of}\ names)
       E^{\text{L}} \vdash id \text{ not shadowed}
       \frac{\Delta, E, E^{\text{L}} \vdash pat_1 : t_1 \rhd E_1^{\text{L}} \dots \Delta, E, E^{\text{L}} \vdash pat_n : t_n \rhd E_n^{\text{L}}}{\text{disjoint doms}\left(E_1^{\text{L}}, \dots, E_n^{\text{L}}\right)}
\frac{\Delta, E, E^{\text{L}} \vdash id \ pat_1 \dots pat_n : p \ t\_args \rhd E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}}{\Delta, E, E^{\text{L}} \vdash id \ pat_1 \dots pat_n : p \ t\_args \rhd E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}}
CHECK_PAT_AUX_IDENT_CONSTR
                                                                \frac{E, E^{\text{L}} \vdash x \text{ not ctor}}{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ l_2 \ : t \rhd \{x \mapsto t\}} \quad \text{CHECK\_PAT\_AUX\_VAR}
```

```
\overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)}^i
                              \Delta, E, E^{\perp} \vdash pat_i : t_i \triangleright E_i^{\perp}
                             \begin{array}{l} \operatorname{\mathbf{disjoint}}\operatorname{\mathbf{doms}}\,(\,\overline{E_i^{\scriptscriptstyle \mathrm{L}}}^{\,i}\,)\\ \operatorname{\mathbf{duplicates}}\,(\,\overline{x_i}^{\,i}\,)\,=\,\emptyset \end{array}
                              \overline{\Delta, E, E^{\text{L}} \vdash \langle | \overline{id_i = pat_i \ l_i}^i; ?| \rangle : p \ t\_args \rhd \uplus \overline{E_i^{\text{L}}}^i} \quad \text{CHECK\_PAT\_AUX\_RECORD}
                    \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_1 : t \, \rhd \, E_1^{\mathrm{L}} \quad \dots \quad \Delta, E, E^{\mathrm{L}} \vdash \mathit{pat}_n : t \, \rhd \, E_n^{\mathrm{L}}
                    \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
                   \mathbf{length}\left(pat_1 \dots pat_n\right) = num
           \overline{\Delta, E, E^{\text{L}} \vdash [|pat_1; \dots; pat_n|] : \_\text{vector } num \ t \triangleright E_1^{\text{L}} \uplus \dots \uplus E_n^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_VECTOR}
\Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_1: \_\textbf{vector} \; \textit{ne}_1 \; t \, \rhd \, E^{\text{\tiny L}}_1 \quad \dots \quad \Delta, E, E^{\text{\tiny L}} \vdash \textit{pat}_n: \_\textbf{vector} \; \textit{ne}_n \; t \, \rhd \, E^{\text{\tiny L}}_n
disjoint doms (E_1^L, \ldots, E_n^L)
\frac{ne' = ne_1 + \dots + ne_n}{\Delta, E, E^{\mathsf{L}} \vdash [|pat_1 \dots pat_n|] : \_\mathbf{vector} \ ne' \ t \, \triangleright \, E^{\mathsf{L}}_1 \, \uplus \, \dots \, \uplus \, E^{\mathsf{L}}_n}
                                                                                                                                                                                                                                                               CHECK_PAT_AUX_VECTOR
                      \Delta, E, E^{\mathsf{L}} \vdash pat_1 : t_1 \triangleright E_1^{\mathsf{L}} \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash pat_n : t_n \triangleright E_n^{\mathsf{L}}
                      \frac{\text{disjoint doms}\left(E_{1}^{\text{L}}, \dots, E_{n}^{\text{L}}\right)}{\Delta, E, E^{\text{L}} \vdash (pat_{1}, \dots, pat_{n}) : t_{1} * \dots * t_{n} \triangleright E_{1}^{\text{L}} \uplus \dots \uplus E_{n}^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_TUP}
                           \Delta \vdash t \mathbf{ok}
                           \Delta, E, E^{\text{L}} \vdash pat_1 : t \triangleright E_1^{\text{L}} \quad \dots \quad \Delta, E, E^{\text{L}} \vdash pat_n : t \triangleright E_n^{\text{L}}
                          \frac{\textbf{disjoint doms}\left(E_{1}^{\text{L}}, \dots, E_{n}^{\text{L}}\right)}{\Delta, E, E^{\text{L}} \vdash \left[pat_{1}; \dots; pat_{n};^{?}\right] : \_\textbf{list} \ t \vartriangleright E_{1}^{\text{L}} \uplus \dots \uplus E_{n}^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_LIST}
                                                                   \frac{\Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}}}{\Delta, E, E_1^{\text{L}} \vdash (pat) : t \vartriangleright E_2^{\text{L}}} \quad \text{CHECK\_PAT\_AUX\_PAREN}
                                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash \mathit{pat}_1 : t \vartriangleright E_2^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash \mathit{pat}_2 : \_\textbf{list} \ t \vartriangleright E_3^{\text{\tiny L}} \end{array}
                                                              \mathbf{disjoint \, doms}\, (\underline{E_2^{\scriptscriptstyle \mathrm{L}}, E_3^{\scriptscriptstyle \mathrm{L}}})
                                            \frac{}{\Delta,E,E_1^{\rm L} \vdash \mathit{pat}_1 :: \mathit{pat}_2 : \_\mathit{list} \; t \, \triangleright \, E_2^{\rm L} \uplus E_3^{\rm L}} \quad \text{CHECK\_PAT\_AUX\_CONS}
                                                                            \frac{\vdash lit: t}{\Delta, E, E^{\perp} \vdash lit: t \triangleright \{\}} \quad \text{CHECK\_PAT\_AUX\_LIT}
                                                                       E, E^{\text{L}} \vdash x \text{ not ctor}
                           \frac{E,E + x \text{ notetor}}{\Delta, E, E^{\text{L}} \vdash x \ l + num : \_\text{num} \ \triangleright \{x \mapsto \_\text{num} \}}
                                                                                                                                                                                     CHECK_PAT_AUX_NUM_ADD
  E \vdash id \mathbf{field}
                                                  Check that the identifier is a permissible field identifier
                                                                                            E^{\text{F}}(x) \triangleright f \text{\_}desc
                                                                     \overline{\langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \vdash x \ \mathit{l}_{1} \ \mathit{l}_{2} \, \mathbf{field}}
                                                                                                                                                                      ID_FIELD_EMPTY
                                                                                            E^{\mathrm{M}}(x) \triangleright E
                                                                                           x \not\in \mathbf{dom}(E^{\mathrm{F}})
                                                                                           E \vdash \overline{y_i^l}^i z^l l_2  field
                                                               \frac{E \vdash y_i^{\iota}. \ \ z^{\iota} \ l_2 \, \mathbf{field}}{\langle E^{\scriptscriptstyle \mathrm{M}}, E^{\scriptscriptstyle \mathrm{P}}, E^{\scriptscriptstyle \mathrm{F}}, E^{\scriptscriptstyle \mathrm{X}} \rangle \vdash x \ l_1. \, \overline{y_i^{l}.}^i \ z^l \ l_2 \, \mathbf{field}} \quad \text{id_Field\_cons}
  E \vdash id \mathbf{value}
                                                     Check that the identifier is a permissible value identifier
                                                                   \frac{E^{\mathbf{X}}(x) \vartriangleright v\_desc}{\langle E^{\mathbf{M}}, E^{\mathbf{P}}, E^{\mathbf{F}}, E^{\mathbf{X}} \rangle \vdash x \ l_1 \ l_2 \ \mathbf{value}} \quad \text{id\_value\_empty}
```

```
E^{\mathrm{M}}(x) \triangleright E
                                                                                                    x \not\in \mathbf{dom}(E^{X})
                                                                                                  E \vdash \overline{y_i^l}^i \ z^l \ l_2  value
                                                                    \frac{\textit{$_L \vdash y_i^{\text{\tiny $c$}}$. } \textit{$z^{\text{\tiny $c$}}$ $l_2$ } \textbf{value}}{\langle E^{\text{\tiny $M$}}, E^{\text{\tiny $P$}}, E^{\text{\tiny $F$}}, E^{\text{\tiny $X$}} \rangle \vdash x \textit{$l_1$. } \overline{y_i^l}.^i \textit{$z^l$ $l_2$ } \textbf{value}} \quad \text{\tiny ID\_VALUE\_CONS}
       \Delta, E, E^{\text{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Typing expressions, collecting typeclass and index constraints
                                                                            \Delta, E, E^{\text{\tiny L}} \vdash exp\_aux: t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                         \Delta, E, E^{\perp} \vdash exp\_aux \ l : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} CHECK_EXP_ALL
    \Delta, E, E^{L} \vdash exp\_aux : t \triangleright \Sigma^{C}, \Sigma^{N}
                                                                                                                    Typing expressions, collecting typeclass and index constraints
                                                                        \frac{E^{\mathrm{L}}(x) \rhd t}{\Delta, E, E^{\mathrm{L}} \vdash x \ l_1 \ l_2 : t \rhd \{\,\}, \{\,\}} \quad \text{CHECK\_EXP\_AUX\_VAR}
                                                                                                                                                                             CHECK_EXP_AUX_NVAR
                                                                      \overline{\Delta, E, E^{\text{L}} \vdash N : num \triangleright \{\}, \{\}}
                                   E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                   E \vdash id value
                                   E \vdash id \text{ value}
\Delta, E \vdash \underbrace{\text{ctor } id : t\_multi \rightarrow p \ t\_args} \triangleright (x \text{ of } names)
CHECK\_EXP\_AUX\_CTOR
                                                                                   E^{\mathrm{L}} \vdash id \text{ not shadowed}
                                                                                   E \vdash id value
                                                                              \frac{\Delta, E \vdash \mathbf{val} \, id : t \, \triangleright \, \Sigma^{\mathcal{C}}}{\Delta, E, E^{\mathsf{L}} \vdash id : t \, \triangleright \, \Sigma^{\mathcal{C}}, \{\,\}} \quad \text{CHECK\_EXP\_AUX\_VAL}
                               \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                               \mathbf{disjoint}\,\mathbf{doms}\,(E_1^{\scriptscriptstyle{\mathrm{L}}},\,\dots,E_n^{\scriptscriptstyle{\mathrm{L}}})
         \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{fun} \ pat_{1} \dots pat_{n} \rightarrow exp \ l : \mathbf{curry} \left( (t_{1} * \dots * t_{n}), u \right) \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_FN}
                                                 \frac{\overline{\Delta, E, E^{\scriptscriptstyle L} \vdash pat_i : t \vartriangleright E_i^{\scriptscriptstyle L}{}^i}}{\Delta, E, E^{\scriptscriptstyle L} \uplus E_i^{\scriptscriptstyle L} \vdash exp_i : u \vartriangleright \Sigma^{\scriptscriptstyle \mathcal{C}}{}_i, \Sigma^{\scriptscriptstyle \mathcal{N}}{}_i{}^i}
                                                                                                                                                                                                                             CHECK_EXP_AUX_FUNCTION
      \overline{\Delta, E, E^{\text{L}} \vdash \mathbf{function}} \mid^{?} \overline{pat_{i} \rightarrow exp_{i} \, l_{i}}^{i} \, \mathbf{end} : t \rightarrow u \, \triangleright \, \overline{\Sigma^{\mathcal{C}}_{i}}^{i}, \overline{\Sigma^{\mathcal{N}_{i}}}^{i}
                                                            \begin{array}{l} \Delta, E, E^{\text{\tiny L}} \vdash exp_1 : t_1 \rightarrow t_2 \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\text{\tiny L}} \vdash exp_2 : t_1 \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                     \frac{1}{\Delta,E,E^{\text{L}} \vdash exp_1 \ exp_2 : t_2 \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2} \quad \text{Check_exp_aux_app}
                                            \begin{array}{l} \Delta, E, E^{\mathrm{L}} \vdash (ix) : t_1 \to t_2 \to t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\mathrm{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
                                            \Delta, E, E^{\mathrm{L}} \vdash exp_2 : t_2 \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
\overline{\Delta, E, E^{\text{L}} \vdash exp_1 \ ix \ l \ exp_2 : t_3 \vartriangleright \Sigma^{\mathcal{C}}_1 \ \cup \ \Sigma^{\mathcal{C}}_2 \ \cup \ \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \ \cup \ \Sigma^{\mathcal{N}}_2 \ \cup \ \Sigma^{\mathcal{N}}_3} \quad \text{CHECK\_EXP\_AUX\_INFIX\_APP1}
                                                \begin{array}{l} \Delta, E, E^{\text{\tiny L}} \vdash x : t_1 \rightarrow t_2 \rightarrow t_3 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \\ \Delta, E, E^{\text{\tiny L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \end{array}
\frac{\Delta, E, E^{\text{L}} \vdash exp_2 : t_2 \rhd \Sigma^{\mathcal{C}_3}, \Sigma^{\mathcal{N}_3}}{\Delta, E, E^{\text{L}} \vdash exp_1 `x`l \ exp_2 : t_3 \rhd \Sigma^{\mathcal{C}_1} \cup \Sigma^{\mathcal{C}_2} \cup \Sigma^{\mathcal{C}_3}, \Sigma^{\mathcal{N}_1} \cup \Sigma^{\mathcal{N}_2} \cup \Sigma^{\mathcal{N}_3}}
                                                                                                                                                                                                                                      CHECK_EXP_AUX_INFIX_APP2
                                   \overline{\Delta, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)}^i
                                   \Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i
                                   duplicates (\overline{x_i}^i) = \emptyset
                                   names = \{ \overline{x_i}^i \}
                          \Delta, E, E^{\text{L}} \vdash \langle | \overline{id_i = exp_i \ l_i}^i \ ;^? \ l | \rangle : p \ t\_args \, \triangleright \, \overline{\Sigma^{\mathcal{C}}_i}^i, \overline{\Sigma^{\mathcal{N}}_i}^i \quad \text{CHECK\_EXP\_AUX\_RECORD}
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\overline{\Delta}, E \vdash \mathbf{field}\ id_i : p\ t\_args \rightarrow t_i \triangleright (x_i \ \mathbf{of}\ names)^i
                                              \Delta, E, E^{\text{L}} \vdash exp_i : t_i \triangleright \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i
                                              \mathbf{duplicates} (\, \overline{x_i}^{\, i} \,) = \emptyset
                                              \Delta, E, E^{\mathsf{L}} \vdash exp : p \ t\_args \rhd \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'}
\Delta, E, E^{\text{\tiny L}} \vdash \langle | \textit{exp} \ \textbf{with} \ \overline{id_i = \textit{exp}_i \ l_i}^i \ ; ^? \ l | \rangle : \textit{p} \ \textit{t\_args} \ \vartriangleright \ \underline{\Sigma^{\mathcal{C}'}} \ \cup \ \overline{\Sigma^{\mathcal{C}}_i}^i \ , \underline{\Sigma^{\mathcal{N}'}} \ \cup \ \overline{\Sigma^{\mathcal{N}_i}}^i
                      \Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \triangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n
                      length (exp_1 \dots exp_n) = num
\overline{\Delta, E, E^{\text{L}} \vdash [|exp_1; \dots; exp_n|] : \_\textbf{vector} \ num \ t \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \dots \, \cup \, \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \, \cup \, \dots \, \cup \, \Sigma^{\mathcal{N}}_n}}
                                       \Delta, E, E^{\mathrm{L}} \vdash exp : \_\mathbf{vector} \ ne' \ t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                      \vdash Nexp \leadsto ne
                                                                                                                                                                                        CHECK_EXP_AUX_VECTORGET
                            \overline{\Delta, E, E^{\text{\tiny L}} \vdash exp.(Nexp) : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \ \cup \ \{ne\langle ne'\}\}}
                                                  \Delta, E, E^{L} \vdash exp : \_vector ne' t \triangleright \Sigma^{C}, \Sigma^{N}
                                                  \vdash Nexp_1 \leadsto ne_1
                                                  \vdash Nexp_2 \leadsto ne_2
                                                  ne = ne_2 + (-ne_1)
                                                                                                                                                                                                                                           CHECK_EXP_AUX_VECTORSUB
\overline{\Delta, E, E^{\text{L}} \vdash exp.(Nexp_1..Nexp_2) : \_vector\ ne\ t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \cup \{ne_1 \langle ne_2 \langle ne' \} \}
                                          E \vdash id \mathbf{field}
                                          \Delta, E \vdash field id : p \ t\_args \rightarrow t \triangleright (x \ \textbf{of} \ names)
                                          \Delta, E, E^{L} \vdash exp : p \ t\_args \triangleright \Sigma^{C}, \Sigma^{N}
                                                                                                                                                                                             CHECK_EXP_AUX_FIELD
                                                                 \Delta, E, E^{L} \vdash exp.id : t \triangleright \Sigma^{C}, \Sigma^{N}
                                                                     \overline{\Delta, E, E^{\text{\tiny L}} \vdash pat_i : t \vartriangleright E_i^{\text{\tiny L}}}^i
                                                                    \frac{1}{\Delta, E, E^{\mathsf{L}} \uplus E_{i}^{\mathsf{L}} \vdash exp_{i} : u \rhd \Sigma^{\mathsf{C}}_{i}, \Sigma^{\mathsf{N}_{i}}}^{\mathsf{L}}}{\Delta, E, E^{\mathsf{L}} \vdash exp : t \rhd \Sigma^{\mathsf{C}'}, \Sigma^{\mathsf{N}'}}
\Delta, E, E^{\text{L}} \vdash \mathbf{match} \ exp \ \mathbf{with} \ |^{?} \ \overline{pat_{i} \rightarrow exp_{i} \ l_{i}}^{i} \ l \ \mathbf{end} : u \vartriangleright \Sigma^{\mathcal{C}'} \cup \ \overline{\Sigma^{\mathcal{C}_{i}}}^{i}, \Sigma^{\mathcal{N}'} \cup \ \overline{\Sigma^{\mathcal{N}_{i}}}^{i}  CHECK_EXP_AUX_CASE
                                                                   \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                   \Delta, E \vdash typ \leadsto t
                                                        \frac{1}{\Delta, E, E^{\text{L}} \vdash (exp: typ): t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_TYPED}
                                                            \Delta, E, E_1^{\mathrm{L}} \vdash letbind \triangleright E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                            \Delta, E, E_1^{\mathrm{L}} \uplus E_2^{\mathrm{L}} \vdash exp: t \triangleright \Sigma^{\overline{\mathcal{C}}}_2, \Sigma^{\overline{\mathcal{N}}}_2
                         \frac{1}{\Delta, E, E^{\text{L}} \vdash \textbf{let} \ letbind \ \textbf{in} \ exp: t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_LET}
\frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t_1 \rhd \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad \dots \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t_n \rhd \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash (exp_1, \dots, exp_n) : t_1 * \dots * t_n \rhd \Sigma^{\mathcal{C}}_1 \cup \dots \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup \dots \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_TUP}
        \Delta \vdash t \mathbf{ok}
 \frac{\Delta, E, E^{\text{L}} \vdash exp_1 : t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\text{L}} \vdash exp_n : t \vartriangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\text{L}} \vdash [exp_1; \ ..; exp_n \ ;^?] : \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_LIST}
                                                                  \Delta, E, E^{\mathrm{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                \frac{\cdot}{\Delta, E, E^{\text{L}} \vdash (exp) : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_PAREN}
                                                                   \Delta, E, E^{\mathrm{L}} \vdash exp : t \triangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                   \frac{1}{\Delta, E, E^{\mathrm{L}} \vdash \mathbf{begin} \ exp \ \mathbf{end} : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_EXP\_AUX\_BEGIN}
                                                                      \Delta, E, E^{\mathsf{L}} \vdash exp_1 : \_\mathbf{bool} \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                      \Delta, E, E^{\mathsf{L}} \vdash exp_2 : t \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                                                      \Delta, E, E^{\mathsf{L}} \vdash exp_3 : t \triangleright \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3
                                                                                                                                                                                                                                                                       CHECK\_EXP\_AUX\_IF
\overline{\Delta, E, E^{\scriptscriptstyle L} \vdash \mathbf{if} \ exp_1 \, \mathbf{then} \ exp_2 \, \mathbf{else} \ exp_3 : t \, \triangleright \, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2 \, \cup \, \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_1 \, \cup \, \Sigma^{\mathcal{N}}_2 \, \cup \, \Sigma^{\mathcal{N}}_3}
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\Delta, E, E^{\perp} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1
                                                                    \Delta, E, E^{\mathrm{L}} \vdash exp_2 : \_\mathbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                           \frac{1}{\Delta, E, E^{\text{L}} \vdash exp_1 :: exp_2 : \_\textbf{list} \ t \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2} \quad \text{CHECK\_EXP\_AUX\_CONS}
                                                                                          \frac{ \vdash \mathit{lit} : t}{\Delta, E, E^{\mathtt{l}} \vdash \mathit{lit} : t \, \triangleright \, \{\,\}, \{\,\}} \quad \mathsf{CHECK\_EXP\_AUX\_LIT}
                               \Delta \vdash t_i \mathbf{ok}^i
                               \Delta, E, E^{\text{L}} \uplus \{\, \overline{x_i \mapsto t_i}^{\,i} \,\} \vdash \exp_1 : t \, \rhd \, \Sigma^{\mathcal{C}}_{\,1}, \Sigma^{\mathcal{N}}_{\,1}
                                \Delta, E, E^{\text{L}} \uplus \{ \overline{x_i \mapsto t_i}^i \} \vdash exp_2 : \_bool \triangleright \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2
                                disjoint doms (E^{\mathrm{L}}, \{\overline{x_i \mapsto t_i}^i\})
                                E = \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle
                                \overline{x_i \not\in \mathbf{dom}(E^{\mathrm{X}})}^{i}
                                                                                                                                                                                                                                               CHECK_EXP_AUX_SET_COMP
                  \overline{\Delta, E, E^{\text{\tiny L}} \vdash \{exp_1 | exp_2\} : \_\mathbf{set} \ t \rhd \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_1 \cup \Sigma^{\mathcal{N}}_2}
                                                                     \begin{array}{l} \Delta, E, E_1^{\scriptscriptstyle L} \vdash \overline{qbind_i}^i \rhd E_2^{\scriptscriptstyle L}, \Sigma^{\mathcal{C}}_1 \\ \Delta, E, E_1^{\scriptscriptstyle L} \uplus E_2^{\scriptscriptstyle L} \vdash exp_1 : t \rhd \Sigma^{\mathcal{C}}_2, \Sigma^{\mathcal{N}}_2 \\ \Delta, E, E_1^{\scriptscriptstyle L} \uplus E_2^{\scriptscriptstyle L} \vdash exp_2 : \_-\mathbf{bool} \ \rhd \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_3 \end{array}
\Delta, \overline{E, E_1^{\text{L}} \vdash \{exp_1 | \mathbf{forall} \ \overline{qbind_i}^i | exp_2\}} : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2 \cup \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \cup \Sigma^{\mathcal{N}}_3
            \Delta \vdash t \mathbf{ok}
 \frac{\Delta, E, E^{\mathsf{L}} \vdash exp_1 : t \triangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \quad .. \quad \Delta, E, E^{\mathsf{L}} \vdash exp_n : t \triangleright \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_n}{\Delta, E, E^{\mathsf{L}} \vdash \{exp_1; ...; exp_n;^?\} : \_\mathbf{set} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup ... \cup \Sigma^{\mathcal{C}}_n, \Sigma^{\mathcal{N}}_1 \cup ... \cup \Sigma^{\mathcal{N}}_n} \quad \text{CHECK\_EXP\_AUX\_SET}
                                                      \begin{array}{l} \Delta, E, E_{1}^{\text{\tiny L}} \vdash \overline{qbind_{i}}^{i} \, \triangleright \, E_{2}^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_{1} \\ \Delta, E, E_{1}^{\text{\tiny L}} \uplus E_{2}^{\text{\tiny L}} \vdash exp : \_\textbf{bool} \, \, \triangleright \, \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2} \end{array}
                                                                                                                                                                                                                                          CHECK_EXP_AUX_QUANT
                                \frac{}{\Delta,E,E_{1}^{\text{\tiny L}} \vdash q \ \overline{qbind_{i}}^{i} .exp: \_\textbf{bool} \ \triangleright \Sigma^{\mathcal{C}}_{1} \cup \Sigma^{\mathcal{C}}_{2},\Sigma^{\mathcal{N}}_{2}}
                                                                   \begin{array}{l} \Delta, E, E_{1}^{\text{\tiny L}} \vdash \mathbf{list} \, \overline{qbind_{i}}^{i} \rhd E_{2}^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_{1} \\ \Delta, E, E_{1}^{\text{\tiny L}} \uplus E_{2}^{\text{\tiny L}} \vdash exp_{1} : t \rhd \Sigma^{\mathcal{C}}_{2}, \Sigma^{\mathcal{N}}_{2} \\ \Delta, E, E_{1}^{\text{\tiny L}} \uplus E_{2}^{\text{\tiny L}} \vdash exp_{2} : \_\mathbf{bool} \; \rhd \Sigma^{\mathcal{C}}_{3}, \Sigma^{\mathcal{N}}_{3} \end{array}
 \underline{\Delta, E, E_1^{\text{L}} \vdash [exp_1 | \mathbf{forall} \ \overline{qbind_i}^i | exp_2] : \_\mathbf{list} \ t \triangleright \Sigma^{\mathcal{C}}_1 \cup \Sigma^{\mathcal{C}}_2 \cup \Sigma^{\mathcal{C}}_3, \Sigma^{\mathcal{N}}_2 \cup \Sigma^{\mathcal{N}}_3}
    \Delta, E, E_1^{\text{L}} \vdash qbind_1 ... qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}} Build the environment for quantifier bindings, collecting typeclass cons
                                                                         \Delta, E, E^{\text{L}} \vdash \triangleright \{\}, \{\} CHECK_LISTQUANT_BINDING_EMPTY
                                          \Delta \vdash t \mathbf{ok}
                                          \Delta, E, E_1^{\mathsf{L}} \uplus \{x \mapsto t\} \vdash \ \overline{qbind_i}^{\ i} \, \rhd \, E_2^{\mathsf{L}}, \Sigma^{\mathcal{C}}{}_1
                                    \frac{\mathbf{disjoint}\,\mathbf{doms}\,(\{x\mapsto t\},E_{2}^{\mathtt{L}})}{\Delta,E,E_{1}^{\mathtt{L}}\vdash x\,l\,\overline{qbind_{i}}^{i}}\triangleright\{x\mapsto t\}\uplus E_{2}^{\mathtt{L}},\Sigma^{\mathcal{C}}_{1}
                                                                                                                                                                                                            CHECK_LISTQUANT_BINDING_VAR
                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat : t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp : \_\mathbf{set} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                              \Delta, E, E_1^{\mathrm{L}} \uplus E_3^{\mathrm{L}} \vdash \overline{qbind_i}^i \rhd E_2^{\mathrm{L}}, \Sigma^{\mathcal{C}}_2
                                              disjoint doms (E_3^{\scriptscriptstyle 
m L}, E_2^{\scriptscriptstyle 
m L})
          \frac{\Delta E_{1} \cup E_{2} \cup E_{3}}{\Delta, E, E_{1}^{\text{L}} \vdash (pat \, \mathbf{IN} \, exp) \, \overline{qbind_{i}}^{\, i} \, \rhd \, E_{2}^{\text{L}} \uplus E_{3}^{\text{L}}, \Sigma^{\mathcal{C}}_{1} \, \cup \, \Sigma^{\mathcal{C}}_{2}}
                                                                                                                                                                                                                             CHECK_LISTQUANT_BINDING_RESTR
                                          \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp: \_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                          \Delta, E, E_1^{\text{\tiny L}} \uplus E_3^{\text{\tiny L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_2
                                          disjoint doms (E_3^{\scriptscriptstyle \rm L},E_2^{\scriptscriptstyle \rm L})
                                                                                                                                                                                                                           CHECK_LISTQUANT_BINDING_LIST_RESTR
\Delta, E, E_1^{\mathrm{L}} \vdash (\mathit{pat}\,\mathbf{MEM}\,\mathit{exp})\,\overline{\mathit{qbind}_i}^i \, \rhd \, E_2^{\mathrm{L}} \uplus E_3^{\mathrm{L}}, \Sigma^{\mathcal{C}}{}_1 \, \cup \, \Sigma^{\mathcal{C}}{}_2
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CHECK_QUANT_BINDING_EMPTY
                                                                                                         \overline{\Delta, E, E^{\text{L}} \vdash \text{list} \triangleright \{\}, \{\}}
                                                                              \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \vdash pat: t \vartriangleright E_3^{\text{\tiny L}} \\ \Delta, E, E_1^{\text{\tiny L}} \vdash exp: \_\_\textbf{list} \ t \vartriangleright \Sigma^{\mathcal{C}}_1, \Sigma^{\mathcal{N}}_1 \end{array}
                                                                             \begin{array}{l} \Delta, E, E_1^{\text{\tiny L}} \uplus E_3^{\text{\tiny L}} \vdash \overline{qbind_i}^i \rhd E_2^{\text{\tiny L}}, \Sigma^{\mathcal{C}}_2 \\ \textbf{disjoint doms} \left(E_3^{\text{\tiny L}}, E_2^{\text{\tiny L}}\right) \end{array}
      \Delta, E, E_1^{\text{L}} \vdash \textbf{list} \left( pat \, \textbf{MEM} \, exp \right) \overline{qbind_i}^i \, \triangleright \, E_2^{\text{L}} \uplus E_3^{\text{L}}, \Sigma^{\mathcal{C}}_1 \, \cup \, \Sigma^{\mathcal{C}}_2  Check_quant_binding_restr
      \Delta, E, E^{L} \vdash funcl \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
                                                                                                                                                                                                                          Build the environment for a function definition clause, collecting typec
                                                                    \begin{array}{l} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} \quad \dots \quad \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus \dots \ \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                                      disjoint doms (E_1^L, \ldots, E_n^L)
                                                                    \Delta, E \vdash typ \leadsto u
\overline{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n \ : typ = exp \ l_2 \rhd \{x \mapsto \mathbf{curry} \ ((t_1 \ast \dots \ast t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                       \begin{array}{lll} \Delta, E, E^{\mathrm{L}} \vdash pat_{1} : t_{1} \rhd E_{1}^{\mathrm{L}} & \dots & \Delta, E, E^{\mathrm{L}} \vdash pat_{n} : t_{n} \rhd E_{n}^{\mathrm{L}} \\ \Delta, E, E^{\mathrm{L}} \uplus E_{1}^{\mathrm{L}} \uplus & \dots & \uplus E_{n}^{\mathrm{L}} \vdash exp : u \rhd \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                       disjoint doms (E_1^L, \ldots, E_n^L)
\overline{\Delta, E, E^{\text{L}} \vdash x \ l_1 \ pat_1 \dots pat_n = exp \ l_2 \triangleright \{x \mapsto \mathbf{curry} \ ((t_1 * \dots * t_n), u)\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_FUNCL\_NOANNOT}
     \Delta, E, E_1^{\text{L}} \vdash letbind \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for a let binding, collecting typeclass and index con
                                                                                                         \begin{array}{l} \Delta, E, E_1^{\text{L}} \vdash pat : t \vartriangleright E_2^{\text{L}} \\ \Delta, E, E_1^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array}
                                                                                                         \Delta, E \vdash typ \leadsto t
                                                                    \frac{-1}{\Delta, E, E_1^{\mathsf{L}} \vdash pat : typ = exp \ l \rhd E_2^{\mathsf{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}
                                                                                                                                                                                                                                                                                                             CHECK_LETBIND_VAL_ANNOT
                                                                         \begin{array}{l} \Delta, E, E_{1}^{\text{L}} \vdash pat : t \vartriangleright E_{2}^{\text{L}} \\ \Delta, E, E_{1}^{\text{L}} \vdash exp : t \vartriangleright \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \\ \hline \Delta, E, E_{1}^{\text{L}} \vdash pat = exp \ l \vartriangleright E_{2}^{\text{L}}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} \end{array} \quad \text{CHECK\_LETBIND\_VAL\_NOANNOT}
                                                                                       \frac{\Delta, E, E_1^{\mathsf{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}}{\Delta, E, E_1^{\mathsf{L}} \vdash funct\_aux \ l \rhd \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}} \quad \text{CHECK\_LETBIND\_FN}
        \Delta, E, E^{\text{L}} \vdash rule \triangleright \{x \mapsto t\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}} Build the environment for an inductive relation clause, collecting typed
                                                                                                                                                                 \Delta \vdash t_i \mathbf{ok}^i
                                                                                                                                                                 E_2^{\rm L} = \{ \overline{y_i \mapsto t_i}^i \}
                                                                                                                                                                \begin{array}{l} \Delta, E, E_{1}^{\mathtt{L}} \uplus E_{2}^{\mathtt{L}} \vdash exp' : \_\mathbf{bool} \quad \triangleright \ \Sigma^{\mathcal{C}'}, \Sigma^{\mathcal{N}'} \\ \Delta, E, E_{1}^{\mathtt{L}} \uplus E_{2}^{\mathtt{L}} \vdash exp_{1} : u_{1} \triangleright \Sigma^{\mathcal{C}}_{1}, \Sigma^{\mathcal{N}}_{1} \quad .. \quad \Delta, E, E_{1}^{\mathtt{L}} \uplus E_{2}^{\mathtt{L}} \vdash exp_{n} : u_{n} \triangleright \Sigma^{\mathcal{C}}_{n}, \Sigma^{\mathcal{N}}_{n} \end{array}
\overline{\Delta, E, E_1^{\text{L}} \vdash id^? \mathbf{forall} \, \overline{y_i \, l_i}^i \cdot exp' \Longrightarrow x \, l \, exp_1 \dots exp_n \, l' \, \triangleright \, \{x \mapsto \mathbf{curry} \, ((u_1 * \dots * u_n), \_\mathbf{bool} \, )\}, \Sigma^{\mathcal{C}'} \cup \Sigma^{\mathcal{C}}_1 \cup \dots \cup \mathcal{C}_n \cup
          xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E^{\mathrm{P}}
                                                                                                                                                                       Extract the type constructor information
                                                                                                                                 tnvars^l \leadsto tnvs
                                                                                                                                  \Delta, E \vdash typ \leadsto t
                                                                                                                                  \mathbf{duplicates}\left(\mathit{tnvs}\right) = \emptyset
                                                                                                                                  \mathbf{FV}(t) \subset tnvs
                                                                                                                                \overline{y_i}. i \ x \not\in \mathbf{dom}(\Delta)
                                                                                                                                                                                                                                                                                                                                                                                         CHECK_TEXP_TC_ABBREV
           \overline{\overline{y_i}^i, \Delta, E \vdash \mathbf{tc} \ x \ l \ tnvars^l = typ \rhd \{\overline{y_i.}^i \ x \mapsto tnvs \ .t\}, \{x \mapsto \overline{y_i.}^i \ x\}}
```

Build the environment for quantifier bindings, collecting typeclass

 $\Delta, E, E_1^{\text{L}} \vdash \mathbf{list} \ qbind_1 ... \ qbind_n \triangleright E_2^{\text{L}}, \Sigma^{\mathcal{C}}$

```
tnvars^l \leadsto tnvs
                                                      \mathbf{duplicates}\left(\mathit{tnvs}\right) = \emptyset
                                                      \overline{y_i} x \notin \mathbf{dom}(\Delta)
                                                                                                                                                            CHECK_TEXP_TC_ABSTRACT
          \overline{\overline{y_i}^i, \Delta, E_1 \vdash \mathbf{tc} \, x \, l \, tnvars^l \, \triangleright \{\overline{y_i}^i \, x \mapsto tnvs \}, \{x \mapsto \overline{y_i}^i \, x\}}
                                                                               tnvars^l \leadsto tnvs
                                                                               duplicates(tnvs) = \emptyset
                                                                               \overline{y_i}^i x \not\in \mathbf{dom}(\Delta)
                                                                                                                                                                                                                            CHECK_TEXP_TC_REC
\overline{y_i^{\ i}, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l \ = \langle |x_1^l : typ_1; \dots; x_i^l : typ_j ; ?| \rangle} \rhd \{ \overline{y_i.}^i \ x \mapsto tnvs \}, \{ x \mapsto \overline{y_i.}^i \ x \}
                                                                             tnvars^l \leadsto tnvs
                                                                             \mathbf{duplicates}\left(tnvs\right) = \emptyset
                                                                             \overline{y_i}^i x \notin \mathbf{dom}(\Delta)
                                                                                                                                                                                                                       CHECK\_TEXP\_TC\_VAR
\overline{y_i^{\ i}, \Delta_1, E \vdash \mathbf{tc} \ x \ l \ tnvars^l = |? \ ctor\_def_1| \dots | ctor\_def_j \rhd \{\overline{y_i}.^i \ x \mapsto tnvs \}, \{x \mapsto \overline{y_i}.^i \ x\}}
     xs, \Delta_1, E \vdash \mathbf{tc} td_1 ... td_i \triangleright \Delta_2, E^{\mathsf{P}}
                                                                                            Extract the type constructor information
                                                         \overline{xs, \Delta, E \vdash \mathbf{tc} \rhd \{\,\}, \{\,\}} \quad \text{CHECK\_TEXPS\_TC\_EMPTY}
                    xs, \Delta_1, E \vdash \mathbf{tc} \ td \triangleright \Delta_2, E_2^{\mathrm{P}}
                    xs, \Delta_1 \uplus \Delta_2, E \uplus \langle \{\}, E_2^P, \{\}, \{\} \rangle \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_3, E_3^P
                    \mathbf{dom}\left(E_{2}^{\mathrm{P}}\right)\cap\,\mathbf{dom}\left(E_{3}^{\mathrm{P}}\right)=\emptyset
                                xs, \Delta_1, E \vdash \mathbf{tc} \ td \ \overline{td_i}^i \rhd \Delta_2 \uplus \Delta_3, E_2^{\mathrm{P}} \uplus E_3^{\mathrm{P}} Check_texps_tc_abbrev
     \Delta, E \vdash tnvs \ p = texp \triangleright \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle Check a type definition, with its path already resolved
                                                                                                                                   CHECK_TEXP_ABBREV
                                                    \overline{\Delta, E \vdash tnvs \ p = typ \, \triangleright \, \langle \{ \, \}, \{ \, \} \rangle}
                                  \overline{\Delta, E \vdash typ_i \leadsto t_i}^i
                                  names = \{ \overline{x_i}^i \}
                                  \mathbf{duplicates}\left(\,\overline{x_{i}}^{\,i}\,\right) = \,\emptyset
                                  \overline{\mathbf{FV}(t_i)} \subset tnvs
                                  E^{F} = \{ \overline{x_i \mapsto \langle \text{ forall } tnvs.p \to t_i, (x_i \text{ of } names) \rangle}^i \}  CHECK_TEXP_REC
                                           \Delta, E \vdash tnvs \ p = \langle | \ \overline{x_i^l : typ_i}^i \ ; ? | \rangle \rhd \langle E^F, \{ \ \} \rangle
                          \overline{\Delta, E \vdash typs_i \leadsto t\_multi_i}
                           names = \{ \overline{x_i}^i \}
                          \mathbf{duplicates}\,(\,\overline{x_i}^{\,i}\,)=\,\emptyset
                          \overline{\mathbf{FV}\left(t_{-}multi_{i}\right)\,\subset\,tnvs}
                          E^{\mathbf{X}} = \{ \overline{x_i \mapsto \langle \mathbf{\ forall} \ tnvs.t\_multi_i \rightarrow p, (x_i \mathbf{\ of} \ names) \rangle}^i \}
                                                                                                                                                                            CHECK_TEXP_VAR
                                            \Delta, E \vdash tnvs \ p = |? \overline{x_i^l \text{ of } typs_i}^i \rhd \langle \{ \}, E^{\mathbf{X}} \rangle
     xs, \Delta, E \vdash td_{1...td_{n} \triangleright \langle E^{F}, E^{X} \rangle}
                                                                                                                          CHECK_TEXPS_EMPTY
                                                               \overline{\overline{y_i}^i, \Delta, E \vdash \triangleright \langle \{\}, \{\} \rangle}
                                tnvars^l \leadsto tnvs
                                \Delta, E_1 \vdash tnvs \overline{y_i}^i x = texp \triangleright \langle E_1^F, E_1^X \rangle
                                \begin{array}{l} \overline{y_i}^{\,i}, \Delta, E \vdash \overline{td_j}^{\,j} \, \triangleright \, \langle E_2^{\scriptscriptstyle \mathrm{F}}, E_2^{\scriptscriptstyle \mathrm{X}} \rangle \\ \mathbf{dom} \, (E_1^{\scriptscriptstyle \mathrm{X}}) \, \cap \, \mathbf{dom} \, (E_2^{\scriptscriptstyle \mathrm{X}}) = \emptyset \end{array}
                                \mathbf{dom}\,(E_1^{\mathsf{F}})\,\cap\,\mathbf{dom}\,(E_2^{\mathsf{F}})=\emptyset
        \overline{y_i}^i, \Delta, \overline{E} \vdash x \ l \ tnvars^l \ = \ texp \ \overline{td_j}^j \ \triangleright \ \langle E_1^{\scriptscriptstyle \mathrm{F}} \uplus E_2^{\scriptscriptstyle \mathrm{F}}, E_1^{\scriptscriptstyle \mathrm{X}} \uplus E_2^{\scriptscriptstyle \mathrm{X}} \rangle Check_texps_cons_concrete
```

```
\frac{\overline{y_i}^i, \Delta, E \vdash \overline{td_j}^j \rhd \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle}{\overline{y_i}^i, \Delta, E \vdash x \ l \ tnvars^l \ \overline{td_j}^j \rhd \langle E^{\mathrm{F}}, E^{\mathrm{X}} \rangle} \quad \text{CHECK\_TEXPS\_CONS\_ABSTRACT}
   \delta, E \vdash id \leadsto p Lookup a type class
                                                                              \frac{\delta(p) \triangleright xs}{\delta, E \vdash id \leadsto p} \quad \text{Convert\_class\_all}
   I \vdash (p \ t) \mathbf{IN} \, \mathcal{C}
                                                 Solve class constraint
                                                                                                                                                       SOLVE_CLASS_CONSTRAINT_IMMEDIATE
\overline{I \vdash (p \alpha) \mathbf{IN} (p_1 tnv_1) ... (p_i tnv_i) (p \alpha) (p'_1 tnv'_1) ... (p'_j tnv'_j)}
                 (p_1 tnv_1) ... (p_n tnv_n) \Rightarrow (p t) \mathbf{IN} I
               I \vdash (p_1 \sigma(tnv_1)) \mathbf{IN} \mathcal{C} \quad .. \quad I \vdash (p_n \sigma(tnv_n)) \mathbf{IN} \mathcal{C}I \vdash (p \sigma(t)) \mathbf{IN} \mathcal{C}
                                                                                                                                                        SOLVE_CLASS_CONSTRAINT_CHAIN
   I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C} Solve class constraints
                             \frac{I \vdash (p_1 \ t_1) \mathbf{IN} \, \mathcal{C} \quad .. \quad I \vdash (p_n \ t_n) \mathbf{IN} \, \mathcal{C}}{I \vdash \{(p_1 \ t_1), \dots, (p_n \ t_n)\} \triangleright \mathcal{C}} \quad \text{SOLVE\_CLASS\_CONSTRAINTS\_ALL}
   \Delta, I, E \vdash val\_def \triangleright E^{\mathbf{X}} Check a value definition
                                          \Delta, E, \{\} \vdash letbind \triangleright \{\overline{x_i \mapsto t_i}^i\}, \Sigma^{\mathcal{C}}, \Sigma^{\mathcal{N}}
I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                          \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
                 \frac{}{\Delta, I, E_1 \vdash \mathbf{let} \, \tau^? \, letbind \, \triangleright \, \{ \, \overline{x_i \mapsto \langle \, \mathbf{forall} \, tnvs. \mathcal{C} \Rightarrow t_i, \mathbf{let} \rangle^{\, i} \, \}} \quad \text{CHECK\_VAL\_DEF\_VAL}
                                     \overline{\Delta, E, E^{\mathsf{L}} \vdash funcl_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}_i}}^{i}
I \vdash \Sigma^{\mathcal{C}} \triangleright \mathcal{C}
                                      \overline{\mathbf{FV}\left(t_{i}\right) \subset \mathit{tnvs}}
                                      \mathbf{FV}\left(\mathcal{C}\right) \subset \mathit{tnvs}
       \frac{\mathbf{compatible\,overlap}\,(\,\overline{x_i \mapsto t_i}^{\,i}\,)}{E^{\mathrm{L}} = \{\,\overline{x_i \mapsto t_i}^{\,i}\,\}} \qquad \text{CHECK\_VAL\_DEF\_RECFUN}}{\Delta, I, E \vdash \mathbf{let\,rec}\,\tau^?\,\overline{funcl_i}^{\,i}} \triangleright \{\,\overline{x_i \mapsto \langle\,\mathbf{forall}\,tnvs.\mathcal{C} \Rightarrow t_i, \mathbf{let}\rangle}^{\,i}\,\}}
    \Delta, (\alpha_1, ..., \alpha_n) \vdash t instance Check that t be a typeclass instance
                                                                                                                             CHECK\_T\_INSTANCE\_VAR
                                                                 \overline{\Delta,(\alpha) \vdash \alpha \text{ instance}}
                                      \overline{\Delta,(\alpha_1,\ldots,\alpha_n)} \vdash \alpha_1 * \ldots * \alpha_n  instance
                                                                                                                                                        CHECK\_T\_INSTANCE\_TUP
                                                   \overline{\Delta,(\alpha_1,\alpha_2) \vdash \alpha_1 \to \alpha_n \, \mathbf{instance}} \quad \text{CHECK\_T\_INSTANCE\_FN}
                                              \frac{\Delta(p) \, \triangleright \, \alpha_1' \mathinner{\ldotp\ldotp} \alpha_n'}{\Delta, (\alpha_1, \mathinner{\ldotp\ldotp\ldotp}, \alpha_n) \vdash p \, \alpha_1 \mathinner{\ldotp\ldotp} \alpha_n \, \mathbf{instance}} \quad \text{Check\_tlinstance\_tc}
   \overline{z_j}^j, D_1, E_1 \vdash def \triangleright D_2, E_2 Check a definition
```

```
\overline{z_i}^j, \Delta_1, E \vdash \mathbf{tc} \, \overline{td_i}^i \rhd \Delta_2, E^{\mathrm{P}}
                           \frac{\overline{z_{j}}^{j}, \Delta_{1} \uplus \Delta_{2}, E \uplus \langle \{\}, E^{P}, \{\}, \{\} \rangle \vdash \overline{td_{i}}^{i} \rhd \langle E^{F}, E^{X} \rangle}{\overline{z_{j}}^{j}, \langle \Delta_{1}, \delta, I \rangle, E \vdash \mathbf{type} \, \overline{td_{i}}^{i} \, l \rhd \langle \Delta_{2}, \{\}, \{\} \rangle, \langle \{\}, E^{P}, E^{F}, E^{X} \rangle}
                                              \frac{\Delta, I, E \vdash val\_def \rhd E^{\mathbf{X}}}{\overline{z_{i}}^{j}, \langle \Delta, \delta, I \rangle, E \vdash val\_def \ l \rhd \epsilon, \langle \{ \}, \{ \}, \{ \}, E^{\mathbf{X}} \rangle} \quad \text{CHECK\_DEF\_VAL\_DEF}
                                      \overline{\Delta, E_1, E^{\mathsf{L}} \vdash rule_i \triangleright \{x_i \mapsto t_i\}, \Sigma^{\mathcal{C}}_i, \Sigma^{\mathcal{N}}_i}
                                       I \vdash \overline{\Sigma^{\mathcal{C}_i}}^i \rhd \mathcal{C}
                                       \overline{\mathbf{FV}(t_i) \subset tnvs}
                                       \mathbf{FV}(\mathcal{C}) \subset tnvs
                                      \mathbf{compatible}\,\mathbf{overlap}\,(\,\overline{x_i\mapsto t_i}^{\,i}\,)
                                       E^{L} = \{ \overline{x_i \mapsto t_i}^i \}

\frac{E_{2} = \langle \{\}, \{\}, \{\}, \{\overline{x_{i}} \mapsto \langle \mathbf{forall} \ tnvs.\mathcal{C} \Rightarrow t_{i}, \mathbf{let} \rangle^{i} \} \rangle}{\overline{z_{i}}^{j}, \langle \Delta, \delta, I \rangle, E_{1} \vdash \mathbf{indreln} \tau^{?} \overline{rule_{i}}^{i} \ l \rhd \epsilon, E_{2}} \quad \text{CHECK_DEF_INDRELN}

                                                                                  \overline{z_j}^j x, D_1, E_1 \vdash defs \triangleright D_2, E_2
\overline{z_j}^j, D_1, E_1 \vdash \mathbf{module} \ x \ l_1 = \mathbf{struct} \ defs \ \mathbf{end} \ l_2 \triangleright D_2, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle
         \frac{E_1(id) \triangleright E_2}{\overline{z_i}^j, D, E_1 \vdash \mathbf{module} \ x \ l_1 = id \ l_2 \triangleright \epsilon, \langle \{x \mapsto E_2\}, \{\}, \{\}, \{\} \rangle}
                                                                                                                                                                                                                               CHECK_DEF_MODULE_RENAME
                                       \Delta, E \vdash typ \leadsto t
                                       \mathbf{FV}(t) \subset \overline{\alpha_i}^i
                                      \frac{\mathbf{FV}\left(\overline{\alpha_k'}^k\right) \subset \overline{\alpha_i}^i}{\delta, E \vdash id_k \leadsto p_k}^k
                    \frac{E' = \langle \{ \}, \{ \}, \{ \}, \{x \mapsto \langle \operatorname{\mathbf{forall}} \overline{\alpha_i}^i. \overline{(p_k \alpha_k')}^k \Rightarrow t, \operatorname{\mathbf{val}} \rangle \} \rangle}{\overline{z_j}^j, \langle \Delta, \delta, I \rangle, E \vdash \operatorname{\mathbf{val}} x \ l_1 : \operatorname{\mathbf{forall}} \overline{\alpha_i \ l_i''}^i. \overline{id_k \alpha_k' \ l_k'}^k \Rightarrow \mathit{typ} \ l_2 \rhd \epsilon, E'} \quad \text{CHECK_DEF_SPEC}
                     \frac{\Delta, E_1 \vdash typ_i \leadsto t_i}{\mathbf{FV}(t_i) \subset \alpha^i}^i
                      p = \overline{z_i}^j x
                     E_2 = \langle \{\}, \{x \mapsto p\}, \{\}, \{\overline{y_i \mapsto \langle \mathbf{forall} \, \alpha. (p \, \alpha) \Rightarrow t_i, \mathbf{method} \rangle}^i \} \rangle
                     \delta_2 = \{ p \mapsto \overline{y_i}^{\ i} \}
                     p \not\in \mathbf{dom}(\delta_1)
    \overline{z_j}^j, \langle \Delta, \delta_1, I \rangle, \overline{E_1 \vdash \mathbf{class} (x \ l \ \alpha \ l'')} \overline{\mathbf{val} \ y_i \ l_i : typ_i \ l_i}^i \ \mathbf{end} \ l' \ \triangleright \ \langle \{ \ \}, \delta_2, \{ \ \} \rangle, E_2
                                                                                                                                                                                                                                                                             CHECK_DEF_CLASS
```

$$\begin{split} E &= \langle E^{\mathrm{M}}, E^{\mathrm{P}}, E^{\mathrm{F}}, E^{\mathrm{X}} \rangle \\ \Delta, E &\vdash typ' \leadsto t' \\ \Delta, \left(\overline{\alpha_i}^i\right) \vdash t' \text{ instance} \\ tnvs &= \overline{\alpha_i}^i \\ \textbf{duplicates} \left(tnvs\right) &= \emptyset \\ \overline{\delta, E \vdash id_k \leadsto p_k}^k \\ \textbf{FV} \left(\overline{\alpha_k'}^k\right) &\subset tnvs \\ E(id) &\triangleright p \\ \delta(p) &\triangleright \overline{z_j}^j \\ \overline{\Delta, I \cup I_2, E \vdash val_def_n \triangleright E_n^{\mathrm{X}}}^n \\ \textbf{disjoint doms} \left(\overline{E_n^{\mathrm{X}}}^n\right) \\ \overline{E^{\mathrm{X}}(x_k)} &\triangleright \langle \textbf{forall } \alpha''.(p \, \alpha'') \Rightarrow t_k, \textbf{method} \rangle^k \\ \left\{ \overline{x_k} &\mapsto \langle \textbf{forall } tnvs. \ \Rightarrow \{\alpha'' \mapsto t'\}(t_k), \textbf{let} \rangle^k \right\} &= \overline{E_n^{\mathrm{X}}}^n \\ \overline{x_k}^k &= \overline{z_j}^j \\ I_3 &= \left\{ \overline{(p_k \, \alpha_k')} \Rightarrow (p \, t')^k \right\} \\ \left(p \left\{ \overline{\alpha_i} &\mapsto \alpha_i'''^i \right\}(t')) \not\in I \end{split}$$

 $\frac{\overline{z_{j}^{j}}, \langle \Delta, \delta, I \rangle, E \vdash \mathbf{instance forall}}{\overline{z_{i}^{j}}^{i}, \overline{id_{k} \alpha_{k}^{\prime} l_{k}^{\prime\prime}}^{i}} \xrightarrow{k} (id \ typ^{\prime}) \overline{val} \underline{-def_{n} \ l_{n}}^{n} \ \mathbf{end} \ l^{\prime} \rhd \langle \{ \}, \{ \}, I_{3} \rangle, \epsilon}$

CHECK_DEF_

 $\overline{z_j}^j$, D_1 , $E_1 \vdash defs \triangleright D_2$, E_2 Check definitions, given module path, definitions and environment

$$\overline{z_{j}}^{j}, D, E \vdash \triangleright \epsilon, \epsilon \qquad \text{CHECK_DEFS_EMPTY}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash def \triangleright D_{2}, E_{2}$$

$$\overline{z_{j}}^{j}, D_{1} \uplus D_{2}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash def;;_{i}^{?} \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{2} \uplus D_{3}, E_{2} \uplus E_{3}$$

$$E_{1}(id) \triangleright E_{2}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \uplus E_{2} \vdash \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}$$

$$\overline{z_{j}}^{j}, D_{1}, E_{1} \vdash \mathbf{open} \ id \ l;;_{i}^{?} \overline{def_{i};;_{i}^{?}}^{i} \triangleright D_{3}, E_{3}$$

$$\text{CHECK_DEFS_OPEN}$$

Definition rules: 145 good 0 bad Definition rule clauses: 437 good 0 bad