

# SAX-PAC (Scalable And eXpressive PAccket Classification)

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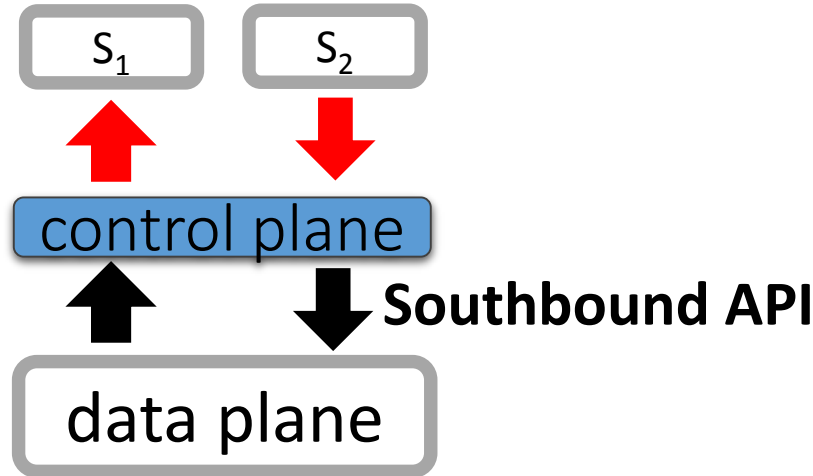
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# Outline

- Current state of the art in packet classification
- Impact of structural properties on representation efficiency
- Classifiers as Boolean expressions
- Proposed solutions
- Evaluation
- Summary and future work

# Representation of expressiveness on data plane



$$\begin{aligned} R_1 &= ([1, 3], [4, 31], [1, 28]) \\ R_2 &= ([4, 4], [2, 30], [4, 27]) \\ R_3 &= ([7, 9], [5, 21], [3, 18]) \end{aligned} \quad \begin{array}{c} \uparrow \\ \text{priority} \end{array}$$

# SW-based vs. TCAM-based solutions

SW-based:  $N = 4$  rules  $K = 2$  fields ~~prefixes~~ ranges

$$\begin{aligned} R_1 &= (100*, 001*) \\ R_2 &= (1010, 0001) \\ R_3 &= (000*, ****) \\ R_4 &= (001*, ****) \end{aligned}$$

Memory	Lookup time
$O(N)$	$O(\log^{k-1} N)$
$O(N^k)$	$O(\log N)$

TCAM-based:  $N = 3$  rules  $K = 3$  prefixes ~~ranges~~

$$\begin{aligned} R_1 &= ([1, 3], [4, 31], [1, 28]) \\ R_2 &= ([4, 4], [2, 30], [4, 27]) \\ R_3 &= ([7, 9], [5, 21], [3, 18]) \end{aligned}$$

Encoding	#TCAM entries
Binary	42+28+50=120
Gray	24+8+32=64

# Order-independence

If the rules of a classifier do not  
“intersect”, their order is not important.

$$\begin{aligned} R_1 &= ([1, 3], [4, 31], [1, 28]) \\ R_2 &= ([4, 4], [2, 30], [4, 27]) \\ R_3 &= ([7, 9], [5, 21], [3, 18]) \end{aligned}$$

- Example: prefixes of the same length
- Implicit creation of order-dependence for service policies

	cisco1	cisco2	cisco3	fw	ipc	acl
Order-independent rules	120	249	329	39962	48294	49779
Total	148	269	364	45723	49840	49870
Order-independent %	81	93	90	87	97	99

# Exploiting order-independence

- Adding new fields keep order-independence
- At most one rule is matched and it can be false-positive
- We can reduce space by skipping new fields

$$R_1 = ([1, 3], [4, 31])$$

$$R_2 = ([4, 4], [2, 30])$$

$$R_3 = ([7, 9], [5, 21])$$

$$R_1^{+1} = ([1, 3], [4, 31], [1, 28])$$

$$R_2^{+1} = ([4, 4], [2, 30], [4, 27])$$

$$R_3^{+1} = ([7, 9], [5, 21], [3, 18])$$

#Fields	Bin Encoding	Gray Encoding
2	6+7+10=23	6+4+8=18
3	42+28+50=120	24+8+32=64

incoming  
packet

$p=(4, 7, 5)$

lookup

$R_1$	[1,3]	[4,31]	[1,28]
$R_2$	[4,4]	[2,30]	[4,27]
$R_3$	[7,9]	[5,21]	[3,18]

false  
positive  
test

$R_2$	[4,4]	[2,30]	[4,27]
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result

$R_2$	[4,4]	[2,30]	[4,27]
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$p=(4, 2, 2)$

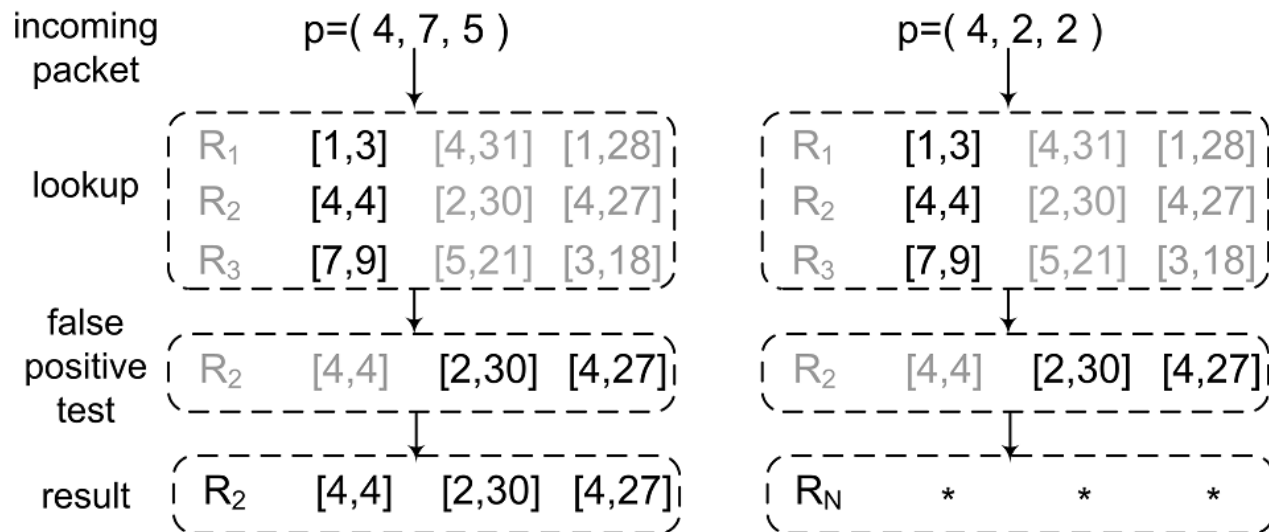
$R_1$	[1,3]	[4,31]	[1,28]
$R_2$	[4,4]	[2,30]	[4,27]
$R_3$	[7,9]	[5,21]	[3,18]

$R_2$	[4,4]	[2,30]	[4,27]
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$R_N$	*	*	*
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# Fields subset minimization (FSM)

**Problem 1:** Find a maximal subset  $M$  of fields of an order-independent classifier  $K$  s.t.  $K^{-M}$  is order-independent

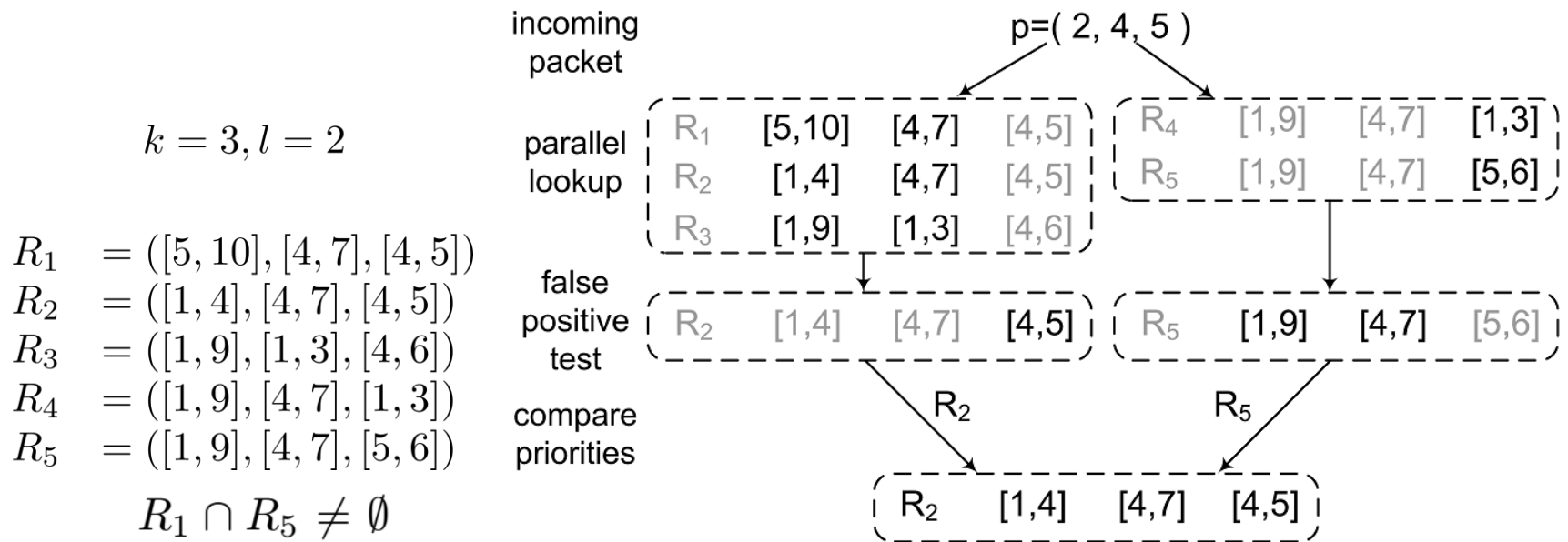


Fields	Binary Encoding	Gray Encoding
{1,2,3}	42+28+50=120	24+8+32=64
{1,2}	6+7+10=23	6+4+8=18
{1}	2+1+2=5	2+1+2=5

# Multi-group Representation

A classifier  $K$  is irreducible or order-dependent?

**Problem 2:** Given a classifier  $K$  on  $k$  fields and  $0 < l < k$ . Find an assignment of rules to a minimal number of disjoint groups s.t. every group is order-independent on  $l$  fields.





# Multi-group representation of rules subset

Number of groups  $>$  supported level of parallelism  $\beta$ ?

**Problem 3:** Given a classifier  $K$  on  $k$  fields,  $0 < l < k$ , and  $\beta > 0$ .

Find an assignment of a maximal subset of rules to  $\beta$  disjoint groups  
s.t. every group is order-independent on  $l$  fields.

$$k = 3, l = 2$$

$$\begin{aligned} R_1 &= ([5, 9], [4, 4], [4, 4]) \\ R_2 &= ([2, 4], [5, 7], [5, 5]) \\ R_3 &= ([2, 3], [1, 4], [4, 6]) \\ R_4 &= ([1, 5], [1, 7], [1, 3]) \\ R_5 &= ([1, 9], [1, 7], [1, 6]) \end{aligned}$$

$$\{R_1, R_2, R_3, R_4\}$$

Order-independent part ( $I$ )

$$\begin{bmatrix} R_1 & [5,9] & [4,4] & [4,4] \\ R_2 & [2,4] & [5,7] & [5,5] \\ R_3 & [2,3] & [1,4] & [4,6] \end{bmatrix} \quad \begin{bmatrix} R_4 & [1,5] & [1,7] & [1,3] \end{bmatrix}$$

Order-independent part ( $I$ )

$$\begin{bmatrix} R_1 & [5,9] & [4,4] & [4,4] \\ R_2 & [2,4] & [5,7] & [5,5] \\ R_4 & [1,5] & [1,7] & [1,3] \end{bmatrix}$$

The rest ( $D$ )

$$\begin{bmatrix} R_5 & [1,9] & [1,7] & [1,6] \end{bmatrix}$$

The rest ( $D$ )

$$\begin{bmatrix} R_3 & [2,3] & [1,4] & [4,6] \\ R_5 & [1,9] & [1,7] & [1,6] \end{bmatrix}$$

Max order-independent set on 3 fields

# Classifiers as Boolean expressions

$$R_1 = (100*, 001*)$$

$$R_2 = (1010, 0001)$$

$$R_3 = (000*, ***)$$

$$R_4 = (001*, ****)$$

$$(x_1 \wedge \bar{x}_2 \wedge \bar{x}_3 \wedge \bar{x}_5 \wedge \bar{x}_6 \wedge x_7) \vee$$

$$(x_1 \wedge \bar{x}_2 \wedge x_3 \wedge \bar{x}_4 \wedge \bar{x}_5 \wedge \bar{x}_6 \wedge \bar{x}_7 \wedge x_8) \vee$$

$$(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee$$

$$(\bar{x}_1 \wedge \bar{x}_2 \wedge x_3)$$

**MinDNF:** For a given Boolean function  $f$ , find a minimal size of DNF for  $f$

**MinDNF:**

$$R_1 = (100*, 001*)$$

$$R_2 = (1010, 0001)$$

$$R_{34} = (00 **, ***)$$

**FSM with per  
field resolution:**

$$R_1^{-1} = (100*)$$

$$R_2^{-1} = (1010)$$

$$R_3^{-1} = (000*)$$

$$R_4^{-1} = (001*)$$

**FSM with per  
bit resolution:**

$$R_1^{-6} = (10)$$

$$R_2^{-6} = (11)$$

$$R_3^{-6} = (00)$$

$$R_4^{-6} = (01)$$

# Exact algorithm

FSM is NPC (reduction from SetCover)

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**Algorithm**     $\text{FSMBINSEARCH}(k, \min, \max)$

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```
1:  $m = \lfloor \frac{\min + \max}{2} \rfloor$ 
2: if  $\min = \max$  then return  $m$ 
3: for  $M \subseteq \{1, \dots, k\}, |M| = m$  do
4:   if  $\text{ISORDERINDEPENDENT}(M)$  then
5:      $\min = m$ 
6:   return  $\text{FSMBINSEARCH}(k, \min, \max)$ 
7:  $\max = m - 1$ 
8: return  $\text{FSMBINSEARCH}(k, \min, \max)$ 
```

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**Theorem:**  $\text{FSMBinSearch}(k, 0, k - 1)$  runs in time  $O(k2^{k-1}N^2)$

# Approximate algorithms

$$\begin{array}{rcl} & S_1 & S_2 & S_3 \\ R_1 & = & ([1, 3], [4, 31], [1, 28]) \\ R_2 & = & ([4, 4], [2, 30], [4, 27]) \\ R_3 & = & ([7, 9], [5, 21], [28, 31]) \end{array}$$

Heuristics: algorithms for *SetCover* and *MaxSetCoverage*

$$U = \{(i, j) | i < j, i, j \in [1, N]\} = \{(1, 2), (1, 3), (2, 3)\}$$

Define  $k$  sets  $S_1, \dots, S_k$  (one per field) to cover  $U$

$S_i, 1 \leq i \leq k$ , contains all pairs of rules that do not intersect in this field

$$S_1 = \{(1, 2), (1, 3), (2, 3)\} \quad S_2 = \emptyset \quad S_3 = \{(2, 3)\}$$

**Theorem:** *FSM* is reducible to *SetCover* in  $O(kN^2)$  time with approximation factor  $2 \ln(N) + 1$

# Evaluation

6 classifiers with real parameters (see paper for more examples)

#		Max OI part 5 fields			Multi-group representation									
	Total				1-field groups					2-field groups				
	rules	OI size	FSM	{0,1}	all	95%	99%	$\leq 2$	$\leq 5$	all	95%	99%	$\leq 2$	$\leq 5$
1	584	538	0,1,3,4	406	15	8	13	2	8	10	4	7	3	4
2	269	249	0,1,4	246	4	2	3	1	1	2	2	2	0	0
3	364	329	0,1,3,4	324	7	3	5	2	4	4	2	3	1	14
4	49870	49779	0,1,4	49768	16	1	1	6	9	12	1	1	5	7
5	47276	44178	0,1,3,4	43819	67	5	13	19	32	39	2	5	10	20
6	48885	48826	0,1,2,4	48755	20	3	3	15	15	12	1	1	5	9

# Summary

- Semantically equivalent: more fields imply less efficient representation
- Representation with false-positive: more fields – more efficient representation
- Structural properties can significantly improve time-space trade-off
- No restrictions on representation of every group (we define only additional abstraction layer)

# Ongoing and future work

- Consider special cases as a FIB representation

G. Retvari et al., **Compressing IP Forwarding Tables: Towards Entropy Bounds and Beyond** SIGCOMM 13

- Identify additional structural properties
- Composition of structural properties
- Application to neighboring fields: data bases, program optimization, etc.

Thank you