Geosphere: Consistently Turning MIMO Capacity into Throughput

Konstantinos Nikitopoulos

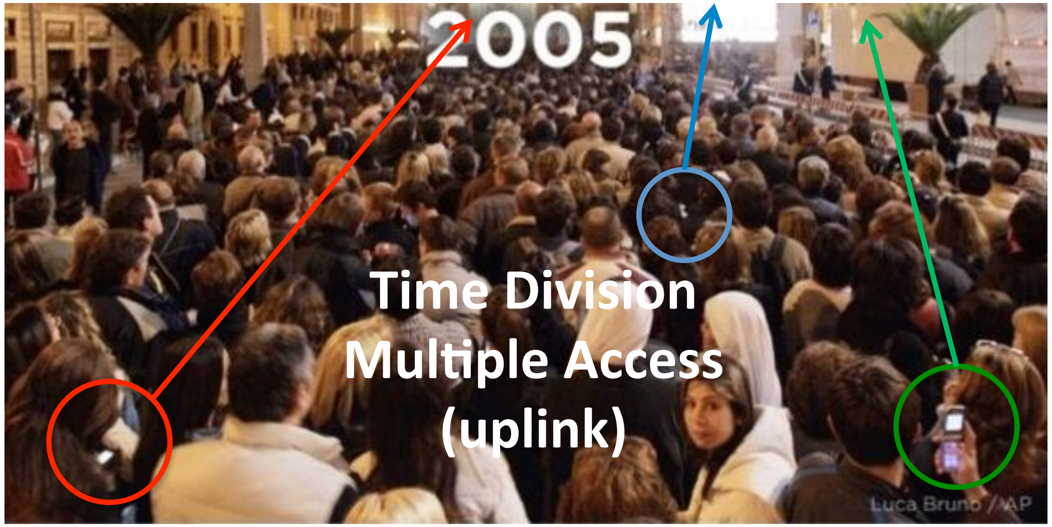
5G Innovation Centre
University of Surrey

Juan Zhou, Ben Congdon , Kyle Jamieson

Department of Computer Science
University College London

SIGCOMM, Chicago 2014

Need to Scale Wireless Capacity…



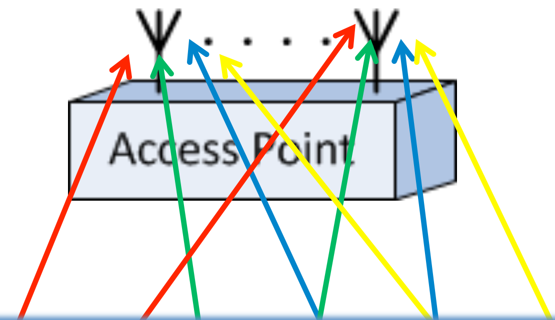
Time Division
Multiple Access
(uplink)

Increasing the number of users results in a reduction in *per-user* throughput

How can we scale capacity?

Photo credit: www.cultofmac.com

MIMO with Spatial Multiplexing



Question: How can we most efficiently demultiplex the mutually interfering information streams?

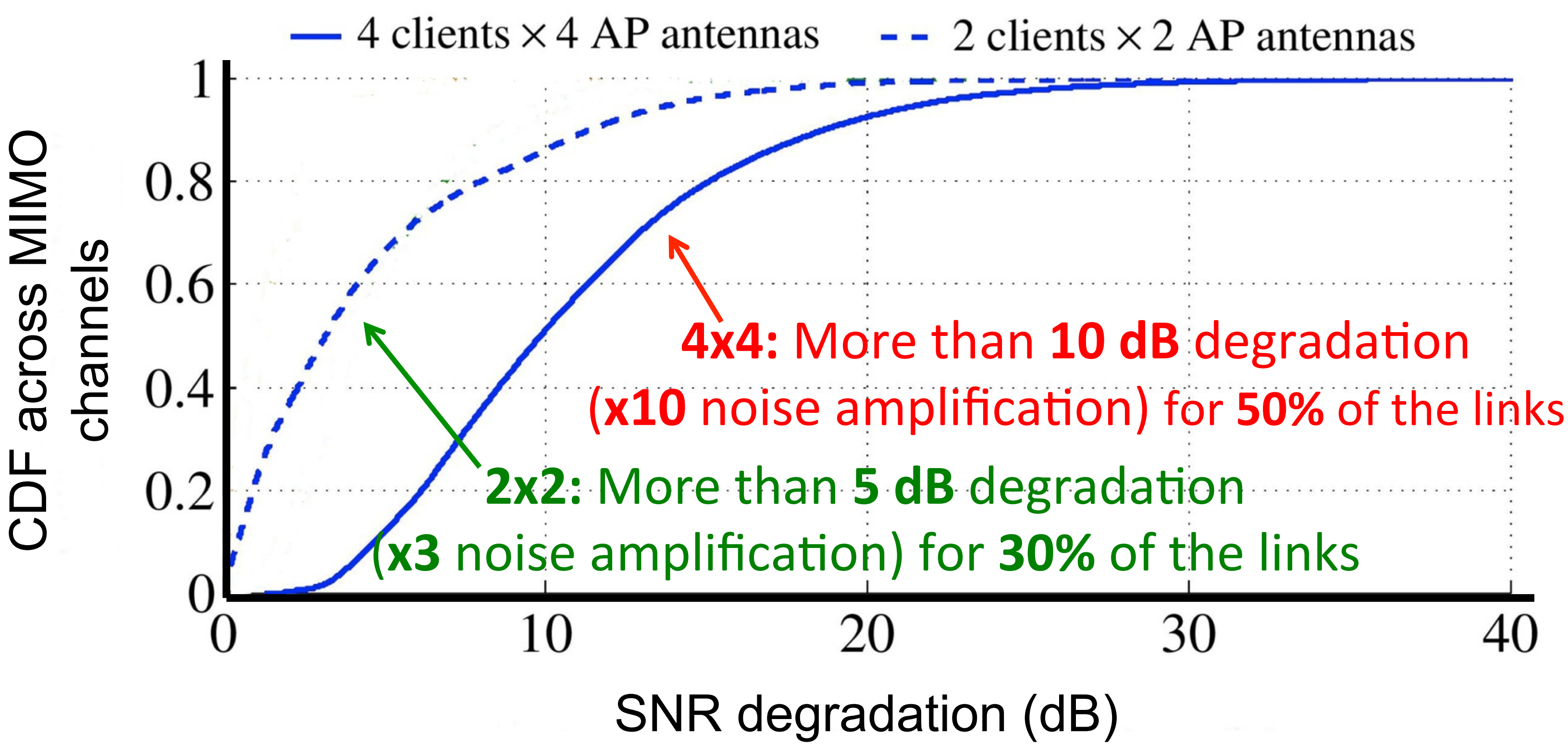
Photo credit: www.cultofmac.com

Motivation

- **Problem 1:**

Zero-forcing (e.g., [SAM, Mobicom ’09], [Bigstation, Sigcomm ’13]) suffers as APs get more antennas.

Motivation: Zero-forcing suffers



Motivation

❑ **Problem 1:**

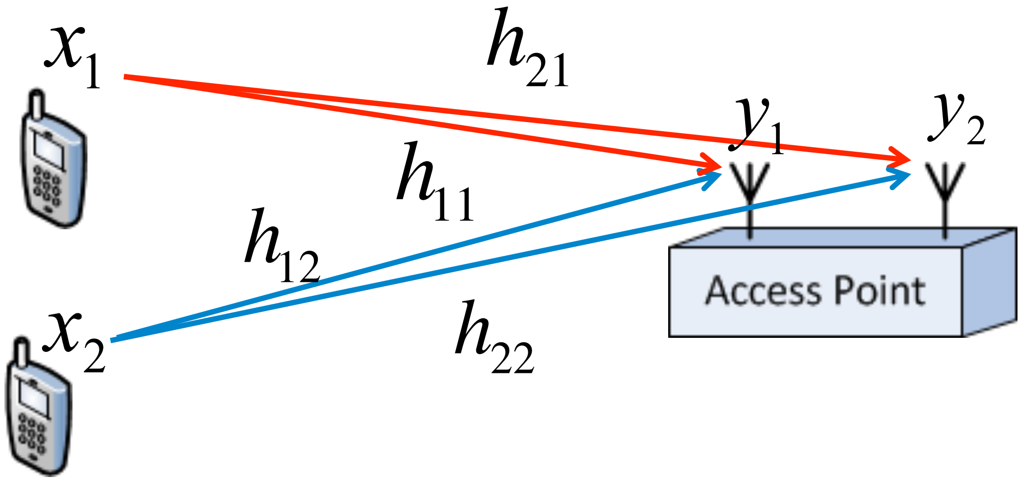
Zero-forcing (e.g., [SAM, Mobicom '09], [Bigstation, Sigcomm '13]) suffers as APs get more antennas.

Geosphere: Enables optimal detection at a reasonable complexity by employing geometrical reasoning.

❑ **Problem 2:**

Optimal solutions are very computationally complex and, therefore, cannot scale to high transmission rates.

Zero-Forcing Amplifies Noise

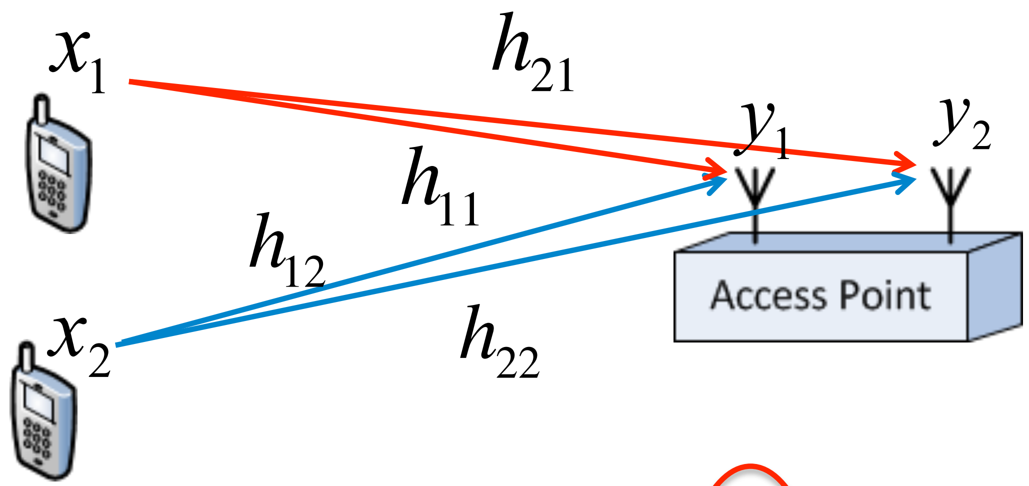


The Noiseless Case:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Leftrightarrow \mathbf{Y} = \mathbf{HX}$$

The Zero-Forcing solution is: $\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{H}^{-1}\mathbf{Y}$

Zero-Forcing Amplifies Noise



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Leftrightarrow \mathbf{Y} = \mathbf{HX} + \mathbf{N}$$

With noise Zero-Forcing gives $\hat{\mathbf{X}} = \mathbf{H}^{-1}\mathbf{Y} = \mathbf{H}^{-1}(\mathbf{HX} + \mathbf{N})$

$$\hat{\mathbf{X}} = \mathbf{X} + \underbrace{\mathbf{H}^{-1}\mathbf{N}}_{\text{Noise amplification}}$$

Maximum-Likelihood Detection and Sphere Decoding

$$\hat{\mathbf{x}} = \arg \min_{\text{possible } \mathbf{x}} \left\| \mathbf{y} - \mathbf{H}\mathbf{x} \right\|^2$$

- Minimizes detection errors
- Finding the ML solution by exhaustive search is impractical

Sphere Decoder uses QR decomposition to transform the problem into

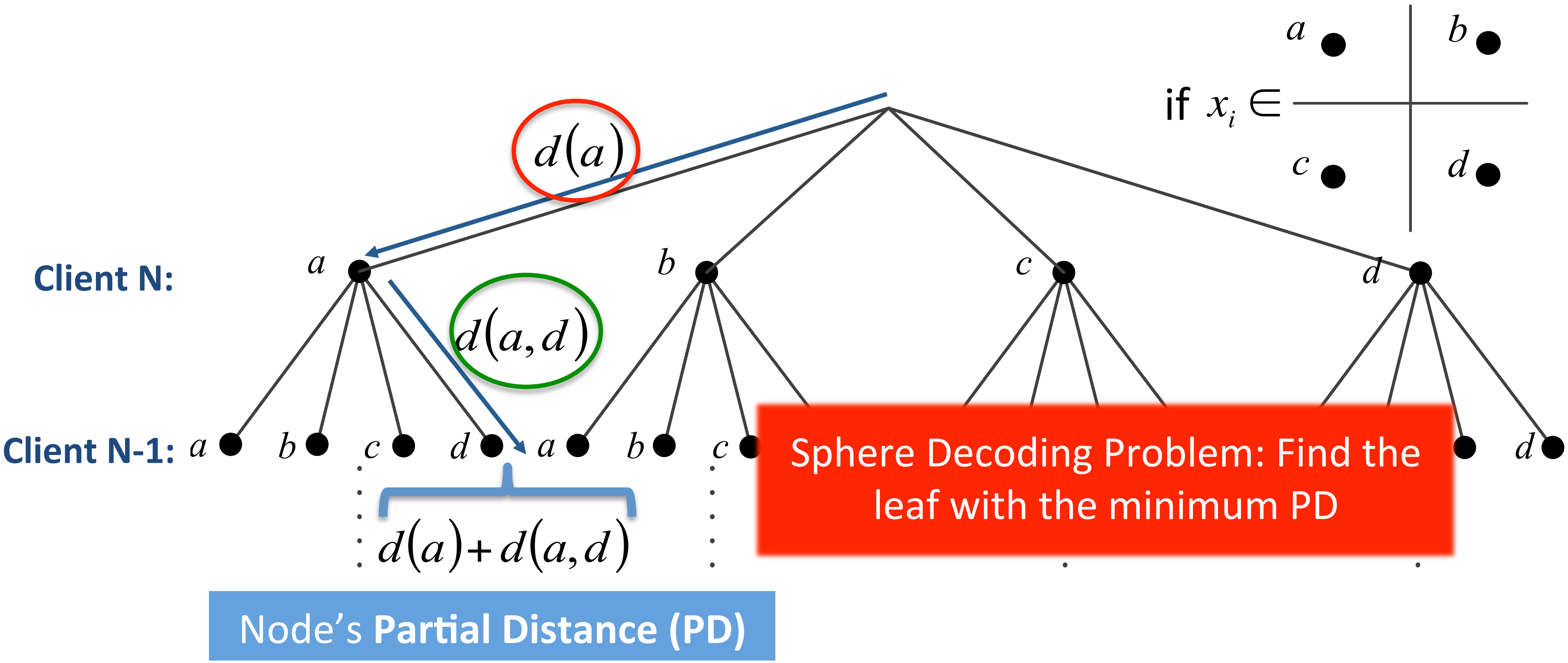
$$\hat{\mathbf{x}} = \arg \min_{\text{possible } \mathbf{x}} \underbrace{\left\| \mathbf{y}' - \mathbf{R}\mathbf{x} \right\|^2}$$

$$\left\| \begin{bmatrix} y'_1 \\ \vdots \\ y'_N \end{bmatrix} - \begin{bmatrix} R_{11} & \cdots & R_{1N} \\ 0 & \ddots & \vdots \\ 0 & 0 & R_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \right\|^2 \begin{array}{l} \rightarrow d(x_N, \ldots, x_1) \\ \rightarrow d(x_N, x_{N-1}) \\ \rightarrow d(x_N) \end{array}$$

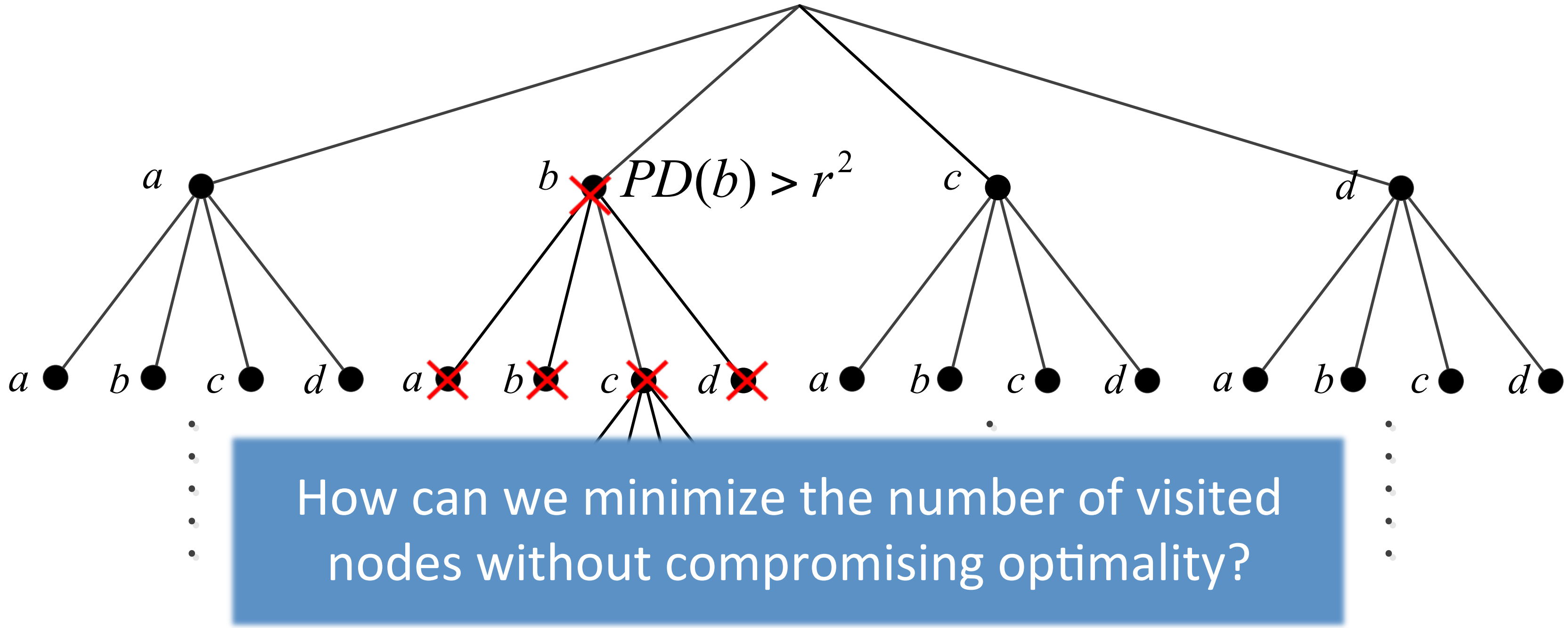
Maximum-Likelihood Detection and Sphere Decoding

Therefore, the ML problem transforms to:

$$\hat{\mathbf{x}} = \min_{x_i \in \{a,b,\ldots\}} \left\{ d(x_N) + d(x_N, x_{N-1}) + \ldots \right\}$$



Maximum-Likelihood Detection and Sphere Decoding

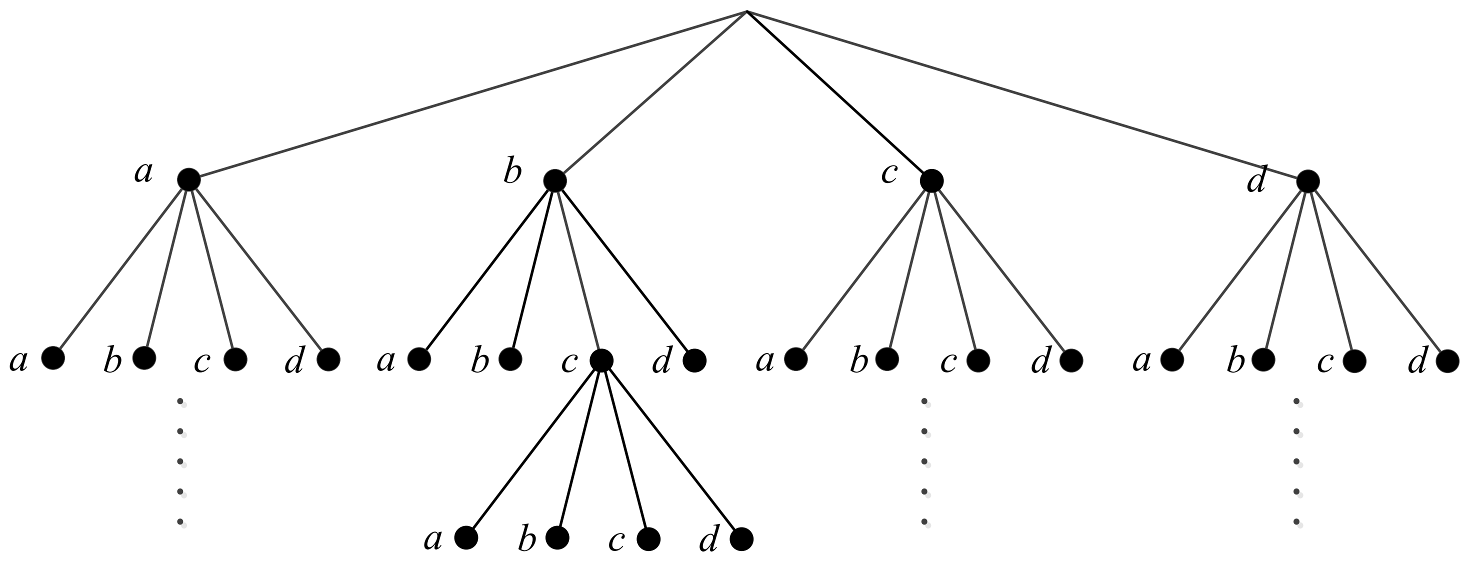


How can we minimize the number of visited nodes without compromising optimality?

- To avoid exhaustive search, original SDs search just a subset of tree nodes (with $PD<r^2$).
- Such approaches cannot guarantee the ML solution.

Geosphere's (and ETH-SD's[1]) tree traversal and pruning

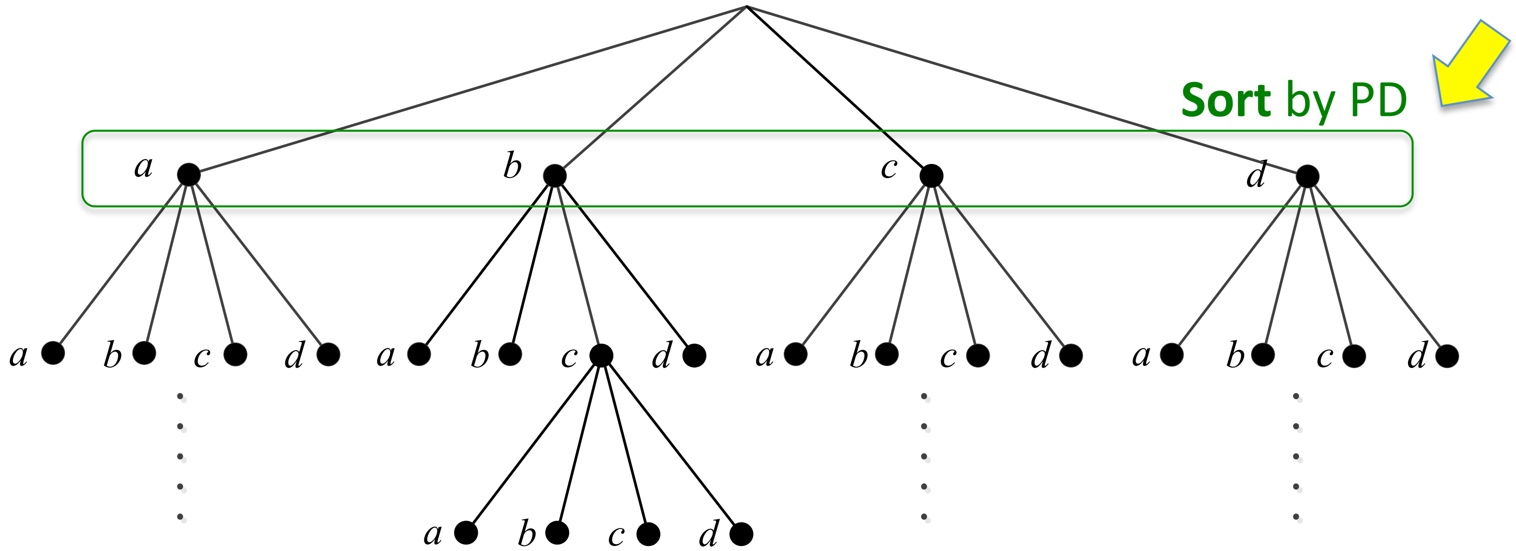
Example: 3x3 system with four element constellation (= 64 tree nodes)



[1] Burg, Andreas, et al. "VLSI implementation of MIMO detection using the sphere decoding algorithm." *IEEE Journal of Solid-State Circuits,* 40.7 (2005): 1566-1577.

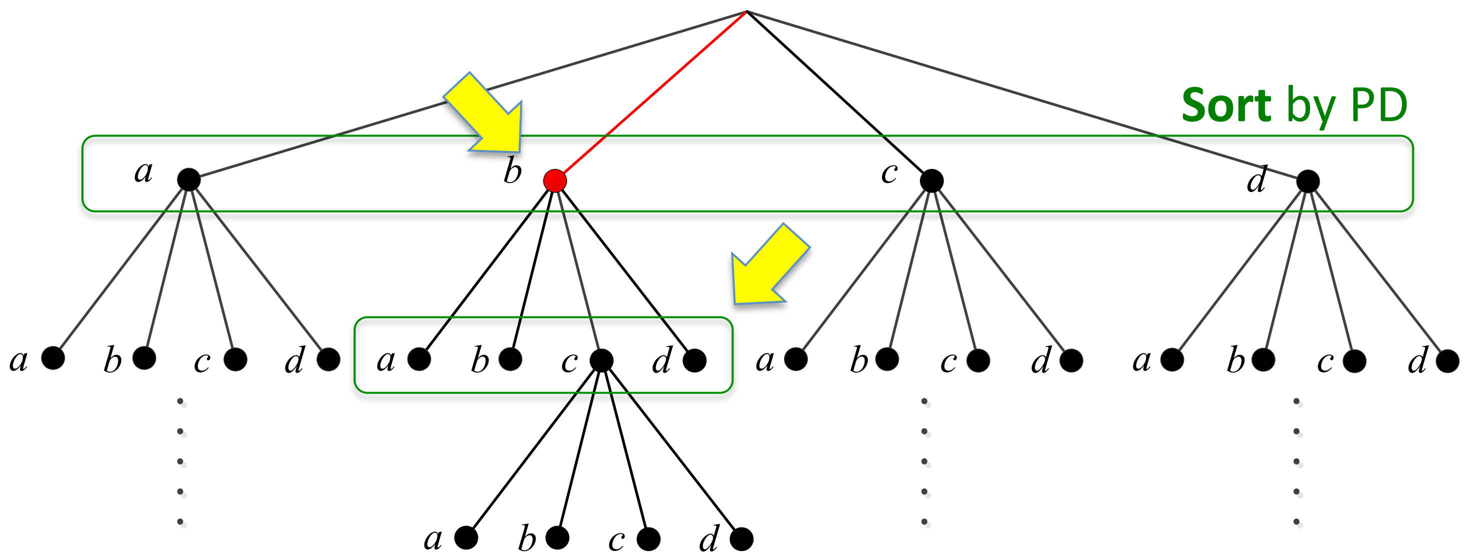
Geosphere's (and ETH-SD's) tree traversal and pruning

$r^2 = +\infty$



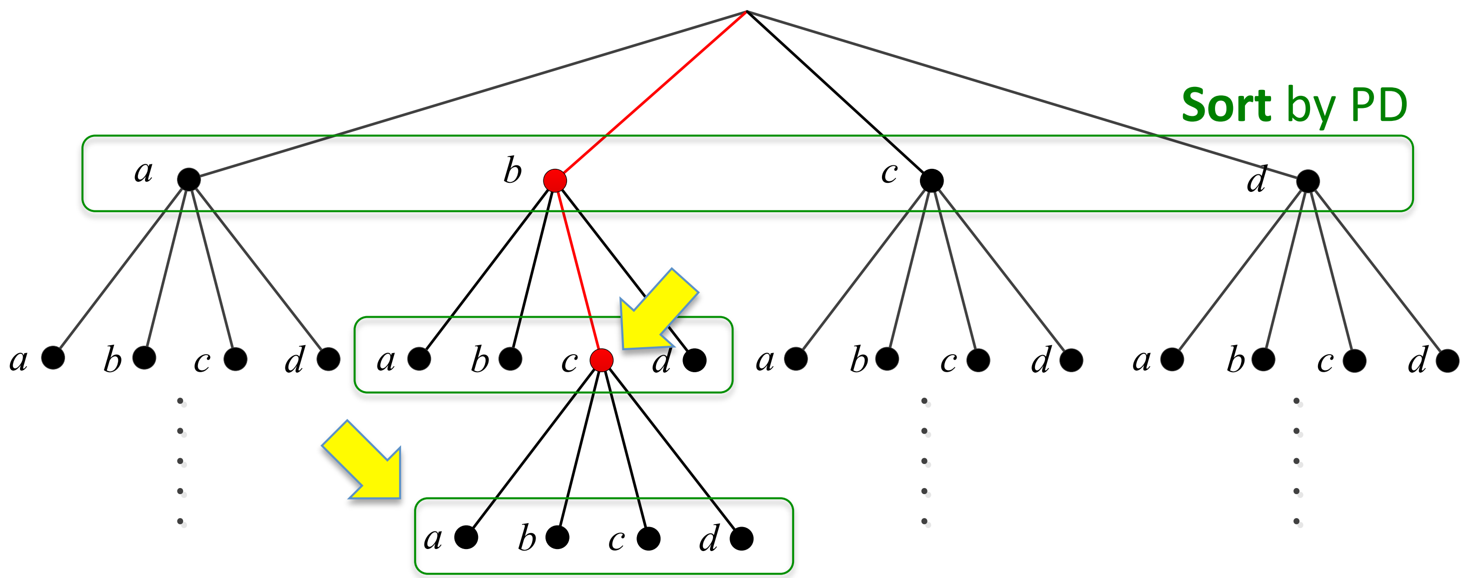
Geosphere's (and ETH-SD's) tree traversal and pruning

$r^2 = +\infty$



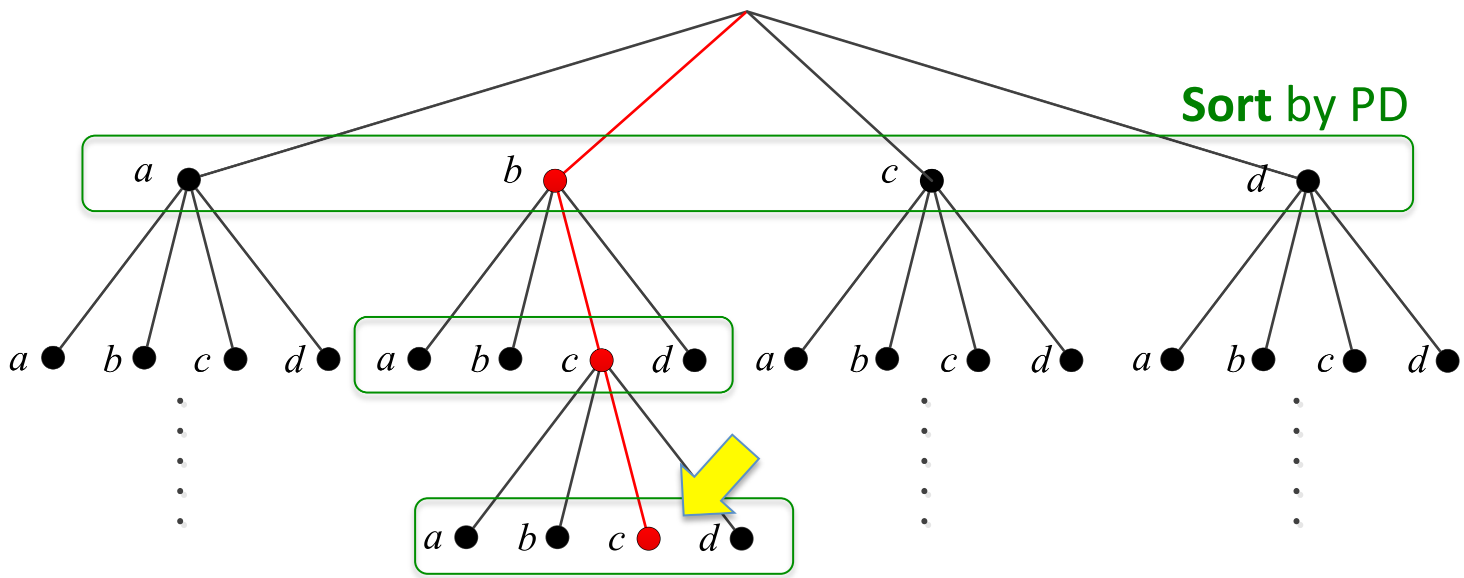
Geosphere's (and ETH-SD's) tree traversal and pruning

$r^2 = +\infty$



Geosphere's (and ETH-SD's) tree traversal and pruning

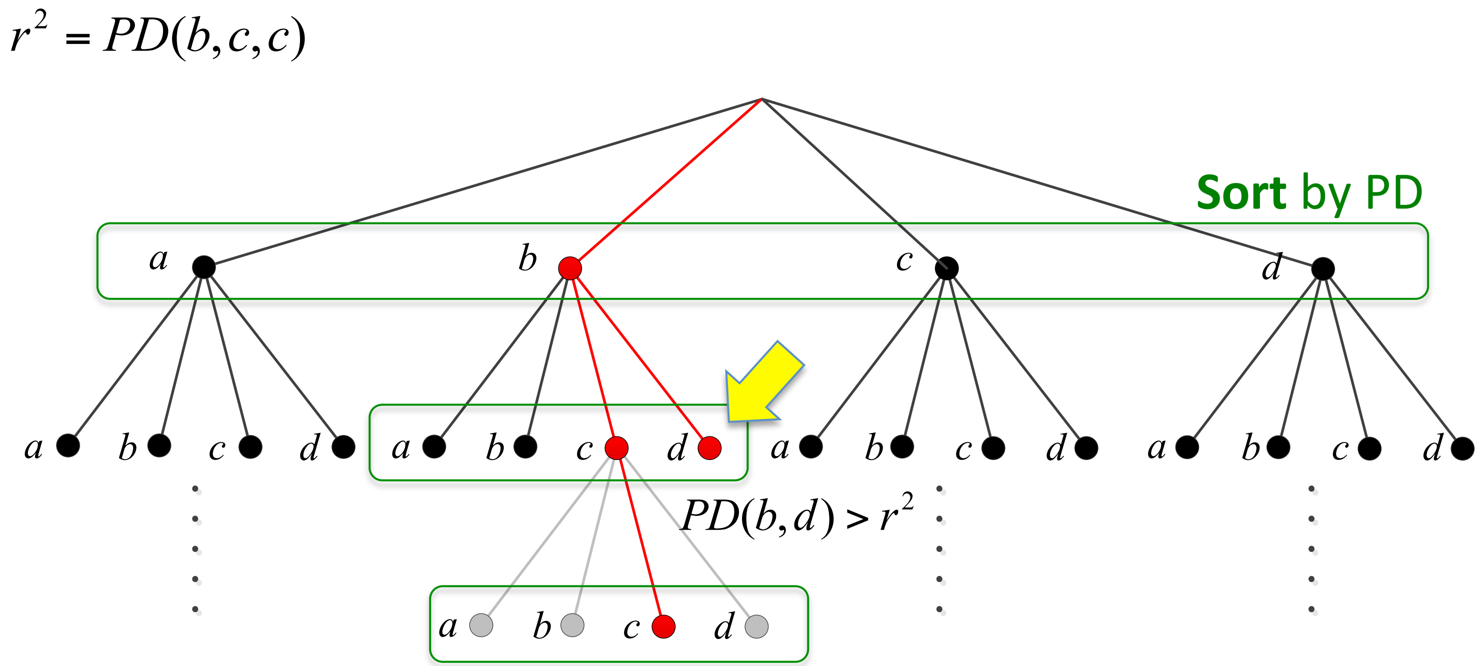
$\cancel{r^2 = +\infty}$



Radius update

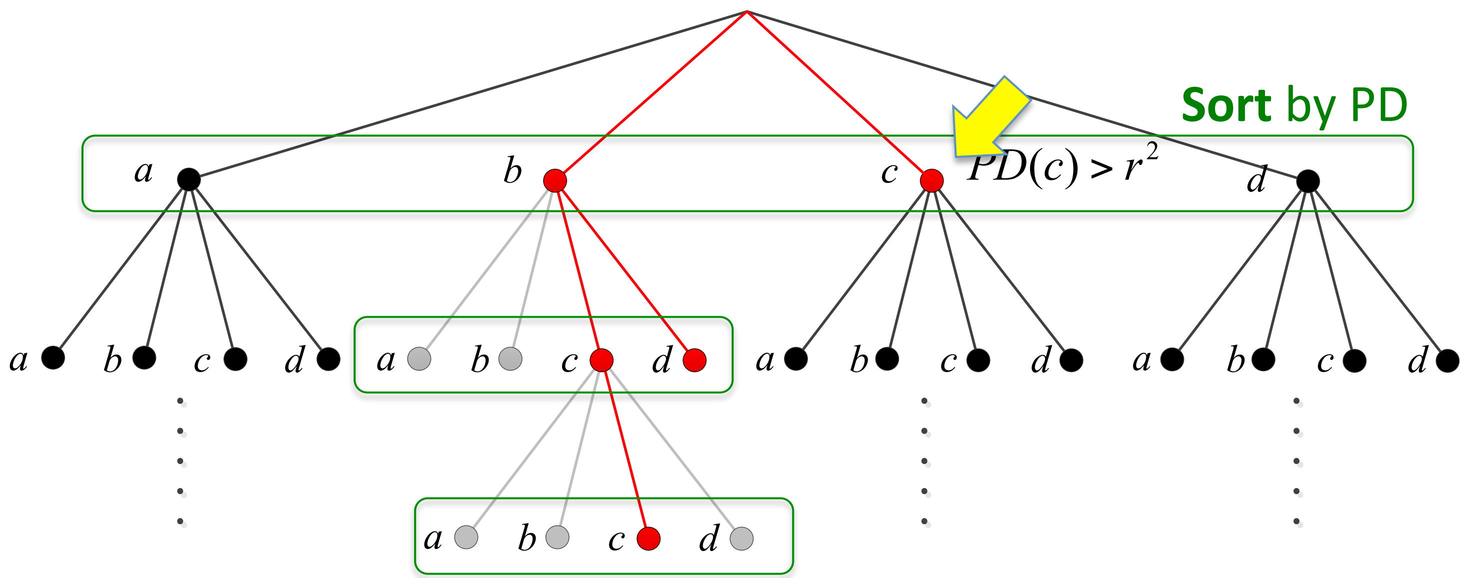
$$r^2 = PD(b,c,c)$$

Geosphere's (and ETH-SD's) tree traversal and pruning



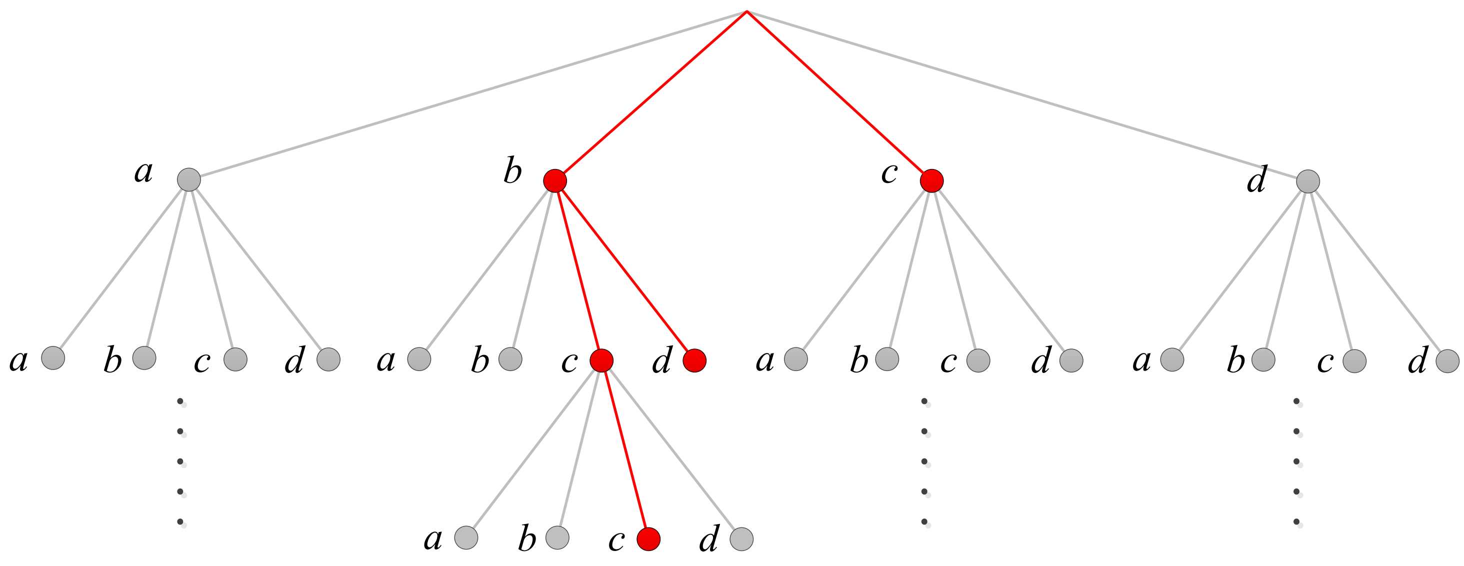
Geosphere's (and ETH-SD's) tree traversal and pruning

$r^2 = PD(b,c,c)$



Geosphere's (and ETH-SD's) tree traversal and pruning

$r^2 = PD(b,c,c)$

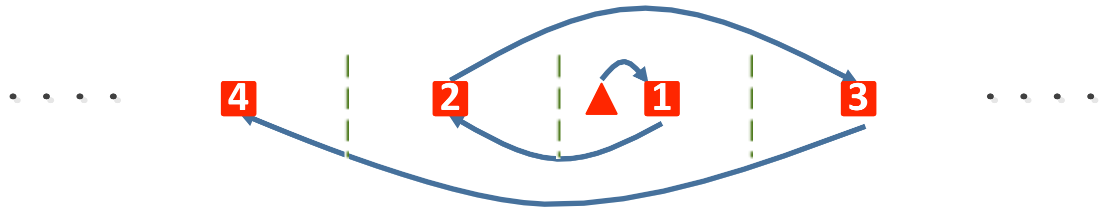


We can find the ML solution by visiting **only 5 nodes**

How can we minimize sorting complexity?

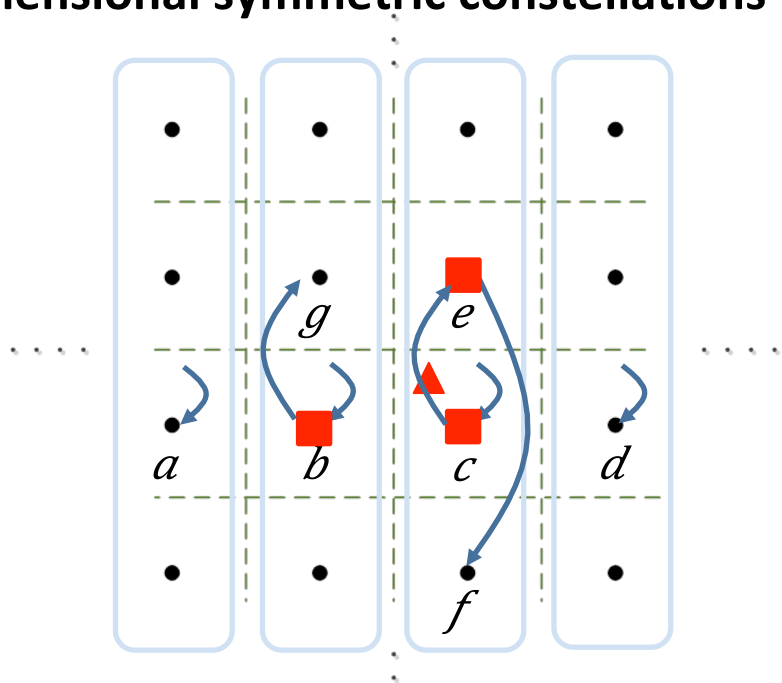
Traditional Sorting and PD calculations

Single Dimensional Constellations



Visiting **3** nodes requires **3** PD calculations

Dense two-dimensional symmetric constellations



Visiting **3** nodes requires **(constellation size)$^{1/2}$+2** PD calculations

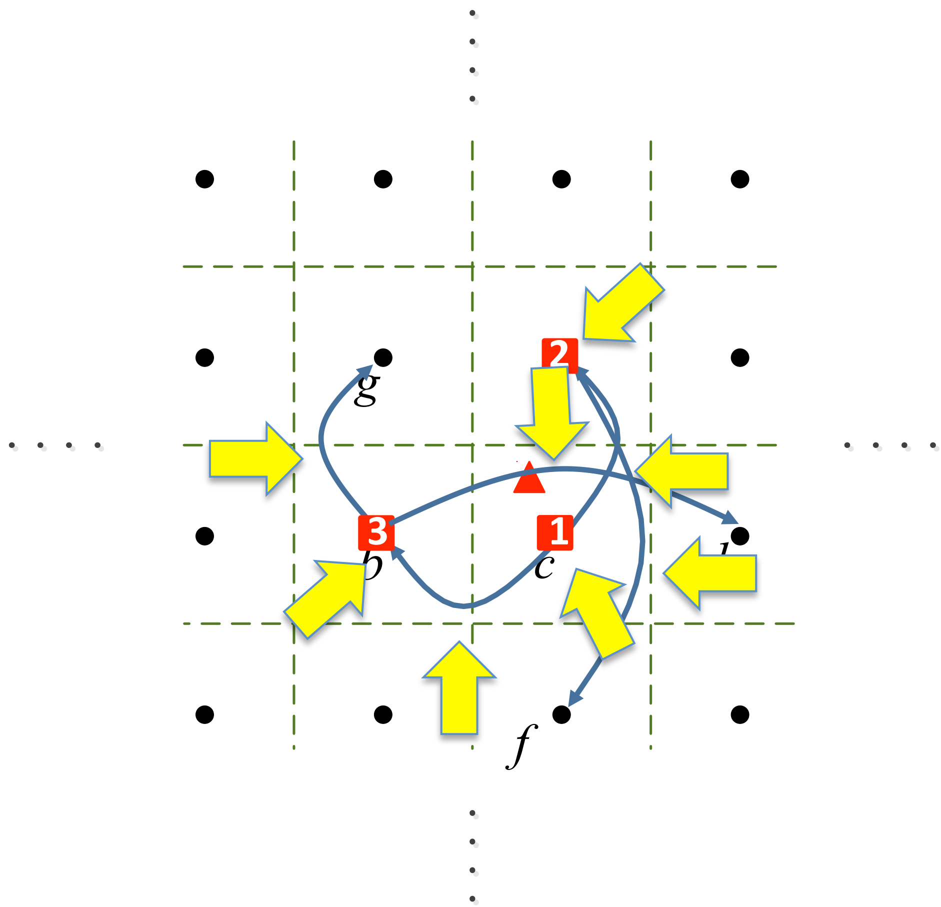
Half distance between symbols

Transmitted Symbol

Received Signal

Selected Symbol

Geosphere's 2D zig-zag



Half distance between symbols

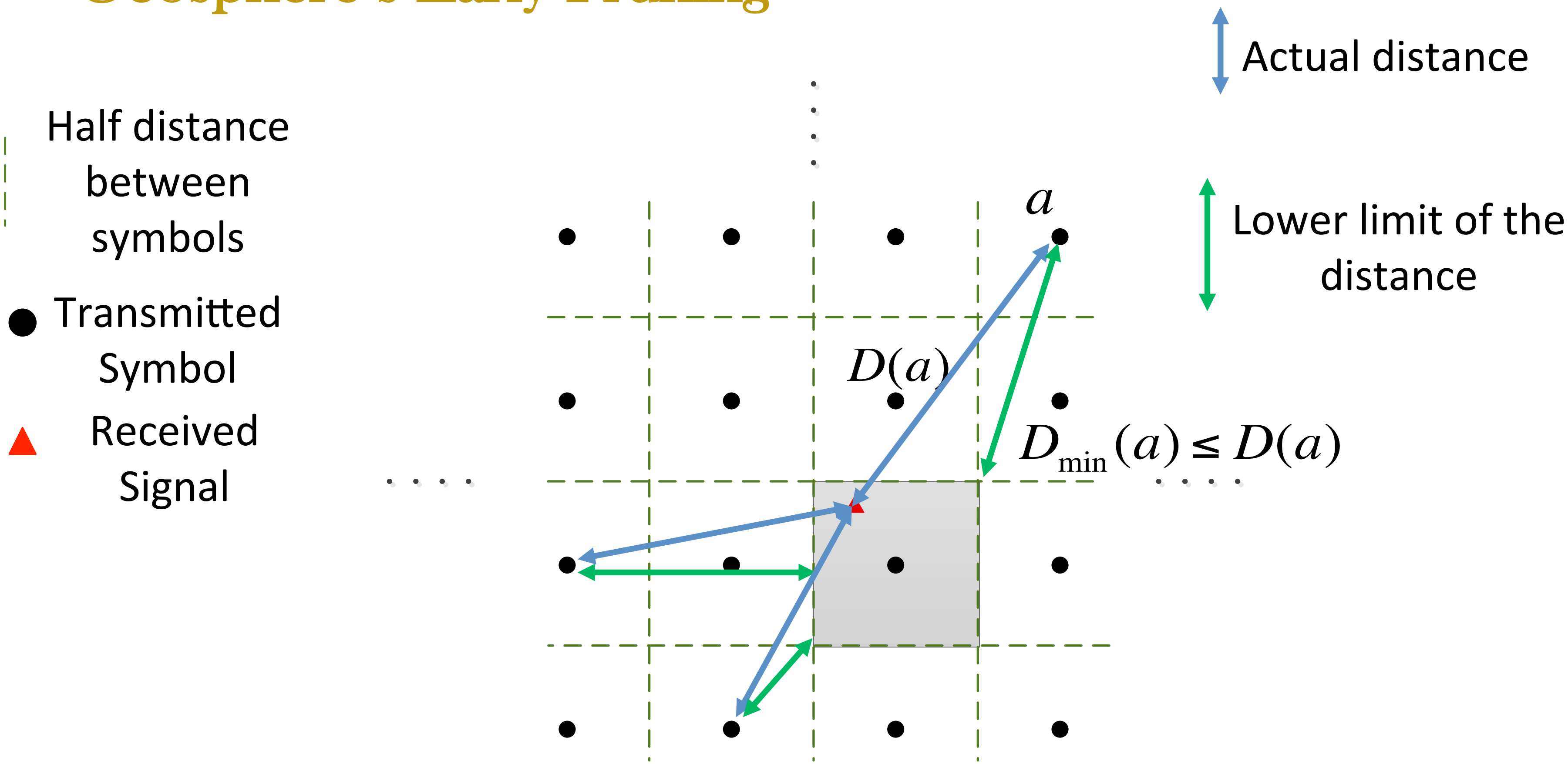
Transmitted Symbol

Received Signal

Selected Symbol

Visiting **3** nodes requires **4** PD calculations

Geosphere's Early Pruning



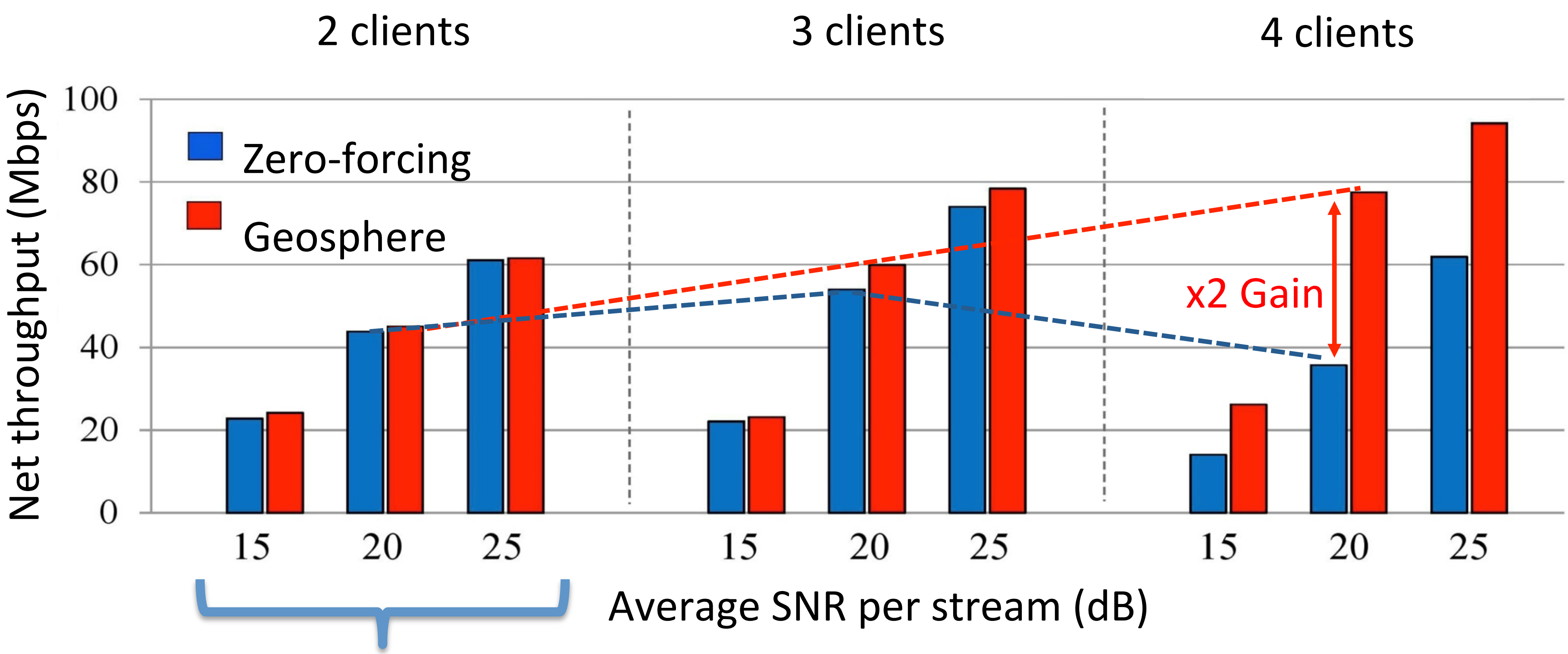
$D_{\min}$ can be pre-calculated for all constellation symbols (function of QAM geometry)

We can avoid PD calculations by first checking $D_{\min}$ meets the pruning criterion.

Evaluation

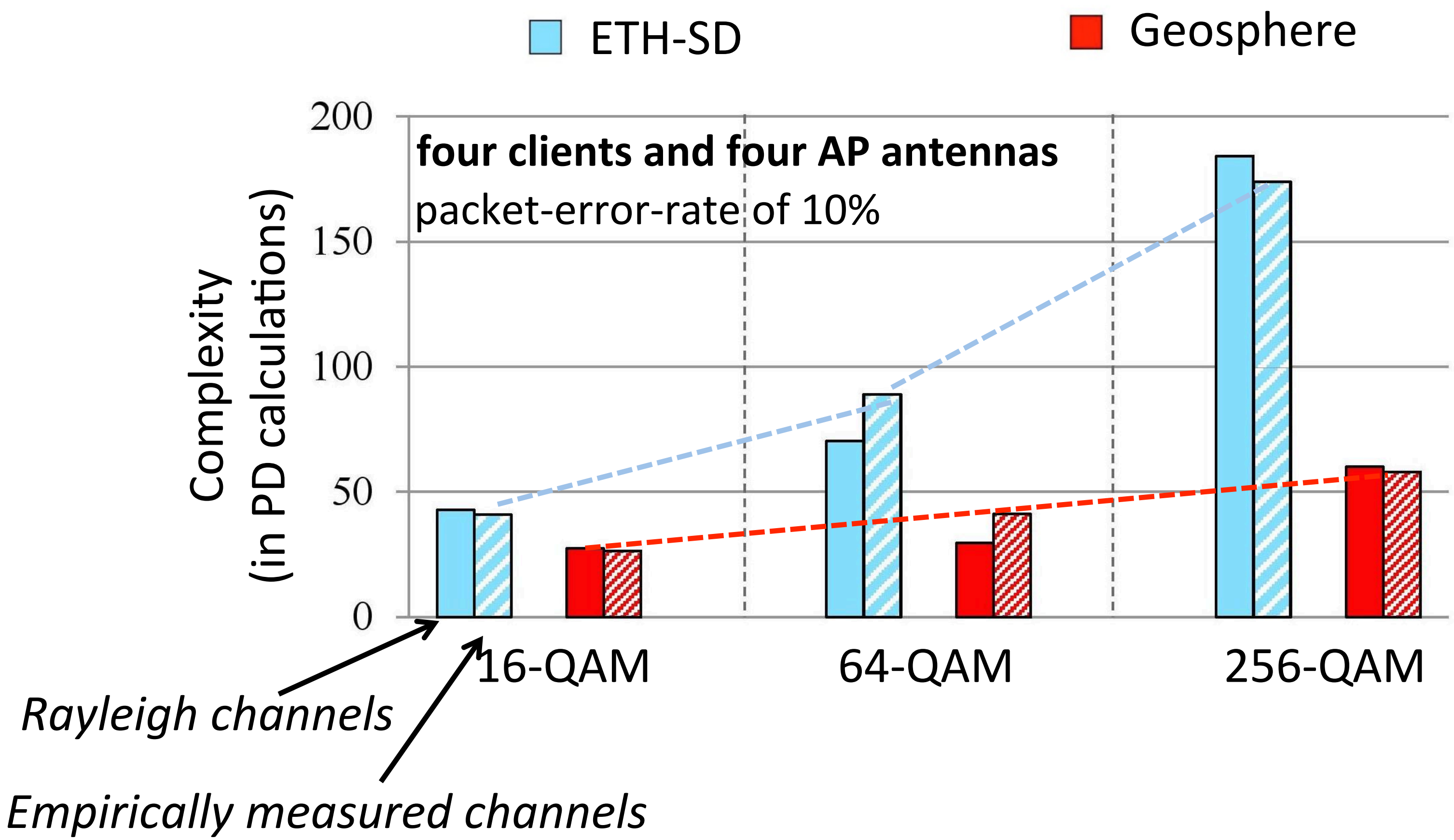
- Both by using:
  - WARP-based **testbed** in indoor (office) environment
    (5GHz band, 20MHz bandwidth, 64-OFDM)
  - **Simulations**
    (using Rayleigh and empirically measured channels)
- We compare Geosphere:
  - With **Zero-forcing** for **throughput**
  - With **ETH-SD** for **complexity**

Geosphere's Throughput Gains for 4 AP Antennas



ZF is less suboptimal when
we sacrifice throughput

Computational Complexity Gains for 4 AP Antennas



For two clients the complexity is
~3 PD calculations across all QAM constellations!

Related Work

- The sphere decoding literature is very rich[1].
- However, already proposed approaches
  - Cannot efficiently support for very dense symbols constellations **&**,
  - Guarantee optimal performance **&,**
  - Efficiently adjust their complexity according to the MIMO channel utilization.

[1]"SD Sequence determination" IEEE SARNOFF '09], ["K-Best SD" IEEE JSAC '06/10, IEEE ISCAS '08, IEEE TVLSI '09],["Fixed Complexity SD", IEEE TCOM '08], "Probabilistic Pruning" IEEE TSP '08]…

Conclusions

- Low complexity detection become **highly suboptimal** when increasing the number of antennas.
- ML detection allows scaling capacity in MIMO networks but it very **complex**.
- Geosphere **enables ML detection pragmatic systems** with dense constellations
- Future research will be focused in extending Geosphere to
  - Shannon capacity achieving **soft-receiver processing**
  - **Large MIMO systems**