In a first task we should estimate the average decay length of K^+ using data (100'000 measurements of decay lengths, (dl_i)) of an already conducted experiment (see figure 1), where also a beam of particles with a fixed momentum of 75 GeV/c was used. However, the beam consisted only of 16% of K^+. The remaining 84% consisted of π^+ (with a known average decay length of $\beta\gamma c\tau_\pi$ = 4.188 km = $adl_{\pi+}$).

Figure 1 (Histogram)

To estimate the average decay length of K^+ (adl_{K+}) we used the method of maximum likelihood with the following equation, which was subsequently maximized:

$$\ln\left(L(dl_i|adl_{K^+})\right) = \sum_{i=1}^{N} \ln\left(\frac{0.84}{adl_{\pi^+}} * e^{\frac{-dl_i}{adl_{\pi^+}}} + \frac{0.16}{adl_{K^+}} * e^{\frac{-dl_i}{adl_{K^+}}}\right) \tag{1.1}$$

The equation is constructed by two exponential distributions, weighted with the corresponding probability that a particle belongs to this distribution (with this parameters). Implementing this resulted in a maximum likelihood for an average decay length of K^+ of (562 ± 10) m (see figure 2).

The error on dl_{K^+} was calculated by

$$\ln(L(dl_{K^+} \pm \sigma)) = \ln L(dl_{K^+}) - \frac{1}{2} \tag{1.2}$$

Figure 2

From this τ_{K^+} can be calculated by

$$\tau_{K^+} = \frac{adl_{K^+}}{\beta\gamma c}, \tag{1.3}$$

where $\beta = |p_K|/E_K$ and $\gamma = E_K/m_K$ with $E = \sqrt{m^2 + |p|^2}$, $|p| = \sqrt{p_x^2 + p_y^2 + p_z^2} = 75\ GeV/c$, and $m_{K^+} = (493.677 \pm 0.016) * 10\text{^}6\ eV$.

This resulted in a maximum likelihood for τ_{K^+} at (1.235 ± 0.022) * 10^{-8} s. Compared to the literature value of (1.2380±0.0020) * 10^{-8} s this results seems reasonable.