Designing a particle physics experiment

Data analysis 2017 – Group project IV

Michele Atzeni, Arno Gadola, Olaf Steinkamp, Andreas Weiden

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Motivation

In this project you perform a simulation study to optimize the layout of a simple particle physics experiment. The experiment should study decays of charged kaons (K^+) into a charged pion (π^+) and a neutral pion (π^0):

$$K^+ \to \pi^+ \pi^0 \,.$$

The K^+ is a composite particle that consist of an up quark and a strange antiquark. It is unstable and one of its decay modes is that into a pair of pions. The π^+ consists of an up quark and a down antiquark, the π^0 is a mixture of up and down quark/antiquark pairs. The π^+ and π^0 are also unstable (the mean lifetime of the π^+ is roughly twice that of the K^+, while the mean lifetime of the π^0 is in fact several orders of magnitude shorter), but for the sake of simplicity we will neglect this in the simulation study and assume that the π^+ and π^0 do not decay.

1 Setup

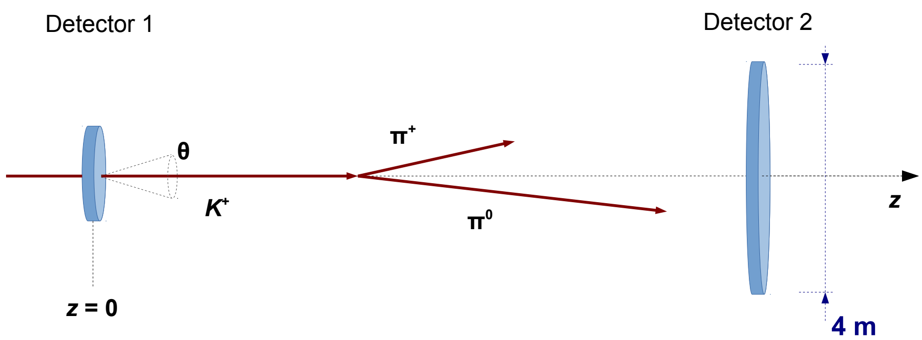

Figure 1: Sketch of the setup.

A sketch of the planned setup is shown in Fig. 1. The experiment uses a beam of K^+ with fixed momentum of $p_K = 75$ GeV/c and consists of two detectors. The first detector is used to detect the incoming K^+ and the second detector is used to detect the two pions emitted in the decay. This second detector has a circular cross section with a diameter of 4 m and is centred on the beam axis. The purpose of your study is to determine the optimal distance between the two detectors at which the acceptance of the experiment (i.e. the fraction of K^+ decays for which both pions go into the downstream detector) is as large as possible.

The hall in which the experiment should be set up is 300 m long. It could be expanded if your simulation proves that this would significantly improve the acceptance, but this would be very expensive.

2 Determination of the average decay length of the K^+

First, you want to determine the average decay length of the K^+ from data recorded in a previous experiment. This experiment also made use of a beam of particles with fixed momentum of 75 GeV/c, but the beam consisted to 16% of K^+ and to 84% of π^+ and the experiment could not distinguish between decays of K^+ and π^+. The file `dec_lengths.dat` contains 100'000 decay lengths measured in this experiment. Knowing the composition of the beam, and knowing that the π^+ has an average decay length of $\beta\gamma c\tau_\pi = 4.188$ km at the given beam energy, perform a fit to the data in order to extract the average decay length of the K^+. Compute the uncertainty on your result, and check if your result is compatible with the the known mean lifetime of the K^+, which you can look up on the web page of the Particle Data Group (pdg.lbl.gov).

3 Beam parallel to the z axis

At first, assume that the K^+ travel exactly along the z axis. Generate a sample of $K^+ \to \pi^+\pi^0$ decays as follows:

(a). Generate the position of the K^+ decay vertex according to an exponential distribution with the average decay length that you have determined above;

(b). Generate the decay angles of the π^+ and π^0 isotropically distributed in the K^+ rest frame;

(c). Boost the four-vectors of the two pions into the laboratory frame as described in the Appendix below.

Finally, determine for how many of the generated decays both pions fly into the acceptance of the downstream detector. Do this for different z positions of the downstream detector and determine the z position that maximizes the number of such events.

4 Divergent beam

Repeat the study assuming now that the K^+ beam has a finite angular divergence. Simulate this by generating K^+ flight directions according to a Gaussian profile with a standard deviation of $\sigma_\theta = 1$ mrad, where θ is the angle between the z axis and the K^+ flight direction.

Appendix: K^+ decay and Lorentz transformation

It is convenient to describe the particles in terms of four-vectors:

$$P = (E, p_x, p_y, p_z) \ ,$$

where $E = \sqrt{m^2 + |p|^2}$, $|p| = \sqrt{p_x^2 + p_y^2 + p_z^2}$ and m is the rest mass of the particle (you can look up the rest masses of the K^+, π^+ and π^0 on the web page of the Particle Data Group (pdg.lbl.gov). The total four-vector is conserved in the decay:

$$P_K = P_{\pi^+} + P_{\pi^0} \ .$$

From momentum conservation it follows that the momenta of the two pions are produced back-to-back in the rest frame of the K^+, i.e. that their momenta in this frame have equal magnitude and point in opposite directions. The angular distribution of the two pions in the K^+ rest frame is isotropic.

If the K^+ travels parallel to the z axis, the four-momenta of the pions in the laboratory frame are related to those in the K^+ rest frame, denoted with a *, as

$$\begin{bmatrix} E \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} E^* \\ p_x^* \\ p_y^* \\ p_z^* \end{bmatrix} ,$$

where

$$\beta = |p_K|/E_K$$

and

$$\gamma = E_K/m_K$$

If the K^+ does not travel parallel to the z axis, you have to modify the directions of the two pions accordingly by performing a rotation.