from this we know where the decays are happening.

Assuming the following quantities (TODO add pdg data link oder wikipedia link): [TODO find values] [TODO show gamma, beta claculation...]

$$
\begin{aligned}
m_{K^+} &= \\
m_{pi^+} &= \\
m_{pi^0} &= \\
\beta &= \\
\gamma &=
\end{aligned}
$$

In the $K^+$ rest frame we now have a momentum of 0 and a rest energy $m_K^+$. The $K^+$ now decays into a $\pi^+$ and a $\pi^0$ with momenta $p_{\pi^+}$, $p_{\pi^0}$ in oposing directions. The momenta must have the same magnitude and $p_{\pi^+} = -p_{\pi^0}$ .

The magnitude of the momentum is calculated with the energy of lost mass in the decay. In the following equations $p$ is used as absolute scalar.

$$
\begin{aligned}
E_{\pi^+} &= \frac{m_{K^+}^2 + m_{pi^+}^2 - m_{pi^0}^2}{2m_{K^+}} && (1) \\
p_{\pi^+} &= \sqrt{E_{\pi^+}^2 - m_{pi^+}^2} && (2) \\
p_{\pi^+} &= p_{\pi^0} && (3) \\
& && (4)
\end{aligned}
$$

We use isotropically distributed unit vetors in the simulation (see AW). We multiply them with $p_{\pi^0}$ to get the momenta for $\pi^0$ in the $K^+$ rest frame. Tho get the momenta of $\pi^+$ we take the same distribution and flip signs. We then build the four vectors $P_\pi^*$.

$$
P_\pi^* = \begin{bmatrix} E_\pi^* \\ p_{\pi,x}^* \\ p_{\pi,y}^* \\ p_{\pi,z}^* \end{bmatrix} = \begin{bmatrix} \sqrt{m_\pi^2 + ||p_\pi||^2} \\ p_{\pi,x}^* \\ p_{\pi,y}^* \\ p_{\pi,z}^* \end{bmatrix} \tag{5}
$$

Back to the lab rest frame, we boost them with the boost matrix. This boost matrix performs a boost along the z-axis.

$$
P_\pi = \begin{bmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{bmatrix} \cdot P_\pi^* \tag{6}
$$

$P_\pi$ is the four-vector in the lab frame. $P_\pi^*$ is the four-vector in the kaon rest frame.

## 0.1 With scattering

When we use a beam that is not parallel to the z-axis, we need to perform a rotation to the momentum of the four-vector before and after the boost. The rotation after the boost is the same that would rotate $e_z$ to align with the position vector. the one before is the inverse rotation.

Since the momentum of the four-vectors $P_\pi$ of the pions are isotropically distributed in the $K^+$ rest frame, we can omit the first rotation and still end up with a an isotropical distribution.

The operation performed with the angle $\alpha$ around y-axis and $\beta$ around x-axis:

$$R(\alpha, \beta) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \tag{7}$$

We apply $R(\alpha, \beta)$ to the momentum after we have extracted the momentum of the four-vector $P_\pi$.

# 1 Hits

To determine the number of decays for which both pions hit the detector, we first define a distance $z_d$ from source to detector. And a radius $r$ of the detector. The impulses in the lab frame are $\overrightarrow{p_{\pi^+}}$ and $\overrightarrow{p_{\pi^0}}$. And decay position $\overrightarrow{x} = (x, y, z)$. We want to find the number of double hits.

1. Drop all the decay entries where $z > z_d$.
2. Do the following for $\overrightarrow{p} = \overrightarrow{p_{\pi^+}}$ or $\overrightarrow{p_{\pi^0}}$.
   (a) Calculate $s$ such that $\overrightarrow{q} = \overrightarrow{x} + s \cdot \overrightarrow{p}$ lands in the plane of $z_d$.
   (b) Drop the pion if it's outside the detector. $(x + q_x)^2 + (y + q_y)^2 > r^2$.
3. Drop the decay if at least one of the two pions was dropped.
4. Count the number of decays left.

# 2 Determine The optimal detector distance $z_d$

We will now determine the optimal distance such that the acceptance of is maximized. Acceptance is defined as follows:

$$P(z_d) = \frac{\text{n decays for which both pions hit}}{\text{n decays simulated}} \tag{8}$$

To maximize the acceptance we run the experiment about 500 times with $N = 10^6$ kaons generated. First we simulate a beam aligned to the z axis. Second we generate a beam that scatters at the source in a gaussian distribution with $\sigma_\theta = 0.001$ rad.

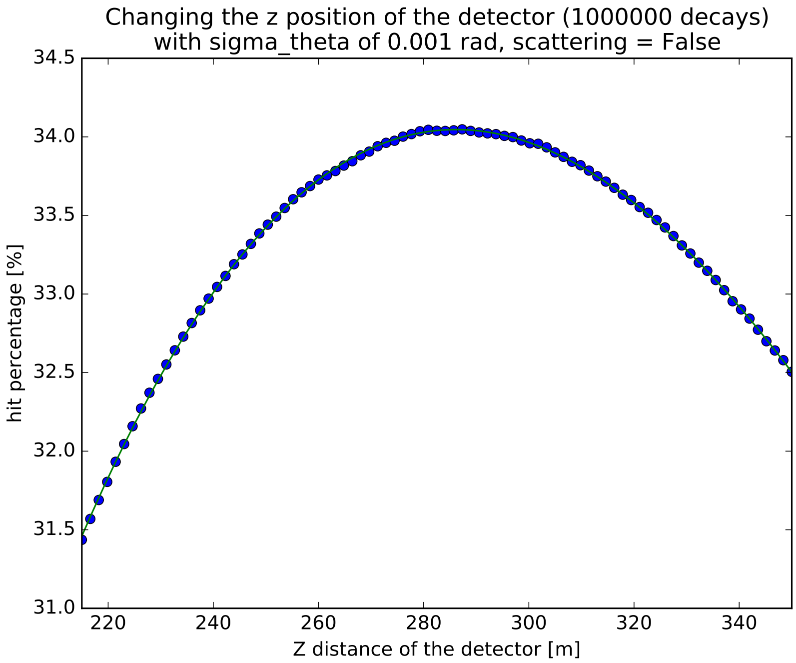


Figure 1: Sample output of decay_simulation_6.py

To determine the optimal z distance, we implemented the algorithm described in 1. We let it run for z values in a range from 215 to 350 meters in 85 evenly sized steps. The algorithm generates 85 double hit counts in an array. We divide by N to get the probabilities. Then we fit it with a polynom of degree 3 and determine the maximum of that polynom. To show it graphically, we will show a plot. The simulation was run once with $N = 10^6$ decays.

We run the simulation about 500 times. Fitting the maximum and saving it. The maxima are then put into a histogram. the average and standard deviation of the maximas are listed below. The standard deviation gives a measure of the statistical uncertainty of the simulation.

[TODO add units]

**Aligned Beam**

$$\begin{aligned} <z> &= 285.5 \\ \sigma_z &: 0.7 \end{aligned}$$

**Scattered Beam**

$$\begin{aligned} <z> &= 280.7 \\ \sigma_z &: 0.7 \end{aligned}$$

To calculate the final uncertainty we used the relative error on the average decay length from chapter [TODO add reference]. The relative errors get quadratically added.

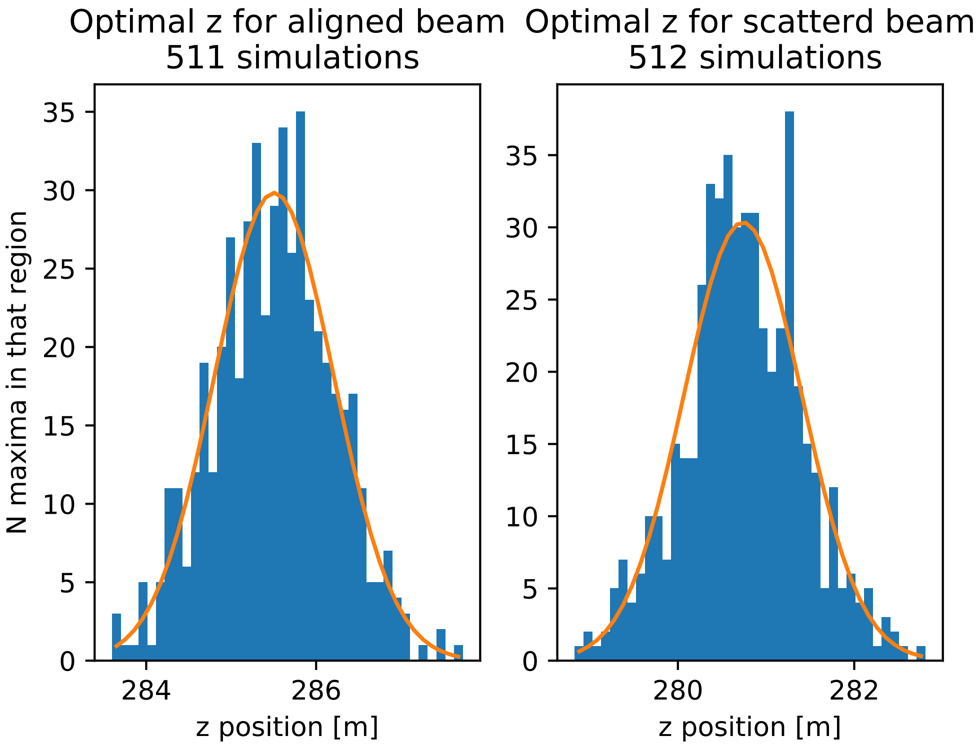


Figure 2: [TODO find good title]

$$m_z = \sigma_z \cdot \sqrt{\left(\frac{\text{uncertanty on adl}}{\text{adl}}\right)^2 + \left(\frac{\sigma_z}{z}\right)^2} \tag{9}$$

The final optimal values for z are:

$$z_{\text{aligned}} = (286 \pm 5)m$$
$$z_{\text{scattered}} = (281 \pm 5)m$$

The error does not matter that much, since a change of 5 meters around the maximum does maximally change the acceptance by 0.5 percent as seen in figure 1.