Simulating Exponential Distributions

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Synopsis

R is loaded with powerful methods for simulating, modeling and analyzing data. In this document I give a basic simulation of exponential data and compare that data with a theoretical model.

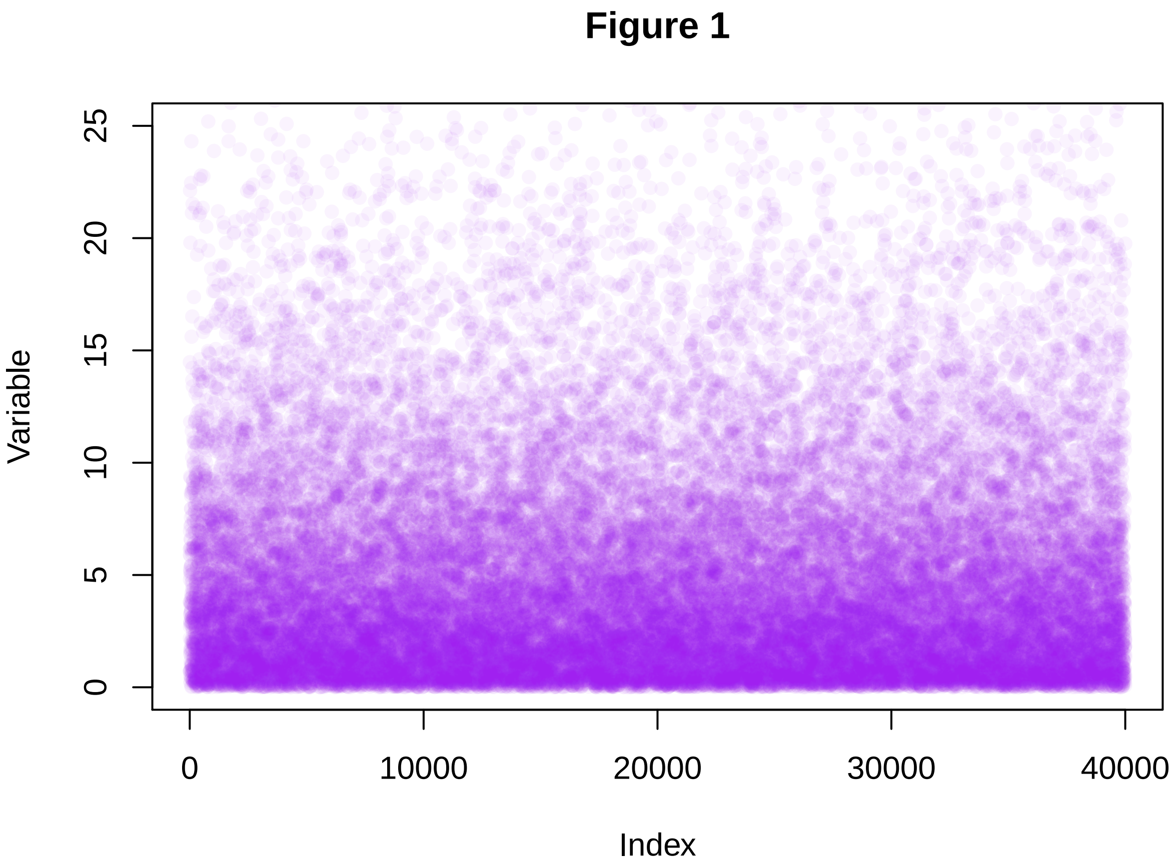
Simulation

We start by simulating 1000 trials with 40 observations each. This is accomplished using a for loop and the rexp() function. The means and standard deviations for each trial are saved in respective data frames.

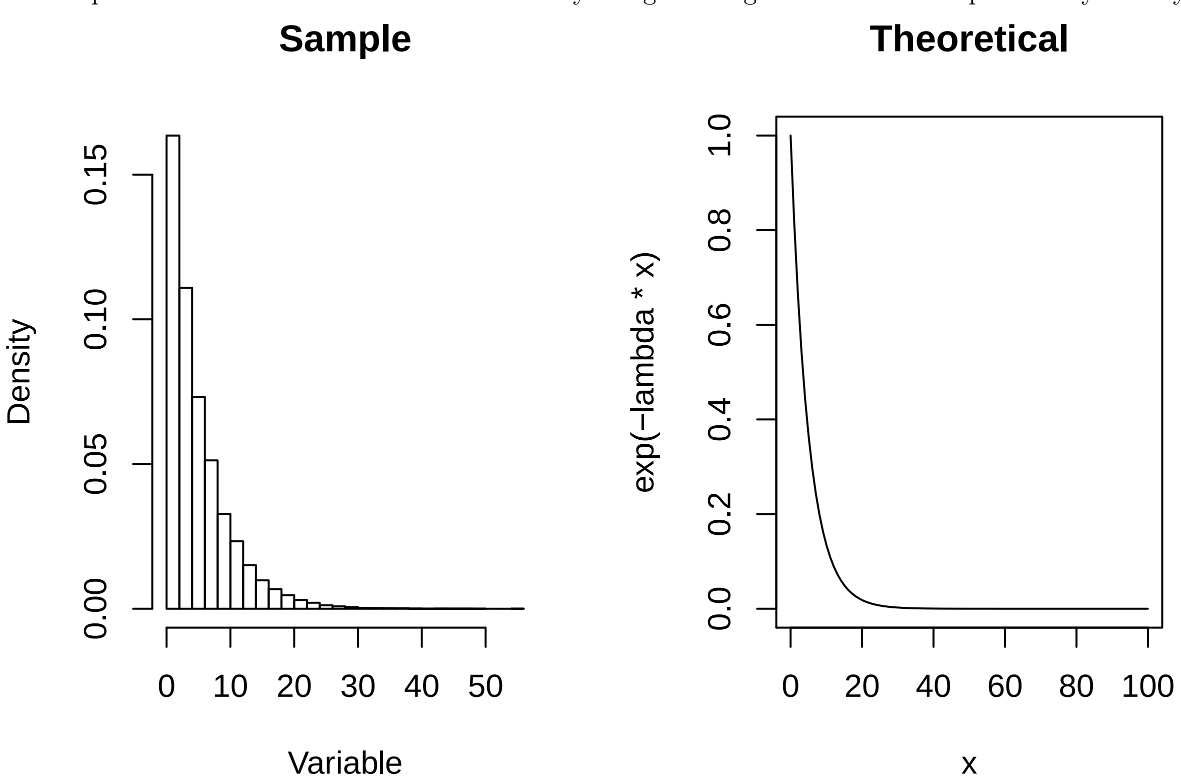
```
lambda <- .2
n <- 40
s <- 1000

mns = NULL
sd = NULL
dat = NULL
for (i in 1:s) {
       set.seed(i)
       dat = c(dat, rexp(n, lambda))
       mns = c(mns, mean(rexp(n, lambda)))
       sd = c(sd, sd(rexp(n, lambda)))
}
```

Raw Data

The raw data as a scatter plot comes out of the simulation looking like figure 1. We can see that the data becomes more and more concentrated around 0 which indicates an exponential distribution.



The Exponential distribution can be visualized by using a histogram to model the probability density.



Validating Model

In this section we evaluate the simulated exponential data against the theoretical model.

Sample Mean vs Theoretical Mean

Using the data we can calculate the simulated and theoretical means.

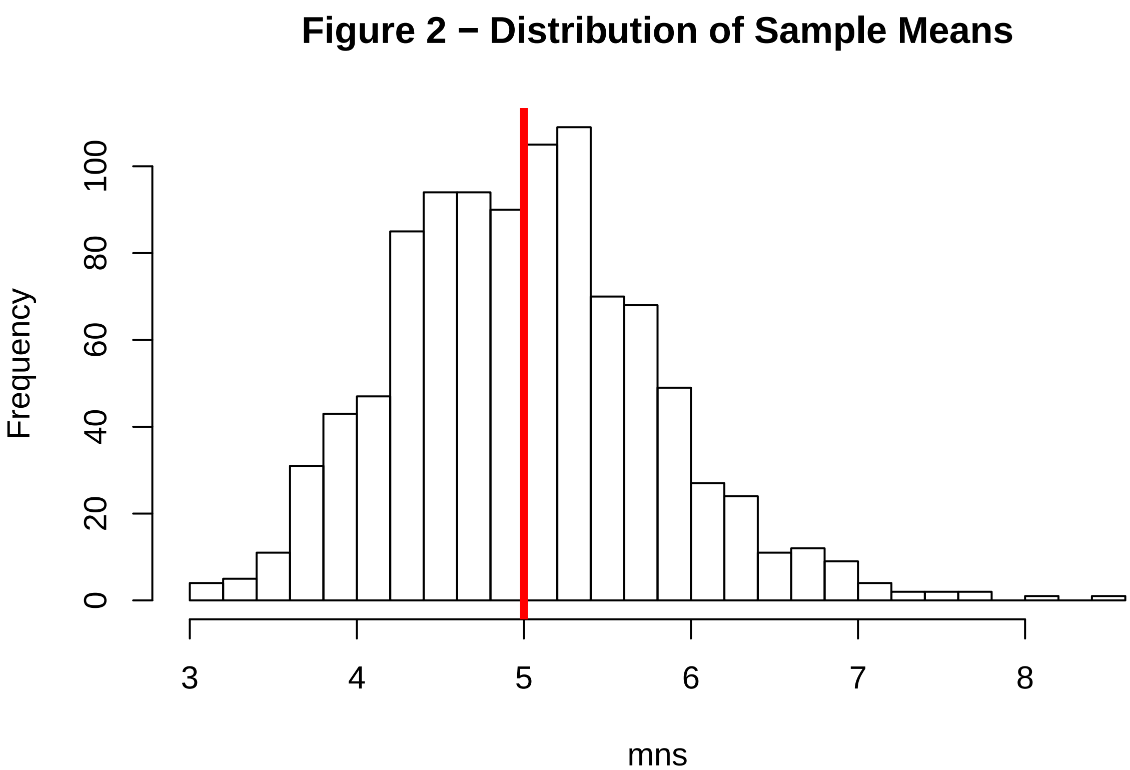
```
sample_mean = mean(mns)
sample_mean
```

```
## [1] 5.011243
```

```
theor_mean = 1/lambda
theor_mean
```

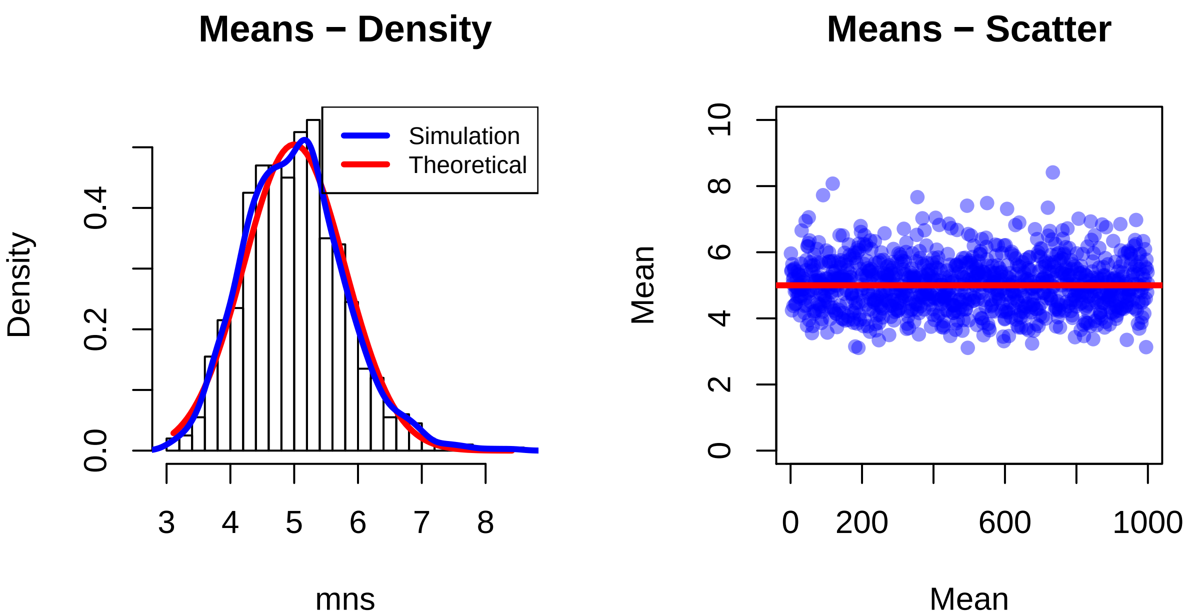
```
## [1] 5
```

```
hist(mns, 22, main = "Figure 2 - Distribution of Sample Means")
abline(v=5,lw=4,col="red")
```



The results show that the sample mean is off by only .011. In the figure below we can further see that the distirbution of the means is approximately normal.

Figure 3 − Distribution of Sample Means



Sample Variance vs Theoretical Variance

Next we calculate the standard deviations and variances.

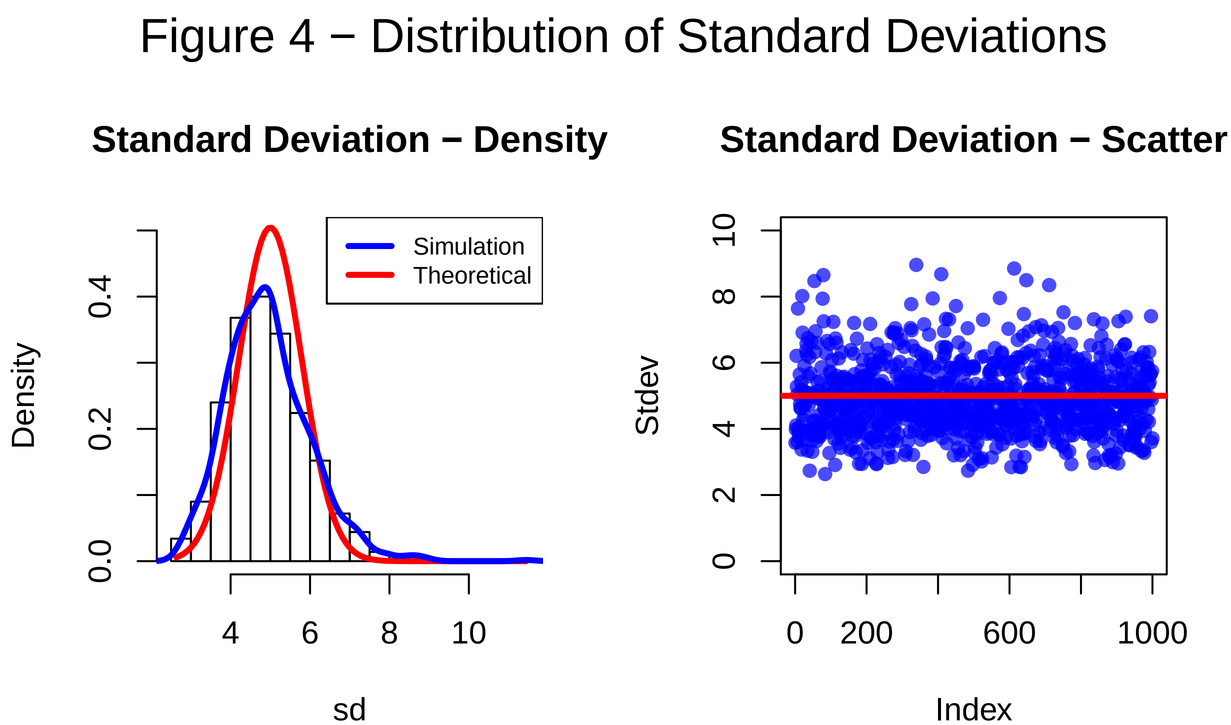
```
sample_var <- var(mns)
sample_var
```

```
## [1] 0.60981
```

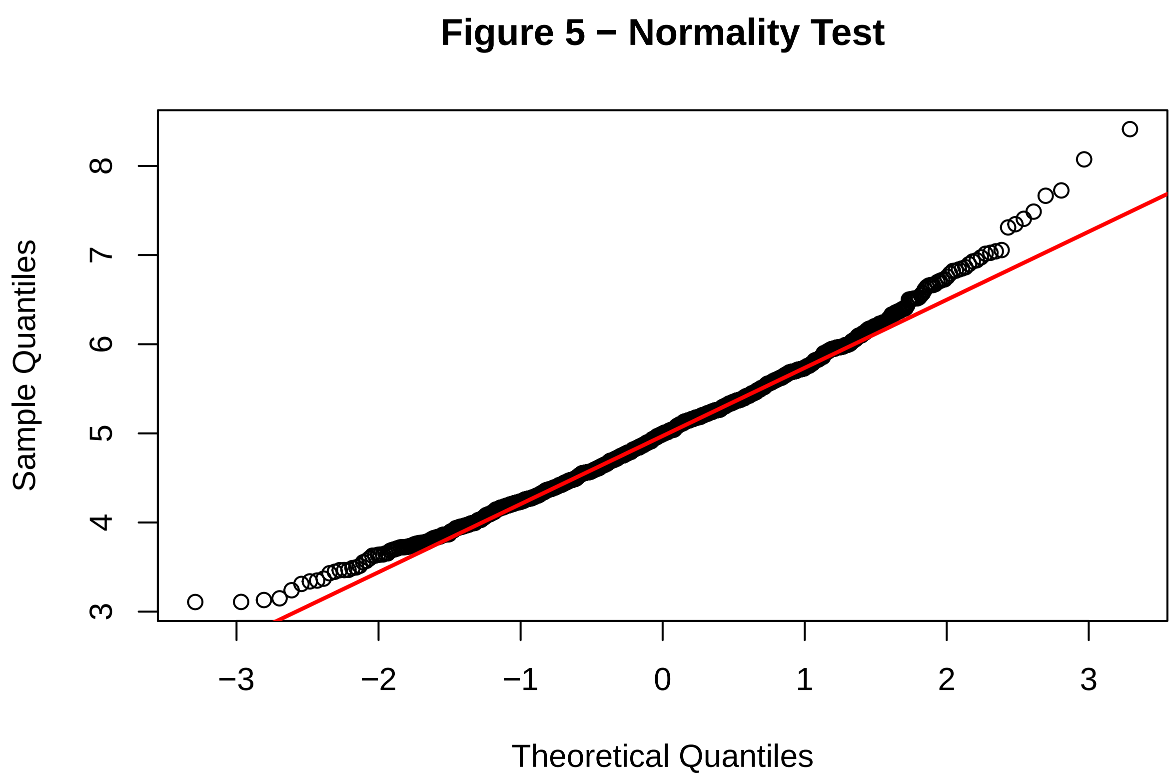
```
theor_var <- (1/lambda^2)/n
theor_var
```

```
## [1] 0.625
```

The variances match up closely. Below is a chart comparing theoretical standard deviation distribution versus the standard deviation of the sample data.



Finally, we can test the normality using a q-q plot.



Appendix - Plot Codes

```
plot(dat, pch = 16, col = adjustcolor("purple", alpha=0.05), ylim = c(0,25), ylab = "Variable", main =
```

```
sample_mean = mean(mns)
sample_mean

theor_mean = 1/lambda
theor_mean

hist(mns, 22, main = "Figure 2 - Distribution of Sample Means")
abline(v=5,lw=4,col="red")
```

```
#FIGURE 3
par(mfrow=c(1,2))

#Generating histogram as a density function
hist(mns, 22, freq = FALSE)

#Fitting the theoretical line
xfit <- seq(min(mns), max(mns), length = 100)
yfit <- dnorm(xfit, mean = 1/lambda, sd = 1/lambda/sqrt(n))
lines(xfit, yfit, pch=22, lty = 1, lw = 3, col = "red")

#Fitting the line to the data
lines(density(mns), lw = 3, col = "blue")

#Adding a legend
legend('topright', c("Simulation", "Theoretical"), col=c("blue", "red"), lw=c(3,3), cex = .75)

#plotting the data on the mean for the 1000 simulations
plot(mns, main = "Mean", ylab = "Mean", xlim = c(0,s), ylim = c(0,10), xlab = "Mean", pch = 16, col = a
abline(h=5, col = "red", lw =3)
```

```
par(mfrow=c(1,2), oma=c(2,0,2,0))
hist(sd, 22, freq = FALSE, ylim = c(0,.5), main = "Standard Deviation - Density")

#Fitting the theoretical line
xfit <- seq(min(sd), max(sd), length = 100)
yfit <- dnorm(xfit, mean = 1/lambda, sd = 1/lambda/sqrt(n))
lines(xfit, yfit, pch=22, lty = 1, lw = 3, col = "red")

#Fitting the line to the data
lines(density(sd), lw = 3, col = "blue")

#Adding a legend and title
legend('topright', c("Simulation", "Theoretical"), col=c("blue", "red"), lw=c(3,3), cex = .75)

mtext("Figure 4 - Distribution of Standard Deviations", outer = TRUE, cex = 1.5)

plot(sd, main = "Standard Deviation - Scatter", ylab = "Stdev", xlim = c(0,s), ylim = c(0,10),pch = 16,
abline(h=5, col = "red", lw =3)
```

```
qqnorm(mns, main= "Figure 5 - Normality Test")
```

```
qqline(mns, col="red", lw = "2")
```

Check out my website at: http://www.ryantillis.com/