Can growth take place while reducing emissions? The role of energy mix*

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Abstract

Do the macroeconomic effects of a carbon tax differ between countries according to the primary energy source? We answer this question using a theoretical model of directed technical change and empirically test the main results. We find four main results: (i) in the absence of subsidies, carbon taxes have a negative effect on economic growth; (ii) this negative effect is a decreasing function of the proportion of clean energy sources; (iii) subsidies for clean inputs have a positive effect on economic growth; and (iv) the magnitude of the positive effect grows with the proportion of clean energy sources. The empirical results are consistent with the predictions of the theoretical model, indicating that environmental policies should consider the initial state of the energy mix and the impact of the transition to clean sources on economic growth.

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1 Introduction

Tackling climate change without harming economic growth is a major challenge for countries today. Although carbon pricing is increasingly recognized as a key policy tool to incentivize emissions reductions, its macroeconomic impact remains a subject of debate. Experiences with carbon taxes vary significantly between countries, raising questions about the factors driving these differences. For example, Norway, with its substantial hydroelectric resources and a high initial share of clean energy in its energy mix, has implemented a carbon tax with relatively limited adverse effects on its economy. Conversely, countries heavily reliant on fossil fuels, such as South Africa, face greater economic adjustment challenges due to their dependence on carbon-intensive industries. This divergence underscores the critical role of a nation's "energy mix"[1] in shaping the macroeconomic consequences of carbon pricing. The energy mix includes both "clean" sources (e.g., hydro, nuclear, solar, and wind power) and "dirty" sources, primarily fossil fuels (e.g., coal, oil, and natural gas). Different energy mixes imply varying degrees of dependence on carbon-intensive industries and disparate adaptation costs associated with transitioning to a low-carbon economy.

This study explores the heterogeneous effects of the energy mix on the macroeconomic impact of carbon taxes, focusing on how the ratio between clean and dirty energy sources shapes GDP growth in the short and long run. Building on the Schumpeterian growth model developed by Acemoglu *et al.* (2012a), we derive theoretical predictions about the role of environmental policy and the energy mix in influencing aggregate production and growth. The model distinguishes between two sectors, clean and dirty, and incorporates environmental policies that affect the direction of technological change. Using this framework, we extend the analysis by formulating testable propositions related to income and growth. These propositions are quantitatively validated using empirical methodologies, specifically event studies and local projections, to capture the dynamic responses of economic output to

[1] the composition of energy sources used for production and consumption

carbon taxes in countries with varying energy profiles.

The findings indicate that, in the absence of targeted clean energy subsidies, the introduction of a carbon tax leads to a short-term contraction in economic growth, especially in economies that are heavily dependent on fossil fuels. However, as highlighted by Meckling *et al.* (2017) in the context of policy sequencing for decarbonization, combining carbon pricing with complementary policies can be essential. When the tax is accompanied by subsidies, the effect can be less harmful in the short run and become more favorable as energy sources transition to cleaner alternatives in the long run. Based on these findings, a key policy implication of this study is that allocating a portion of the revenue generated by the carbon tax to subsidize clean energy sources could help mitigate the adverse economic effects of environmental policy. Consistent with previous research[2], the findings highlight that environmental policies can be temporary if they successfully redirect innovation towards cleaner technologies and increase the share of renewable energy sources in the long run, as innovations respond endogenously to changes in policy incentives.

There is a considerable body of research examining how carbon taxes influence macroeconomic results, though the conclusions remain somewhat uncertain. It is important to highlight that most of these investigations focus on developed nations, resulting in a relatively limited understanding of the effects in developing countries. Bernard *et al.* (2018) and Metcalf & Stock (2020) find no adverse impact of the carbon tax on aggregate GDP growth or employment in British Columbia or in 31 European countries. However, Yamazaki (2017) shows that while British Columbia's carbon tax does not reduce employment in general, it does lead to declines in the most carbon-intensive and trade-sensitive sectors, while increasing employment in the cleaner service and health industries. Similarly, studies examining the distributive effects of the carbon tax on households yield varied conclusions. While some suggest regressiveness, as the tax raises costs for carbon-intensive goods and shifts factor prices (Mueller & Steiner, n.d.; Rausch *et al.*, 2011), and income inequality can significantly

[2] Fried (2018), Acemoglu *et al.* (2012a)

influence these distributional outcomes (Andersson J (2020) on Sweden's gasoline tax), other research offers a more nuanced perspective. For instance, Sterner (2012), analyzing transport fuel taxes across seven European countries, found the evidence for regressivity to be weak, concluding that the tax is approximately proportional, particularly when considering lifetime income rather than annual income.

The literature explicitly examining the interaction between the national energy source composition and the macroeconomic impacts of climate policies remains relatively limited. While some studies touch upon related aspects, few directly test how the energy mix modulates aggregate outcomes from policies such as carbon taxes. For instance, Papageorgiou *et al.* (2017) estimated that the substitution elasticity between clean and dirty energy inputs significantly exceeds unity, suggesting favorable conditions for green growth transitions. Similarly, Matsumoto (2022), exploring household-level responses in Japan, found that a carbon tax incentivized a shift towards gas and away from electricity, indicating micro-level adjustments in energy consumption patterns. More directly addressing macroeconomic heterogeneity, recent work by Känzig & Konradt (2023) provides empirical evidence from Europe's carbon pricing initiatives. The study finds that in countries with a more "brown" energy matrix, the increase in energy prices is stronger. Despite these initial insights, a comprehensive analysis integrating theoretical channels, such as directed technical change, with empirical validation across diverse country contexts, particularly focusing on how the energy mix shapes both short-term and long-term growth effects of carbon taxes, is still needed.

However, none of the authors have examined how different combinations of energy sources may influence the macroeconomic effects of implementing climate policies, such as the carbon tax, in the short and long run.

This article is also related to a large and growing literature on the environment, resources, and directed technical change. Early contributions (Golosov *et al.*, 2011; Nordhaus, 1993; Stern, 2007) focused on developing theoretical models of the climate-economy relationship, such as the DICE model, which extends the Ramsey growth model. Economists have since advanced growth theories that incorporate environmental constraints (Acemoglu *et al.*, 2012b; Laffont & Martimort, 2009; Romer & Romer, 2010; Torres, 2021), with particular emphasis on how innovation and directed technological change can support long-term sustainable growth. For instance, Popp (2002) finds that high energy prices incentivize cost-saving innovations in the air conditioning industry, and Aghion *et al.* (2012) conducts a similar analysis in the automobile sector. This work also complements the theoretical literature on optimal carbon taxation, such as Belfiori (2021), which emphasizes the centrality of carbon pricing over renewable subsidies unless externalities exist.

While numerous macroeconomic models examine the broader implications of climate policies—particularly those integrating environmental constraints within frameworks of directed technological change—the specific relationship between energy source composition, economic growth, and climate policy remains underexplored. This study addresses this gap by empirically validating our model and offering new insights into how various combinations of energy sources shape the macroeconomic effects of climate policies, with a focus on carbon taxes and subsidies for clean energy.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model, outlining the propositions on the impact of a carbon tax and subsidy on both aggregate and sectoral production, as well as growth rates. Section 3 outlines the empirical methodologies used to estimate the correlation between the carbon tax and GDP growth, employment rates, and variations associated with different energy mix compositions. Section 4 delves into the presentation and discussion of the empirical findings, along with their implications. Finally, section 6 provides concluding remarks.

2 Theoretical model

Building on the directed technological change model with environmental constraints by Acemoglu *et al.* (2012a) (AABH from now on), we analyze an economy where a final good is produced using substitutable inputs from "clean" and "dirty" energy sectors. Innovation occurs in both sectors through patents for intermediate goods, with mobile labor ensuring uniform wages.

We investigate how carbon taxes and clean energy subsidies impact GDP and growth, focusing on how the energy matrix composition influences this relationship. The magnitude of these effects depends on two factors: (i) the technological development of the clean and dirty sectors, both initially and over time, and (ii) the policy design, particularly if tax revenue is used to subsidize clean goods and promote innovations.

2.1 Model with an environmental policy

Environmental policy must ensure that the demand for dirty inputs takes into account the environmental cost of an additional unit of inputs. We model a policy that combines a tax on the dirty sector (τ) and a subsidy on the clean sector (q). This policy mix can generate a decrease in the dirty sector accompanied by growth in welfare. We assume that the subsidy rate is a linear function of the tax rate, $q = \tau \cdot \phi$. This assumption allows us to analyze the heterogeneous effects of a carbon tax depending on whether, in addition to the tax, there are subsidies for clean technologies. For simplicity, we do not impose a balanced budget, nor do we allow for changes in tax rates or subsidies over time. These simplifying assumptions do not affect the model's qualitative results.

Households

Each country is inhabited by a continuum of households, consisting of workers and scientists, who can freely switch sectors without incurring adjustment costs. The households have the following preferences:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \tag{1}$$

where C_t represents the consumption of the final good at time t, S_t denotes the quality of the environment, and $\rho > 0$ is the discount rate. The environmental quality, $S_t \in [0, \hat{S}]$, where $\hat{S}$ is the baseline level of environmental quality without pollution [3]. Environmental quality degrades due to the production of dirty inputs at a rate $\xi > 0$ but regenerates at a natural rate $\delta > 0$. Finally, in the event of an environmental disaster, it collapses to $S_t = 0$. Therefore, evolution of environmental quality can be expressed by the following law of motion:

$$S_{t+1} = min\left[\hat{S}, (1+\delta)S_t - \xi Y_{dt}\right] \tag{2}$$

Final Good

There is a unique final good, Y_t, produced competitively using "clean" and "dirty" inputs (depending on the primary energy source required) Y_c and Y_d.

$$Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $\epsilon \in (0, +\infty)$ is the elasticity of substitution between the two sectors. If the inputs are (gross) substitutes, $\epsilon > 1$, then any final good production can be obtained from alternative clean energies. For example, renewable energy, provided it can be stored and transported efficiently, may replace energy derived from fossil fuels (Popp, 2002). On the contrary, if the two inputs are (gross) complements, $\epsilon < 1$, then it is impossible to produce without fossil fuels. Final good producers choose the quantity of each input to maximize profits, solving

[3] The quality of the environment absent any human pollution.

the following problem:

$$\max_{Y_{dt}, Y_{ct}} \{Y_t - (1+\tau)P_{dt}Y_{dt} - (1-\phi\tau)P_{ct}Y_{ct}\}$$

where Y_{dt} and Y_{ct} represent the quantities of dirty and clean inputs, respectively, and P_{dt} and P_{ct} denote the prices of these inputs. The parameter τ captures the carbon tax rate imposed on dirty inputs, while $\phi\tau$ represents a subsidy rate for clean inputs. Using the final good as the numeraire, the price of inputs with the environmental policy is:

$$\left[((1+\tau)P_{dt})^{1-\epsilon} + ((1-\tau\phi)P_{ct})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1 \tag{4}$$

The tax is charged on the price paid for each unit of the dirty inputs demanded, while the subsidy is applied as a discount for each unit of clean inputs purchased. Consequently, the relative demand for dirty inputs declines due to the tax, whereas the demand for clean inputs increases as the subsidy reduces their effective prices:

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{P_{dt} \cdot (1+\tau)}{P_{ct} \cdot (1-\tau\phi)}\right)^{\epsilon} \tag{5}$$

Clean and Dirty Intermediate Inputs

The two inputs, Y_c and Y_d are produced competitively.[4] Using L_{jt} labor, A_{jit} the quality of machine i in the sector j, and x_{jit} a continuum of sector-specific machines (intermediates). Thus, the optimization problem faced by producers in both sectors involves maximizing profits through the optimal allocation of labor and machines.

$$\max_{x_{jit}, L_{jt}} \left\{P_{jt} L_{jt}^{1-\alpha} \left(\int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \cdot di\right) - w_{jt}L_{jt} - \int_0^1 p_{jit}x_{jit} \cdot di\right\}$$

[4] In this version, we do not consider the depletion of fossil resources. Although fossil fuel reserves are finite, historical prices have not followed the predictions of the Hotelling model, and scarcity constraints are less relevant in the context of climate change targets (Fried, 2018).

From the first-order conditions we obtain the demand for machines and labor in each sector:

$$x_{jit} = \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \text{ and } L_{jt} = \left(\frac{(1-\alpha)P_{jt}}{w_{jt}}\right)^{\frac{1}{\alpha}} A_{jit}^{\frac{1-\alpha}{\alpha}} x_{jit}. \tag{6}$$

In line with AABH we assume that machines are produced at marginal cost equal to α^2 under monopolistic competition and sold at price p_{jit}, taking into account the demand for machines x_{jit}. Therefore the profits of the monopolists, π_{jit}, are given by: $\pi_{jit} = (p_{jit} - \alpha^2)x_{jit}$. So, replacing the demand for machines, the profits of the monopolist are:

$$\pi_{jit} = (p_{jit} - \alpha^2) \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \tag{7}$$

As a result, each monopolist sets a price $p_{jt} = \frac{1}{\alpha}$, identical for all i. Thus, replacing the price of machine p_{jt} in equation 6, the optimal demand for machines in each sector is obtained:

$$x_{jit} = \alpha^{\frac{2}{1-\alpha}} A_{jit} L_{jt} \left(P_{jt}\right)^{\frac{1}{1-\alpha}} \tag{8}$$

Combining the previous results, we obtain the quantities of inputs produced in each sector as follows:

$$Y_{jt} = \alpha^{\frac{2\alpha}{1-\alpha}} A_{jt} L_{jt} \left(P_{jt}\right)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

where $A_{jt} = \int_0^1 A_{jit} di$.

Technological change

At the beginning of every period all intermediate goods within a sector starts with the average level of productivity of the previous period. Successful scientists invent a better version of the machine i in sector j and increases the quality of the machine by a factor γ. The probability of success is $\eta_j \in (0,1)$ and depends positively on the investment on research and development (R&D), R_{jt} and inversely on the desired productivity A_{jit}: $\eta_{jt} = \lambda \left(\frac{R_{jt}}{A_{jit}}\right)^{\sigma}$. When an innovation is unsuccessful $(1-\eta_j)$, the sector's productivity is equal to that of the previous period, A_{jt-1}.

$$A_{jt} = \begin{cases} \gamma A_{jt-1} & \text{if successful } (\eta_j) \\ A_{jt-1} & \text{if not successful} (1-\eta_j) \end{cases}$$

The problem for entrepreneurs is to maximize the expected profits of innovating: $\max_{R_{jt}} \{\eta_{jt} P_{Ajit} - R_{jt}\}$. From the first-order condition and given that the price of the patent equals the net profits of the machine producer $P_{Ajit} = \pi_{jit}$, we derive the probability of innovating in each sector:

$$\eta_{jt} = 2(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} L_{jt} \tag{10}$$

Equilibrium

Taking the input production, equation 9, and the relative input demand, equation 5, the ratio between clean and dirty production is expressed in the equation 11.

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{P_{ct}}{P_{dt}}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{ct}}{A_{dt}} \frac{L_{ct}}{L_{dt}} = \left(\frac{P_{dt} \cdot (1+\tau)}{P_{ct} \cdot (1-\tau\phi)}\right)^{\epsilon} \tag{11}$$

It shows that the environmental policy has no direct impact on production costs, but it increases the demand for clean goods and decrease the demand for dirty inputs, generating incentives for labor reallocation. On the supply side of this model, three forces determine the relative benefits of innovation: the *price effect,* the *market size effect*, and the *direct effect of productivity.* In the following lines, we see how the tax and subsidy affect each of these. Price effect: the less technologically advanced sector has higher prices. As the technological level of the clean sector improves, the price of clean goods decreases. In equilibrium, subsidies for clean production raise the prices of both clean and dirty inputs, while taxes on dirty production lower them (see appendix A.2). However, since the percentage changes in prices are proportional, environmental policies like subsidies and taxes do not affect the relative price ratio between clean and dirty inputs.

$$\frac{P_{ct}}{P_{dt}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{(1-\alpha)} \tag{12}$$

By substituting the relative prices into equation 11 (see details in the appendix 7), we derive the allocation of labor in equilibrium.

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{(1-\alpha)(\epsilon-1)} \tag{13}$$

The introduction of environmental policy, holding all other factors constant, increases wages in the clean sector and reduces them in the dirty sector. This wage differential drives a reallocation of labor towards clean input production. Taking into account the relative prices (eq. 12) and labor (eq.13), the relative production of clean inputs compared to dirty in equilibrium is given by:

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\epsilon(1-\alpha)} \tag{14}$$

We can now evaluate the comparative likelihood of successful innovations within both sectors. Using the equation of probability of innovation (eq. 10), substituting relative prices (eq. 12) and labor (eq. 13), we get the relative probability of innovation in terms of productivity and environmental policies.

$$\frac{\eta_{ct}}{\eta_{dt}} = \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} \tag{15}$$

Where $\varphi = (\epsilon - 1)(1 - \alpha)$. The probability of innovation is positively influenced by its technological level and negatively by the technological level of the other sector. In the absence of environmental policies, innovation is always greater in the sector with the highest technological level, which typically leads to innovation concentrated in the polluting sector. Implementing a carbon tax along with a subsidy enhances innovation in the clean sector while lowering it in the dirty sector. From equation 15, we can establish that the tax required to shift innovation towards the clean sector must meet the following condition:

$$\tau > \frac{\left(\frac{A_{dt}}{A_{ct}}\right)^{\frac{\varphi-1}{\epsilon}} - 1}{1 + \phi\left(\frac{A_{dt}}{A_{ct}}\right)^{\frac{\varphi-1}{\epsilon}}} \tag{16}$$

Equation 16 indicates that the greater the technological gap between the clean and dirty sectors, the larger the subsidy and the tax required to redirect resources to the clean sector. Furthermore, the subsidy reduces the necessary tax rate.

If the environmental policy is strong enough, the inequality in equation 16 holds, then the growth rate of the clean sector is higher and, in the long run total output is equal to the output of the clean sector. Notice that equations 3 and 14 imply that the final good output can be written in the following form:

$$Y_t = Y_{ct}\left(1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right)^{\frac{\epsilon}{\epsilon-1}} \text{ and } Y_t = Y_{dt}\left(1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\right)^{\frac{\epsilon}{\epsilon-1}}$$

Therefore, as $\frac{A_{dt}}{A_{ct}}$ decreases $\frac{Y_{ct}}{Y_t}$ grows and $lim_{\frac{A_{dt}}{A_{ct}}\to 0}\frac{Y_{ct}}{Y_t} = 1$.

2.2 Level effect of environmental policy

In this subsection, we extend the model to determine the static effect of environmental policies on aggregate production. We also examine the heterogeneity according to the relationship between the productivity of the sectors.

2.2.1 Effect on sectoral production

Replacing the prices (A8) and labor (A7) in equation 9, we obtain the output of the two sectors in terms of productivities, as well as the tax and subsidy rates.

$$
\begin{aligned}
Y_{ct} &= \alpha^{\frac{2\alpha}{1-\alpha}} \cdot (1+\tau)^{\epsilon} A_{ct}^{\epsilon(1-\alpha)} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \\
Y_{dt} &= \alpha^{\frac{2\alpha}{1-\alpha}} \cdot (1-\phi\tau)^{\epsilon} A_{dt}^{\epsilon(1-\alpha)} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}}
\end{aligned} \tag{17}
$$

Equation A9 indicates that the tax and the subsidy affect the production of both inputs. Now, in order to identify the direction and magnitude of the effect of the environmental policy, we derive the production with respect to τ.

$$
\begin{aligned}
\frac{\partial \log(Y_{ct})}{\partial \tau} =& \frac{\epsilon}{1+\tau} - \frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{-\epsilon}}{(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}}\right) \\
& - \epsilon\left(\frac{(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon}}\right)
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\frac{\partial \log(Y_{dt})}{\partial \tau} =& -\frac{\epsilon\phi}{1-\phi\tau} - \frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{-\epsilon}}{(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}}\right) \\
& - \epsilon\left(\frac{(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon}}\right)
\end{aligned} \tag{19}
$$

From equations 18 and 19:

- The effect of a tax on dirty inputs is greater (more positive or less negative) for the clean sector, i.e., $\frac{\partial \log(Y_{ct})}{\partial \tau} > \frac{\partial \log(Y_{dt})}{\partial \tau}$. In other words, a tax on dirty inputs generates a sectoral redistribution in favor of the clean sector at the expense of the dirty sector.

- For low levels of relative productivity of the clean sector, $\frac{A_{ct}}{A_{dt}}$, an increase in the tax rate results in an increase in the production of clean inputs. Specifically, if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}$, then $\frac{\partial \log(Y_{ct})}{\partial \tau} > 0$.

- For high levels of relative productivity of the clean sector, an increase in the tax rate leads to a decrease in the production of dirty inputs. Specifically, if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}$, then $\frac{\partial \log(Y_{dt})}{\partial \tau} < 0$. Therefore, if fiscal policy is strong enough to generate this transition, it will also have a negative effect on the production of dirty inputs. In particular, if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} > 1$, then $\frac{\partial \log(Y_{dt})}{\partial \tau} < 0$.

- The second derivative with respect to the relative productivity of the clean sector, $\frac{A_{ct}}{A_{dt}}$, is always negative, and exactly the same for both sectors: $\frac{\partial^2 \log(Y_{jt})}{\partial \tau \partial \frac{A_{ct}}{A_{dt}}} < 0$ for $j = \{d, c\}$.[5] This implies that increasing the relative productivity of the clean sector negatively affects the impact of the tax on output for both sectors.

2.2.2 Effect on aggregate production

From the previous subsection, we know that environmental policy generates a reallocation of labor to the clean sector and a redistribution of inputs in the same direction. Using the production function A9 and equation 3, it is possible to analyze the effect of taxes and subsidies on aggregate income.

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} \cdot \left((1+\tau)^{\epsilon-1} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon-1} A_{dt}^{\varphi}\right)^{\frac{\epsilon}{\epsilon-1}} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \tag{20}$$

Taking logs and derivatives, we obtain the effect of the tax on final production, and the implications are presented in Proposition 1.

$$\begin{aligned} \frac{\partial \log(Y_t)}{\partial \tau} =& \frac{\epsilon}{(1+\tau)}\left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}\right) \\ & - \frac{\alpha}{(1-\alpha)}\frac{1}{(1+\tau)}\left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{-\epsilon}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}}\right) \end{aligned} \tag{21}$$

[5] See the detailed procedure in the appendix 7

rearranging,

$$
\begin{aligned}
\frac{\partial \log(Y_t)}{\partial \tau} =& \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{1}{(1+\tau)}\epsilon \left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}} \right) \\
& - \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{1}{(1+\tau)} \frac{\alpha}{(1-\alpha)} \left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}} \right) \\
& - \phi \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2} \frac{1}{(1+\tau)}\epsilon \left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{\left(\frac{1+\tau}{1-\phi\tau}\right)}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}} \right) \\
& + \phi \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} \frac{1}{(1+\tau)} \frac{\alpha}{(1-\alpha)} \left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}} \right)
\end{aligned} \tag{22}
$$

Notice that the sum of the terms third and fourth is positive. Therefore, the introduction of subsidies reduces the negative effect of the carbon tax.

Proposition 1: *In the absence of subsidies, a tax on dirty production that promotes the energy transition has a negative effect on aggregate production,* $\frac{\partial \log(Y_t)}{\partial \tau} < 0$. *However, this negative impact is mitigated by the subsidy for clean inputs.*

Proof:

Claim 1*:* If $\phi = 0$ then $\frac{\partial \log(Y_t)}{\partial \tau} < 0$. From equation 22 it follows that if $\phi = 0$ then

$$
\begin{aligned}
\frac{\partial \log(Y_t)}{\partial \tau} =& \frac{\epsilon}{(1+\tau)} \left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{\epsilon-1}} - \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{\epsilon}} \right) \\
& - \frac{\alpha}{(1-\alpha)} \frac{1}{(1+\tau)} \left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{-(\epsilon-1)}} \right)
\end{aligned}
$$

and

$$\frac{\partial \log(Y_t)}{\partial \tau} = -\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{1}{(1+\tau)} \left(\frac{\epsilon\tau\left(\frac{1}{1+\tau}\right)^{\epsilon}}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{\epsilon-1}\right)\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{\epsilon}\right)} + \frac{\frac{\alpha}{(1-\alpha)}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{-(\epsilon-1)}} \right)$$

Therefore $\frac{\partial \log(Y_t)}{\partial \tau} < 0$.

Claim 2: The negative impact is mitigated by the subsidy for clean inputs.

Proof:

It follows directly from equation 22.

The objective of environmental policy, in this context, is to ensure that the productivity of the clean sector is at least as high as that of the polluting sector, so that production factors flow into the clean sector and technological progress is greater in this sector. We have just seen that carbon taxes have a negative effect on income levels. In the following lines, we demonstrate that, over the course of the transition, this negative effect diminishes as the relative productivity of the clean sector increases. Specifically, as long as the productivity of the clean sector is lower than that of the polluting sector, the negative impact of the environmental policy decreases as the relative productivity of the clean sector grows. Additionally, once the productivity of the two sectors is equal, the effect of environmental policy on income becomes positive.

Proposition 2: *If the elasticity of substitution is high enough, $\epsilon > \frac{\alpha}{1-\alpha}$, then along the energy transition ($Ac < Ad$), the negative effect of environmental policy on income levels is a decreasing function of the relative size of the clean sector, and when the productivity of the clean sector reaches that of the dirty sector ($Ac = Ad$), the effect of environmental policy on income levels becomes positive.*

Proof: *See appendix 7 for the complete proofs of Proposition 2. Two main claims are established:*

Claim 1: If $\epsilon > \frac{\alpha}{1-\alpha}$ and $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < 1$, then the combined effect of the tax rate change and the productivity ratio between sectors results in a positive increase in aggregate output growth, specifically $\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} > 0$. This implies that, under this condition, growth is enhanced when the clean sector's productivity relative to the dirty sector is low.

Claim 2: If $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} = 1$ and $\epsilon > \frac{\alpha}{1-\alpha}$, then the direct effect of the tax on output is positive, meaning $\frac{\partial \log(Y_t)}{\partial\tau} > 0$. This suggests that when both sectors have equal productivity, the tax application benefits output growth.

The results presented in this section suggest that the implementation of environmental policies can initially generate negative economic impacts. However, the extent of these effects is closely linked to the relative productivity of the clean sector. When combined with subsidies, the imposition of taxes can yield positive outcomes for income levels, with the magnitude of these effects diminishing as the clean sector becomes more productive. This occurs because taxes reduce incentives for innovation within the dirty sector, thereby curbing its growth. Consequently, as the clean sector expands relative to the dirty sector, the overall negative impact of taxes on aggregate growth is mitigated.

2.3 Effect on economic growth

In this section, we analyze the effect of a climate policy on growth. Using the equation 3, it is possible to present the growth rates of the economy as a function of the growth of the two sectors and their relative size:

$$\frac{\Delta Y_t}{Y_t} = \left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \cdot \frac{\Delta Y_{ct}}{Y_{ct}} + \left(\frac{Y_{dt}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \cdot \frac{\Delta Y_{dt}}{Y_{dt}} \tag{23}$$

The effect of a carbon tax and subsidy on the total growth rate can be decomposed into two: (i) the effect on the sectoral distribution of income, $\frac{\partial \frac{Y_{ct}}{Y_t}}{\partial \tau} > 0$ and $\frac{\partial \frac{Y_{dt}}{Y_t}}{\partial \tau} < 0$; (ii) the effect on the growth rate of the two sectors, $\frac{\partial \frac{\Delta Y_{ct}}{Y_t}}{\partial \tau} > 0$ and $\frac{\partial \frac{\Delta Y_{dt}}{Y_t}}{\partial \tau} < 0$. Using equation 23 we can derive the effects of taxes and subsidies on the growth rate of the economy.

2.3.1 Effect on sectoral share of production

First, note that the initial share of the production of each sector relative to the total production can be expressed as follows:

$$\begin{aligned}
\frac{Y_{ct}}{Y_t} &= \left[1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}} \\
\frac{Y_{dt}}{Y_t} &= \left[1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}}
\end{aligned} \tag{24}$$

Equation 25 indicates that τ generates a re composition of the product in favor of the clean sector $\frac{\partial\left(\frac{Y_{ct}}{Y_t}\right)}{\partial \tau} > 0$ and against the dirty sector $\frac{\partial\left(\frac{Y_{dt}}{Y_t}\right)}{\partial \tau} < 0$:

$$\begin{aligned}
\frac{\partial\left(\frac{Y_{ct}}{Y_t}\right)}{\partial \tau} &= \epsilon \left[1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi} \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2} \cdot \frac{1+\phi}{(1+\tau)^2}. \\
\frac{\partial\left(\frac{Y_{dt}}{Y_t}\right)}{\partial \tau} &= -\epsilon \left[1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}-1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-2} \cdot \frac{1+\phi}{(1-\phi\tau)^2}
\end{aligned} \tag{25}$$

2.3.2 Effect on sector growth rate

Using equation (A9) it is possible to derive the growth rate of the two sectors.

$$\frac{\Delta Y_{ct}}{Y_{ct}} = \alpha\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}}\right) - \varphi\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}}\right) + \epsilon(1-\alpha)\frac{\Delta A_{ct}}{A_{ct}}$$

$$\frac{\Delta Y_{dt}}{Y_{dt}} = \alpha\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}}\right) - \varphi\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}}\right) + \epsilon(1-\alpha)\frac{\Delta A_{dt}}{A_{dt}} \tag{26}$$

Therefore, $\frac{\Delta Y_{ct}}{Y_{ct}} - \frac{\Delta Y_{dt}}{Y_{dt}} = \epsilon(1-\alpha)\gamma(\eta_{ct} - \eta_{dt})$.

Proposition 3: *If* $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ *and* $\varphi > 1$ *then environmental policy has a positive effect on the aggregate growth rate.*

Proof:

From equation 14 it follows that if $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ then $\frac{Y_{ct}}{Y_{dt}} > 1$. Similarly, from 15 it follows that if $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} > 1$ then $\eta_{ct} > \eta_{dt}$.

Claim 1: $\frac{Y_{ct}}{Y_{dt}} > 1$ *and* $\eta_{ct} > \eta_{dt}$

- *Case 1:* If $\frac{A_{ct}}{A_{dt}} > 1$ and $\varphi > 1$ then $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ and $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} > 1$.
- *Case 2:* If $\frac{A_{ct}}{A_{dt}} < 1$ and $\varphi > 1$ then $\left(\frac{A_{ct}}{A_{dt}}\right)^{\phi-1} > \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}$ so if $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ then $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} > 1$.

Therefore, $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ and $\varphi > 1$ imply $Y_t^c > Y_t^d$ and $\eta_{ct} > \eta_{dt}$.

Claim 2: $\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} > \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau}$

From equation 26, $\frac{\Delta Y_{ct}}{Y_{ct}} - \frac{\Delta Y_{dt}}{Y_{dt}} = \epsilon(1-\alpha)\gamma(\eta_{ct} - \eta_{dt})$ so, $\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} - \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau} = \epsilon(1-\alpha)\frac{\partial(\eta_{ct}-\eta_{dt})}{\partial\tau}$ which implies that $\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} > \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau}$.

Claim 3: $\frac{\partial\left(\frac{\Delta Y_t}{Y_t}\right)}{\partial\tau} > \left(\frac{\partial\left(\left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}}\right)}{\partial\tau}\varphi\gamma(\eta_{ct}-\eta_{dt})\right) + \left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}}\left(\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} - \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau}\right)$. From claims 1, 2 and 3 it follows that if $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_t^d}\right)^{\varphi} > 1$ and $\varphi > 1$, then $\frac{\partial\left(\frac{\Delta Y_t}{Y_t}\right)}{\partial\tau} > 0$ and $\frac{\partial\left(\frac{\Delta Y_t}{Y_t}\right)}{\partial\tau} > 0$.

2.4 Discussion

The impact of a carbon tax alone on the income level is generally negative; it depends on two key factors: (i) the initial share of the clean sector in total production, and (ii) the relative productivity of this sector. As the productivity of the clean sector increases, environmental policies can have a positive effect on overall economic growth, shifting the balance toward a more sustainable growth trajectory. Additionally, a subsidy on clean inputs helps mitigate the negative effect of the carbon tax on the income level. The effect of the subsidy on economic growth is likely to be positive. In particular, once the clean sector becomes predominant, if the elasticity of substitution is high enough, environmental policy accelerates economic growth. To provide a quantitative understanding of the implications of the energy mix, we report the results of a parameter calibration using different initial values of clean and dirty goods production. First, we illustrate the static effect on aggregate output. This exercise highlights the effect of a tax on aggregate production and growth at different rates of production of clean and dirty inputs. We use the same parameters of Acemoglu *et al.* (2012a): $\gamma = 1$, $\alpha = 0.3$, and $\epsilon = 3$ and different values for the subsidy ϕ . These figures corroborate Propositions 1 and 2, showing that the effect of the tax without a subsidy is negative for the level of aggregate production. However, when the environmental policy is accompanied by a subsidy, the negative effect diminishes as the productivity of the clean sector increases relative to the dirty sector. With a tax rate of 0.9 and a subsidy above 0.33, the environmental policy can have a positive effect on the level of aggregate production. On average, the countries analyzed derive 20% of their energy consumption from clean sources, corresponding to a clean-to-dirty energy ratio of 0.25.

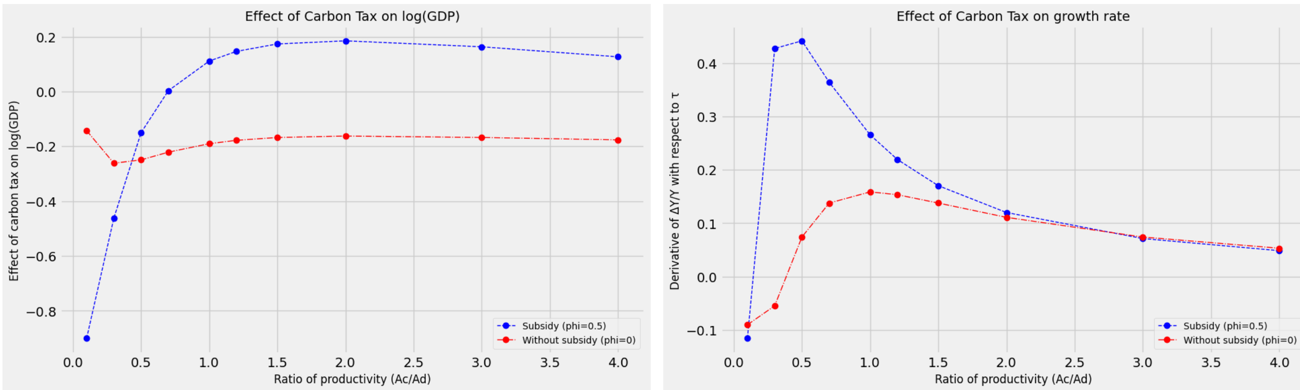


Figure 1: Static and dynamic effects of carbon taxation on economic growth

In figure 1 we illustrate the dynamic effect of the environmental policy. As predicted by propositions 1, 2 and 3, first, there is a jump in the income level which size and direction depend on the relative productivity of the clean sector, the elasticity of substitution between inputs and the size of the subsidy. Then, as the energy transition advances economic growth accelerates, the growth rate increases.

In summary, a fiscal policy that combines taxes and subsidies to steer the economy towards a productive transformation aimed at decarbonization impacts both the income level and the growth rate. The effect on income levels tends to be negative, though less pronounced for economies with a cleaner energy mix. Conversely, the effect on the growth rate is positive, as environmental policy stimulates the expansion of the sector with the highest contribution to output. In the following section, we test the key implications of the theoretical model: (i) The negative impact of a carbon tax is more pronounced for economies with a dirtier energy matrix, (ii) the adverse effect of the carbon tax diminishes over time as the economy undergoes decarbonization, and for economies with a cleaner energy mix, environmental policy ultimately accelerates growth.

3 Data and Empirical Strategy

3.1 Data

To empirically analyze the relation of a carbon tax on income and GDP growth, we use data from the World Bank Group, Energy Consumption data are from the International Energy Agency, Carbon pricing data are from World Carbon Pricing Database.

We use a yearly data panel from a sample of 66 countries, of which 23 had implemented a carbon tax. The sample covers the period from 1990 to 2020. Table 1 presents descriptive statistics of the variables of interest and the sources of the databases. The outcome variables studied are GDP growth (%) and the employment rate measured as the number of employees over an economically active population (%). Table 3 in the annex presents in detail the characteristics of the carbon tax for each of the countries studied. It can be seen that since 2010 the adoption of carbon taxes in the countries has increased. The table also shows the carbon tax's monetary value as well as the percentage of emissions covered by the tax.

Table 1: Description of the main outcome variables.

Variable	**Mean**	**Median**	**Std. Dev.**	**Source**
Real GDP (millions US$ constant 2017)	1026887	258975	2511953	Penn World Table
Crecimiento del PIB (anual %)	2.86%	2.99%	4.33%	Data WorldBank
GDP per capita (current US$)	9.384	9.532	1.143	Data WorldBank
Employment rate (% total labor)	92.30%	92.94%	4.56%	Data WorldBank
Population, total	49035868	9771437	165987702	Data WorldBank
Primary energy consumption (TWh)	1589	324	4390	Our World in Data
Clean energy fraction* (% total consumption)	14%	9%	16%	International Energy Agency
Clean electricity fraction* (% total consumption)	37%	32%	31%	International Energy Agency
Countries	66			
Observations	2044			

**Primary sources of clean energy are hydro, nuclear, solar, and wind power.*

To apply the theoretical model to real-world data, we use the share of primary energy consumed by each source as a proxy for the initial rates of production of clean and dirty inputs, Y_c and Y_d. We categorized the sample of countries based on the share of primary energy consumed by each source, dividing them into those with a low-carbon intensity energy mix and those with a high-carbon intensity energy mix. The term "low-carbon intensity" is used to describe the energy consumption of hydro, nuclear, solar, and wind sources. These sources emit lower levels of carbon than traditional fossil fuels. Conversely, the term "high-carbon intensity" is used to describe energy generated from the combustion of fossil fuels, such as coal, oil, natural gas, and biofuels. Figure 2 shows the share of energy from low-carbon intensity sources by countries.

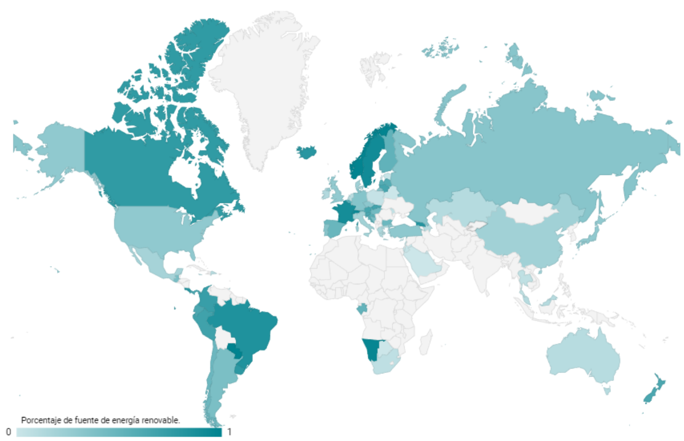


Figure 2: Map of countries according to the share of clean energy sources

The countries with a share of clean energy higher than average (14%), at the time of implementation of the carbon tax, constitute the database of countries with a "low-carbon intensity" energy matrix. Similarly, the countries that had a clean energy share lower than average (14%) at the time of implementing the carbon tax constitute the database of countries with a "high-carbon intensity" energy matrix. Table 2 presents the statistics of the outcome variables for each sample, the full sample of 66 countries, and the sample of countries with polluting and clean energy mix.

3.2 Empirical strategy

In this paper, we estimate the effect of introducing a carbon tax on GDP growth, according to the primary energy sources of consumption, to validate the corollaries defined in the theoretical model. For this, we use the event study method with the estimators proposed by Callaway & Sant'Anna (2021), Sun & Abraham (2021), and TWFE (Two Way Fixed Effects).

Event Study is used to estimate the effect of introducing a carbon tax on GDP growth rate. In this approach, we aim to estimate the effect on GDP which is not associated with its historical economic growth. We assume that changes in GDP not predicted by historical GDP growth in the country itself, nor by current and past international economic shocks, are exogenous. Several studies have used the event study strategy to analyze the effects of regulatory changes on carbon prices, energy, and stock prices (Mansanet-Bataller & Pardo (2009); Fan *et al.* (2017); Bushnell *et al.* (2013), among others).

We introduce the following assumptions, which capture the effect of carbon tax on GDP: *Treatment timing.* The treatment time assumption refers to a scenario in which there are several periods and countries implement a carbon tax at any time within those periods. Once a country implements the tax, it remains in treatment for the remainder of the period. This assumption implies that the timing of the implementation of the tax is unrelated to other factors such as GDP that may influence the outcome, meaning that it is considered exogenous. In other words, the timing of tax implementation is independent of other factors and is not influenced by them.

No-anticipation assumption. The implementation of the carbon tax does not affect the path of GDP outcomes before the treatment period. In other words, the counterfactual outcome paths for GDP in periods before the treatment period would have been the same whether or not the carbon tax had been implemented at some point in the future. Similarly, the treatment assignment does not depend on the potential GDP outcomes in any period.

Parallel trends. The parallel trends assumption in the context of a staggered events study with a carbon tax as the treatment and GDP as the outcome variable would imply that in the absence of the carbon tax, the trends in GDP would be parallel across the treated and control groups. In this study, any differences in the post-treatment outcomes between the two groups can be attributed to the treatment (i.e., the carbon tax) and not to pre-existing differences in the trends of GDP.

3.2.1 Heterogeneous Effects

Proposition 2 states that the effect of a carbon tax on the economy's growth rate is negative if the polluting sector's share in final output exceeds a critical level relative to the share of the clean sector. On the other hand, it indicates that the carbon tax promotes economic growth if the share of the clean sector in final production surpasses a critical level relative to the polluting sector. To test this hypothesis empirically, we use the Event Study strategy and examine two sub samples of countries. The first sub-sample consists of "polluting countries," where the share of clean sources is below the country average. The second sub-sample includes "clean countries", where the share of clean sources exceeds the country average (above 14%).

In this model, we consider the year in which the carbon tax was introduced as year 0. We then define the periods before ($t < 0$) and after ($t > 0$) the introduction of the carbon tax, and we align time t=0 for all countries in the treatment group. We assume that the evolution of the potential outcome in the absence of the treatment can be decomposed into a time-fixed effect. Based on this assumption, we estimate the average dynamic effect of introducing a carbon tax on GDP growth ($Y_{c,t}$) in country c and year t. To conduct our analysis, we employ equation 27.

$$Y_{c,t} = \beta_1 \sum_{-T \leq r \leq T}^{r \neq 0} 1\left[CarbonTax_{c,t} = r\right] + \Phi_c + \Phi_t + \epsilon_{c,t} \tag{27}$$

where $Y_{c,t}$ is the GDP growth rate. β_1 measures the average dynamic effect of a carbon tax in the sample of countries. When the outcome variable is the GDP growth rate this estimator tests the *proposition 2*. Also include α_c country fixed effects c for unobserved country-specific characteristics and Φ_t time fixed effects to capture other policy and time-varying resource price shocks, among other changes that may occur over time.

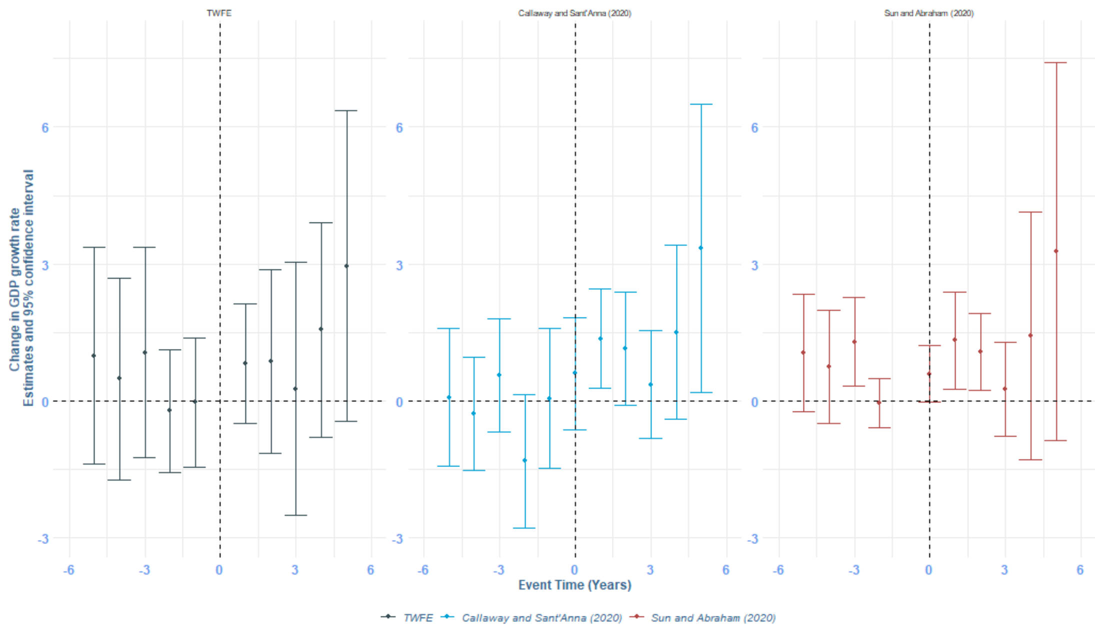
4 Results

4.1 Implications of carbon tax on growth rate and employment

Figure 3 presents the effect of implementing a carbon tax on the annual GDP growth rate using the database of 66 countries. Implementing a carbon tax is associated with an increase in the GDP growth rate in the early years of the climate policy, and is maintained when using different model specifications. Table 5 presents the estimators of the average effect of implementing a carbon tax on GDP growth over the next ten years. It is observed that implementing the carbon tax is associated with a 1.5 percentage point growth in GDP one year after the policy, this effect is significant using the Sun et al. specification (Column 3). Using the TWFE and Callaway specification, implementing the carbon tax is associated with 0.9 and 0.6 percentage point growth in GDP, respectively, in the year after adopting the policy. The effect under the Callaway methodology is consistent with the results from the literature in which the effect of the carbon tax between the first and second year is associated with an increase in the GDP growth rate of 0.5 percentage points, in the literature implementing a carbon tax is not associated with adverse effects on the GDP growth rate.

This result implies that if a country hypothetically implements a carbon tax it can be expected to, on average, experience an increase in the growth rate of 1 percentage point in the following year after implementing the policy.

Figure 3: Implications of carbon tax on GDP growth rate.



4.2 Heterogeneous effects according to the composition of the energy matrix

In this section, we empirically estimate how the effect of introducing a carbon tax on the GDP growth rate and the employment rate varies according to the composition of the countries' energy matrix, i.e., according to the share of primary energy sources in final consumption. To test the corollaries derived from the theoretical model we use the share of energy consumed from clean and polluting sources as a proxy for the initial share of the clean and polluting sectors in the final product.

4.2.1 A carbon tax in polluting countries.

To test Corollary 1, it would be expected that using the sample of polluting countries (whose consumption of energy from fossil and biofuel sources is higher than the sample average), the effect of introducing a carbon tax on the annual GDP growth rate may be negative.

Figure 4: Annual GDP growth (%) in countries with a polluting energy matrix.

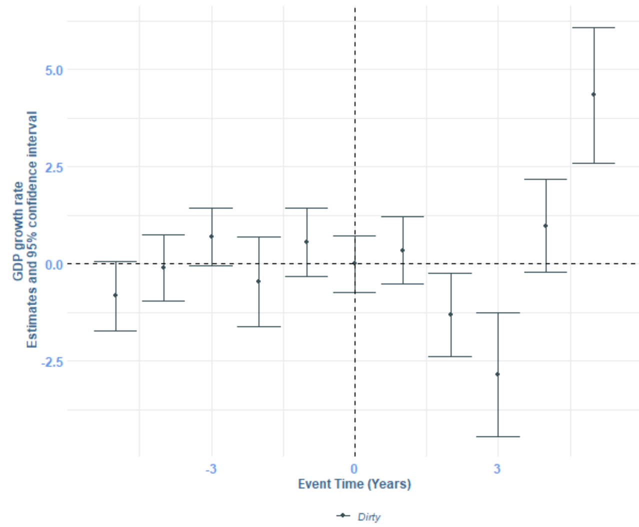


Figure 4 presents the coefficients of the average cumulative effect of the carbon tax on the growth rate in polluting countries, and on the y-axis, in addition to indicating the magnitude, the 95% confidence intervals are presented. It is observed that implementing the carbon tax in countries with a polluting energy matrix is associated with a reduction in the growth rate in the second and third years the In the second year the cumulative effect of implementing the carbon tax is -1.3 percentage points of annual GDP, in the third year the effect is -2.8 percentage points. This effect can be explained by the cost of discouraging the innovation of technologies that require polluting energy. In the long term, a positive effect on the annual GDP growth rate may be due to countries adopting new clean technologies.

4.2.2 A carbon tax in clean countries.

Corollary 1 implies that introducing a carbon tax favors the annual GDP growth rate in countries with a cleaner-than-average energy matrix. The effects reported in table 4 show the estimators for countries with a clean and polluting energy matrix, columns 2 and 4 respectively. The results suggest that if hypothetically a country with a clean energy matrix implements a carbon tax it can be expected to experience an increase in the growth rate of 1.8 percentage points in the following year after implementing the policy. This result validates the stipulation in Corollary 1 that the higher the initial share of the clean sector in final production, the greater the positive effect of the carbon tax. In this case, we approximate the clean sector's share of final energy consumption to the share of clean primary energy sources.

Figure 5: GDP growth (%) in countries with clean energy matrix.

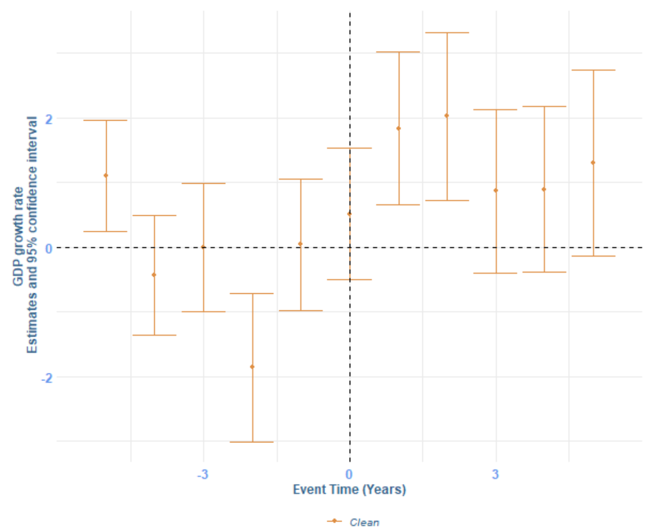


Figure 5 shows that the effect of a carbon tax on the GDP growth rate is positive for countries whose primary energy comes from clean sources. This result suggests that introducing a carbon tax is associated with an increase in annual GDP of 1.4 percentage points in the first five years after implementing the policy, and this effect is significant. The results suggest that the higher the share of clean sources at the time of implementing the climate policy, the more favorable the effect of the tax on the GDP growth rate.

Figure 6 presents the estimated dynamic effects of carbon tax implementation on GDP growth rate, analyzed across different subsamples of countries. These subsamples are defined by increasing thresholds of clean primary energy participation in their energy mix, progressing from top-left to bottom-right panels. Observing the figure, a clear trend emerges: as the proportion of clean energy in a country's energy mix increases (moving towards the bottom-right panels), the estimated effect of the carbon tax on GDP growth rate in the years immediately following implementation (particularly years 1 and 2) shifts from a more negative impact towards a more positive one. This suggests that the composition of a country's energy sources significantly moderates the short-term economic impacts of carbon taxation.

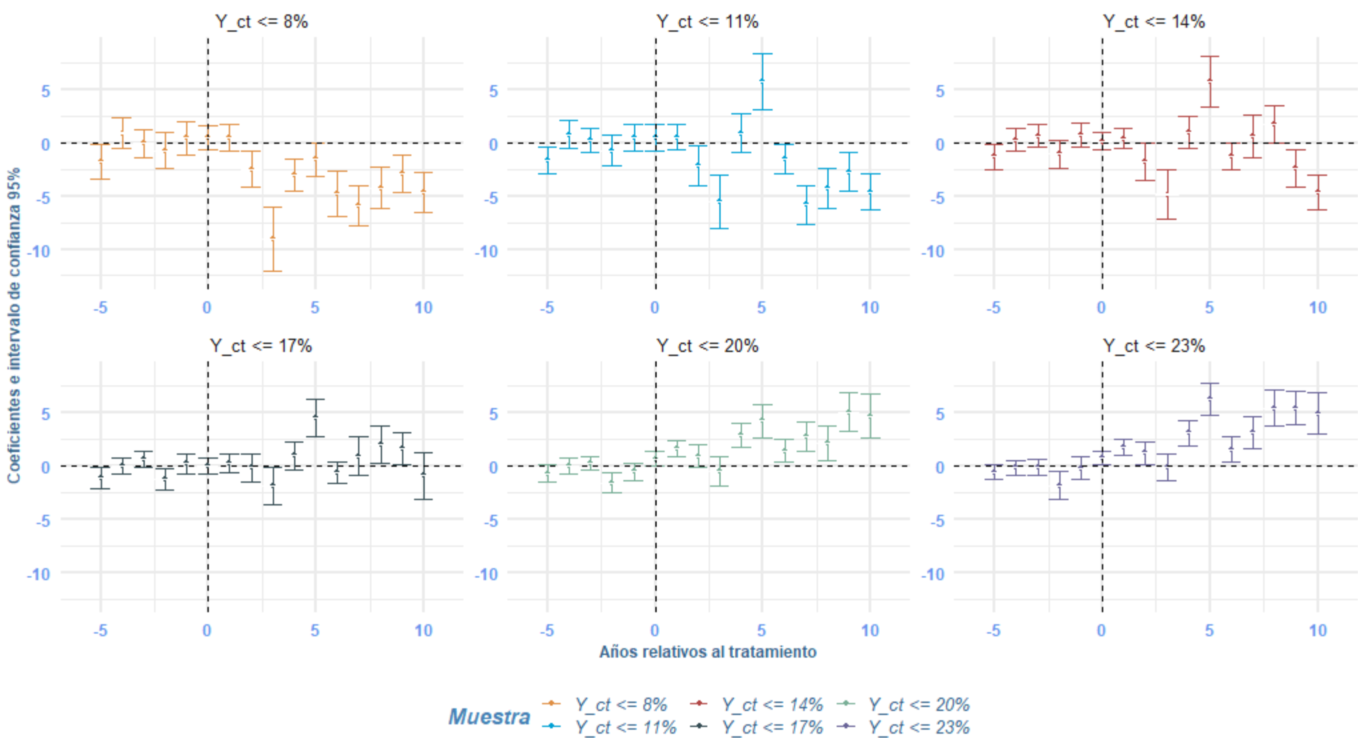


Figure 6: Effect of the carbon tax at different thresholds of the participation of energy sources.

Note: This figure displays the estimated effect of carbon taxes on GDP growth rate for different subsamples of countries. Each subplot represents a subsample defined by a threshold on the proportion of clean primary energy sources in the country's energy mix (Y_ct). The threshold for clean primary energy participation increases from top-left to bottom-right, ranging from $Y\_ct \leq 8\%$ in the top-left panel to $Y\_ct \leq 23\%$ in the bottom-right panel.

5 Robustness exercises

To verify whether the previous results were robust, we performed several econometric exercises. we used the sources of electricity generation, in exchange for the energy matrix, as they can approximate the share of the sectors (clean and polluting) in the final production.

We assess the impact of the carbon tax on growth rates by examining different samples of polluting and clean countries. The goal is to identify the threshold at which the carbon tax's effect shifts from negative (for polluting countries) to positive. We defined various cut-off points Θ based on the proportion of clean energy sources in the total energy consumption. Specifically, we considered scenarios where the sample of polluting countries does not exceed certain percentages of clean energy sources. The findings on the carbon tax's impact on GDP growth, relative to these cut-off points for clean and polluting samples, are presented in 6. When the sample of polluting countries includes less than 8% clean energy (representing the most polluting countries), the effect is highly negative. Conversely, when the sample reaches 23% clean energy, the effect becomes positive. The carbon tax is beneficial for GDP growth in countries with an energy matrix comprising at least 17% clean energy.

5.1 Effect using electricity mix

The electricity mix is composed of the set of sources available to generate the electricity consumed within a country. Electricity unlike energy can be generated entirely by renewable sources, therefore, for this exercise, we divide the sample of polluting and clean countries based on the 37% share of clean sources. That is, if the country generates more than 37% of electricity from sources such as solar, wind, hydro, and nuclear, it is considered clean, and would be part of the sub-sample of clean countries, otherwise it would belong to the sub-sample of polluting countries.

Figure 7: Effect of carbon tax on GDP growth rate in countries with polluting electricity mix.

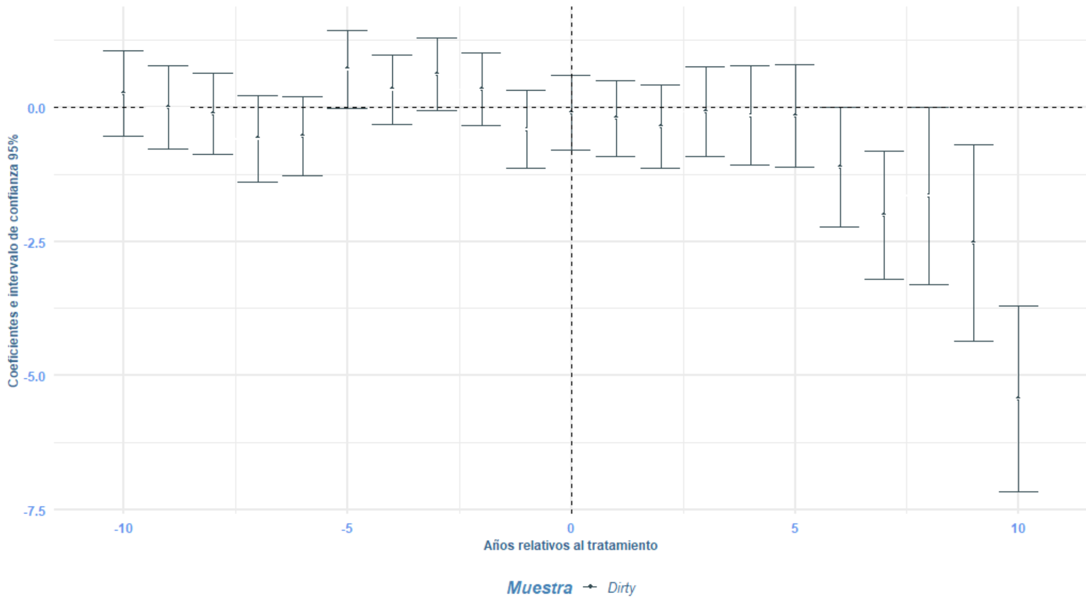
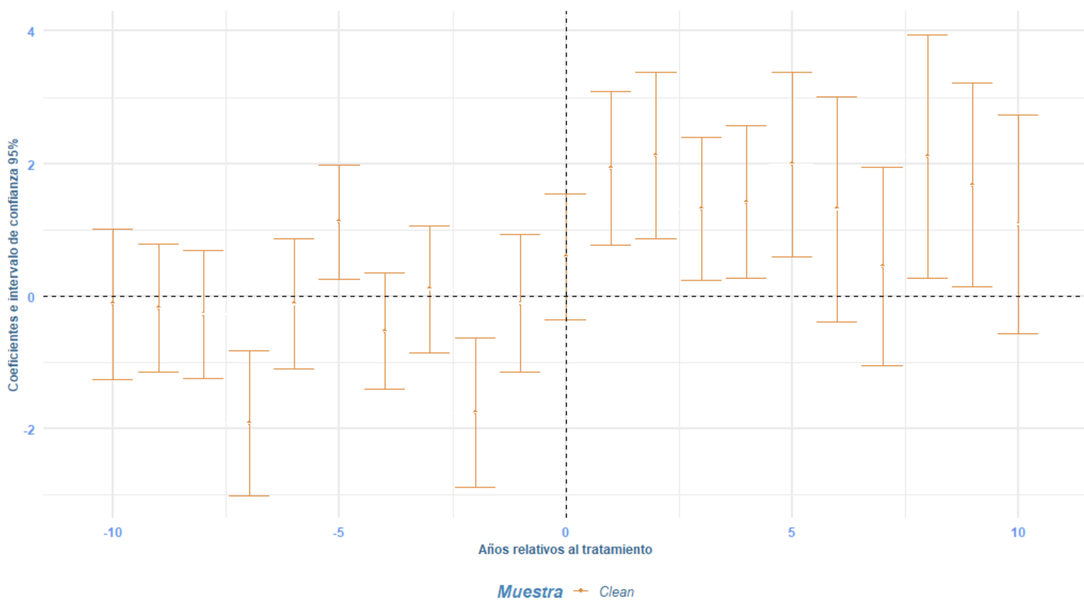


Table 6 presents the estimators of the effect of the carbon tax on the GDP growth rate according to the electricity matrix. The coefficients estimated with the electricity matrix are similar in magnitude and direction to those estimated with the energy matrix, however, these results are significant in more periods unlike those estimated with the energy matrix. Figure 7 presents the coefficients of the effect of the carbon tax on the growth rate using the sample of countries with a polluting electricity matrix. The carbon tax is associated with negative growth rates, this effect is larger in magnitude than the one calculated with the sample of countries divided according to the energy matrix. On the other hand, the figure 8 shows similar results to those obtained with the sample of clean countries using the energy matrix. For countries with a clean electricity matrix, the effect is slightly positive, increasing the growth rate by 0.6 percentage points in the first 5 years after implementing the climate policy.

Figure 8: Effect of carbon tax on GDP growth rate in countries with clean electricity mix.



6 Conclusions

This study provides robust evidence that the macroeconomic impact of a carbon tax is significantly influenced by the composition of a country's energy mix. By examining the proportion of energy generated from fossil fuels and low-carbon-intensity sources, we highlight the heterogeneity in outcomes following the implementation of a carbon tax. In economies heavily reliant on fossil fuels, the introduction of a carbon tax may lead to a short-term decline in GDP growth, as predicted by the theoretical model. However, the long-term trajectory suggests that growth can recover, particularly as the share of clean energy increases or energy efficiency improves, validating proposition 1 of the model.

Conversely, in countries where energy production relies primarily on low-carbon sources, the imposition of a carbon tax may positively impact GDP growth in the short term, with minimal or no negative effects on employment. This suggests that countries with cleaner energy mixes are better positioned to absorb the initial economic costs of carbon pricing and can even experience economic benefits from the transition to cleaner production.

Our findings also indicate that the adverse effects on GDP growth in high-carbon-intensity economies tend to dissipate over time. This is due to a shift in demand away from polluting goods, which incentivizes innovation and expansion in the clean energy sector. As this sector grows, it eventually overtakes the polluting industries, allowing the economy to return to its pre-tax growth trajectory. The transition is marked by a reallocation of labor and capital towards cleaner technologies, driven by the carbon tax's effect of increasing the relative cost of polluting goods.

The study also supports the idea that a carbon tax can serve as an effective policy tool not only for reducing carbon emissions but also for fostering long-term clean economic growth. As the tax increases the costs of production in high-emission sectors, it simultaneously encourages greater productivity and innovation in the clean sector, ultimately transforming the economic structure towards sustainability.

A key policy implication derived from our model is the strategic use of carbon tax revenues. In the early stages of the transition, it is important to reinvest these revenues in the development and scaling of clean technologies, enabling the clean energy sector to meet growing demand. This reinvestment can mitigate the short-term negative impact on economic growth, facilitating a smoother transition to a low-carbon economy.

In conclusion, the study underscores the importance of considering a country's energy mix when designing carbon taxes and other climate policies. Tailoring these policies to national contexts can optimize their economic and environmental effectiveness, minimizing transitional costs while accelerating the shift towards sustainable development.

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Supplementary Tables

Table 4: Effect of carbon tax on GDP growth rate, according to energy matrix

Effect on GDP growth rate				
Period	Estimator	Standard error	Estimator	Standard error
	Panel A. Clean Countries		*Panel B. Dirty Countries*	
-4	-0.4407	(0.4502)	-0.1065	(0.4174)
-3	-0.006	(0.5013)	0.7044	(0.3898)
-2	-1.8644*	(0.5847)	-0.4588	(0.5817)
-1	0.0342	(0.4994)	0.5648	(0.4513)
0	0.5112	(0.5215)	-0.0016	(0.3975)
1	1.8328*	(0.6228)	0.3466	(0.4201)
2	2.024*	(0.6683)	-1.3099*	(0.5608)
3	0.8623	(0.6068)	-2.8622*	(0.764)
4	0.8891	(0.6215)	0.9814	(0.5686)
5	1.3018	(0.733)	4.3461*	(0.9005)
6	0.7619	(0.872)	-0.4006	(0.5029)

Note: Panel A shows the results for the sample of cleaner countries, which consists of 25 countries, while Panel B presents the results for the more polluting countries, which consists of 41 countries.

Table 2: Descriptive statistics of the samples

Variable	**Mean**	**Median**	**Std. Dev.**	**Mean**	**Median**	**Std. Dev.**
	All countries					
	With carbon tax			without carbon tax		
GDP real (millions 2017US$)	850686.90	393482.88	1082446.42	1121275.39	166135.61	3006622.00
GDP per capita (current US$)	9.89%	10.10%	0.94%	9.11%	9.15%	1.15%
GDP growth (annual %)	2.482	2.683	3.694	3.058	3.254	4.630
Employment rate (% total labor)	92.07%	92.85%	4.75%	92.43%	93.02%	4.45%
Primary energy consumption (TWh)	1094.63	474.96	1379.31	1855.63	254.93	5334.58
Clean electricity fraction (%)	47%	46%	33%	31%	24%	29%
Clean energy fraction (%)	23%	18%	20%	9%	4%	11%
Countries	23			43		
	Countries with a low carbon intensity energy mix					
	With carbon tax			without carbon tax		
GDP real (millions 2017US$)	609238.88	322000.91	721705.56	654034.94	161623.54	1096710.94
GDP per capita (current US$)	10.02%	10.17%	0.91%	9.26%	9.44%	1.04%
GDP growth (annual %)	2.17	2.57	3.28	2.35	2.67	3.47
Employment rate (% total labor)	92.14%	92.70%	4.51%	90.99%	91.68%	4.23%
Primary energy consumption (TWh)	841.80	351.14	1103.98	748.81	232.82	1166.34
Clean electricity fraction (%)	70%	71%	21%	60%	60%	16%
Clean energy fraction (%)	36%	32%	17%	23%	23%	8%
Countries	13			12		
	Countries with a high carbon intensity energy mix					
	With carbon tax			without carbon tax		
GDP real (millions 2017US$)	1164569.32	579572.91	1359162.31	1302519.86	168376.80	3459270.47
GDP per capita (current US$)	9.72%	9.86%	0.95%	9.06%	9.07%	1.19%
GDP growth (annual %)	2.89	2.83	4.14	3.33	3.57	4.98
Employment rate (% total labor)	91.97%	93.96%	5.06%	93.06%	93.67%	4.39%
Primary energy consumption (TWh)	1423.29	1010.65	1614.45	2289.49	277.86	6198.64
Clean electricity fraction (%)	16%	12%	15%	19%	10%	24%
Clean energy fraction (%)	6%	5%	6%	3%	1%	5%
Countries	10			31		

Note: "Clean electricity fraction" includes electricity from renewables and nuclear. "Clean energy fraction" encompasses clean electricity and direct renewable energy use. Units are as indicated in the variable names, with GDP in millions of 2017 US$ and primary energy consumption in TWh.

Table 3: Characteristics of the carbon tax in the countries analyzed.

Jurisdiction	**Sectors**	**Fuels**	**Tax Point**	**Year**	**Emissions**	**Covered**	**Price**
Argentina	Most	Liquid, gas	Prod., distr.	2018	441	20%	6
Canada	Most	Fossil	Reg. distr.	2019	817	22%	32
Chile	Power, ind.	Fossil	Users	2017	149	39%	5
Colombia	Most	Liquid, gas	Sellers, importers	2017	190	24%	5
Denmark	Build., transp.	Fossil	Distr.	1992	63	35%	28
Estonia	Power, ind.	Thermal	Users	2000	28	6%	2
Finland	Ind., transp.	Fossil excl. peat	Distr.	1990	112	36%	72.8
France	Ind., build., transp.	Fossil	Distr.	2014	488	35%	52
Iceland	Most (ETS)	Liquid, gas	Prod., distr.	2010	5	55%	35
Ireland	Most (ETS)	Fossil	Distr.	2010	65.6	49%	39
Japan	Most	Fossil	Prod., distr.	2012	1345	75%	3
Latvia	Ind., power	Fossil excl. peat	Distr.	2004	18	3%	14
Liechtenstein	Ind., build., transp.	Fossil	Distr.	2008	0	26%	101
Mexico	Power, ind., etc.	All excl. nat. gas	Prod., distr.	2014	822	23%	3
Norway	Most (ETS)	Liquid, gas	Prod., distr.	1991	75	66%	69
Poland	Most (ETS)	Fossil	Users	1990	429	4%	0.08
Portugal	Ind., build.	Fossil	Distr.	2015	81	29%	28
Singapore	Power, ind.	Fossil	Facility ops.	2019	56	80%	4
Slovenia	Build., transp.	Fossil	Distr.	1996	21	50%	20
South Africa	Ind., build., etc.	Not spec.	Users	2019	640	80%	9
Spain	F-gases	Not spec.	First entry	2014	367	3%	18
Sweden	Transp., build. (ETS)	Fossil	Distr.	1991	111	40%	137
Switzerland	Ind., build., etc.	Fossil	Distr.	2008	55	33%	101
United Kingdom	Power	Fossil	Users	2013	583	23%	25
Ukraine	Ind., build.	Fossil	Users	2011	312	71%	0.3

Note: Emissions in MtCO_2e; coverage as share of national GHG emissions; prices in US$.
Source: World Bank's State and Trends of Carbon Pricing report.

Table 5: Effect of carbon tax on GDP growth rate.

Effect on GDP growth rate			
Period	TWFE	Sun et. al.	Callaway et. al.
-4	0.548 (1.1334)	0.7931 (0.625)	-0.151 (0.6237)
-3	1.1046 (1.1724)	1.297** (0.4847)	0.5186 (0.6281)
-2	-0.1466 (0.6834)	0.0445 (0.2604)	-1.2647 (0.7162)
-1	-0.0484 (0.7365)	0.6092. (0.2996)	-0.0448 (0.7143)
0	0.9087 (0.6737)	1.543** (0.572)	0.6128 (0.6395)
1	1.0405 (1.047)	1.326*** (0.4413)	1.5846** (0.5919)
2	0.4668 (1.4567)	0.5076 (0.5355)	1.3926* (0.5856)
3	1.7464 (1.2217)	1.707 (1.363)	0.605 (0.6133)
4	3.1444 (1.7416)	3.575 (2.085)	1.7872. (0.9386)
5	0.7838 (1.0323)	1.027 (0.6695)	3.651* (1.6256)
6	0.7748 (1.4977)	1.021 (1.232)	1.1508 (0.6942)
Fixed-Effects			
Country	Yes	Yes	No
Year	Yes	Yes	No
S.E.:Clustered	Country	Country	Country
Observations	2288	2288	2288

Note: This table presents estimators of the average effect of implementing a carbon tax on GDP growth rate over a ten-year event window. Columns TWFE, Sun et al., and Callaway et al. show the estimated effect using different econometric specifications. The values represent the percentage point change in GDP growth rate associated with the implementation of a carbon tax.

Table 6: Effect of carbon tax on GDP growth rate, according to electricity matrix composition.

Effect on GDP growth rate				
Period	Estimator	Standard error	Estimator	Standard error
	Panel A. Clean Countries		*Panel B. Dirty Countries*	
-4	-0.5326	(0.4392)	-0.1527	(0.4358)
-3	0.1009	(0.4992)	0.7496	(0.4114)
-2	-1.7654*	(0.5608)	-0.4363	(0.5449)
-1	-0.111	(0.5012)	0.4642	(0.4953)
0	0.5908	(0.4877)	-0.0125	(0.4074)
1	1.9277*	(0.599)	0.2502	(0.4246)
2	2.1262*	(0.6625)	-1.4785*	(0.569)
3	1.317*	(0.566)	-2.956*	(0.8038)
4	1.4174*	(0.6158)	0.8687	(0.6318)
5	1.9908**	(0.7196)	4.2354*	(0.9454)
6	1.3091	(0.8273)	-0.5926	(0.5206)

Note: This table shows the estimated effect of carbon taxes on GDP growth rate, differentiated by the electricity matrix composition of countries. Panel A presents results for countries with a clean electricity matrix, while Panel B shows results for countries with a dirty electricity matrix. The estimators are similar to those obtained using the energy matrix, with some differences in significance. For countries with a dirty electricity matrix, the effect tends to be negative, whereas for countries with a clean electricity matrix, the effect is slightly positive, increasing the growth rate by approximately 0.6 percentage points in the first five years after carbon tax implementation.

7 Appendix

Intermediate Inputs

From the first-order conditions is obtained the demand for machines and labor in each sector,

$$x_{jit} = \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \quad \text{and} \quad L_{jt} = \left(\frac{(1-\alpha) P_{jt}}{w_{jt}}\right)^{\frac{1}{\alpha}} A_{jit}^{\frac{1-\alpha}{\alpha}} x_{jit} \tag{A1}$$

where w_{jt} denotes the wage paid for each unit of labor hired and p_{jit} is the price that the producer of inputs must pay for each machine used.

Producers of intermediate goods maximize profits by knowing the demand function they face,

$$\max_{x_{ji}} \{p_{jit} x_{jit} - x_{jit}\} \tag{A2}$$

Machines are produced at marginal cost ν under monopolistic competition and, sold at price p_{jt}, taking into account the demand for machines x_{jit} in the sector in which they are used. Therefore the profits of the monopolists, π_{jt}, are given by: $\pi_{jt} = (p_{jit} - \nu) x_{jit}$. So, replacing the demand for machines, the profits of the monopolist are:

$$\pi_{jt} = (p_{jit} - \nu) \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \tag{A3}$$

Following Acemoglu *et al.* (2012a), we normalize $\nu = \alpha^2$, so, each monopolist sets a price $p_{jit} = \frac{1}{\alpha}$. Thus, replacing the price of machine $p_{jit} = \frac{1}{\alpha}$ in equation A1, the optimal demand for machines and the profits of intermediate goods in each sector can be written as:

$$x_{jit} = \alpha^{\frac{2}{1-\alpha}} A_{jit} L_{jt} (P_{jt})^{\frac{1}{1-\alpha}} \quad \text{and} \quad \pi_{jt} = (1-\alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} A_{jit-1} L_{jt} \tag{A4}$$

and the quantities of inputs produced in sector j are:

$$Y_{jt} = \alpha^{\frac{2\alpha}{1-\alpha}} A_{jt} L_{jt} \left(P_{jt}\right)^{\frac{\alpha}{1-\alpha}} \tag{A5}$$

Combining this equation A3 and replacing ν, the equilibrium profits of machine producers can be written as:

$$\pi_{jt} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} A_{jit-1} L_{jt} \tag{A6}$$

Factors of production in equilibrium

From the profit-maximization problem of the producer of machines, assuming the total labor supply is normalized to one, such that $L_{ct} + L_{dt} = 1$, and given that equilibrium wages are equal, $w_{ct} = w_{dt}$, we can substitute the price index (equation 4) to express the equilibrium labor allocation in each sector as follows:

$$\begin{aligned} L_{ct} &= \frac{(1+\tau)^{\epsilon} A_{ct}^{\varphi}}{(1-\tau\phi)^{\epsilon} A_{ct}^{\varphi} + (1+\tau)^{\epsilon} A_{dt}^{\varphi}} \\ L_{dt} &= \frac{(1-\tau\phi)^{\epsilon} A_{dt}^{\varphi}}{(1-\tau\phi)^{\epsilon} A_{ct}^{\varphi} + (1+\tau)^{\epsilon} A_{dt}^{\varphi}} \end{aligned} \tag{A7}$$

where $\varphi = (\epsilon - 1)(1-\alpha)$. Additionally, the equilibrium prices can be determined as follows:

$$\begin{aligned} P_{ct} &= \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\tau\phi)^{-(\epsilon-1)} A_{dt}^{\varphi}\right)^{\frac{1}{\epsilon-1}}}{A_{ct}^{(1-\alpha)}} \\ P_{dt} &= \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\tau\phi)^{-(\epsilon-1)} A_{dt}^{\varphi}\right)^{\frac{1}{\epsilon-1}}}{A_{dt}^{(1-\alpha)}} \end{aligned} \tag{A8}$$

It is important to note that subsidies for clean production increase the prices of both inputs, while taxes on dirty production decrease them. However, because the percentage change is proportional for both prices, the relative price ratio remains unchanged. Regarding labor equilibrium, higher productivity and the carbon tax in the clean sector lead to a greater allocation of labor to that sector. Furthermore, as the clean energy subsidy increases, the labor share in the clean sector expands, even if the technological level in the clean sector is relatively low.

Replacing the prices (eq. A7), and labor (eq. A8), in equation 9, I can get the output of two sectors in terms of productivity for each sector.

$$
\begin{aligned}
Y_{ct} &= \alpha^{\frac{2\alpha}{1-\alpha}} \cdot (1+\tau)^{\epsilon} A_{ct}^{\epsilon(1-\alpha)} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \\
Y_{dt} &= \alpha^{\frac{2\alpha}{1-\alpha}} \cdot (1-\phi\tau)^{\epsilon} A_{dt}^{\epsilon(1-\alpha)} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}}
\end{aligned} \tag{A9}
$$

Proof Proposition 2:

Claim 1: *If* $\epsilon > \frac{\alpha}{1-\alpha}$ *and* $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < 1$ *then* $\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} > 0$

From 22,

$$
\begin{aligned}
\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} =& (1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon-1}\epsilon\left(\frac{(1+\tau)^{-1} - \phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon-1}\right]^2}\right) \\
&- (1-\phi\tau)^{\epsilon-1}(1+\tau)^{\epsilon-1}\epsilon\left(\frac{(1-\phi\tau) - (1+\tau)\phi}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon}\right]^2}\right) \\
&- (1+\tau)^{-(\epsilon-1)}(1-\phi\tau)^{-(\epsilon-1)}\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-1} - \phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}\right]^2}\right)
\end{aligned}
$$

rearranging,

$$\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} = \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\epsilon\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right]^2}\right)$$
$$+\frac{1}{(1+\tau)^2}\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\epsilon\left(\frac{2\tau\phi}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right]^2}\right)$$
$$-\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+1\right]^2}\right)$$

grouping terms to simplify the expression,

$$\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} = \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\left((1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}\right)$$
$$\left(\frac{\epsilon}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right]^2}-\frac{\frac{\alpha}{1-\alpha}}{\left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+1\right]^2}\right)$$
$$+\frac{1}{(1+\tau)^2}\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\epsilon\left(\frac{2\tau\phi}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right]^2}\right)$$

Notice that if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < 1$ then if $\left(1-\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right)\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < \left(1-\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right)$ so $\frac{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right]^2}{\left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+1\right]^2} < 1$.

Now, since $\frac{\epsilon}{\frac{\alpha}{1-\alpha}} > 1$, this implies $\frac{\epsilon}{\frac{\alpha}{1-\alpha}} > \frac{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right]^2}{\left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+1\right]^2}$, so $\frac{\epsilon}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right]^2} - \frac{\frac{\alpha}{1-\alpha}}{\left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+1\right]^2} > 0$ and $\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} > 0$.

Claim 2 *: If $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} = 1$ and $\epsilon > \frac{\alpha}{(1-\alpha)}$ then $\frac{\partial \log(Y_t)}{\partial\tau} > 0$*

If $\epsilon > \frac{\alpha}{(1-\alpha)}$ then

$$\frac{\partial \log(Y_t)}{\partial \tau} > \frac{\epsilon}{(1+\tau)}\left[\left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}\right) - \left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{-\epsilon}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}}\right)\right] \tag{A10}$$

Therefore, if the following condition holds then $\frac{\partial \log(Y_t)}{\partial \tau} > 0$:

$$\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} > \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}} + \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{-1}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)} + 1}$$

If $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} = 1$ then the condition becomes

$\frac{1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}}{1+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} > \frac{1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}}{1+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}} + \frac{\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)} - \phi\left(\frac{1-phi\tau}{1+\tau}\right)^{-1}}{\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}+1}$ rearranging,

$\frac{1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2} - \left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)} + \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{-1}}{1+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} > \frac{1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}}{1+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}$ multiplying both sides by $1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}$,

$$1 - \left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)} + \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{-1} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2} > \frac{\left(1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right)^2}{1+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}$$

Notice that $1 - \left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)} > \frac{\left(1-\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right)^2}{1+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}$ and $\left(\frac{1-\phi\tau}{1+\tau}\right)^{-1} > \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}$

Therefore if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} = 1$ and $\epsilon > \frac{\alpha}{(1-\alpha)}$ then $\frac{\partial \log(Y_t)}{\partial \tau} > 0$

Second derivative of the effect of the tax and relative productivities on sectoral production

$$\frac{\partial \log(Y_t)}{\partial \tau} = \left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}\right)\frac{1}{(1+\tau)}\epsilon\left(\frac{\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon} - \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right)\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right)}\right)$$
$$- \left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\right)\frac{1}{(1+\tau)}\frac{\alpha}{(1-\alpha)}\left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}}\right)$$

so If $\phi = 0$ then

$$\frac{\partial \log(Y_t)}{\partial \tau} = \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{1}{(1+\tau)} \left(\epsilon \frac{\left(\frac{1}{1+\tau}\right)^{\epsilon} - \left(\frac{1}{1+\tau}\right)^{\epsilon-1}}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{\epsilon-1}\right)\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{\epsilon}\right)} - \frac{\alpha}{(1-\alpha)} \left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1}{1+\tau}\right)^{-(\epsilon-1)}}\right)\right.$$

The second derivative with respect to the relative productivity of the clan sector $\frac{A_{ct}}{A_{dt}}$ is always negative and exactly the same, for both sectors Y_{ct} and Y_{dt}.

$$\begin{aligned} \frac{\partial^2 \log(Y_{jt})}{\partial \tau \partial \frac{A_{ct}}{A_{dt}}} = &- \frac{\alpha}{1-\alpha} \left(\frac{(1+\tau)^{-\epsilon}(1-\phi\tau)^{-(\epsilon-1)} + (1+\tau)^{-(\epsilon-1)}\phi(1-\phi\tau)^{-\epsilon}}{\left((1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}\right)^2}\right) \\ &- \epsilon \left(\frac{(1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon} + (1+\tau)^{\epsilon}\phi(1-\phi\tau)^{\epsilon-1}}{\left((1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon}\right)^2}\right) \end{aligned} \tag{A11}$$

Second derivative of the effect of the tax and relative productivities on final good production

The analysis of the second derivative with respect to the relative productivity of the clean sector allows us to prove the second part of Proposition 2.

$$
\begin{aligned}
\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} =& \epsilon\left(\frac{(1+\tau)^{\epsilon-2}(1-\phi\tau)^{\epsilon-1}-(1+\tau)^{\epsilon-1}\phi(1-\phi\tau)^{\epsilon-2}}{\left[(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon-1}\right]^2}\right)\\
&-\epsilon\left(\frac{(1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon}-(1+\tau)^{\epsilon}\phi(1-\phi\tau)^{\epsilon-1}}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon}\right]^2}\right)\\
&-\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-\epsilon}(1-\phi\tau)^{-(\epsilon-1)}-(1+\tau)^{-(\epsilon-1)}\phi(1-\phi\tau)^{-\epsilon}}{\left[(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{-(\epsilon-1)}\right]^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} =& (1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon-1}\epsilon\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon-1}\right]^2}\right)\\
&-(1-\phi\tau)^{\epsilon-1}(1+\tau)^{\epsilon-1}\epsilon\left(\frac{(1-\phi\tau)-(1+\tau)\phi}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon}\right]^2}\right)\\
&-(1+\tau)^{-(\epsilon-1)}(1-\phi\tau)^{-(\epsilon-1)}\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{-(\epsilon-1)}\right]^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} =& (1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon-1}\epsilon\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon-1}\right]^2}\right)\\
&+(1-\phi\tau)^{\epsilon-1}(1+\tau)^{\epsilon-1}\epsilon\left(\frac{2\phi\tau}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon}\right]^2}\right)\\
&-(1+\tau)^{-(\epsilon-1)}(1-\phi\tau)^{-(\epsilon-1)}\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{-(\epsilon-1)}\right]^2}\right)
\end{aligned}
$$

Therefore, (i) $\lim_{\frac{A_{ct}}{A_{dt}}\to\infty} \frac{\partial^2 \log(Y_t)}{\partial\tau\partial\frac{A_{ct}}{A_{dt}}^{\varphi}} = 0$ (ii) If $\epsilon > \frac{\alpha}{1-\alpha}$, $\frac{1-\phi}{\phi} > \tau$ and $\frac{(1-\phi\tau)^{-(\epsilon-1)}-(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon-1}-(1+\tau)^{1-\epsilon}} > (\frac{A_{ct}}{A_{dt}})^{\varphi}$ then $\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\frac{A_{ct}}{A_{dt}}^{\varphi}} > 0$

Now, given that $(1+\tau) > 1$ and $(1-\phi\tau) < 1$

$$\begin{aligned}
\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} >&(1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon-1}\epsilon\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon-1}\right]^2}\right) \\
&+(1-\phi\tau)^{\epsilon-1}(1+\tau)^{\epsilon-1}\epsilon\left(\frac{2\phi\tau}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon-1}\right]^2}\right) \\
&-(1+\tau)^{-(\epsilon-1)}(1-\phi\tau)^{-(\epsilon-1)}\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{-(\epsilon-1)}\right]^2}\right)
\end{aligned}$$

so

$$\begin{aligned}
\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} >&(1+\tau)^{\epsilon-1}(1-\phi\tau)^{\epsilon-1}\epsilon\left(\frac{2\phi\tau+(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-\phi\tau)^{\epsilon-1}\right]^2}\right) \\
&-(1+\tau)^{-(\epsilon-1)}(1-\phi\tau)^{-(\epsilon-1)}\frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1}}{\left[(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+(1-phi\tau)^{-(\epsilon-1)}\right]^2}\right)
\end{aligned}$$

Therefore if $(1+\tau)^{-(\epsilon-1)}(1-\phi\tau)^{-(\epsilon-1)} < (1+\tau)^{(\epsilon-1)}(1-\phi\tau)^{(\epsilon-1)}$, $2\phi\tau+(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1} > 0$ and $(1+\tau)^{-1}-\phi(1-\phi\tau)^{-1} < 0$ then $\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} > 0$ then $\frac{\partial^2 \log(Y_t)}{\partial\tau\partial\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} > 0$