Can growth take place while reducing emissions?

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Abstract

Do the macroeconomic effect of a carbon tax differ between countries, according to the primary energy source? I answer this question with a theoretical model of directed technical change and test empirically the main results. I find four main results: (i) In the absence of subsidies, carbon taxes have a negative effect on economic growth, (ii) this negative effect is a decreasing function of the proportion of clean energy sources. (iii) subsidies for clean inputs have a positive effect on economic growth, and (iv) the magnitude of this effect grows with the proportion of clean energy sources. The empirical results are consistent with the predictions of the theoretical model and suggest that policymakers could consider this relationship between the energy mix and the economic effect of a carbon tax when creating environmental regulations.

JEL Classification Numbers: O1, O4, Q4, Q5, O57.

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1 Introduction

One of the greatest challenges that countries face is combating climate change without adversely affecting economic growth. Policies such as carbon pricing and clean energy subsidies create incentives to switch to renewable energy sources and adopt energy-saving technologies. This transition incurs adaptation costs for both workers and producers of goods and services (Nordhaus (1991); Pearce (1991)), which can result in negative impacts on employment and economic growth (Känzig, 2023). However, these effects can vary according to the characteristics of individual countries, such as their energy consumption patterns and the sources of energy required for production, commonly referred to as the energy mix.

This study examines the heterogeneous effects of energy mix, specifically investigating whether the impact of introducing a carbon tax on GDP growth and employment rates varies across countries based on their energy sources. To this end, I extend the model of Acemoglu et al. (2012a), which focuses on directed technological change where new innovations drive growth, and entities that fail to innovate tend to become obsolete and disappear. In this extended model, I differentiate between two sectors "clean" and "dirty". I incorporate an environmental policy that imposes a tax on dirty goods and provides a subsidy on clean goods. Unlike the original model, I consider the initial participation of these sectors and derive two effects of environmental policy on GDP: static and dynamic. These propositions are subsequently validated using an empirical strategy of event studies, analyzing data from 66 countries from 1990 to 2022.

The findings indicate that carbon taxes without the direct use of revenues for clean energy initiatives have a negative effect on economic growth in the short run, particularly if the energy sources rely heavily on fossil fuels. However, if the tax is accompanied by subsidies for clean energy, the effect can be positive in the short run and becomes more favorable as energy sources transition to cleaner alternatives in the long run. Beyond this analysis and results, this paper contributes more broadly to the discourse on achieving economic growth while reducing CO2 emissions, emphasizing the crucial role of energy sources.

There is substantial literature on the effects of a carbon tax on macroeconomic outcomes, but the findings are unclear. Bernard *et al.* (2018) and Metcalf & Stock (2020) do not find adverse impacts of the carbon tax on aggregate GDP growth or employment in British Columbia, nor in 31 countries in Europe. However, Yamazaki (2017) finds that while British Columbia's carbon tax does

not have an adverse effect on employment overall, at the sectoral level the most carbon-intensive and trade-sensitive industries see employment fall, while the clean service and health industries increase employment. Likewise, several authors have studied the distributive effects of the carbon tax on households and have found evidence of regressiveness, by increasing the cost of carbon-intensive products and by changing factor prices (Rausch et al. (2011) and Mueller & Steiner (n.d.)). A study by Andersson J (2017) highlights that the Swedish carbon tax, by increasing gasoline prices, has regressive effects, especially when measured against annual income. Furthermore, income inequality plays an important role in determining the distributional impact of these taxes.

The literature on the relationship between energy sources, the carbon tax, and climate policies is relatively limited. Some studies, such as Papageorgiou et al. (2017) estimated the elasticity of substitution between clean and dirty energy inputs within the energy aggregate significantly exceeds unity, which is a favorable condition for promoting green growth. Similarly, Matsumoto (2022), exploring the effect of the carbon tax on the energy source mixes of Japanese households, found that increasing the carbon tax leads to a higher percentage of households using gas and a reduced percentage using electrification. However, none of the authors have examined how different combinations of energy sources may influence the macroeconomic effects of implementing climate policies, such as the carbon tax, in the short and long run.

This article is also related to a large and growing literature on the environment, resources, and directed technical change. The first contributions (Nordhaus (1993); Stern (2007) and Golosov et al. (2011)) concentrated on the development of theoretical models of climate change and economy, for example, the DICE model that extends the Ramsey growth model. Several economists have developed new theories of economic growth integrating the environmental constraint (Acemoglu et al. (2012b), Romer & Romer (2010) and Laffont & Martimort (2009), Torres (2021)). In particular, some authors have analyzed theoretically and empirically how innovations and directed technology drive long-term sustainable growth. For example, Popp (2002) demonstrates that high energy prices encourage cost-saving innovations in the air conditioning industry, and Aghion et al. (2012), carried out a similar exercise in the automobile sector.

While there are numerous macroeconomic models, such as those integrating environmental constraints within the context of directed technological change, that explore the broader implications of climate policies. However, as far as I know, the specific relationship between the composition of energy sources, economic growth, and climate policies has not been thoroughly examined. This

study aims to fill this gap by empirically demonstrating the model's validity and providing new insights into how different combinations of energy sources influence the macroeconomic effects of implementing climate policies, particularly carbon taxes and subsidies for clean energy.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model, outlining the propositions regarding the impact of a carbon tax and subsidy on aggregate and sectoral production, as well as growth rates. Section 3 outlines the empirical methodologies employed to estimate the correlation between the carbon tax and GDP growth rate, employment rate, and the variations based on the energy mix composition. Section 4 delves into the presentation and discussion of the empirical findings, along with their implications. Finally, Section 6 provides concluding remarks.

2 Theoretical model

In this model, a final good is created by combining inputs from two sectors, one that uses "clean" energy sources and another that uses "dirty" sources. These inputs are substitute goods, which means they can be used interchangeably to produce the final good. Innovation can occur in both the clean and dirty sectors, with intermediate goods purchasing patents. Labor is assumed to be mobile, so there are no differentiated wages. However, if production is done with dirty sources, the environmental quality degrades, which can lead to an environmental disaster and make production impossible. Taxes can be used to incentive innovation in the clean sector, thereby preventing environmental degradation and ensuring long-term production.

Following the model of directed technological change with environmental constraints developed by Acemoglu *et al.* (2012a), I explore how a carbon tax and a subsidy affects GDP growth, focusing on how this impact varies depending on the share of primary energy sources. Based on the model's specifications, I propose a hypothesis and three corollaries that can be empirically tested to determine the impact of a carbon tax on economic growth.

2.1 Conceptual framework

Environmental policy must ensure that the demand for dirty inputs takes into account the environmental cost of an additional unit of inputs. We model a policy that combines a tax on the dirty sector (τ) and a subsidy on the clean sector (q). This policy mix can generate a decrease in the dirty sector accompanied by growth in welfare.

$$q = \tau \cdot \phi_t \tag{1}$$

We assume a balanced budget with an environmental policy, tax revenue must be equal to subsidy spending. As a result, the subsidy to the clean sector must be equal to the carbon tax rate multiplied by the ratio of dirty to clean input production. However, this is an extreme assumption, since in many countries public spending on clean technologies is much higher than the collection of taxes on dirty technologies. For this reason, from now on, we consider (ϕ) is an exogenous parameter.

2.1.1 Final good production

There is a unique final good, Y_t , produced competitively using "clean" and "dirty" inputs (depending on the primary energy source required) Y_c and Y_d .

$$Y_t = \left(Y_{ct}^{\frac{\epsilon - 1}{\epsilon}} + Y_{dt}^{\frac{\epsilon - 1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon - 1}} \tag{2}$$

where $\epsilon \in (0, +\infty)$ is the elasticity of substitution between the two sectors. If the inputs are (gross) substitutes, $\epsilon > 1$, then any final good production can be obtained from alternative clean energies. For example, renewable energy, provided it can be stored and transported efficiently, may replace energy derived from fossil fuels (Popp, 2002). On the contrary, if the two inputs are (gross) complements, $\epsilon < 1$, then it is impossible to produce without fossil fuels.

Each input is produced symmetrically, in perfect competition at a price P_{jt} . Using the final good as the numeraire, the price of the inputs that the producer of the final good must pay is:

$$\left[((1+\tau)P_{dt})^{1-\epsilon} + ((1-\tau\phi)P_{ct})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} = 1$$
 (3)

The tax is charged on the price paid by the producer of the final good for each unit of the dirty inputs demanded, and $\tau\phi$ is the subsidy rate received by the producer of final goods for each unit of the clean inputs purchased. So the relative demand for dirty and clean inputs are:

$$\left(Y_{dt}^{\frac{\epsilon-1}{\epsilon}} + Y_{ct}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \times Y_{dt}^{-\frac{1}{\epsilon}} = (1+\tau)P_{dt}
\left(Y_{dt}^{\frac{\epsilon-1}{\epsilon}} + Y_{ct}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \times Y_{ct}^{-\frac{1}{\epsilon}} = (1-\tau\phi)P_{ct}$$
(4)

On the demand side, a carbon tax raises the prices of final dirty goods, therefore, the willingness to pay for them falls. The subsidy has the opposite effect, it increases their demand, and, all else equal, it increases the price that producers of final goods are willing to pay.

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{P_{dt} \cdot (1+\tau)}{P_{ct} \cdot (1-\tau\phi)}\right)^{\epsilon} \tag{5}$$

Equation 5 allows us to determine relative production based on pricing, taxes, and subsidies. The relative input demand is decreasing in their relative supply.

2.1.2 Intermediate inputs production

The two inputs, Y_c and Y_d are produced using labor L_{jt} , where $j \in (c, d)$ denotes the sector, and a continuum of sector-specific machines (intermediates) x_{jit} . The production of inputs is given by:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \cdot di$$
 (6)

 A_{jit} is the quality of machine i, and x_{jit} is the quantity of this machine i used in sector j at time t.

Given that the market operates under conditions of perfect competition, the optimization problem faced by producers in both sectors involves maximizing profits through the optimal allocation of labor and machines.

$$\max_{x_{jit}, L_{jt}} \left\{ P_{jt} L_{jt}^{1-\alpha} \left(\int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \cdot di \right) - w_{jt} L_{jt} - \int_0^1 p_{jit} x_{jit} \cdot di \right\}$$
 (7)

where w_{jt} denotes the wage paid for each unit of labor hired in each sector j and p_{jit} is the price that the producer of inputs must pay for each machine i used. Machines are of different quality and technological progress occurs as the quality of machines increases, so the productivity of a sector is the average productivity of machines in that sector.

From the first-order conditions is obtained the demand for machines in each sector,

$$x_{jit} = \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \tag{8}$$

and, the demand for labor,

$$L_{jt} = \left(\frac{(1-\alpha)P_{jt}}{w_{jt}}\right)^{\frac{1}{\alpha}} A_{jit}^{\frac{1-\alpha}{\alpha}} x_{jit}$$

$$\tag{9}$$

Machines are produced at marginal cost ν under monopolistic competition and, sold at price p_{jt} , taking into account the demand for machines x_{jit} in the sector in which they are used. Therefore the profits of the monopolists, π_{jt} , are given by: $\pi_{jt} = (p_{jit} - \nu)x_{jit}$. So, replacing the demand for machines, the profits of the monopolist are:

$$\pi_{jt} = (p_{jit} - \nu) \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt}$$
(10)

Following Acemoglu et al. (2012a), we normalize $\nu = \alpha^2$, so, each monopolist sets a price $p_{jt} = \frac{1}{\alpha}$.

Thus, replacing the price of machine p_{jit} in equation 8, the optimal demand for machines in each sector is obtained:

$$x_{jit} = \alpha^{\frac{2}{1-\alpha}} A_{jit} L_{jt} \left(P_{jt} \right)^{\frac{1}{1-\alpha}} \tag{11}$$

and the quantities of inputs produced in sector j are:

$$Y_{jt} = \alpha^{\frac{2\alpha}{1-\alpha}} A_{jt} L_{jt} \left(P_{jt} \right)^{\frac{\alpha}{1-\alpha}} \tag{12}$$

Combining this equation 10 and replacing ν , the equilibrium profits of machine producers can be written as:

$$\pi_{jt} = (1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} A_{jit-1} L_{jt}$$

$$\tag{13}$$

2.1.3 Technological change

At the beginning of every period, each scientist decides whether to direct her research to clean or dirty technology. Also every period all intermediate goods, within a sector, begin with the same level of productivity, which is given by the average productivity of the previous period. Market clearing for scientists working on machines in sector $j \in (c, d)$ at time t by s_{jt} , takes the form $s_{ct} + s_{dt} \leq 1$.

A successful scientist, who has invented a better version of the machine i in sector j increases the quality of the machine by a factor $1 + \gamma$ and happens with probability $\eta_j \in (0,1)$. When an innovation is unsuccessful $(1 - \eta_j)$, the sector's productivity is equal to that of the previous period. A_{jt-1} .

$$A_{jt} = \begin{cases} \gamma A_{jt-1} & \text{if successful } (\eta_j) \\ A_{jt-1} & \text{if not successful} (1 - \eta_j) \end{cases}$$

Innovation requires research, a costly activity that uses the final good as input. The probability η_{jt} that an innovation occurs in sector j any period t depends positively on the amount of research expenditure R_t and depends inversely on the desired productivity A_{jit} . The reason is that as technology advances it becomes more complex and thus harder to improve upon. The probability function of success is:

$$\eta_{jt} = \lambda \left(\frac{R_{jt}}{A_{jit}}\right)^{\sigma} \tag{14}$$

The parameter λ reflects the productivity of research expenditure, and the elasticity parameter σ lies between zero and one. The problem for entrepreneurs is to maximize the probability of innovating η_{jt} in each sector j, which depends on the price of the patent P_{Ajit} , and R&D investment R_{jt} , that is,

$$\max_{\eta_{jt}} \left\{ \eta_{jt} P_{Ajit} - R_{jt} \right\} \tag{15}$$

From the first-order condition, we extract the correlation between resources and desired productivity alongside the probability of success, substituting equation 14. Given that the price of the patent equals the net profits of the machine producer $P_{Ajit} = \pi_{jit}$ (equation 13), then the probability of innovating in each sector is:

$$\eta_{jt} = 2(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} L_{jt}$$
(16)

2.1.4 Equilibrium

Taking the input quantities equation 12 and the relative input demand equation 5, the ratio between clean and dirty production can be written as

$$\frac{Y_{ct}}{Y_{dt}} = \frac{P_{ct}^{\frac{\alpha}{1-\alpha}}}{P_{dt}^{\frac{\alpha}{1-\alpha}}} \frac{A_{ct}}{A_{dt}} \frac{L_{ct}}{L_{dt}} = \frac{(P_{dt} \cdot 1 + \tau)^{\epsilon}}{(P_{ct} \cdot 1 - \tau \phi)^{\epsilon}}$$

$$(17)$$

Equation 17 shows that the tax has no impact on production costs, but it does affect the demand for final goods in each sector. Tax, in particular, increases the demand for clean goods while decreasing the demand for dirty ones.

In order to determine the relative price of inputs between sectors, the demand for machines equation 8 is substituted into the demand for labor equation 9, and since equilibrium wages are equal $w_{ct} = w_{dt}$, we obtain the following:

$$w_t = \left((1+\tau)^{-(\epsilon-1)} A_{ct}^{(1-\alpha)(\epsilon-1)} + (1-\phi\tau)^{-(\epsilon-1)} A_{dt}^{(1-\alpha)(\epsilon-1)} \right)^{\frac{1}{(1-\alpha)(\epsilon-1)}(1-\alpha)} \cdot (1-\alpha) \tag{18}$$

Therefore, the relative price of each input is:

$$\frac{P_{ct}}{P_{dt}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{(1-\alpha)} \tag{19}$$

Equation 19 implies that environmental policy does not affect the price ratio of the same period. However, the technological level of the sector is inversely related to its price. The price of clean goods decreases as the clean sector's technological level rises, if $A_c > A_d$ then $P_c < P_d$. This suggests that demand for clean goods could increase as their price becomes more affordable when compared to dirty goods.

By substituting the relative prices from equation 19 into equation 17, we can derive the relative labor demand as expressed in equation 20.

$$\frac{L_{ct}}{L_{dt}} = \frac{(1+\tau)^{\epsilon} A_{dt}^{(1-\alpha)(\epsilon-1)}}{(1-\phi\tau)^{\epsilon} A_{ct}^{(1-\alpha)(\epsilon-1)}}$$
(20)

Consequently, the tax on dirty goods and the subsidy on clean goods, all else constant, increase the wage in the clean sector and reduce it in the dirty sector. This, in turn, results in a rise in labor associated with clean intermediate goods $L_{ct} > L_{dt}$, indicating a labor reallocation towards the clean sector. On the other hand, labor reallocation generates an increase in the relative supply of clean goods, which pressures their relative price downward, so that the direct effect on prices is offset by the indirect effect of labor reallocation. This incentivizes intermediate goods producers to increase their output. However, as we will see later on, labor reallocation increases incentives for innovation in the clean sector and reduces incentives for innovation in the dirty sector, so that over time, the productivity of the clean sector grows more rapidly and its relative price decreases.

To ensure the equilibrium of the labor market, it is necessary for labor demand to equal the total labor supply, $L_{ct} + L_{dt} \leq 1$, and using equations, 18, 19 and 20. The equilibrium prices and labor in each sector:

$$L_{ct} = \frac{(1+\tau)^{\epsilon} A_{ct}^{(1-\alpha)(\epsilon-1)}}{(1-\tau\phi)^{\epsilon} A_{ct}^{(1-\alpha)(\epsilon-1)} + (1+\tau)^{\epsilon} A_{dt}^{(1-\alpha)(\epsilon-1)}}$$

$$L_{dt} = \frac{(1-\tau\phi)^{\epsilon} A_{dt}^{(1-\alpha)(\epsilon-1)}}{(1-\tau\phi)^{\epsilon} A_{ct}^{(1-\alpha)(\epsilon-1)} + (1+\tau)^{\epsilon} A_{dt}^{(1-\alpha)(\epsilon-1)}}$$
(21)

$$P_{ct} = \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{(1-\alpha)(\epsilon-1)} + (1-\tau\phi)^{-(\epsilon-1)} A_{dt}^{(1-\alpha)(\epsilon-1)} \right)^{\frac{1}{\epsilon-1}}}{A_{ct}^{(1-\alpha)}}$$

$$P_{dt} = \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{(1-\alpha)(\epsilon-1)} + (1-\tau\phi)^{-(\epsilon-1)} A_{dt}^{(1-\alpha)(\epsilon-1)} \right)^{\frac{1}{\epsilon-1}}}{A_{dt}^{(1-\alpha)}}$$
(22)

Note that subsidies for clean production raise the prices of both inputs, while taxes on dirty production lower the prices of both sectors. However, because the percentage change is the same for both prices, the price ratio remains unaltered. In terms of labor equilibrium, both technological productivity and the value of the carbon tax in the clean sector increase the workforce employed in this sector. Additionally, the clean sector's labor market expands as the subsidy rises $\phi \tau$, even when the clean sector's technological level is low.

Taking the price index of inputs relative to the final good of this economy (equation 19), the relative production of clean goods compared to dirty goods is given by:

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{1+\tau}{1-\tau\phi}\right)^{\epsilon} \left(\frac{A_{ct}}{A_{dt}}\right)^{\epsilon(1-\alpha)} \tag{23}$$

This suggests that implementing environmental policies raises the clean sector's production share while decreasing the dirty sector's share.

Replacing in equation 12, the prices 22, and labor 21 which are expressed in terms of productivity for each sector, we can get the output of two sectors. Where $\varphi = (1 - \alpha)(\epsilon - 1)$.

$$Y_{ct} = \alpha^{\frac{2\alpha}{1-\alpha}} \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)} A_{dt}^{\varphi} \right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} (1+\tau)^{\epsilon} A_{ct}^{\epsilon(1-\alpha)}$$

$$Y_{dt} = \alpha^{\frac{2\alpha}{1-\alpha}} \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)} A_{dt}^{\varphi} \right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} (1-\phi\tau)^{\epsilon} A_{dt}^{\epsilon(1-\alpha)}$$
(24)

Since the labor in clean sector increase $L_{ct} > L_{dt}$, due to the tax and the subsidy, then the production in clean sector in greater than in the dirty ones $Y_{ct} > Y_{dt}$. Equations 24 also imply that the production of dirty input is decreasing in the tax τ and ϕ .

Finally, by utilizing equations 24 and 2, it is possible to express the final product

$$Y_{t} = \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)} A_{dt}^{\varphi} \right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \left((1+\tau)^{\epsilon-1} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon-1} A_{dt}^{\varphi} \right)^{\frac{\epsilon}{\epsilon-1}}$$
(25)

Now we can analyze the productivity growth rate in terms of productivity and environmental policies. Combining equation 27 with equations 21 and 22

$$\eta_{ct} = 2(1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \frac{(1+\tau)^{\epsilon} A_{ct}^{\varphi-1}}{((1-\phi\tau_t)^{\epsilon} A_{ct}^{\varphi\varphi}(1+\tau)^{\epsilon} A_{dt}^{\varphi})^{\frac{\varphi-1}{\varphi}}}
\eta_{dt} = 2(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \frac{(1-\phi\tau_t)^{\epsilon} A_{dt}^{\varphi-1}}{((1+\tau)^{\epsilon} A_{ct}^{\varphi}(1-\phi\tau_t)^{\epsilon} A_{dt}^{\varphi})^{\frac{\varphi-1}{\varphi}}}$$
(26)

The equation 26 shows that the productivity growth of each sector depends positively on its own technological level and negatively on the technological level of the other sector. The introduction of a carbon tax and a subsidy increases productivity in the clean sector and decreases it in the dirty sector.

$$\frac{\eta_{ct}}{\eta_{dt}} = \frac{(1+\tau_t)^{\epsilon} A_{ct}^{(1-\alpha)(\epsilon-1)-1}}{(1-\phi\tau_t)^{\epsilon} A_{dt}^{(1-\alpha)(\epsilon-1)-1}}$$

$$(27)$$

In the absence of any environmental policy the relative benefits of innovating are always greater in the sector with the highest technological level, under initial conditions $A_{d0} > A_{c0}$ innovations occurs in the dirty sector. So the tax may increase the production of clean inputs, making the labor force in this sector more productive, increase the probability of innovating in the clean sector rather than the dirty.

$$\tau > \frac{\left(\frac{A_{dt}}{A_{ct}}\right)^{\frac{(1-\alpha)(\epsilon-1)-1}{\epsilon}} - 1}{\phi\left(\frac{A_{dt}}{A_{ct}}\right)^{\frac{(1-\alpha)(\epsilon-1)-1}{\epsilon}} + 1} \tag{28}$$

Equation 28 shows that the technological gap between the dirty sector and the clean sector is a direct function of the ratio of dirty to clean production (ϕ_t). Also shows that the more the clean sector lags behind the dirty, the larger the subsidy must be to direct R&D resources to the clean sector. Since the subsidy is composed of the ratio of clean and dirty production and the tax, the tax should be greater. This tax rate makes the technological level of the clean sector grow faster than in the dirty sector.

2.2 Static effect of environmental policy

Previously we determined the variables of Prices, Labor, Production and Productivity growth rates in the two sectors in equilibrium. Next we analyze the static effect of a climate policy on sectoral production.

2.2.1 Effect on sectoral production

Equation 24 can be rewritten as:

$$Y_{ct} = \alpha^{\frac{2\alpha}{1-\alpha}} \frac{\left(A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}\right)^{\frac{\alpha}{\varphi}}}{A_{ct}^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon} A_{dt}^{\varphi}} \frac{1}{(1+\tau)^{\frac{\alpha}{1-\alpha}}} A_{ct}^{\epsilon(1-\alpha)}$$

$$Y_{dt} = \alpha^{\frac{2\alpha}{1-\alpha}} \frac{\left(\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} A_{ct}^{\varphi} + A_{dt}^{\varphi}\right)^{\frac{\alpha}{\varphi}}}{\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}} \frac{1}{(1-\phi\tau)^{\frac{\alpha}{1-\alpha}}} A_{dt}^{\epsilon(1-\alpha)}$$

$$(29)$$

If clean inputs and dirty inputs are perfect substitutes, $\epsilon > \frac{\alpha}{1-\alpha}$, exists a combination of tax and subsidy such that, $\frac{1+\tau}{1-\phi\tau} < \left(\epsilon\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2\epsilon}}$, a tax on dirty inputs generates an increase in clean production.

$$\ln(Y_{ct}) = \frac{2\alpha}{1-\alpha} \ln(\alpha) + \frac{\alpha}{\varphi} \ln\left(A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}\right) - \ln\left(A_{ct}^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon} A_{dt}^{\varphi}\right) - \frac{\alpha}{1-\alpha} \ln(1+\tau) + \epsilon(1-\alpha) \ln(A_{ct})$$

$$\ln(Y_{dt}) = \frac{2\alpha}{1-\alpha} \ln(\alpha) + \frac{\alpha}{\varphi} \ln\left(\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} A_{ct}^{\varphi} + A_{dt}^{\varphi}\right) - \ln\left(\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}\right) - \frac{\alpha}{1-\alpha} \ln(1-\phi\tau) + \epsilon(1-\alpha) \ln(A_{dt})$$

$$(30)$$

Now notice that the effect of the tax on the relative productivity of the clean sector is positive $\frac{\partial \ln(Y_{ct})}{\partial \tau} > 0$ and on the dirty sector it is negative $\frac{\partial \ln(Y_{dt})}{\partial \tau} < 0$.

$$\frac{\partial \ln(Y_{ct})}{\partial \tau} = \frac{\alpha}{\varphi} \frac{\left(\epsilon - 1\right) \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon - 2} A_{dt}^{\varphi} \frac{(1+\phi)}{(1-\phi\tau)^{2}}}{A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon - 1} A_{dt}^{\varphi}} + \frac{\epsilon \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon - 1} A_{dt}^{\varphi} \frac{(1+\phi)}{(1+\tau)^{2}}}{A_{ct}^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon} A_{dt}^{\varphi}} - \frac{\alpha}{1-\alpha} \frac{1}{1+\tau}$$

$$\frac{\partial \ln(Y_{dt})}{\partial \tau} = -\frac{\alpha}{\varphi} \frac{\left(\epsilon - 1\right) \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon - 2} A_{ct}^{\varphi} \frac{(1+\phi)}{(1-\phi\tau)^{2}}}{\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon - 1} A_{ct}^{\varphi} + A_{dt}^{\varphi}} - \frac{\epsilon \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon - 1} A_{ct}^{\varphi} \frac{(1+\phi)}{(1+\tau)^{2}}}{\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}} + \frac{\alpha\phi}{1-\alpha} \frac{1}{1-\phi\tau}$$
(31)

$$\frac{\partial \ln(Y_{ct})}{\partial \tau} = \frac{\alpha}{(1-\alpha)} \frac{\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-2} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi} \left(\frac{(1+\phi)}{(1-\phi\tau)^{2}}\right)}{1+\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}} + \frac{\epsilon \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi} \left(\frac{(1+\phi)}{(1-\phi\tau)^{2}}\right)}{1+\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}} - \frac{\alpha}{1-\alpha} \frac{1}{1+\tau}$$

$$\frac{\partial \ln(Y_{dt})}{\partial \tau} = -\frac{\alpha}{(1-\alpha)} \frac{\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2} \left(\frac{(1+\phi)}{(1-\phi\tau)^{2}}\right)}{\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} + \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}} - \frac{\epsilon \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \left(\frac{(1+\phi)}{(1-\phi\tau)^{2}}\right)}{\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} + \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}} + \frac{\alpha\phi}{1-\alpha} \frac{1}{1-\phi\tau}$$
(32)

Assuming a reasonable elasticity of substitution between clean and dirty inputs, $\epsilon > \frac{(2-\alpha)}{(1-\alpha)}$, and considering the income elasticity of intermediate goods, $\alpha \cong 1/3$, the findings suggest that if subsidy and tax rates are at or below 17.6%, the tax on dirty goods positively influences the production of clean goods. In essence, the implementation of taxes and subsidies induces a sectoral income reallocation favoring the clean sector. With these reasonable assumptions, this reallocation results in increased income for the clean sector. Furthermore, if the dirty sector exhibits relatively low productivity, it will contract in response to the environmental policy.

Analysis of the relationship between relative productivity and sectoral outputs is shown below.

i)
$$\lim_{\substack{A_{dt} \\ A_{ct}} \to 0} \frac{\partial \ln(Y_{ct})}{\partial \tau} \approx -\frac{\alpha}{1-\alpha} \frac{1}{1+\tau}$$

ii)
$$\lim_{\substack{A_{dt} \\ A_{ct}} \to 0} \frac{\partial \ln(Y_{dt})}{\partial \tau} = \frac{\alpha \phi (1 - \phi \tau) - (\alpha + \epsilon)(1 + \phi)}{(1 - \alpha)(1 - \phi \tau)^2}$$

iii)
$$\lim_{\substack{A_{dt} \to \infty \\ A_{ct}} \to \infty} \frac{\partial \ln(Y_{ct})}{\partial \tau} \approx \frac{\alpha}{1-\alpha} \left(\frac{1+\phi}{(1-\phi\tau)^2} \left(\frac{1+\tau}{1-\phi\tau} \right)^{\epsilon-2} \right) + \epsilon \left(\frac{1+\phi}{(1-\phi\tau)^2} \left(\frac{1-\phi\tau}{1+\tau} \right)^{\epsilon-1} \right) - \frac{\alpha}{1-\alpha} \frac{1}{1+\tau}$$

iv)
$$\lim_{\frac{A_{dt}}{A_{ct}} \to \infty} \frac{\partial \ln(Y_{dt})}{\partial \tau} \approx \frac{\alpha \phi}{1 - \alpha} \frac{1}{1 - \phi \tau}$$

The combination between the tax and subsidy affects dirty production according to the combination between relative productivity, $\frac{A_{dt}}{A_{ct}}$:

v)
$$\lim_{\substack{A_{dt} \\ A_{ct}} \to 0} \frac{\partial^2 \ln(Y_{dt})}{\partial \phi \partial \tau} \approx \frac{\alpha}{1-\alpha} \frac{\tau}{(1-\phi\tau)^2}$$

vi)
$$\lim_{\substack{A_{dt} \\ A_{ct}} \to \infty} \frac{\partial^2 \ln(Y_{dt})}{\partial \phi \partial \tau} \approx -\epsilon \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \frac{(1+\tau)}{(1-\phi\tau)^3}$$

2.2.2 Effect on aggregate production

$$Y_{t} = \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)} A_{dt}^{\varphi} \right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \left((1+\tau)^{\epsilon-1} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon-1} A_{dt}^{\varphi} \right)^{\frac{\epsilon}{\epsilon-1}}$$
(33)

Using the equation 33 to estimate the aggregate effect on productivity, we calculate the logarithm and differentiate with respect to τ :

$$\frac{\partial \log Y_t}{\partial \tau} = -\frac{\alpha}{1-\alpha} \cdot \frac{A_{ct}^{\varphi}(1+\tau)^{-(\epsilon)} - \phi A_{dt}^{\varphi}(1-\phi\tau)^{-(\epsilon)}}{(1+\tau)^{-(\epsilon-1)}A_{ct}^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}A_{dt}^{\varphi}}
-\epsilon \cdot \frac{A_{ct}^{\varphi}(1+\tau)^{\epsilon-1} - \phi A_{dt}^{\varphi}(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon}A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon}A_{dt}^{\varphi}}
+\epsilon \cdot \frac{A_{ct}^{\varphi}(1+\tau)^{\epsilon-2} - \phi A_{dt}^{\varphi}(1-\phi\tau)^{\epsilon-2}}{(1+\tau)^{\epsilon-1}A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon-1}A_{dt}^{\varphi}}$$
(34)

$$\frac{\partial \log Y_t}{\partial \tau} = -\frac{\alpha}{1-\alpha} \cdot \frac{A_{ct}^{\varphi}(1+\tau)^{-(\epsilon)} - \phi A_{dt}^{\varphi}(1-\phi\tau)^{-(\epsilon)}}{(1+\tau)^{-(\epsilon-1)}A_{ct}^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}A_{dt}^{\varphi}} + \epsilon \cdot \left[\frac{A_{ct}^{\varphi}(1+\tau)^{\epsilon-2} - \phi A_{dt}^{\varphi}(1-\phi\tau)^{\epsilon-2}}{(1+\tau)^{\epsilon-1}A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon-1}A_{dt}^{\varphi}} - \frac{A_{ct}^{\varphi}(1+\tau)^{\epsilon-1} - \phi A_{dt}^{\varphi}(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon}A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon}A_{dt}^{\varphi}} \right]$$
(35)

Dividing the numerator and denominator by A_{dt}^{φ} and organizing:

$$\frac{\partial \log Y_t}{\partial \tau} = -\frac{\alpha}{(1-\alpha)} \frac{1}{(1+\tau)} \cdot \left[\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}}{\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}} \right]
+ \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{\epsilon}{(1+\tau)} \left[\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}} \right]
+ \frac{\epsilon\phi}{(1-\phi\tau)} \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon} \left[\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}} - \frac{\left(\frac{1-\phi\tau}{1+\tau}\right)^{-1}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} \right]$$
(36)

Proposition 1: A tax on dirty production generates a negative effect on the aggregate product. The size of this effect (i) depends on the magnitude of the clean production subsidy (ϕ) , and (ii) depends positively on the relative productivity of the two sectors. When there is no subsidy $(\phi = 0)$ the effect of the tax is always negative, so $\frac{\partial \log Y_t}{\partial \tau} < 0$; The greater the subsidy, the more positive the effect of environmental policy is. As long as clean inputs and dirty inputs are perfect substitutes, and the tax is accompanied by a subsidy on clean goods, the effect on aggregate production can be positive to the extent that the relative productivity of the clean sector increases:

• Negative when the relative productivity of the clean sector tends to zero,

$$\lim_{\substack{\frac{A_{ct}}{A_{Jt}} \to \infty}} \frac{\partial \log Y_t}{\partial \tau} = -\frac{\alpha}{(1-\alpha)} \frac{1}{(1+\tau)}$$
(37)

positive when the relative productivity of the clean sector tends to infinity,

$$\lim_{\substack{\frac{A_{ct}}{A_{dt}} \to 0}} \frac{\partial \log Y_t}{\partial \tau} = \frac{\alpha}{(1-\alpha)} \frac{1}{(1+\tau)} \phi\left(\frac{1+\tau}{1-\phi\tau}\right)$$
(38)

$$\frac{\partial^{2} \log Y_{t}}{\partial \left(\frac{A_{ct}}{A_{dt}}\right) \partial \tau} = -\frac{\alpha}{(1-\alpha)} \frac{1}{1+\tau} \frac{\varphi\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} \left[\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} + \phi\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\right]}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}\right)^{2}} + \frac{\epsilon}{1+\tau} \left[\frac{\varphi\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} \left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} - \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right]}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right)^{2}}\right) + \frac{\epsilon\phi}{1-\phi\tau} \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon} \left[\frac{\varphi\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} \left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} - \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right]}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right)^{2}}\right) \tag{39}$$

2.3 Dynamic effect of environmental policy on growth

The results of the previous section imply that environmental policy can have a negative impact once it is implemented. However, the magnitude of this effect depends on two things: (i) the size of the subsidies, (ii) the relative productivity of the clean sector. The effect of taxes on the level of income can be positive in combination with subsidies; without subsidies the effect will always be negative but its magnitude is reduced as the relative productivity of the clean sector increases. This happens because the tax discourages research in the dirty sector and, therefore, reduces its growth. For this reason, the smaller the relative size of the dirty sector, the smaller the negative effect of the tax on aggregate growth. The effect of the subsidies, in turn, is to reallocate work toward the clean sector and increase research in the clean sector. If the productivity of the dirty sector is high relative to that of the clean sector, the reallocation of labor reduces the aggregate product. However, the higher the relative productivity of the clean sector, the smaller the negative effect on aggregate growth. Furthermore, the greater the share of the clean sector, the greater the effect of productivity growth in this sector on the aggregate product. Using the equation 2 it is possible to present the growth rates of the economy as a function of the growth of the two sectors and of their relative size:

$$\frac{\Delta Y_t}{Y_t} = \left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon - 1}{\epsilon}} \cdot \frac{\Delta Y_{ct}}{Y_{ct}} + \left(\frac{Y_{dt}}{Y_t}\right)^{\frac{\epsilon - 1}{\epsilon}} \cdot \frac{\Delta Y_{dt}}{Y_{dt}} \tag{40}$$

The effect of a carbon tax/subsidy on the total growth rate can be decomposed into two: (i) the effect on the sectoral distribution of income, $\frac{\partial \frac{Y_{ct}}{Y_t}}{\partial \tau} > 0$ and $\frac{\partial \frac{Y_{dt}}{Y_t}}{\partial \tau} < 0$; (ii) the effect on the growth rate of the two sectors, $\frac{\partial \frac{\Delta Y_{ct}}{Y_t}}{\partial \tau} > 0$ and $\frac{\partial \frac{\Delta Y_{dt}}{Y_t}}{\partial \tau} < 0$.

$$\frac{\partial \frac{\Delta Y_{t}}{Y_{t}}}{\partial \tau} = \left[\frac{\partial \left(\left(\frac{Y_{ct}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \right)}{\partial \tau} \frac{\Delta Y_{ct}}{Y_{ct}} + \left(\frac{Y_{ct}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}} \right)}{\partial \tau} \right] + \left[\frac{\partial \left(\left(\frac{Y_{dt}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \right)}{\partial \tau} \frac{\Delta Y_{dt}}{Y_{dt}} + \left(\frac{Y_{dt}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}} \right)}{\partial \tau} \right] \\
\frac{\partial \frac{\Delta Y_{t}}{Y_{t}}}{\partial \phi} = \left[\frac{\partial \left(\left(\frac{Y_{ct}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \right)}{\partial \phi} \frac{\Delta Y_{ct}}{Y_{ct}} + \left(\frac{Y_{ct}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}} \right)}{\partial \phi} \right] + \left[\frac{\partial \left(\left(\frac{Y_{dt}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \right)}{\partial \phi} \frac{\Delta Y_{dt}}{Y_{dt}} + \left(\frac{Y_{dt}}{Y_{t}} \right)^{\frac{\epsilon - 1}{\epsilon}} \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}} \right)}{\partial \phi} \right] \right] \tag{41}$$

2.3.1 Effect on sectoral share of production

Using equations 23 and 2 the final production can be expressed as following:

$$Y_{t} = Y_{ct} \left[1 + \left(\frac{1+\tau}{1-\phi\tau} \right)^{\epsilon-1} \left(\frac{A_{ct}}{A_{dt}} \right)^{\varphi} \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$Y_{t} = Y_{dt} \left[1 + \left(\frac{1-\phi\tau}{1+\tau} \right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}} \right)^{\varphi} \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$(42)$$

So, if $\epsilon > 1$ when $\lim_{\frac{A_{ct}}{A_{dt}} \to \infty}$ then $Y_t = Y_{ct}$ and when $\lim_{\frac{A_{ct}}{A_{dt}} \to 0}$ then $Y_t = Y_{dt}$

The initial share of the production of each sector relative to the total production is given by:

$$\frac{Y_{ct}}{Y_t} = \left[1 + \left(\frac{1 - \phi \tau}{1 + \tau} \right)^{\epsilon - 1} \left(\frac{A_{dt}}{A_{ct}} \right)^{\varphi} \right]^{\frac{\epsilon}{\epsilon - 1}}$$

$$\frac{Y_{dt}}{Y_t} = \left[1 + \left(\frac{1 + \tau}{1 - \phi \tau} \right)^{\epsilon - 1} \left(\frac{A_{ct}}{A_{dt}} \right)^{\varphi} \right]^{\frac{-\epsilon}{\epsilon - 1}}$$
(43)

Equation 43 indicates that environmental policy generates a recomposition of the product in favor of the clean sector $\frac{\partial \frac{Y_{ct}}{Y_t}}{\partial \left(\frac{1+\tau}{1-\phi\tau}\right)} > 0$ and against the dirty sector $\frac{\partial \frac{Y_{dt}}{Y_t}}{\partial \left(\frac{1+\tau}{1-\phi\tau}\right)} < 0$.

$$\frac{\partial \frac{Y_{ct}}{Y_t}}{\partial \left(\frac{1+\tau}{1-\phi\tau}\right)} = \epsilon \frac{Y_{ct}}{Y_t} \frac{\left(\frac{1+\tau}{1-\phi\tau}\right)^{-\epsilon} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}}{1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}}$$

$$\frac{\partial \frac{Y_{dt}}{Y_t}}{\partial \left(\frac{1+\tau}{1-\phi\tau}\right)} = -\epsilon \frac{Y_{dt}}{Y_t} \frac{\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-2} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} \tag{44}$$

Now we see that the effect of the tax is positive for the relative production of the clean sector $\frac{\partial \left(\frac{Y_{ct}}{Y_t}\right)}{\partial \tau} > 0$ and has a negative effect on the relative production of the dirty sector $\frac{\partial \left(\frac{Y_{dt}}{Y_t}\right)}{\partial \tau} < 0$:

$$\frac{\partial \left(\frac{Y_{ct}}{Y_t}\right)}{\partial \tau} = \epsilon \left[1 + \left(\frac{1 - \phi\tau}{1 + \tau}\right)^{\epsilon - 1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi} \right]^{\frac{-\epsilon}{\epsilon - 1} - 1} \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi} \left(\frac{1 - \phi\tau}{1 + \tau}\right)^{\epsilon - 2} \cdot \frac{1 - \phi - \phi\tau}{(1 + \tau)^2}.$$

$$\frac{\partial \left(\frac{Y_{dt}}{Y_t}\right)}{\partial \tau} = -\epsilon \left[1 + \left(\frac{1 + \tau}{1 - \phi\tau}\right)^{\epsilon - 1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \right]^{\frac{-\epsilon}{\epsilon - 1} - 1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \left(\frac{1 + \tau}{1 - \phi\tau}\right)^{\epsilon - 2} \cdot \frac{1 + \phi + \tau\phi}{(1 - \phi\tau)^2}.$$
(45)

To summarize, taxes and subsidies cause a recomposition of income in favor of the clean industry. Under acceptable assumptions, this recomposition occurs as the clean sector's productivity rises, and if the dirty sector's productivity is low, it decreases in response to environmental policies.

2.3.2 Effect on sector growth rate

Using equation (24) it is possible to derive the growth rate of the two sectors.

$$\frac{\Delta Y_{ct}}{Y_{ct}} = \alpha \left(\frac{A_{ct}^{\varphi} \frac{\Delta A_{ct}}{A_{ct}} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi} \frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}} \right) - \varphi \left(\frac{A_{ct}^{\varphi} \frac{\Delta A_{ct}}{A_{ct}} \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi} \frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + A_{dt}^{\varphi}} \right) + \epsilon (1-\alpha) \frac{\Delta A_{ct}}{A_{ct}}$$

$$\frac{\Delta Y_{dt}}{Y_{dt}} = \alpha \left(\frac{A_{ct}^{\varphi} \frac{\Delta A_{ct}}{A_{ct}} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi} \frac{\Delta A_{dt}}{A_{dt}}}{A_{dt}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1} A_{dt}^{\varphi}} \right) - \varphi \left(\frac{A_{ct}^{\varphi} \frac{\Delta A_{ct}}{A_{ct}} \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi} \frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + A_{dt}^{\varphi}} \right) + \epsilon (1-\alpha) \frac{\Delta A_{dt}}{A_{dt}}$$

$$(46)$$

From the equation 46 one deduces that $\frac{\Delta Y_{ct}}{Y_{ct}} - \frac{\Delta Y_{dt}}{Y_{dt}} = \epsilon (1 - \alpha) \gamma (\eta_{ct} - \eta_{dt})$.

Proposition 2: Environmental policy has a positive effect on the aggregate growth rate, if $\left(\frac{1+\tau}{1-\phi}\right)^{\epsilon} \left(\frac{A_t^c}{A_t^d}\right)^{\phi} > 1$ and $\varphi > 1$.

Proof

i)
$$Y_{ct} > Y_{dt}$$
 and $\eta_{ct} > \eta_{dt}$

 $\begin{aligned} &\textbf{Case 1:} \ \, \frac{A_t^c}{A_t^d} > 1. \ \, \left(\frac{1+\tau}{1-\phi}\right)^\epsilon \left(\frac{A_t^c}{A_t^d}\right)^\phi > 1 \ \, \text{and} \ \, \left(\frac{1+\tau}{1-\phi}\right)^\epsilon \left(\frac{A_t^c}{A_t^d}\right)^{\phi-1} > 1. \ \, \text{Thus,} \ \, Y_t^c > Y_t^d \ \, \text{and} \ \, \eta_t^c > \eta_t^d. \end{aligned}$

ii)
$$\frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial \tau} > \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial \tau} \text{ and } \frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial \phi} > \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial \phi} > 0$$
Then, from equation $46 \frac{\Delta Y_{ct}}{Y_{ct}} - \frac{\Delta Y_{dt}}{Y_{dt}} = \epsilon(1-\alpha)\gamma(\eta_{ct}-\eta_{dt}) \text{ one deduces that, } \frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial \tau} - \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial \tau} = \epsilon(1-\alpha)\frac{\partial (\eta_{ct}-\eta_{dt})}{\partial \tau} \text{ and } \frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial \phi} - \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial \phi} = \epsilon(1-\alpha)\frac{\partial (\eta_{ct}-\eta_{dt})}{\partial \phi}$
These results imply that $\frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial \tau} > \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial \tau} \text{ and } \frac{\partial \left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial \phi} > \frac{\partial \left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial \phi} > 0.$

From i), ii) and iii) it follows that if $\left(\frac{1+\tau}{1-\phi}\right)^{\epsilon} \left(\frac{A_t^c}{A_t^d}\right)^{\phi} > 1$ and $\varphi > 1$, then $\frac{\partial \left(\frac{\Delta Y_t}{Y_t}\right)}{\partial \tau} > 0$ and $\frac{\partial \left(\frac{\Delta Y_t}{Y_t}\right)}{\partial \tau} > 0$.

Corollary: The effect of a carbon tax on growth rate increases with (i) the initial proportion of the clean sector in production and (ii) the clean sector's growth rate which in turns depends on their productivity. As the relative productivity of the clean sector rises, so does the clean growth rate, and environmental policy positively affects the overall growth rate.

To provide a quantitative understanding of the implications of the energy mix, I report the results of a parameter calibration using different initial values of clean and dirty goods production. This exercise highlights the effect of a tax on different rates of production of clean and dirty inputs in their initial state. I use Acemoglu *et al.* (2012a) parameters, where $\gamma = 2$, $\alpha = 0.3$, and $\epsilon = 3$. I only considered one period, namely period t + 1, after one year of implementing a carbon tax.

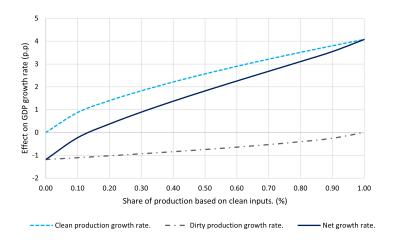


Figure 1: Representation of the effect of the carbon tax on the growth rate (g_t/τ) .

Data and Empirical Strategy 3

3.1Data

To empirically analyze the relation of a carbon tax on GDP growth and employment rates, I use a yearly data panel from a sample of 66 countries, of which 23 had implemented a carbon tax. The sample covers the period from 1990 to 2020. Table 1 presents descriptive statistics of the variables of interest and the sources of the databases. The outcome variables studied are GDP growth (%) and the employment rate measured as the number of employees over an economically active population (%). Table 2 in the annex presents in detail the characteristics of the carbon tax for each of the countries studied. It can be seen that since 2010 the adoption of carbon taxes in the countries has increased. The table also shows the carbon tax's monetary value as well as the percentage of emissions covered by the tax.

Table 1: Description of the main outcome variables.

Variable	Mean	Median	Std. Dev.	Source	
Real GDP (millions US\$ constant 2017)	1026887	258975	2511953	Penn World Table	
Crecimiento del PIB (anual %)	2.86%	2.99%	4.33%	Data WorldBank	
GDP per capita (current US\$)	9.384	9.532	1.143	Data WorldBank	
Employment rate (% total labor)	92.30%	92.94%	4.56%	Data WorldBank	
Population, total	49035868	9771437	165987702	Data WorldBank	
Primary energy consumption (TWh)	1589	324	4390	Our World in Data	
Clean energy fraction* (% total consumption)	14%	9%	16%	International Energy Agency	
Clean electricity fraction* (% total consumption)	37%	32%	31%	International Energy Agency	
Countries	66				
Observations	2044				
*Primary sources of clean energy are hydro, nuclear, solar, and wind power.					

To apply the theoretical model to real-world data, I use the share of primary energy consumed by each source as a proxy for the initial rates of production of clean and dirty inputs, Y_c and Y_d . I categorized the sample of countries based on the share of primary energy consumed by each source, dividing them into those with a low-carbon intensity energy mix and those with a high-carbon intensity energy mix. The term "low-carbon intensity" is used to describe the energy consumption of hydro, nuclear, solar, and wind sources. These sources emit lower levels of carbon than traditional

fossil fuels. Conversely, the term "high-carbon intensity" is used to describe energy generated from the combustion of fossil fuels, such as coal, oil, natural gas, and biofuels. Figure 2 shows the share of energy from low-carbon intensity sources by countries.

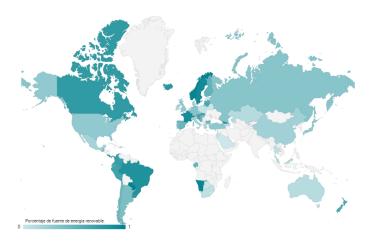


Figure 2: Map of countries according to the share of clean energy sources

The countries with a share of clean energy higher than average (14%), at the time of implementation of the carbon tax, constitute the database of countries with a "low-carbon intensity" energy matrix. Similarly, the countries that had a clean energy share lower than average (14%) at the time of implementing the carbon tax constitute the database of countries with a "high-carbon intensity" energy matrix. Table 3 presents the statistics of the outcome variables for each sample, the full sample of 66 countries, and the sample of countries with polluting and clean energy mix.

The graph 3 shows the relationship between the share of clean energy in the energy mix and GDP growth in countries with and without a carbon tax. The relationship has a slight negative slope, indicating that countries with a higher share of clean energy have slightly lower GDP growth rates. It is important to note, however, that this does not imply that countries with carbon taxes have the lowest growth rates.

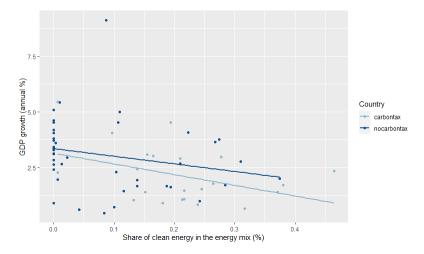


Figure 3: The relationship between GDP growth and the proportion of clean energy sources in the energy mix

3.2 Empirical strategy

In this paper, I estimate the effect of introducing a carbon tax on macroeconomic variables, according to the primary energy sources of consumption, to validate the corollaries defined in the theoretical model. For this, I use the event study method with the estimators proposed by Callaway & Sant'Anna (2021), Sun & Abraham (2021) and TWFE (Two Way Fixed Effects).

Event Study is used to estimate the effect of introducing a carbon tax on macroeconomic variables: GDP growth rate and employment rate. In this approach, I aim to estimate the effect on GDP which is not associated with its historical economic growth. I assume that changes in GDP not predicted by historical GDP growth in the country itself, nor by current and past international economic shocks, are exogenous. Several studies have used the event study strategy to analyze the effects of regulatory changes on carbon prices, energy, and stock prices (Mansanet-Bataller & Pardo (2009); Fan et al. (2017); Bushnell et al. (2013), among others).

We introduce the following assumptions, which capture the effect of carbon tax on GDP and employment rate: Treatment timing. The treatment time assumption refers to a scenario in which there are several periods and countries implement a carbon tax at any time within those periods. Once a country implements the tax, it remains in treatment for the remainder of the period. This assumption implies that the timing of the implementation of the tax is unrelated to other factors such as GDP that may influence the outcome, meaning that it is considered exogenous. In other

words, the timing of tax implementation is independent of other factors and is not influenced by them.

No-anticipation assumption. The implementation of the carbon tax does not affect the path of GDP or labor force outcomes before the treatment period. In other words, the counterfactual outcome paths for GDP and employment rate in periods before the treatment period would have been the same whether or not the carbon tax had been implemented at some point in the future. Similarly, the treatment assignment does not depend on the potential GDP or employment rate outcomes in any period.

Parallel trends. The parallel trends assumption in the context of a staggered events study with a carbon tax as the treatment and GDP and employment rate as the outcome variables would imply that in the absence of the carbon tax, the trends in GDP and employment rate would be parallel across the treated and control groups. In this study, any differences in the post-treatment outcomes between the two groups can be attributed to the treatment (i.e., the carbon tax) and not to pre-existing differences in the trends of GDP and employment rate.

3.2.1 Effect of a carbon tax in countries with high- and low-carbon energy sources.

Proposition 2 states that the effect of a carbon tax on the economy's growth rate is negative if the polluting sector's share in final output exceeds a critical level relative to the share of the clean sector. On the other hand, it indicates that the carbon tax promotes economic growth if the share of the clean sector in final production surpasses a critical level relative to the polluting sector. To test this hypothesis empirically, I use the Event Study strategy and examine two sub samples of countries. The first sub-sample consists of "polluting countries," where the share of clean sources is below the country average. The second sub-sample includes "clean countries," where the share of clean sources exceeds the country average (above 14%).

In this model, I consider the year in which the carbon tax was introduced as year 0. I then define the periods before (t < 0) and after (t > 0) the introduction of the carbon tax, and I align time t=0 for all countries in the treatment group. I assume that the evolution of the potential outcome in the absence of the treatment can be decomposed into a time-fixed effect. Based on this assumption, I estimate the average dynamic effect of introducing a carbon tax on the GDP and employment growth $(Y_{c,t})$ in country c and year t. To conduct our analysis, I employ equation 47.

$$Y_{c,t} = \beta_1 \sum_{-T \le r \le T}^{r \ne 0} 1 \left[CarbonTax_{c,t} = r \right] + \Phi_c + \Phi_t + \epsilon_{c,t}$$

$$\tag{47}$$

where $Y_{c,t}$ is the GDP growth rate and employment rate. β_1 measures the average dynamic effect of a carbon tax in the sample of countries. When the outcome variable is the GDP growth rate this estimator tests the *proposition 2*. Also include α_c country fixed effects c for unobserved country-specific characteristics and Φ_t time fixed effects to capture other policy and time-varying resource price shocks, among other changes that may occur over time.

4 Results

4.1 Implications of carbon tax on growth rate and employment

Figure 4 presents the effect of implementing a carbon tax on the annual GDP growth rate using the database of 66 countries. Implementing a carbon tax is associated with an increase in the GDP growth rate in the early years of the climate policy, and is maintained when using different model specifications. Table 4 presents the estimators of the average effect of implementing a carbon tax on GDP growth over the next ten years. It is observed that implementing the carbon tax is associated with a 1.5 percentage point growth in GDP one year after the policy, this effect is significant using the Sun et al. specification (Column 3). Using the TWFE and Callaway specification, implementing the carbon tax is associated with 0.9 and 0.6 percentage point growth in GDP, respectively, in the year after adopting the policy. The effect under the Callaway methodology is consistent with the results from the literature in which the effect of the carbon tax between the first and second year is associated with an increase in the GDP growth rate of 0.5 percentage points, in the literature implementing a carbon tax is not associated with adverse effects on the GDP growth rate.

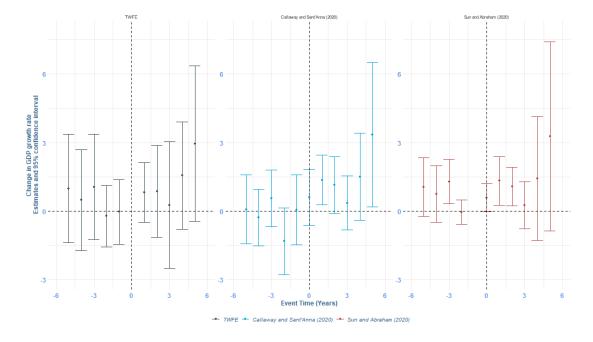


Figure 4: Implications of carbon tax on GDP growth rate.

This result implies that if a country hypothetically implements a carbon tax it can be expected

to, on average, experience an increase in the growth rate of 1 percentage point in the following year after implementing the policy.

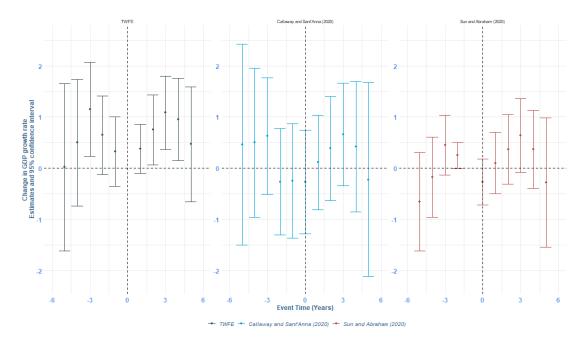


Figure 5: Implications of the carbon tax on employment rate.

Figure 5 presents the results of the effect of the carbon tax on the employment rate using the whole sample. There is evidence of a decrease in the employment rate after the third and fourth year, but there is no statistical significance in any of the estimators 6. It could be argued that the carbon tax is not associated with changes in the employment rate in the first three years after the policy.

4.2 Heterogeneous effects according to the composition of the energy matrix

In this section, I empirically estimate how the effect of introducing a carbon tax on the GDP growth rate and the employment rate varies according to the composition of the countries' energy matrix, i.e., according to the share of primary energy sources in final consumption. To test the corollaries derived from the theoretical model I use the share of energy consumed from clean and polluting sources as a proxy for the initial share of the clean and polluting sectors in the final product.

4.2.1 A carbon tax in polluting countries.

To test Corollary 1, it would be expected that using the sample of polluting countries (whose consumption of energy from fossil and biofuel sources is higher than the sample average), the effect of introducing a carbon tax on the annual GDP growth rate may be negative.

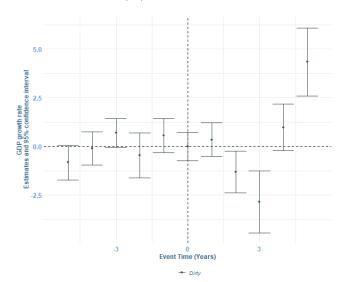
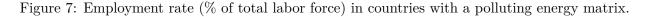
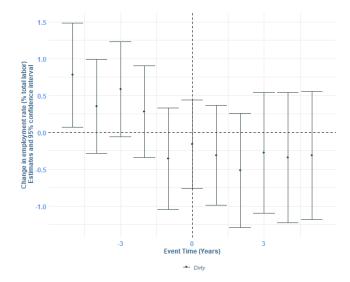


Figure 6: Annual GDP growth (%) in countries with a polluting energy matrix.

Figure 6 presents the coefficients of the average cumulative effect of the carbon tax on the growth rate in polluting countries, and on the y-axis, in addition to indicating the magnitude, the 95% confidence intervals are presented. It is observed that implementing the carbon tax in countries with a polluting energy matrix is associated with a reduction in the growth rate in the second and third years the In the second year the cumulative effect of implementing the carbon tax is -1.3 percentage points of annual GDP, in the third year the effect is -2.8 percentage points. This effect can be explained by the cost of discouraging the innovation of technologies that require polluting energy. In the long term, a positive effect on the annual GDP growth rate may be due to countries adopting new clean technologies.





Similarly, I estimate the effect of implementing a carbon tax on the employment rate of the countries according to the composition of the energy matrix. Figure ?? shows that the carbon tax is not associated with any effect in the first 5 years, however, from the sixth period onwards, a drop in the employment rate is observed. Although the theoretical model does not explain the effect on employment, this effect can be explained by the fact that the labor force is not sufficiently trained to migrate from the polluting sector to the clean sector, which requires a cost and time of adaptation. The cost of training the skills required for clean production is higher if the share of the polluting sector in final production is predominant. Table 7 shows the estimators and standard error for the countries according to the source of energy generation.

4.2.2 A carbon tax in clean countries.

Corollary 1 implies that introducing a carbon tax favors the annual GDP growth rate in countries with a cleaner-than-average energy matrix.

The effects reported in table 5 show the estimators for countries with a clean and polluting energy matrix, columns 2 and 4 respectively. The results suggest that if hypothetically a country with a clean energy matrix implements a carbon tax it can be expected to experience an increase in the growth rate of 1.8 percentage points in the following year after implementing the policy. This result validates the stipulation in Corollary 1 that the higher the initial share of the clean sector in final production, the greater the positive effect of the carbon tax. In this case, I approximate the

clean sector's share of final energy consumption to the share of clean primary energy sources.

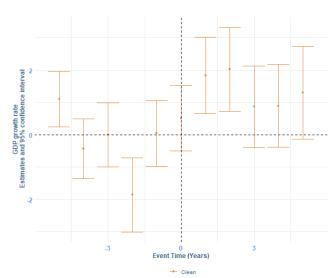
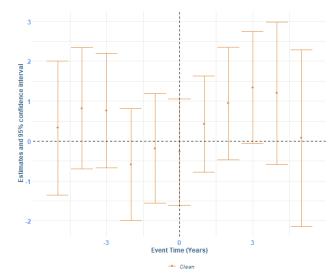


Figure 8: GDP growth (%) in countries with clean energy matrix.

Figure 8 shows that the effect of a carbon tax on the GDP growth rate is positive for countries whose primary energy comes from clean sources. This result suggests that introducing a carbon tax is associated with an increase in annual GDP of 1.4 percentage points in the first 5 years after implementing the policy, this effect is significant. The result suggests that the higher the share of clean sources at the time of implementing the climate policy, the effect of the tax on the GDP growth rate will be favorable.

On the other hand, as shown in the figure 9, implementing a carbon tax is associated with a slight increase in the employment rate in the third period of implementing the policy, but this effect is not significant.

Figure 9: Employment rate (percentage of total workforce) in countries with clean energy matrix.



5 Robustness exercises

To verify whether the previous results were robust, I performed several econometric exercises. First, I estimated the effect of the carbon tax on GDP using different samples of polluting and clean countries, based on the thresholds of the share of energy sources. Second, I used the sources of electricity generation, in exchange for the energy matrix, as they can approximate the share of the sectors (clean and polluting) in the final production. Third, I estimate the effect of the tax using two samples of countries based on the magnitude of the carbon tax in 2020 and the proportion of emissions covered by the tax.

5.1 Exercise with Various Samples

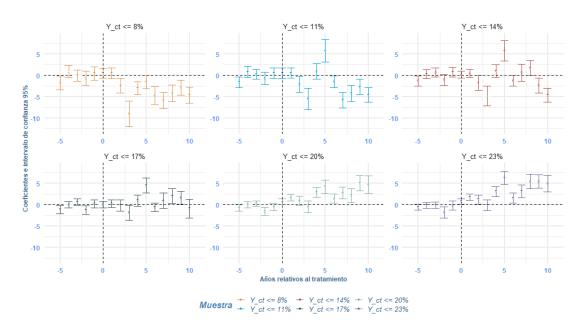


Figure 10: Effect of the carbon tax at different thresholds of the participation of energy sources.

I assess the impact of the carbon tax on growth rates by examining different samples of polluting and clean countries. The goal is to identify the threshold at which the carbon tax's effect shifts from negative (for polluting countries) to positive. I defined various cut-off points Θ based on the proportion of clean energy sources in the total energy consumption. Specifically, I considered scenarios where the sample of polluting countries does not exceed certain percentages of clean energy sources. The findings on the carbon tax's impact on GDP growth, relative to these cut-off points for

clean and polluting samples, are presented in 10. When the sample of polluting countries includes less than 8% clean energy (representing the most polluting countries), the effect is highly negative. Conversely, when the sample reaches 23% clean energy, the effect becomes positive. The carbon tax is beneficial for GDP growth in countries with an energy matrix comprising at least 17% clean energy.

5.2 Effect using electricity mix

The electricity mix is composed of the set of sources available to generate the electricity consumed within a country. Electricity unlike energy can be generated entirely by renewable sources, therefore, for this exercise, I divide the sample of polluting and clean countries based on the 37% share of clean sources. That is, if the country generates more than 37% of electricity from sources such as solar, wind, hydro, and nuclear, it is considered clean, and would be part of the sub-sample of clean countries, otherwise it would belong to the sub-sample of polluting countries.

Figure 11: Effect of carbon tax on GDP growth rate in countries with polluting electricity mix.

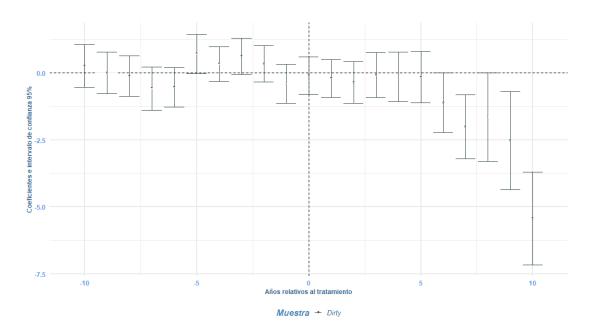
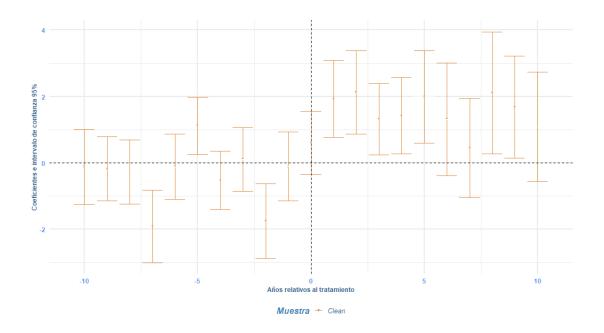


Table 8 presents the estimators of the effect of the carbon tax on the GDP growth rate according to the electricity matrix. The coefficients estimated with the electricity matrix are similar in magnitude and direction to those estimated with the energy matrix, however, these results are significant in more periods unlike those estimated with the energy matrix. Figure 11 presents the

coefficients of the effect of the carbon tax on the growth rate using the sample of countries with a polluting electricity matrix. The carbon tax is associated with negative growth rates, this effect is larger in magnitude than the one calculated with the sample of countries divided according to the energy matrix. On the other hand, the figure 12 shows similar results to those obtained with the sample of clean countries using the energy matrix. For countries with a clean electricity matrix, the effect is slightly positive, increasing the growth rate by 0.6 percentage points in the first 5 years after implementing the climate policy.

Figure 12: Effect of carbon tax on GDP growth rate in countries with clean electricity mix.



6 Conclusions

The findings of the study suggest that the macroeconomic impact of a carbon tax is closely tied to the primary source of energy. To measure this heterogeneity, the study used the proportion of energy generated from fossil fuels and low-carbon-intensity sources. In countries where energy production is primarily from fossil fuels, the implementation of a carbon tax may lead to a short-term slowdown in GDP growth and a long-term reduction in employment rates. These empirical findings confirm proposition 1 of the theoretical model. However, in the long term, economic growth will recover as the share of clean energy sources increases or energy efficiency improves.

On the other hand, in countries where energy production is primarily from low-carbon intensity sources, the implementation of a carbon tax can boost GDP growth in the short term and even have no negative or statistically significant effect on employment.

Furthermore, the study found that the negative impact on GDP growth in high-carbon-intensity economies tends to diminish in the long term. This is because the reduced demand for polluting goods encourages innovation in the clean sector, leading to its growth and the eventual surpassing of the polluting sector. As the clean sector grows and becomes dominant, the economy can return to its pre-tax growth trend.

According to the model, a carbon tax increases the cost of final goods produced from polluting sources, reducing the demand for these goods and increasing the demand for clean goods. As a result, the production of dirty goods tends to decrease, while the productivity of the clean sector improves. Additionally, the carbon tax increases labor participation in the clean sector, as companies shift towards cleaner production methods to avoid the tax. Overall, the model suggests that a carbon tax can be an effective policy tool for reducing carbon emissions and promoting clean economic growth and some recommendations can be introduced to minimize the cost of transition.

The model highlights the importance of reusing tax revenue from the carbon tax, particularly in the early stages of the transition, it is suggested that it be allocated to the development of clean technologies until reaching the point at which this sector can grow driven. for the demand. This can minimize the negative effect of the carbon tax on economic growth. These findings underscore the importance of considering the energy mix when designing effective climate policies, such as carbon taxes.

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A Annex

Table 2: Characteristics of the carbon tax in the countries analyzed.

Jurisdictions Argentina Canada Chile Colombia Denmark Estonia Frinland Frinland Frinland Ireland Japan Latvia Latvia Liedtenstein Mexico Norway Poland Portugal Singapore Slovenia South Africa Spain Sweden Sweden Sweden Syaine	
Sectors covered All sectors with some exemptions. All sectors with some exemptions. All sectors with some exemptions. Power and industry sectors. All sectors with some exemptions. Buildings and transport, exempt sectors covered by ETS. Power and industry sectors. Industry, buildings, and transport (not public) sectors. Industry, buildings, and transport (not public) sectors. All sectors but sectors covered by EU ETS are exempt. All sectors but sectors covered by EU ETS are exempt. All sectors with some exemptions. Industry and power sectors, exempt sector covered by ETS. Industry, power, buildings and transport sector Power, industry, transport, buildings, waste, forestry sectors. All sectors but EU ETS are exempt. All sectors but EU ETS are exempt. Industry, buildings and transport sector buildings and transport sector Endustry, power, buildings and transport sector Fluorinated GHG emissions (HFCs, PFCs, and SF6) Transport and buildings, exempt sectors covered by ETS. Industry, power, buildings and transport sectors but EU ETS. Industry, power and buildings sectors	
Fossil fuels covered Liquid and gaseous fossil fuels Fossil fuels Fossil fuels Fossil fuels Liquid and gaseous fossil fuels Fossil fuels Fossil fuels Fossil fuels Fossil fuels Liquid and gaseous fossil fuels Fossil fuels Liquid and gaseous fossil fuels	
Point of Taxation Producers, distributors and importers. Registered distributors of the fossil fuels. Users of the fassil fuels. Sellers and importers of the fossil fuels. Sellers and importers and importers. Users of the fossil fuels. Sellers and importers. Users of the fossil fuels. Distributors and importers. Distributors and importers. Producers, distributors and importers. Distributors and importers. Distributors and importers. Distributors and importers. Producers, distributors and importers. Producers, distributors and importers. Distributors and importers. Producers, distributors and importers. Producers, distributors and importers. Users of the fossil fuels. Distributors and importers. Users of the fossil fuels. The first entry of all F-gases. Distributors and importers. Users of the fossil fuels. The first entry of all F-gases. Distributors and importers. Users of the fossil fuels.	
Year 2018 2019 2017 2017 1992 2000 1990 2014 2010 2010 2010 2012 2004 2012 2004 2018 2019 1991 1991 1991 1996 2014 1991 1996 2015 2014 1991 1996 2015 2014 2015 2016	
GHG emissions 441 MtCO2e 149 MtCO2e 149 MtCO2e 139 MtCO2e 28 MtCO2e 28 MtCO2e 28 MtCO2e 112 MtCO2e 1345 MtCO2e 1345 MtCO2e 1345 MtCO2e 18 MtCO2e 26 MtCO2e 27 MtCO2e 27 MtCO2e 28 MtCO2e 28 MtCO2e 29 MtCO2e 21 MtCO2e 25 MtCO2e 26 MtCO2e 26 MtCO2e 27 MtCO2e 27 MtCO2e 28 MtCO2e 28 MtCO2e 28 MtCO2e 28 MtCO2e 28 MtCO2e 38 MtCO2e	
GHG covered 20% 22% 22% 39% 6% 8 86% 466% 49% 25% 25% 25% 40% 8 80% 25% 23% 25% 25% 25% 25% 25% 25% 25% 25% 25% 25	
Price US85/tCO2e US83/tCO2e	

Table 3: Descriptive statistics of the samples

Variable	Mean	Median	Std. Dev.	Mean	Median	Std. Dev.
			All con	untries		
	W	ith carbon t	ax	wit	hout carbon	tax
GDP real (millions 2017US\$)	850686.90	393482.88	1082446.42	1121275.39	166135.61	3006622.00
GDP per capita (current US\$)	9.89%	10.10%	0.94%	9.11%	9.15%	1.15%
GDP growth (annual %)	2.482	2.683	3.694	3.058	3.254	4.630
Employment rate (% total labor)	92.07%	92.85%	4.75%	92.43%	93.02%	4.45%
Primary energy consumption (TWh)	1094.63	474.96	1379.31	1855.63	254.93	5334.58
clean electricity fraction (% total consumption)	47%	46%	33%	31%	24%	29%
Clean energy fraction (% total consumption)	23%	18%	20%	9%	4%	11%
Countries	23			43		
	Countries with a low carbon intensity energy mix					
	W	ith carbon t	ax	without carbon tax		
GDP real (millions 2017US\$)	609238.88	322000.91	721705.56	654034.94	161623.54	1096710.94
GDP growth (annual %)	10.02%	10.17%	0.91%	9.26%	9.44%	1.04%
GDP per capita (current US\$)	2.17	2.57	3.28	2.35	2.67	3.47
Employment rate (% total labor)	92.14%	92.70%	4.51%	90.99%	91.68%	4.23%
Primary energy consumption (TWh)	841.80	351.14	1103.98	748.81	232.82	1166.34
clean electricity fraction (% total consumption)	70%	71%	21%	60%	60%	16%
Clean energy fraction (% total consumption)	36%	32%	17%	23%	23%	8%
Countries	13			12		
	Countries with a high carbon intensity energy mix					
	W	ith carbon t	ax	wit	hout carbon	tax
GDP real (millions 2017US\$)	1164569.32	579572.91	1359162.31	1302519.86	168376.80	3459270.47
GDP growth (annual %)	9.72%	9.86%	0.95%	9.06%	9.07%	1.19%
GDP per capita (current US\$)	2.89	2.83	4.14	3.33	3.57	4.98
Employment rate (% total labor)	91.97%	93.96%	5.06%	93.06%	93.67%	4.39%
Primary energy consumption (TWh)	1423.29	1010.65	1614.45	2289.49	277.86	6198.64
clean electricity fraction (% total consumption)	16%	12%	15%	19%	10%	24%
Clean energy fraction (% total consumption)	6%	5%	6%	3%	1%	5%
Countries	10			41		

Table 4: Effect of carbon tax on GDP growth rate.

Efecto sobre la tasa de crecimiento del PIB.				
Periodo	TWFE	Sun et. al.	Callaway et. al.	
-10	2.0511. (1.0538)	2.072* (0.9154)	-0.066 (0.4674)	
-9	2.2377. (1.101)	2.341*** (0.6307)	$0.2848 \ (0.7686)$	
-8	$1.963\ (1.0627)$	2.098** (0.7346)	-0.2447 (0.7355)	
-7	$0.3101\ (1.2907)$	$0.4184\ (0.9856)$	-1.6744* (0.7573)	
-6	$0.9138\ (1.1936)$	$0.9745 \ (0.8854)$	$0.552 \ (0.9452)$	
-5	$0.9333 \ (1.1927)$	$0.9629 \ (0.65)$	-0.0054 (0.7334)	
-4	$0.548\ (1.1334)$	$0.7931\ (0.625)$	-0.151 (0.6237)	
-3	$1.1046 \ (1.1724)$	1.297** (0.4847)	$0.5186 \ (0.6281)$	
-2	-0.1466 (0.6834)	$0.0445 \ (0.2604)$	-1.2647 (0.7162)	
-1	-0.0484 (0.7365)	$0.6092. \ (0.2996)$	-0.0448 (0.7143)	
1	$0.9087 \; (0.6737)$	1.543** (0.572)	$0.6128 \; (0.6395)$	
2	$1.0405 \ (1.047)$	1.326*** (0.4413)	1.5846** (0.5919)	
3	$0.4668 \; (1.4567)$	$0.5076 \ (0.5355)$	1.3926* (0.5856)	
4	$1.7464 \ (1.2217)$	1.707 (1.363)	$0.605 \ (0.6133)$	
5	3.1444 (1.7416)	3.575 (2.085)	1.7872. (0.9386)	
6	$0.7838\ (1.0323)$	$1.027 \ (0.6695)$	3.651* (1.6256)	
7	0.7748 (1.4977)	1.021 (1.232)	1.1508 (0.6942)	
8	2.1321 (1.2189)	2.584. (1.358)	$1.1404 \ (0.9535)$	
9	1.4368 (1.0582)	2.047 (1.153)	2.676** (0.9565)	
10	$1.075 \ (1.6154)$	2.086 (1.238)	2.1011* (0.9394)	
Fixed-Effects				
Country	Yes	Yes	No	
year	Yes	Yes	No	
S.E.:Clustered	Country	Country	Country	
Observations	2288	2288	2288	

Table 5: Effect of carbon tax on GDP growth rate, according to energy matrix

Effect on GDP growth rate						
Periodo	Estimador	Error estándar	Estimador	Error estándar		
Panel A. Países limpios			Panel B. P	Panel B. Países sucios		
-10	-0.3345	(0.6734)	-0.8265	(0.4406)		
-9	-0.2288	(0.5453)	1.2703	(1.2058)		
-8	-0.3621	(0.5072)	-0.1283	(1.2226)		
-7	-1.499**	(0.5559)	-1.739*	(0.5033)		
-6	-0.323	(0.5076)	0.7477	(0.5209)		
-5	1.1018*	(0.4524)	-0.831	(0.4793)		
-4	-0.4407	(0.4502)	-0.1065	(0.4174)		
-3	-0.006	(0.5013)	0.7044	(0.3898)		
-2	-1.8644*	(0.5847)	-0.4588	(0.5817)		
-1	0.0342	(0.4994)	0.5648	(0.4513)		
0	0.5112	(0.5215)	-0.0016	(0.3975)		
1	1.8328*	(0.6228)	0.3466	(0.4201)		
2	2.024*	(0.6683)	-1.3099*	(0.5608)		
3	0.8623	(0.6068)	-2.8622*	(0.764)		
4	0.8891	(0.6215)	0.9814	(0.5686)		
5	1.3018	(0.733)	4.3461*	(0.9005)		
6	0.7619	(0.872)	-0.4006	(0.5029)		
7	0.3903	(0.8541)	0.0547	(0.6629)		
8	1.661	(0.9432)	1.936*	(0.85)		
9	1.0672	(0.7778)	1.5139	(0.8188)		
10	0.2729	(0.7947)	2.3459.	(1.1834)		

Table 6: Effect of carbon tax on employment rate.

Effect on the employment rate.				
Periodo	TWFE	Sun et. al.	Callaway et. al.	
-10	2.6611 (1.5807)	2.447** (0.8509)	0.1847 (1.4704)	
-9	$2.3939 \ (1.5619)$	2.142*(0.8659)	-0.3068 (1.3768)	
-8	2.6395. (1.3626)	2.387** (0.8485)	$0.2855 \ (1.4232)$	
-7	$2.3171\ (1.2668)$	2.059*** (0.6812)	-0.3256 (1.446)	
-6	$1.5681\ (1.0975)$	1.287*** (0.4159)	-0.6085 (1.5235)	
-5	$1.0559 \ (1.0219)$	0.7722*(0.3419)	-0.5214 (1.4776)	
-4	$0.9203\ (0.9383)$	0.6388** (0.24)	-0.1021 (1.6909)	
-3	$0.8444 \ (0.8389)$	0.5714*** (0.1596)	-0.0635 (1.6412)	
-2	$0.2272\ (0.5176)$	0.1457. (0.0749)	-0.3946 (1.6765)	
-1	$0.0151\ (0.3191)$	-0.1022 (0.2043)	-0.151 (1.585)	
1	-0.0006 (0.3809)	$0.0791 \ (0.4367)$	-0.0862 (1.454)	
2	-0.5649 (0.7229)	$0.0704\ (0.6796)$	0.1468 (1.6043)	
3	-1.1109 (0.8925)	-0.1436 (0.8014)	0.157 (1.4507)	
4	-1.7576 (1.2235)	-0.6067 (1.029)	-0.0456 (1.6202)	
5	-1.8548 (1.3877)	-0.9074 (1.147)	-0.4825 (2.0502)	
6	-1.0674 (1.5969)	-0.4448 (1.358)	-0.7684 (2.0155)	
7	-0.9602 (2.1663)	-0.4497 (1.913)	-0.2878 (2.1902)	
8	-1.7954 (2.512)	-1.31 (2.309)	-0.2794 (2.4654)	
9	-2.809(2.975)	-1.994 (2.812)	-1.1433 (3.5648)	
10	-3.142 (3.1873)	-2.287 (3.054)	-1.8144 (2.3919)	
Fixed-Effects				
Country	Yes	Yes	No	
year	Yes	Yes	No	
S.E.:Clustered	Country	Country	Country	
Observations	2288	2288	2288	

Table 7: Effect of the carbon tax on the employment rate, according to the energy matrix.

Efecto sobre la tasa de empleo				
Periodo	Estimador	Error estándar	Estimador	Error estándar
	Panel A. Países limpios		Panel B. Países sucios	
-10	-0.8827	(2.1563)	0.2963	(0.3754)
-9	1.5237	(1.8727)	0.0085	(0.4002)
-8	-0.3595	(1.6389)	-0.0913	(0.366)
-7	0.1159	(1.4556)	-0.6016	(0.3824)
-6	0.2858	(1.3498)	-0.5876	(0.3664)
-5	0.3304	(0.9291)	0.7763*	(0.3335)
-4	0.8198	(0.7706)	0.353	(0.2958)
-3	0.7605	(0.7469)	0.5847	(0.3161)
-2	-0.5885	(0.7845)	0.2812	(0.3423)
-1	-0.1869	(0.7433)	-0.3589	(0.3377)
0	-0.2718	(0.7126)	-0.161	(0.3283)
1	0.424	(0.6439)	-0.3109	(0.3616)
2	0.9435	(0.6758)	-0.5194	(0.4048)
3	1.3398.	(0.7078)	-0.2784	(0.3922)
4	1.1973	(0.8432)	-0.3419	(0.4487)
5	0.0748	(1.1492)	-0.3159	(0.4802)
6	-0.5212	(1.5708)	-1.3156*	(0.5209)
7	0.3221	(1.0348)	-2.1817*	(0.6321)
8	0.0513	(0.9423)	-1.7983*	(0.7957)
9	-0.3719	(1.0621)	-2.5975**	(0.9207)
10	0.1126	(1.1674)	-5.5129*	(0.9173)

Table 8: Effect of carbon tax on GDP growth rate, according to electricity matrix composition.

Efecto sobre la tasa de crecimiento del PIB				
Periodo	Estimador	Error estándar	Estimador	Error estándar
	Panel A. Países limpios		Panel B. Países sucios	
-10	-0.1204	(0.5823)	-0.8602*	(0.4063)
-9	-0.1837	(0.5054)	1.4112	(1.2276)
-8	-0.2729	(0.5652)	-0.0702	(1.2763)
-7	-1.9222*	(0.5846)	-1.6589*	(0.4992)
-6	-0.1248	(0.4972)	0.8153	(0.5344)
-5	1.1171**	(0.4247)	-0.8765	(0.5163)
-4	-0.5326	(0.4392)	-0.1527	(0.4358)
-3	0.1009	(0.4992)	0.7496	(0.4114)
-2	-1.7654*	(0.5608)	-0.4363	(0.5449)
-1	-0.111	(0.5012)	0.4642	(0.4953)
0	0.5908	(0.4877)	-0.0125	(0.4074)
1	1.9277*	(0.599)	0.2502	(0.4246)
2	2.1262*	(0.6625)	-1.4785*	(0.569)
3	1.317*	(0.566)	-2.956*	(0.8038)
4	1.4174*	(0.6158)	0.8687	(0.6318)
5	1.9908**	(0.7196)	4.2354*	(0.9454)
6	1.3091	(0.8273)	-0.5926	(0.5206)
7	0.4475	(0.8145)	-0.1533	(0.7561)
8	2.1054*	(0.8762)	1.6759.	(0.8269)
9	1.6742*	(0.7858)	1.4351	(0.7959)
10	1.081	(0.7777)	2.1487	(1.2084)