TRIGONOMETRY

HANDWRITTEN NOTES

Designed with ♥
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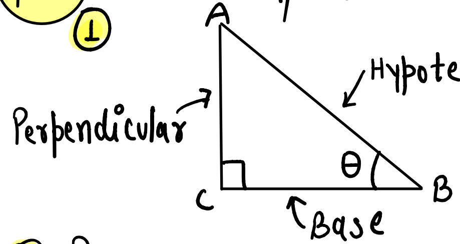
[Standard और Basic → दोनों की strategy इसी Notes वाली video में बताई है]

K³B ⇒ ① Some important terms related to Triangles (हाँ हाँ तुम्हें सब आता है फिर भी :P)



$\theta \rightarrow$ a greek letter (किसी Angle का नाम के लिए use होता है।)

② Pythagorus Theorem: $\boxed{(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2}$

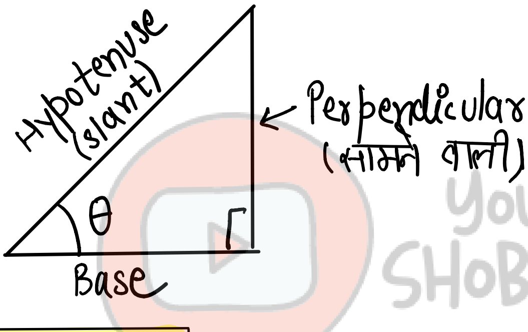
for ex. in above Δ, $(AB)^2 = (AC)^2 + (BC)^2$

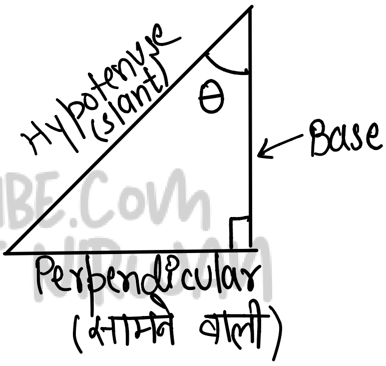
*** ③ Hypotenuse, Base, और Perpendicular depend करता है कि किस angle के Respect में देखा जा रहा है।

→ Slant वाली side हमेशा Hypotenuse होगी।

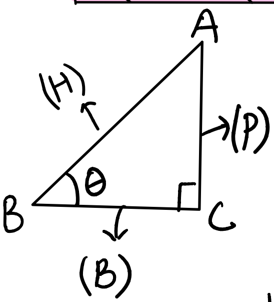
→ जिस Angle से देख रहे हो उसके सामने वाली side Perpendicular होगी।

→ बची हुई side is Base





TRIGONOMETRIC RATIOS



$\frac{AB}{BC}, \frac{BC}{AC}, \frac{AC}{AB}, \frac{AB}{AC}$ ------ } ratios of sides of a Δ.

और अगर इन ratios को नाम दे दे तो उसे Trigonometric Ratios कहते हैं

H → Hypotenuse
B → Base
P → Perpendicular

**

$\sin\theta = \frac{P}{H}$ opp $\csc\theta = \frac{H}{P}$ (cosecθ)

$\cos\theta = \frac{B}{H}$ opp $\sec\theta = \frac{H}{B}$

$\tan\theta = \frac{P}{B}$ opp $\cot\theta = \frac{B}{P}$

$\sin\theta = \frac{1}{\text{cosec}\,\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta}$

or, $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ or, $\sec\theta = \frac{1}{\cos\theta}$ or, $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

K³B ① $\sin\theta \neq (\sin)\times(\theta)$ [product नहीं है!]

↳ Pronounced as "sin of angle θ" not "sin into θ" ✗

eg:- $\sin(A+B) \neq \sin A + \sin B$

② $\sin^2 A \equiv (\sin A)^2$
$\cos^2 A \equiv (\cos A)^2$

③ $(\sin A)^{-1} \neq \sin^{-1} A$

↳ $(\sin A)^{-1}$ means $1/\sin A$ but $\sin^{-1} A$ is called sin inverse A.

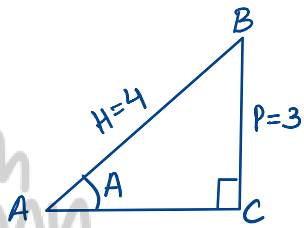
④ $\sin\theta$ की value -1 से 1 के बीच में ही होगी ।

mathematically, $\sin\theta \in [-1, 1]$ → channel के video lecture में और अच्छी तरह समझाया है।

LP: Given $\sin A = \frac{3}{4}$, calculate all other trigonometric ratios.

sol:- **Concept:** अगर हमें कोई भी एक trigonometric ratio की value पता है तो हमें बाकी सबकी भी पता चल जाएगी।

given, $\sin A = \frac{3}{4} \begin{matrix} \to P \\ \to H \end{matrix}$ $\therefore P = 3$ and $H = 4$



by pytha, $H^2 = B^2 + P^2$

$\Rightarrow (4)^2 = B^2 + (3)^2$

$\Rightarrow 16 - 9 = B^2 \Rightarrow B^2 = 7$

$B = +\sqrt{7}$ or $-\sqrt{7}$ → ✗ (because side cannot be -ve).

$\therefore \boxed{B = \sqrt{7}}$

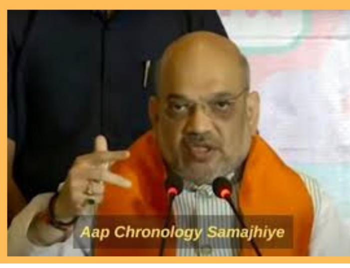
अब H, B, P तीनों पता हैं → बस हो गया सवाल!

Now, $\sin A = \frac{P}{H} = \frac{3}{4}$ $\quad \operatorname{cosec} A = \frac{4}{3}$

$\cos A = \frac{B}{H} = \frac{\sqrt{7}}{4}$ $\quad \sec A = \frac{4}{\sqrt{7}}$

$\tan A = \frac{P}{B} = \frac{3}{\sqrt{7}}$ $\quad \cot A = \frac{\sqrt{7}}{3}$

- **Pehle H,B aur P nikalenge,**
- **Fir Saare trigonometric ratios ki value nikalenge,**
- **Fir jiski value question poochega vo nikal denge**



K^3B ⇒ Ex-8.1 के ज़्यादातर questions ऐसी ही हैं, पहले सारी trigonometric ratios निकाल लो then जिस भी expression की value पूछी हो वो निकाल लो; for eg ऊपर वाले question में further like $\sin^2 A + \cos^2 A$, $\sin^2 A - \cot^2 A$, $\frac{\sin A}{\cot A}$ etc i.e. simply value put कर दो

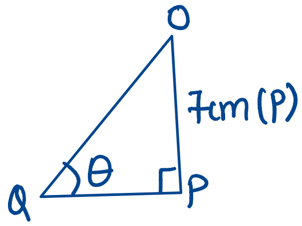
LP:- In ΔOPQ, right angled at P, OP = 7cm and OQ − PQ = 1cm. Determine values of $\sin Q$ and $\cos Q$.

Concept: जब अजीब से question में कुछ समझ ना आए तो Pytha लगेगी।

Sol: Given: $OP = \text{Perp.} = 7\,cm$

$OQ - PQ = 1\,cm$

$\boxed{OQ = 1 + PQ}$ — (I)



To find: $\sin Q, \cos Q$

Now by pytha, $OQ^2 = OP^2 + QP^2$

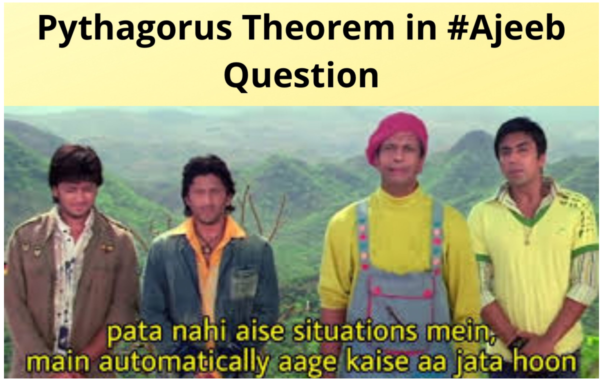
$OQ^2 = (7)^2 + (QP)^2$

$(1+PQ)^2 = 49 + (PQ)^2$ (from (I))

$1 + \cancel{(PQ)^2} + 2(PQ) = 49 + \cancel{(PQ)^2}$

$2PQ = 48$

$\boxed{PQ = 24}$



putting in (I), $OQ = 1 + 24$

$= 25$

$\therefore$ Perp (P) $= 7\,cm$

$H = 25$

$B = 24$

$\therefore \sin\theta = \frac{P}{H} = \frac{7}{25}$ & $\cos\theta = \frac{B}{H} = \frac{24}{25}$

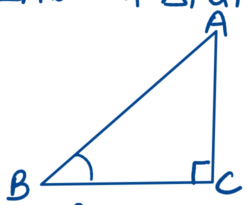
**LP:- If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

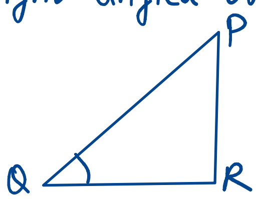
Sol:- Given: $\angle B, \angle Q \rightarrow$ acute angles

$\sin B = \sin Q$

To prove: $\angle B = \angle Q$

Proof: let $\triangle ABC$ & $\triangle PQR$ are two right angled triangle.





$\sin B = \frac{AC}{AB}$ $\qquad$ $\sin Q = \frac{PR}{PQ}$

ATQ, $\sin B = \sin Q$

$\frac{AC}{AB} = \frac{PR}{PQ}$

दोनों के perpendicular $\frac{AC}{PR} = \frac{AB}{PQ} = K$ (let) दोनों के Hypo

Hint → बस यहीं से Base के Ratio भी इसी ratio के बराबर हो जाए।

$BC = \sqrt{AB^2 - AC^2}$ & $QR = \sqrt{PQ^2 - PR^2}$ [by pytha]

Now, $\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{K^2(PQ)^2 - K^2(PR)^2}}{\sqrt{PQ^2 - PR^2}} = K\frac{\cancel{\sqrt{PQ^2 - PR^2}}}{\cancel{\sqrt{PQ^2 - PR^2}}} \Rightarrow \underline{\underline{K}}$

So, $\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore$ by similarity of Δs concept: $\Delta ACB \sim \Delta PRQ$

$\therefore$ $\boxed{\angle B = \angle Q}$ Hence Prooved

★★

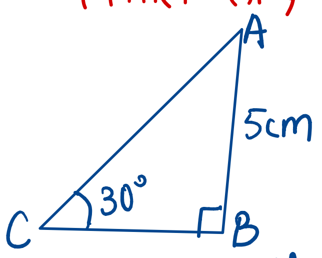
$\angle\theta$	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	N.D.
$\operatorname{cosec}\theta$	N.D.	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D.
$\cot\theta$	N.D	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Very Important Table #RattaMaarLo

ये कुछ standard results हैं कुछ खास angles के लिए जिनकी value direct questions में आएंगी। for example – Ex-8.2 का Q1, Q2 पूरा ऐसे ही है, value डालो answer पाओ।

LP: In ΔABC, right angled at B, AB = 5 cm, $\angle ACB = 30°$. Determine the length of sides BC and AC.

Sol: Concept: जो दो चीजें दी हैं, उनमें standard result use करके तीसरी value निकाल लो, पूरा question then बनता रहेगा।



Given:- $P = 5cm$
$\angle ACB = 30°$ or $C = 30°$

To find: BC & AC

Now, $C = 30°$
taking sin both sides
$\sin C = \sin 30°$

$\frac{P}{H} = \frac{1}{2}$

$H = P(2) \Rightarrow (5)(2)$

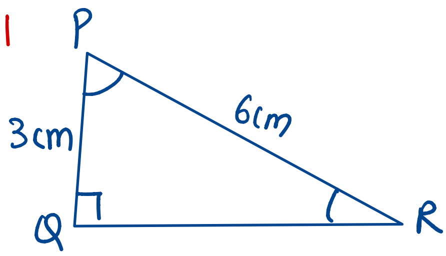
$\therefore \boxed{H = 10}$

why ? sin क्यू लिया ?
ans: P → किसमें आता है ?
↳ sin में, tan में
इसलिए sin लिया।
(tan भी ले सकते थे।)

by pytha, $H^2 = P^2 + B^2$
$100 = 25 + B^2 \Rightarrow \boxed{B = \sqrt{75}}$

Q: In ΔPQR, right angled at Q, PQ = 3cm and PR = 6cm. Determine $\angle QPR$ and $\angle PRQ$.

Sol: Concept: वही पिछले वाला, बस थोड़ा घुमा दिया।



Given: H = 6cm (for both angle)
For $\angle P$, B = 3cm
For $\angle R$, P = 3cm

To find, $\angle P$ & $\angle R$

Concept:- पहले कोई एक angle पकड़ो, let say P
For P we have, base & Hypo; Now B & H किसमें आता है?
↳ cos में।

$\therefore \cos P = \frac{B}{H}$

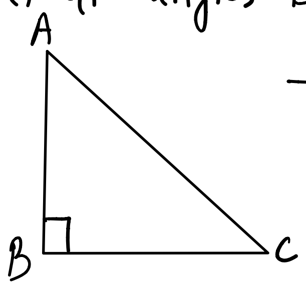
$\cos P = \cancel{\frac{3}{6}}_2 \Rightarrow \cos P = \frac{1}{2}$, or $\cos P = \cos 60°$
$\therefore \boxed{P = 60°}$

Now, $\angle P + \angle R + \angle Q = 180°$ (sum of all angles of Δ)
$60° + \angle R + 90° = 180°$
$\boxed{\angle R = 30°}$

V. Imp

Complementary Angles

↳ ऐसे दो angles जिनके sum 90 हो।



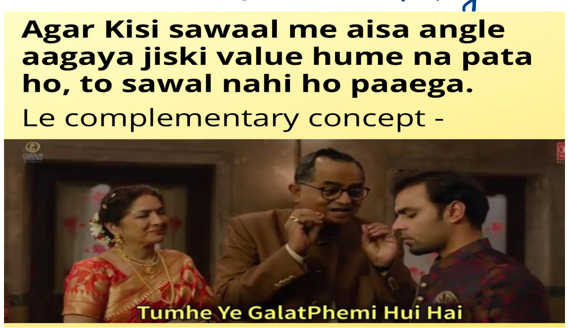
→ For a right angled Δ, $A + B + C = 180°$
$A + 90 + C = 180$
$\boxed{A + C = 90°}$

तो right angled Δ में complementary angles मिलते हैं और trigo में तो हम Right angled Δ ही पढ़ते हैं!

Some Imp. formulaes:-

$\sin(90-\theta) = \cos\theta$	$\tan(90-\theta) = \cot\theta$	$\sec(90-\theta) = \operatorname{cosec}\theta$
$\cos(90-\theta) = \sin\theta$	$\cot(90-\theta) = \tan\theta$	$\operatorname{cosec}(90-\theta) = \sec\theta$

K³B ⇒ **Q** Complementary concept का काम क्या है?
↳ जब भी कोई ऐसा angle दिया हो जो की standard angles में से नहीं है, तो वहाँ ये काम आएगा।



LP :- Evaluate $\dfrac{\tan 26^\circ}{\cot 64^\circ}$?

** **Concept :-** पहली बात तो दोनों में से कोई भी standard angle नहीं है।
∴ किसी भी एक में complementary का concept लगा दो।

Sol :-

$$\tan 26^\circ = \tan(90-64)$$

$$= \cot 64$$

$$\therefore \frac{\tan 26}{\cot 64} = \frac{\cancel{\cot 64}}{\cancel{\cot 64}} \Rightarrow \underline{\underline{1}}$$

} Exercise में ज्यादातर questions ऐसे ही हैं, कि किसी भी एक में concept लगाओ then simplify होता दिखेगा।

Same type Q → eg 11, Ex-8.3 → 1, 2, 7

LP :- If $\sin 3A = \cos(A-26)$, find A.

Concept :- दो Trigonometric ratios बराबर दे रखे हो और तीसरा निकालना हो तो भी complementary का concept लगेगा। (किसी एक को change कर दो।)

Sol :- $\sin 3A = \sin(+90-90+3A)$.

$$= \sin(90-(90-3A)) \Rightarrow \cos(90-3A)$$

$[\because \sin(90-A) = \cos A]$

$$\therefore \sin 3A = \cos(A-26)$$

$$\Rightarrow \cos(90-3A) = \cos(A-26)$$

$$\therefore 90-3A = A-26$$

$$\boxed{A = 29^\circ}$$

↳ Same concept पर Ex-8.3 → Q 3, 4, 5, 6

TRIGONOMETRIC IDENTITIES

↳ पूरी की पूरी exercise (infact सारे questions जो आगे आएंगे) वो इन 3 formulaes पर ही based हैं :-

**

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

K³B :- अब जितने भी Important Questions हैं उन्हें मैंने कुछ Types में बाँट दिया है। और साथ में मैंने ये भी बताया कि किस Type के question को देखकर क्या click करना चाहिए।

⊛ ⇒ जब भी किसी Question में अटक जाओ तो सारे Types को recall करना।

TYPE # 1 : Direct formula वाले :-

LP :- $\dfrac{\sin^2 63 + \sin^2 27}{\cos^2 17 + \cos^2 73^\circ}$, Evaluate

Sol:- $\dfrac{\sin^2 63 + [\sin(90-63)]^2}{\cos^2 17 + [\cos(90-17)]^2} = \dfrac{\overbrace{\sin^2 63 + \cos^2 63}^{\sin^2\theta+\cos^2\theta=1}}{\underbrace{\cos^2 17 + \sin^2 17}_{\sin^2\theta+\cos^2\theta=1}} \Rightarrow \dfrac{1}{1} \Rightarrow 1$

Ex-8.4 → Q3 → (ii)
Q4 → (i) (iv)
Q5 → (vii) (x)
} Type #1 questions.

Type #2 : Converting or Expressing in terms of one trigonometric Ratio:-

LP:- Express the trigonometric ratio sin A, sec A and tan A in term of cot A.

**Concept :- जिसमे trigonometric ratio मे convert करना है, ये याद करो कि वो trignometric ratio कौनसे formulae में आता है।

for eg here → cot A कौनसे formulae मे दिखता है?
↳ $\text{cosec}^2 A - \cot^2 A = 1$
तो इसे सबसे पहले use करेंगे।

Sol^n
$\text{cosec}^2 A = 1 + \cot^2 A$
$\Rightarrow \text{cosec} A = \sqrt{1+\cot^2 A}$ — (I)
$\Rightarrow \dfrac{1}{\sin A} = \sqrt{1+\cot^2 A} \Rightarrow \boxed{\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}}$ — (II)

Now, $\sin^2 A + \cos^2 A = 1 \Rightarrow \cos A = \sqrt{1-\sin^2 A} \Rightarrow \sqrt{1 - \dfrac{1}{1+\cot^2 A}}$ — (III)

now $\sec A = \dfrac{1}{\cos A} = \dfrac{1}{\sqrt{1-\sin^2 A}}$

$\Rightarrow \boxed{\sec A = \dfrac{1}{\sqrt{1 - \dfrac{1}{1+\cot^2 A}}}}$ — (IV)

and $\boxed{\tan A = \dfrac{1}{\cot A}}$ — (V)

$\underbrace{\text{I}, \text{II}, \text{III}, \text{IV} \ \& \ \text{V}}_{\text{Answer}}$

**TYPE # 3 : TanA/ cotA को sinA, cosA मे बदलकर करना :-

concept : जब भी question containing tanA/cotA को sinA/cosA/cosecA/secA की terms में proove करना हो तो tanA/cotA को sinA, cosA मे convert करो।

LP:- Proove that $\dfrac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \dfrac{\sec\theta+1}{\sec\theta-1}$

Sol:- LHS:- $\dfrac{\dfrac{\sin\theta}{\cos\theta}+\sin\theta}{\dfrac{\sin\theta}{\cos\theta}-\sin\theta} = \dfrac{\dfrac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}{\dfrac{\sin\theta-\sin\theta\cos\theta}{\cos\theta}}$

$$= \frac{\cancel{\sin\theta}(1+\cos\theta)}{\cancel{\sin\theta}(1-\cos\theta)}$$

$$= \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}} \Rightarrow \frac{\frac{\sec\theta+1}{\cancel{\sec\theta}}}{\frac{\sec\theta-1}{\cancel{\sec\theta}}} \Rightarrow \frac{\sec\theta+1}{\sec\theta-1}$$

$= RHS$

LP:- $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\,\mathrm{cosec}\,\theta$

Sol:- LHS: $\dfrac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \dfrac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}} \Rightarrow \dfrac{\sin\theta/\cos\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \dfrac{\cos\theta/\sin\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta}\cdot\sin\theta}{\sin\theta-\cos\theta} + \frac{\frac{\cos\theta}{\sin\theta}\cdot\cos\theta}{\cos\theta-\sin\theta}$$

↳ अगर l.cm लेना है तो यहाँ minus common लेना होगा।

$$\Rightarrow \frac{\sin^2\theta/\cos\theta}{\sin\theta-\cos\theta} - \frac{\cos^2\theta/\sin\theta}{\sin\theta-\cos\theta}$$

$$\Rightarrow \frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$\Rightarrow \frac{1}{(\sin\theta-\cos\theta)}\,\frac{\sin^3\theta-\cos^3\theta}{(\sin\theta\cos\theta)}$$

$$[a^3-b^3 = (a-b)(a^2+b^2+ab)]$$

$$\Rightarrow \frac{\cancel{(\sin\theta-\cos\theta)}\,[\overbrace{\sin^2\theta+\cos^2\theta}^{=1}+\sin\theta\cdot\cos\theta]}{\cancel{(\sin\theta-\cos\theta)}\,(\sin\theta\cdot\cos\theta)}$$

$$\Rightarrow \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$\Rightarrow \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}+1$$

$\Rightarrow 1+\sec\theta\,\mathrm{cosec}\,\theta$ RHS Hence Proved

LP:- $(\sec A+\tan A)(1-\sin A) =$

(A) $\sec A$ (B) $\sin A$ (C) $\mathrm{cosec}\,A$ (D) $\cos A$

Sol

$$\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$\Rightarrow \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$\Rightarrow \frac{1-\sin^2 A}{\cos A} \Rightarrow \frac{\cos^{\cancel{2}} A}{\cancel{\cos A}}$$

$$\left[\begin{array}{l}\sin^2A+\cos^2A=1\\ \therefore 1-\sin^2A=\cos^2A\end{array}\right]$$

$\Rightarrow \cos A$

↳ (D) ✓

LP:- $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

LHS:- $\left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2 \Rightarrow \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{(1)^2-\cos^2\theta}$ $\left[\begin{array}{l}\sin^2\theta+\cos^2\theta=1\\ \sin^2\theta = 1-\cos^2\theta\end{array}\right]$

$= \dfrac{(1-\cos\theta)^{\cancel{2}}}{\cancel{(1-\cos\theta)}(1+\cos\theta)}$

$\Rightarrow \dfrac{1-\cos\theta}{1+\cos\theta}$ RHS Hence Prooved

Type #4 : Rationalise:-

LP: $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

sol: **LHS:** $\sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}} \Rightarrow \sqrt{\dfrac{(1+\sin A)^2}{(1)^2-\sin^2 A}}$

$\Rightarrow \sqrt{\dfrac{(1+\sin A)^2}{\cos^2 A}} \Rightarrow \dfrac{1+\sin A}{\cos A} \Rightarrow \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$\Rightarrow \dfrac{1}{\cos A} + \tan A$

$\Rightarrow \sec A + \tan A$ = RHS

Hence Prooved

Type #5: LHS आगे ना solve हो पाए तो RHS को Simplify करो:

LP:- $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

↳ जो Proove करना है उसमे sin/cos लाना है मतलब secA को तो तोड़ना होगा।

sol: **LHS:** $\dfrac{1+\dfrac{1}{\cos A}}{\dfrac{1}{\cos A}} = \dfrac{\dfrac{\cos A+1}{\cancel{\cos A}}}{\dfrac{1}{\cancel{\cos A}}}$

$\Rightarrow \cos A + 1$ —(I)

अब ये आगे नही solve हो रहा तो RHS करेंगे।

RHS $\dfrac{(1)^2-\cos^2 A}{1-\cos A} \Rightarrow \dfrac{\cancel{(1-\cos A)}(1+\cos A)}{\cancel{(1-\cos A)}}$

$\Rightarrow 1+\cos A$ —— (II)

We can see (I) = (II) i.e. LHS = RHS

Hence Prooved

LP:- $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

Sol: उल्टा उल्टा

LHS $\left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$\Rightarrow \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)$

$\Rightarrow \dfrac{\cos^2 A}{\cancel{\sin A}} \times \dfrac{\sin^2 A}{\cancel{\cos A}} \Rightarrow \sin A \cdot \cos A$ — I

RHS $\dfrac{1}{\tan A + \cot A} \Rightarrow \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}} \Rightarrow \dfrac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \Rightarrow \sin A \cdot \cos A$ — II

I = II i.e. LHS = RHS Hence Proved

TYPE #6: m/n/p/q type of Question:-

LP:- If $\tan\theta + \sin\theta = m$ & $\tan\theta - \sin\theta = n$, show $m^2 - n^2 = 4\sqrt{mn}$

Sol: LHS: $m^2 - n^2 \Rightarrow (m+n)(m-n)$

$= (\tan\theta + \cancel{\sin\theta} + \tan\theta - \cancel{\sin\theta}) \cdot (\cancel{\tan\theta} + \sin\theta - \cancel{\tan\theta} + \sin\theta)$

$\Rightarrow 2\tan\theta \cdot 2\sin\theta$

$\Rightarrow 4\tan\theta \cdot \sin\theta$ ⟶ इससे आगे solve करना मुश्किल ∴ RHS करो।

RHS ⇒ $4\sqrt{mn}$

$m = \dfrac{\sin\theta}{\cos\theta} + \sin\theta \Rightarrow \sin\theta\left(\dfrac{1+\cos\theta}{\cos\theta}\right)$

$\Rightarrow \tan\theta\,(1+\cos\theta)$

$\left[\dfrac{\sin\theta}{\cos\theta} = \tan\theta\right]$

$n = \dfrac{\sin\theta}{\cos\theta} - \sin\theta \Rightarrow \sin\theta\left(\dfrac{1-\cos\theta}{\cos\theta}\right)$

$\Rightarrow \tan\theta\,(1-\cos\theta)$

Now, $4\sqrt{mn} = 4\sqrt{(\tan\theta)(1+\cos\theta)(\tan\theta)(1-\cos\theta)}$

$= 4\sqrt{\tan^2\theta\,(1-\cos^2\theta)}$ ⟶ $\sin^2\theta$

$= 4\tan\theta\sin\theta$

$=$ LHS Hence Proved

TYPE #7: Miscellenious:-

LP:- If $2\sin\theta - 4\cos\theta = 0$, find $\dfrac{\tan^2\theta + 1}{\sec\theta}$

Sol:- $\cancel{2}\sin\theta = \cancel{4}^{\,2}\cos\theta$ ⟶ this was main imp. starting step

$\boxed{\tan\theta = 2}$

Now, $\tan\theta = \frac{2 \to P}{1 \to B}$

by pytha, $H^2 = (2)^2 + (1)^2$

$\boxed{H = \sqrt{5}}$

$\therefore \sec\theta = \frac{H}{B} = \frac{\sqrt{5}}{1}$

Now, $\frac{\tan^2\theta + 1}{\sec\theta} \Rightarrow \frac{(2)^2 + 1}{\sqrt{5}} \Rightarrow \frac{5}{\sqrt{5}}$ → Rationalising $\Rightarrow \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$\Rightarrow \sqrt{5}$

LP: Proove that $\cos^4\theta - \cos^2\theta = \sin^4\theta - \sin^2\theta$

sol: Firstly making the question as:

$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$ → same power वाले same side, main starting step.

Now, LHS $\Rightarrow (\cos^2\theta)^2 - (\sin^2\theta)^2$

$\Rightarrow \underbrace{(\cos^2\theta + \sin^2\theta)}_{=1}(\cos^2\theta - \sin^2\theta)$

$\Rightarrow \cos^2\theta - \sin^2\theta \Rightarrow$ RHS Hence Prooved

LP: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

concept: कुछ ना समझ आए और adding वाली equation है तो LCM ही ले लो।

LHS $\frac{(\cos A)(\cos A) + (1+\sin A)^2}{(1+\sin A)(\cos A)} \Rightarrow \frac{\overbrace{\cos^2 A + 1 + \sin^2 A}^{=1} + 2\sin A}{(1+\sin A)\cos A}$

$\Rightarrow \frac{2 + 2\sin A}{(1+\sin A)\cos A} \Rightarrow \frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cos A}$

$\Rightarrow 2\sec A = $ RHS Hence Prooved

Pyqs of this chapter also in description of same video.
Love You all