Pair of
Linear Equations in
Two Variables.

Linear Equations in Two Variables

An equation that can be put in the form

$$\mathbf{ax + by + c = 0},$$

where a, b and c are real numbers and a, b not both equal to zero is called a linear equation in two variables namely x and y. ($\mathbf{a^2 + b^2 \neq 0}$)

Examples: $2x + 3y = 5$

$x - 2y - 3 = 0$

and $x - 0y = 2$, i.e., $x = 2$

The solution for such an equation is a pair of values, one for x and one for y which further makes the two sides of an equation equal.

For example, let us substitute $x = 1$ *and* $y = 1$ *in the left hand side (LHS) of the* equation $2x + 3y = 5$. *Then*

$$\text{LHS} = 2(1) + 3(1) = 2 + 3 = 5,$$

which is equal to the right hand side (RHS) of the equation.

Therefore, $x = 1$ *and* $y = 1$ *is a solution of the equation* $2x + 3y = 5$.

Now let us substitute $x = 1$ *and* $y = 7$ *in the equation* $2x + 3y = 5$.

Then,

$$\text{LHS} = 2(1) + 3(7) = 2 + 21 = 23$$

which is not equal to the RHS.

Therefore, $x = 1$ *and* $y = 7$ *is* ***not a solution of the equation.***

Geometrical meaning of solution of a Linear Equations in Two Variables

Geometrically, what does this mean? It means that the point (1, 1) lies on the line representing the equation $2x + 3y = 5$, *and the point (1, 7) does not lie on it. So,* ***every* solution of the equation is a point on the line representing it.**

In fact, this is true for any linear equation, that is, **each solution (*x, y*) *of a* linear equation in two variables, *$ax + by + c = 0$, corresponds to a point on the* line representing the equation, and vice versa.**

1. Liner Equation

- The most general form of a linear equations is

  $ax+by+c = 0$

  Where a,b,c, are real numbers.

  and a^2+b^2 is not equal to zero

Pair of linear equations in two variables

- Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is-

  $a_1x+b_1y+c_1 = 0$

  $a_2x+b_2y+c_2= 0$

Where a_1 a_2 b_1 b_2 c_1 c_2 are real numbers

Graphical representation of…

We have studied that the geometrical (i.e., graphical) representation of **a linear equation** in two variables is a straight line.

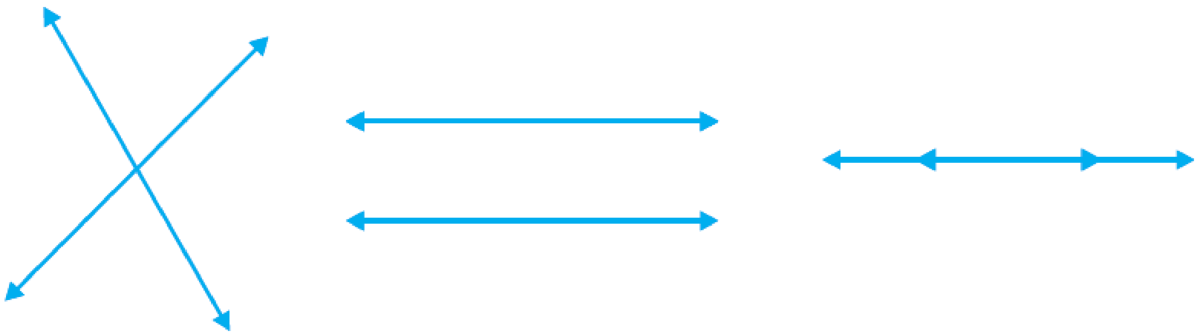
Can you now suggest what **a pair of linear equations** in two variables will look like, geometrically?

There will be two straight lines, both to be considered together.

Graphical representation of…

If we draw two lines in a plane, only one of the following three possibilities can happen:

- The two lines will intersect at one point.
- The two lines will not intersect, i.e., they are parallel.
- The two lines will be coincident.



(a) (b) (c)

Graphical representation of…

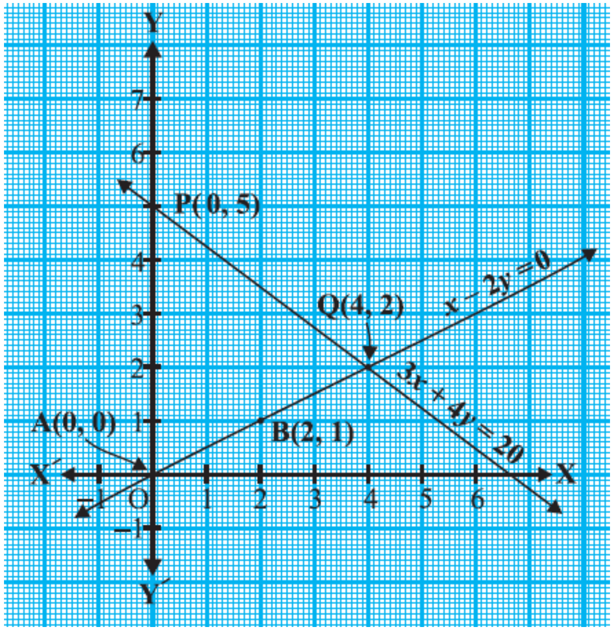
Example: $x - 2y = 0$ -----------(1)

$3x + 4y = 20$ --------(2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

x	0	2
$y = \frac{x}{2}$	0	1

x	0	$\frac{20}{3}$	4
$y = \frac{20 - 3x}{4}$	5	0	2



Graphical representation of…

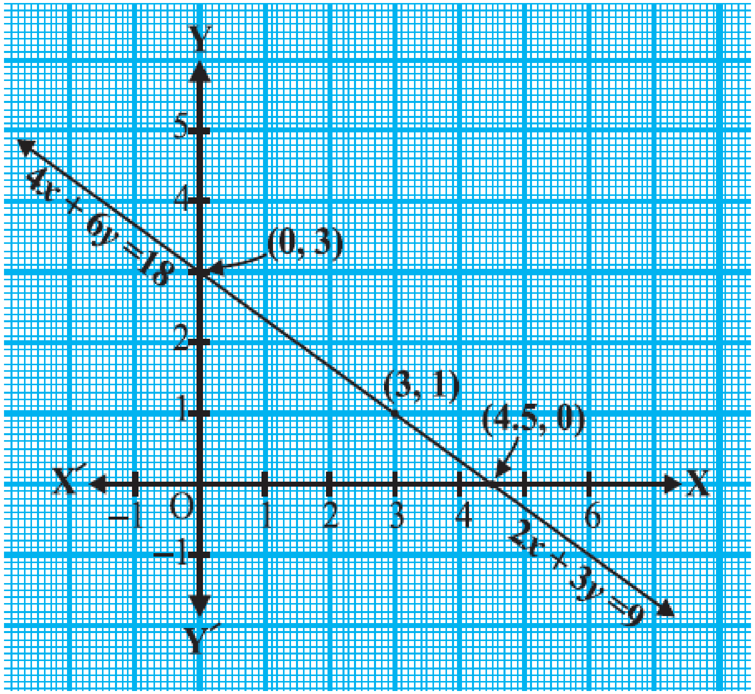
Example: $2x + 3y = 9$ ----------(1)

$4x + 6y = 18$ --------(2)

To represent these equations graphically we need at least two solutions for each equation.

x	0	4.5
$y = \dfrac{9-2x}{3}$	3	0

x	0	3
$y = \dfrac{18-4x}{6}$	3	1



Graphical representation of…

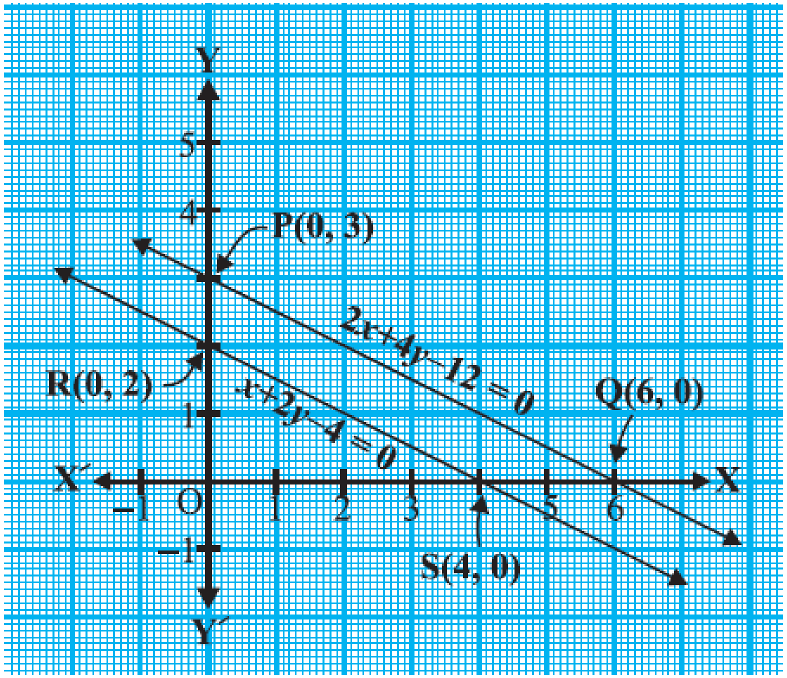
Example: $x + 2y - 4 = 0$ ---------(1)

$2x + 4y - 12 = 0$ -------(2)

To represent these equations graphically we need at least two solutions for each equation.

x	0	4
$y = \frac{4-x}{2}$	2	0

x	0	6
$y = \frac{12-2x}{4}$	3	0



Representation and Solution

- A pair of linear equations in two variables can be represented, and solved, by the :
- (i) Graphical method.
- (ii) Algebraic method.

Graphical Method

- The graph of pair of linear equations in two variables is represented by two lines.
- (i) If the lines intersect at a point, then that point, gives the unique solution of the two equations. In this case, the pair of equations is consistent.
- (ii) If the lines coincide, then there are infinitely many solutions – each point on the line being a solution. In this case, the pair of equations is dependent (consistent).
- (iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is inconsistent.

Graphical Method

Draw the graphs of the following equations:

$$x + y = 5, x - y = 5$$

(*i*) **Find the solution of the equations from the graph.**

(*ii*) **Shade the triangular region formed by the lines and the *y*-axis.** **[CBSE 2011]**

$$x + y = 5 \quad ...(i)$$

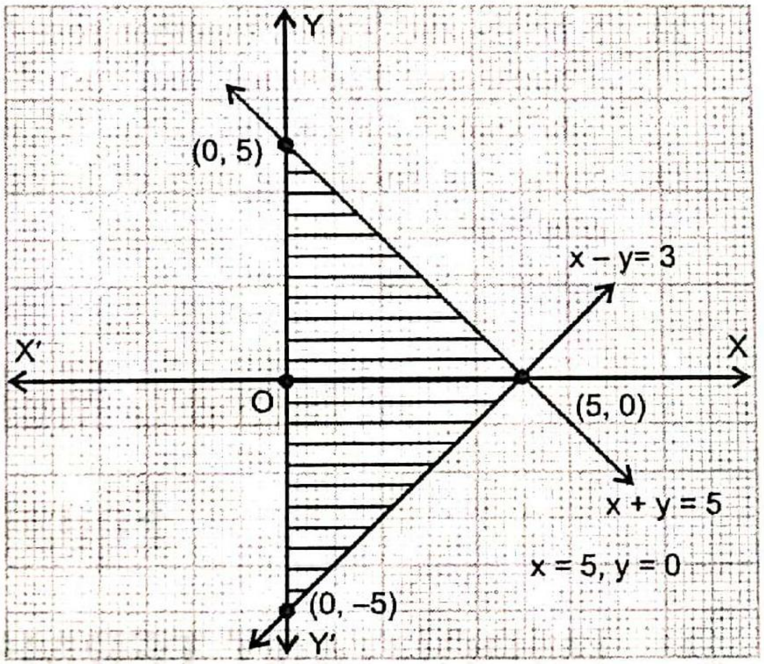
Table for Eq. (*i*) is

x	5	0	3
y	0	5	2

$$x - y = 5 \quad ...(ii)$$

Table for eq. (*ii*)

x	5	0	3
y	0	–5	–2



(*i*) Point (5, 0) is common in both the lines. Hence, $x = 5$ and $y = 0$ is the solution of the equations.

(*ii*) The triangle is shaded in the diagram.

Graphical Method

Solve the equations graphically:

$$2x + y = 2; 2y - x = 4$$

What is the area of the triangle formed by the two lines and the line $y = 0$? **[CBSE 2011]**

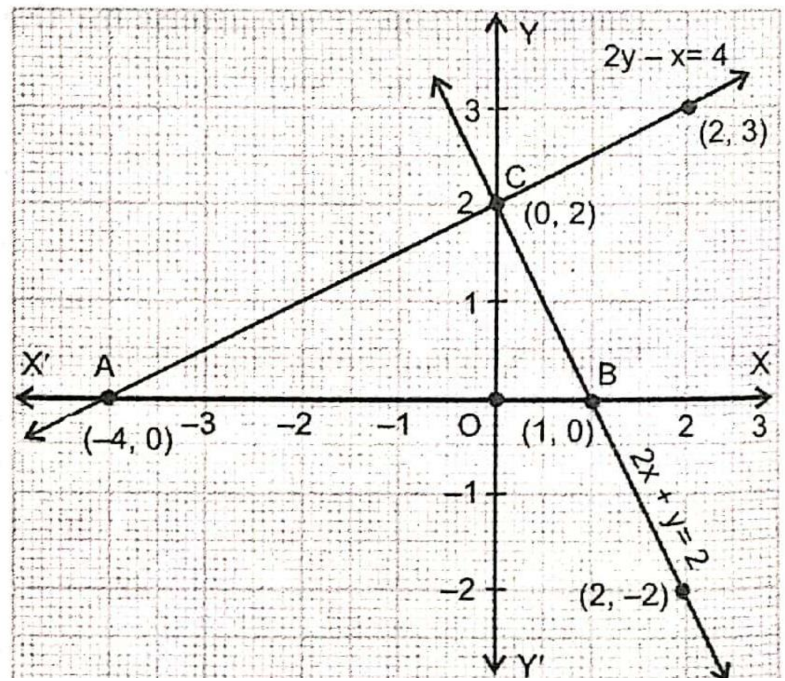
$2x + y = 2$ ... (i), $2y - x = 4$ ... (ii)

From (i), $2x + y = 2$

x	1	0	2
y	0	2	–2

From (ii), $2y - x = 4$

x	0	–4	2
y	2	0	3



$$\text{Area } \Delta = \frac{1}{2} AB \times CO$$

$$= \frac{1}{2} \times 5 \times 2$$

$= 5$ square units.

Graphical Method

Draw the graphs of the equations

$$4x - y - 8 = 0 \text{ and } 2x - 3y + 6 = 0$$

Also, determine the vertices of the triangle formed by the lines and x-axis.

$4x - y - 8 = 0$ and

$$-y = -4x + 8,$$

$$\Rightarrow \quad y = 4x - 8,$$

Solution table for $4x - y - 8 = 0$ is

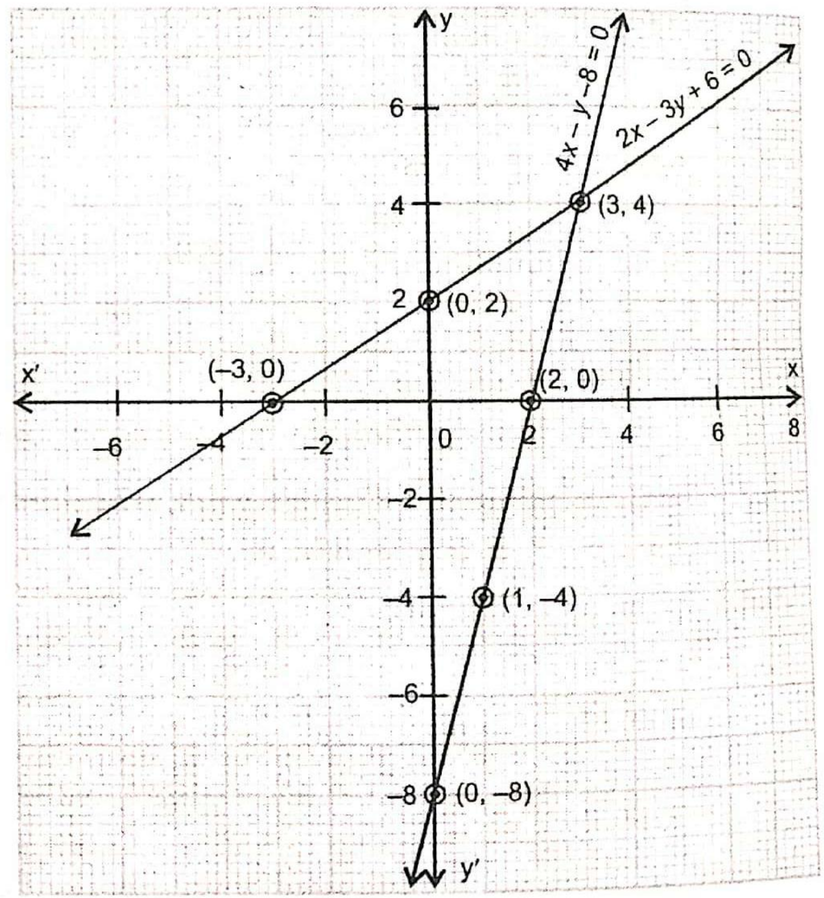
x	0	1	2
y	–8	–4	0

$2x - 3y + 6 = 0; \quad -3y = -2x - 6$

$3y = 2x + 6$

Solution table for $2x - 3y + 6 = 0$ is

x	0	3	–3
y	2	4	0



Vertices of the triangle formed by lines and x-axis are (2, 0), (3, 4) and (–3, 0).

Graphical Method

Draw the graphs of the equations

$$4x - y - 8 = 0 \text{ and } 2x - 3y + 6 = 0$$

Also, determine the vertices of the triangle formed by the lines and x-axis.

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Solution table for $4x - y - 8 = 0$ is

x	0	1	2
y	-8	-4	0

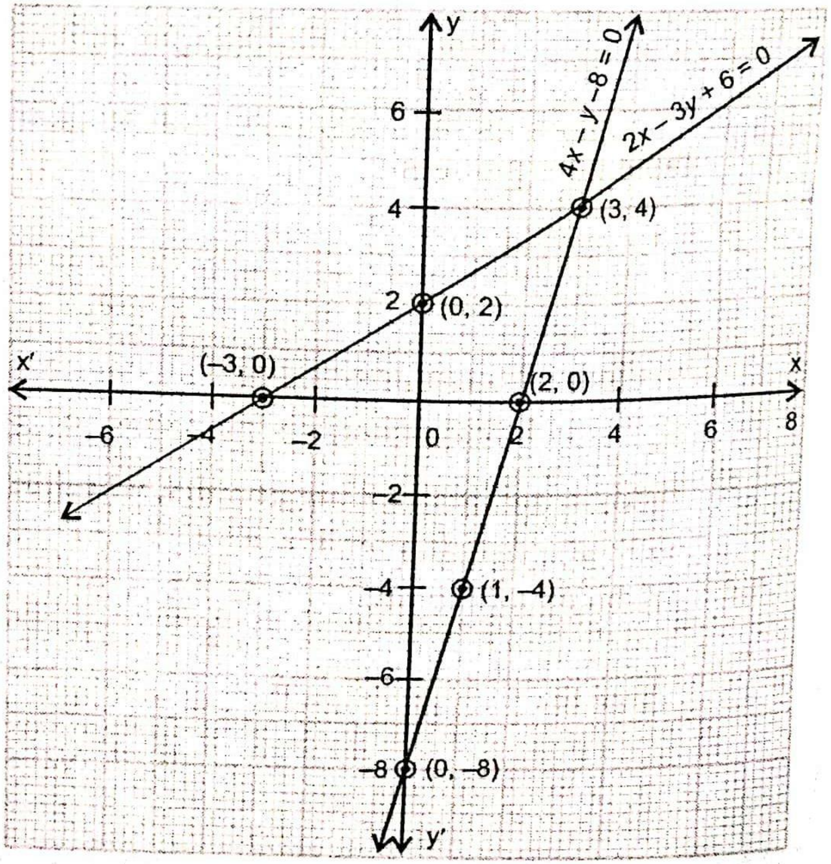
$2x - 3y + 6 = 0;\quad -3y = -2x - 6$

$3y = 2x + 6$

Solution table for $2x - 3y + 6 = 0$ is

x	0	3	-3
y	2	4	0

Vertices of the triangle formed by lines and x-axis are $(2, 0)$, $(3, 4)$ and $(-3, 0)$.



Graphical Method

Solve the following system of linear equations graphically:

$$3x - 2y - 1 = 0; \quad 2x - 3y + 6 = 0$$

Shade the region bounded by the lines and x-axis.

$3x - 2y - 1 = 0 \Rightarrow -2y = -3x + 1$

$\Rightarrow \; 2y = 3x - 1 \;\Rightarrow\; y = \dfrac{3x-1}{2}$

The solution table for $3x - 2y - 1 = 0$

x	0	1	–1
y	–1/2	1	–2

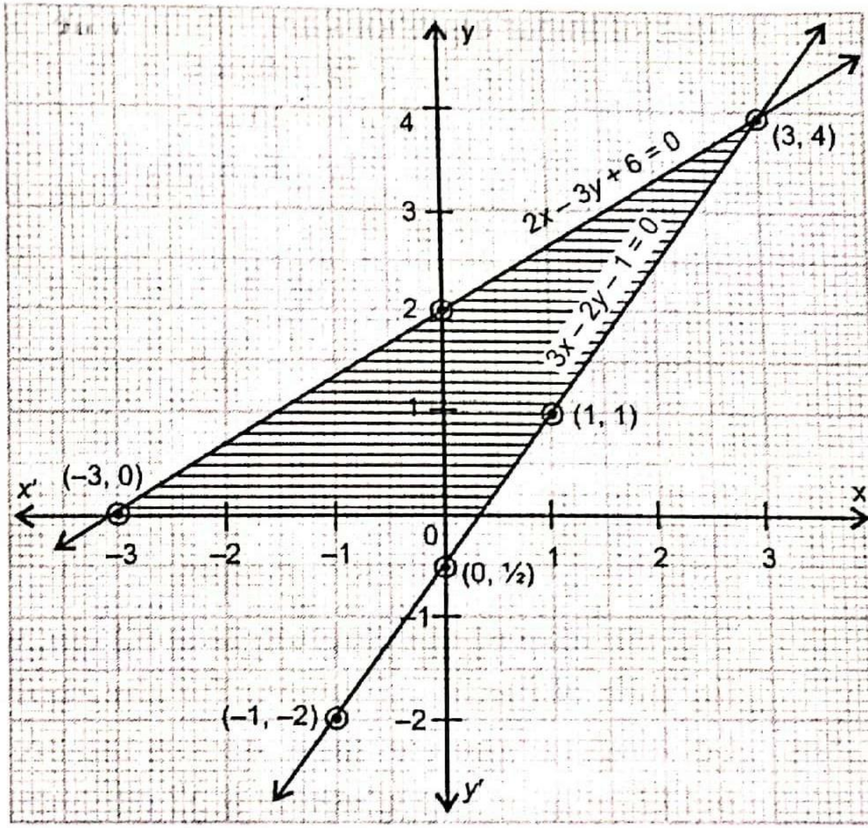
and $2x - 3y + 6 = 0$

$\Rightarrow \; -3y = -2x - 6 \Rightarrow 3y = 2x + 6$

$\Rightarrow \; y = \dfrac{2x+6}{3}$

The solution table for $2x - 3y + 6 = 0$

x	0	3	–3
y	2	4	0



Plotting the points on graph, we get the shaded region.

Algebraic Methods:

- The following methods for finding the solution (s) of a pair of linear equations :
- (i) Substitution Method
- (ii) Elimination Method
- (iii) Cross-multiplication Method.

Method of substitution

- (i) Find y in terms of x from one of the two equations.
- (ii) Substitute this value of y in the other equation. Solve this equation for x.
- (iii) Substitute this value of x in any of the given equations and sove it for
Alternatively we may find x in terms of y. Then we shall get value of y in step (ii) above and value of x in step (iii) above.

Method of elimination

- (i) Multiply the given equations by some suitable constants so as to make the coefficients of one of the variables numerically equal.
- (ii) Add or subtract according as the like terms having same coefficients are opposite in sign or of the same sign respectively.
- (iii) Solve the equation obtained in step (ii) in one variable.
- (iV) Substitute the value of this variable in any of the two equations and solve for the second variable.

Solving word problems

- To solve a word problem, the following steps are suggested:
- (i) The problem should be read carefully.
- (ii) The unknowns in the problem should be denoted by x and y.
- (iii) Translate the word problem to algebraic equations.
- (iV) Solve the equations to get the values of unknown variables x and y.
- (V) Write the answer in the desired form.

Cross multiplication method

- The Cross multiplication rule to solve the pair of equations

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2= 0$ is given by

$x/b_1c_2-b_2c_1 = Y/c_1a_2-c_2a_1 = 1/a_1b_2-a_2b_1$

or

$x= b_1c_2-b_2c_1 /a_1b_2-a_2b_1$

and

$y=c_1a_2-c_2a_1 /a_1b_2-a_2b_1$.

How to find nature of equation

Let $a_1x+b_1y+c_1 = 0$ and
$a_2x+b_2y+c_2= 0$

- (a) Then, the pair of equation has *no Solution* if
  a1/a2 = b1/b2 ≠ c1/c2

- (b The pair of equations has *Unique Solution* if
- a1/a2 ≠ b1/b2
- (c) The pair of equations has *an infinite number of Solution* if
  a1/a2 = b1/b2 = c1/c2