Classification Metrics

Evaluation Metrics - Video

Data $\begin{bmatrix} \text{Cat} \\ \text{Dog} \end{bmatrix}$ (Labels) + $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix}_{3 \times 4}$ (Feat)

Agenda:

1. Accuracy : Problems
2. Confusion Matrix
3. Precision / Recall
4. F1 Score

1. Accuracy

Accuracy $= \frac{C}{C+W} = \frac{C}{T}$

Dumb Model :- Always Predict 0 (Non-Spam)

Test (Few rows) → Pred = 0, Pred = 0, Pred = 0 → 100% Accuracy

Production (Real) → Pred = 0, Pred = 1, Pred = 0 → 90% Accuracy

Acc (Dumb Model) $= \frac{90}{100} = \boxed{90\%}$ → Misleading acc.

Problem : Accuracy :

1. Imbalanced data : Acc. is about useless
2. Acc. doesn't give $\boxed{\text{Performance}}$

2. Confusion Matrix

Binary : Yes/Success / Positive / Non-Zero

Binary : No/Failure / Negative / Zero

Predictions

	Non-Spam	Spam
Actual Non-Spam	✓	✗
Actual Spam	✗	✓

Actual

Predicted	0	1
0	TN	FP
1	FN	TP

True Positive

True Negative

False Positive

False Negative

Actual Positive (1) → TP, FN

Actual Negative (0) → TN, FP

N

Predicted Negatively (Predicted 0)

TN + FN

P

Predicted Positively (Predicted 1)

TP + FP

T P

Is your prediction ?

1. True
2. False

TP : actual = 1 , Pred = 1 } correct pred. ✓✓

TN : actual = 0 , Pred = 0

FP : actual = 0 , Pred = 1 } Errors (Type 1 error)

FN : actual = 1 , Pred = 0

Q. Dumb Model : Confusion Matrix ?

Test Data : (Total 100 rows)

90 Non-Spam (0)

10 Spam (1)

Dumb Prediction

Always : Non-Spam

(Always 0)

(Majority)

Actual

Predicted	0	1
0	90	0
1	10	0

→ 90 True Negatives & 10 False Negatives

(N) Actually → TN+FP

90 + 0

(P) Actually → FN+TP

10 + 0

$\hat{P}$ = Predicted Positive

$\hat{N}$ = Predicted Negative

Q. Confusion Matrix : Ideal Model ?

90 Non-Spam

10 Spam

Actual

Predicted	0	1
0	90	0
1	0	10

Accuracy from CM → Sum of diagonal / Total

$\Rightarrow \frac{TP + TN}{TP + TN + FP + FN} \Rightarrow \frac{\text{Sum of Diagonal}}{\text{Total}}$

3. Precision

1. Precision = When model predicts positive, how often is it correct ?
2. Actually positive prediction rate from your strategy → FP

Total Pos. Pred. are our Judgements.

Actually → FP

Precision → $\boxed{TP / T_{Pred}}$

How many times dumb strategy not of our use Predicted Positive (Spam)

Actual

Predicted	0	1
0	TN	FN
1	FP	TP

→ $\frac{TP}{\hat{P}} = \frac{TP}{TP+FP}$

Predicted → 5 spam emails

out of 5 emails only 2 actually spam.

$P_r = \frac{TP}{TP+FP} = \frac{2}{5} = 40\%$

TP: 2 FP: 3

Precision $\in [0, 1]$

Low on Precision

Spam → $\boxed{\text{Not Spam}}$

Fraudulent → $\boxed{\text{Not Fraud}}$

$\Rightarrow \frac{TP}{TP} = 1$

4. Recall

1. Actually → Actually positive or Correct ones (FP)
2. Actually → Positive Actually Correct ones Actually → (FP+FN)

(Negatives)

T : Correct

N : Not Correct

Actually → FN ✓✓

Predicted → TP ✓✓

→ Recall = $\frac{TP}{P}$

How many Correct Actually Actual Correctly.

Total Actual Correct Actually Actual Correctly.

Recall = $\frac{TP}{P}$

$\boxed{\text{Recall} = \frac{TP}{TP+FN}}$

Actual Positive

Actual ✓✓ Correct Total → $\boxed{P = TP + FN}$

Image Recall $\in [0, 1]$

Fraud Case.

Fraud → 1 Fraud → 0 : Non-Fraud

$\boxed{TP}$ $\boxed{FN}$

Good FP : Not Fraud, Predicted as Fraud.

Bad FN : A Fraud Transac. Predicted as Not.

$\boxed{\text{Recall}}$

5. More metrics c.m.

Predictivity

↑ TPR (True Positive Rate) → $\frac{TP}{P} = \frac{TP}{TP+FN}$

Sensitivity

↑ TNR (True Negative Rate) → $\frac{TN}{N} = \frac{TN}{TN+FP}$

↓ FPR (False Positive Rate) → $\frac{FP}{N} = \frac{FP}{TN+FP}$

↓ FNR (False Negative Rate) → $\frac{FN}{P} = \frac{FN}{TP+FN}$

6. F1 Score

(1) $\hat{P}$: ability to enjoy the time.

(2) N : ability (intended to enjoy)

Trade-off FP : "Bad debt" → loss } Both are Imp.

and FN : opportunity loss → loss

	P_r	R_c
M1	0.5	0.8
M2	0.85	0.10
M3	0.7	0.9

"Cannot choose"

Simple Avg → $\frac{P_r + R_c}{2}$

	P_r	R_c	Arth Mean $\left(\frac{P_r+R_c}{2}\right)$
M1	0.7	0.2	0.7
M2	0.9	0.5	0.7

Harmonic Mean $HM(5,9) = \frac{2xy}{x+y}$

→ HM > Arth Mean

$HM(P, R) = \frac{2xy}{x+y} = \frac{2}{\frac{1}{x} + \frac{1}{y}}$

$\boxed{F1\text{-Score} = \frac{2 \cdot P \cdot R}{P + R}}$

	P_r	R_c	Arth Mean	F1-Score = HM(Pr,Rc)
M1	0.7	0.2	0.7	0.7
M2	0.9	0.5	0.7	0.64

Performance

$F_1 = 1$ $P_r = 0$ $R_c = 1$

$F_1 = 0$ $F_1 = 0$

Range $F_1 \in [0, 1]$

$\boxed{P_r = 1, R_c = 1 \Rightarrow F_1 = 1}$

$2^{10} = \boxed{\text{Wrong}}$ $\boxed{\text{Right}}$ ★

$\frac{1}{y} = \frac{1}{\frac{x_1 + x_2 + x_3 + \dots + x_n}{n}}$

$= \left[\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}\right]$ div. of Arth Mean.

$\boxed{\text{Harmonic Mean}}$

$\boxed{\text{Harmonic Mean} \le \text{Arithmetic Mean}}$