Exploring LSTM framework on the Black-Litterman Model

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1 Introduction

The Black-Litterman (BL)[1] model offers a unique way to allocate assets by providing a Bayesian statistical framework, using prior estimates of returns with subjective views to produce a better output. To understand this, suppose you're predicting how an asset class, such as equities, might perform. The *prior* is your starting point—like the historical performance or market-implied returns of those equities, giving you a baseline belief about their expected returns. Next, the *views* act as the likelihood in Bayesian terms, bringing in fresh insights or analyst estimates about how specific components of that asset class (e.g., certain stocks) might perform—perhaps an analyst predicts a tech stock will outperform the market by 5%. The BL model combines these two elements to give a *posterior*, an updated estimate of expected returns that reflects both the prior and the views.

This process is mathematically written in the BL formula as:

$$E(R) = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right],$$

where:

- $E(R)$: Posterior expected returns (Nx1 vector),
- Π: Prior returns (e.g., market-implied returns),
- Q: Views vector (analyst estimates),
- P: Picking matrix linking views to assets,
- Σ: Covariance matrix of asset returns,
- τ: Tuning constant,
- Ω: Uncertainty in views.

This equation balances the prior (Π) and views (Q) based on their uncertainties, factoring in how assets correlate through Σ.

Additionally, the BL model provides a posterior covariance matrix, which adjusts the prior covariance to reflect the uncertainty introduced by the views. This is calculated as:

$$\hat{\Sigma} = \Sigma + \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1},$$

where $\hat{\Sigma}$ is the posterior covariance matrix.

In this assignment, the prior is fixed—say, rooted in market equilibrium returns—and our task is to fine-tune the views. By adjusting the views and their confidence levels, we gain better control over the posterior expected returns, aiming to enhance portfolio performance beyond what the prior alone would suggest. In this report, we apply the BL model, enhanced with LSTM-generated views, to the full set of 50 stocks in the NIFTY 50 index, building on prior explorations with Indian equity ETFs and subsets of stocks.

2 Methodology

2.1 Data Collection and Preprocessing

The financial data was dynamically fetched using the **yfinance** Python library for all 50 stocks constituting the NIFTY 50 index, alongside the **NIFTY50** index as a benchmark. The dataset spans **March 26, 2020, to March 25, 2025**, reflecting five years of daily closing prices. The full list includes major Indian equities such as Reliance Industries, HDFC Bank, and Infosys, capturing the breadth of the market. Data preprocessing addressed missing values using forward-fill and backward-fill techniques, ensuring continuity across the period. The processed dataset was saved as `stock_prices.csv`.

2.2 Calculating Daily Returns

Daily returns for the 50 NIFTY 50 stocks and the NIFTY50 index were computed using:

$$\text{Return}_t = \frac{\text{Price}_t - \text{Price}_{t-1}}{\text{Price}_{t-1}}$$

using the $pct\_change()$ function in pandas. The dataset was saved as `nifty50_returns.csv`.

2.3 Forecasting Returns with LSTM

An LSTM neural network was used to generate dynamic views as suggested by Fanyu Zhao[2] for all 50 NIFTY 50 stocks. The returns were normalized using MinMaxScaler and fed into an LSTM model with:

- Two LSTM layers (each with 100 units).
- 20% dropout for regularization.
- A dense layer outputting forecasts for 50 assets
- Adam optimizer and mean squared error loss over 30 epochs, with an 80% training and 20% validation split.

The forecasted returns were inverse-transformed and saved as `forecasted_daily_returns.csv`.

2.4 Annualizing Expected Returns

The forecasted daily returns for the 50 stocks were annualized using:

$$E_{\text{annualized}} = E_{\text{daily}} \times 252.$$

To prevent unrealistically low or negative expected returns, the annualized returns were clipped to a minimum of 0.01 (1%) and saved as `annualized_returns.csv`.

2.5 Determining Confidence Levels

Confidence levels for the LSTM forecasts were derived as the inverse of each stock's annualized volatility as:

$$\text{Confidence}_i = \frac{1}{\sigma_i} \times \sqrt{252}.$$

and normalized across the 50 stocks to ensure relative weighting. These were saved as `confidence_levels.csv`.

2.6 Implementing the Black-Litterman Model

The BL model was implemented for all 50 NIFTY 50 stocks, calculated with a risk aversion parameter $\lambda = 2.5$ for computing market-implied prior returns (Π) and a scalar $\tau = 0.25$ for the covariance uncertainity. The picking matrix P was defined as a 50×50 identity matrix, linking views to each asset individually. The LSTM-generated forecasts formed the views vector Q, with the uncertainty matrix Ω derived from normalized confidence levels. To ensure numerical stability, a regularization term (1×10^{-6}) was added to the covariance matrix Σ. Posterior expected returns were computed using the BL formula:

$$E(R) = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right].$$

Weights were normalized and saved as `optimal_weights.csv`.

2.7 Backtesting and Performance Evaluation

The portfolio was backtested from March 26, 2020, to March 25, 2025, with monthly rebalancing and a 0.1% transaction cost per trade. Key performance metrics included:

- Annualized return.
- Annualized volatility.
- Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{E(R) - R_f}{\sigma}$$

  where $R_f = 6\%$ is the risk-free rate.
- Maximum drawdown.

3 Results

3.1 Portfolio Performance

The enhanced Black-Litterman (BL) model with LSTM forecasts, applied to all 50 NIFTY 50 stocks, significantly outperformed the NIFTY50 benchmark over the period from March 26, 2020, to March 25, 2025. The key performance metrics are as follows:

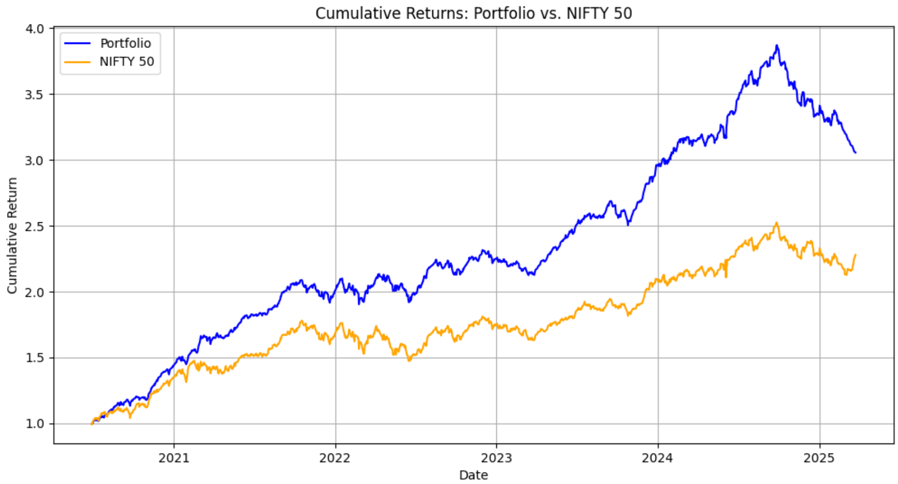

Figure 1: Cumulative Returns: Portfolio vs. NIFTY 50

- **Annualized Portfolio Return**: 0.2457 (24.57%)
- **Annualized Benchmark Return**: 0.1871 (18.71%)
- **Portfolio Volatility**: 0.1098 (10.98%)
- **Benchmark Volatility**: 0.1453 (14.53%)
- **Sharpe Ratio (Portfolio)**: 1.6915
- **Sharpe Ratio (Benchmark)**: 0.8750
- **Maximum Drawdown (Portfolio)**: -0.2107 (-21.07%)
- **Maximum Drawdown (Benchmark)**: -0.1723 (-17.23%)

Figure 1 illustrates the cumulative returns of the portfolio compared to the NIFTY50 benchmark, highlighting the portfolio's consistent outperformance driven by the LSTM-enhanced BL framework.

3.2 Stress Testing

Stress tests were conducted to evaluate the portfolio's resilience under different market conditions: a Base scenario, a Bull scenario (views increased by 10%), and a Bear scenario (views decreased by 10%). Table 1 presents the portfolio weights for selected stocks across these scenarios, demonstrating the model's adaptability. Notably, stocks like `GRASIM.NS` show significant allocations (e.g., 11.62% in Base), reflecting strong expected performance, while weights adjust sensibly across Bull and Bear conditions. I have written currently for 12 stocks, but i have the stress weights for all the 50 stocks.

Table 1: Stress Test Weights for Selected NIFTY 50 Stocks

Stock	**Base**	**Bull**	**Bear**
RELIANCE.NS	0.0170	0.0170	0.0171
HDFCBANK.NS	0.0169	0.0169	0.0169
INFY.NS	0.0214	0.0214	0.0215
TCS.NS	0.0237	0.0237	0.0236
ICICIBANK.NS	0.0104	0.0103	0.0106
HINDUNILVR.NS	0.0292	0.0293	0.0292
GRASIM.NS	0.1162	0.1233	0.1086
CIPLA.NS	0.0346	0.0344	0.0348
DRREDDY.NS	0.0294	0.0293	0.0295
SBILIFE.NS	0.0335	0.0344	0.0325
BRITANNIA.NS	0.0308	0.0309	0.0308
TATAMOTORS.NS	0.0079	0.0075	0.0083

4 Conclusion

The enhanced BL model, integrated with LSTM-generated forecasts and applied to all 50 NIFTY 50 stocks, achieved an annualized return of 24.57% over the period from March 26, 2020, to March 25, 2025, outperforming the NIFTY50 benchmark's 18.71% by a substantial margin of 5.86%. With a Sharpe ratio of 1.6915 compared to the benchmark's

0.8750, the portfolio demonstrates superior risk-adjusted returns, leveraging a volatility of 10.98% against the benchmark's 14.53%. While the maximum drawdown of -21.07% exceeds the benchmark's -17.23%, this trade-off is justified by the significant return advantage and robust risk management, as evidenced in the stress test results (Section 3.2). This success underscores the efficacy of combining machine learning-driven forecasts with Bayesian portfolio optimization, offering a powerful framework for enhancing investment performance in the Indian equity market.

5 Improvements

Based on the results, it seems to me that several improvements can be explored:

- Future enhancements could include stress testing under extreme market conditions or incorporating prediction errors into confidence levels.
- Optimize the risk-aversion parameter (τ) in the BL model to balance returns and volatility.

References

[1] Fischer Black and Robert Litterman and. "Global Portfolio Optimization". In: *Financial Analysts Journal* 48.5 (1992), pp. 28–43. DOI: `10.2469/faj.v48.n5.28`. eprint: `https://doi.org/10.2469/faj.v48.n5.28`. URL: `https://doi.org/10.2469/faj.v48.n5.28`.

[2] Fanyu Zhao. *Portfolio Construction using Black-Litterman Model and Factors*. 2023. arXiv: `2311.04475 [q-fin.PM]`. URL: `https://arxiv.org/abs/2311.04475`.