

Tutorial - 08

UIACSOO9

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a) $L_1 = \{ wcw \mid w \text{ is in } (a/b)^*\}$

→ Given language is not CFG, because there is no midpoint as no PDA can be designed to accept it.

e.g. $s \xrightarrow{} \underline{abaab} c \underline{abaab} \xrightarrow{} \text{no midpoint possible}$

b) What is left recursive grammar? Remove left recursion from following.

i) $E \xrightarrow{} E + T \mid T \Rightarrow E \xrightarrow{} T E'$
 $T \xrightarrow{} T x F \mid F \quad E' \xrightarrow{} + T E' \mid \epsilon$
 $F \xrightarrow{} id \quad T \xrightarrow{} F T'$
 $\qquad\qquad\qquad T' \xrightarrow{} x F T' \mid \epsilon$
 $\qquad\qquad\qquad F \xrightarrow{} id$

ii) $S \xrightarrow{} (L) \mid a \quad \rightarrow \quad \begin{cases} S \xrightarrow{} (L) \mid a \\ L \xrightarrow{} S L' \end{cases}$
 $L \xrightarrow{} L S \mid S \quad \curvearrowleft \quad \begin{cases} L \xrightarrow{} S L' \mid \epsilon \\ S \xrightarrow{} (L) \mid a \end{cases}$
 $\qquad\qquad\qquad \begin{cases} L \xrightarrow{} (L) L' \mid a L' \\ L' \xrightarrow{} S L' \mid \epsilon \end{cases}$

iii) $S \xrightarrow{} A \quad \rightarrow \quad S \xrightarrow{} A$
 $A \xrightarrow{} Aa \mid Aa \mid aB \mid ac \quad A \xrightarrow{} aBA' \mid aCA'$
 $B \xrightarrow{} bBC \mid f \quad A' \xrightarrow{} dA' \mid eA' \mid \epsilon$
 $\qquad\qquad\qquad B \xrightarrow{} bBC \mid f$

iv) $X \xrightarrow{} XSB \mid Sa \mid b \quad \rightarrow \quad X \xrightarrow{} SSgX' \mid bX'$
 $S \xrightarrow{} gB \mid Xa \mid a \quad X' \xrightarrow{} SBX' \mid \epsilon$
 $\qquad\qquad\qquad S \xrightarrow{} XgS' \mid aS'$
 $\qquad\qquad\qquad S' \xrightarrow{} BS' \mid \epsilon$

$$\begin{array}{l}
 \text{e)} \quad S \rightarrow aB \mid a \mid Sd \mid Se \quad \rightarrow \quad S \rightarrow aBS' \mid a \mid S \\
 \quad \quad B \rightarrow bBc \mid f \quad \quad \quad \quad S' \rightarrow dS' \mid eS' \mid \epsilon \\
 \quad \quad C \rightarrow g \quad \quad \quad \quad B \rightarrow bBc \mid f \\
 \quad \quad \quad \quad \quad \quad \quad C \rightarrow g
 \end{array}$$

Left recursion \rightarrow a grammar is left recursive if it has a non-terminal A such that there is a derivation $A \rightarrow A\alpha$.

- \rightarrow Top down parser can't handle them as left most symbol is never a non-terminal, it will be stuck in an infinite loop.
- \rightarrow A simple rule for direct left recursion elimination.

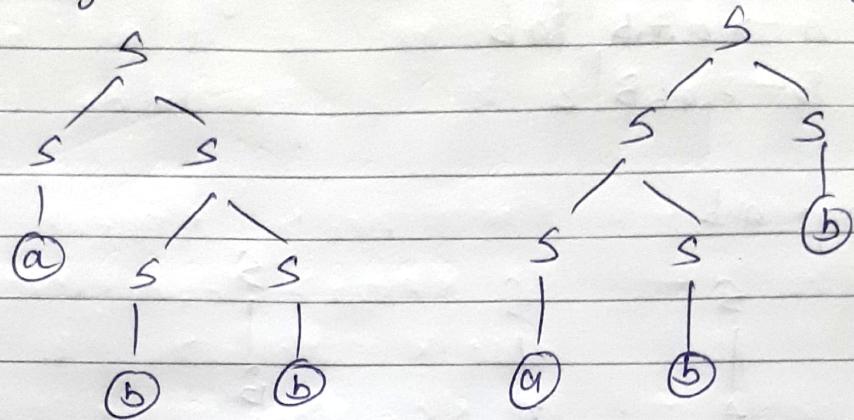
$$\begin{array}{ll}
 A \rightarrow A\alpha \mid B & \text{or } A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_n \mid \beta_1 \mid \dots \mid \beta_m \\
 \hookrightarrow A \rightarrow BA' & \hookrightarrow A \rightarrow B_1A' \mid B_2A' \mid \dots \mid B_mA' \\
 A' \rightarrow \alpha A' \mid \epsilon & A' \rightarrow \alpha_1A' \mid \alpha_2A' \mid \dots \mid \alpha_mA' \mid \epsilon
 \end{array}$$

Q-2 Define ambiguity. Is following grammar ambiguous or not? Justify your answer and correct grammar if necessary.

- \rightarrow Ambiguity \rightarrow We call a grammar ambiguous if a string of terminal symbols can be reached by two different derivation sequences i.e., it can have more than one parse tree.
- \rightarrow It makes design of parser difficult because we don't know which parse tree will be discovered.

$$\begin{array}{l}
 \text{D) } S \rightarrow SS \\
 \quad \quad S \rightarrow a \\
 \quad \quad S \rightarrow b
 \end{array}$$

By deriving abb we can say it is ambiguous



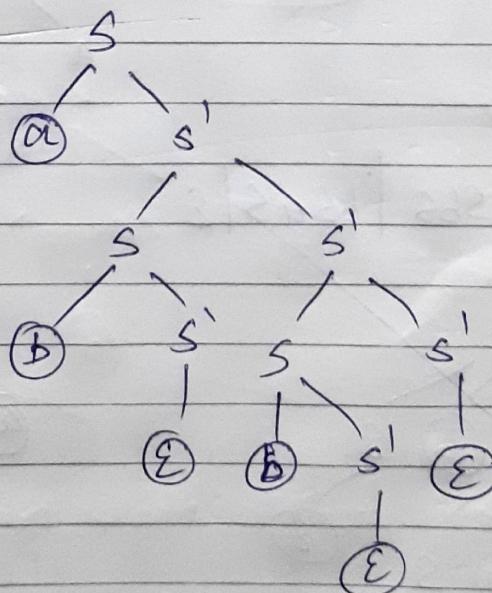
Remove left recursion

$$\therefore S \rightarrow aS' \mid bS'$$

$$S' \rightarrow SS' \mid \epsilon$$

now the above grammar is un-ambiguous.

Deriving abb



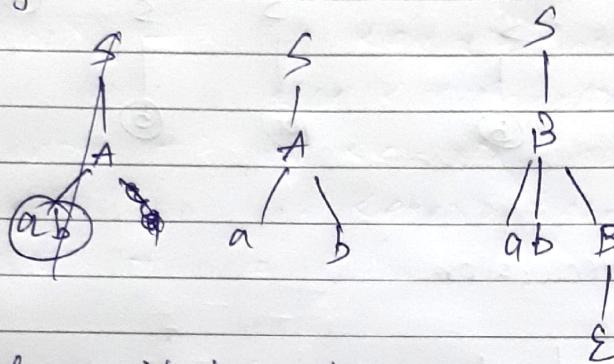
is the only possible parse tree possible.

$$2) \quad S \rightarrow A \mid B$$

$$A \rightarrow aAb \mid ab$$

$$B \rightarrow abB \mid \epsilon$$

String ab

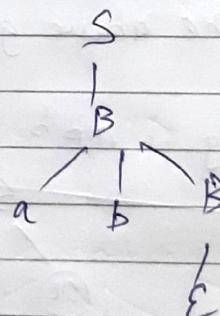


Therefore it is ambiguous.

$$S \rightarrow aAb \mid B$$

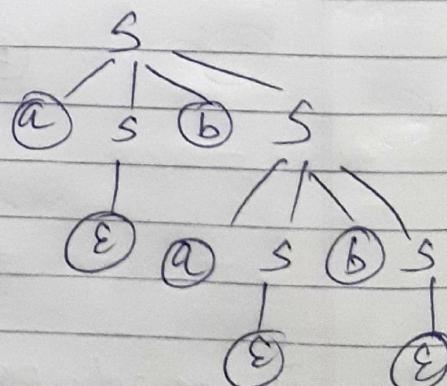
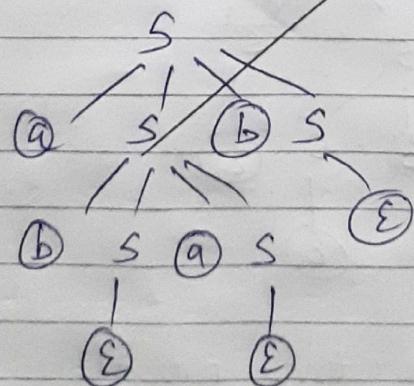
$$A \rightarrow aAb \mid ab$$

$$B \rightarrow abB \mid \epsilon$$



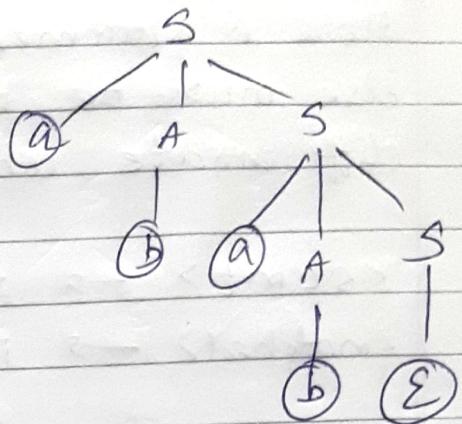
$$3) \rightarrow S \rightarrow aSbS \mid bSaS \mid \epsilon$$

abab

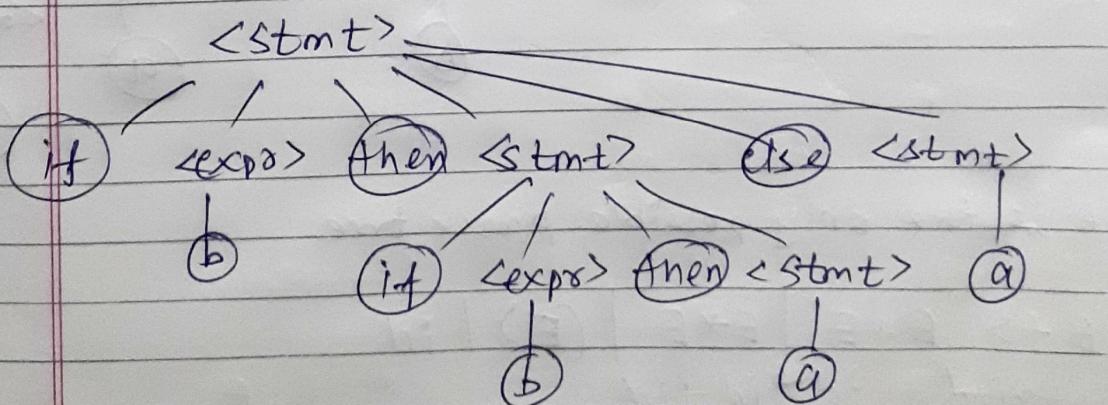
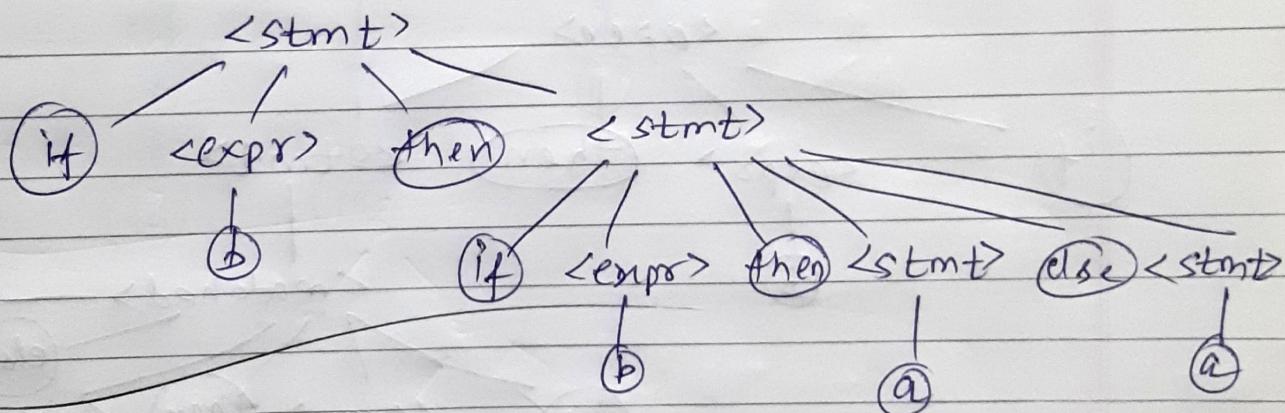


Hence grammar is ambiguous

$$\begin{array}{l} S \rightarrow aAS \mid bBS \mid \varepsilon \\ A \rightarrow aAb \mid b \\ B \rightarrow bBb \mid a \end{array}$$



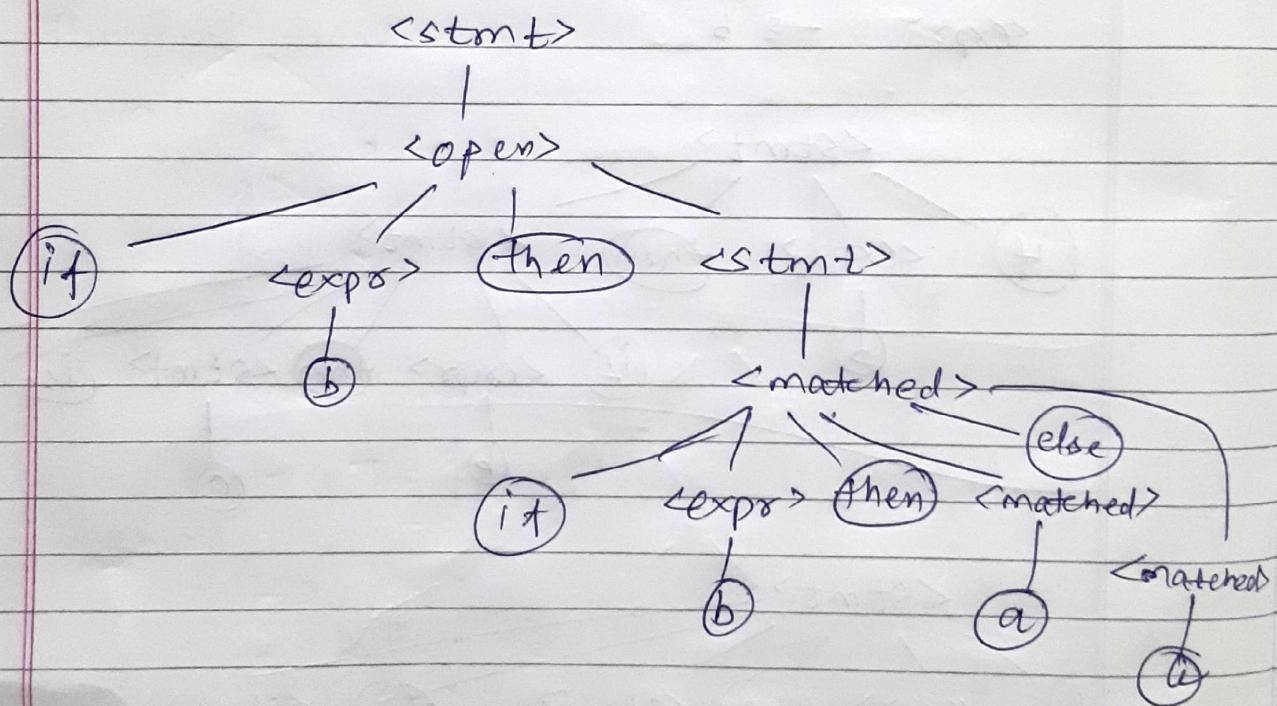
4) $\langle \text{stmt} \rangle \rightarrow \begin{cases} \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt} \rangle \\ \quad \quad \quad | \\ \quad \quad \quad \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt} \rangle \text{ else } \langle \text{stmt} \rangle \\ \quad \quad \quad | \\ \quad \quad \quad \text{a} \\ \quad \quad \quad | \\ \langle \text{expr} \rangle \rightarrow b \end{cases}$



\vdash Grammar is ambiguous, for string

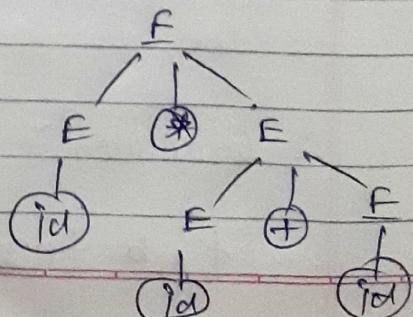
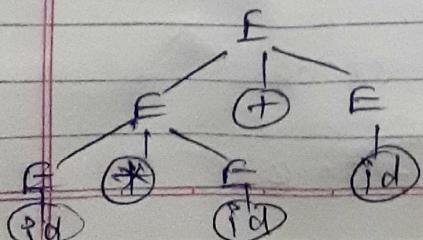
if b then if b than a else a

Here a statement appearing between then and else must be matched as it will be difficult to differentiate which 'if' following 'else' is for.

$$\begin{aligned}
 <\text{stmt}> &\rightarrow <\text{matched}> | <\text{open}> \\
 <\text{matched}> &\rightarrow \text{if } <\text{expr}> \text{ then } <\text{matched}> \\
 &\quad \text{else } <\text{matched}> | a \\
 <\text{open}> &\rightarrow \text{if } <\text{expr}> \text{ then } <\text{stmt}> \\
 &\quad | \text{if } <\text{expr}> \text{ then } <\text{matched}> \text{ else } <\text{open}> \\
 <\text{expr}> &\rightarrow b
 \end{aligned}$$


It is the only possible parse tree

$$\begin{array}{l}
 (5) \quad E \rightarrow E+E \mid E \times E \mid (E) \mid \text{id} \\
 \rightarrow \text{strand id} \neq \text{id+id}
 \end{array}$$



$$\begin{array}{l} E \rightarrow E + E \mid F \\ F \rightarrow F * F \mid G \\ G \rightarrow id \mid (E) \end{array}$$

Here \oplus will have less precedence than $*$

