

Codechef Problem : QNUMBER

Brij Raj Kishore

1 Type 1 Query

Given N , you have to answer Q queries, in each query you will be given a number K , you have to find count of common divisors of N and K

$$N \leq 10^{12}$$

$$K \leq 10^{12}$$

$$Q \leq 10^5$$

12 3

5

8

6

$Q1 : 1\{1\}$

$Q2 : 3\{1, 2, 4\}$

$Q3 : 4\{1, 2, 3, 6\}$

Naive Approach :

1. Generate list of divisors of N (can be done in $\mathcal{O}(\sqrt{N})$)
2. For each query : For each divisor d of N , if it also divides K , then cnt ++

Complexity Per Query : $\mathcal{O}(\sqrt[3]{a})$

Second Approach :

1. Calculate GCD between N and K , let it be G
2. Find number of divisors of G

Complexity Per Query : $\mathcal{O}(\log(\max(N, K)) + \sqrt{G})$

Next Approach :

Find and store Prime factorization of N

$$1800 = \{\{2, 3\}, \{3, 2\}, \{5, 2\}\}$$

There are no more than $\log(N)$ primes factors for N

1. Using each prime p in prime factorization of N , factorize K
 $1800 = \{\{2, 3\}, \{3, 2\}, \{5, 2\}\}$
 $200 = \{\{2, 3\}, \{3, 0\}, \{5, 1\}\}$
2. For each prime find the minimum count, and calculate total divisors
 $\{\{2, 3\}, \{3, 0\}, \{5, 1\}\}$
 $(3 + 1) \times (0 + 1) \times (1 + 1)$

Overall Complexity : $\mathcal{O}((\log N)^2)$

2 Type 2 Query

For given K , find number of divisors of N which are multiple of K .

$$K = P_1^{a_1} \times P_2^{a_2} \times P_3^{a_3} \dots P_m^{a_m}$$

Observation 1 : Let d be a multiple of K , then all primes which exist in prime factorization of K must also exist in d and for each prime their power in d must be atleast as much as in K

$$K = 60 = \{\{2, 2\}, \{3, 1\}, \{5, 1\}\}$$

$$M = 180 = \{\{2, 2\}, \{3, 2\}, \{5, 1\}\}$$

Observation 2 : Let d be a multiple of K and divides N , then

1. d cannot have any prime which is not present in N
Example :
 $d = 2 \times 3 \times 3 \times 5$
 $N = 2 \times 3 \times 3 \times 7$
2. let prime P is present in d with count x , and in N with count y then $x \leq y$, in other words y is upper bound.

Example :

$$d = 2 \times 3 \times 3 \times 5$$

$$N = 2 \times 3 \times 3 \times 5 \times 5$$

$$\text{Let } N = 2 \times 3 \times 3 \times 3 \times 5 \times 5$$

$$K = 2 \times 3$$

Choices of d :

$$2 \ 3 \ 5$$

$$1 \ 3 \ 3$$

$$d1 = 2 \times 3 \times 5 \times 5$$

$$d2 = 2 \times 3 \times 3$$