Number Theory : Common Divisors Query

Brij Raj Kishore

Given N, you have to answer Q queries, in each query you will be given a number K, you have to find count of common divisors of N and K
$N \leq 10^{12}$
$K \leq 10^{12}$
$Q \leq 10^5$
12 3
5
8
6
$Q1 : 1\{1\}$
$Q2 : 3\{1,2,4\}$
$Q3 : 4\{1,2,3,6\}$

Naive Approach :

1. Generate list of divisors of N (can be done in $\mathcal{O}(\sqrt{N})$)
2. For each query : For each divisor d of N, if it also divides K, then cnt ++

Complexity Per Query : $\mathcal{O}(\sqrt[3]{a})$

Second Approach :

1. Calculate GCD between N and K, let it be G
2. Find number of divisors of G

Complexity Per Query : $\mathcal{O}(\log(max(N,K)) + \sqrt{G})$

Next Approach :

Find and store Prime factorization of N
$1800 = \{\{2,3\},\{3,2\},\{5,2\}\}$
There are no more than $\log(N)$ primes factors for N

1. Using each prime p in prime factorization of N, factorize K
   $1800 = \{\{2,3\},\{3,2\},\{5,2\}\}$
   $200 = \{\{2,3\},\{3,0\},\{5,1\}\}$
2. For each prime find the minimum count, and calculate total divisors
   $\{\{2,3\},\{3,0\},\{5,1\}\}$
   $(3+1)\times(0+1)\times(1+1)$

Overall Complexity : $\mathcal{O}((\log N)^2)$