Extreme Fidelity Computational Electromagnetic Analysis in the Supercomputer Era

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***Abstract*—The computational solution of modern engineering electromagnetics problems has pushed current methods and tools to their limits presenting a need for more advanced high-fidelity modeling and solution techniques. This paper investigates a high-performance, geometry-aware, domain decomposition method for the parallel and scalable solution of time-harmonic Maxwell's equations. The technical ingredients introduced by this work are a volume-based, multi-trace integral equation formulation, and a surface-based, interior-penalty discontinuous Galerkin formulation. The three major outcomes of this research are a reduction in the condition number for the iterative matrix solution of very large geometries, a flexible framework for modeling and discretization, and parallel algorithms that scale well with increasing problem size. This algorithm can solve electromagnetic problems with one billion degrees of freedom in a reasonable timeframe. More capabilities of these techniques are demonstrated through two full-scale, real-world examples.**

Many complex electromagnetic engineering problems cannot be solved without the aid of computational tools. In many real-world applications the number of degrees of freedom required for an accurate solution exceeds the capabilities of most traditional algorithms and computer architectures. Indeed, the use of computational tools has aided in advanced antenna design [1], [2], stealth technologies [3], [4], and integrated circuits [5].

In this paper, we investigate the complexity of electromagnetic engineering problems from three fronts: composite material structures, multi-scale problems, and extreme-scale problems. The unmanned aerial vehicle (UAV) in Figure 1 is modeled with several material parameters which we solve using a multi-trace integral equation formulation [6]. The aircraft carrier, also in Figure 1, is an extreme-scale model with nearly one billion degrees of freedom and also has many multi-scale features. We employ a non-overlapping domain decomposition method to overcome these challenges [7].

The surface integral equation is a natural choice for the studied problems because all of the unknown currents reside on the surface of the scatterer. To eliminate internal resonances the combined field integral equation formulation defined in Equation 1 is used.

$$(1-\alpha)\,\mathbf{j} - \mathcal{C}_\alpha\left(\mathbf{j};\partial\Omega\right) = \alpha\mathbf{e}^{inc} + (1-\alpha)\,\mathbf{j}^{inc} \tag{1}$$

where the definition of $\mathcal{C}_\alpha$ is found in [4]

Recently, a non-overlapping integral equation domain decomposition method has been developed [7]. The continuity of

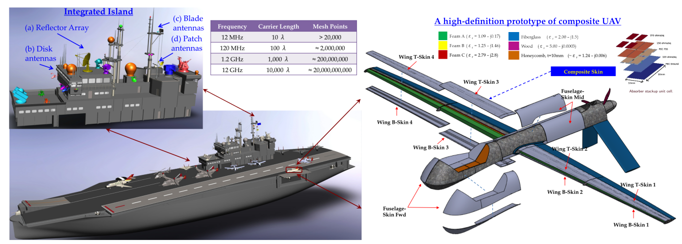

Fig. 1. High-fidelity models for representative EM problems

the surface current across sub-domain boundaries is enforced by a skew-symmetric interior penalty discontinuous Galerkin formulation, given by Equation 2.

$$\begin{aligned}&\frac{1}{4\imath k_0}\sum_{\mathcal{C}_{mn}\in\mathcal{C}}\left\langle [\![\mathbf{j}]\!]_{mn}, \sum_{n=1}^{2}\Psi_F\left(\nabla_\tau\cdot\mathbf{v}_n;\mathcal{S}_n\right)\right\rangle_{\mathcal{C}_{mn}}\\ &+\frac{\beta}{2k_0}\sum_{\mathcal{C}_{mn}\in\mathcal{C}}\left\langle [\![\mathbf{j}_{mn}]\!], [\![\mathbf{v}_{mn}]\!]\right\rangle_{\mathcal{C}_{mn}} = 0\end{aligned} \tag{2}$$

The jump operator is defined as: $[\![\mathbf{j}]\!]_{mn} := \hat{\mathbf{t}}_{mn}\cdot\mathbf{j}_m - \hat{\mathbf{t}}_{mn}\cdot\mathbf{j}_n$ on $\mathcal{C}_{mn}$.

The skew-symmetric interior penalty formulation is useful for extreme-, multi-scale geometries, such as the aircraft carrier in the previous section. To conquer the material complexities presented by the UAV we introduce a multi-trace formulation, as shown in Equation 3.

$$\begin{aligned}&\alpha\mathbf{e}_m + (1-\alpha)\,\overline{\eta}_m\mathbf{j}_m - \overline{\eta}_m\mathcal{C}_\alpha^{k_m}\left(\mathbf{j}_m;\partial\Omega_m\right)\\ &-\hat{\mathbf{n}}_m\times\mathcal{C}_{(1-\alpha)}^{k_m}\left(\mathbf{e}_m\times\hat{\mathbf{n}}_m;\partial\Omega_m\right) = \mathbf{y}_m^{inc} \text{ on } \partial\Omega_m\end{aligned} \tag{3}$$

To combine these two formulations together we first decompose a large domain into subdomains with uniform material properties, and then further decompose those subdomains according to computational needs. To do this, the multi-trace interior penalty term is employed, shown in Equation 4.

$$\begin{aligned}&\frac{1}{4\imath k_0}\sum_{\mathcal{C}_{mn}\in\mathcal{C}}\left\langle [\![\mathbf{e}_m\times\hat{\mathbf{n}}_m]\!]_{mn}, \sum_{n=1}^{2}\Psi_F\left(\nabla_\tau\cdot\mathbf{v}_{mn};\mathcal{S}_{mn}\right)\right\rangle_{\mathcal{C}_{mn}}\\ &+\frac{\beta}{2k_0}\sum_{\mathcal{C}_{mn}\in\mathcal{C}}\left\langle [\![\mathbf{e}_m\times\hat{\mathbf{n}}_m]\!]_{mn}, [\![\mathbf{v}_m]\!]_{mn}\right\rangle_{\mathcal{C}_{mn}} = 0\end{aligned} \tag{4}$$

The interior penalty and multi-trace formulations are both suitable for Schwarz preconditioning [8].where $\mathbf{u}_m = \mathbf{A}_m \mathbf{x}_m$. It has been demonstrated that the preconditioned system results in a uniformly confined eigenspectrum with respect to an increasing problem size and discretization parameters [7].

The proposed mathematical advancements enable a parallel framework suitable for HPC architectures. From the design of the model to the postprocessing stage, all steps have been parallelized, as shown in Figure 2.

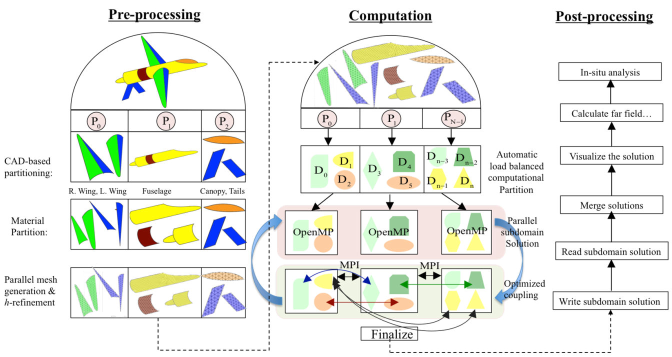

Fig. 2. Parallel Simulation Framework

Our first example is plane wave scattering from a composite material UAV at 10 GHz; the plane wave is directed at the nose of the plane. First, we decompose the model into a collection of components, then those components are further decomposed into sub-regions with uniform material properties. The model is discretized, and then the entire computational domain is decomposed into 81 sub-domains using automatic graph partitioning [9]. The computational sub-domains are solved using a data-parallel iterative solution technique, and finally the surface current is plotted in Figure 3.

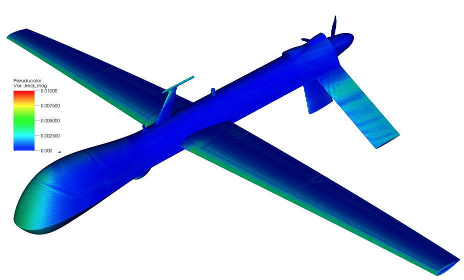

Fig. 3. Surface current on a composite UAV at 10 GHz

The next example we will study is plane wave scattering from an extreme-scale aircraft carrier. The size of the aircraft carrier is given in Figure 1. At most frequencies of interest this geometry is very electrically large. There are also many multi-scale features of this geometry.

Once the discretization of the geometry is made, the entire computational domain is split into 224 sub-domains. Each computational sub-domain acts as a work unit within a data-parallelism framework. The simulation results are presented in Figure 4. In this paper we have presented a domain decomposition method for reducing the complexity of many

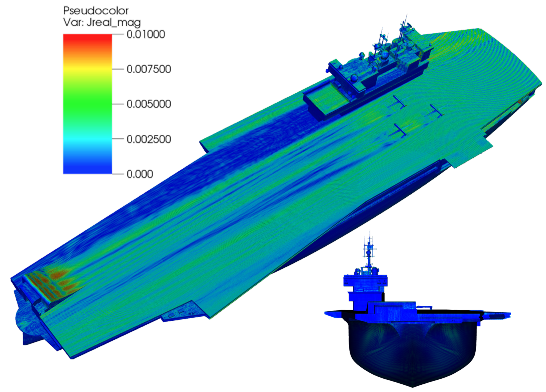

Fig. 4. Surface current on a composite UAV at 2 GHz

real-world electromagnetics engineering problems. We have provided a framework with which the complexity of composite material structures can be reduced by introducing an additional trace on the boundary between sub-regions. Additionally, we have demonstrated the capabilities of these algorithms to solve extreme-and multi-scale problems on large high performance computing architectures. Future work will include developing algorithms more adaptable to different computing environments, such as cloud computing, as well as pushing the boundaries of the scale of problems that can be solved.

Acknowledgments

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