sqrt(-i)

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This is a simple demonstration of the beauty of math. One day I was interested in revisiting Euler's formula for complex numbers so I asked myself what the $\sqrt{-\imath}$ was equal to. Here are my findings:

$$z = \sqrt{-\imath} \tag{1}$$

Using Euler's formula (stated below) we can begin the simplification process.

$$\exp \imath\theta = \cos\theta + \imath \sin\theta \tag{2}$$

Let's start with $z = -\imath$. This is along the negative x-axis, and therefore we have $\theta = \frac{3\pi}{2}$. To check:

$$\exp \imath\frac{3\pi}{2} = \cos\frac{3\pi}{2} + \imath \sin\frac{3\pi}{2} = 0 - \imath \tag{3}$$

Since we have the exponential representation of $\imath$ we can now take its square root, apply Euler's formula, and obtain a representation for $\sqrt{-\imath}$.

$$\sqrt{\exp \imath\frac{3\pi}{2}} = \exp \imath\frac{3\pi}{4} = \cos\frac{3\pi}{4} + \imath \sin\frac{3\pi}{4} = \sqrt{\frac{1}{2}}\,(-1 + \imath) \tag{4}$$

Therefore, $\sqrt{-\imath} = \sqrt{\frac{1}{2}}\,(-1 + \imath)$