Unconstrained Inverse Kinetics · Week II

CCO · Constraint Continous Optimization

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0.1 Introduction

The assignment is the describe and implement the Levenberg-Marquardt algorithm and compare this to last weeks Newton's method. Interesting comparisons could be based on number of iterations used, the precision of the solution found and the algorithms robustness.

A: Levenberg-Marquardt Algorithm

The Levenberg-Marquardt algorithm has a lot of similarities with Newton's method. Both are iterative root-finding algorithms and both compute a new direction in each step. However, where Newton's method computes the new direction using a simple Taylor approximation, Levenberg-Marquardt uses a combination of approximation and deepest descent. Intuitively, the algorithm uses long steps as long as this approach results in improved values. When a long step does no longer work, the algorithm gradually reduces the step size.

The Levenberg-Marquardt update value is defined as:

$$\Delta = -(J^T J - \lambda I)^{-1} J^T r$$

Where J is the jacobian matrix, λ is the step modifier, r is the residual vector and Δ is the next modification of the approximation. When the step modifier λ gets large, it will dominate the inverted matrix, such that:

$$\Delta \approx -(-\lambda I)^{-1} J^T r =$$

$$-\frac{1}{\lambda} I J^T r$$

It is now clear why λ is a step modifier. A large λ implies a small Δ. Also, λ determines whether $J^T J$ or $-\lambda I$ should dominate the direction. In the latter case, the direction is determined by J^T, that is the first order partial derivatives. This is the same direction as the steepest descent method would have chosen and it is this trick, that is the key to the convergence of Levenberg-Marquardt. Since the λ value is multiplied by a positive factor when no better approximation is available, the method will eventually find a better point closer to the goal.

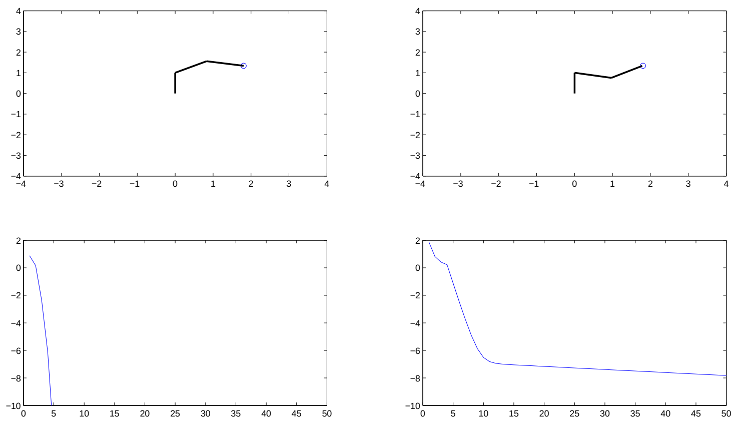
B: Comparison to Week1 Implementation

Explanation of Pictures

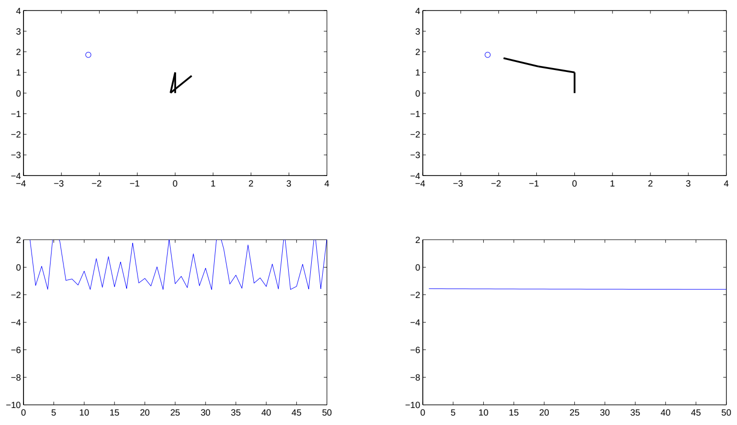
Each picture contains four plots. Here described from left to right, row by row.

1. The plot in the upper left corner shows the solution chosen by Newton's method.
2. The plot in the upper right corner shows the solution chosen by the Levenberg-Marquardt method.

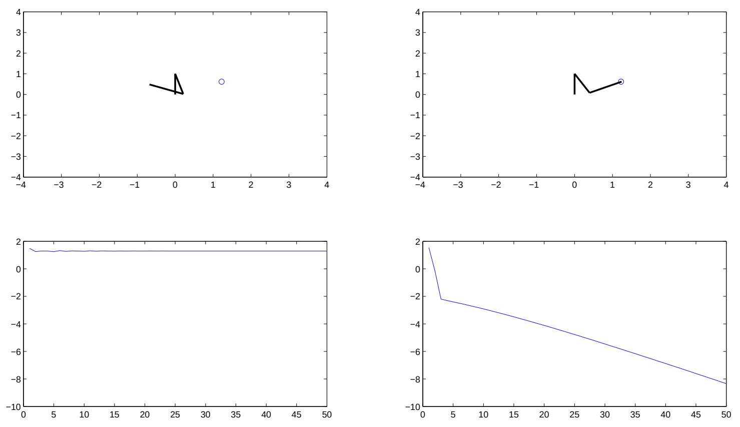
3. The plot in the lower left corner shows the logarithmic error, for each iteration, for Newton's method.
4. The plot in the lower right corner shows the logarithmic error for each iteration, for Levenberg-Marquardt.



This graph shows the two algorithm's solutions to the same goal. Both algorithms finds a solution that seems close to the wanted point, however the error graphs clearly show that Newton's method is both faster and more precise. In fact, Newton's method is very fast in this case. Notice how Levenberg-Marquardt converges quadratically at first and then turn linear when steepest descent kicks in.



This is a case where the goal point lies outside the reach of the arm. It is not possible to actually reach it. It is here the steepest descent trick of Levenberg-Marquardt comes into play. While this ensures a solution in the right direction, Newton's method fails to find one. This suggests that Levenberg-Marquardt is more robust. Note however, that steepest descent is quite slow.



This is an interesting plot. The point chosen lies within reach of the arm, but the initial guess is far from it (all to the left). This fools Newton's method, that doesn't seem to get anyway in the 50 iterations. Levenberg-Marquardt finds the solution with something that looks like a linear convergence rate. The bend it its curve suggest is most likely a flip to steepest descent.

Implementation

This section contains the modified pseudo code and MatLab code for this week's programming case.

Pseudo Code for LM

This is my pseudo code for the Levenberg-Marquardt algorithm. In the implementation, the inner for-loop has been replaced by a while-loop.

```
def LevenbergMarquardt(f, goal):
  ld = 0.0001, c = 2, x = 0
  endPoint = f(x)
  oldError = || goal - endPoint ||
  for i = 1 to N:
    dt = LMupdate(ld, goal - endPoint)
    newEndPoint = f(x + dt)
    newError = || goal - newEndPoint ||
    if newError >= oldError
      ld = ld * c
      REJECT
    else
      ld = ld / c
      oldError = newError
      x = x + dt
      ACCEPT
    fi
  rof
fed
```

MatLab Code

LMupdate This is my MatLab code for the helper function `LMUpdate`:

```
function [ delta ] = LMupdate(lambda, J, r)
  tJ = transpose(J);
  delta = -inv(tJ*J - lambda * eye(size(J))) * tJ * r;
end
```

Levenberg-Marquardt This is my MatLab code for the Levenberg-Marquardt algorithm:

```
function [ angles ] = levenberg_marquardt( goal, t, angles )
%
% input
%     g      : Goal position/vector (specified in homogenuous coordinates)
%     t      : A vector of fixed rod-link vectors
%     angles : A vector with joint angles
%     e      : Position/vector (specified in homogenuous coordinates)
% output
%     angles : The updated pose which will reach the specified goal position.

e  = [0;0;1];
endPoint = f(t, angles);

iter = zeros(1,50);
err  = zeros(1,50);

lambda   = 0.0001;

oldError = dot(goal-endPoint,goal-endPoint);

for i = 1:50
    % Compute new angles, endpoint and error
    r = goal - endPoint;
    J = jacobian(t, angles, e);

    % Keep doubling lambda until error is reduced
    newError = oldError + 1;
    while newError >= oldError
        % REJECT
        lambda = lambda * 2;
        delta = LMupdate(lambda, J, r);
        newAngles   = angles + delta;
        newEndPoint = f(t, newAngles);
        error = goal-newEndPoint;
        newError = dot(error, error);
    end

    % ACCEPT
    oldError = newError;
    angles   = newAngles;
    endPoint = newEndPoint;
    lambda = lambda / 2;

    % Remember observed error
    err(i)  = log(oldError);
    iter(i) = i;
end

subplot(2,2,4);
plot(iter, err);
axis([0 50 -10 2]);
subplot(2,2,1);

end
```

Run I also changed the code in the file "run.m" to get support more advanced figures with 4 plots on each:

```
% Make sure we got a clean environment to work in
close all;
clear all;

% Setup a default configuration
t      = [ 0 0 0; 1 1 1];
angles = [pi/4; pi/4; pi/4];

% Try to find end-effector position
e = f(t, angles);

% Get x and y coordinates
x = e(1);
y = e(2);

% Verify if f worked as we expected
if ( (x  + 1.7071) > 0.001 )
    error('x-test failed.');
end

if (  (y  - 1.7071) > 0.001 )
    error('y-test failed.');
```

```
end

figure(1);

clf;
hold on;
title('Inverse Kinematic Chain');
xlabel('X');
ylabel('Y');

subplot(2,2,1);
draw_chain( t, angles );
subplot(2,2,2);
draw_chain( t, angles );

theaxis = [-4 4 -4 4];
angles = ones(size(angles));
for n = 1:20
    % Newton
    subplot(2,2,1);

    [gx,gy] = ginput(1);
    plot(gx,gy,'bo');

    new_angles = nonlinear_newton([gx; gy; 1],t,angles);
    subplot(2,2,1);
    draw_chain( t, new_angles );

    axis(theaxis);

    % Levenberg
    subplot(2,2,2);
    plot(gx,gy,'bo');
    angles = levenberg_marquardt([gx; gy; 1],t,angles);
    subplot(2,2,2);
    draw_chain( t, angles );

    axis(theaxis);
end
hold off;
```

newton_gauss And lastly, I updated my Newton's method code from last week to draw error plots:

```
function [ angles ] = nonlinear_newton( g, t, angles )
%
% input
%     g      : Goal position/vector (specified in homogenuous coordinates)
%     t      : A vector of fixed rod-link vectors
%     angles : A vector with joint angles
%     e      : Position/vector (specified in homogenuous coordinates)
% output
%     angles : The updated pose which will reach the specified goal position.

e  = [0;0;1];
ep = f(t, angles);

iter = zeros(1,50);
err  = zeros(1,50);
for i=1:50
    da = pinv(jacobian(t, angles, e))*(g - ep);
    angles = angles + da;
    ep = f(t, angles);
    error = g-ep;
    err(i)  = log(dot(error,error));
    iter(i) = i;
end

subplot(2,2,3);
plot(iter, err);
axis([0 50 -10 2]);

end
```