Selection Problem · Week III

Data Structures: Theory and Practice

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1) Exercise 20.3-4

I am to describe the following recurrence describing the space usage of a van Emde Boas tree:

$$S(K) = (1 + \sqrt{K}) \cdot S(\sqrt{K}) + O(\sqrt{K})$$

The representations are:

- $S(\sqrt{K})$: The space usage of vEB-tree of size $\sqrt{K}$.

- $(1 + \sqrt{K})$: The number of sub-trees the current tree contains. This is 1 for the summary tree and the $\sqrt{K}$ is for the sub-trees.

- $O(\sqrt{K})$: The space usage of the current tree-node containing min, max and u and possibly references to the summary and sub-trees. However, this should not be necessary.

Next, I am to show, that the recurrence results in a running time of $O(n)$. I prove this by complete (strong) induction:
Assume that $S(K) = O(K)$ for all $K < N$. Then

$$S(N) = (1 + \sqrt{N}) \cdot O(\sqrt{N}) + O(\sqrt{N}) \Rightarrow$$

$$S(N) = O(\sqrt{N}) + \sqrt{N} \cdot O(\sqrt{N}) + O(\sqrt{N}) \Rightarrow$$

$$S(N) = O(N) + O(\sqrt{N}) \Rightarrow$$

$$S(N) = O(N)$$

2) Problem 20-1: van Emde Boas Trees

a: What happens when inserting an element that already exists?

Short answer: Nothing. The routine will recursively search through the tree, looking for the element. At some point, the base case of $u = 2$ will occur (a 2-vEB tree is the base case). Here we simply set the bit representing the number being inserted ignoring that the bit is already set. This does not change the state of the tree in any way.

b: What happens when deleting a non-existing element?

Short answer: Nothing. Again the routine searches the tree for the element x. However, since the element is not in the tree, the base case will either be min = max = nil (empty tree) or min = max $\neq$ nil (1 element, not x). In the first case nothing happens. In the latter, a non-existing element is "deleted" by setting a zero-bit to zero: nothing happens.