Research summary

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I am a Master's student working with Dr. Speissegger primarily in the Zermelo-Fraenkel (ZF) scheme of set theory. Despite being a natural domain in which mathematics can be done with fair efficiency and rigor, set theory by itself is quite esoteric. Rather, its utility lies in its ability to lend itself to various other mathematical fields as a result of its constructive nature. By omitting and adding axioms from and to ZF/ZFC, set theory is able to model various regimes in which interesting mathematics may be done (see [Bag20]).

As an example, one considers the o-minimal structures (which I have worked with briefly in summer 2019), which fail to model the Infinity and Choice axioms of ZFC (among others; in the general case), and hence act as a fairly natural domain in which to obtain "nice" analytical and topological results without the emergence of "confounding" elements such as the Banach-Tarski paradox (see [Dri98], and [Rob15]).

It is on account of Gödel's first incompleteness theorem, that ZF set theory lends itself well to the business of adding new axioms (see [Kur92]). As ZF is incomplete, provided that we can establish the independence of a given statement, we may add it to ZF to generate new axiomatizations in which mathematics may take place. Naturally then, the goal of set theory is essentially that of devising new axioms to discover new regimes where "interesting" math may take place, or that of modifying existing axioms to allow for the same.

In my current work, my goal is to understand fundamental independence proofs such as that of AC, CH, and GCH, and the mechanisms behind these proofs such as forcing, and absoluteness. To this end, I have been working through Kunen's text ([Ken92]), which conglomerates the work of set theorists as Cohen, Martin, and Solovay in establishing these independence results. The hope is then that we may be able to conjecture some new independence results which may lead to nicer regimes for certain mathematics to take place in. As well, we hope that we may establish a greater understanding of the properties of models in which these new axioms hold ([Bag20]).

More recently, work has been done in attempting to establish computational methods to search for independence results via quotient presentations (see [GH17]). Although models of ZF do not admit computable quotient presentations (Tennenbaum's phenomenon), there is speculation, that it may be possible to obtain nice quotient representations of set theoretical universes up to constraint by defining a functor from **Set** or **Class** to some other category with computable quotient presentation (see topos theory and [Har+17]). We hope to also contribute to this work, though it is not our primary focus.

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