Search Methods

Fundamentals of AI

Learning Outcomes

- Understand the different types of problem-solving agents
- How to formulate a problem
- Gain an in-depth understanding of basic search algorithms

A Skeletal Version of a problem-solving agent

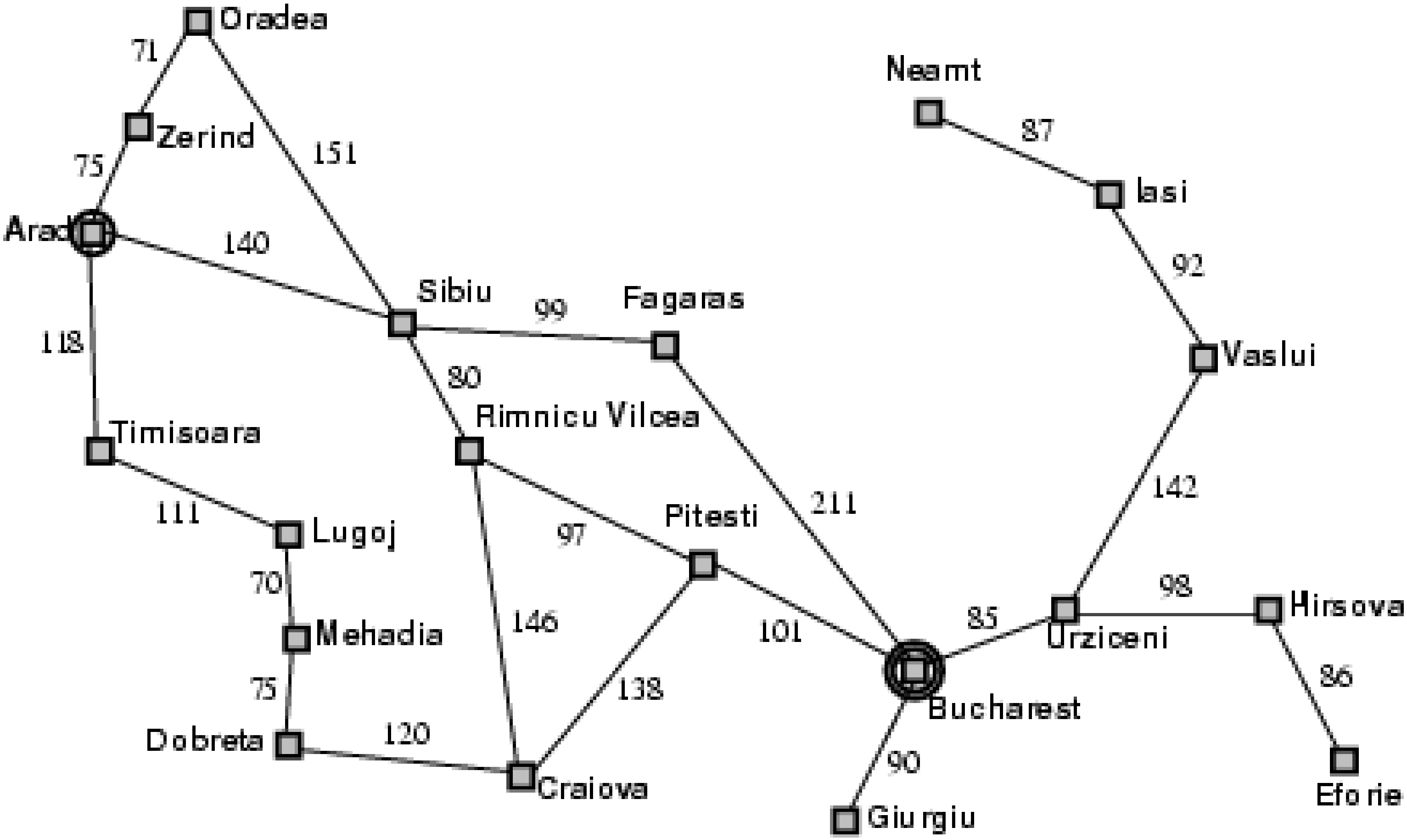
```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then do
       goal ← FORMULATE-GOAL(state)
       problem ← FORMULATE-PROBLEM(state, goal)
       seq ← SEARCH( problem)
   action ← FIRST(seq)
   seq ← REST(seq)
   return action
```

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania

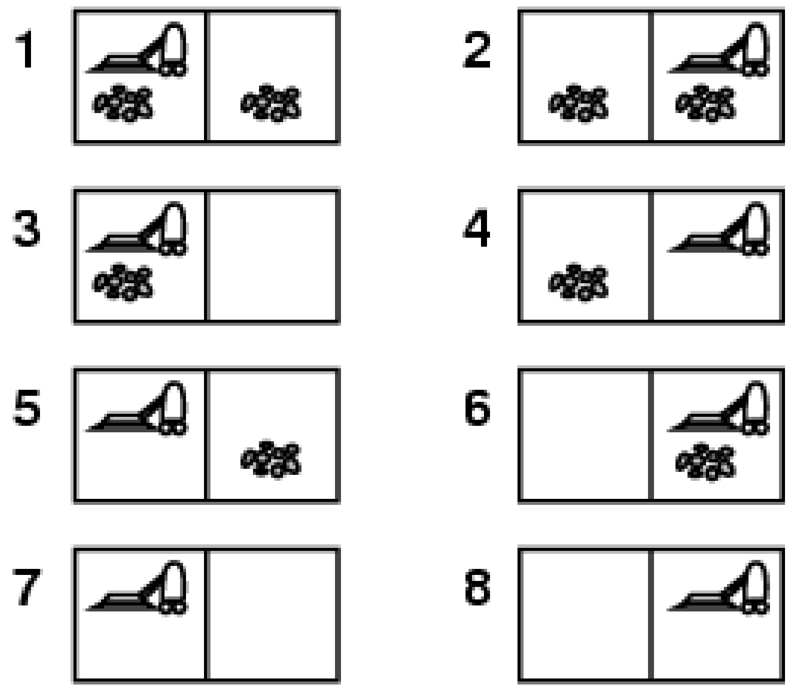


Problem types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave search, execution
- Unknown state space → exploration problem

Example: vacuum world

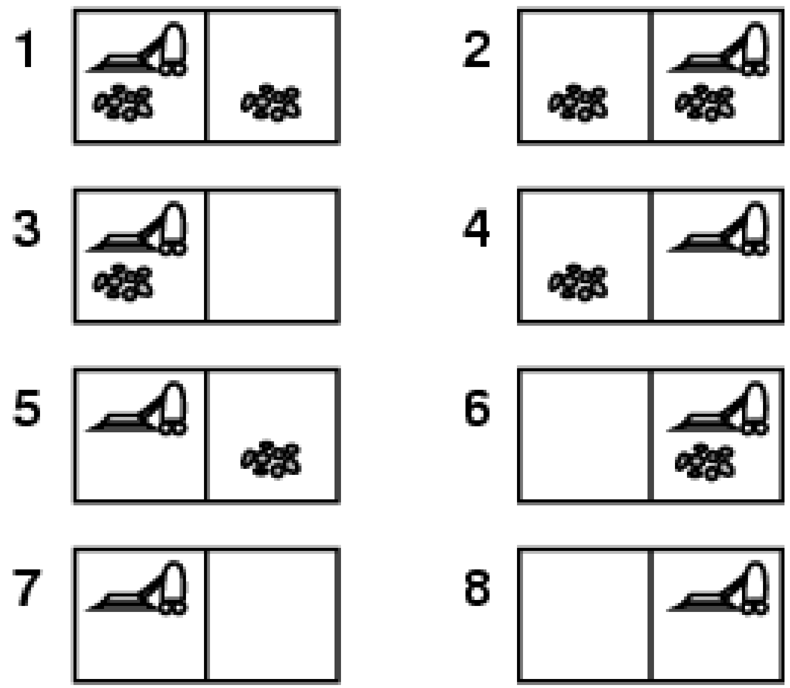
- Single-state, start in #5.
  Solution?



Example: vacuum world

- Single-state, start in #5.
  Solution? *[Right, Suck]*

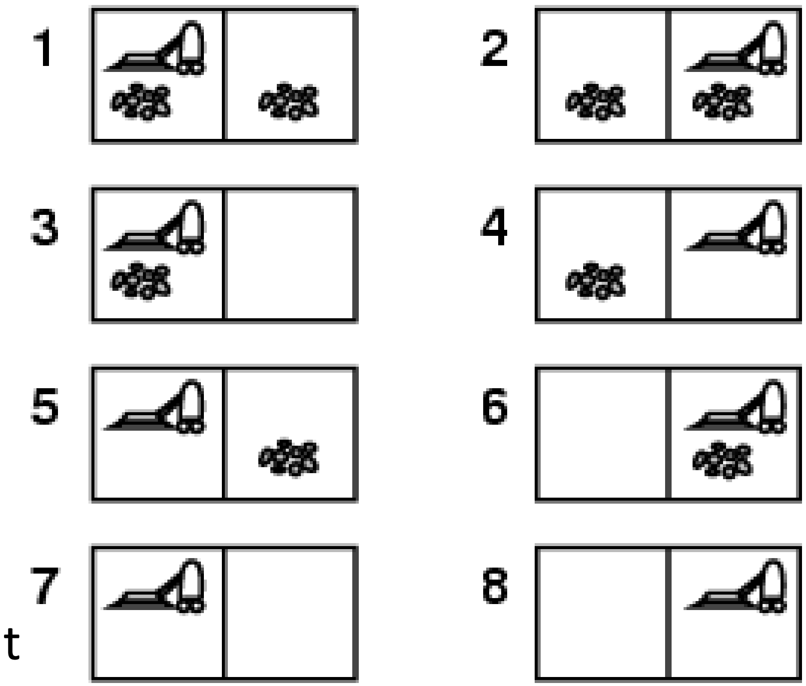
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution?



Example: vacuum world

- Sensorless, start in *{1,2,3,4,5,6,7,8}* e.g., *Right* goes to *{2,4,6,8}*
  Solution?
  [Right,Suck,Left,Suck]

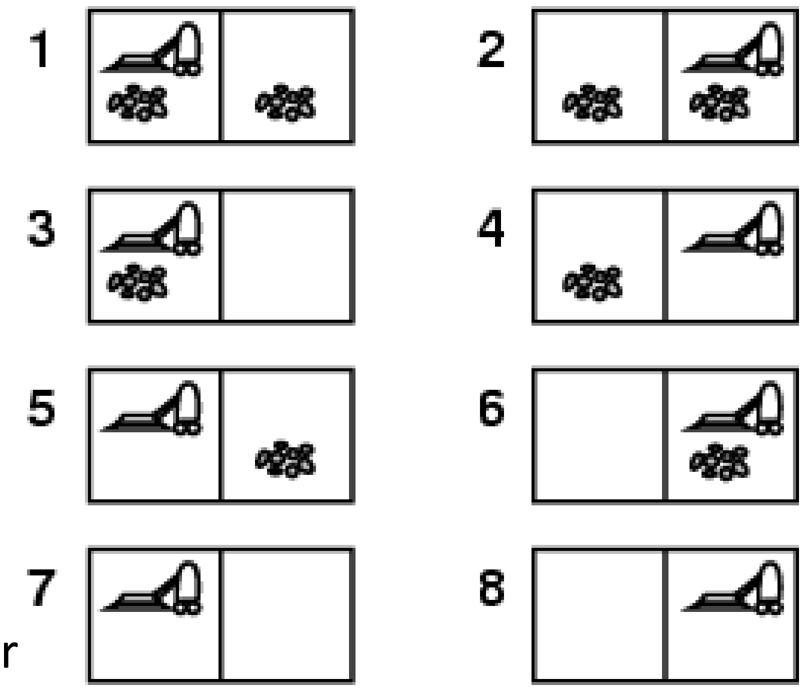
- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at
  - Percept: *[L, Clean],* i.e., start in #5 or #7
    Solution?



Example: vacuum world

- Sensorless, start in *{1,2,3,4,5,6,7,8}* e.g., *Right* goes to *{2,4,6,8}* Solution? *[Right,Suck,Left,Suck]*

- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at cur
  - Percept: *[L, Clean],* i.e., start in #5 or #7 Solution? *[Right, **if** dirt **then** Suck]*



Single-state problem formulation

A problem is defined by four items:

1. initial state e.g., "at Arad"
2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad) = {<Arad → Zerind, Zerind>, … }*
3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0

A solution is a sequence of actions leading from the initial state to a goal state

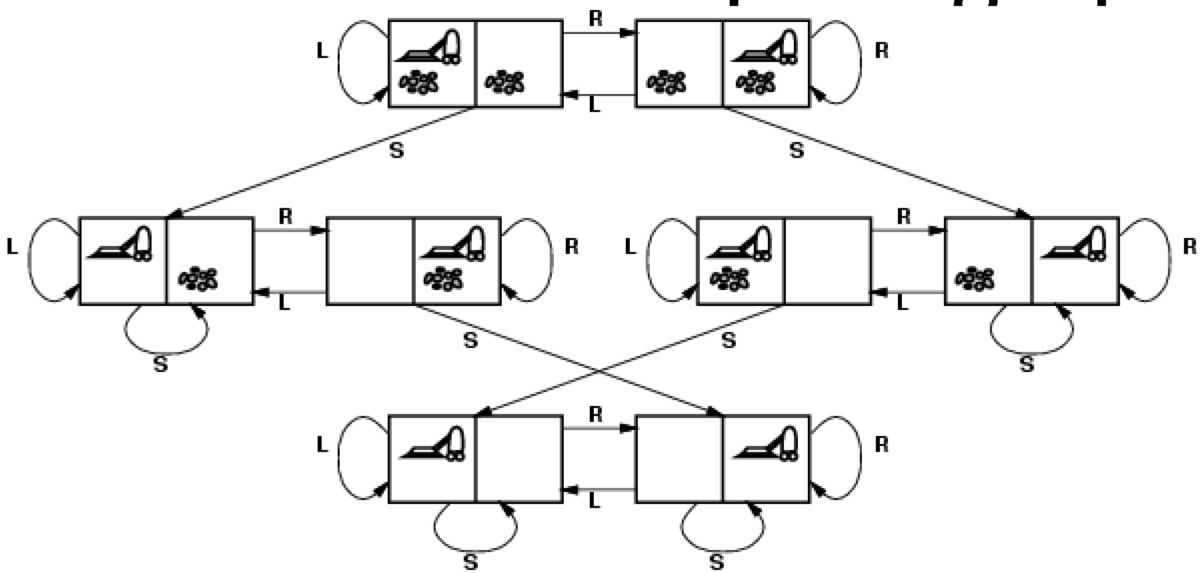
Selecting a state space

- Real world is absurdly complex
  → state space must be abstracted for problem solving
  → anything irrelevant to solving the problem at hand must be left out
  → what remains is the abstract state
- Abstract state space = set of real states relevant to the problem at hand and the actions required to get to the goal state

Q) What is the abstract state space for making a black cup of coffee (without the use of a coffee machine)?

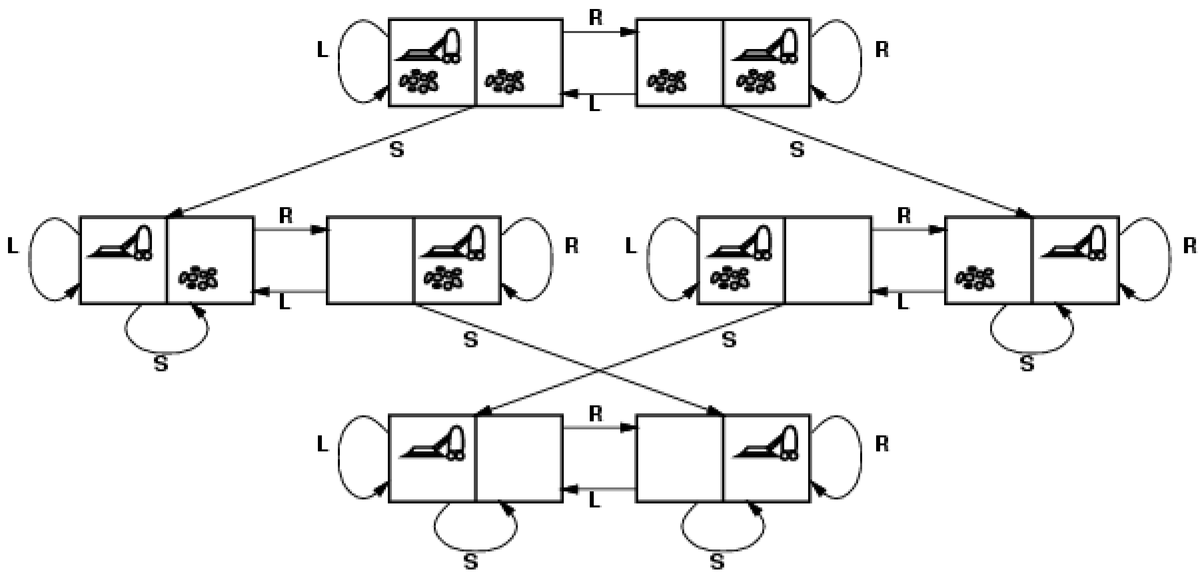
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

Vacuum world state space graph



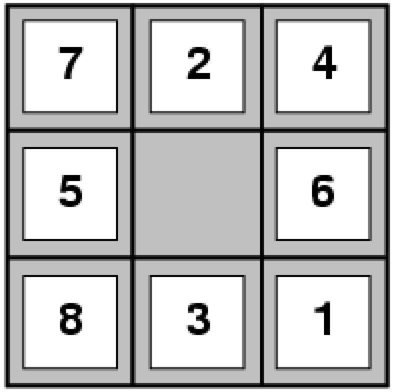
- states?
- actions?
- goal test?
- path cost?

Vacuum world state space graph

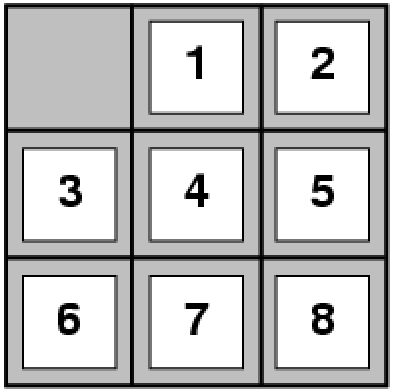


- <u>states?</u> integer dirt and robot location
- <u>actions?</u> *Left*, *Right*, *Suck*
- <u>goal?</u> no dirt at all locations
- <u>cost?</u> 1 per action

Example: The 8-puzzle



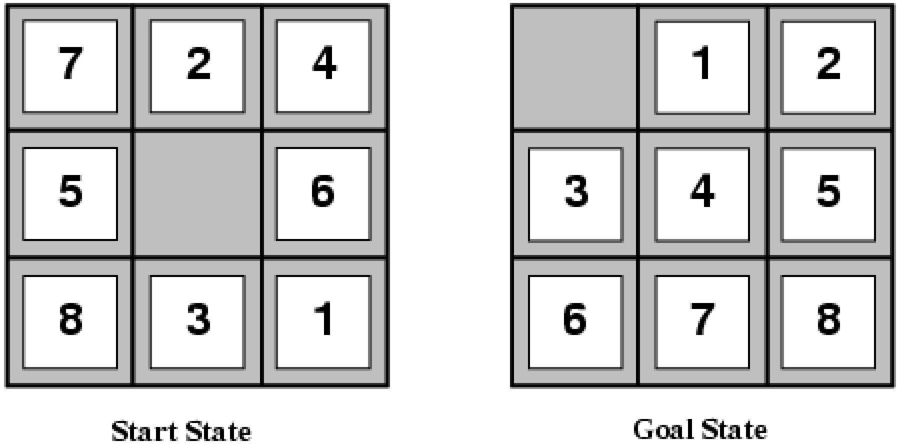
Start State



Goal State

- states?
- actions?
- goal?
- path?

Example: The 8-puzzle

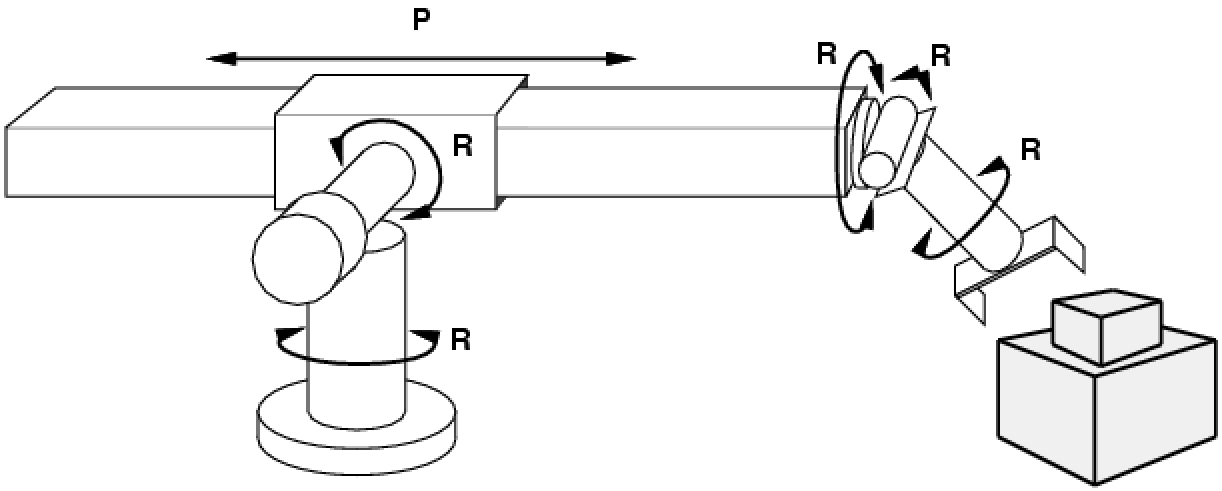


- states? locations of tiles
- actions? move blank left, right, up, down
- goal? = goal state (given)
- path? 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard]

Q) How many states exist?

Example: robotic assembly



- states?: real-valued coordinates of robot's joint angles
- actions?: continuous motions of robot joints
- goal?: complete assembly
- path?: time to complete assembly

Formulating a general solution

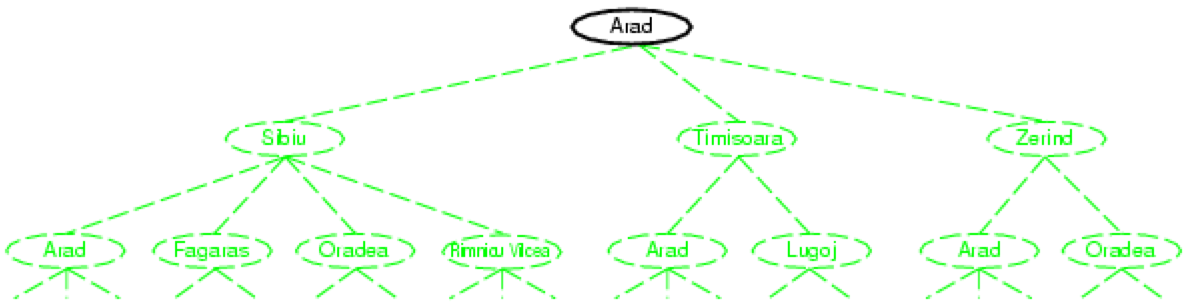
- Basic idea:
  - Explore the state space until the goal state is reached
  - The exploration is done according to a chosen strategy
  - The state space is structured as a tree (a natural structure)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

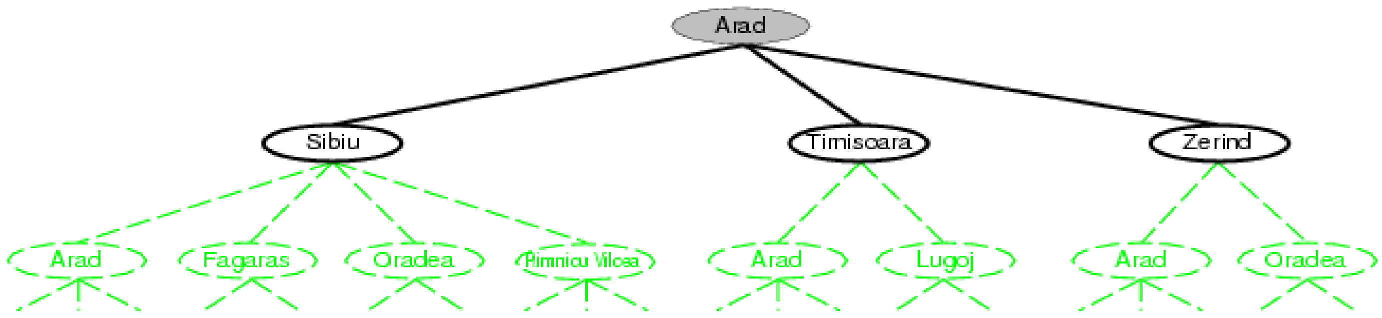
Summary so far

- Problem Characteristics
  - Observable
  - Deterministic
  - Episodic
  - Static
  - Single Agent
- Problem Types
  - – Single State
  - – Sensorless (Conformant)
  - – Contingency
  - – Exploration
- Agent Types
  - Simple Reflex
  - Model-based
  - Goal-based
  - Utility-based
  - Learning
- Uninformed Search Strategies
  - – Breadth-first
  - – Uniform-cost
  - – Depth-first
  - – Depth-limited
  - – Iterative Deepening
  - – Bidirectional

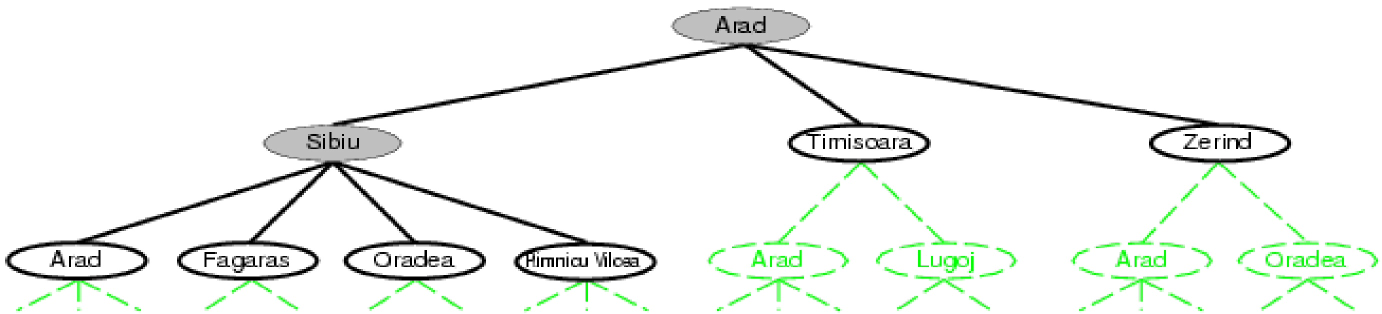
Tree search example



Tree search example



Tree search example



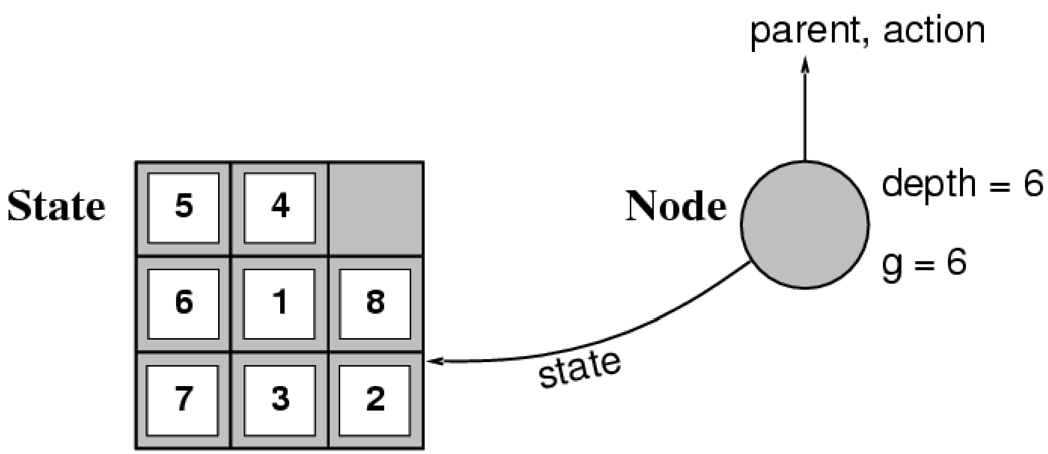
Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Implementation: states vs. nodes, what's the difference?

- A state is a (representation of) a physical configuration
- A node is part of a search tree and includes: state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields and using the `Successor Fn` of the problem to create the corresponding states.

Search strategies

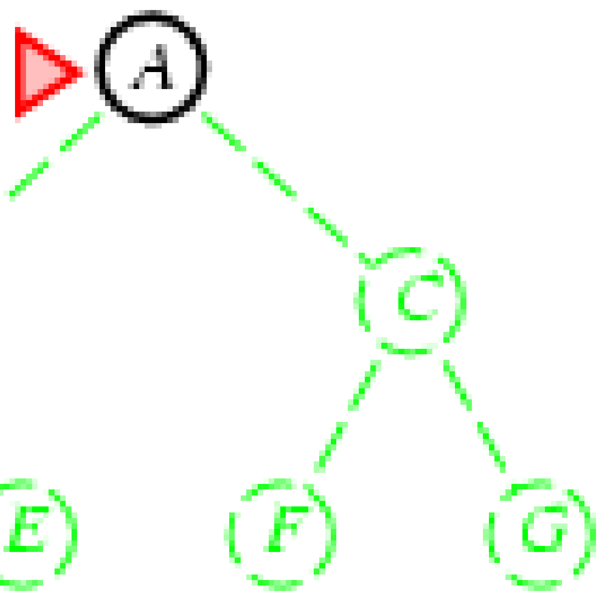
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes searched until solution is found
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search
- Bidirectional

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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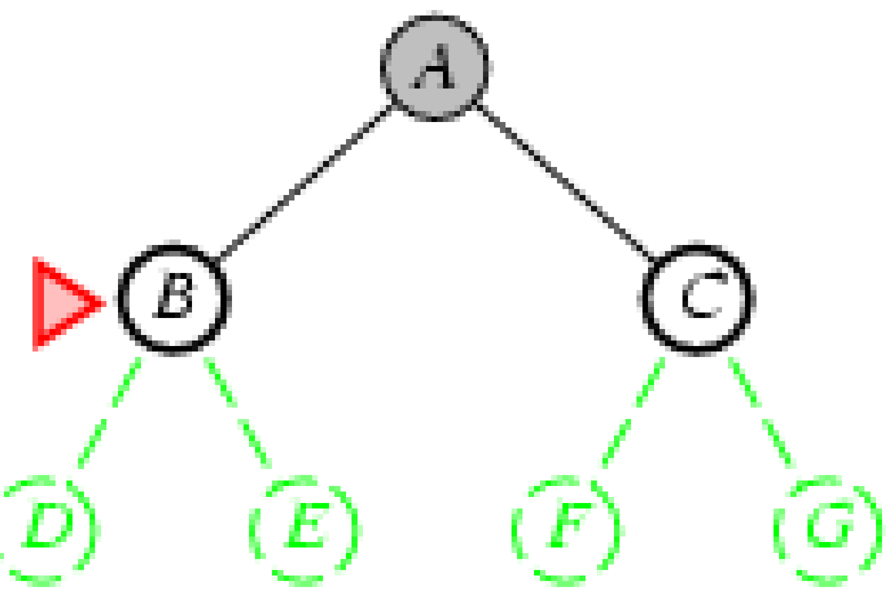

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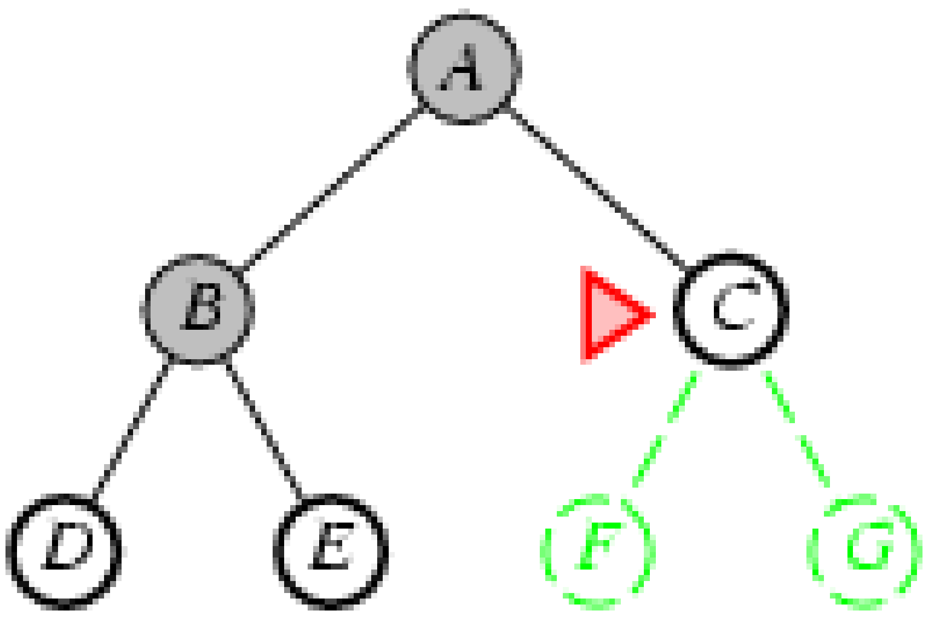

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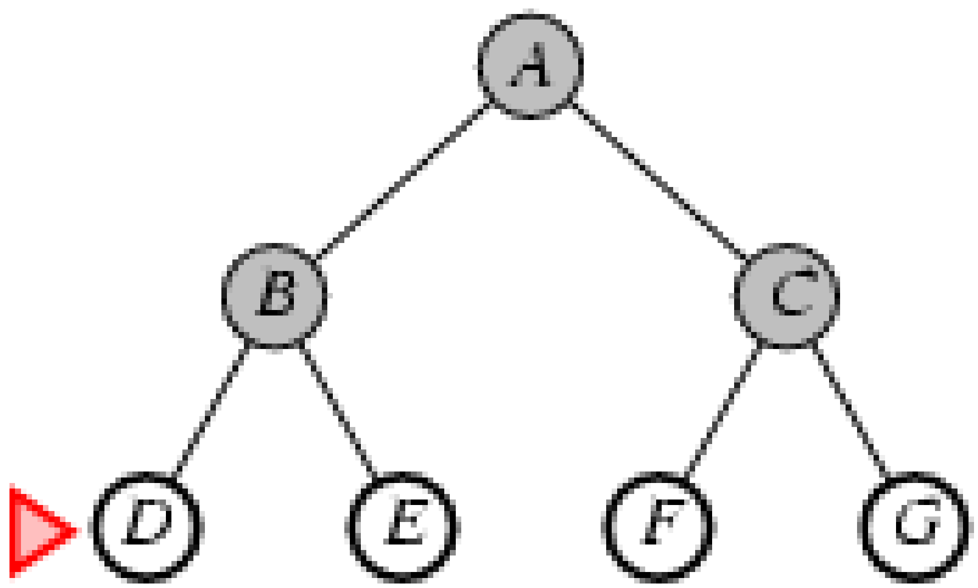


Properties of breadth-first search

- Complete? Yes (if b is finite)
- Time? $b+b^2+b^3+\ldots+b^d = O(b^d)$
- Space? $O(b^d)$ (keeps every node in memory)
- Optimal?
  - Guaranteed to find the *shallowest* goal state
  - Guaranteed to find the *least cost* goal state if path cost is a non decreasing function of depth (true when all actions have uniform cost)
- Space is the fundamental issue when b and d are sufficiently large

Practicalities of BFS on large scale problems

- With the processing speed taken as 1 million nodes per second, and the space required is 1kB per node (realistic assumptions) the execution times and space requirements are:

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

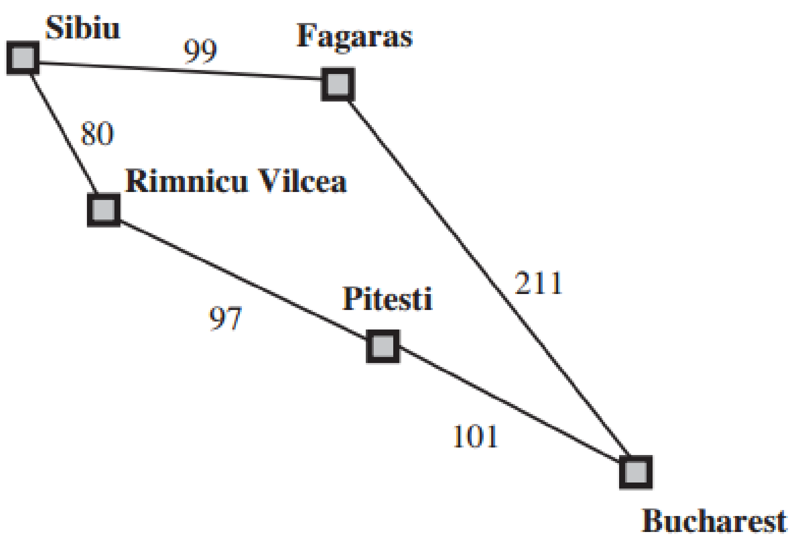
Uniform-cost search

- Expands least-cost unexpanded node
- Implementation:
  - *fringe* = priority queue ordered by path cost
- Equivalent to BFS if step costs all equal; otherwise, is better than BFS
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of $g(n)$

BFS and Uniform Cost Search: which to use?

- Uniform cost search has greater time and space complexity than that of BFS, so only use when the assumption of uniform path costs is not true
- This is generally true in path navigation problems
- Q) How about computer network routing problems?

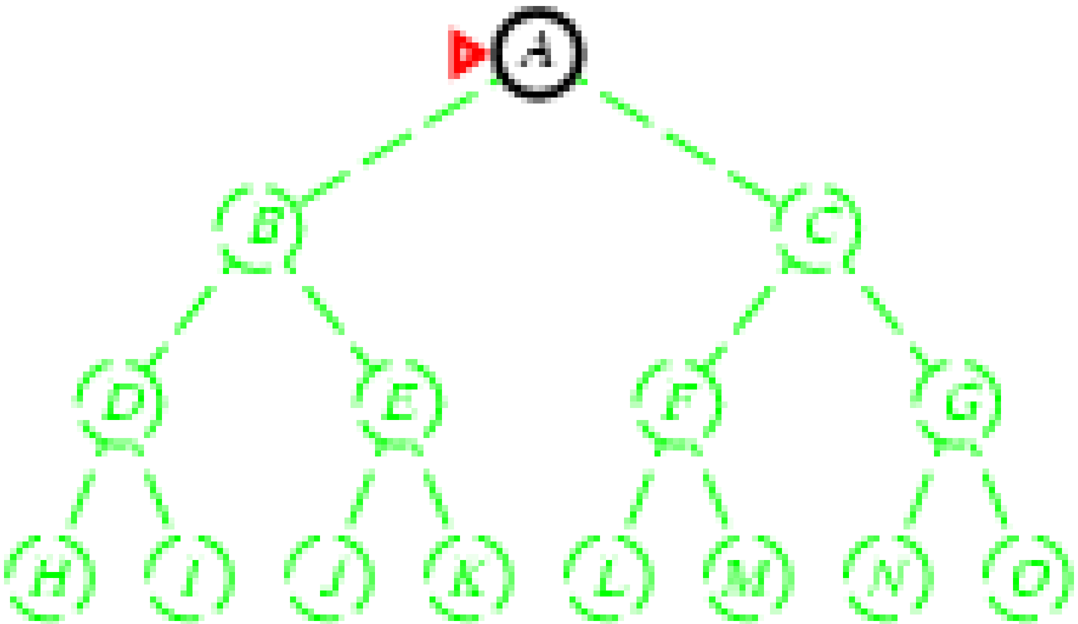
Uniform Cost(UC) Search vs BFS: Example



- Let's say we want to get to Bucharest in Romania, starting from Sibiu
- With UC we expand the Rimnicu Vilcea (RV) node since it has the least path cost when compared to Faragas (F), path cost so far is 80
- From RV we then expand Pitesti (P) given a path cost so far of 80+97=177
- Now F is the node with the least path cost, expanding F gives a path cost of 99+211=310
- We now expand P since it has a lower path cost than 310, giving the optimal path cost to Bucharest of 177+101=278
- Q) What solution will BFS come up with?

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO stack, i.e., put successors at front



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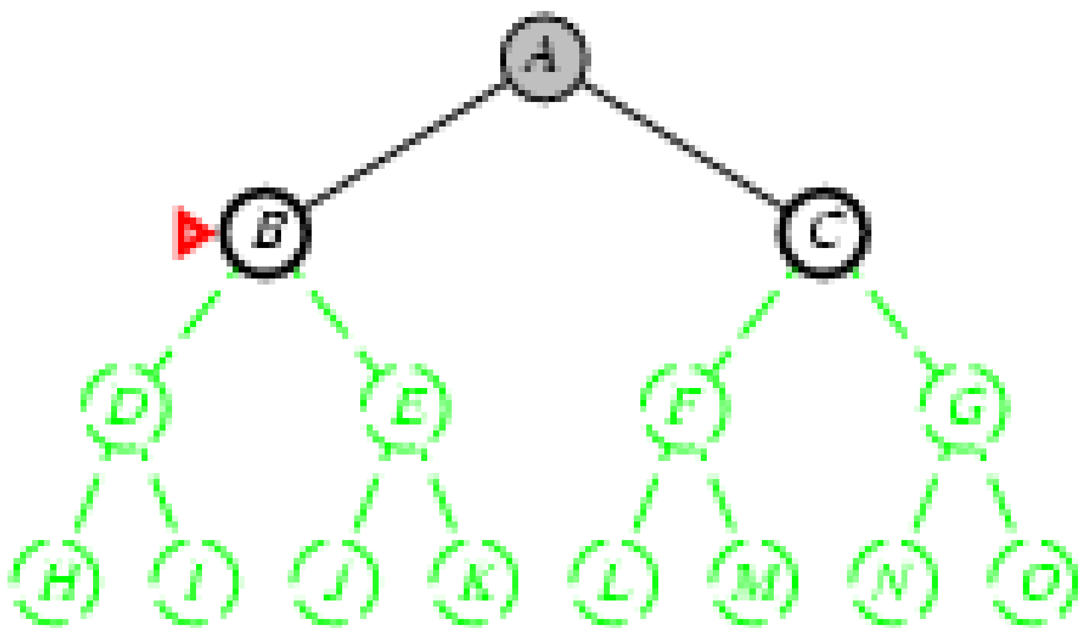

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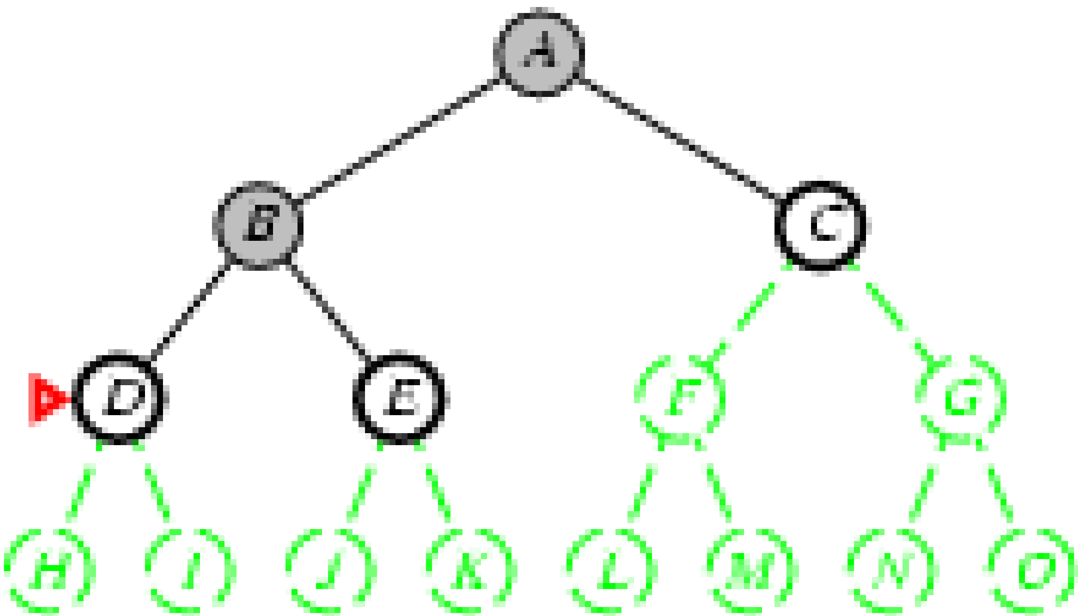

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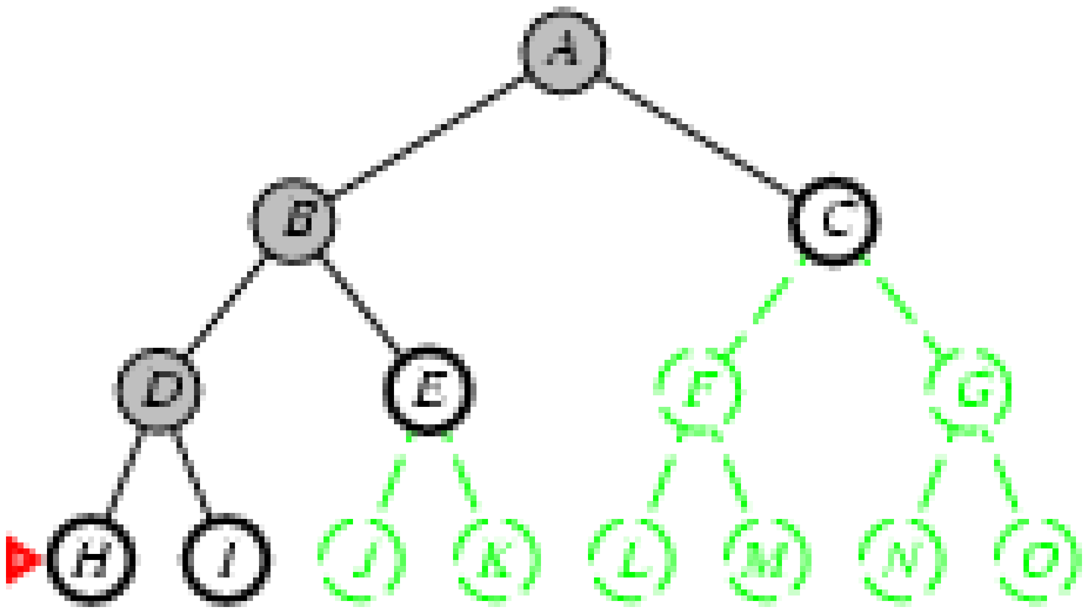

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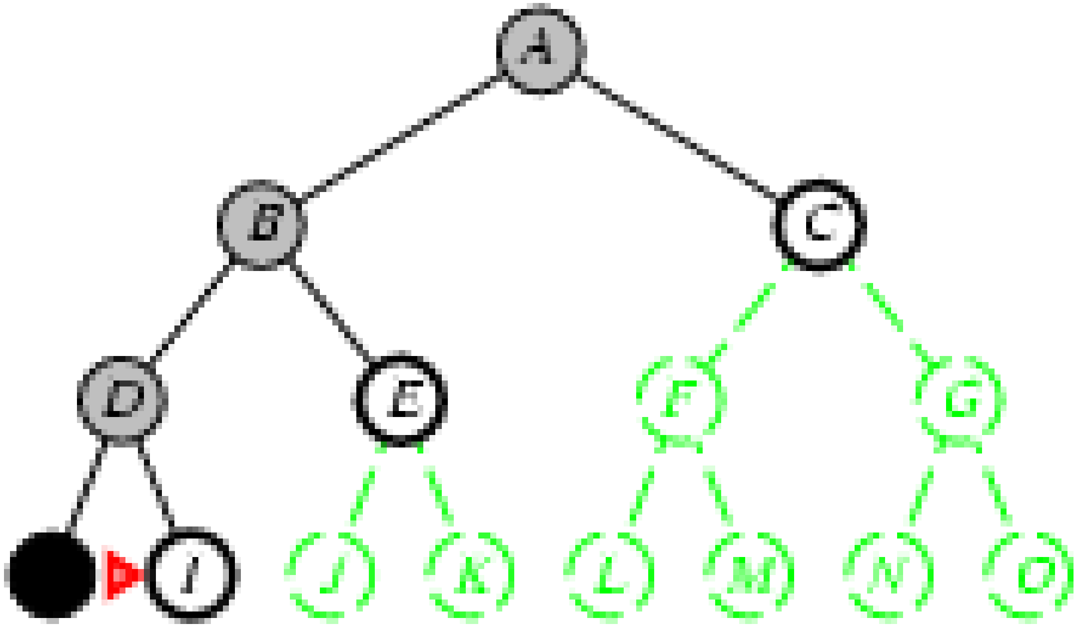

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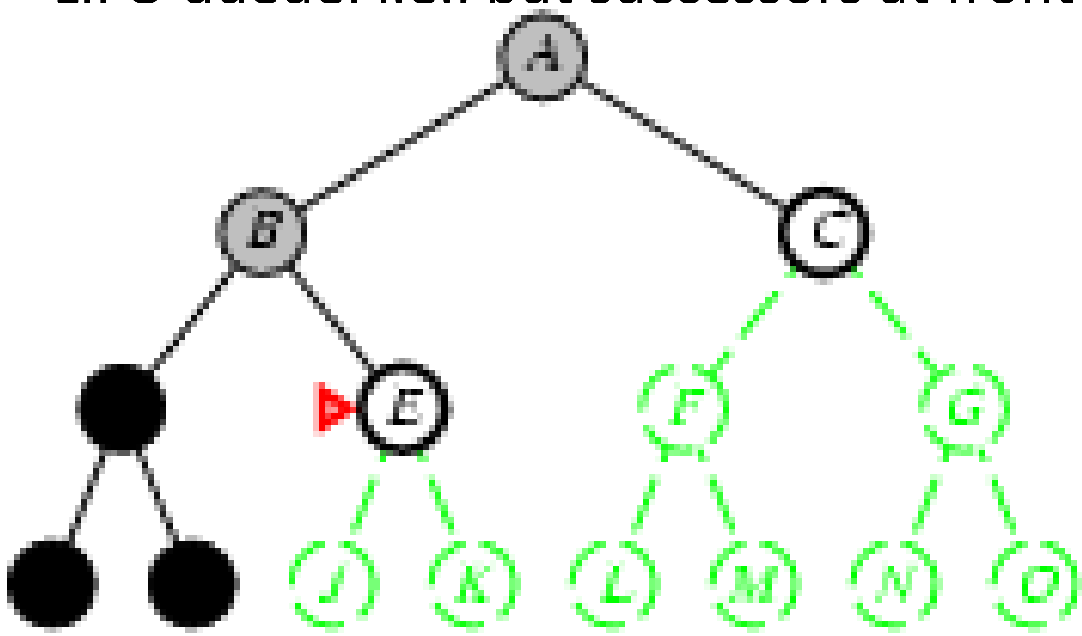

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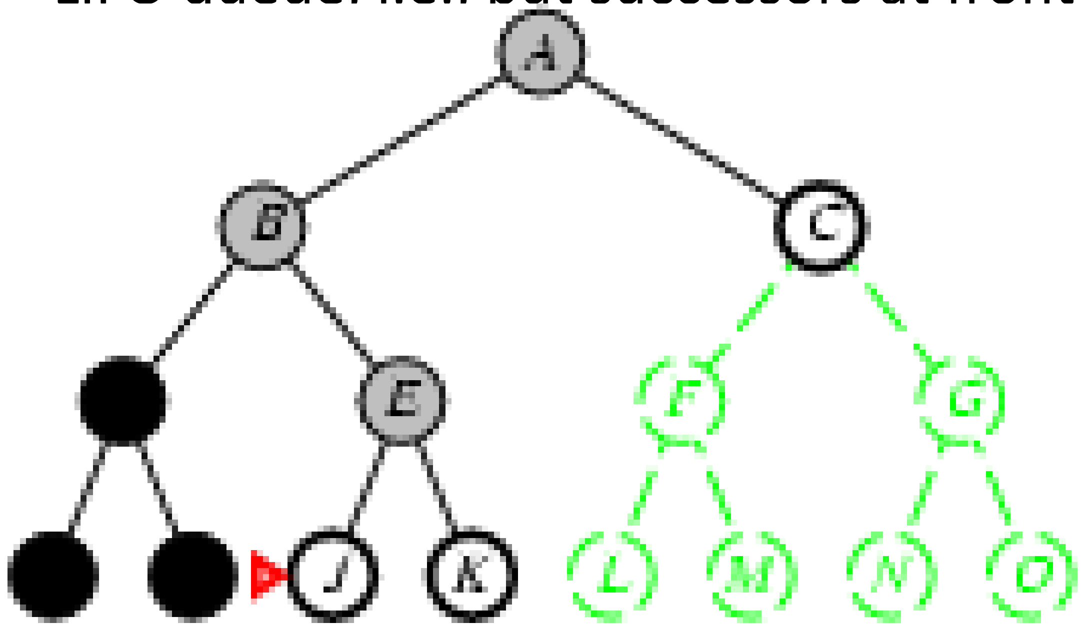

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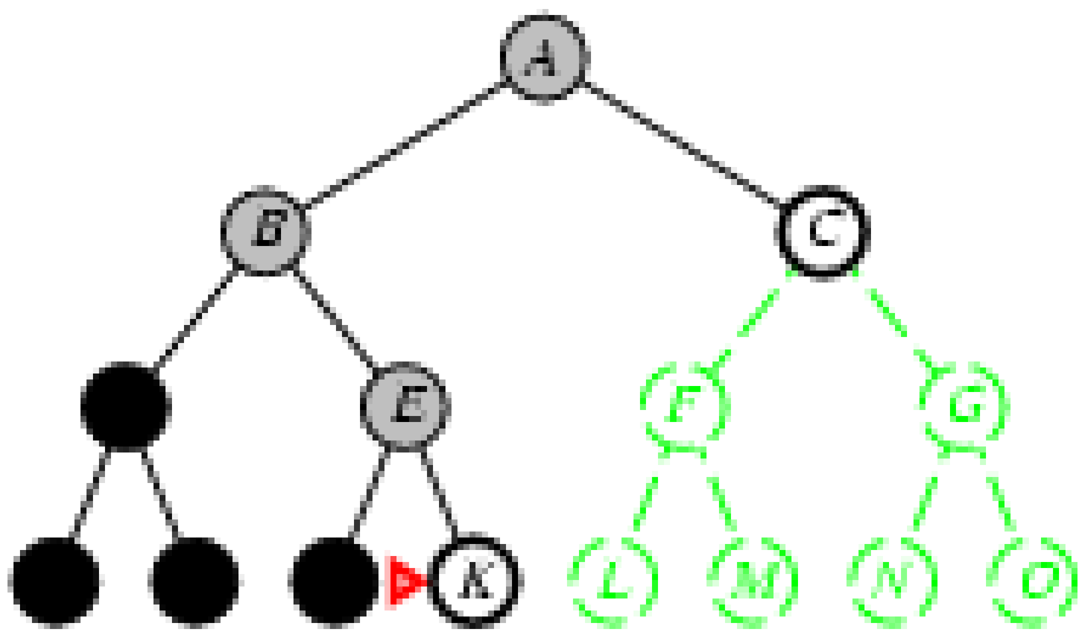

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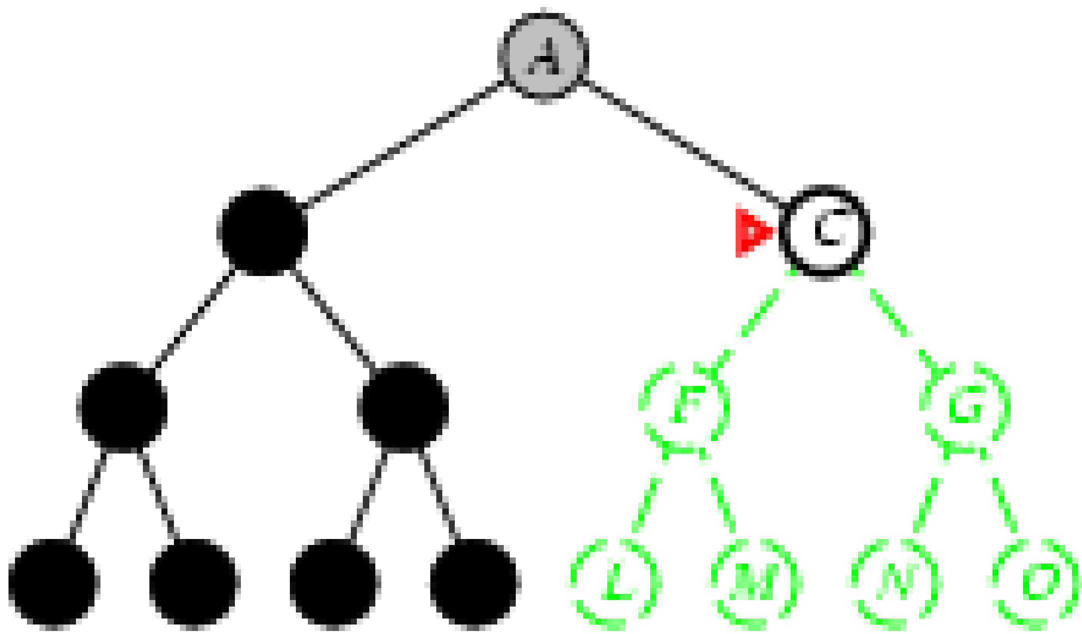

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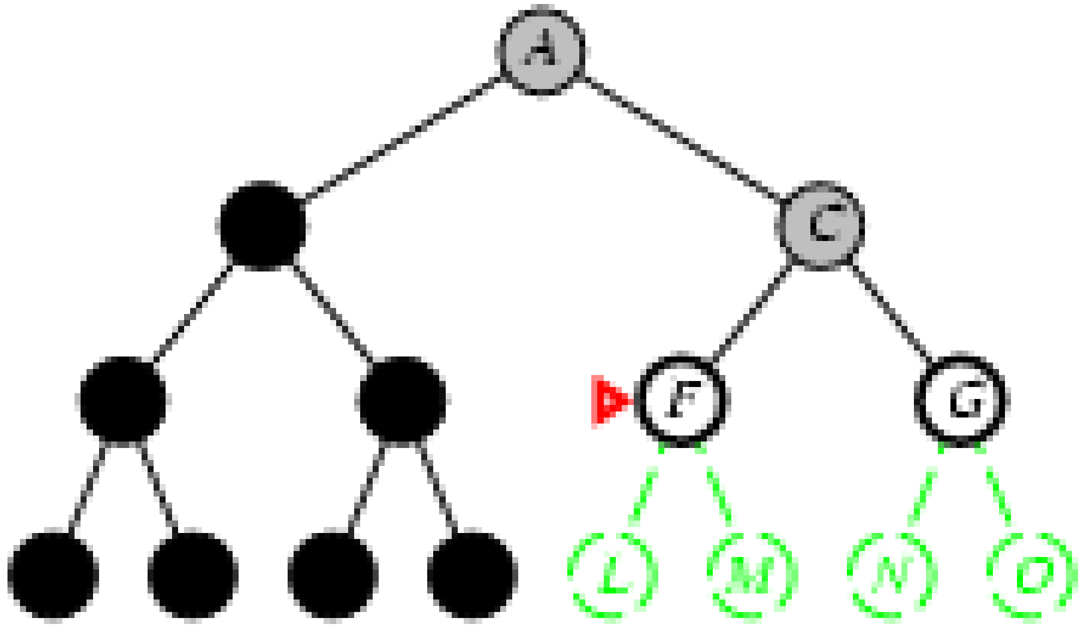

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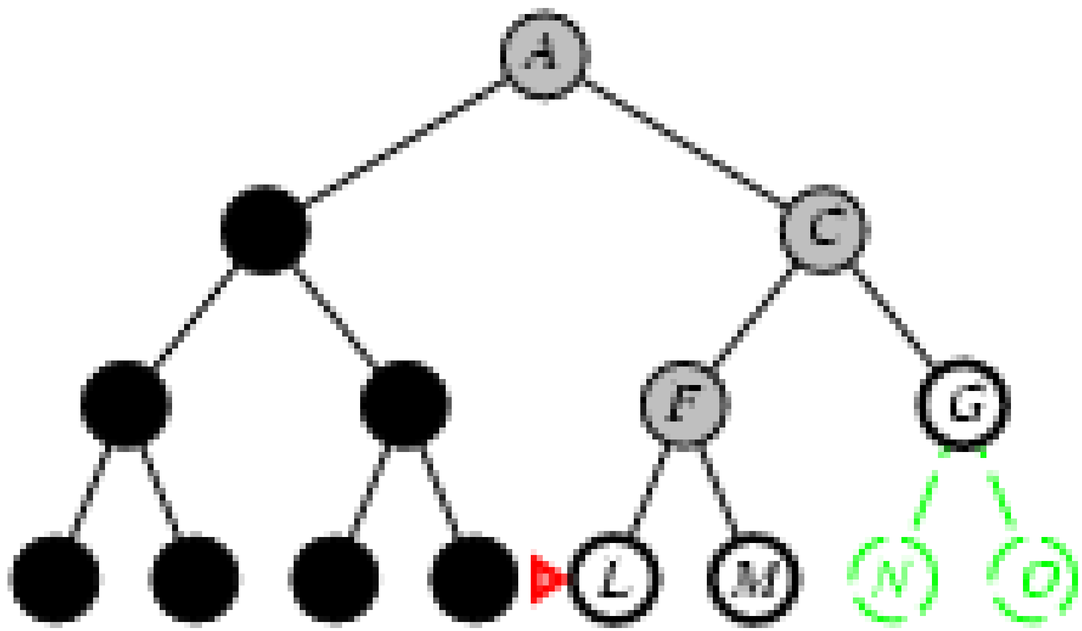

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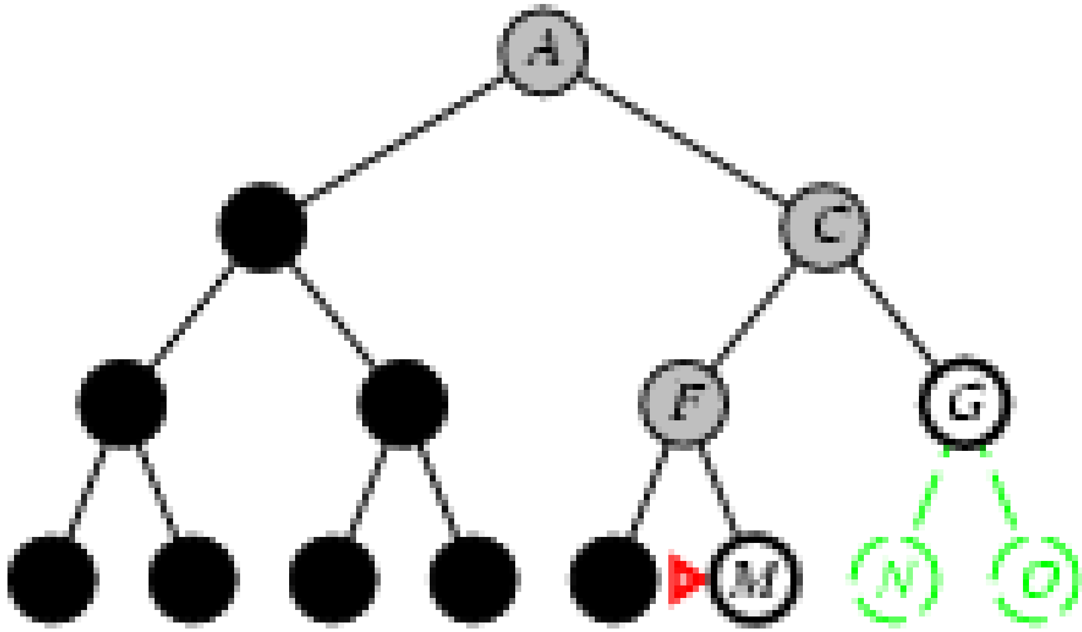


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
    → complete in finite spaces

- Time? $O(b^m)$: terrible if m is much larger than d
  - but if the tree is dense, may be much faster than breadth-first

- Space? $O(bm)$, i.e., linear space!

- Optimal? No

Depth-limited search

= depth-first search with depth limit *l*,

i.e., nodes at depth *l* have no successor

Recursive implementation:

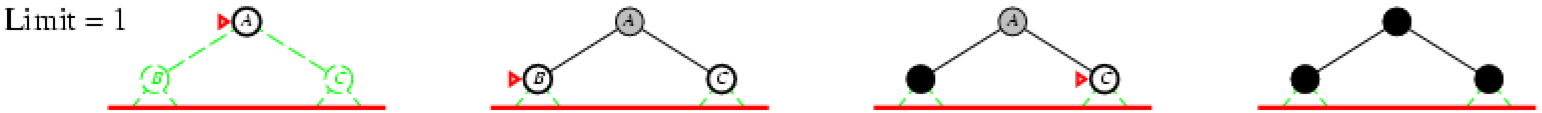
```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

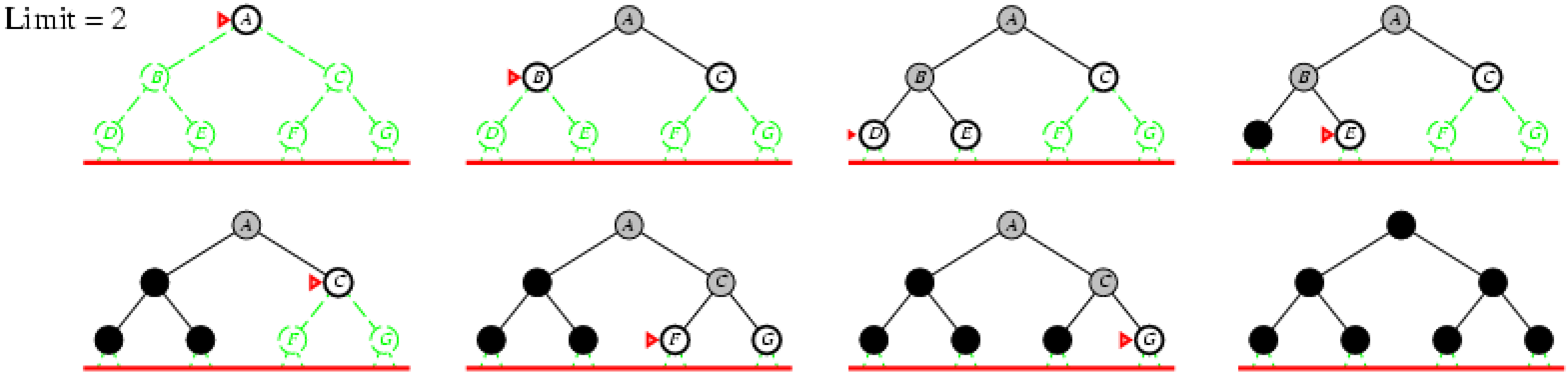
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or fail-
ure
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

Iterative deepening search *l* =0

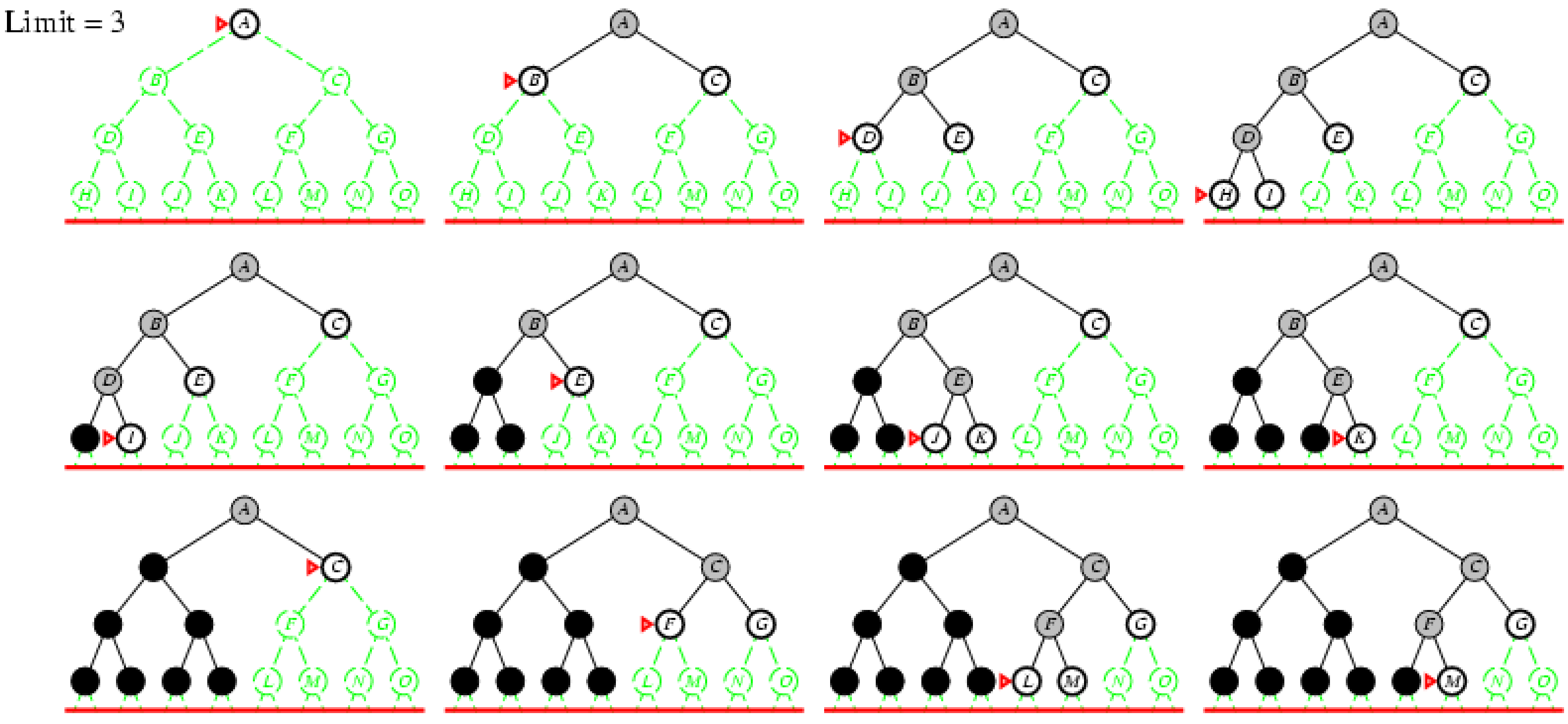
Iterative deepening search *l* =1



Iterative deepening search *l* =2



Iterative deepening search *l* =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{l-2} + b^{l-1} + b^l$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10, d = 5,$
  - N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
  - N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456

- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

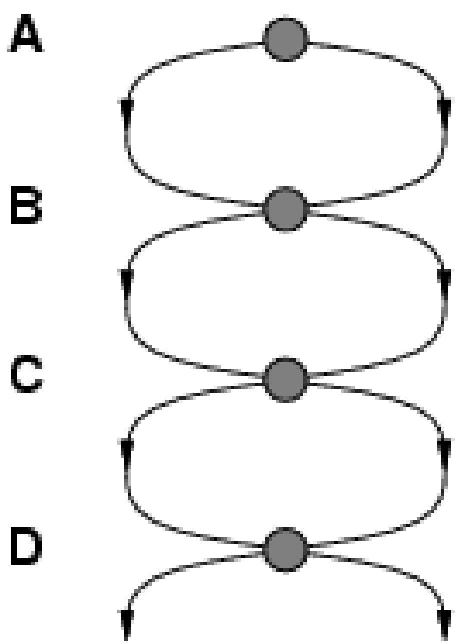
- Complete? Yes
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

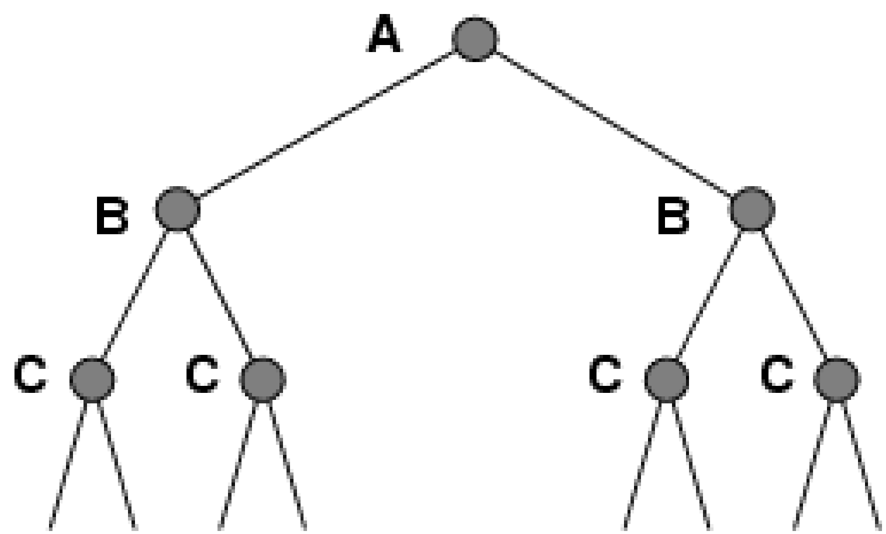
Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!





Graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
      if STATE[node] is not in closed then
          add STATE[node] to closed
          fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Overall Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Exam I Review (1)

- Problem Characteristics
  - Observable
  - Deterministic
  - Episodic
  - Static
  - Single Agent
- Agent Types
  - Simple Reflex
  - Model-based
  - Goal-based
  - Utility-based
  - (Learning)
- Problem Types
  - – Single State
  - – Sensorless (Conformant)
  - – Contingency
  - – Exploration
- Uninformed Search Strategies
  - – Complete?
  - – Optimal?
  - – Time
  - – Space

Exam I Review (2)

Uninformed Search Strategies

- Breadth-first
- Best-first
- Depth-first
- Depth-limited
- Iterative Deepening
- Bidirectional

Uniform-cost

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes