

- ①  $I_R = I_T = I_{in}$  where  $I_R = \frac{V_{in} - V}{R}$  and  $I_T = I_S e^{V_{in}/U_T}$
- so  $\frac{V_{in} - V}{R} = I_S e^{V_{in}/U_T}$   $\Rightarrow V_{in} = I_S e^{V_{in}/U_T} R + V$  eq. 1
- and we know  $V_{on} = U_T \ln(I_{on}/I_S)$  and  $I_{on} = \frac{U_T}{R}$
- so  $N_{on} = U_T \ln\left(\frac{U_T}{RI_S}\right)$

$$\left(\frac{V_{on}}{U_T}\right) = \frac{U_T}{RI_S} \Rightarrow RI_S = U_T e^{-V_{on}/U_T} \text{ eq. 2.}$$

plugging eq. 2 into eq. 1:

$$V_{in} = U_T (e^{-V_{on}/U_T} e^{V_{in}/U_T}) + V \Rightarrow V_{in} = U_T (e^{(V_{in} - V_{on})/U_T}) + V$$

- b) when  $V < V_{on}$  by a few  $U_T$ ;  $e^{(V - V_{on})/U_T} \approx 0$ , so

$$V_{in} \approx V$$

$$\text{and } I = I_S e^{V_{in}/U_T} \text{ where } V = V_{in}$$

$$\text{so } I = I_S e^{V_{in}/U_T}$$

- c) when  $V > V_{on}$  i.e.  $e$ -term becomes very large

$$\text{so } V_{in} \approx U_T e^{V - V_{on}/U_T}$$

$$\text{and } I = \frac{V_{in}}{R} \text{ b/c } V \text{ is negligible compared to } V_{in}$$

②

- a) if  $V_1 < V_2$  by several  $U_T$ :  
using the second equation  
 $V = V_1 - U_T \log(1 + e^{(V_1 - V_2)/U_T})$   
 $e^{(V_2 - V_1)/U_T} \approx 0$  and  $\log(1 + 0) = 0$   
therefore

$$V = V_1 - U_T(0) = V_1$$

if  $V_1 > V_2$  by several  $U_T$ :  
using the first equation  
 $V = -U_T \log(e^{-V_1/U_T} + e^{-V_2/U_T})$   
 $e^{-V_1/U_T} \approx 0$

therefore

$$V = -U_T \log(e^{-V_2/U_T}) = -U_T \cdot \frac{-V_2}{U_T} = V_2$$

$$V = V_2$$

- b)  $V_{in} = U_T e^{(V_{on} - V)/U_T} + V$  from problem 1a  
solve for  $V$ :  $(V_{on} - V)/U_T$   
 $V = V_{in} - U_T e^{(V_{on} - V)/U_T}$

which is very similar to the SMF (soft minimum function)

so  $V = V_{in}$  } so when  $V_{in} < V_{on}$ ,  $V \approx V_{in}$ .

$V_1 = V_{in}$  } when  $V_{in} > V_{on}$ ,  $V \approx V_{on}$ ;

$V_2 = V_{on}$

or, precisely  $\Rightarrow V = -U_T \ln(e^{V_{in}/U_T} + e^{-V_{on}/U_T})$

$$c) I = \frac{V}{R}$$

$$\text{so } I = \frac{V_{in} - V}{R} = \frac{V_{in} - V_{in} - U_T \ln(1 + e^{(V_{in} - V_{on})/U_T})}{R}$$

$$I = \frac{-U_T \ln(1 + e^{(V_{in} - V_{on})/U_T})}{R}$$

when  $V_{in} < V_{on}$   $\Rightarrow I = 0$

when  $V_{in} > V_{on}$  then  $\ln(1 + e^{(V_{in} - V_{on})/U_T}) \approx \ln(e^{(V_{in} - V_{on})/U_T})$

$$\ln(e^{(V_{in} - V_{on})/U_T}) = \frac{V_{in} - V_{on}}{U_T} \Rightarrow I = \frac{-V_{in}}{R}$$

$$d) V = U_T \ln\left(\frac{I}{I_S}\right) = -U_T \ln(e^{-V_{in}/U_T} + e^{-V_{on}/U_T})$$

$$I = I_S (e^{+V_{in}/U_T} + e^{+V_{on}/U_T})$$

when  $V_{in} < V_{on}$   $\Rightarrow I = I_S e^{V_{on}/U_T}$

when  $V_{in} > V_{on}$   $\Rightarrow I = I_S e^{V_{in}/U_T}$