Documentation of gravity-powered generator

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1 Initial goals and planning

The github repository for this project can be cloned from https://github.com/bringsyrup/GravityPowered.git

The goal is to co-design a gravity-powered generator that can power lighting in an operating theater for a sustained time, at least 20 minutes, before needing to be reset. The python scripts (found in the repository link above) written based on this document aim to play with and eventually optimize the system.

It is important to have the technicians and engineers in Uganda co-design the final system so that they may repair and improve the system independently.

2 Visualizing the basic system

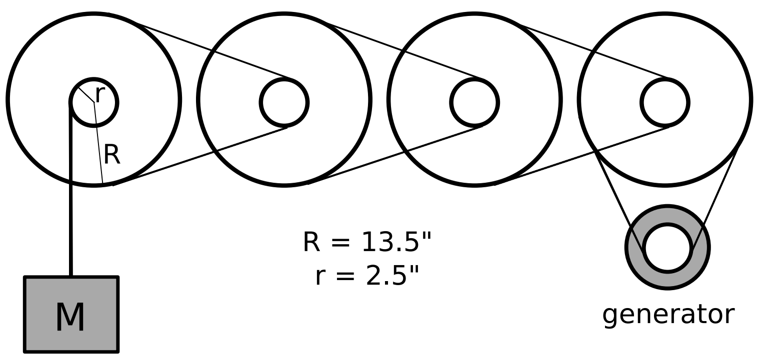

Figure 1: A simplified design of a gravity-powered generator with a gear ratio of 850:1 (due to 5 gear steps). The layout in the figure is for the sake of clear viewing.

The gear ratio of the system in Fig. **??** is 850:1, calculated as follows:

A gear ratio, GR, is defined such that from one radius, r, to the next, R, the ratio is

$$GR = R^{n-1} : r^{n-1}$$

where n is the number of transitions from r to R.

In Fig. **??**, $n = 5$ and $R = 13.5$" and $r = 2.5$", therefore:

$$GR = 13.5^4 : 2.5^4 = 850 : 1$$

It is important to recognize a few key physical facts about the system. In no way does this system produce a power advantage. The mechanical power remains constant throughout each step in the gear-up system, assuming no frictional losses. The mechanical advantage of this system comes from a trade-off of rotational velocity for torque: as rotational velocity increases due to the gear ratio, torque decreases, thus conserving the energy of the system. this is easily seen in the equation:

$$P_{mechanical} = \tau\omega$$

3 Playing with the system: calculating required mass and distance given power load, desired run-time, and number of gear-ups

Input power load P_e, desired run-time t, and number of gear-ups, n, can be used to calculate the required mass, m and vertical drop, d.

The mass m can be calculated in as follows:

$$\begin{aligned}
\tau_m &= Fr = Mgr \\
M &= \frac{\tau_m}{gr} \qquad (1)
\end{aligned}$$

where τ_m and F are the torque and force exerted by m on the first wheel

Due to the gear ratio we also have:

$$\tau_m = \tau_g \left(\frac{R}{r}\right)^{n-1} \qquad (2)$$

where τ_g is the torque on the generator. Substituting Eq. 2 into Eq. 1 yields:

$$m = \tau_g \frac{R^{n-1}}{gr^n} \qquad (3)$$

The torque τ_g and rpm s required to power a given load P_e can be calculated from characterizing the system's generator (see Fig. **??** in the Appendix for example). Data for power load P_e versus generator efficiency E can be used to find the required mechanical power P_m:

$$E = \frac{P_e}{P_m} \longrightarrow P_m = \frac{P_e}{E}$$

Data for P_m versus rotational velocity ω for varying P_e loads can be used to calculate τ_g:

$$P_m = \tau_g\omega \longrightarrow \tau_g = \frac{P_m}{\omega}$$

The vertical drop d of m can be calculated from ω and t:

$$\begin{aligned}
v_m &= \frac{d}{t} \\
d &= v_m t \qquad (4)
\end{aligned}$$

where v_m is the velocity of mass M. The gear ratio informs us that:

$$v_m = v_g \left(\frac{r}{R}\right)^{n-1} \qquad (5)$$

where $v_g = r\omega$. Substituting Eq. 5 into Eq. 4:

$$d = v_g t \left(\frac{r}{R}\right)^{n-1} \qquad (6)$$

the challenge is taking data for P_m. We have no inexpensive method for measuring torque (dynamometers are usually close to \$2,000), so the equation $P_m = \tau_g \omega$ can't be used for collecting data. Alternatives equations include:

$$P_m = \phi_m I_e \omega \ *$$

where ϕ_m is the magnetic flux of the generator and I_e is the current, or

$$P_m = P_e + n I_e^2 R_e \ **$$

where n is the number of phases in the generator and R_e is the resistance in each phase.

Regardless of the method used to calculate P_m, until the required data is collected from the appropriate generators in Uganda, I will use fake data or data taken from locally available generators.

* http://www.arrakis.nl/reports/GeneratorsTest.pdf

** http://people.ucalgary.ca/~aknigh/electrical_machines/synchronous/design/power_torque.html

4 Optimizing the system for power efficiency from maximum mass, set vertical distance, and desired run-time

Here I will discuss the methods used to calculate the optimal power load for a given system from efficiency characterization data under mass, distance, and time constraints.

Given constrained max mass m_{max}, distance d, and time t, the maximum mechanical power of the system, P_{max} is calculated:

$$P_{max} = m_{max} g \frac{d}{t}$$

where g is the gravitational constant.

Next, given the best-fit polynomials (see Fig. **??** in appendix for example), the range from $0 - P_{max}$ is used to calculate the maximum power load, $P_{load,max}$. In turn, the range from $0 - P_{load,max}$ is used to get the range of possible efficiencies. Finally, the maximum (optimal) efficiency in that range, E_{opt}, is returned and used to get the optimal power load, $P_{load,opt}$. But we still need to calculate the corresponding optimal mass. We can use the same equation we used to calculate the P_{max}, solving for mass instead of power, but first we need to get the corresponding optimal mechanical power, which is simply $P_{opt} = \frac{P_{load,opt}}{E_{opt}}$. Now:

$$m_{opt} = P_{opt} \frac{t}{gd}$$

The number of gears-ups, n, required for the optimal load and mass is calculated as follows:

$$\begin{aligned} P_{opt} &= \tau_o \omega_o \\ &= m_{opt} g r \frac{r}{R}^{n-1} \omega_f \end{aligned} \tag{7}$$

where τ_o and ω_o are the torque and rotational velocity of the first gear (the gear attached to the mass) and ω_f corresponds to the generator shaft (Fig. **??**).

Algebra yields the following result for n:

$$n = log_{\frac{r}{R}} \frac{P_{opt}}{R \omega_f m_{opt} g}$$

5 Appendix

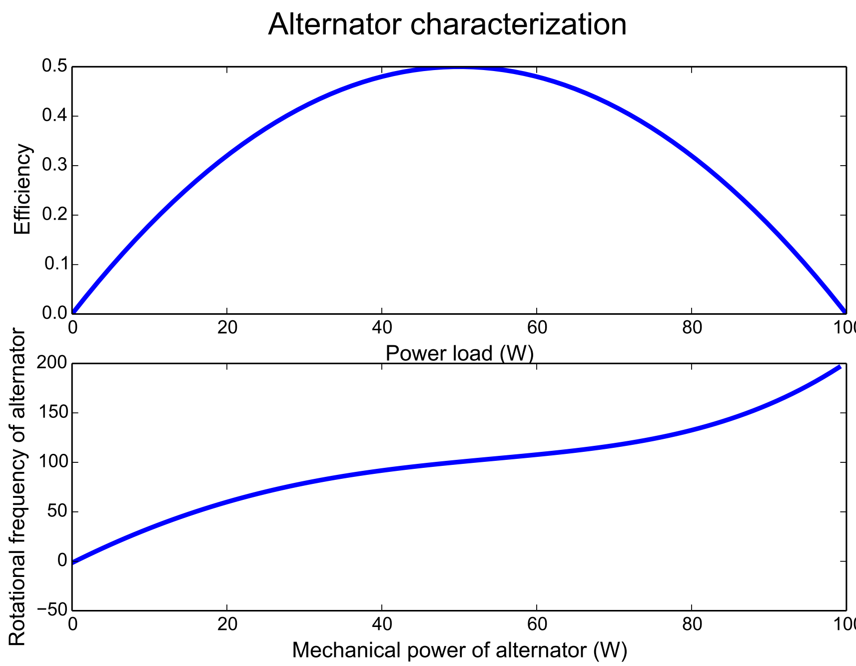

Figure 2: Example best-fit polynomials from fake generator characterization data

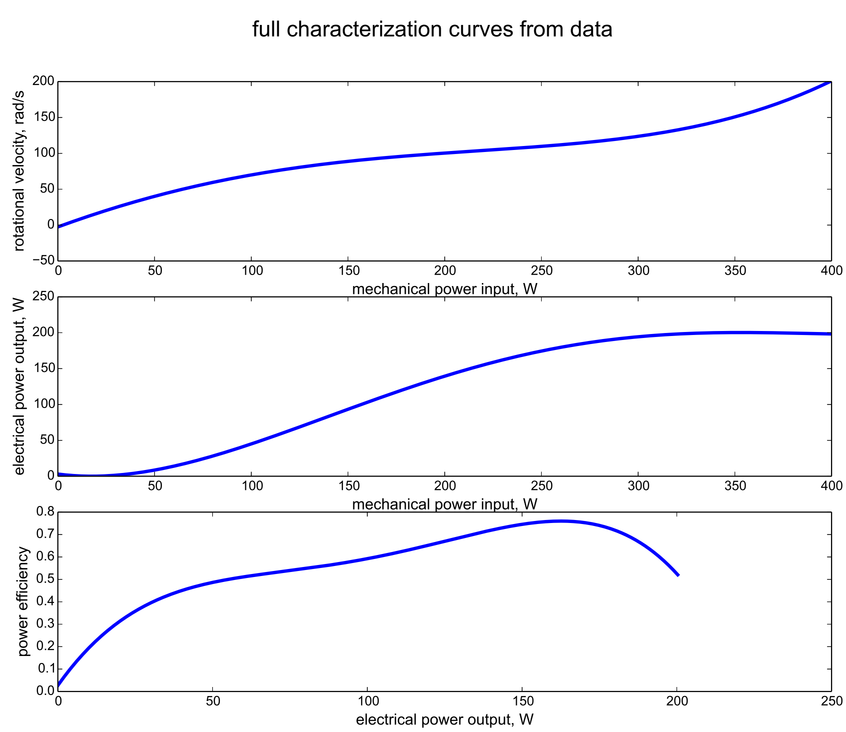

Figure 3: Example best-fit polynomials from fake generator characterization data