Franklin W. Olin College of Engineering

Signals and Systems – Spring 2015

PROBLEM SET 7

Problems

1. Consider a train of unit impulses separated by T time units, given by the following expression

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \tag{1}$$

   a. Sketch a representation of $p(t)$.

   b. Find the Fourier series representation of $p(t)$ with an infinite number of terms.

   c. Let a function $x(t)$ be represented as a Fourier series with an infinite number of terms as follows:

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}k\,t}\,. \tag{2}$$

   Find $X(\omega)$, in terms of C_k.

   d. Using your answer to the previous two parts, find $P(\omega)$.

   e. Sketch $P(\omega)$. How does changing T affect $p(t)$ and $P(\omega)$? Is this what you would expect?

2. Consider an LTI system with an impulse response $h(t)$, input signal $x(t)$ and output $y(t)$. It is known that $H(\omega)$ is the following.

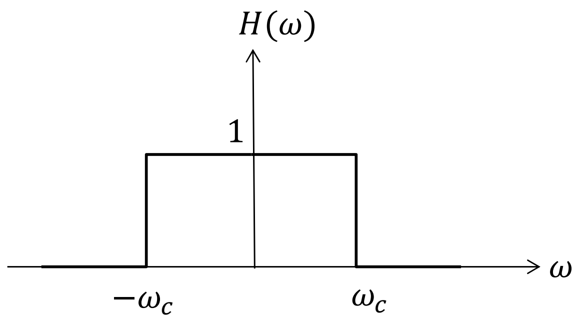

Figure 1: Ideal low pass filter

a. Find $h(t)$.

b. Suppose that $X(\omega)$ is the following. Please sketch $Y(\omega)$.

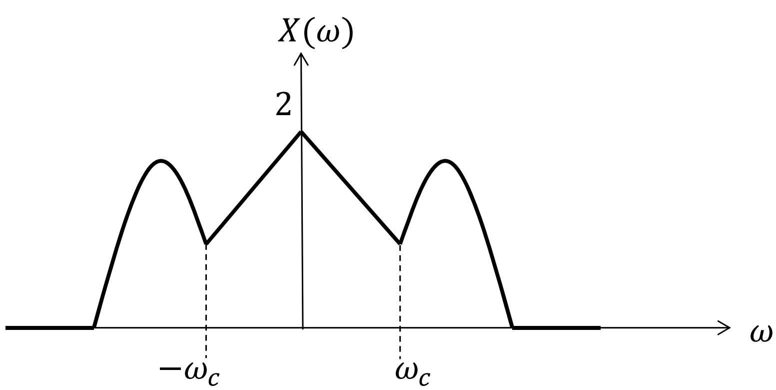

Figure 2: Spectrum of a representative signal.

c. Explain why this LTI system is known as an ideal low-pass filter with cut-off frequency ω_c.

d. Pull code from the GitHub repository sgovindasamy/SigSys2015, and load the SquareWaveFilterExercise.ipynb notebook. Consider the Fourier series representation of the square wave in Figure 3, with $T = 4$. Recall that this square wave has frequency components at integer multiples of $\frac{\pi}{2}$.

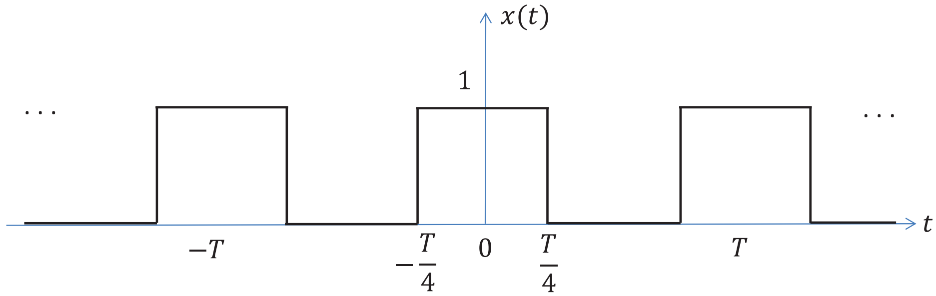

Figure 3: Square wave with period T.

Modify the code in the third cell of the ipython notebook to implment a low-pass filter with cutoff frequency of $\omega_c = 0.75\,\pi$. Run the code to see the filtered version of the square wave. Repeat this for $\omega_c = 1.75\pi$. You should turn in the plots that are generated with both cut-off frequencies.

3. Consider a signal $x(t)$ which is band-limited to the range $[-\omega_M, \omega_M]$. In other words, $X(w) = 0$ for $\omega < -\omega_M$ and $\omega > \omega_M$. Suppose that $X(\omega)$ is given in Figure 4. Let $y(t) = x(t)\cos(\omega_c t)$, where $\omega_c \gg \omega_M$.

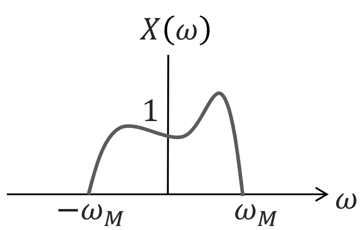

Figure 4: $X(\omega)$.

Please sketch $Y(\omega)$.

Note that multiplying a relatively low bandwidth signal with a high frequency cosine (or sine) wave is the basis of amplitude modulation (AM), which is an old technique for broadcast radio, but is at the heart of almost all modern wireless communications systems. Translating a low frequency signal to a much higher frequency enables us to transmit wireless signals using reasonably sized antennaesince the physical sizes of antennas need to be on the order of magnitude of the wavelength of the signal in order to radiate efficiently. Moreover, by assigning different carrier frequencies (i.e. ω_c) to different users, multiple users (e.g. radio stations), can share the same propagation medium with negligible interference. This is why different radio stations operate at different frequencies.