Franklin W. Olin College of Engineering

Signals and Systems – Spring 2015

PROBLEM SET 8

Problems

1. Consider a signal $x(t)$ which is band-limited to ω_M. $x(t)$ and $X(\omega)$ are shown in Figure 1

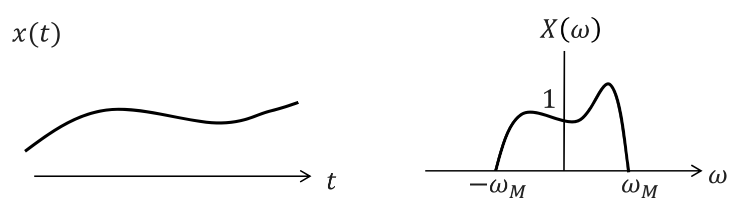

Figure 1: Representative band-limited signal.

Additionally, let $p(t)$ be an impulse train with impulses separated by T_s. In other words,

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s) \tag{1}$$

Further, define

$$x_p(t) = x(t)p(t) \tag{2}$$

Note that the first 5 parts of this problem were covered in class and are thus optional in this homework. You are encouraged (strongly encouraged, if you weren't in class) to do them to reinforce the important concepts that they illustrate. Moreover, they will be used in the latter parts of this problem.

a. Sketch a representation of $x_p(t)$. (Optional)

b. Sketch $P(\omega)$. (Optional)

c. Sketch $X_p(\omega)$. (Optional)

d. What is the relationship between T_s and ω_M which ensures that $X_p(\omega)$ contains all the information present in $X(\omega)$. (Optional)

e. How can you recover $x(t)$ from $x_p(t)$ exactly? (Optional)

f. Consider a new signal $z(t)$ which is shown in Figure 2.

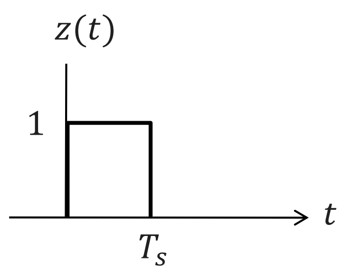

Figure 2: Rectangular pulse.

g. Sketch $x_z(t) = x_p * z(t)$. Note that $x_z(t)$ is a zero-order hold reconstruction of the sampled signals $x_p(t)$, and is a method used in many (particularly lower-cost) Digital-to-analog converters.

h. Sketch $X_z(\omega)$.

i. Sketch $\bar{X}(\omega) = X_z(\omega)H(\omega)$ and $\hat{X}(\omega) = X_p(\omega)H(\omega)$ where $H(\omega)$ is shown in Figure 3 with $\omega_c = \frac{\pi}{T_s}$.

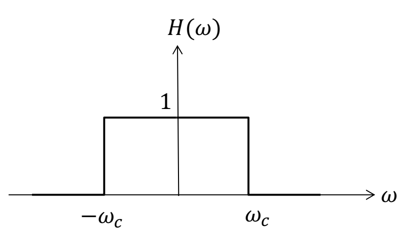

Figure 3: Ideal low-pass filter.

j. How are $\bar{X}(\omega)$ and $\hat{X}(\omega)$ different?

k. What is the ratio of $\bar{X}(\omega_M)$ to $\hat{X}(\omega_M)$, when $\omega_M = \frac{\pi}{T_s}$?

2. Consider a signal $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$. Suppose that $X_1(\omega) = 0$ and $X_2(\omega) = 0$ if $|\omega| > \omega_M$. Assume that $\omega_1 \gg \omega_M$ and $\omega_2 \gg \omega_M$, and $\omega_1 + 2\omega_M < \omega_2$. Suppose that $X_1(\omega)$ and $X_2(\omega)$ are given by the following:

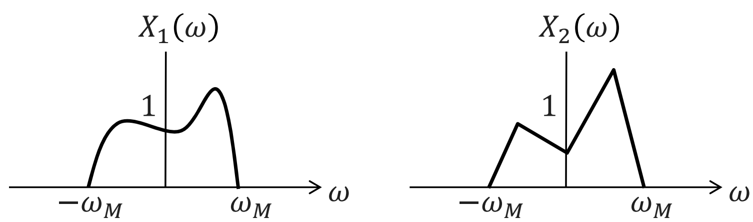

Figure 4: $X_1(\omega)$ and $X_2(\omega)$.

a. Please sketch $Y(\omega)$.

b. Please sketch the Fourier transforms of $y(t)\cos(\omega_1 t)$ and $y(t)\cos(\omega_2 t)$.

c. Using appropriate sketches, describe how you can recover $x_1(t)$ and $x_2(t)$ from $y(t)$. We can view $y(t)$ as the received AM radio signal from two different AM transmitters, one using the frequency ω_1 and ω_2.

3. Consider the RLC circuit in Figure 5. Recall the following.

$$i(t) = C\frac{d}{dt}v_{out}(t) \tag{3}$$

$$v_L(t) = L\frac{d}{dt}i(t) \tag{4}$$

a. Write a differential equation relating $v_{out}(t)$ and $v_{in}(t)$.

b. Treating $v_{in}(t)$ and $v_{out}(t)$ respectively as inputs and output of an LTI system, find an expression for the frequency response $H(\omega)$, of the system.

c. Find an expression for the magnitude of $H(\omega)$.

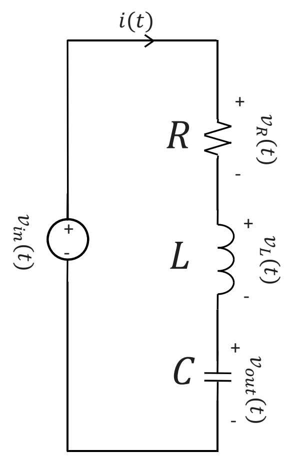

Figure 5: RLC System

d. As a function of R, L and C, find the value of ω that maximizes $|H(\omega)|$. Hint: remember calculus!

e. Plot the magnitude and phase of the frequency response for the following sets of values. You should use a log-log scale for the magnitude and a log-linear scale for the phase. You may find the Matplotlib functions `loglog` and `semilogx` useful here.

   i. $C = 10^{-7}F, L = 10^{-2}H, R = 400\Omega$.

   ii. $C = 10^{-7}F, L = 10^{-2}H, R = 50\Omega$.