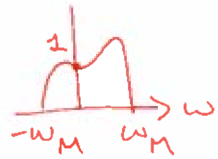
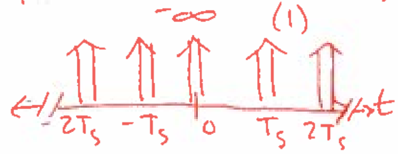
PS08 — Ruby S.

note to Ry: there are far too many pictures in this pset to bother with latex. sorry :)!

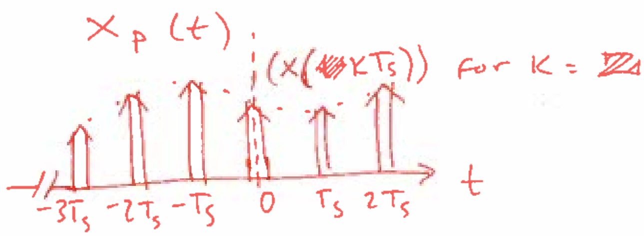
① $x(t)$ — $X(\omega)$ — $p(t) = \sum_{-\infty}^{\infty} \delta(t - kT_s)$ for $k = \mathbb{Z}$





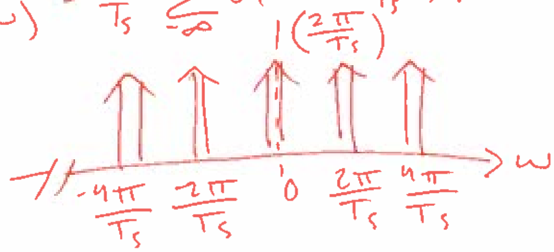
also $x_p(t) = x(t)\,p(t)$

a)

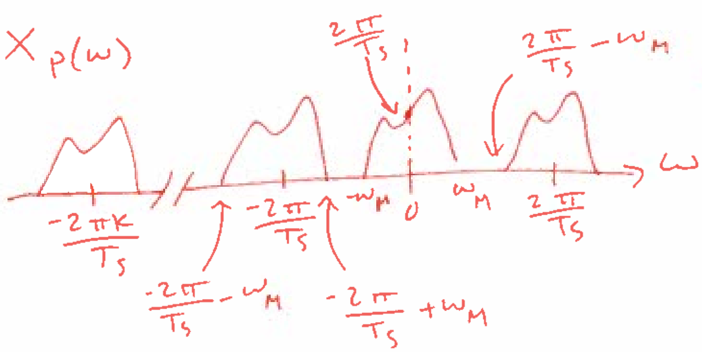


b)

$$P(\omega) = \frac{2\pi}{T_s} \sum_{-\infty}^{\infty} \delta\left(t - \frac{2\pi k}{T_s}\right) \text{ for } k = \mathbb{Z}$$



c)



*assuming $\frac{\pi}{T_s} > \omega_M$

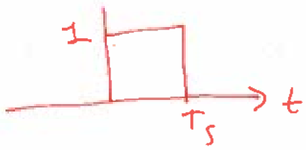
$\therefore T_s < \frac{\pi}{\omega_M}$

d) there must be no overlap of the signal copies,

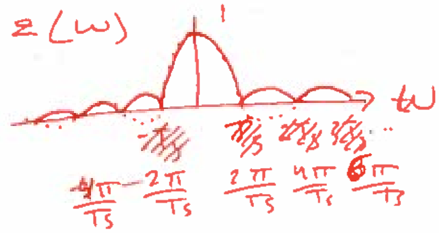
so $\frac{2\pi}{T_s} > 2\omega_M \rightarrow \boxed{T_s < \frac{\pi}{\omega_M}}$

e) you can recover the original signal by filtering out all but the copy centered at $t = 0$ with a low-pass filter where the cutoff $f_c \rightarrow \omega_M < f_c < \frac{2\pi}{T_s} - \omega_M$

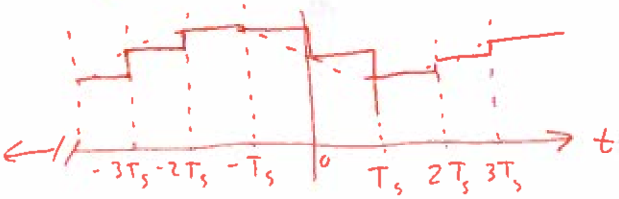
f) $z(t)$



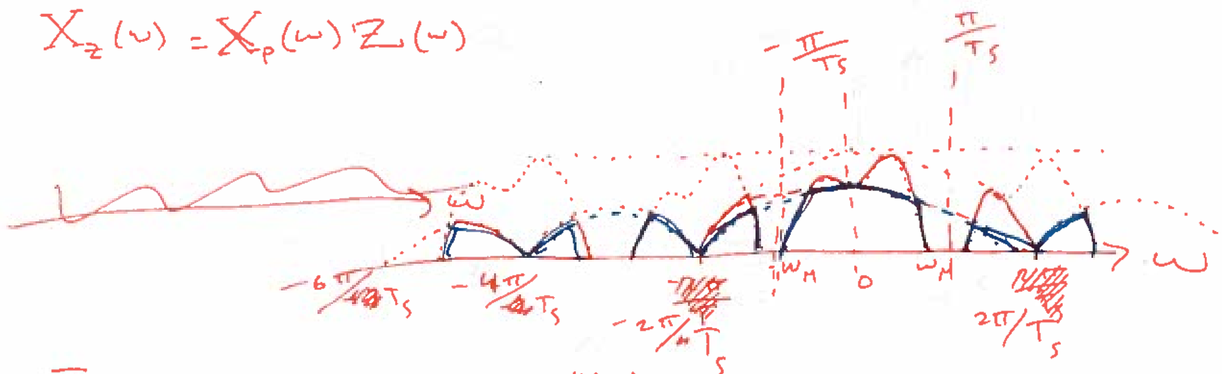
$Z(\omega)$



g) $x_z(t) = x_p * z(t)$

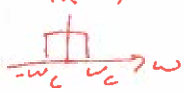


h) $X_z(\omega) = X_p(\omega) Z(\omega)$

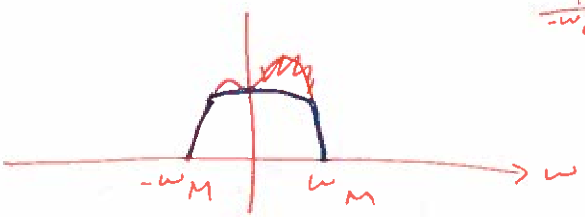


i) $\bar{X}(\omega) = X_z(\omega) H(\omega)$

$H(\omega)$



$\hat{X}(\omega) = X_p(\omega) H(\omega)$

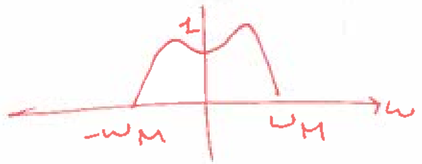


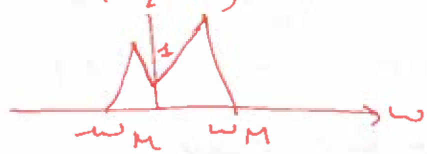
j) $\bar{X}(\omega)$ and $\hat{X}(\omega)$ have different ~~amp~~ amplitudes at lower frequencies.

k) $\dfrac{\bar{X}\left(\frac{\pi}{T_s}\right)}{\hat{X}\left(\frac{\pi}{T_s}\right)} = 1$

② $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

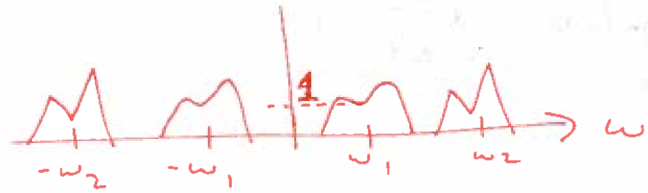
$X_1(\omega) = 0$ & $X_2(\omega) = 0$ if $|\omega| > \omega_M$ and $\omega_1 >> \omega_M$ and $\omega_2 >> \omega_M$ and $\omega_1 + 2\omega_M < \omega_2$





a) $Y(\omega) = X_1(\omega)(\pi\delta(t-\omega_1) + \pi\delta(t+\omega_2))$

$+ X_2(\omega)(\pi\delta(t-\omega_2) + \pi\delta(t+\omega_2))$

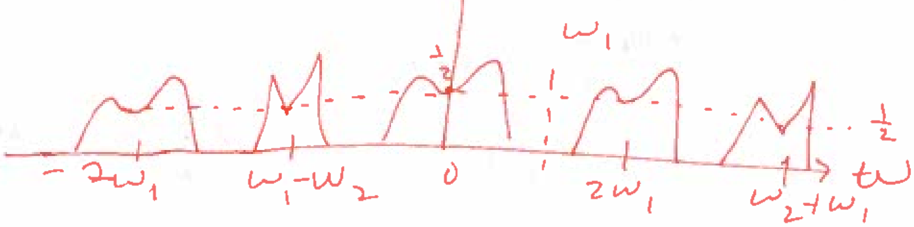


b) $y(t)\cos(\omega_1 t)$ and $y(t)\cos(\omega_2 t)$

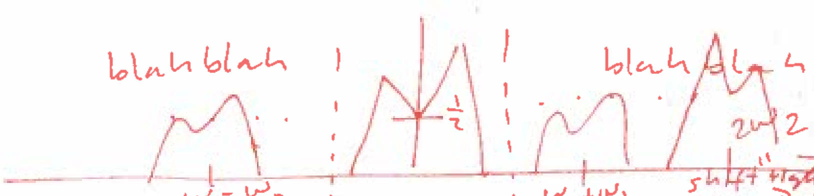
$F(y(t)\cos(\omega_1 t))$

$= F(x_1(t)(\frac{1}{2}\cos(0) + \frac{1}{2}\cos(2\omega_1 t)) + x_2(t)(\frac{1}{2}\cos((\omega_2-\omega_1)t + \frac{1}{2}\cos((\omega_2+\omega_1)$

shifts $\frac{1}{2}x_1(t)$ to center!



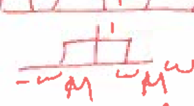
$F(y(t)\cos(\omega_2 t))$ → shifts the second signal to center!



shift "right" → ← shift "left"

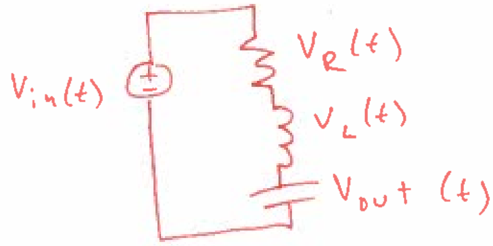
c) - shift sig 1 or sig 2 →

- filter centered signal w/



- inverse fourier transform using sinc function!

3 RLC circuit



$$i(t) = C\frac{dV_{out}}{dt}$$

$$V_L(t) = L\frac{di(t)}{dt}$$

$$V_R(t) = Ri(t)$$

a) $V_{in} = V_{out} + V_R + V_L$

$$= V_{out} + RC\frac{dV_{out}}{dt} + LC\frac{dd\,V_{out}}{dt^2}$$

$$\boxed{V_{out} = V_{in} - RC\frac{dV_{out}}{dt} - LC\frac{dd\,V_{out}}{dt^2}}$$

b) say $V_{in} = e^{j\omega t}$

$$H(\omega)e^{j\omega t} = V_{out}$$

$$\cancel{e^{j\omega t}}H(\omega) = \cancel{e^{j\omega t}} - (j\omega \cancel{e^{j\omega t}} RC - j^2\omega^2 \cancel{e^{j\omega t}} LC)H(\omega)$$

$$H(\omega) = 1 - H(\omega)RCj\omega + H(\omega)LC\omega^2$$

$$0 = 1 - H(\omega)(RCj\omega + LC\omega^2 - 1)$$

$$\boxed{H(\omega) = \frac{1}{RCj\omega + (LC\omega^2 - 1)}}$$

c) $$\boxed{|H(\omega)| = \frac{1}{\sqrt{(RC\omega)^2 + (LC\omega^2 - 1)^2}}}$$

phase response

$$\phi\ \angle H(\omega) = \cancel{\tan}\angle(1/[RCj\omega + (LC\omega^2 - 1)])$$

$$= \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{RCj\omega}{LC\omega^2 - 1}\right)$$

$$\phi = -\tan^{-1}\left(\frac{RCj\omega}{LC\omega^2 - 1}\right)$$ ← is that right? it produces a weird phase response graph...

d) $(RC\cancel{j}\omega)^2 + (LC\omega^2 - 1)^2 = 0$

$$R^2C^2\omega^2 + L^2C^2\omega^4 - 2LC\omega^2 + 1 = 0$$

$$(L^2C^2)\omega^4 + (R^2C^2 - 2LC)\omega^2 + 1 = 0$$

whatever, it's where ever the denominator = 0

b/c $\lim_{x \to 0}\left(\frac{a}{bx}\right) = \infty$