P1

$$Y(s) = \int_{-\infty}^{\infty} (1 - e^{-t})\, u(t)\, e^{-st}\, dt$$

$$= \mathcal{L}\{u(t)\} - \mathcal{L}\{e^{-st} u(t)\} = \frac{1}{s} - \frac{1}{s+1}$$

$$= \frac{s+1}{s^2+s} - \frac{s}{s^2+s} = \frac{+1}{s^2+s} = \boxed{\frac{+1}{s(s+1)}}$$

$$\dot{y} + y = x$$

$$sY(s) + Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$\mathcal{L}\{u(t)\} = \mathcal{L}\{x(t)\}$$

$$X(s) = \frac{1}{s}$$

$$\boxed{Y(s) = \frac{1/s}{s+1} = \frac{1}{s^2+s}}$$

P2

a) want $\dfrac{Y(s)}{Y_{sp}(s)}$

$$E(s) = Y_{sp} - Y$$

$$X(s) = E \cdot K$$

$$Y(s) = H \cdot X$$

$$X = E \cdot K$$

$$= (Y_{sp} - Y)K$$

$$= (Y_{sp} - HX)K$$

$$X + KHX = KY_{sp}$$

$$X(1+KH) = KY_{sp}$$

$$\frac{X}{Y_{sp}} = \frac{K}{1+KH}$$

$$\frac{\frac{Y}{H}}{Y_{sp}} = \frac{K}{1+KH}$$

$$\boxed{\frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH}}$$

DC gain

$$\lim_{s \to 0} \frac{KH}{1+KH}$$

$$\lim_{s \to 0} \frac{K_I/s\, H(s)}{1 + K_I/s\, H(s)}$$

$$\lim_{s \to 0} \frac{K_I H(s)}{s + K_I H(s)} = \boxed{1 \leftarrow \text{DC gain}}$$

hope, the DC gain doesn't depend on K_I

(B) $$\frac{Y(s)}{Y_{sp}(s)} = \frac{K \frac{1/\tau}{s+1/\tau}\left(\cancel{s+\frac{1}{\tau}}\right)\tau}{1+K\frac{1/\tau}{s+1/\tau}\left(\cancel{s+\frac{1}{\tau}}\right)\tau} = \frac{K}{(s+\frac{1}{\tau})\tau + K}$$

$$H(s) = \frac{Y(s)}{Y_{sp}(s)} = \frac{K}{s\tau + 1 + K}$$

$$= \frac{K_I/s\,(s)}{s\tau + 1 + K_I/s\,(s)} = \frac{K_I}{s^2\tau + s + K_I}$$

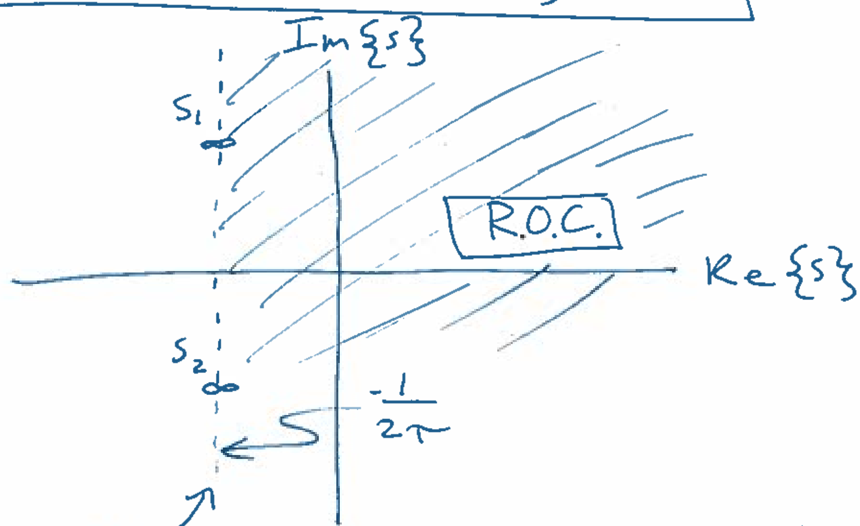
$$s^2\tau + s + K_I = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot K_I}}{2\tau} = \frac{-1}{2\tau} \pm \frac{\sqrt{1-4K_I\tau}}{2\tau}$$

if $K_I \gg 1/\tau \rightarrow 4K_I \gg \frac{4}{\tau} \rightarrow 4K_I\tau \gg 4$

and s is imaginary ~~for both + and -~~ ...

$\boxed{\text{there are 2 imaginary poles}}$



- real component is the same for both poles!
- imaginary is $\pm \frac{\sqrt{1-4K_I\tau}}{2\tau}$

P4

A

$$Y(s) = X(s)H(s) \rightarrow X(s) = U(s)$$

$$= \mathcal{L}(u(t))\,H(s)$$

$$= \frac{1}{s}\;\frac{1}{s^2 - 0.01s + 1}$$

$$Y(s) = \frac{1}{s^3 - 0.01s^2 + s}$$

Just kidding, I didn't need to do that. the plots for p4 are in p4.ipynb

B proportional control

$$K(s) = K_p$$

$$H(s) = \frac{K(s)H(s)}{1 + K(s)H(s)} = \frac{K_p\left(\frac{1}{s^2 - 0.01s + 1}\right)\left(\frac{s^2 - 0.01s + 1}{1}\right)}{1 + K_p\left(\frac{1}{s^2 - 0.01s + 1}\right)\left(\frac{s^2 - 0.01s +}{1}\right.}$$

$$= \frac{K_p}{s^2 - 0.01s + 1 + K_p}$$

C $K(s) = \frac{K_p}{s}$ ← integral control

$(K(t) = \int K_p\ ?)$

$$H(s) = \frac{\frac{K_p}{s}\left(\frac{1}{s^2 - 0.01s + 1}\right)}{1 + \frac{K_p}{s}\left(\frac{1}{s^2 - .01s + 1}\right)} = \frac{\frac{K_p}{s}}{s^2 - 0.01s + 1 + \frac{K_p}{s}}$$

$$= \frac{K_p}{s^3 - 0.01s^2 + s + K_p}$$

D $K(s) = K_p s$ ← differential control

$$H(s) = \frac{K_p}{s - 0.01 + \frac{1}{s} + K_p} = \frac{K_p}{s^2 - 0.01s + 1 + K_p s}$$

$$= \frac{K_p}{s^2 + s(K_p - 0.01) + 1}$$