Franklin W. Olin College of Engineering

Signals and Systems – Spring 2015

Problem set 6

Problems

1. Earlier in the course, you saw how the recorded audio signal of a gun being fired in a shooting range can be convolved with a violin recording to approximate how the violin would sound if played in a shooting range. Please explain this using what you know about the impulse and impulse responses.

2. Consider a simple model of an echo channel. Suppose that the output of the echo channel is $y(t)$ and the input is $x(t)$, and the input and output are related as follows:

$$y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)\,.$$

   Explain why it is reasonable to call this an echo channel and find an expression for the impulse response of this system, and sketch it.

3. a. Find the Fourier series representation for the square wave in Figure 1. You may find the identity $\sin(\theta) = \frac{1}{2j}e^{j\theta} - \frac{1}{2j}e^{-j\theta}$ useful here.

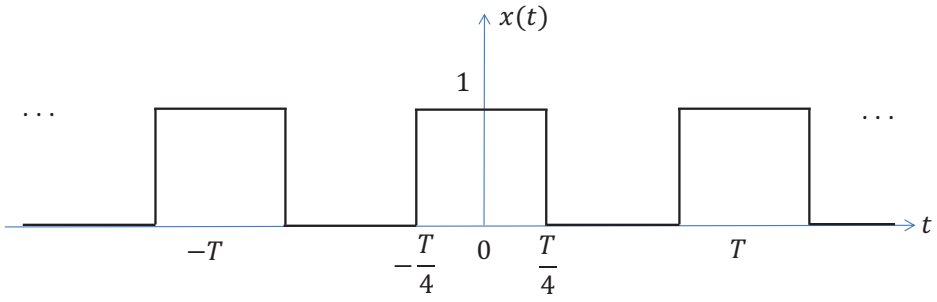

Figure 1: Square wave with period T.

b. Using a computer, plot the Fourier series representation of the square wave in the previous part with fundamental period $T = 4$, and for 5, 17, and 257 terms in the Fourier series. For clarity, you should plot them in separate subplots/plots. Note that the function $\frac{\sin(\pi x)}{\pi x}$ is called the *sinc* function and is available in the numpy package in python.

c. Describe what you see in the Fourier series representation, at the discontinuous points of the square wave, i.e. the points where the square wave goes from 1 to 0 and 0 to 1. How can you reconcile this with (10) in the Fourier series notes? Note: what you should observe is a manifestation of the Gibbs phenomenon, which is caused by the inability of the Fourier series to produce accurate representations of periodic signals at points of discontinuity.

4. a. Suppose that $x(t)$ is a periodic signal with fundamental period T, and has a Fourier series representation with coefficients C_k. Consider a new signal, $y(t) = x(t-T_1)$, where $|T_1| < T$. Thus $y(t)$ is a delayed version of $x(t)$. Find the Fourier series coefficients for $y(t)$ in terms of C_k. Feel free to use symbolic math software to solve this if you wish.

b. Using your answer above, find the Fourier series coefficients for the triangle wave in Figure 2. Verify that your answer is correct by modifying and running the code for the Fourier series of the triangle wave that you used in class. Please turn in a listing of your code and a plot of the triangle wave you generated.

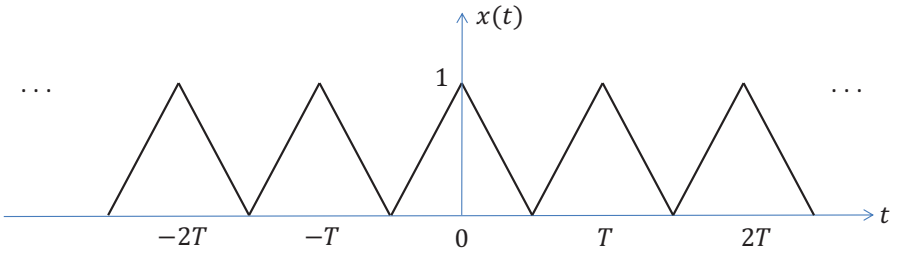

Figure 2: Triangle wave with period T.