Lab 6: Negative Resistance

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Abstract

In this lab we built a 'negative resistor' and proved that it behaved as such by plotting voltage versus current and observing that current and voltage have an inverse relationship in said plot.

1 The Circuit

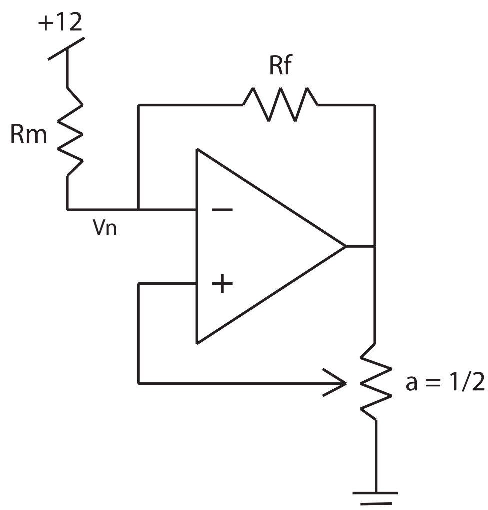

Figure 1: Circuit Diagram of negative resistor in series with R_m

1.1 quantitative analysis

In order to prove that the resistance is 'negative', the configuration of the op-amp in fig.1 must be analyzed for V_n where $a = \frac{1}{2}$.

First we must write V_-, V_+, and V_o in terms of our variables:

$$V_o = Vn - R_f i_n \tag{1}$$
$$V_- = V_n \tag{2}$$
$$V_+ = aV_o = a\left(V_n - R_f i_n\right) \tag{3}$$
$$\tag{4}$$

For section I:

$$
\begin{aligned}
\text{We assume...} & \\
V_+ &= V_- && (5)\\
a\left(V_n - R_f i_n\right) &= V_n && (6)\\
R_f i_n &= \frac{aV_n - V_n}{a} && (7)\\
R_f i_n &= V_n \frac{a-1}{a} && (8)\\
V_n &= \frac{-a}{1-a} R_f i_n && (9)\\
V_n &= -R_f i_n && (10)\\
\text{...when} & \\
-V_s < & V_o < V_s && (11)\\
-V_s < & V_n - R_f i_n < V_s && (12)\\
-V_s + R_f i_n < & V_n < V_s + R_f i_n && (13)\\
-V_s + R_f V_n * \frac{a-1}{aR_f} < & V_n < V_s + R_f V_n \frac{a-1}{aR_f} && (14)\\
-V_s - \frac{1-a}{a} V_n < & V_n < V_s - \frac{1-a}{a} V_n && (15)\\
-aV_s < & V_n < aV_s && (16)\\
-6 < & V_n < 6 && (17)\\
& && (18)
\end{aligned}
$$

For section II:

When...

$$V_o = V_s \tag{19}$$
$$V_s = V_n - R_f i_n \tag{20}$$
$$i_n = \frac{V_s - V_n}{R_f} \tag{21}$$

or solving for V_n, which will be useful later:

$$V_n = V_s + R_f i_n \tag{22}$$

...we assume

$$V_+ > V_- \tag{23}$$
$$a\left(V_n - i_n R_f\right) > V_n \tag{24}$$
$$V_n > \frac{-a}{1-a} R_f i_n \tag{25}$$
$$\text{but we know from line 22 } V_n = V_s + R_f i_n \tag{26}$$
$$V_s + R_f i_n > \frac{-a}{1-a} R_f i_n \tag{27}$$
$$\frac{1-a}{1-a} R_f i_n + \frac{a}{1-a} R_f i_n > -V_s \tag{28}$$
$$\frac{R_f i_n}{1-a} > -V_s \tag{29}$$
$$i_n > \frac{a-1}{R_f} V_s \tag{30}$$
$$i_n > \frac{-6}{R_f} \tag{31}$$
$$\tag{32}$$

For section III:

We can just borrow from section II and flip the signs:

We assume...

$$V_+ < V_- \tag{33}$$
$$i_n < \frac{6}{R_f} \tag{34}$$

...when

$$V_o = -V_s \tag{35}$$
$$i_n = \frac{V_n + V_s}{R_f} \tag{36}$$

or

$$V_n = R_f i_n - V_s \tag{37}$$
$$\tag{38}$$

2 Results

Now we need to make use of the quantitative analysis. We can plot a theoretical graph of V_n versus i_n and plot experimental points over the graph to compare the two. The results are in fig. 2 below.

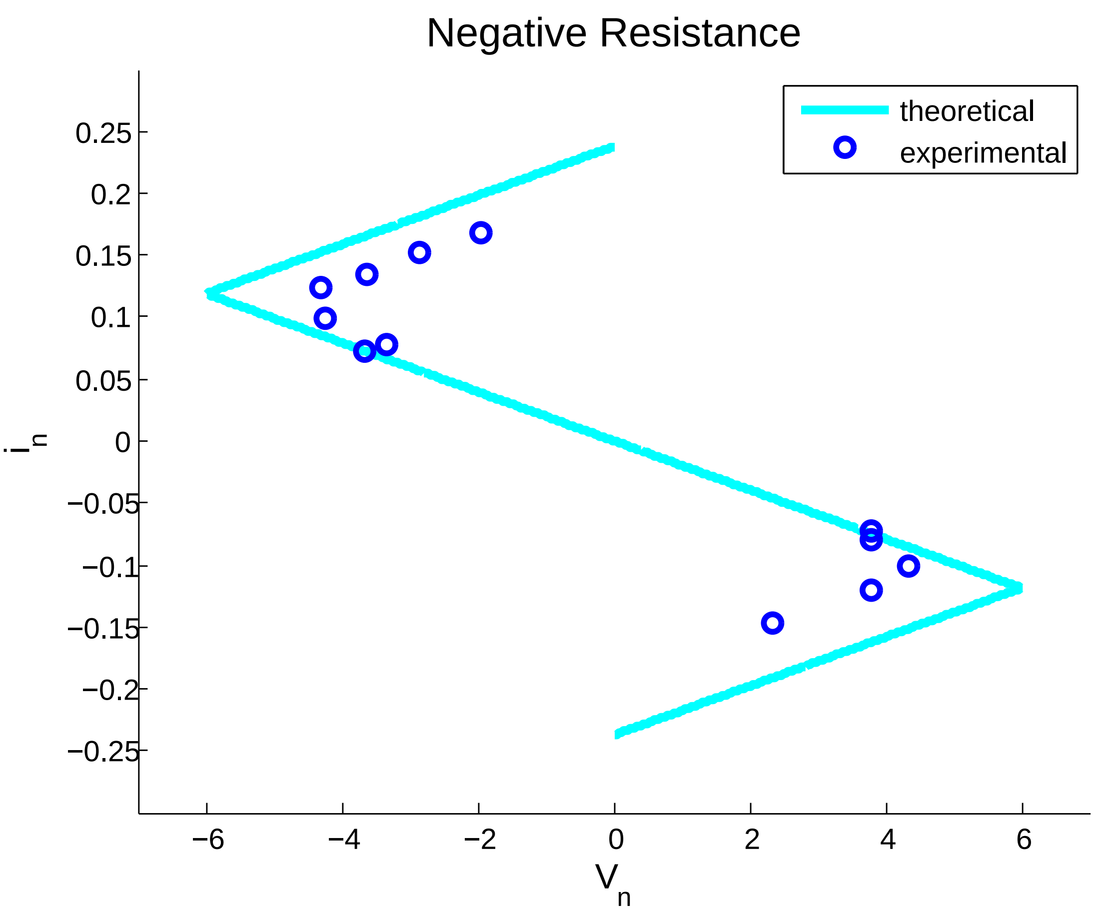

Figure 2: negative resistance graph

3 Qualitative Analysis

As you can see, the experimental data comes pretty close to theoretical in region I on the interval $-4 < V_n < 4$, and the slopes in regions II and III look accurate. But the op-amp was obviously not able to output enough voltage. Indeed, OPA's rails only reach about plus or minus 10 Volts as apposed to a theoretical plus or minus 12 Volts. This explains why the data 'falls short' of the theoretical.

3.1 A brief explanasion of the methods used to collect and implement the data

Strategic values of R_m were applied to the circuit and the resulting values of V_n were recorded. The current i_n was then found by using the equations $i_n = \frac{V_s - V_n}{R_m}$ when $V_s = 12$ and $i_n = \frac{V_s + V_n}{R_m}$ when $V_s = -12$.