

Robot Velocity Based Path Planning Along Bezier Curve Path

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Abstract. This paper presents a path planning method considering physical limits for two-wheeled mobile robots (TMRs). A convolution operator is used to generate the center velocity trajectory to travel the distance along predefined Bezier curve while considering the physical limits. The trajectory gained through convolution does not consider the rotating angle of TMR, so we present a transformational method for the center velocity trajectory along the planned path as a function of time for the parameter of the Bezier curve.

Keywords: TMR, Bezier curve, Convolution, Physical Limits

1 Introduction

TMRs are recently widely used as cleaning robots and intelligent service robots; thus, extensive research is underway on trajectory planning to minimize energy and optimize traveling time as well as to resolve issues regarding smooth traveling toward the desired destinations in workspaces [1-3].

If the physical limits of TMR during path planning and trajectory generation are considered, potential damage to TMR can be reduced, and trajectory tracking accuracy can be improved [3]. To this end, trajectory planning methods of center velocity of TMR using a convolution operator have been suggested that consider physical limits of TMR in workspaces [2, 4]. However, the method does not consider a heading angle—part of TMR's kinematics. Instead, they consider only the translational velocity and paths as opposed to the center of TMR in Cartesian coordinates. In a path planning for TMR, a smooth path planning method that considers the initial and final heading angles is a basic goal.

When generating a trajectory of TMR, path planning methods have been studied for TMR arriving in a desired posture at a final destination based on a starting position and heading angle using a Bezier curve [5].

In this study, we first generate a path based on a Bezier curve in order to build smooth path while considering heading angle. A convolution operator is used to generate the velocity for the robot center to travel the planned path. In this process, the velocity trajectory can be generated while considering the maximum velocity and

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acceleration. The velocity trajectory gained through convolution is a trajectory along the path which a robot travels given distance that does not consider the rotating angle of TMR. In order to consider a rotating angle of TMR, a transformational method is presented that consists of segmented paths along designed Bezier curve with the central velocity generated through convolution. Finally, the trajectory obtained through transformational process can be used for TMR to smoothly follow the planned path while staying within the physical limits. In the convolution and transformation process, the sampling should be conducted. Tracking errors between the parametric Bezier curve and the trajectory generated through transformation process are analyzed according to the sampling time.

2 Bezier Curve based Path Planning

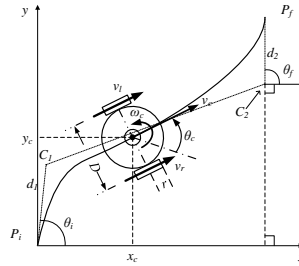


Fig. 1. Kinematics of TMR and Bezier Curve-based path planning

A curved trajectory is commonly generated using Bezier curve [5]. As shown in Fig. 1, a trajectory is generated using a Bezier curve consisting of $P_i(A_0, B_0)$, $P_f(A_3, B_3)$, and control points $C_1(A_1, B_1)$ and $C_2(A_2, B_2)$. An equation for the Bezier curve is calculated using C_1 and C_2 and it is given below in equation (1).

$$x(u) = A_0(1-u)^3 + 3A_1u(1-u)^2 + 3A_2u^2(1-u) + A_3u^3, \quad (1-a)$$

$$y(u) = B_0(1-u)^3 + 3B_1u(1-u)^2 + 3B_2u^2(1-u) + B_3u^3, \quad (1-b)$$

In equation (1), u is an arbitrary parametric value in $0 \leq u \leq 1$ and can be used to generate a smooth curve from a starting point to a target point: a more precise Bezier curve with a smaller increase. The path given by equation (1) does not consider velocity and it is only parameterized by u .

3 Convolution based Trajectory Planning Following Bezier Curve

There have been research results that the path generation method may use convolution operator to create a central velocity trajectory of TMR for smooth path generation while satisfying physical limits [2,3].

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Let a Bezier-curve-based path as shown in Fig. 1 that considers the rotating angle using a constant value u be as $\rho(u)$. The traveling distance is calculated as in equation (2). The moving distance is used to generate the central velocity trajectory for the robot in convolution [2,3]. The curved distance B_d along the path $\rho(u)$ from P_i to P_f as in Fig. 1, is calculated as follows:

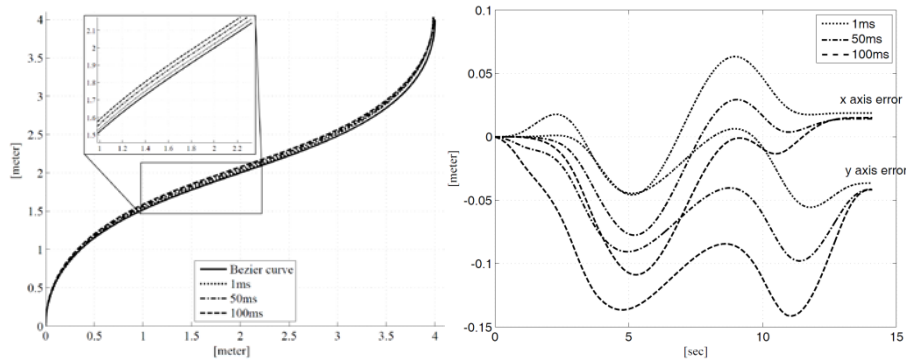
$$B_d = \sum_{u=0}^1 \Delta\rho(u). \quad (2)$$

The calculated B_d is an actual moving distance along the path designed with Bezier curve which has smooth curve. To generate the center velocity trajectory of TMR using convolution, the moving distance S is thus used as an input value. If the center velocity trajectory, $v_c(t)$ is generated to have the traveling distance $S = B_d$ the trajectory using the advantages of convolution while considering velocity limits. Here, v_i , v_f , v_{max} and the sampling time can be arbitrarily set according to the specifications of TMR [2, 3].

However, the generated velocity trajectory of $v_c(t)$ becomes the central velocity trajectory of TMR to travel the moving distance S while not considering the directions. In other words, assume that for an any position, $(x(u_i), y(u_i))$, the robot travels at the velocity of $v_c(t_i)$ in Fig. 1. the subsequent position can be moved to an entirely different position depending on the rotating angle θ_i . In order to consider positions in task space that depend on velocities in paths with rotating angles, the parameter $u(t)$ of Bezier curve for the distance during the sampling time should be determined and is calculated as shown in equation (3). The trajectory $\rho(u(t))$ with the rotating angle can be obtained by inputting the determined $u(t)$ in equation (1). In $\rho(u(t))$, as the sampling time is shorter, the path can more accurately follow $\rho(u)$ generated by constant parameter value u .

$$u(t) = \frac{\sum_{t=0}^{t_0+t_1+t_2} v_c(t)}{B_d}, \quad (3)$$

where, t_1 , t_2 are travelling time according to first and second convolutions, respectively, and $u(t)$ represents the parameter of Bezier curve depending on the central velocity with $0 \leq u(t) \leq 1$.



- (a) Trajectories driven by central velocities (b) Tacking errors along the Bezier curve

Fig. 2. Trajectories and tracking errors according to the sampling time

The trajectory generated by using $u(t)$ satisfies the maximum velocity allowed by the physical limits of TMR while following the curved path with heading angles. Fig. 2(a) shows the trajectories based on central velocity according to the sampling times, which satisfied the physical limits along the Bezier curve path. In this figure, the distance between positions of the trajectory is the travelling distance driven by the central velocity function in a sampling time. The results showed that the generated trajectory was depending on the central velocity function's trajectory. Fig. 2(b) represents the tracking error between the Bezier curve and the trajectory generated according to sampling time of 1ms, 50ms, 10ms. The error becomes larger as high angular velocity and long sampling time.

4 Conclusions

A velocity trajectory generation method that enables a TMR to travel smoothly along curved path while staying within the actuator's physical limits was proposed. The path planning method proposed in this paper can be utilized for a path planning with optimized travelling time and an energy-efficient path planning that considers the limited battery power of a running robot.

We examined tracking errors according to the sampling time when convolution and transformation process. For smooth control, the effect of sampling time should be considered.

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References

1. Craig, J.J.: Introduction to Robotics, Prentice-Hall (2005)
2. Lee, G., Yi, B.J., Kim, D., and Choi, Y.: New Robotic Motion Generation using Digital Convolution with Physical System Limitation. In: IEEE Conference on Decision and Control and European Control Conference, pp. 698--703 (2011)
3. Lee, G., Kim, D. and Choi, Y.: Faster and Smoother Trajectory Generation considering Physical System Limits under Discontinuously Assigned Target Angles. In: IEEE International Conference on Mechatronics and Automation, pp. 1196--1201 (2012)
4. Lepetic, M., Klancar, G., Skrjanc, I., Matko, D., and Potocnik, B.: Time optimal path planning considering acceleration limits, Robotics and Automation Systems, vol. 45, pp. 199--210 (2003)
5. Jolly, K.G., Sreerama Kumar, R., Vijayakumar, R.: A Bezier curve based path planning in a multi-agent robot soccer system without violating the acceleration limits. Robotics and Automation Systems, vol. 57, pp. 23--33 (2009)