Wave Mathematics

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INTRODUCTION

Algebra began with arithmetic such as $1+1=2$, which was invented numerous times by different ancient civilisations for accountancy, exchange of valuables and currencies.

Soon after, angles, shapes, early polar coordinates and pi was discovered due to the requirements of complex construction of buildings and also the observation of space, stars and planets.

Mathematical Physics has traversed vastly since, with exciting new topics such as Relativity and Quantum Physics.

SEQUENTIAL SUMMATION OF NORMAL DISTRIBUTIONS

If normal distributions are summed together they become a continuous wave. If they are summed together where their means are located at $1/4$ and $3/4$ of the total length of the wave then the wave is approximately sinusoidal. (Check this) This provides an insight into the true properties of waves and how they form.

Hence we start with a single quantity. If that quantity duplicates itself with a certain standard deviation error of sigma after an infinite amount of duplication, a distribution is formed. A single quantity from that distribution may shift out of phase by 2 sigma and continue the duplication process in that new space-time. Hence a second identical distribution forms which is connected to the original distribution. This forms a sequence of distributions otherwise known as a wave.

WAVE HARMONICS

During 500 BC, Pythagoras discovered the principles of Harmonics while venturing through Ancient Egypt.

The principles of Harmonics were discovered by Pythagoras During 500 BC, P during travels to Egypt and throughout the ancient world. Pythagoras first began to teach at the age of 50. His theories were first introduced to the western world in Plato's Timeaus, Chapter 6, The Soul of the World in the form of the Tetraktys. A translation in the 15th Century spurred on the Renaissance. It resurfaced during the 16th century with the astronomer Kepler and further with Newton in the 17th Century. The work of the great 19th century physicist Hermann Helmholz applied the scientific method to tone and fueled a misconception that Microtonality and Harmonics are the same discipline. At about the same time as Helmholz, Albert Thimus a native of Cologne, Germany translating latin and greek texts rediscovered the Lambdoma which is the basis for Harmonics. Based on Thimus, Harmonic Symbolism in Antiquity - published 1866 (Harmonikale Symbolik des Alterthums), Hans Kayser a student of Thimus, also from Cologne, wrote a series of works, that organize and demonstrate Harmonic principles. His most important work is the Handhook of Harmonics - Zurich 1950 (Lehrbuch der Harmonik). This work organizes Harmonics for a formal study. Fields examined by Kayser include but are not limited to: philosophy, mathematics, astronomy, architecture, grammar, crystallography, color mixing, botany and microtonality. From this perspective microtonality is in the domain of Harmonics. The lineage is also clear with composer Harry Partch being one of the first practicing harmonists in the 20th Century when new Pythagoreans surfaced within the Arts and Sciences.(Re write everything before this in this section)

Lambdoma

The main focus of Harmonics is the study of a three dimensional model called the Lambdoma. The Lambdoma is constructed by combining the ratios of the arithmetic, harmonic and geometric series into one form. Ernest G. McClain (USA) in his book The Pythagorean Plato - Nicholas-Hays, York Beach, Maine 1978, confirmed that the Lambdoma was recognized in antiquity by findings that were concealed within the Marriage Allegory of Plato's Republic. (Re write everything before this in this section)

The Lambdoma matrix:

$$LM = \begin{bmatrix} 1 & 1/2 & 1/3 & ... & 1/n \\ 2 & 1 & 2/3 & ... & 2/n \\ 3 & 3/2 & 1 & ... & 3/n \\ ... & ... & ... & ... & ... \\ n & n/2 & n/3 & ... & 1 \end{bmatrix} \tag{1}$$

This lambdoma matrix is based upon string length; with its inverse frequency ratio matrix they comprise the primary tools for Harmonics—the study of ratio and proportion. All ratios to the right of the 1/1 diagonal are less than 1. Even times even ratios and their reciprocals are shown in color. If you imagine a string length placed in the 1/16 column and extended one row above that column, then drawing a line through the 1/2 ratio vector will cut the string in half. Notice that drawing a line through the 1/2 ratio vector does not end at 1/1, but rather above 1/1 at the 0/0 origin. Also, drawing a line to the left of the diagonal through any ratio vector starting at the 0/0 origin extends the string length by the ratio vector amount. Please note that a division or extension of string length only works in an equally spaced grid. Try it on graph paper. The string length lambdoma matrix is constructed with rows of the overtone series and columns of the undertone series. The slider adjusts the index of the lambdoma matrix from 16 to 1.(Re write everything before this in this section)

Standing wave harmonics (2D)

Let's work out the relationships among the frequencies of these modes. For a wave, the frequency is the ratio of the speed of the wave to the length of the wave: f = v/. Compared with the string length L, you can see that these waves have lengths 2L, L, 2L/3, L/2. We could write this as 2L/n, where n is the number of the harmonic.

The fundamental or first mode has frequency f1 = v/1 = v/2L, The second harmonic has frequency f2 = v/2 = 2v/2L = 2f1 The third harmonic has frequency f3 = v/3 = 3v/2L = 3f1, The fourth harmonic has frequency f4 = v/4 = 4v/2L = 4f1, and, to generalise,

The nth harmonic has frequency fn = v/n = nv/2L = nf1. (Re write everything before this in this section)

Polar wave harmonics (2D)

Polar wave harmonics is the Polar perspective of traditional wave harmonics.

Spherical wave harmonics (3D)

Spherical harmonics was first founded by Laplace

WAVE AND POLAR WAVE ARITHMETIC

Similar to normal sinusoidal waves, quantum waves will also undergo constructive and destructive interference when summed together which will result in a new wave forming. If two identical waves in phase are summed, then constructive interference will cause the peaks and troughs to increase by a power of two. If two identical waves out of phase are summed, then the wave will undergo complete destructive interference with 0 energy remaining. The following arrows can be used to denote identical waves moving in different directions up $\rotatebox{90}{$\rightsquigarrow$}$, down $\rotatebox{90}{$\leftsquigarrow$}$, left $\leftsquigarrow$, right $\rightsquigarrow$, left-right $\leftrightsquigarrow$, up-down $\rotatebox{90}{$\leftrightsquigarrow$}$, polar left spiral $\circlearrowleft$ or right spiral $\circlearrowright$ and spherical polar $\bigstar$. There are three required variables which are amplitude A, wavelength λ and frequency f. To denote these properties of a wave we will use the following notation when conducting equations involving more than one type of wave: ${}^{A}_{\lambda}\rotatebox{90}{$\rightsquigarrow$}(f)$.

Notation

The chosen notation has been decided in order to simplify mathematical expressions involving waves in Cartesian, Polar and Spherical Polar scenarios.

$$k = \frac{2\pi}{\lambda} \tag{2}$$

$${}^{A}_{\lambda} \rightsquigarrow (f) = A\exp(i(kx - \frac{w}{f})) \tag{3}$$

$${}^{A}_{\lambda} \leftsquigarrow (f) = A\exp(-i(kx + \frac{w}{f})) \tag{4}$$

$${}^{A}_{\lambda} \rotatebox{90}{$\rightsquigarrow$} (f) = A\exp(i(ky - \frac{w}{f})) \tag{5}$$

$${}^{A}_{\lambda} \rotatebox{90}{$\leftsquigarrow$} (f) = A\exp(-i(ky + \frac{w}{f})) \tag{6}$$

Note that $C = 2\pi r$ and so $\lambda = 2\pi r$ as the wavelength covers a full circle with frequency f hence,

$${}^{A}_{\lambda} \circlearrowright (f) = -x^2 - y^2 + r^2 = -x^2 - y^2 + (\lambda/2\pi f)^2 \tag{7}$$

$${}^{A}_{\lambda} \circlearrowleft (f) = x^2 + y^2 - r^2 = x^2 + y^2 - (\lambda/2\pi f)^2 \tag{8}$$

$${}^{A}_{\lambda} \circlearrowright (f) = -{}^{A}_{\lambda} \circlearrowleft (f) \tag{9}$$

$${}^{A}_{\lambda} \circlearrowright (f) + {}^{A}_{\lambda} \circlearrowleft (f) = {}^{A}_{\lambda} \bigstar (f) \tag{10}$$

Note that the Spherical Polar wave has a net zero effect on any Polar wave at the Spherical Polar wave boundary. That is true if the Polar wave is centered with respect to the Spherical Polar wave and has the same frequency and wavelength. This is how equation 9 and 11 can coexist and so,

$${}^{A}_{\lambda} \circlearrowright (f) + {}^{A}_{\lambda} \bigstar (f) = {}^{A}_{\lambda} \circlearrowright (f) \tag{11}$$

where n is any real number,

$${}^{A}_{\lambda} \circlearrowright (f) + {}^{nA}_{\lambda} \bigstar (f) = {}^{nA}_{\lambda} \circlearrowright (f) \tag{12}$$

In equation 12 one observes how the Spherical Polar wave acts like an amplification transformation in all directions within a 3D space of size $nA \times nA$.

$$ {}^{m}_{\lambda}\circlearrowright(f) + {}^{n}_{\lambda}\bigstar(f) = {}^{nm}_{\lambda}\circlearrowright(f) \tag{13}$$

$$ {}^{A}_{\lambda}\leftrightsquigarrow(f) = A\exp(i(kx - \frac{w}{f})) - A\exp(i(kx - \frac{w}{f})) = \pm^{A}_{\lambda}\circlearrowright(f) \tag{14}$$

$$ {}^{A}_{\lambda}\rotatebox{90}{$\leftrightsquigarrow$}(f) = A\exp(i(ky - \frac{w}{f})) - A\exp(i(ky - \frac{w}{f})) = \pm^{A}_{\lambda}\circlearrowright(f) \tag{15}$$

$$ {}^{A}_{\lambda}\bigstar(f) = A(x^2 + y^2 + z^2)/(\lambda/2\pi f)^2 \tag{16}$$

Arithmetic examples

For example, a wave propagating right summed with a wave propagating right will result in double the amplitude,

$$ \rightsquigarrow + \rightsquigarrow =^{2}\rightsquigarrow . \tag{17}$$

A wave propagating right sequenced with a wave propagating right will result in double the wavelength,

$$ \rightsquigarrow, \rightsquigarrow =_{2}\rightsquigarrow . \tag{18}$$

A wave propagating right sequenced with a wave propagating left three times will result in alternating waves,

$$ 3(\rightsquigarrow, \leftsquigarrow) = \rightsquigarrow\leftsquigarrow\rightsquigarrow\leftsquigarrow\rightsquigarrow\leftsquigarrow =_{3}\leftrightsquigarrow =_{3}\circlearrowright . \tag{19}$$

$$ 3(\leftsquigarrow, \rightsquigarrow) = \leftsquigarrow\rightsquigarrow\leftsquigarrow\rightsquigarrow\leftsquigarrow\rightsquigarrow = -_{3}\leftrightsquigarrow =_{3}\circlearrowleft . \tag{20}$$

A wave propagating right summed with a wave propagating left five times will result in a left-right wave with amplitude 5,

$$ 5(\rightsquigarrow + \leftsquigarrow) = 5(\leftsquigarrow + \rightsquigarrow) =^{5}\leftrightsquigarrow =^{5}\circlearrowright . \tag{21}$$

A wave propagating up summed with a wave propagating down sequenced five times will result in a up-down wave with wavelength 5,

$$ 5(\rotatebox{90}{$\rightsquigarrow$}, \rotatebox{-90}{$\rightsquigarrow$}) =_{5}\rotatebox{90}{$\leftrightsquigarrow$} =_{5}\circlearrowright . \tag{22}$$

$$ 5(\rotatebox{-90}{$\rightsquigarrow$}, \rotatebox{90}{$\rightsquigarrow$}) = -_{5}\rotatebox{90}{$\leftrightsquigarrow$} =_{5}\circlearrowleft . \tag{23}$$

A wave propagating right summed with a wave propagating left, up and down six times will result in a polar wave with amplitude 6,

$$ 6(\rightsquigarrow + \leftsquigarrow + \rotatebox{90}{$\rightsquigarrow$} + \rotatebox{-90}{$\rightsquigarrow$}) = 6(\leftrightsquigarrow + \rotatebox{90}{$\leftrightsquigarrow$}) =^{6}\bigstar . \tag{24}$$

A wave propagating right summed with a wave propagating up will result in an angled wave,

$$ \rightsquigarrow + \rotatebox{90}{$\rightsquigarrow$} = \rotatebox{90}{$\rightsquigarrow$}\rightsquigarrow . \tag{25}$$

WAVE CALCULUS

Limits of Cartesian waves

Infinitely very small amplitude of waves become infinitely thin lines the length of a wavelength λ hence they can be approximated to be continuous lines of zero space. If wavelength also tends to zero then it becomes a singularity otherwise known as a single point.

$$ lim_{A\to 0,\lambda\to 0}({}^{A}_{\lambda}\rightsquigarrow(f)) = 0 \tag{26}$$

Limits of polar waves

Infinitely very small amplitude of polar waves or spherical polar waves become infinitely small points hence they can be approximated to be singularities or areas of zero space otherwise known as a single point.

$$ lim_{A\to 0}({}^{A}_{\lambda}\circlearrowright(f)) = 0 \tag{27}$$

$$ lim_{A\to 0}({}^{A}_{\lambda}\bigstar(f)) = 0 \tag{28}$$

Very large amplitude polar waves can be approximated as either x = 0 for all y and y = 0 for all x when frequency is 2 or tend to all of space as frequency tends to infinity. Whenever frequency tends to 0 or infinity the polar wave tends to the shape of a circle.

$$ lim_{A\to\infty}({}^{A}_{\lambda}\bigstar(2)) = \begin{cases} y = 0, z = 0 & \forall x \epsilon \mathbb{R} \\ x = 0, z = 0 & \forall y \epsilon \mathbb{R} \end{cases} \tag{29}$$

Polar wave is an area of space which does not fit a traditional Cartesian graph as within a polar wave, space becomes very wave-like and will be very similar to polar coordinates. Hence a new type of graph is required which incorporates wave like but also Cartesian behaviour.

A polar wave observed from the outside is similar to zero as anything that goes towards it compresses and disappears, as visualised in a polar plot as radius decreases. However when observed from the inside a polar wave appears to be like infinity as anything that proceeds away expands radially outwards.

Hence by using mathematical limits we obtain the following:

$$lim_{f\to 1\ or\,\infty}({}^{A}_{\lambda}\bigstar(f)) = (x^2 + y^2 + z^2)/A^2 \tag{30}$$

$$ {}^{A}_{\lambda}\bigstar(f) = (x^2 + y^2 + z^2)/\rho^2 \tag{31}$$

where,

$$\rho = Ae^{2f(x+y+z)i} \tag{32}$$

$$x = \rho\cos\lambda\theta\sin f\phi \tag{33}$$

$$y = \rho\sin\lambda\theta\sin f\phi \tag{34}$$

$$z = \rho\cos f\phi \tag{35}$$

A special case is when $f = 2$:

Integral wave Calculus

Polar wave integration

Spherical polar wave integration

NETWORK WAVE ARITHMETIC

A network is formed when at least two nodes are connected by an edge. This section will analyse how polar and spherical polar waves can be modelled as nodes while waves can be modelled as edges within a network graph.

CONCLUSION

[1] J Earman, M Friedman. *The meaning and status of Newton's law of inertia and the nature of gravitational forces*. (1973). The University of Chicago Press Journals.

[2] D Breuer, S Labrosse, T Spohn. *Thermal evolution and magnetic field generation in terrestrial planets and satellites*. (2010). Springer.

[3] A.P. Vanden Berg, D.A. Yuen, G. Beebe, M.D. Christiansen. *The dynamical impact of electronic thermal conductivity on deep mantle convection of exosolar planets*. (2010). Elsevier.