# Integration Formulas:

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| **The Power Rule:** | $\int a\,dx = ax + c$ <br> $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ <br> $\int (ax+b)^n\,dx = \frac{1}{a} * \frac{(ax+b)^{n+1}}{n+1}$ |
| **Rational Functions:** | $\int \frac{1}{x}\,dx = \ln x + C$ <br> $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$ |
| **Exponential Functions:** | $\int e^x\,dx = e^x + C$ <br> $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$ <br> $\int a^{bx+d}\,dx = \frac{a^{bx+d}}{b\ln a} + C$ |
| **Trig Functions:** | $\int \sin x\,dx = -\cos x + C$ <br> $\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + C$ <br> $\int \cos x\,dx = \sin x + C$ <br> $\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + C$ <br> $\int \csc x \cot x\,dx = -\csc x + C$ <br> $\int \sec x \tan x\,dx = \sec x + C$ <br> $\int \tan x\,dx = \ln\lvert\sec x\rvert + C = -\ln\lvert\cos x\rvert + C$ <br> $\int \cot x\,dx = \ln\lvert\sin x\rvert + C = -\ln\lvert\csc x\rvert + C$ <br> $\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$ <br> $\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + C$ |
| **Integration By Parts:** | $\int u\,dv = uv - \int v\,du$ <br> $\int uv\,dw = uvw - \int vw\,du - \int uw\,dv$ |

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| **Integration By Parts: "Related Formulas"** | $\int \boldsymbol{e^{ax}\sin(bx)\,dx} = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx] + C$ <br> $\int \boldsymbol{e^{ax}\cos(bx)\,dx} = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx] + C$ <br> $\int \boldsymbol{x^n \ln x\,dx} = \frac{x^{n+1}}{n+1}\ln x - \frac{x^{n+1}}{(n+1)^2} + C \quad n \neq 1$ <br> $\int \boldsymbol{x^n e^{ax}\,dx} = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$ |
| **Trig Substitution:** | $\int \boldsymbol{\sqrt{a^2-u^2}\,du} = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}sin^{-1}\left(\frac{u}{a}\right) + C$ <br> $Trig\ Sub:\quad \boldsymbol{u = a\sin\theta \quad du = a\cos\theta\ d\theta}$ <br> $\int \boldsymbol{\sqrt{a^2+u^2}\,du} = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left\|u+\sqrt{u^2+a^2}\right\| + C$ <br> $Trig\ Sub:\quad \boldsymbol{u = a\tan\theta \quad du = a\,sec^2(\theta)\,d\theta}$ <br> $\int \boldsymbol{\sqrt{u^2-a^2}\,du} = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left\|u+\sqrt{u^2-a^2}\right\| + C$ <br> $Trig\ Sub:\quad \boldsymbol{u = a\sec\theta \quad du = a\sec\theta\tan\theta\ d\theta}$ |
| **Reduction Formulas:** | $\int \boldsymbol{sin^2(x)dx} = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C = \frac{1}{2}(x - \sin x\cos x) + C$ <br> $\int \boldsymbol{cos^2(x)dx} = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C = \frac{1}{2}(x + \sin x\cos x) + C$ <br> $\int \boldsymbol{tan^2(x)dx} = \tan x - x + C \qquad \int \boldsymbol{cot^2(x)dx} = -\cot x - x + C$ <br> $\int \boldsymbol{sin^n(x)dx} = -\frac{1}{n}sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int sin^{n-2}(x)\,dx$ <br> $\int \boldsymbol{cos^n(x)dx} = \frac{1}{n}cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int cos^{n-2}(x)\,dx$ <br> $\int \boldsymbol{tan^n(x)dx} = \frac{1}{n-1}tan^{n-1}(x) - \int tan^{n-2}(x)dx \quad n \neq 1$ |

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| **Reduction Formulas:** | $\int \boldsymbol{cot^n(x)dx} = -\frac{1}{n-1} cot^{n-1}(x) - \int cot^{n-2}(x)dx \quad n \neq 1$<br><br>$\int \boldsymbol{sec^n(x)dx} = \frac{1}{n-1} sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int sec^{n-2}(x)dx \;\; n \neq 1$<br><br>$\int \boldsymbol{csc^n(x)dx} = -\frac{1}{n-1} csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int csc^{n-2}(x)dx \;\; n \neq 1$ |
| **Integration of Logs:** | $\int \boldsymbol{\log_d(ax+b)\,dx} = \frac{ax+b}{a}\log_d\left\|\frac{ax+b}{e}\right\| + C$ |
| **Inverse Trig:**<br>**"Related Formulas"** | $\int \boldsymbol{\frac{1}{a^2+u^2}du} = \frac{1}{a}tan^{-1}\left(\frac{u}{a}\right) + C = -\frac{1}{a}cot^{-1}\left(\frac{u}{a}\right) + C$<br><br>$\int \boldsymbol{\frac{1}{\sqrt{a^2-u^2}}du} = sin^{-1}\left(\frac{u}{a}\right) + C = -cos^{-1}\left(\frac{u}{a}\right) + C$<br><br>$\int \boldsymbol{\frac{1}{u\sqrt{u^2-a^2}}du} = \frac{1}{a}sec^{-1}\left(\frac{u}{a}\right) + C = -\frac{1}{a}csc^{-1}\left(\frac{u}{a}\right) + C$ |
| **Inverse Trig Formulas:** | $\int \boldsymbol{sin^{-1}(u)\,du} = u\,sin^{-1}(u) + \sqrt{1-u^2} + C$<br><br>$\int \boldsymbol{cos^{-1}(u)\,du} = u\,cos^{-1}(u) - \sqrt{1-u^2} + C$<br><br>$\int \boldsymbol{tan^{-1}(u)\,du} = u\,tan^{-1}(u) - \ln\sqrt{1+u^2} + C$<br><br>$\int \boldsymbol{cot^{-1}(u)\,du} = u\,cot^{-1}(u) + \ln\sqrt{1+u^2} + C$<br><br>$\int \boldsymbol{sec^{-1}(u)\,du} = u\,sec^{-1}(u) - \ln\left\|u + \sqrt{u^2-1}\right\| + C$<br><br>$\int \boldsymbol{csc^{-1}(u)\,du} = u\,csc^{-1}(u) + \ln\left\|u + \sqrt{u^2-1}\right\| + C$ |

# Table of Integrals:

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| Form: $\boldsymbol{u^2 \pm a^2}$ | $\int \frac{\mathbf{1}}{\boldsymbol{u^2 - a^2}}\, \boldsymbol{du} = \frac{1}{2a}\ln\left\|\frac{u-a}{u+a}\right\| + C \quad Note\text{: } \|u-a\| = \|a-u\|$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{a^2 - u^2}}\boldsymbol{du} = \frac{1}{2a}\ln\left\|\frac{u+a}{u-a}\right\| + C = \frac{1}{2a}\ln\left\|\frac{a+u}{a-u}\right\| + C$ <br> $\int \frac{\mathbf{1}}{\sqrt{\boldsymbol{u^2 \pm a^2}}}\, \boldsymbol{du} = \ln\left\|u + \sqrt{u^2 \pm a^2}\right\| + C$ <br> $\int \frac{\sqrt{\boldsymbol{u^2 \pm a^2}}}{\boldsymbol{u^2}}\, \boldsymbol{du} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln\left\|u + \sqrt{u^2 \pm a^2}\right\| + C$ <br> $\int \frac{\boldsymbol{u^2}}{\sqrt{\boldsymbol{u^2 \pm a^2}}}\, \boldsymbol{du} = \frac{1}{2}\left[u\sqrt{u^2 \pm a^2} \mp a^2 \ln\left\|u + \sqrt{u^2 \pm a^2}\right\|\right] + C$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{u^2}\sqrt{\boldsymbol{u^2 \pm a^2}}}\, \boldsymbol{du} = \mp\frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$ <br> $\int \frac{\mathbf{1}}{(\boldsymbol{u^2 \pm a^2})^{3/2}}\, \boldsymbol{du} = \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$ <br> $\int \frac{\boldsymbol{u^2}}{\sqrt{\boldsymbol{a^2 - u^2}}}\, \boldsymbol{du} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} sin^{-1}\left(\frac{u}{a}\right) + C$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{u^2}\sqrt{\boldsymbol{a^2 - u^2}}}\, \boldsymbol{du} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$ <br> $\int \frac{\sqrt{\boldsymbol{a^2 - u^2}}}{\boldsymbol{u^2}}\, \boldsymbol{du} = -\frac{\sqrt{a^2 - u^2}}{u} - sin^{-1}\left(\frac{u}{a}\right) + C$ |
| Form: $\sqrt{\boldsymbol{a + bu}}$ | $\int \frac{\boldsymbol{u}}{\sqrt{\boldsymbol{a + bu}}}\, \boldsymbol{du} = \frac{2bu - 4a}{3b^2}\sqrt{a + bu} + C$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{u}\sqrt{\boldsymbol{a + bu}}}\, \boldsymbol{du} = \frac{1}{\sqrt{a}}\ln\left\|\frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}}\right\| + C \quad a > 0$ <br> $\int \frac{\sqrt{\boldsymbol{a + bu}}}{\boldsymbol{u}}\, \boldsymbol{du} = 2\sqrt{a + bu} + a\int \frac{1}{u\sqrt{a + bu}}du$ |

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| **Form**: $\mathbf{1/(a+bu)}$ | $\int \frac{\boldsymbol{u}}{\boldsymbol{a+bu}}\,\boldsymbol{du} = \frac{u}{b} - \frac{a}{b^2}\ln\lvert a+bu\rvert + C$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{u(a+bu)}}\,\boldsymbol{du} = \frac{1}{a}\ln\left\lvert\frac{u}{a+bu}\right\rvert + C$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{u^2(a+bu)}}\,\boldsymbol{du} = \frac{b}{a^2}\ln\left\lvert\frac{a+bu}{u}\right\rvert - \frac{1}{au}$ <br> $\int \frac{\boldsymbol{u^2}}{\boldsymbol{a+bu}}\,\boldsymbol{du} = \frac{1}{2b}u^2 - \frac{a}{b^2}u + \frac{a^2}{b^3}\ln\lvert a+bu\rvert + C$ <br> $\int \frac{\boldsymbol{u^2}}{\boldsymbol{(a+bu)^2}}\,\boldsymbol{du} = \frac{1}{b^3}\left[bu - \frac{a^2}{a+bu} - 2a\ln\lvert a+bu\rvert\right] + C$ |
| **Form:** $\mathbf{1/(a+bu+cu^2)}$ | $\int \frac{\mathbf{1}}{\boldsymbol{a+bu+cu^2}}\boldsymbol{du} = \frac{2}{\sqrt{4ac-b^2}}tan^{-1}\left(\frac{2cu+b}{\sqrt{4ac-b^2}}\right) + C \quad if\ b^2 < 4ac$ <br> $\int \frac{\mathbf{1}}{\boldsymbol{a+bu+cu^2}}\boldsymbol{du} = \frac{1}{\sqrt{b^2-4ac}}\ln\left\lvert\frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}}\right\rvert + C\ \ if\ b^2 > 4ac$ |