# Braid Groups

(*) composition?

$\sigma_i$ permutations

The *n-strand braid groups* $B_n$ has the presentation:

$$B_n = \langle \sigma_1, \ldots, \sigma_{n-1} \mid \sigma_i\sigma_j = \sigma_j\sigma_i \text{ for } |i-j| \geq 2, \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \rangle.$$

## 1 Mapping class groups

Let $\mathcal{S}$ an oriented compact surface, possibly with boundary, and $\mathcal{P}$ be a finite set of distinguished interior points of $\mathcal{S}$.

The *mapping clas group* $\mathcal{MCG}(\mathcal{S}, \mathcal{P})$ of the surface $\mathcal{S}$ relative to $\mathcal{P}$ is the group of all isotopy classes of orientation-preserving homeomorphisms $\psi : \mathcal{S} \to \mathcal{S}$ satisfying $\psi_{|\partial S} = id$ and $\psi(\mathcal{P}) = \mathcal{P}$.

**Proposition 1.1.** *There is an isomorphism $B_n \cong \mathcal{MCG}(D_n)$, where $D_n$ is the disk $D^2$ with $n$ regularly spaced points.*

Dibujo?

## Pure braids

Definition of pure braid.

**Theorem 1.2.** *The pure braid group $P_n$ is biorderable for all $n \geq 1$.*

## Order in $B_n$