Braid Groups

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The n-strand braid groups B_n has the presentation:

$$B_n = \langle \sigma_1, \ldots, \sigma_{n-1} \mid \sigma_i\sigma_j = \sigma_j\sigma_i \text{ for } |i-j| \geq 2, \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \rangle.$$

Pure braids

Two words β_1 and β_2 represent the same braid $\Leftrightarrow \beta_1\beta_2^{-1} = 1$.

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Dibujo de pure braid

Theorem

The pure braid group P_n is biorderable for all $n \geq 1$.

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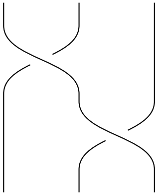

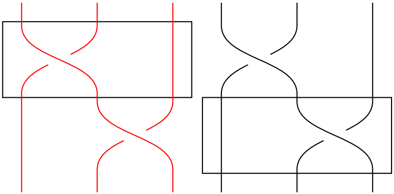

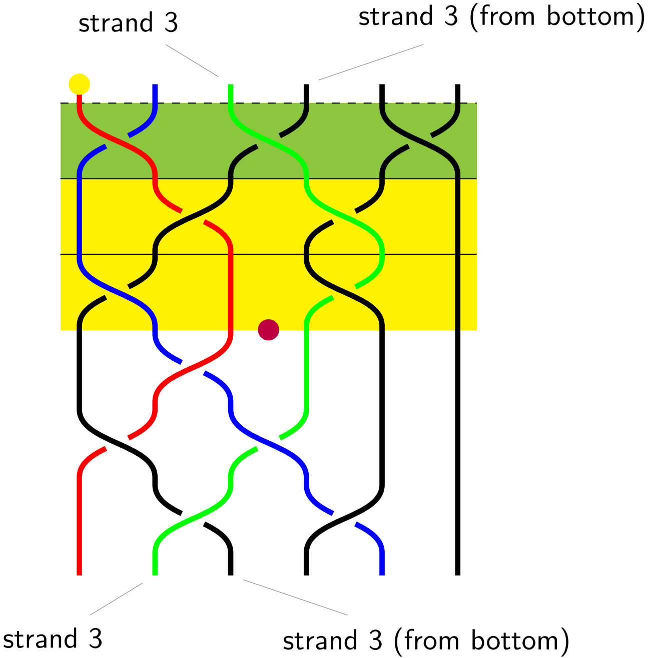
strand 3
strand 3 (from bottom)
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Mapping class groups

Definition

Let $\mathcal{S}$ an oriented compact surface, possibly with boundary, and $\mathcal{P}$ be a finite set of distinguished interior points of $\mathcal{S}$.
The mapping clas group $\mathcal{MCG}(\mathcal{S}, \mathcal{P})$ of the surface $\mathcal{S}$ relative to $\mathcal{P}$ is the group of all isotopy classes of orientation-preserving homeomorphisms $\psi : \mathcal{S} \to \mathcal{S}$ satisfying $\psi_{|\partial \mathcal{S}} = id$ and $\psi(\mathcal{P}) = \mathcal{P}$.

Proposition

There is an isomorphism $B_n \cong \mathcal{MCG}(D_n)$, where D_n is the disk D^2 with n regularly spaced points.

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Order in B_n

σ-positive
subword property

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Thanks.