

# Braid Groups

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The  $n$ -strand braid groups  $B_n$  has the presentation:

$$B_n = \langle \sigma_1, \dots, \sigma_{n-1} \mid \sigma_i \sigma_j = \sigma_j \sigma_i \text{ for } |i-j| \geq 2, \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \rangle.$$

# Pure braids

Two words  $\beta_1$  and  $\beta_2$  represent the same braid  $\Leftrightarrow \beta_1\beta_2^{-1} = 1$ .

## Definition

Dibujo de pure braid

## Theorem

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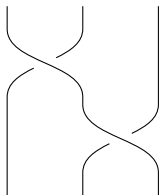
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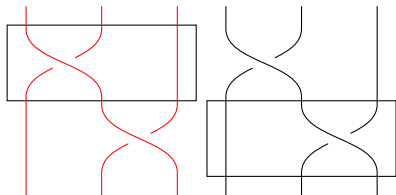
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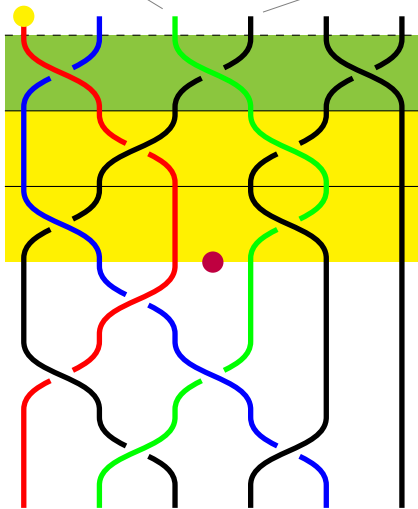
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strand 3

strand 3 (from bottom)



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# Mapping class groups

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Let  $\mathcal{S}$  an oriented compact surface, possibly with boundary, and  $\mathcal{P}$  be a finite set of distinguished interior points of  $\mathcal{S}$ .

The mapping class group  $\mathcal{MCG}(\mathcal{S}, \mathcal{P})$  of the surface  $\mathcal{S}$  relative to  $\mathcal{P}$  is the group of all isotopy classes of orientation-preserving homeomorphisms  $\psi : \mathcal{S} \rightarrow \mathcal{S}$  satisfying  $\psi|_{\partial\mathcal{S}} = id$  and  $\psi(\mathcal{P}) = \mathcal{P}$ .

## Proposition

There is an isomorphism  $B_n \cong \mathcal{MCG}(D_n)$ , where  $D_n$  is the disk  $D^2$  with  $n$  regularly spaced points.

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# Order in $B_n$

$\sigma$ -positive  
subword property

Proposition

Thanks.