STUDY OF COULOMB'S LAW

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1. Abstract

In this series of experiments, a virtual lab was conducted in which a hanging sphere was suspended, and its force body diagram was diagrammed and analyzed. Further, this sphere is placed in close proximity to a neighboring sphere attached to a rod, and their interactions were studied, specifically the magnitude of the Coulomb's force the suspended ball experienced was computed. It was found that the electric field at a distance r from the rod of length L and with charge Q is given by:

$$\vec{E} = \frac{kQ}{r\sqrt{r^2(L/2)^2}}\hat{i}$$

In the final portion of the lab, Coulombs law was used to determine the discharge rate of a series of suspended spheres. We observed that when the system is at equilibrium,the electrostatic force acting on each ball can be approximated by:

$$F^{coul} \approx \frac{mgR}{2L}$$

Where m is the mass of the ball, R the distance between the two balls, L the length of the string by which the spheres are suspended, and $g = 9.8\text{m/s}^2$ is the gravitational constant at Earth's surface.

2. Introduction

One experimental law of physics that is fundamental in understanding electrostatic forces is Coulombs law:

$$|\vec{F}^E_{12}| = k_e \frac{|q_1 q_2|}{r^2_{12}} \tag{1}$$

$$\vec{F}^E_{12} = k_e \frac{|q_1 q_2|}{r^2_{12}}\hat{r}_{12} \tag{2}$$

This mathematical rule states that the magnitude of the force is the product of two charged particles, designated as q_1 and q_2 in units of Coulombs (unit of charge), and Coulombs constant (designated as k_e with the unit Nm^2/C^2) divided by the distance between the two particles squared, which is designated as r_{12} with the unit m^2. It is noteworthy that Coulombs constant is a proportionality constant that relates different electric variables.

With respect to the implications of Coulombs law, in this particular lab, the electrostatic force exerted by a virtual charged sphere suspended by a rope on a neighboring, virtual charged sphere attached to a rod. It is noteworthy that the sphere suspended by the rope is held at one

3. Electric Field Due to a Line of Charge

In this virtual lab, we wanted to measure the electrostatic force a charged rod exerts on a hanging test charge. (a hanging charged sphere). In order to measure the electric field strength created by the rod at different radii, we hanged a ball with 35nC of charge and measured how much the ball was displaced when a charged rod of length 0.7104m was brought near it. Given the displacement of the ball and the ball's mass, we were able to determine the electric force acting on the ball. Given the charge of the ball, we were able to determine what the electric field magnitude due to the rod at the position of the ball. The results of our measurements are shown in Figure 1.

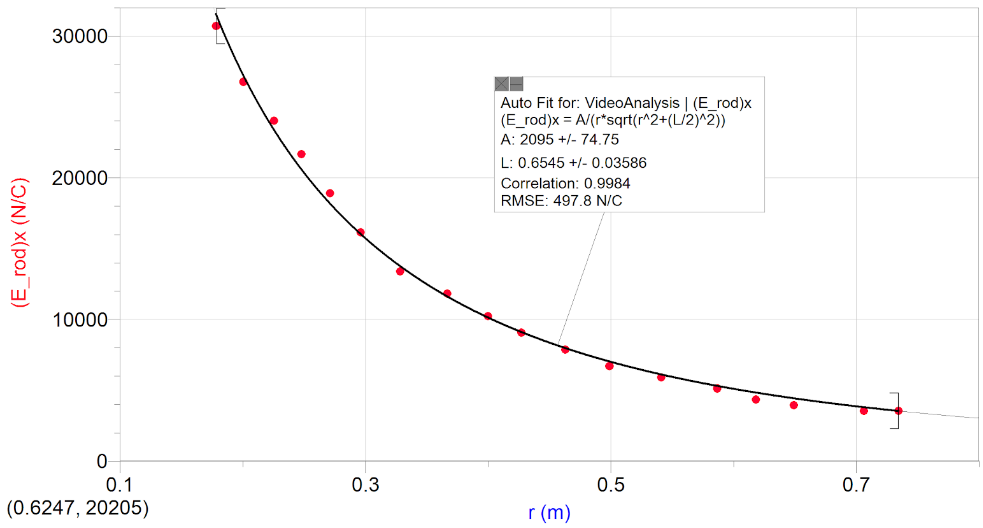

Figure 1. The electric field of the virtual rod (N/C) plotted against its distance from the charged sphere (m).

Note that the line of best fit is given by :

$$x = \frac{A}{r\sqrt{r^2 + (L/2)^2}}$$

where $A \approx 2095$, L is the length of the rod and r is the distance between the rod and the ball.

We are given that $A \approx kQ_{rod}$. From this we can derive the charge present on the rod. We predict that the rod will likely have a charge magnitude that is significantly greater than the hanging charge, since we obtained charges greater than 35nC by friction in previous experiments. To obtain the exact value of the charge on the rod, we compute:

$$\begin{aligned} 2095 \text{ Nm}^2/C &\approx kQ_{rod} \\ \frac{2095 \text{ Nm}^2/C}{k} &\approx Q_{rod} \\ Q_{rod} &\approx 0.233\mu\text{C} \end{aligned}$$

And indeed we see that $Q_{rod} > Q_{ball}$. We would also like a mathematical derivation of the equation for the line of best fit, so that we may better understand the electric field generated by a thin rod.

Since we only know how to derive the electric field generated by a point-particle, we orient the rod such that it lies on the y-axis on an x-y plain and then divide the rod into infinitesimally small pieces of length dy, such that each piece acts like a point-particle. Then, the charge of that particle would be $dy \cdot \lambda$, where $\lambda = Q_{rod}/L$ is the charge density of the rod.

With this information, we can conclude from our definition of an Electric Field for a point particle at a distance R, that

$$dE = k\frac{dQ}{R^2} = \frac{k\lambda dy}{R^2}$$

where dE is an infinitesimal piece of the electric field produced by the rod and dQ is the corresponding charge of the piece of the rod creating the field.

Now note that due to symmetry, the field at a particle placed at the midpoint of the rod (as the ball was placed) would have no y-component, since the repulsive force coming from the top and bottom parts of the rod would cancel in the y-direction. Thus, the only components would be in the x-direction. Therefore we can compute:

$$\begin{aligned}
E &= 2\int_0^{L/2} \cos(\theta)\frac{k\lambda dy}{R^2} \\
&= 2k\lambda \int_0^{L/2} \cos(\theta)\frac{dy}{R^2} \\
&= 2k\lambda \int_0^{L/2} \cos(\theta)\frac{dy}{r^2+y^2} && \text{Pythagorean Theorem} \\
&let \quad y = r\tan(\theta) \\
&\quad\; dy = r\sec^2(\theta)d\theta \\
\implies E &= 2k\lambda \int_{\theta_0}^{\theta_1} cos(\theta)\frac{r\sec^2(\theta)d\theta}{r^2\sec^2(\theta)} \\
E &= \frac{2k\lambda}{r}\int_{\theta_0}^{\theta_1} cos(\theta) \\
&= \frac{2k\lambda}{r}\sin(\theta_1) \\
&= \frac{k\lambda}{r}\cdot\frac{L}{\sqrt{r^2+(L/2)^2}} \\
E &= \frac{kQ_{rod}}{r\sqrt{r^2(L/2)^2}} \\
\vec{E} &= \frac{kQ_{rod}}{r\sqrt{r^2(L/2)^2}}\hat{\imath}
\end{aligned}$$

Where the last step is justified since the field will have non-zero magnitude only along the $\hat{\imath}$ direction (positive x-axis). And thus we have derived the same formula determined empirically for the electric field generated by a thin rod.

4. Discharge Rate

In this virtual lab, we wanted to measure the rate of electrical discharge of an object. To do so, we derived equations needed to determine the amount of charge on two balls as a function of their separation.

Figure 5 shows the free-body diagram used to visualize the derivation of the electrostatic force acting one of the balls. The derivation is as follows: In a state of equilibrium, we require that the vector sum of all the forces be 0. Also, note that the electrostatic force $\vec{F^{coul}}$ is horizontal and thus points only in the $\hat{i}$ direction. Therefore, the sum of the gravitational force $\vec{F^{grav}}$ and the tension force $\vec{F^{tens}}$ in the $\hat{j}$ direction must be zero. So for $R =$ distance between the two balls, $L =$ length of the strings, $m =$ mass of each ball, and $g = 9.8\text{m/s}^2$ we can derive:

$$\begin{aligned}
\vec{F^{tens}} \cdot \hat{j} &= mg \\
\cos(\theta) F^{tens} &= mg \\
F^{tens} &= \frac{mg}{\cos(\theta)} \\
\implies \quad F^{coul} &= F^{tens} \cdot \hat{i} = \frac{mg}{\cos(\theta)} \sin(\theta) \\
&= mg \tan(\theta) \\
F^{coul} &\approx \frac{mgR}{2L}
\end{aligned}$$

where the last step is justified since for $L >> R/2$ and $\theta << 0$, $tan(\theta) \approx \sin(\theta) = (R/2)/L$.

Also note that R can be given by $R = |X_1 - X_2|$ where X_1 is the location of ball 1 and X_2 is the location of ball 2. Using these formulas, we were able to obtain the data in Figures 2-4.

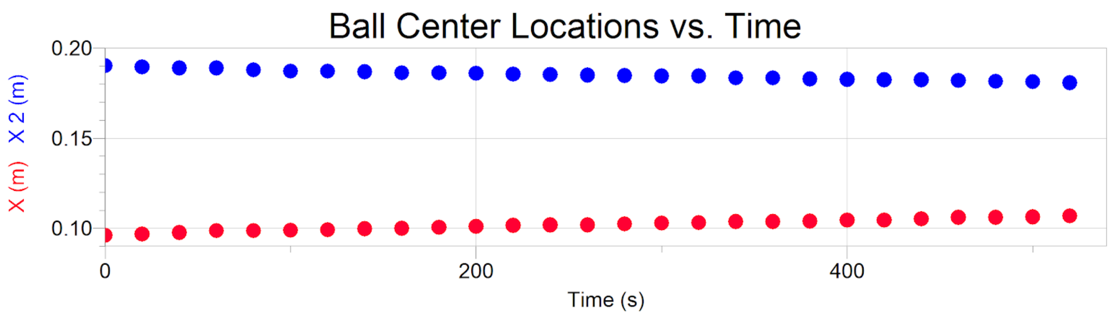

FIGURE 2. The position of the center location of both ball 1 and ball 2, denoted as X (m) and X 2 (m) respectively, versus time (s).

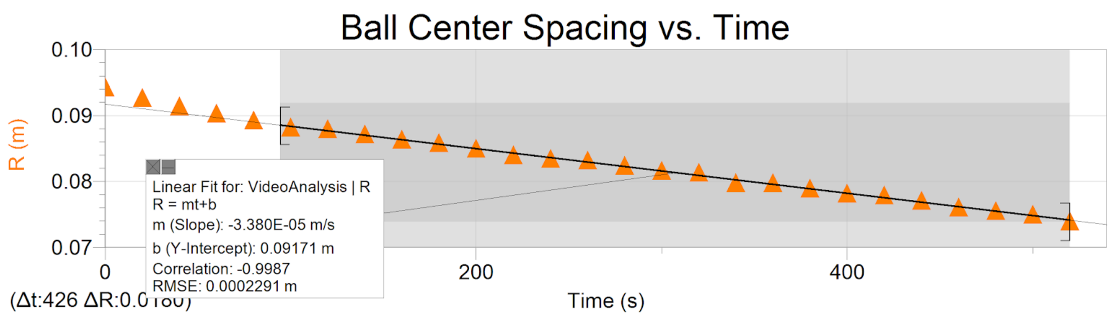

FIGURE 3. The separation of the centers of both ball 1 and ball 2, denoted as R (m), versus time (s).

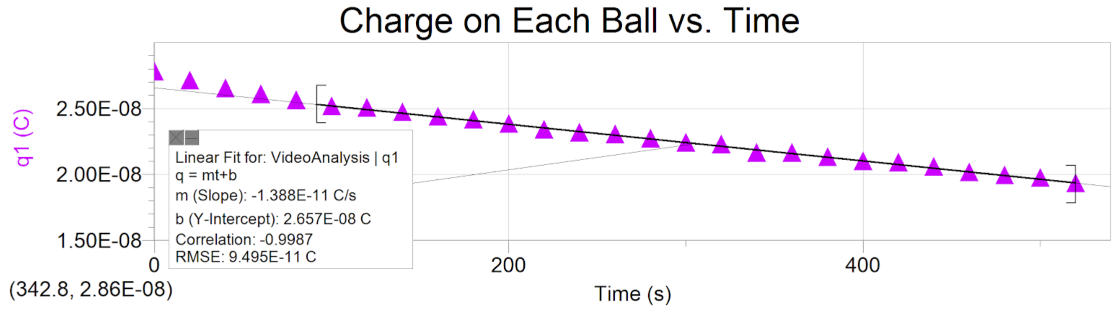

FIGURE 4. The charge on ball 1 (C) versus time (s). Given that the mass and charge of the two balls were nearly identical, only ball 1 was used for analysis.

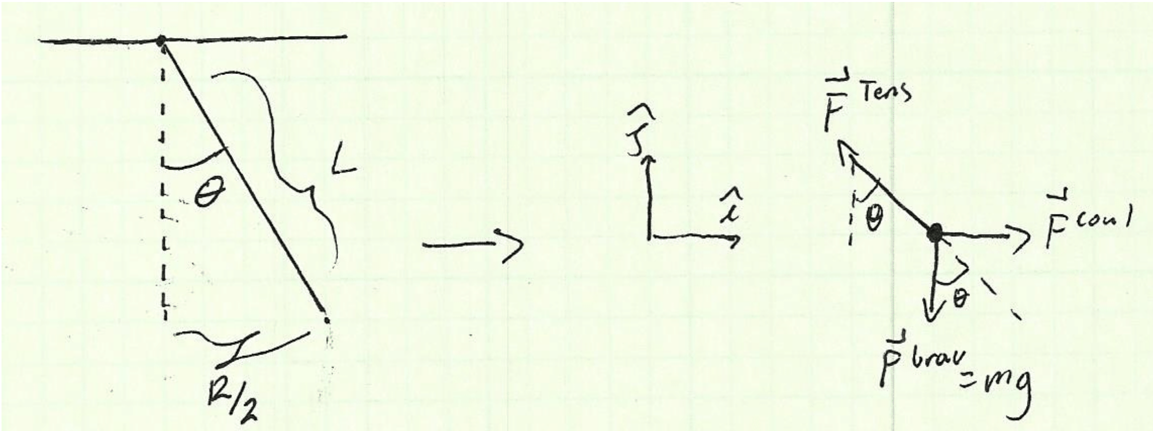

FIGURE 5. Free-body diagram of one of the charged balls suspended from a string

The linear regression line given in Figure 3 allows us to predict the time at which the two balls will touch. Given that the distance R at any time t is given by $R = -3.380 \times 10^{-5}t + 0.09171$, we can predict that $R = 0 \implies t \approx 2713$s which corresponds approximately 45 minutes.

Figure 4 shows that the charge on each ball also decreases with time. This result is not surprising, since the charged balls come closer together with time, which indicates that the system is not in equilibrium (if it was, we would observe that the balls eventually remain stationary without touching). The most likely explanation for the loss of charge on each ball is that their charge is lost to the moisture in the air.

However, note that the charge initially decreases faster than the regression line (prior to 100s). This indicates that there may be another drain for the charge which is significant only when the charges on a given ball is very large. This additional discharge may be due to the non-conducting string. While the charge is very high, the difference in charge between the ball and the string may cause electrons to move along the surface of the string, which would cause the more rapid charge loss that was observed when $t \leq 100$s.