CAPACITORS: WIRING PARALLEL AND IN SERIES

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1. Abstract

In this lab, the effective capacitance of three virtual capacitors, labeled A, B, and C, was studied when these provided capacitors were both wired in series as well as parallel to one another. These observations were experimentally verified by arranging three green ceramic disc capacitors labeled A, B, and C in series as well as parallel to one another. Using a Digital Multi-meter, it was determined that the expected and actual capacitance (nF) of each arrangement had minimal discrepancies. In the second portion of this lab, a virtual capacitor constructed from two large, parallel conducting plates was studied. It is noteworthy that this virtual capacitor was the subject of a movie clip integrated with an interactive Logger Pro file. Particularly, the change in voltage spread across each plate was analyzed as the plates were moved further apart from one another. It was found that as the distance between the plates increased, the voltage reading proportionally increased.

2. Introduction

A capacitor is a system of any two conductors that are separated by an insulator. Each of the conductors in a capacitor carry net excess charge that are equal in magnitude but are opposing charges. The capacitance, the ability of a system to store charge which is mathematically denoted as C, can be expressed as the following:

$$C = \frac{Q}{V}$$

where Q is the magnitude of excess charge possessed by each conductor and V is the potential difference (potential) across both conductors.

Gauss Law can be implemented to demonstrate that in an ideal parallel plate capacitor (a depiction of which is shown in Figure 1), capacitance is directly related to the area A of the plates. Further, Gauss Law displays that the capacitance of this ideal parallel plate system is inversely related to the distance d between the two plates:

$$C = \frac{\kappa \epsilon_0 A}{d}$$

where κ is the dielectric constant that is dependent upon the properties of the insulator between the two conductive plates, and ϵ_0 is the electric constant, often referred to as "permittivity".

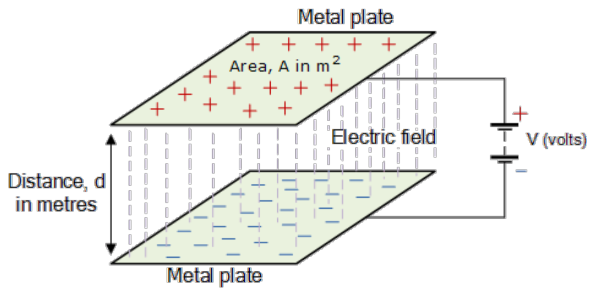

FIGURE 1. Ideal plate capacitor

We often connect capacitors to each other in series and in parallel. A depiction of capacitors in parallel can be seen in Figure 2.

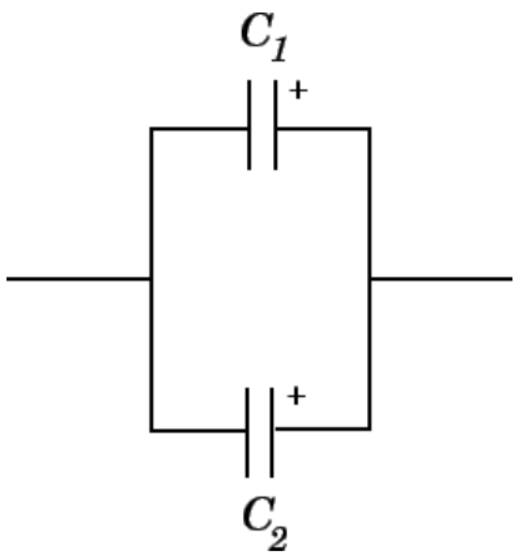

FIGURE 2. Capacitors in parallel. Photo credit [1]

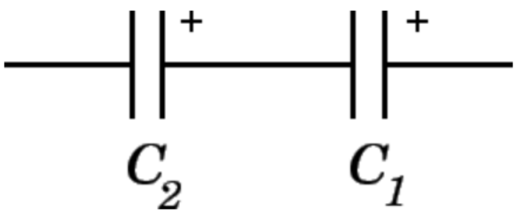

FIGURE 3. Capacitors in series. Photo credit [1]

In this case we know that the voltage across the two capacitors is equal[1] and thus we can derive :

$$\begin{aligned} C_{total} &= \frac{Q_{total}}{V} \\ &= \frac{Q_1 + Q_2}{V} \\ &= \frac{Q_1}{V} + \frac{Q_2}{V} \\ &= C_1 + C_2 \end{aligned}$$

Figure 3 shows two capacitors connected in series. In this case we know that the excess charge Q across both capacitors must be equal [1], and we similarly compute:

$$\begin{aligned}\frac{1}{C_{total}} &= \frac{V}{Q}\\ &= \frac{V_1+V_2}{Q}\\ &= \frac{V_1}{Q}+\frac{V_2}{Q}\\ &= \frac{1}{C_1}+\frac{1}{C_2}\end{aligned}$$

In summary, we can have the following four relations:

(1)	Definition of capacitance	$C = \frac{Q}{V}$
(2)	For an ideal capacitor	$C = \frac{\kappa\epsilon_0 A}{d}$
(3)	For capacitors in paralel	$C_{total} = C_1 + C_2$
(4)	For capacitors in series	$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$

3. Experimental confirmation of equations 2 and 3

In this section of the experiment we seek to gain empirical confirmation of equations (2) and (3) derived in the introduction.

In order to do this, we measured the capacitance of three capacitors A,B, and C. The results of these measurements can be seen in Table 1

Capacitor	Measured Capacitance (nF)
A	105.8
B	99.9
C	121.1

Table 1. Capacitance of capacitors A,B, and C.

After taking these measurements, we used equation (3) predicted the capacitance of all parallel circuits we could make using the three capacitors, then we built the circuits and measured the actual capacitance. This process was repeated for circuits in series with equation (4), and the results are seen in Tables 2 and 3, respectively.

Capacitors	Theoretical Capacitance(nF)	Measured Capacitance(nF)	% Error
A and B	205.7	205	0.341
A and C	226.9	226	0.398
B and C	221	220	0.455
A, B and C	326.8	327	0.0612

Table 2. Capacitance of A,B, and C connected in parallel.

Capacitor	Theoretical Capacitance(nF)	Measured Capacitance(nF)	% Error
A and B	51.4	51.2	0.391
A and C	56.5	56.5	0
B and C	54.7	54.4	0.552
A, B and C	36.1	35.9	0.557

Table 3. Capacitance of A,B, and C connected in series.

As expected, our predictions are very close to our measured values, with a maximum error of 0.455% for the parallel condition, and 0.557% for the series condition. With such small errors, we can clearly say that we have strong evidence in favor of equations (3) and (4) and that our small errors are likely due to measurement inaccuracies.

References

[1] URL: https://farside.ph.utexas.edu/teaching/302l/lectures/node46.html.