SI 7: Modular Arithmetic

1. Prove or disprove the following statement: $\forall n \in \mathbb{N}, \quad n^2 - 79n + 1601$ is prime.

2. Recall Euclid's algorithm for the gcd:

$$\gcd(0, n) = n$$
$$\gcd(m, n) = \gcd(n\%m, m)$$

   Prove the correctness of Euclid's algorithm (prove that it is guaranteed to find the correct result).

3. Prove that $\forall n, m \in \mathbb{N}$:

$$n \cdot m = \gcd(n, m) \cdot \text{lcm}(m, n)$$