SI 4: Function Composition

1 Domain and Co-Domain

1. Provide a function who's domain and co-domain can only be $\mathbb{N}$
2. Consider a function f that returns its input with decimal point truncated, such that $f(0.1) = 0$, $f(\pi) = 3$, $f(2) = 2$. What would the function's "signature" $f : A \to B$ be? (i.e. what would its domain an co-domain be)?

2 Function Composition

3. Define the function in problem (2.) as a composition of other functions using the following functions as building blocks:

   (a) $plus(n, m) = n + m$

   (b) $minus(n, m) = n - m$

   (c) $times(n, m) = n \times m$

   (d) $div(n, m) = n \div m$

   (e) $mod(n, m) = n\%m$ (aka the remainder of dividing n by m).

   Hint: you may define new functions where some of these functions are partially applied. Consider, for example, that you need a function that increments its argument by 5. Then you could define: $plus_5(n) = plus(n, 5)$.

4. Imagine a matrix of size $N \times M$, where the variable $i \in [0, N - 1]$ indexes the rows and $j \in [0, M - 1]$ the columns. A common trick in computer science is to "flatten" such a matrix by using a bijection between the set of indices of the matrix (enumerated by (i, j)) and the set of numbers $[0, (N \times M) - 1]$. Find this bijection and its inverse. *Hint: Notice that you are given that the matrix is of size $N \times M$.*