

SI 4: Function Composition, Nested Induction, and Midterm

1 Function Composition

1. Provide a function whose domain and co-domain can only be \mathbb{N}
2. Consider a function f that returns its input with decimal point truncated, such that $f(0.1) = 0$, $f(\pi) = 3$, $f(2) = 2$. What would the function's "signature" $f : A \rightarrow B$ be? (i.e. what would its domain and co-domain be)?
3. Is this function one-to-one (injective)? Is it onto (surjective)?
4. Define this function as a composition of other functions (hint: it may be useful to come up with your own definition of the division function first, think back to elementary school and remainders)

2 Double Induction

Consider the following function:

$$\begin{aligned}f(1, 1) &= 2 \\f(d^\frown, k) &= f(d, k) + 2(d + k) \\f(d, k^\frown) &= f(d, k) + 2(d + k - 1)\end{aligned}$$

Prove by induction:

$$\begin{aligned}\forall m, n \in \mathbb{N} \mid m, n > 1 \\f(m, n) &= (m + n)^2 - (m + n) - 2n + 2\end{aligned}$$