SI 5: The Proof to end all Proofs (jk you wish)

Consider that for sets $A, B, C \subseteq U$, the following are true:

1. $A \subseteq A$
2. $A \subseteq B$ and $B \subseteq C \implies A \subseteq C$
3. $A = B \iff A \subseteq B$ and $B \subseteq A$
4. $A \subseteq B \cap C \iff A \subseteq B$ and $A \subseteq C$
5. $A \cup B \subseteq C \iff A \subseteq C$ and $B \subseteq C$
6. $A \subseteq B \cup C \iff A \setminus B \subseteq C$

Using these facts and only these facts, prove that intersection distributes over union, i.e prove that:

$$D \cap (E \cup F) = (D \cap E) \cup (D \cap F)$$

Is true $\forall$ subsets D, E, F. Again, you should not mention any facts other than the 6 listed laws.