SI 2 : Lists and More Advanced Induction

1   List Vocabulary Review

For:

$$L = [a, b, c]$$
$$K = [1, 2, 3]$$
$$M = [\text{red, green, blue}]$$

Give the results of the following operations:

1. L++M

2. L++M++K

3. c::K

4. M::K

5. c::b

2   Proofs With Lists

6. Prove: $rev(\text{A++B}) = rev(\text{B}) + +rev(\text{A})$

7. Prove: $rev(rev(\text{K})) =$K

8. For a list L in which no elements repeat, and $i, j \in \mathbb{N}$ such that $i, j < len(L)$ and $i \neq j$, prove that $L_i \neq L_j$

9. Prove: For two lists K and L, if $len(K) = len(L)$, then $len(K++L) = 2len(L)$

3   A More Difficult Inductive Proof

Prove: For a tower of n disks, it takes $2^n - 1$ moves to solve the problem of the Towers of Hanoi.

The problem of the Towers of Hanoi: There are n disks stacked on one of 3 pegs, ordered from smallest disk to largest (smallest disk on the top). The task is to move all the disks to another peg following this set of rules:

- Only one disk may be moved at a time

- No disk may be placed on a peg with a smaller disk underneath it