

LCD, GCD, and Bezout's Theorem

1 Euclid's Algorithm

Euclid's Algorithm for finding GDC is as follows:

$$\begin{aligned} \gcd(0, n) &= n \\ \gcd(m, n) &= \gcd(n \% m, m) \quad \text{for any } m > 0 \end{aligned}$$

Calculate the following GCD's using Euclid's Algorithm, show steps:

1. $\gcd(3, 9)$
2. $\gcd(0, 24)$
3. $\gcd(436, 559)$
4. $\gcd(5, 525)$
5. $\gcd(123456789, 123456790)$

2 Proofs regarding gcd and lcm

Recall the definition of $\text{fib}(n)$

$$\begin{aligned} \text{fib}(0) &= 0 \\ \text{fib}(1) &= 1 \\ \text{fib}(k^{\frown}) &= \text{fib}(k^{\frown}) + \text{fib}(k) \end{aligned}$$

Construct a table of the first 10 values for the following algorithm:

6. $\gcd(\text{fib}(n^{\frown}), \text{fib}(n))$

Prove the Following:

7. $\gcd(\text{fib}(n^{\frown}), \text{fib}(n)) = 1$ for any $n \in \mathbb{N}$
8. $n \cdot m = \gcd(n, m) \cdot \text{lcm}(m, n)$