SI Session 2: Basic Set Theory, Functions, and More Advanced Inductive Proof Writing

1 Set Theory

1.1 Basic Grammar

Define: $A = \{1, 2, 3, \{3, 2\}, 2, 4\}; B = \{1, 2, 3\}; C = \{A, \{B\}, C\}$

1. True or false: $A \in B$
2. True or false: $B \in A$
3. True or false: $B \subseteq A$
4. True or false: $\{A\} \in C$
5. True or false: $\{B\} \in C$
6. True or false: $\emptyset \in C$
7. True or false: $\emptyset \subset C$
8. Something here does not make sense, find it.

1.2 Carteisan Product

Define: $K = \{red, blue, green\}; L = \{1, 2, 3\}$

1. Calculate: $K \times L$
2. Calculate: $L \times K$
3. Calculate:$|K|, |L|, |L \times K|$

2 Functions

2.1 Domain, Co-Domain and Function Composition

1. Define the behavior of a function that takes a $\mathbb{N}$ as input, and outputs into the set A, using the correct notation.
2. Find the Domain and Co-domain: $f(x) = x^2$
3. Define a function that has $\mathbb{I}$ as its domain, and $\mathbb{N}$ as its co-domain.
4. Define a function that has $\mathbb{N}$ as its domain, and $\mathbb{R}$ as its co-domain.
5. Define a function which can only have $\mathbb{N}$ as its domain.
6. You are tasked with defining a function which encodes the location of a cell in a grid of pre-defined length. The length is given as an pair (j, k), and you must encode it into a single natural number. Define the behavior of this function using function composition.

2.2 One-to-One and Onto Functions

For the next section: Indicate whether the function is one to one, onto, both or neither

1. $f(x) = x^2$ for $f : \mathbb{R} \to \mathbb{R}$
2. $f(x) = x^2$ for $f : \mathbb{C} \to \mathbb{I}$
3. $f(x) = x$
4. $f(x) = 2^x$
5. $f(n) = \sum_{k=1}^{n} k$

Define the following:

1. A function that is one to one but not onto
2. A function that is onto but not one to one
3. A function that is neither one to one or onto
4. A function that is one to one and onto

3 Induction

Prove by induction:
$a_1b_1+...+a_nb_n = (a_1-a_2)b_1+(a_2-a_3)(b_1+b_2)+...+(a_{n-1}-a_n)(b_1+...+b_{n-1})+a_n(b_1+...+b_n)$