Modular Arithmetic, Min, Max, and a Review of Strong Induction

1 Modular Arithmetic

For the next few problems you will need to recall a few facts of the Gregorian calendar

Leap years occur every 4th year, except for years ending in 00.
The following months have 30 days: September, April, June, November
February has 28 days on common years, 29 on leap years
All other months have 31 days

Find the day of the week for the following dates

1. 10/22/21
2. 12/10/46
3. 11/9/2118

2 Strong induction

4. Show that for any natural numbers, m and $m \cdot n + 1$ are always relatively prime given $m \neq 1$

3 Min and Max

Consider the following definitions

$$p \leq m \to n \text{ if and only if } min(p, m) \leq n$$
$$m \,\theta\, n \leq p \text{ if and only if } m \leq max(n, p)$$

5. Prove using the definitions of θ and $\to$: $max(m, min(n, p)) = min(max(m, n), max(m, p))$