SI 3: Midterm 1 Review

1 Sets

Consider the following sets:

- $A : \{1, 2, 3, 4, 5\}$
- $B : \{2, 3, 4\}$
- $C : \{\{2\}, \{3\}, \{4\}\}$
- $D : \{Red, Green, Blue\}$

Define the Following:

1. $B \times D$
2. $D \times B$
3. $\mathcal{P}(B)$

Answer True or False for the following:

1. $B \in A$
2. $B \subseteq A$
3. $C \subseteq A$
4. $A \subseteq A$
5. $A \in A$
6. $2 \in C$
7. $A \in \mathbb{N}$
8. $A \subseteq \mathbb{N}$
9. $C \subseteq \mathbb{N}$
10. $\mathcal{P}(B) \in B$
11. $B \in \mathcal{P}(B)$
12. $B \subseteq \mathcal{P}(B)$
13. $\emptyset \in B$
14. $\emptyset \subseteq B$
15. $\emptyset \in \mathcal{P}(B)$
16. $\emptyset \subseteq \mathcal{P}(B)$

2 Functions

2.1 Identifying Co-Domains, Domains, and basic function properties

Identify Domain and Co-Domain

1. $f : A \to B$
2. $f(x) = x^2$
3. $f(x) = \sum_{k=1}^{x} k$
4. $f \circ g$; $f : B \to C; g : A \to B$

Find a function with the following properites:

1. $f : \mathbb{N} \to \mathbb{R}$, 1-1, not onto
2. $f : \mathbb{R} \to \mathbb{N}$, onto, not 1-1
3. $f : \mathbb{N} \to \{True, False\}$, not 1-1, onto
4. $\mathbb{N} \to \{\bullet\}$, not 1-1, onto

Is is possible to make $f(x) = \sqrt{x}$ a total function (i.e. a function defined across its entire Domain)? If so, how?

2.2 Using Function Definitions

$$f(m^\frown) = (m^\frown) + f(m)$$
$$f(0) = 0$$

Find $f(5)$, show steps

You have seen this function before, what does it usually look like?

3 Inductive Proofs

3.1 Fill-in-Blanks

Prove: $m(n+p) = m \cdot n + m \cdot p$

Proof. $\bullet[Basis] : 0(n+p) = 0 \cdot n + 0 \cdot p$

$$\begin{aligned} 0 \cdot n + 0 \cdot p &= 0 + 0 && \text{0 is an annihilator} \\ &= 0 && \rule{4cm}{0.4pt} \\ &= 0(n+p) && \text{0 is an annihilator} \end{aligned}$$

$\bullet[IH] : \exists k \;\; s.t. \;\; k(n+p) = k \cdot n + k \cdot p$
$\bullet[IS]$: We need to show that _______________

$$\begin{aligned} (k^\frown)(n+p) &= (n+p) + k(n+p) && \text{Definition of Multiplication} \\ &= \rule{4cm}{0.4pt} && \text{IH} \\ &= n + k \cdot n + p + k \cdot p && \text{Commutativity of Addition} \\ &= (k^\frown)n + (k^\frown)p && \rule{4cm}{0.4pt} \end{aligned}$$

□

3.2 Writing Proofs

Prove: $\sum_{i=0}^{n} 4^i = \frac{4^{n+1}-1}{3}$ $\qquad \forall n \in \{x \in \mathbb{N} | x > 0\}$