Section 4

Excercise 4.2

In[378]:=

```
Clear[f, f1, f2, euler, Heun, RK, euler,
 eulergraph, Heungraph, RKgraph, eug, heg, rug, ext]
(*Ȑ ᵋ    ừ æ ʪ *)
```

Excercise 4.2(1)

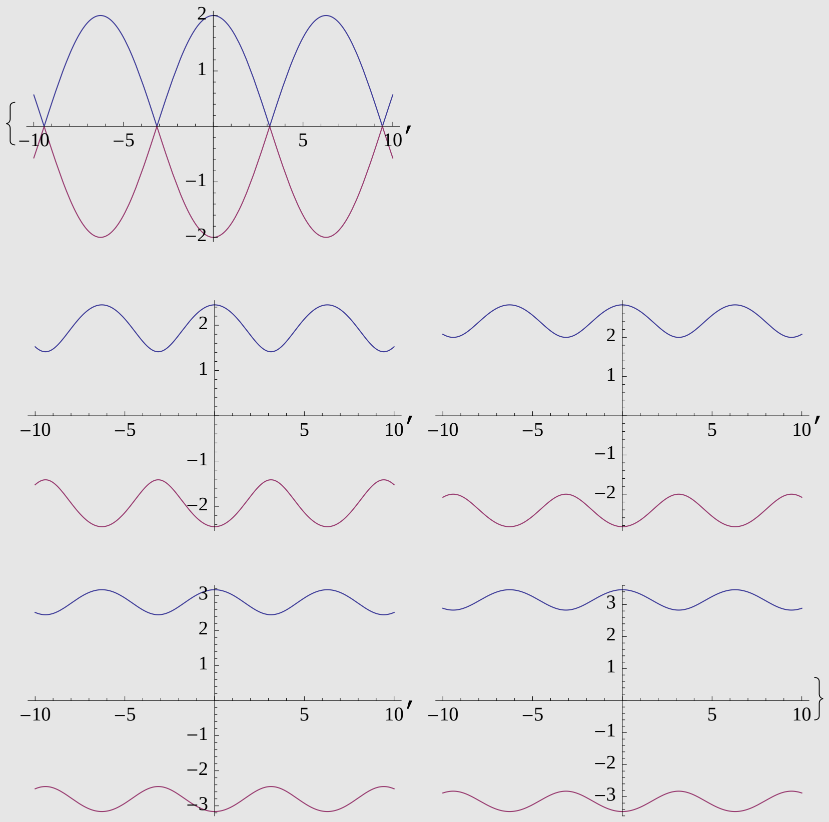
In[379]:=

```
f[x_, t_] := -Sin[x];
f1[x1_, x2_, t_] := x2;
f2[x1_, x2_, t_] := -Sin[x1];
```

In[382]:=

```
f[C_] := Plot[{√2 (C + Cos[x]) , -√2 (C + Cos[x]) }, {x, -10, 10}]
Table[f[c], {c, 1, 5}]
```

Out[383]=



Excercise 4.2(2)

Euler Method

In[384]:=

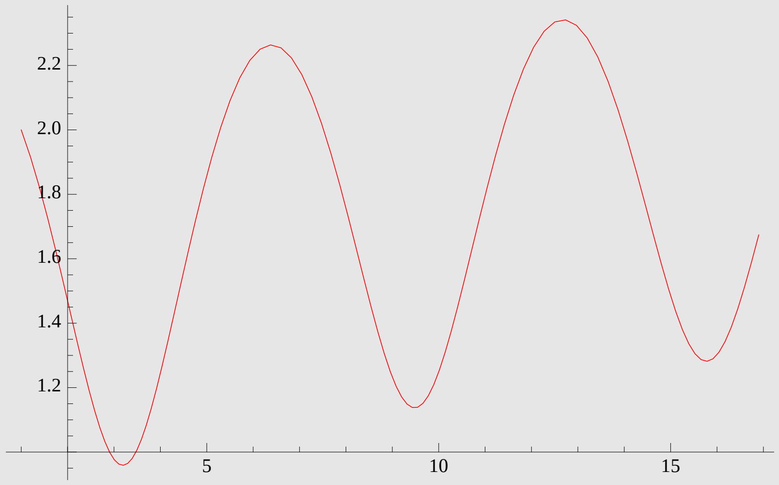
```
euler::nonnegativeint = "Nmax must be a non-negative integer";

euler[dt_, Nmax_, a1_, a2_] :=
  Module[{j = 0, n, x1, x2, t}, If[Or[Nmax ≤ 0, Not[IntegerQ[Nmax]]],
    Message[euler::nonnegativeint], x1[0] := a1; x2[0] := a2;
    x1[j_] := (x1[j]  = x1[j - 1] + dt f1[x1[j - 1], x2[j - 1], t]);
    x2[j_]  := (x2[j]  = x2[j - 1] + dt f2[x1[j - 1], x2[j - 1], t]);
    Table[{x1[j], x2[j]}, {j, 0, Nmax}]
]];
```

In[386]:=

```
eulergraph[dt_, Nmax_, a1_, a2_] := ListLinePlot[euler[dt, Nmax, a1, a2],
   PlotStyle → {Thickness[0.001], RGBColor[1, 0, 0]}, PlotRange → All];
eulergraph[.1, 100, 1, 2]
```

Out[387]=



Heun Method

In[388]:=

```
Heun[dt_, Nmax_, a1_, a2_] := Module[
    {j = 0, n, g1, g2, h1, h2, x1, x2, t}, If[Or[Nmax ≤ 0, Not[IntegerQ[Nmax]]],
     Message[Heun::nonnegativeint], x1[0] := a1; x2[0] := a2;
     g1[j_] := (g1[j] = f1[x1[j], x2[j], t]);
     h1[j_] := (h1[j] = f2[x1[j], x2[j], t]);

     g2[j_] := (g2[j] = f1[x1[j] + dt g1[j], x2[j] + dt h1[j], t + dt]);
     h2[j_] := (h2[j] = f2[x1[j] + dt g1[j], x2[j] + dt h1[j], t + dt]);
                                        dt
     x1[j_]  :=  (x1[j]  = x1[j - 1] + ── (g1[j - 1]  + g2[j - 1]));
                                        2
                                        dt
     x2[j_]  :=  (x2[j]  = x2[j - 1] + ── (h1[j - 1]  + h2[j - 1]));
                                        2
     Table[{x1[j], x2[j]}, {j, 0, Nmax}]
    ]
  ];
```

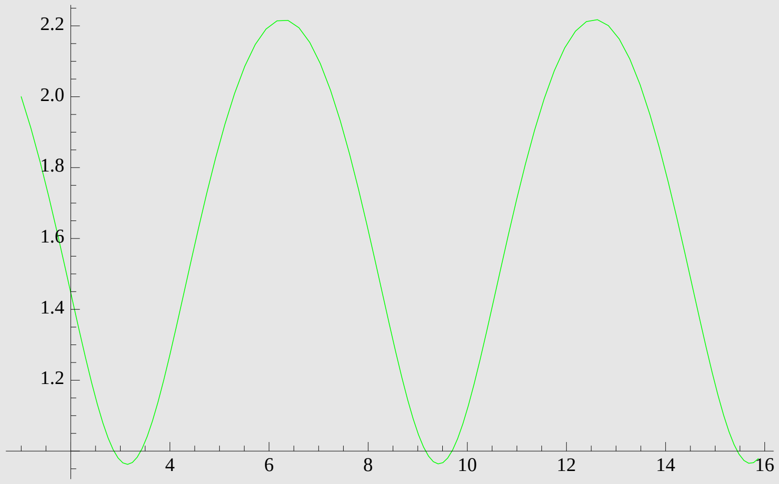
In[389]:=

```
Heungraph[dt_, Nmax_, a1_, a2_] := ListLinePlot[Heun[dt, Nmax, a1, a2],
    PlotStyle → {Thickness[0.001], RGBColor[0, 1, 0]}, PlotRange → All];
```

In[390]:=

```
Heungraph[0.1, 100, 1, 2]
```

Out[390]=



Runge Kutta Method

In[391]:=

```
RK::nonnegativeint = "Nmax must be a non-negative integer";

RK[dt_, Nmax_, a1_, a2_] :=
  Module[{j = 0, n, g1, g2, g3, g4, h1, h2, h3, h4, x1, x2, t},
    If[Or[Nmax ≤ 0, Not[IntegerQ[Nmax]]], Message[RK::nonnegativeint],
     x1[0] := a1; x2[0] := a2;
     g1[j_] := (g1[j] = f1[x1[j], x2[j], t]);
     h1[j_] := (h1[j] = f2[x1[j], x2[j], t]);

     g2[j_] := (g2[j] = f1[x1[j] + dt / 2 g1[j], x2[j] + dt / 2 h1[j], t + dt / 2]);
     h2[j_] := (h2[j] = f2[x1[j] + dt / 2 g1[j], x2[j] + dt / 2 h1[j], t + dt / 2]);

     g3[j_] := (g3[j] = f1[x1[j] + dt / 2 g2[j], x2[j] + dt / 2 h2[j], t + dt / 2]);
     h3[j_] := (h3[j] = f2[x1[j] + dt / 2 g2[j], x2[j] + dt / 2 h2[j], t + dt / 2]);

     g4[j_] := (g4[j] = f1[x1[j] + dt g3[j] , x2[j] + dt h3[j], t + dt]);
     h4[j_] := (h4[j] = f2[x1[j] + dt g3[j] , x2[j] + dt h3[j], t + dt]);
     x1[j_] :=
                                  dt
       (x1[j] = x1[j - 1] +  ── (g1[j - 1] + 2 g2[j - 1] + 2 g3[j - 1] + g4[j - 1]));
                                   6
     x2[j_] := (x2[j] = x2[j - 1] +

            dt
            ── (h1[j - 1] + 2 h2[j - 1] + 2 h3[j - 1] + h4[j - 1]));
             6
     Table[{x1[j], x2[j]}, {j, 0, Nmax}]
    ]
  ];
```

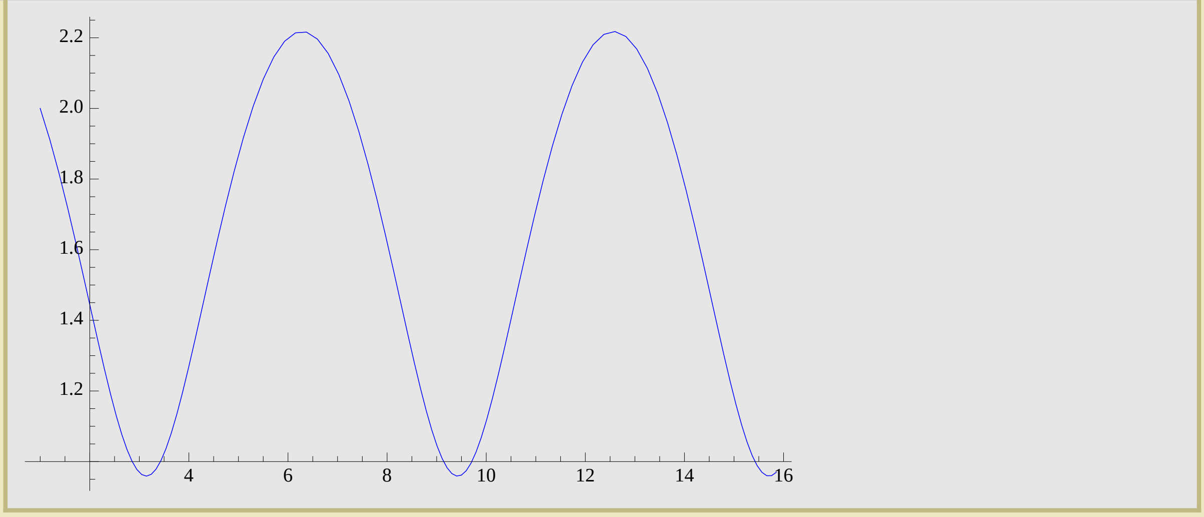
In[393]:=

```
RKgraph[dt_, Nmax_, a1_, a2_] := ListLinePlot[RK[dt, Nmax, a1, a2],
   PlotStyle → {Thickness[0.001], RGBColor[0, 0, 1]}, PlotRange → All];
```

In[286]:=

```
RKgraph[0.1, 100, 1, 2]
```

Out[286]=

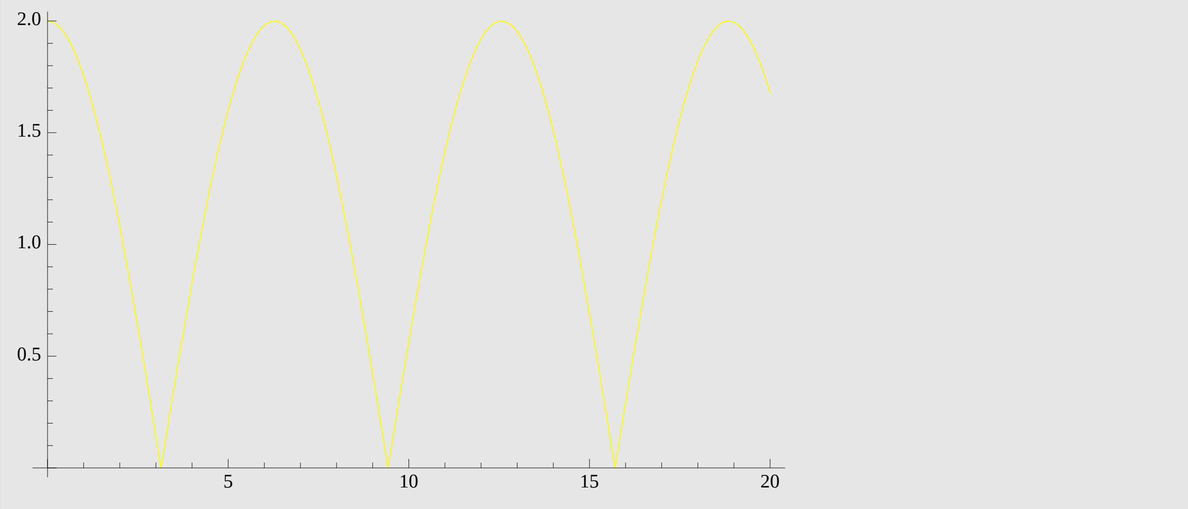


Analytic

In[442]:=

```
asol[a1_, a2_] :=
  DSolve[{x''[t] == f[x[t], t], x[0] == a1, x'[0] == a2}, x[t], t];
agraph[a1_, a2_, tlast_] := Plot[Evaluate[x[t] /. asol[a1, a2]], {t, 0, tlast},
   PlotStyle → {Thickness[0.005], RGBColor[1, 0, 1]}, PlotRange → All];
exactgraph[dt_, Nmax_, a1_, a2_] := Plot[√(2 + 2 Cos[t]) ,
   {t, 0, dt * Nmax}, PlotStyle → RGBColor[1, 1, 0]];
exactgraph[0.1, 200, 1, 2]
```

Out[445]=

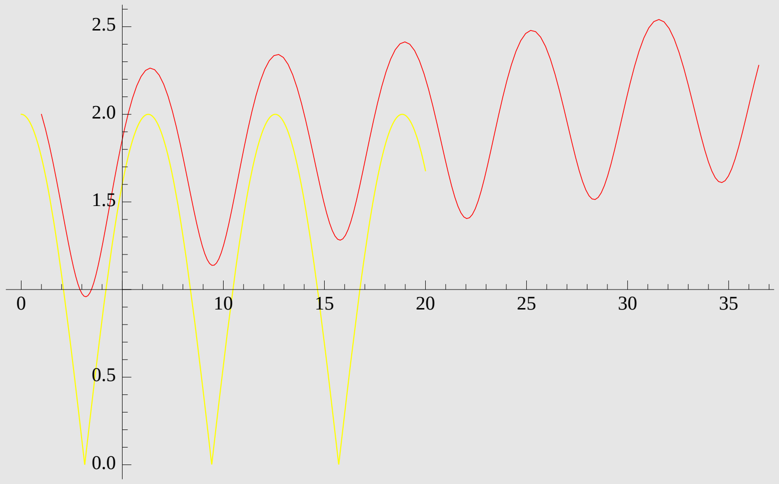


Comparison of Approximation for 3 methods

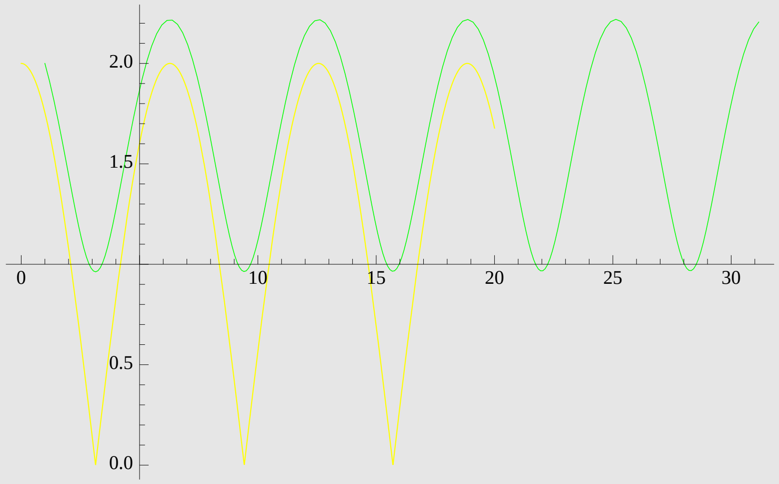
In[512]:=

```
eug = eulergraph[.1, 200, 1, 2];
heg =  Heungraph[.1, 200, 1, 2];
rug = RKgraph[.1, 200, 1, 2];
ext = exactgraph[.1, 200, 1, 2];
Show[eug, ext]
Show[heg, ext]
Show[rug, ext]
Show[eug, heg, rug, ext]
```

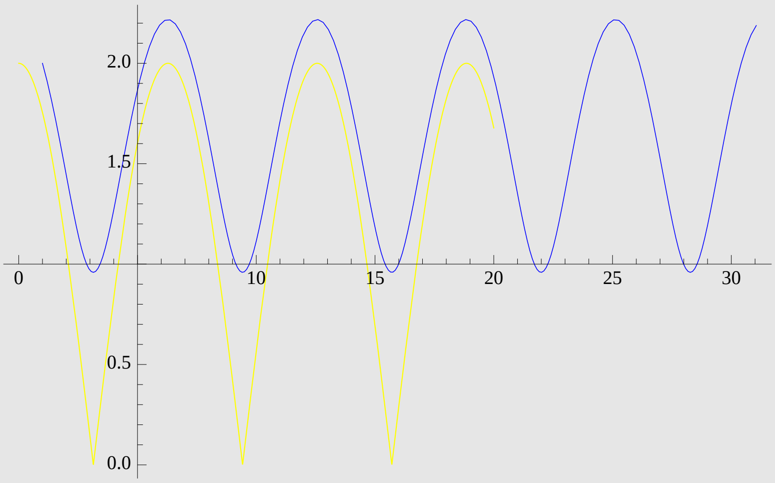
Out[516]=



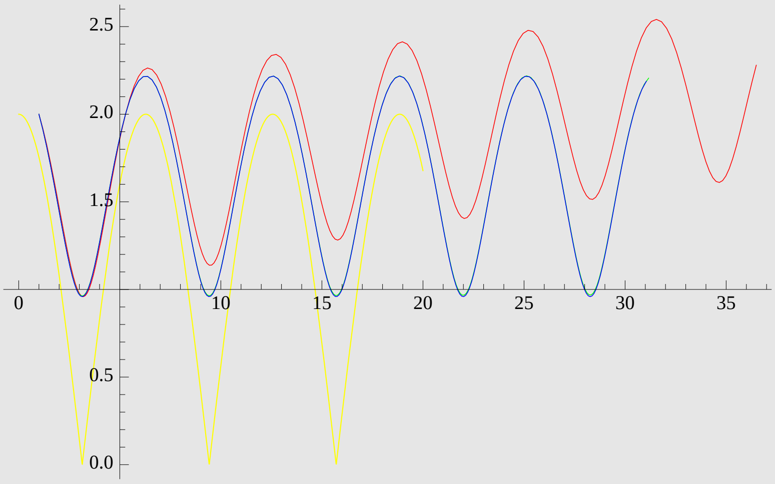
Out[517]=



Out[518]=



Out[519]=

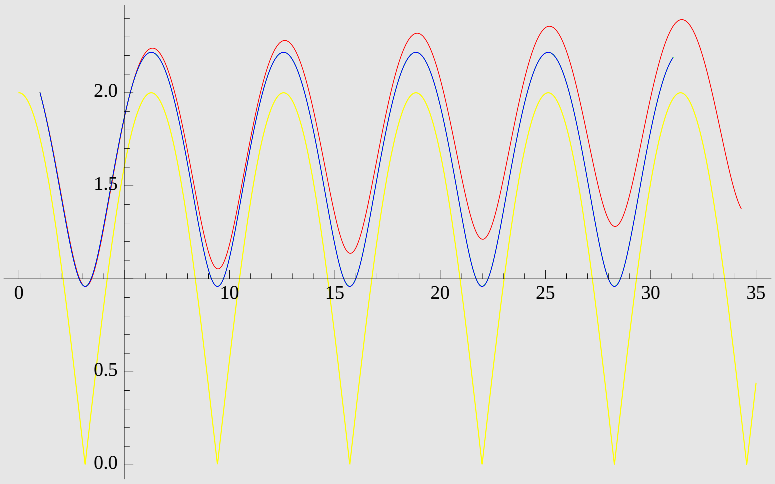


Red : Eulerת Green : Heun, light Green : Runge Kuttaת Yellow : analytic

In[538]:=

```
eug = eulergraph[.05, 400, 1, 2];
heg =  Heungraph[.05, 400, 1, 2];
rug = RKgraph[.05, 400, 1, 2];
ext = exactgraph[.05, 700, 1, 2];
Show[eug, heg, rug, ext]
```

Out[542]=

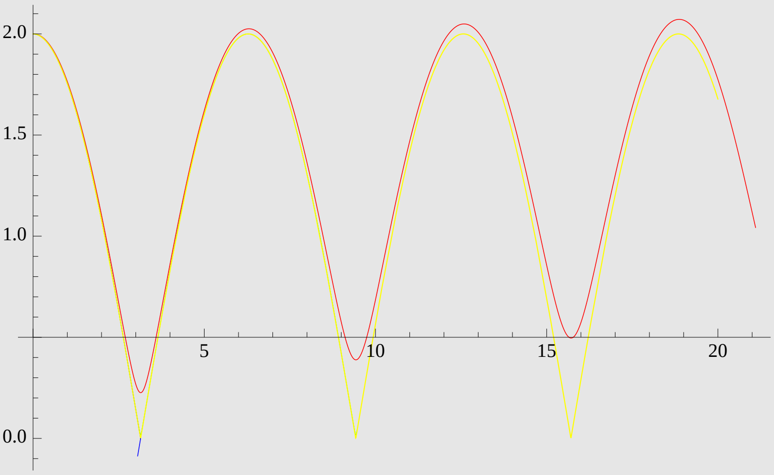


Heun Method Graph and Runge Kutta Graph already superposed with one another. Euler Method has still bad approximation.

In[543]:=

```
eug = eulergraph[.02, 1000, 0, 2];
heg =  Heungraph[.02, 1000, 0, 2];
rug = RKgraph[.02, 1000, 0, 2];
ext = exactgraph[.02, 1000, 0, 2];
Show[eug, heg, rug, ext]
```

Out[547]=



Euler Method is very slow to approximate.In our problem Heun and Runge -Kutta are showing same level of approximation.