Multiscale Analysis of Complex Time Series: Integration of Chaos and Random Fractal Theory, and Beyond
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CHAPTER 1

INTRODUCTION

Complex systems are usually comprised of multiple subsystems that exhibit both highly nonlinear deterministic and stochastic characteristics and are regulated hierarchically. These systems generate signals that exhibit complex characteristics such as sensitive dependence on small disturbances, long memory, extreme variations, and nonstationarity. A stock market, for example, is strongly influenced by multilayered decisions made by market makers, as well as by interactions of heterogeneous traders, including intraday traders, short-period traders, and long-period traders, and thus gives rise to highly irregular stock prices. The Internet, as another example, has been designed in a fundamentally decentralized fashion and consists of a complex web of servers and routers that cannot be effectively controlled or analyzed by traditional tools of queuing theory or control theory and give rise to highly bursty and multiscale traffic with extremely high variance, as well as complex dynamics with both deterministic and stochastic components. Similarly, biological systems, being heterogeneous, massively distributed, and highly complicated, often generate nonstationary and multiscale signals. With the rapid accumulation of complex data in health sciences, systems biology, nano-sciences, information systems, and physical sciences, it has become increasingly important to be able to analyze multiscale and nonstationary data.

Multiscale signals behave differently, depending upon the scale at which the data are examined. How can the behaviors of such signals on a wide range of scales be simultaneously characterized? One strategy we envision is to use existing theories synergistically instead of individually. To make this possible, appropriate scale ranges where each theory is most pertinent need to be identified. This is a difficult task, however, since different theories may have entirely different foundations. For example, chaos theory is mainly concerned about apparently irregular behaviors in a complex system that are generated by nonlinear deterministic interactions with only a few degrees of freedom, where noise or intrinsic randomness does not play an important role. Random fractal theory, on the other hand, assumes that the dynamics of the system are inherently random. Therefore, to make this strategy work, different theories need to be integrated and even generalized.

The second vital strategy we envision is to develop measures that explicitly incorporate the concept of scale so that different behaviors of the data on varying scales can be simultaneously characterized by the same scale-dependent measure. In the most ideal scenario, a scale-dependent measure can readily classify different types of motions based on analysis of short, noisy data. In this case, one can readily see that the measure will be able not only to identify appropriate scale ranges where different theories, including information theory, chaos theory, and random fractal theory, are applicable, but also to automatically characterize the behaviors of the data on those scale ranges.

The vision presented above dictates the style and the scope of this book, as depicted in Fig. 1.1. Specifically, we aim to build an effective arsenal by synergistically integrating approaches based on chaos and random fractal theory, and going beyond this, to complement conventional approaches such as spectral analysis and machine learning techniques. To make such an integration possible, four important efforts are made:

1. Wavelet representation of fractal models as well as wavelet estimation of fractal scaling parameters will be carefully developed. Furthermore, a new fractal model will be developed. The model provides a new means of characterizing long-range correlations in time series and a convenient way of modeling non-Gaussian statistics. More importantly, it ties together different approaches in the vast field of random fractal theory (represented by the four small boxes under the "Random Fractal" box in Fig. 1.1).

2. Fractal scaling break and truncation of power-law behavior are related to specific features of real data so that scale-free fractal behavior as well as structures defined by specific scales can be simultaneously characterized.

3. A new theoretical framework for signal processing — power-law sensitivity to initial conditions (PSIC) — will be developed, to provide chaos and random fractal theory a common foundation so that they can be better integrated.

4. The scale-dependent Lyapunov exponent (SDLE), which is a variant of the finite-size Lyapunov exponent (FSLE), is an excellent multiscale measure. We shall develop a highly efficient algorithm for calculating it and show that it can readily classify different types of motions, aptly characterize complex behaviors of real-world multiscale signals on a wide range of scales, and, therefore, naturally solve the classic problem of distinguishing low-dimensional chaos from noise. Furthermore, we shall show that the SDLE can effectively deal with nonstationarity and that existing complexity measures can be related to the value of the SDLE on specific scales.

To help readers better understand and appreciate the power of the materials in this book, nearly every significant concept or approach presented will be illustrated by applying it to effectively solve real problems, sometimes with unprecedented accuracy. Furthermore, source codes, written in various languages, including Fortran, C, and Matlab, for many methods are provided together with some simulated and experimental data.

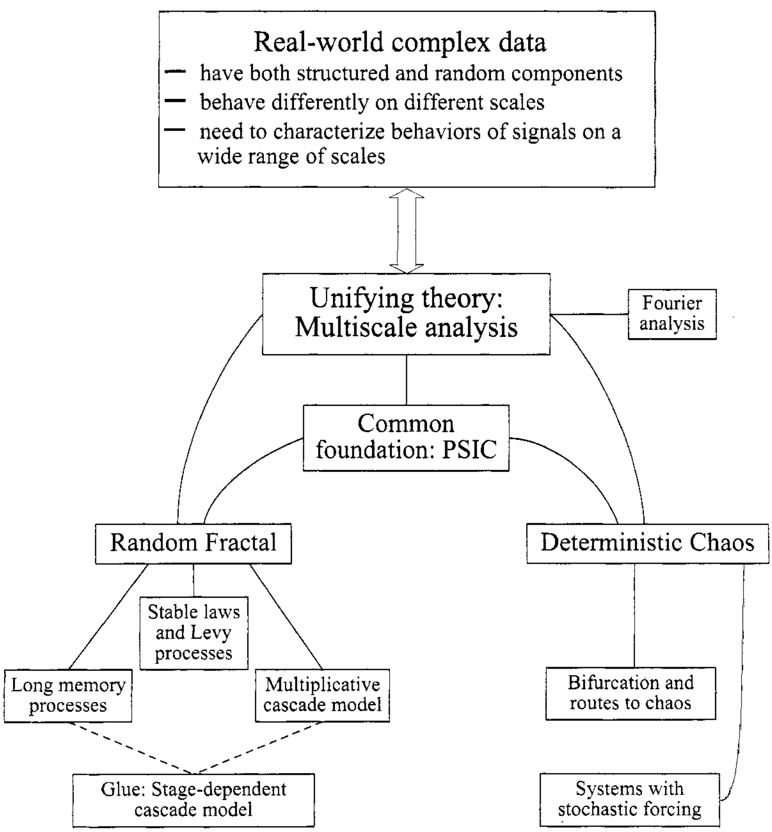

Figure 1.1. Structure of the book.

In the rest of this chapter, we give a few examples of multiscale phenomena so that readers can better appreciate the difficulties and excitement of multiscale signal processing. We then highlight a number of multiscale signal processing problems that will be discussed in depth later in the book. Finally, we outline the structure of the book.

Before proceeding, we note a subtle but important distinction between fractal (and wavelet)-based multiscale analysis methods and multiscale phenomena. As we shall discuss in more detail in Chapter 2, fractal phenomena are situations where a part of an object or phenomenon is exactly or statistically similar to another part or to the whole. Because of this, no specific scales can be defined. Fractal property can thus be considered as a common feature across vastly different scales; such a feature can be considered as a background. While this can be viewed as a multiscale phenomenon, it is in fact one of the simplest. More complicated situations can be easily envisioned. For example, a fractal scaling may be broken at certain spatio-temporal scales determined by the periodicities. Here, the periodicities are part of the multiscale phenomenon but not part of the fractal phenomenon. As we shall see in Chapter 8, exploiting the fractal background and characterizing the fractal scaling break can be extremely powerful techniques for feature identification. In fact, the importance of the fractal feature as a background has motivated us to discuss random fractal theory first.

1.1 EXAMPLES OF MULTISCALE PHENOMENA

Multiscale phenomena are ubiquitous in nature and engineering. Some of them are, unfortunately, catastrophic. One example is the tsunami that occurred in South Asia at Christmas 2004. Another example is the gigantic power outage that occurred in North America on August 14, 2003. It affected more than 4000 megawatts, was more than 300 times greater than mathematical models would have predicted, and cost between $4 billion and $6 billion, according to the U.S. Department of Energy. Both events involved scales that, on the one hand, were so huge that human beings could not easily fathom and, on the other hand, involved the very scales that were most relevant to individual life. Below, we consider six examples. Some of them are more joyful than tsunamis and power outages.

Multiscale phenomena in daily life. When one of the authors, J.B., relocated to Gainesville, Florida, in 2002, he wanted to stay in an apartment with a lake view. Behind the apartment complex he eventually leased, there were numerous majestic cypress trees closely resembling a small wetland forest. The would-be lake was in fact a sinkhole, like many others in the karst topography of Florida. It had a diameter of about half a mile. Nevertheless, it had been dried for a number of years.

The situation completely changed after hurricane Frances struck Florida in September, 2004. During that hurricane season, a formidable phrase often used in the media was "dark clouds the size of Texas." Texas is about twice the size of Florida, so the ratio between the size of the sinkhole behind J.B.'s apartment and the clouds associated with the tropical storm system is on the order of 10^{-3}. Obviously, the sizes of the sinkhole and the cloud system define two very different scales.

During the passage of hurricane Frances, within only three days, the water level in the center of the sinkhole rose 4–5 meters. By the spring of 2005, the sinkhole had fully developed into a beautiful lake ecosystem: wetland plants and trees blossomed; after a few rains, the water swarmed with tiny fishes; each day at around sunset, hundreds of egrets flew back to the lake, calling; dozens of ducks constantly played on the water, generating beautiful water waves; turtles appeared; even alligators came – one day one of them was seen to be killing a snake for food. All these activities occurred on a scale comparable to or much smaller than that of the lake, and therefore much smaller than the size of the clouds accompanying hurricane Frances. In spite of causing devastating destructions to the east coast of Florida, hurricane Frances also replenished and diversified ecosystems in its path. Therefore, although a rather rare and extreme event, hurricane Frances can never be ignored because it made a huge impact on lives long after its passing. An important lesson to learn from this example is that a rare event may not simply be treated as an outlier and ignored.

One of the most useful parameters for characterizing water level change in a river or lake is the Hurst parameter, named after the distinguished hydrologist Hurst, who monitored the water level changes in Niles for decades. The Hurst parameter measures the persistence of correlations. Intuitively, this corresponds to the situation of the sinkhole behind J.B.'s apartment: when it is dry, it can stay dry for years, but with its current water level, it is unlikely to become dry again any time soon. In the past decade, researchers have found that persistent correlation is a prevailing feature of network traffic. Can this feature be ignored when designing or examining the quality of service of a network? The answer is no. We shall have much to say about the Hurst parameter in general and the impact of persistent correlation on network traffic modeling as an example of an application in this book.

Multiscale phenomena in genomic DNA sequences. DNA is a large molecule composed of four basic units called nucleotides. Each nucleotide contains phosphate, sugar, and one of the four bases: adenine, guanine, cytosine, and thymine (usually denoted A, G, C, and T). The structure of DNA is described as a double helix. The two helices are held together by hydrogen bonds. Within the DNA double helix, A and G form two and three hydrogen bonds with T and C on the opposite strand, respectively. The total length of the human DNA is estimated to be 3.2×10^9 base pairs. The most up-to-date estimate of the number of genes in humans is about 20,000 – 25,000, which is comparable to the number of genes in many other species. Typically, a gene is several hundred bases long. This is about $10^{-7} - 10^{-6}$ of the total length of the human genome, comparable to the ratio between the size of an alligator and the size of the clouds accompanying hurricane Frances. There are other, shorter functional units, such as promoters, enhancers,

Figure 1.2. Turbulent flow by Da Vinci.

prohibitors, and so on. Some genes or functional units could repeat, exactly or with slight modifications (called mutations), after tens of thousands of bases. Therefore, genomic DNA sequences are full of (largely static) multiscale phenomena.

Multiscale modeling of fluid motions. Fascinated by the complex phenomena of water flowing and mixing, Leonardo da Vinci made an exquisite portrait of turbulent flow of fluid, involving vortices within vortices over an ever-decreasing scale. See Fig. 1.2 and
http://www.efluids.com/efluids/gallery/gallery_pages/da_vinci_page.htm.
The central theme in multiscale modeling of fluid motions is the determination of what information on the finer scale is needed to formulate an equation for the "effective" behavior on the coarser scale. In fact, this is also the central theme of multiscale modeling in many other fields, such as cloud-resolving modeling for studying atmospheric phenomena on scales much larger than individual clouds, and modeling in materials science and biochemistry, where one strives to relate the functionality of the material or organism to its fundamental constituents, their chemical nature and geometric arrangement.

Multiscale phenomena in computer networks. Large-scale communications networks, especially the Internet, are among the most complicated systems that man has ever made, with many multiscale aspects. Intuitively speaking, these multiscales come from the hierarchical design of a protocol stack, the hierarchical topological architecture, and the multipurpose and heterogeneous nature of the Internet. More precisely, there are multiscales in (1) time, manifested by the prevailing fractal, multifractal, and long-range-dependent properties in traffic, (2) space, essentially due to topology and geography and again manifested by scale-free properties, (3) state, e.g., queues and windows, and (4) size, e.g., number of nodes and number of users. Also, it has been observed that the failure of a single router may trigger routing instability, which may be severe enough to instigate a route flap storm. Furthermore, packets may be delivered out of order or even get dropped, and packet reordering is not a pathological network behavior. As the next-generation Internet applications such as remote instrument control and computational steering are being developed, another facet of complex multiscale behavior is beginning to surface in terms of transport dynamics. The networking requirements for these next-generation applications belong to (at least) two broad classes *involving vastly disparate time scales*: (1) high bandwidths, typically multiples of 10 Gbps, to support bulk data transfers, and (2) stable bandwidths, typically at much lower bandwidths such as 10–100 Mbps, to support interactive, steering, and control operations.

Is there any difference between this example and examples 2 and 3? The answer is yes. In multiscale modeling of fluid motions, the basic equation, the Navier-Stokes equation, is known. Therefore, the dynamics of fluid motions can be systematically studied through a combined approach of theoretical modeling, numerical simulation, and experimental study. However, there is no fundamental equation to describe a DNA sequence or a computer network.

Multiscale phenomena in sea clutter. Sea clutter is the radar backscatter from a patch of ocean surface. The complexity of the signals comes from two sources: the rough sea surface, sometimes oscillatory, sometimes turbulent, and the multipath propagation of the radar backscatter. This can be well appreciated by imagining radar pulses massively reflecting from the wavetip of Fig. 1.3. To be quantitative, in Fig. 1.4, two 0.1 s duration sea clutter signals, sampled with a frequency of 1 KHz, are plotted in (a,b), a 2 s duration signal is plotted in (c), and an even longer signal (about 130 s) is plotted in (d). It is clear that the signal is not purely random, since the waveform can be fairly smooth on short time scales (Fig. 1.4(a)). However, the signal is highly nonstationary, since the frequency of the signal (Fig. 1.4(a,b)) as well as the randomness of the signal (Fig. 1.4(c,d)) change over time drastically. Therefore, naive Fourier analysis or deterministic chaotic analysis of sea clutter may not be very useful. From Fig. 1.4(e), where $X_t^{(m)}$ is the nonoverlapping running mean of X over block size m and X is the sea clutter amplitude data, it can be further concluded that neither autoregressive (AR) models nor textbook fractal models can describe the data. This is because AR modeling requires exponentially decaying autocorrelation (which amounts to $Var(X_t^{(m)}) \sim m^{-1}$, or a Hurst parameter of 1/2; see Chapters 6 and 8), while fractal modeling requires the variation between $Var(X_t^{(m)})$ and m to follow a power law. However, neither behavior is observed in Fig. 1.4(e). Indeed, although extensive work has been done on sea clutter, its nature is still poorly understood. As a result, the important problem of target detection

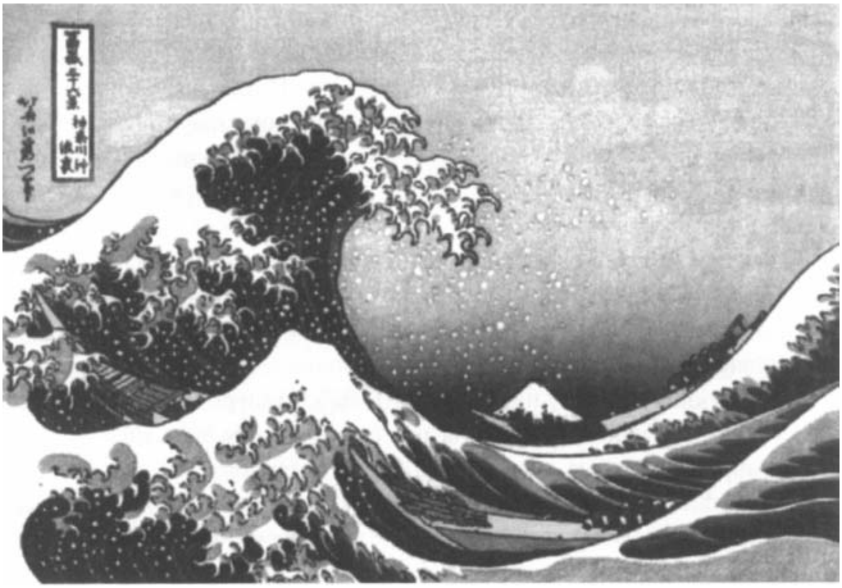

Figure 1.3. Schematic of a great wave (a tsunami, woodblock print from the 19th century Japanese artisit Hokusai). Suppose that our field of observation includes the wavetip of length scale of a few meters. It is then clear that the complexity of sea clutter is mainly due to massive reflection of radar pulses from a wavy and even turbulent ocean surface.

within sea clutter remains a tremendous challenge. We shall return to sea clutter later.

Multiscale and nonstationary phenomena in heart rate variability (HRV). HRV is an important dynamical variable in cardiovascular function. Its most salient feature is spontaneous fluctuation, even if the environmental parameters are maintained constant and no perturbing influences can be identified. It has been observed that HRV is related to various cardiovascular disorders. Therefore, analysis of HRV is very important in medicine. However, this task is very difficult, since HRV data are highly complicated. An example for a normal young subject is shown in Fig. 1.5. Evidently, the signal is highly nonstationary and multiscaled, appearing oscillatory for some period of time (Figs. 1.5(b,d)), and then varying as a power law for another period of time (Figs. 1.5(c,e)). The latter is an example of the so-called $1/f$ processes, which will be discussed in depth in later chapters. While the multiscale nature of such signals cannot be fully characterized by existing methods, the nonstationarity of the data is even more troublesome, since it requires the data to be properly segmented before further analysis by methods derived from spectral analysis, chaos theory, or random fractal theory. However, automated segmentation of complex biological signals to remove undesired components is itself a significant open problem, since it is closely related to, for example, the challenging task of

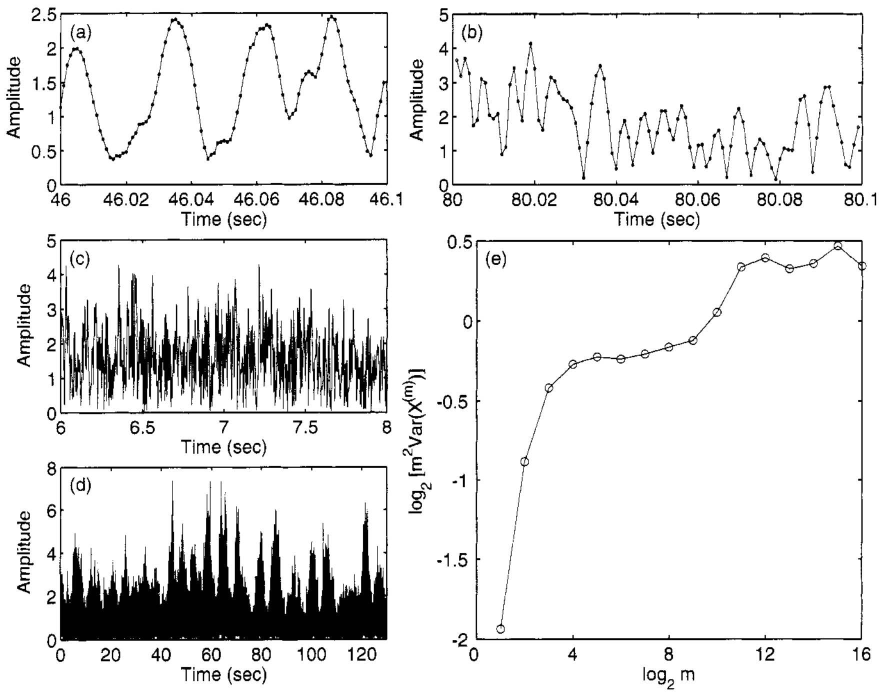

Figure 1.4. (a,b) Two 0.1 s duration sea clutter signals; (c) a 2 s duration sea clutter signal; (d) the entire sea clutter signal (of about 130 s); and (e) $\log_2[m^2Var(X^{(m)})]$ vs. $\log_2 m$, where $X^{(m)} = \{X_t^{(m)} : X_t^{(m)} = (X_{tm-m+1} + \cdots + X_{tm})/m,\ t = 1, 2, \cdots\}$ is the non-overlapping running mean of $X = \{X_t : t = 1, 2, \cdots\}$ over block size m and X is the sea clutter amplitude data. To better see the variation of $Var(X_t^{(m)})$ with m, $Var(X_t^{(m)})$ is multiplied by m^2. When the autocorrelation of the data decays exponentially fast (such as modeled by an AR process), $Var(X_t^{(m)}) \sim m^{-1}$. Here $Var(X_t^{(m)})$ decays much faster. A fractal process would have $m^2Var(X_t^{(m)}) \sim m^{\beta}$. However, this is not the case. Therefore, neither AR modeling nor ideal textbook fractal theory can be readily applied here.

accurately detecting transitions from normal to abnormal states in physiological data.

1.2 EXAMPLES OF CHALLENGING PROBLEMS TO BE PURSUED

In this book, a wide range of important problems will be discussed in depth. As a prelude, we describe 10 of them in this section.

P1: Can economic time series be modeled by low-dimensional noisy chaos? Since late 1980s, considerable efforts have been made to determine whether irregular economic time series are chaotic or random. By analyzing real as well as simulated economic time series using the neural network–based Lyapunov expo-

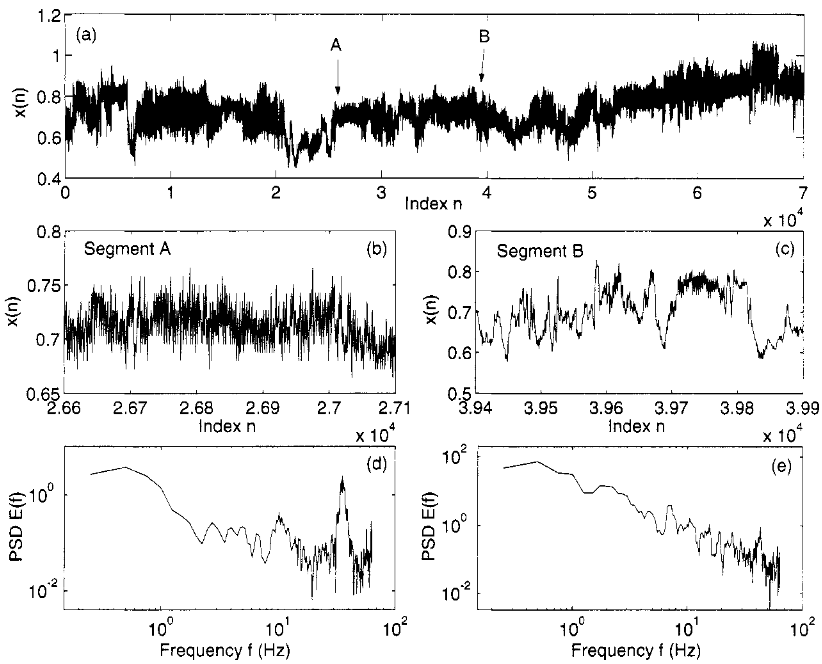

Figure 1.5. (a) The HRV data for a normal subject; (b,c) the segments of signals indicated as A and B in (a); (d,e) power spectral density for the signals shown in (b,c).

nent estimator, a number of recent studies have suggested that the world economy may not be characterized by low-dimensional chaos, since the largest Lyapunov exponent is negative. As will be explained in Chapter 13, the sum of the positive Lyapunov exponents gives a tight upper bound of Kolmogorov-Sinai (KS) entropy. KS entropy characterizes the rate of creation of information in a dynamical system. It is zero, positive, and infinite for regular, chaotic, and random motions, respectively. Therefore, a negative largest Lyapunov exponent in economic time series amounts to negative KS entropy and thus implies regular economic dynamics. But economy is anything but simple! How may we resolve this dilemma? An answer will be provided in Chapter 15.

P2: Network traffic modeling

Data transfer across a network is a very complicated process owing to interactions among a huge number of correlated or uncorrelated users, congestion, routing instability, packet reordering, and many other factors. Traditionally, network traffic is modeled by Poisson or Markovian models. Recently, it has been found that long-range dependence (LRD) is a prevailing feature of real network traffic. However, it is still being debated which type of model, Poisson or Markovian type, LRD, or multifractal, should be used to evaluate the performance of a network. It would be very desirable to develop a general framework that not only includes all the traffic models developed so far as special cases, but also accurately and parsimoniously models real network traffic. Can such a goal be reached? A surprisingly simple answer will be given in Chapter 10.

P3: Network intrusion and worm detection
Enterprise networks are facing ever-increasing security threats from various types of intrusions and worms. May the concepts and methods developed in this book be useful for protecting against these problems? A surprisingly simple and effective solution will be presented in Sec. 13.1.2.

P4: Sea clutter modeling and target detection within sea clutter
Accurate modeling of sea clutter and robust detection of low observable targets within sea clutter are important problems in remote sensing and radar signal processing applications. For example, they may be helpful in improving navigation safety and facilitating environmental monitoring. As we have seen in Fig. 1.4, sea clutter data are highly complicated. Can some concept or method developed in this book provide better models for sea clutter? The answer is yes. In fact, we will find that many of the new concepts and methods developed in the book are useful for this difficult task.

P5: Fundamental error bounds for nano- and fault-tolerant computations
In the emerging nanotechnologies, faulty components may be an integral part of a system. For the system to be reliable, the error of the building blocks has to be smaller than a threshold. Therefore, finding exact error thresholds for noisy gates is one of the most challenging problems in fault-tolerant computations. We will show in Chapter 12 that bifurcation theory offers an amazingly effective approach to solve this problem.

P6: Neural information processing
Mankind's desire to understand neural information processing has been extremely useful in developing modern computing machines. While such a desire will certainly motivate the development of new bio-inspired computations, understanding neural information processing has become increasingly pressing owing to the recent great interest in brain-machine interfaces and deep brain stimulation. In Secs. 8.9.2 and 9.5.2, we will show how the various types of fractal analysis methods developed in the book can help understand neuronal firing patterns.

P7: Protein coding sequence identification in genomic DNA sequences
Gene finding is one of the most important tasks in the study of genomes. Indices that can discriminate DNA sequences' coding and noncoding regions are crucial elements of a successful gene identification algorithm. Can multiscale analysis of genome sequences help construct novel codon indices? The answer is yes, as we shall see in Chapter 8.

P8: Analysis of HRV

We have seen in Fig. 1.5 that HRV data are nonstationary and multiscaled. Can multiscale complexity measures readily deal with nonstationarity in HRV data, find the hidden differences in HRV data under healthy and disease conditions, and shed new light on the dynamics of the cardiovascular system? An elegant answer will be given in Chapter 15.

P9: EEG analysis

Electroencephalographic (EEG) signals provide a wealth of information about brain dynamics, especially related to cognitive processes and pathologies of the brain such as epileptic seizures. To understand the nature of brain dynamics as well as to develop novel methods for the diagnosis of brain pathologies, a number of complexity measures from information theory, chaos theory, and random fractal theory have been used to analyze EEG signals. Since these three theories have different foundations, it has not been easy to compare studies based on different complexity measures. Can multiscale complexity measures offer a unifying framework to overcome this difficulty and, more importantly, to offer new and more effective means of providing a warning about pathological states such as epileptic seizures? Again, an elegant answer will be given in Chapter 15.

P10: Modeling of turbulence

Turbulence is a prevailing phenomenon in geophysics, astrophysics, plasma physics, chemical engineering, and environmental engineering. It is perhaps the greatest unsolved problem in classical physics. Multifractal models, pioneered by B. Mandelbrot, are among the most successful in describing intermittency in turbulence. In Sec. 9.4, we will present a number of multifractal models for the intermittency phenomenon of turbulence in a coherent way.

1.3 OUTLINE OF THE BOOK

We have discussed the purpose and the basic structure of the book in Fig. 1.1. To facilitate our discussions, Chapter 2 is a conceptual chapter consisting of two sections describing fractal and chaos theories. Since fractal theory will be treated formally starting with Chapter 5, the section on fractal theory in Chapter 2 will be quite brief; the section on chaos theory, however, will be fairly detailed because this theory will not be treated in depth until Chapter 13. The rest of the structure of the book is largely determined by our own experience in analyzing complicated time series arising from fields as diverse as device physics, radar engineering, fluid mechanics, geophysics, physiology, neuroscience, vision science, and bioinformatics, among many others. Our view is that random fractals, when used properly, can be tremendously helpful, especially in forming new hypotheses. Therefore, we shall spend a lot of time discussing signal processing techniques using random fractal theory.

To facilitate the discussion on random fractal theory, Chapter 3 reviews the basics of probability theory and stochastic processes. The correlation structure of the latter will be emphasized so that comparisons between stochastic processes with short memory and fractal processes with long memory can be made in later chapters. Chapter 4 briefly discusses Fourier transform and wavelet multiresolution analysis, with the hope that after this treatment, readers will find complicated signals to be friendly. Chapter 5 briefly resumes the discussion of Sec. 2.1 and discusses the basics of fractal geometry. Chapter 6 discusses self-similar stochastic processes, in particular the fractional Brownian motion (fBm) processes. Chapter 7 discusses a different type of fractal processes, the Levy motions, which are memoryless but have heavy-tailed features. Beyond Chapter 7, we focus on various techniques of signal processing: Chapter 8 discusses structure-function–based multifractal technique and various methods for assessing long memories from a time series together with a number of applications. The latter include topics as diverse as network traffic modeling, detection of low observable objects within sea clutter radar returns, gene finding from DNA sequences, and so on. Chapter 9 discusses a different type of multifractal, the multiplicative cascade multifractal. Fractal analysis culminates in Chapter 10, where a new model is presented. This is a wonderful model, since it "glues" together the structure-function–based fractal analysis and the cascade model. When one is not sure which type of fractal model should be used to analyze data, this model may greatly simplify the job. Chapter 11 discusses models for generating heavy-tailed distributions and long-range correlations. In Chapter 12, we switch to a completely different topic — bifurcation theory — and apply it to solve the difficult problem of finding exact error threshold values for fault-tolerant computations. In Chapter 13, we discuss the basics of chaos theory and chaotic time series analysis. In Chapter 14, we extend the discussion in Chapter 13 and consider a new theoretical framework, the power-law sensitivity to initial conditions, to provide chaos theory and random fractal theory with a common foundation. Finally, in Chapter 15, we discuss an excellent multiscale measure — the scale-dependent Lyapunov exponent — and its numerous applications.

Complex time series analysis is a very diverse field. For ease of illustrating the practical use of many concepts and methods, we have included many of our own works, some published, some appearing here for the first time, as examples. This, however, does not mean that our own contribution to this field is very significant. We must emphasize that even though the topics covered in this book are very broad, they are still just a subset of the many interesting issues and methods developed for complex time series analysis. For example, fascinating topics such as chaos control and chaos-based noise reduction and prediction are not touched on at all. Furthermore, we may not have covered all aspects of the chosen topics. Some of the omissions are intentional, since this book is designed to be useful not only for people in the field but also as an introduction to the field — especially to be used as a textbook. Of course, some of the unintentional omissions are due to our ignorance. While we apologize if some of important works are not properly reported here, we hope that readers will search for relevant literature on interesting topics using the literature search method discussed in Chapter 3. Finally, readers are strongly encouraged to work on the homework problems in the end of each chapter, especially those related to computer simulations. Only by doing this can they truly appreciate the beauty as well as the limitation of an unfamiliar new concept.

1.4 BIBLIOGRAPHIC NOTES

In the past two decades, a number of excellent books on chaos and fractal theories have been published. An incomplete list includes [33, 36, 130, 198, 294, 387] on fractal theory, [381,455] on stable laws, [25,45,106,206,250,330,404,409,415,475] on chaos theory, and [1, 249] on chaotic time series analysis. In particular, [249] can be considered complementary to this book. A collection of significant early papers can be found in [331].

In Sec. 1.3, we stated that topics including chaos control, synchronization in chaotic systems, chaos-based noise reduction and prediction would not be touched on in the book. Readers interested in chaos control are referred to the comprehensive book by Ott [330], those interested in chaos synchronization are referred to [338], and those interested in prediction from data are referred to [249] for chaos-based approaches and to [213] for the Kalman filtering approach. For prediction of a dynamical system with known equations but only partial knowledge of initial conditions, we refer readers to [77]. Finally, readers wishing to pursue the physical mechanisms of self-similarity and incomplete self-similarity are strongly encouraged to read the exquisite book by Barenblatt [31].