Multiscale Analysis of Complex Time Series: Integration of Chaos and Random Fractal Theory, and Beyond by Jianbo Gao, Yinhe Cao, Wen-wen Tung and Jing Hu Copyright © 2007 John Wiley & Sons. Inc.

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