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MULTISCALE ANALYSIS OF COMPLEX TIME SERIES

Integration of Chaos and Random
Fractal Theory, and Beyond

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*To our teachers,
Xianyi Zhou of Zhejiang
University,
Yilong Bai and Zhemin
Zheng of Chinese Academy
of Sciences,
Michio Yanai of UCLA,
and to our families*

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PREFACE

Complex interconnected systems, including the Internet, stock markets, and human heart or brain, are usually comprised of multiple subsystems that exhibit highly nonlinear deterministic as well as stochastic characteristics and are regulated hierarchically. They generate signals that exhibit complex characteristics such as nonlinearity, sensitive dependence on small disturbances, long memory, extreme variations, and nonstationarity. A complex system usually cannot be studied by decomposing the system into its constituent subsystems, but rather by measuring certain signals generated by the system and analyzing the signals to gain insights into the behavior of the system. In this endeavor, data analysis is a crucial step. Chaos theory and random fractal theory are two of the most important theories developed for data analysis. Unfortunately, no single book has been available to present all the basic concepts necessary for researchers to fully understand the ever-expanding literature and apply novel methods to effectively solve their signal processing problems. This book attempts to meet this pressing need by presenting chaos theory and random fractal theory in a unified way.

Integrating chaos theory and random fractal theory and going beyond them has proven to be much harder than we had thought, because the foundations for chaos theory and random fractal theory are entirely different. Chaos theory is mainly concerned about apparently irregular behaviors in a complex system that are generated by nonlinear deterministic interactions of only a few numbers of degrees of freedom, where noise or intrinsic randomness does not play an important role,

while random fractal theory assumes that the dynamics of the system are inherently random. After postponing delivery of the book for more than two and half years, we are finally satisfied. The book now contains many new results in Chapters 8–15 that have not been published elsewhere, culminating in the development of a multiscale complexity measure that is computable from short, noisy time series. As shown in Chapter 15, the measure can readily classify major types of complex motions, effectively deal with nonstationarity, and simultaneously characterize the behaviors of complex signals on a wide range of scales, including complex irregular behaviors on small scales and orderly behaviors, such as oscillatory motions, on large scales.

This book has adopted a data-driven approach. To help readers better understand and appreciate the power of the materials in the book, nearly every significant concept or approach presented is illustrated by applying it to effectively solve real problems, sometimes with unprecedented accuracy. Furthermore, source codes, written in various languages, including Java, Fortran, C, and Matlab, for many methods are provided in a dedicated book website, together with some simulated and experimental data (see Sec. A.4 in Appendix A).

This book contains enough material for a one-year graduate-level course. It is useful for students with various majors, including electrical engineering, computer science, civil and environmental engineering, mechanical engineering, chemical engineering, medicine, chemistry, physics, geophysics, mathematics, finance, and population ecology. It is also useful for researchers working in relevant fields and practitioners who have to solve their own signal processing problems.

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