1. Who will be hired?

   "We need more than one person this time. If we hire Adam or Brian and don't hire Chris, then we want Adam, and we will hire Chris if we hire Brian. Otherwise, we will do the opposite."

   Let A = Adam, B = Brian, and C = Chris.

   $((((A \lor B) \land \neg C) \to A) \land (\neg A \to (B \land C))) \lor \neg(((A \lor B) \land \neg C \to A) \land (\neg A \to (B \land C)))$

   $\to$ Elimination:
   $((\neg((A \lor B) \land \neg C) \lor A) \land (\neg\neg A \lor (B \land C))) \lor \neg((\neg((A \lor B) \land \neg C) \lor A) \land (\neg\neg A \lor (B \land C)))$

   DeMorgan's Law:
   $(((\neg(A \lor B) \lor \neg\neg C) \lor A) \land (\neg\neg A \lor (B \land C))) \lor (\neg((\neg(A \lor B) \lor \neg\neg C) \lor A) \lor \neg(\neg\neg A \lor (B \land C)))$

   DeMorgan's Law:
   $((((\neg A \land \neg B) \lor \neg\neg C) \lor A) \land (\neg\neg A \lor (B \land C))) \lor (\neg(((\neg A \land \neg B) \lor \neg\neg C) \lor A) \lor \neg(\neg\neg A \lor (B \land C)))$

   DeMorgan's Law:
   $((((\neg A \land \neg B) \lor \neg\neg C) \lor A) \land (\neg\neg A \lor (B \land C))) \lor ((\neg((\neg A \land \neg B) \lor \neg\neg C) \land \neg A) \lor (\neg\neg\neg A \land \neg(B \land C)))$

   DeMorgan's Law:
   $((((\neg A \land \neg B) \lor \neg\neg C) \lor A) \land (\neg\neg A \lor (B \land C))) \lor (((\neg(\neg A \land \neg B) \land \neg\neg\neg C) \land \neg A) \lor (\neg\neg\neg A \land \neg(B \land C)))$

   DeMorgan's Law:
   $((((\neg A \land \neg B) \lor \neg\neg C) \lor A) \land (\neg\neg A \lor (B \land C))) \lor ((((\neg\neg A \lor \neg\neg B) \land \neg\neg\neg C) \land \neg A) \lor (\neg\neg\neg A \land (\neg B \lor \neg C)))$

   $\neg\neg$ Elimination:
   $((((\neg A \land \neg B) \lor C) \lor A) \land (A \lor (B \land C))) \lor ((((A \lor B) \land \neg C) \land \neg A) \lor (\neg A \land (\neg B \lor \neg C)))$

   Distributive:
   $((((C \lor \neg A) \land (C \lor \neg B)) \lor A) \land ((A \lor B) \land (A \lor C))) \lor (((((A \lor B) \land \neg C) \land \neg A) \lor \neg A) \land ((((A \lor B) \land \neg C) \land \neg A) \lor (\neg B \lor \neg C)))$

   Distributive:
   $(((A \lor (C \lor \neg A)) \land (A \lor (C \lor \neg B))) \land ((A \lor B) \land (A \lor C))) \lor (((\neg A \lor \neg A) \land (\neg A \lor ((A \lor B) \land \neg C))) \land (((\neg B \lor \neg C) \lor \neg A) \land ((\neg B \lor \neg C) \lor ((A \lor B) \land \neg C)))$

Distributive:
$(((A \lor (C \lor \neg A)) \land (A \lor (C \lor \neg B))) \land ((A \lor B) \land (A \lor C))) \lor (((\neg A \lor \neg A) \land (\neg A \lor ((A \lor B) \land \neg C))) \land (((\neg B \lor \neg C) \lor \neg A) \land (((\neg B \lor \neg C) \lor (A \lor B)) \land ((\neg B \lor \neg C) \lor \neg C))))$

Tautologies:
$((C \land (A \lor C \lor \neg B)) \land ((A \lor B) \land (A \lor C))) \lor (((\neg A \land (\neg A \lor ((A \lor B) \land \neg C))) \land (\neg B \lor \neg C \lor \neg A) \land ((\neg C \lor A) \land (\neg B \lor \neg C)))$

Equivalences:
$(C \land (A \lor B) \land (A \lor C)) \lor ((\neg A \land (\neg B \lor \neg C \lor \neg A)) \land ((\neg C \lor A) \land (\neg B \lor \neg C)))$

Distributive:
$((C \land (A \lor B) \land (A \lor C)) \lor (\neg A \land (\neg B \lor \neg C \lor \neg A))) \land ((C \land (A \lor B) \land (A \lor C) \lor ((\neg C \lor A) \land (\neg B \lor \neg C)))$

Equivalences:
$((C \land (A \lor B)) \lor \neg A)) \land ((C \land (A \lor B)) \lor ((\neg C \lor A) \land (\neg B \lor \neg C)))$

Distributive:
$(\neg A \lor C) \land (\neg A \lor (A \lor B)) \land ((C \land (A \lor B)) \lor (\neg C \lor A)) \land ((C \land (A \lor B)) \lor (\neg B \lor \neg C))$

Distributive:
$(\neg A \lor C) \land B \land ((\neg C \lor A) \lor C) \land ((\neg C \lor A) \lor (A \lor B)) \land ((\neg B \lor \neg C) \lor C) \land ((\neg B \lor \neg C) \lor (A \lor B))$

Tautologies:
$(\neg A \lor C) \land B \land A \land (\neg C \lor A \lor B) \land \neg B \land (\neg C \lor A)$

CNF to clauses:
$\{(\neg A, C), B, A, (\neg C, A, B), \neg B, (\neg C, A)\}$

Resolution rule:
$\{A, (\neg C, A, B)\}$

Result:
Adam and Brian were hired.

2. Pink Elephants always like gray elephants
   1) Bonnie is a pink elephant
      $P(B) \land E(B)$
   2) Trish is a grey elephant
      $G(T) \land E(T)$
   3) Bonnie likes some elephant
      $(\exists x)\ (E(x) \to L(B, x))$
   4) All elephants are either gray or pink

$(\forall x)\ (E(x) \rightarrow (G(x) \vee P(x)))$

5) Some elephant likes Trish
$(\exists x)\ (E(x) \rightarrow L(x, T))$
6) Conclusion: there is some pink elephant who likes a gray elephant
$(\exists x)\ (E(x) \wedge P(x) \wedge (\exists y)\ ((E(y) \wedge P(y)) \rightarrow L(x, y)))$
Negated:
$\neg((\exists x)\ (E(x) \wedge P(x) \wedge (\exists y)\ ((E(y) \wedge P(y)) \rightarrow L(x, y))))$

1) $P(B) \wedge E(B)$
$\{P(B), E(B)\}$
2) $G(T) \wedge E(T)$
$\{G(T), E(T)\}$
3) $\exists x\ (E(x) \rightarrow L(B, x))$
$\exists x\ (\neg E(x) \vee L(B, x))$ – $\rightarrow$ Elimination
$\{(\neg E(x), L(B,x))\}$
4) $\forall x\ (E(x) \rightarrow (G(x) \vee P(x)))$
$\forall x\ (\neg E(x) \vee (G(x) \vee P(x)))$ – $\rightarrow$ Elimination
$\forall x\ (\neg E(x) \vee G(x) \vee P(x))$
$\{(\neg E(x), G(x), P(x))\}$

5) $\exists x\ (E(x) \rightarrow L(x, T))$
$\exists x\ (\neg E(x) \vee L(x, T))$ – CNF rule 2
$\{(\neg E(x), L(x, T))\}$

6) Negated conclusion:
$\neg((\exists x)\ (E(x) \wedge P(x) \wedge (\exists y)\ ((E(y) \wedge P(y)) \rightarrow L(x, y))))$
$\neg((\exists x)\ (E(x) \wedge P(x) \wedge (\exists y)\ (\neg(E(y) \wedge G(y)) \vee L(x, y))))$ – $\rightarrow$ Elimination
$\neg((\exists x)\ (E(x) \wedge P(x) \wedge (\exists y)\ (\neg E(y) \vee \neg G(y) \vee L(x, y))))$ – DeMorgan's Law
$(\forall x)\ \neg(E(x) \wedge P(x) \wedge (\exists y)\ (\neg E(y) \vee \neg G(y) \vee L(x, y)))$ – Quantifier equivalence
$(\forall x)\ (\neg E(x) \vee \neg P(x) \vee \neg(\exists y)\ (\neg E(y) \vee \neg G(y) \vee L(x, y)))$ – DeMorgan's Law
$(\forall x)\ (\neg E(x) \vee \neg P(x) \vee (\forall y)\ \neg(\neg E(y) \vee \neg G(y) \vee L(x, y)))$ – Quantifier equivalence
$(\forall x)\ (\neg E(x) \vee \neg P(x) \vee (\forall y)\ (\neg\neg E(y) \wedge \neg\neg G(y) \wedge \neg L(x, y)))$ – DeMorgan's Law
$(\forall x)\ (\neg E(x) \vee \neg P(x) \vee (\forall y)\ (E(y) \wedge G(y) \wedge \neg L(x, y)))$ - $\neg\neg$ Elimination
$(\forall x)\ (\forall y)\ (\neg E(x) \vee \neg P(x) \vee (E(y) \wedge G(y) \wedge \neg L(x, y)))$ – PNF form
$(\forall x)\ (\forall y)\ ((\neg E(x) \vee \neg P(x) \vee E(y)) \wedge (\neg E(x) \vee \neg P(x) \vee G(y)) \wedge (\neg E(x) \vee \neg P(x) \vee \neg L(x, y))))$ – Distributive
$\{(\neg E(x), \neg P(x), E(y)), (\neg E(x),\neg P(x), G(y)), (\neg E(x), \neg P(x), \neg L(x, y))\}$

Resulting clauses: *red means it was used in refutation
1) $\{P(B), E(B)\}$
2) $\{G(T), E(T)\}$
3) $\{(\neg E(x), L(B,x))\}$
4) $\{(\neg E(x), G(x), P(x))\}$
5) $\{(\neg E(x), L(x, T))\}$
6) $\{(\neg E(x), \neg P(x), E(y)), (\neg E(x),\neg P(x), G(y)), (\neg E(x), \neg P(x), \neg L(x, y))\}$

$\{P(B), E(B)\}$ $\{(\neg E(x), \neg P(x), \neg L(x, y))\}$

$\{B/x\}$

$\{\neg L(x, y)\}$ $\{(\neg E(x), L(x, T))\}$

$\{T/y\}$

$\{\neg E(x)\}$ $\{E(T)\}$

$\{T/x\}$

$\perp$

A falsum was reached. Thus, the original conclusion is correct. Therefore, there is some pink elephant who likes a gray elephant.