1. Who will be hired?

“We need more than one person this time. If we hire Adam or Brian and don’t hire Chris, then we want Adam, and we will hire Chris if we hire Brian. Otherwise, we will do the opposite.”

Let A = Adam, B = Brian, and C = Chris.

((((A ∨ B) ⋀ ¬C) → A) ⋀ (¬A → (B ⋀ C))) ∨ ¬(((A ∨ B) ⋀ ¬C → A) ⋀ (¬A → (B ⋀ C)))

→ Elimination:

((¬((A ∨ B) ⋀ ¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C))) ∨ ¬((¬((A ∨ B) ⋀ ¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C)))

DeMorgan’s Law:

(((¬(A ∨ B) ∨ ¬¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C))) ∨ (¬((¬(A ∨ B) ∨ ¬¬C) ∨ A) ∨ ¬(¬¬A ∨ (B ⋀ C)))

DeMorgan’s Law:

((((¬A ⋀ ¬B) ∨ ¬¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C))) ∨ (¬(((¬A ⋀ ¬B) ∨ ¬¬C) ∨ A) ∨ ¬(¬¬A ∨ (B ⋀ C)))

DeMorgan’s Law:

((((¬A ⋀ ¬B) ∨ ¬¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C))) ∨ ((¬((¬A ⋀ ¬B) ∨ ¬¬C) ⋀ ¬A) ∨ (¬¬¬A ⋀ ¬(B ⋀ C)))

DeMorgan’s Law:

((((¬A ⋀ ¬B) ∨ ¬¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C))) ∨ (((¬(¬A ⋀ ¬B) ⋀ ¬¬¬C) ⋀ ¬A) ∨ (¬¬¬A ⋀ ¬(B ⋀ C)))

DeMorgan’s Law:

((((¬A ⋀ ¬B) ∨ ¬¬C) ∨ A) ⋀ (¬¬A ∨ (B ⋀ C))) ∨ ((((¬¬A ∨ ¬¬B) ⋀ ¬¬¬C) ⋀ ¬A) ∨ (¬¬¬A ⋀ (¬B ∨ ¬C)))

¬¬ Elimination:

((((¬A ⋀ ¬B) ∨ C) ∨ A) ⋀ (A ∨ (B ⋀ C))) ∨ ((((A ∨ B) ⋀ ¬C) ⋀ ¬A) ∨ (¬A ⋀ (¬B ∨ ¬C)))

Distributive:

((((C ∨ ¬A) ⋀ (C ∨ ¬B)) ∨ A) ⋀ ((A ∨ B) Λ (A ∨ C))) ∨ (((((A ∨ B) ⋀ ¬C) ⋀ ¬A) ∨ ¬A) ⋀ ((((A ∨ B) ⋀ ¬C) ⋀ ¬A) ∨ (¬B ∨ ¬C)))

Distributive:

(((A ∨ (C ∨ ¬A)) ⋀ (A ∨ (C ∨ ¬B))) ⋀ ((A ∨ B) Λ (A ∨ C))) ∨ (((¬A ∨ ¬A) ⋀ (¬A ∨ ((A ∨ B) ⋀ ¬C))) ⋀ (((¬B ∨ ¬C) ∨ ¬A) ⋀ ((¬B ∨ ¬C) ∨ ((A ∨ B) ⋀ ¬C)))

Distributive:

(((A ∨ (C ∨ ¬A)) ⋀ (A ∨ (C ∨ ¬B))) ⋀ ((A ∨ B) Λ (A ∨ C))) ∨ (((¬A ∨ ¬A) ⋀ (¬A ∨ ((A ∨ B) ⋀ ¬C))) ⋀ (((¬B ∨ ¬C) ∨ ¬A) ⋀ (((¬B ∨ ¬C) ∨ (A ∨ B)) ⋀ ((¬B ∨ ¬C) ∨ ¬C))))

Tautologies:

((C ⋀ (A ∨ C ∨ ¬B)) ⋀ ((A ∨ B) ⋀ (A ∨ C))) ∨ (((¬A ⋀ (¬A ∨ ((A ∨ B) ⋀ ¬C))) ⋀ (¬B ∨ ¬C ∨ ¬A) ⋀ ((¬C ∨ A) ⋀ (¬B ∨ ¬C)))

Equivalences:

(C ⋀ (A ∨ B) ⋀ (A ∨ C)) ∨ ((¬A ⋀ (¬B ∨ ¬C ∨ ¬A)) ⋀ ((¬C ∨ A) ⋀ (¬B ∨ ¬C)))

Distributive:

((C ⋀ (A ∨ B) ⋀ (A ∨ C)) ∨ (¬A ⋀ (¬B ∨ ¬C ∨ ¬A))) ⋀ ((C ⋀ (A ∨ B) ⋀ (A ∨ C) ∨ ((¬C ∨ A) ⋀ (¬B ∨ ¬C)))

Equivalences:

((C ⋀ (A ∨ B)) ∨ ¬A)) ⋀ ((C ⋀ (A ∨ B)) ∨ ((¬C ∨ A) ⋀ (¬B ∨ ¬C)))

Distributive:

(¬A ∨ C) ⋀ (¬A ∨ (A ∨ B)) ⋀ ((C ⋀ (A ∨ B)) ∨ (¬C ∨ A)) ⋀ ((C ⋀ (A ∨ B)) ∨ (¬B ∨ ¬C))

Distributive:

(¬A ∨ C) ⋀ B ⋀ ((¬C ∨ A) ∨ C) ⋀ ((¬C ∨ A) ∨ (A ∨ B)) ⋀ ((¬B ∨ ¬C) ∨ C) ⋀ ((¬B ∨ ¬C) ∨ (A ∨ B))

Tautologies:

(¬A ∨ C) ⋀ B ⋀ A ⋀ (¬C ∨ A ∨ B) ⋀ ¬B ⋀ (¬C ∨ A)

CNF to clauses:

{(¬A, C), B, A, (¬C, A, B), ¬B, (¬C, A)}

Resolution rule:

{A, (¬C, A, B)}

Result:

Adam and Brian were hired.

1. Pink Elephants always like gray elephants
2. Bonnie is a pink elephant

P(B) ⋀ E(B)

1. Trish is a grey elephant

G(T) ⋀ E(T)

1. Bonnie likes some elephant

(∃x) (E(x) → L(B, x))

1. All elephants are either gray or pink

(∀x) (E(x) → (G(x) ∨ P(x)))

1. Some elephant likes Trish

(∃x) (E(x) → L(x, T))

1. Conclusion: there is some pink elephant who likes a gray elephant

(∃x) (E(x) ⋀ P(x) ⋀ (∃y) ((E(y) ⋀ P(y)) → L(x, y)))

Negated:

¬((∃x) (E(x) ⋀ P(x) ⋀ (∃y) ((E(y) ⋀ P(y)) → L(x, y))))

1. P(B) ⋀ E(B)

{P(B), E(B)}

1. G(T) ⋀ E(T)

{G(T), E(T)}

1. ∃x (E(x) → L(B, x))

∃x (¬E(x) ∨ L(B, x)) – → Elimination

{(¬E(x), L(B,x))}

1. ∀x (E(x) → (G(x) ∨ P(x)))

∀x (¬E(x) ∨ (G(x) ∨ P(x))) – → Elimination

∀x (¬E(x) ∨ G(x) ∨ P(x))

{(¬E(x), G(x), P(x))}

1. ∃x (E(x) → L(x, T))

∃x (¬E(x) ∨ L(x, T)) – CNF rule 2

{(¬E(x), L(x, T))}

1. Negated conclusion:

¬((∃x) (E(x) ⋀ P(x) ⋀ (∃y) ((E(y) ⋀ P(y)) → L(x, y))))

¬((∃x) (E(x) ⋀ P(x) ⋀ (∃y) (¬(E(y) ⋀ G(y)) ∨ L(x, y)))) – → Elimination

¬((∃x) (E(x) ⋀ P(x) ⋀ (∃y) (¬E(y) ∨ ¬G(y) ∨ L(x, y)))) – DeMorgan’s Law

(∀x) ¬(E(x) ⋀ P(x) ⋀ (∃y) (¬E(y) ∨ ¬G(y) ∨ L(x, y))) – Quantifier equivalence

(∀x) (¬E(x) ∨ ¬P(x) ∨ ¬(∃y) (¬E(y) ∨ ¬G(y) ∨ L(x, y))) – DeMorgan’s Law

(∀x) (¬E(x) ∨ ¬P(x) ∨ (∀y) ¬(¬E(y) ∨ ¬G(y) ∨ L(x, y))) – Quantifier equivalence

(∀x) (¬E(x) ∨ ¬P(x) ∨ (∀y) (¬¬E(y) ⋀ ¬¬G(y) ⋀ ¬L(x, y))) – DeMorgan’s Law

(∀x) (¬E(x) ∨ ¬P(x) ∨ (∀y) (E(y) ⋀ G(y) ⋀ ¬L(x, y))) - ¬¬ Elimination

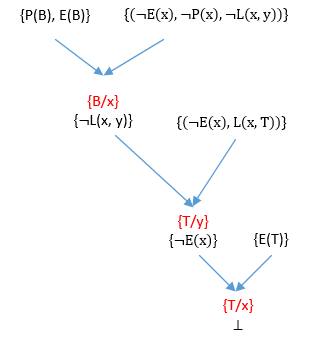
(∀x) (∀y) (¬E(x) ∨ ¬P(x) ∨ (E(y) ⋀ G(y) ⋀ ¬L(x, y))) – PNF form

(∀x) (∀y) ((¬E(x) ∨ ¬P(x) ∨ E(y)) ⋀ (¬E(x) ∨ ¬P(x) ∨ G(y)) ⋀ (¬E(x) ∨ ¬P(x) ∨ ¬L(x, y)))) – Distributive

{(¬E(x), ¬P(x), E(y)), (¬E(x),¬P(x), G(y)), (¬E(x), ¬P(x), ¬L(x, y))}

Resulting clauses: \*red means it was used in refutation

1. {P(B), E(B)}
2. {G(T), E(T)}
3. {(¬E(x), L(B,x))}
4. {(¬E(x), G(x), P(x))}
5. {(¬E(x), L(x, T))}
6. {(¬E(x), ¬P(x), E(y)), (¬E(x),¬P(x), G(y)), (¬E(x), ¬P(x), ¬L(x, y))}



A falsum was reached. Thus, the original conclusion is correct. Therefore, there is some pink elephant who likes a gray elephant.